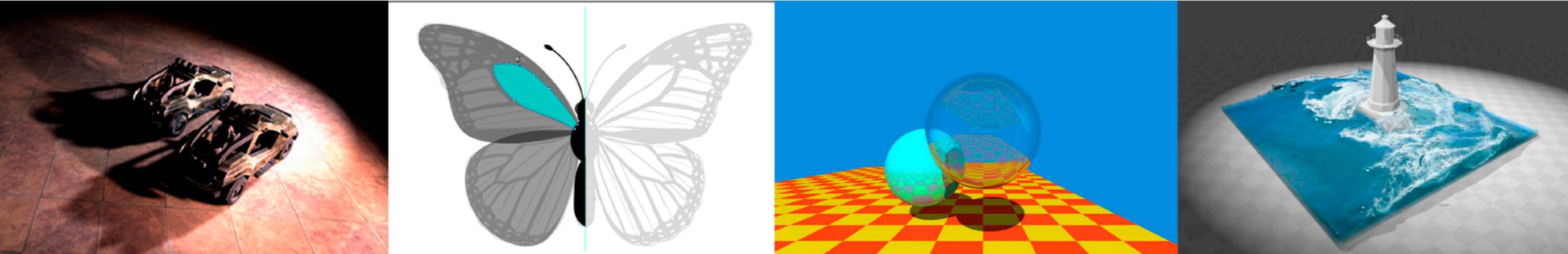


# Introduction to Computer Graphics

AMES101, Lingqi Yan, UC Santa Barbara

## Lecture 15: Ray Tracing 3 (Light Transport & Global Illumination)

光线传播 全局光照



# Announcements

- Homework 5 — 240 submissions so far
- Next two homeworks (**1.5 weeks each**)
  - Homework 6 — acceleration
  - Homework 7 — path tracing (new!)
- Course website has been updated
  - Two more lectures, one more homework (hw7)

# Announcements Cont.

- Why am I always extending the lecture length
  - My CS180 was designed to last 1h to 1.25h
- On the BBS
  - I'd welcome more questions on concepts
- My real-time rendering course
  - Unfortunately has to be internal
  - But will deliver it to GAMES later (maybe summer 2020)
- Again, today's lecture won't be easy

# Last Lectures

- Basic ray tracing
  - Ray generation
  - Ray object intersection
- Acceleration
  - Ray AABB intersection
  - Spatial partitions vs object partitions
  - BVH traversal
- Radiometry

# Today

- Radiometry cont.
- Light transport
  - The reflection equation
  - The rendering equation
- Global illumination
- Probability review

# Reviewing Concepts

Radiant energy  $Q$  [J = Joule] (barely used in CG)

- the energy of electromagnetic radiation

Radiant flux (power)  $\Phi \equiv \frac{dQ}{dt}$  [W = Watt] [lm = lumen]

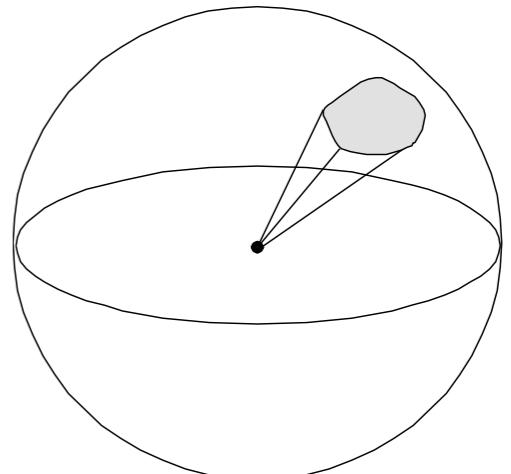
- Energy per unit time

Radiant intensity  $I(\omega) \equiv \frac{d\Phi}{d\omega}$

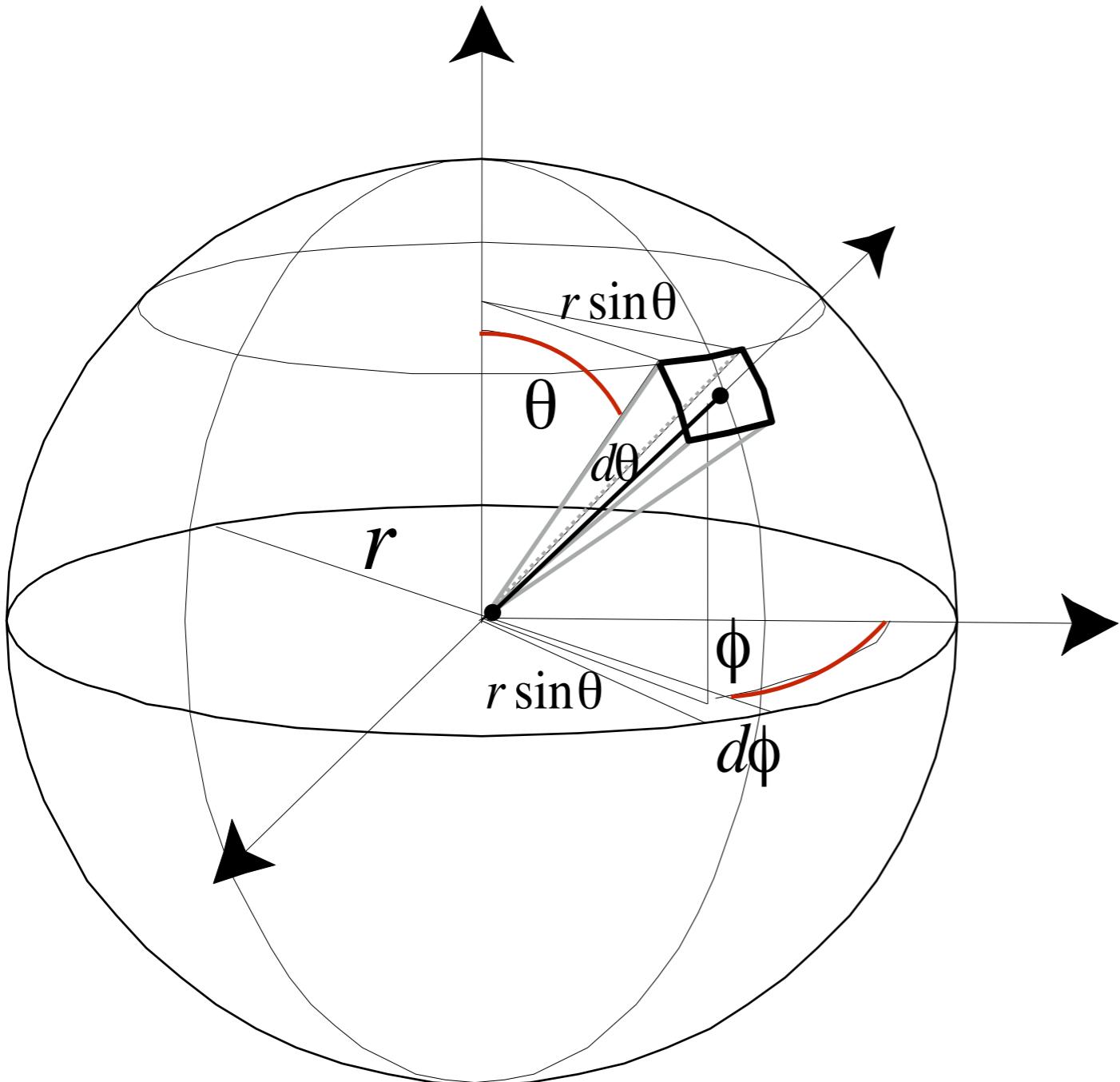
- power per unit solid angle

Solid Angle  $\Omega = \frac{A}{r^2}$

- ratio of subtended area on sphere to radius squared



# Differential Solid Angles



$$\begin{aligned} dA &= (r d\theta)(r \sin \theta d\phi) \\ &= r^2 \sin \theta d\theta d\phi \end{aligned}$$

$$d\omega = \frac{dA}{r^2} = \sin \theta d\theta d\phi$$

# Irradiance

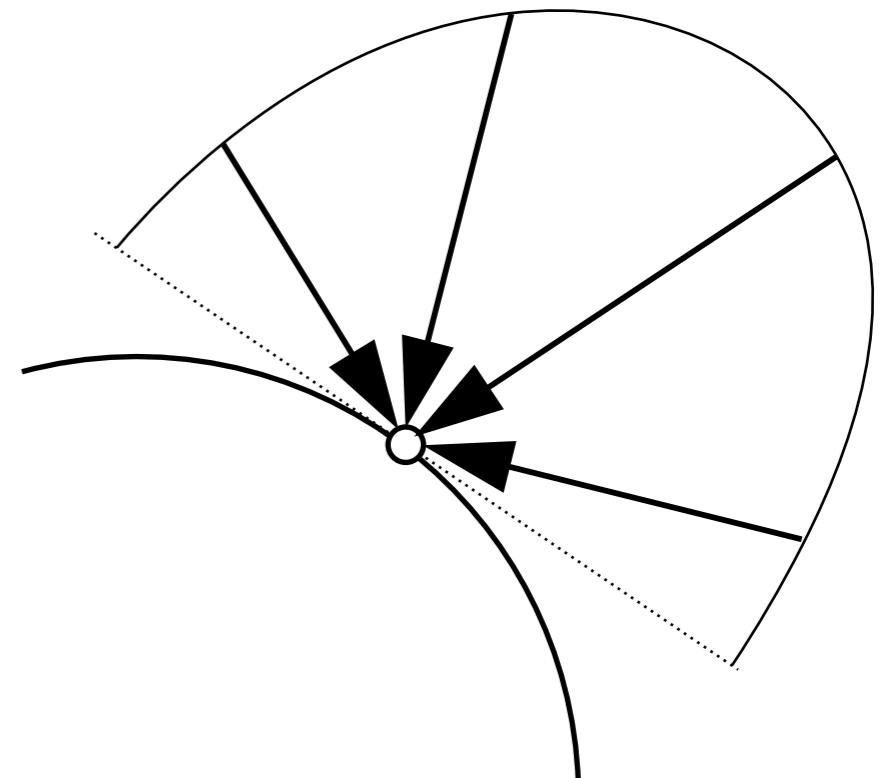
# Irradiance

Definition: The irradiance is **the power per unit area** incident on a surface point.

$$E(\mathbf{x}) \equiv \frac{d\Phi(\mathbf{x})}{dA}$$

面要和光线垂直

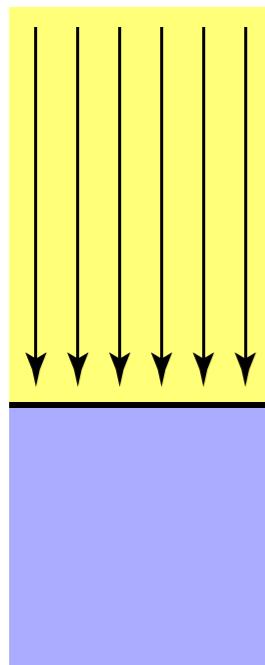
$$\left[ \frac{\text{W}}{\text{m}^2} \right] \left[ \frac{\text{lm}}{\text{m}^2} = \text{lux} \right]$$



# Lambert's Cosine Law

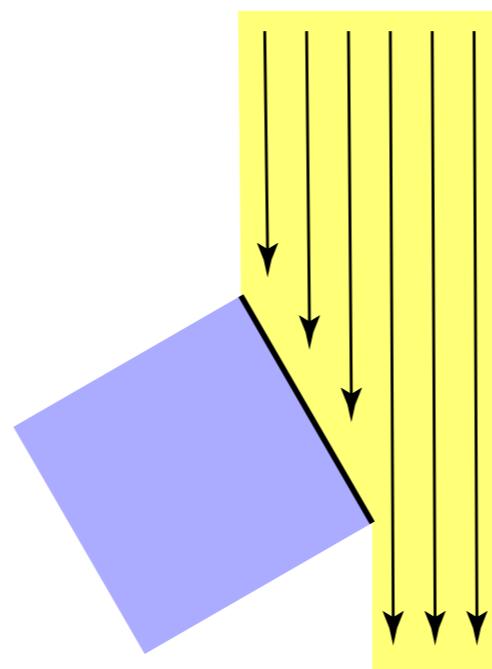
**Irradiance** at surface is proportional to cosine of angle between light direction and surface normal.

(Note: always use a unit area, the cosine applies on  $\Phi$ )



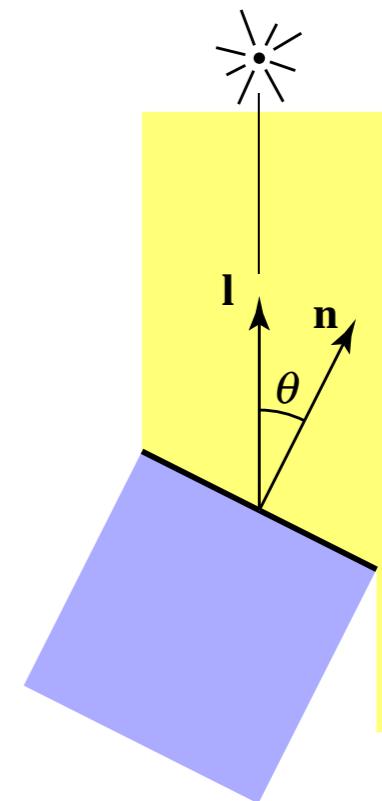
Top face of cube receives a certain amount of power

$$E = \frac{\Phi}{A}$$



Top face of 60° rotated cube receives half power

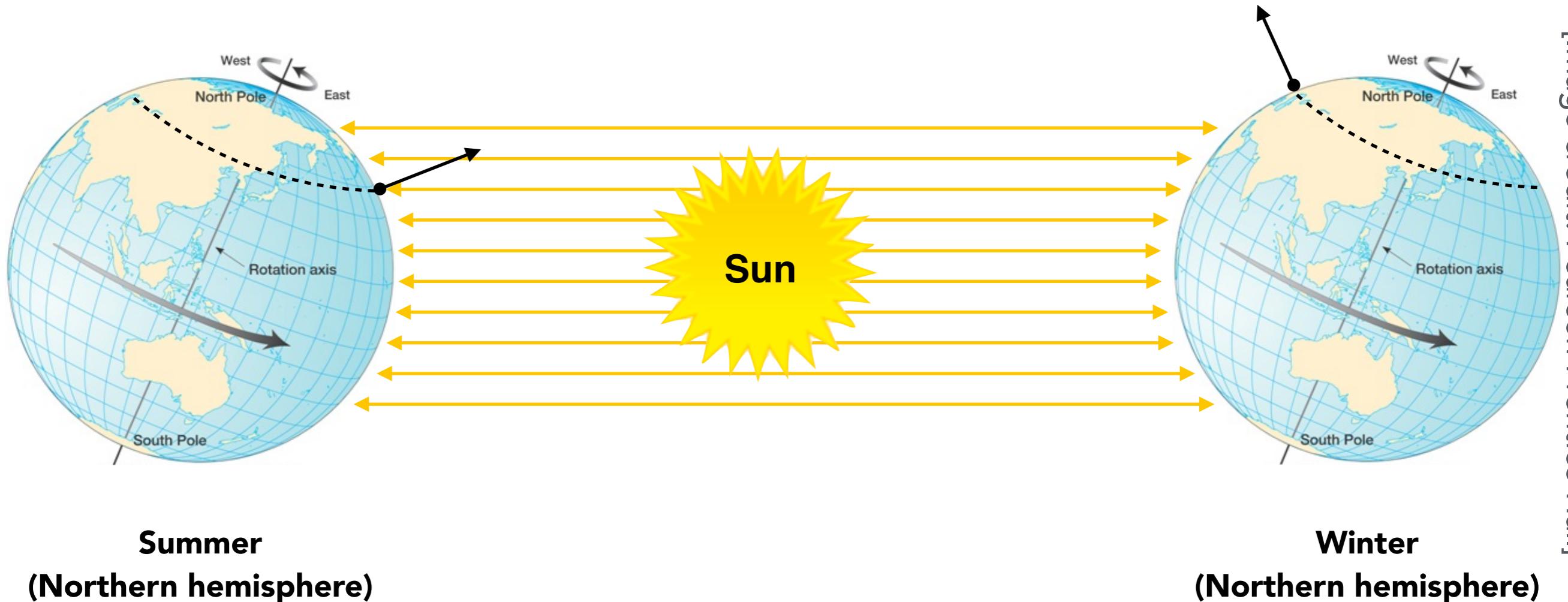
$$E = \frac{1}{2} \frac{\Phi}{A}$$



In general, power per unit area is proportional to  $\cos \theta = l \cdot n$

$$E = \frac{\Phi}{A} \cos \theta$$

# Why Do We Have Seasons?



Earth's axis of rotation:  $\sim 23.5^\circ$  off axis

[Image credit: Pearson Prentice Hall]

# Correction: **Irradiance** Falloff

Assume light is emitting power  $\Phi$  in a uniform angular distribution

Compare irradiance at surface of two spheres:

$$E = \frac{\Phi}{4\pi}$$

$r$

$$E' = \frac{\Phi}{4\pi r^2} = \frac{E}{r^2}$$

# Radiance

# Radiance

Radiance is the fundamental **field quantity** that describes the distribution of light in an environment

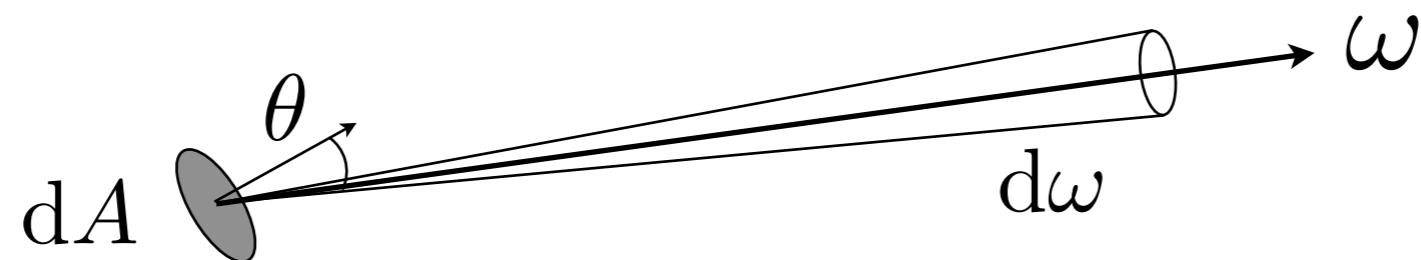
- Radiance is the quantity associated with **a ray**
- Rendering is all about computing radiance



Light Traveling Along A Ray

# Radiance

Definition: The radiance (luminance) is the power emitted, reflected, transmitted or received by a surface, **per unit solid angle, per projected unit area.**



$$L(p, \omega) \equiv \frac{d^2\Phi(p, \omega)}{d\omega dA \cos \theta}$$

$\cos \theta$  accounts for  
projected surface area

$$\left[ \frac{\text{W}}{\text{sr m}^2} \right] \left[ \frac{\text{cd}}{\text{m}^2} = \frac{\text{lm}}{\text{sr m}^2} = \text{nit} \right]$$



# Radiance

Definition: power **per unit solid angle per projected unit area**.

$$L(p, \omega) \equiv \frac{d^2\Phi(p, \omega)}{d\omega dA \cos \theta}$$

Recall

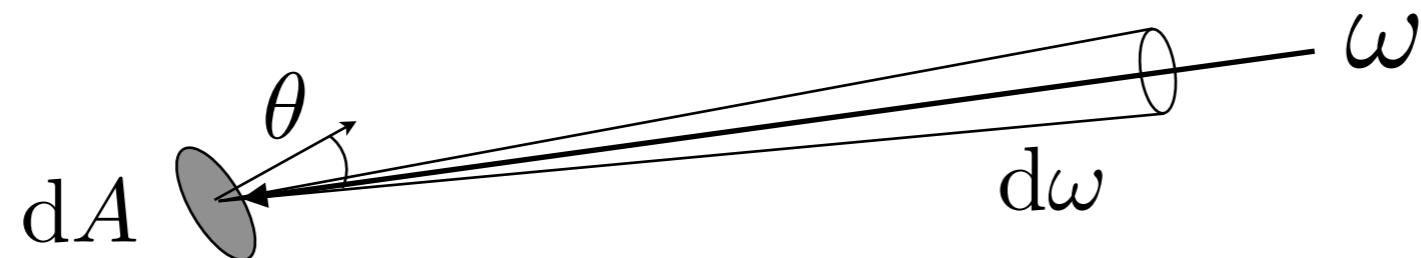
- Irradiance: power per projected unit area
- Intensity: power per solid angle

So

- Radiance: Irradiance per solid angle
- Radiance: Intensity per projected unit area

# Incident Radiance

Incident radiance is the irradiance per unit solid angle arriving at the surface.

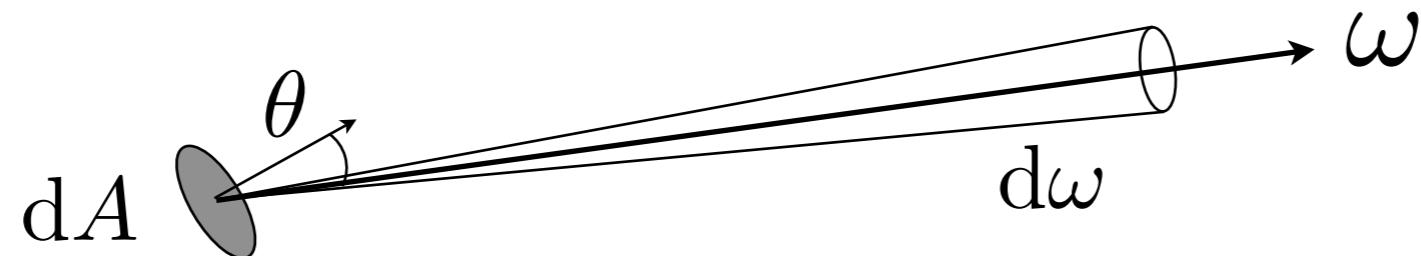


$$L(p, \omega) = \frac{dE(p)}{d\omega \cos \theta}$$

i.e. it is the light arriving at the surface along a given ray (point on surface and incident direction).

# Exiting Radiance

Exiting surface radiance is the intensity per unit projected area leaving the surface.



$$L(p, \omega) = \frac{dI(p, \omega)}{dA \cos \theta}$$

e.g. for an area light it is the light emitted along a given ray (point on surface and exit direction).

# Irradiance vs. Radiance

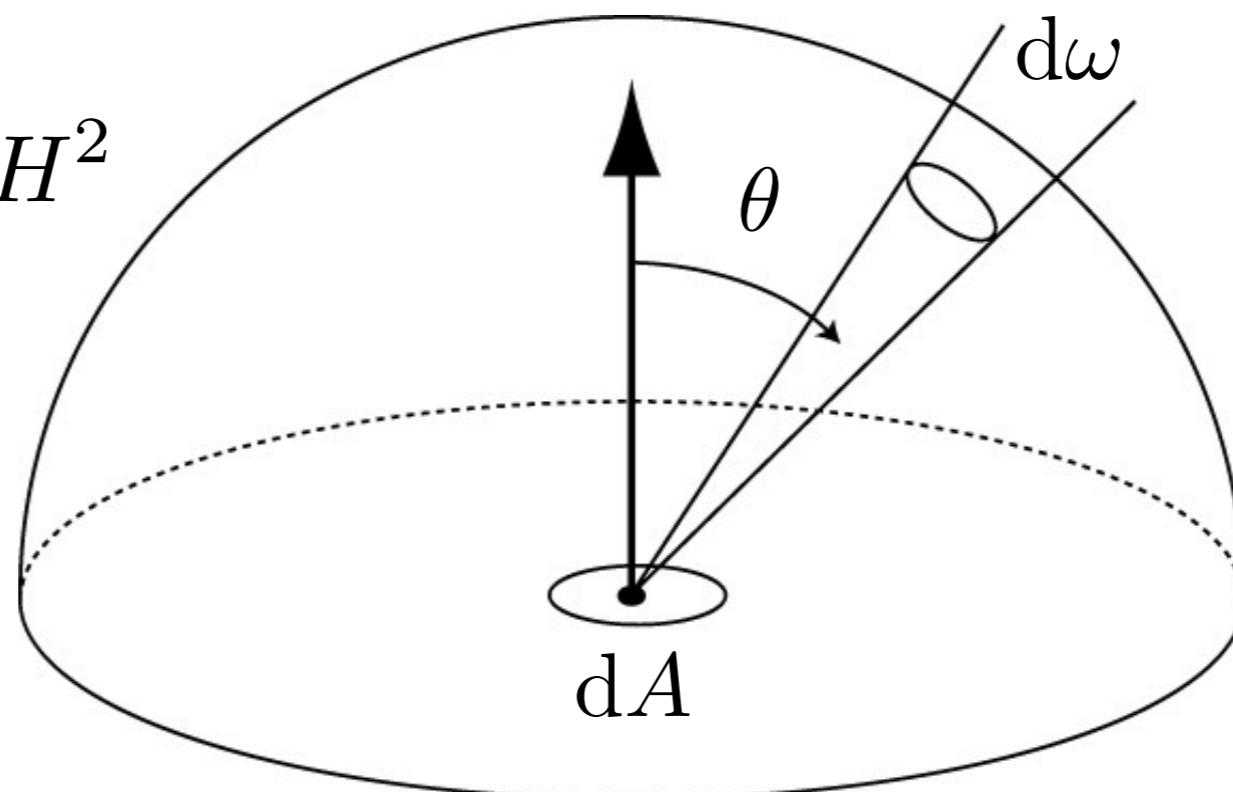
Irradiance: total power received by area  $dA$

Radiance: power received by area  $dA$  from “direction”  $d\omega$

$$dE(p, \omega) = L_i(p, \omega) \cos \theta d\omega$$

$$E(p) = \int_{H^2} L_i(p, \omega) \cos \theta d\omega$$

Unit Hemisphere:  $H^2$



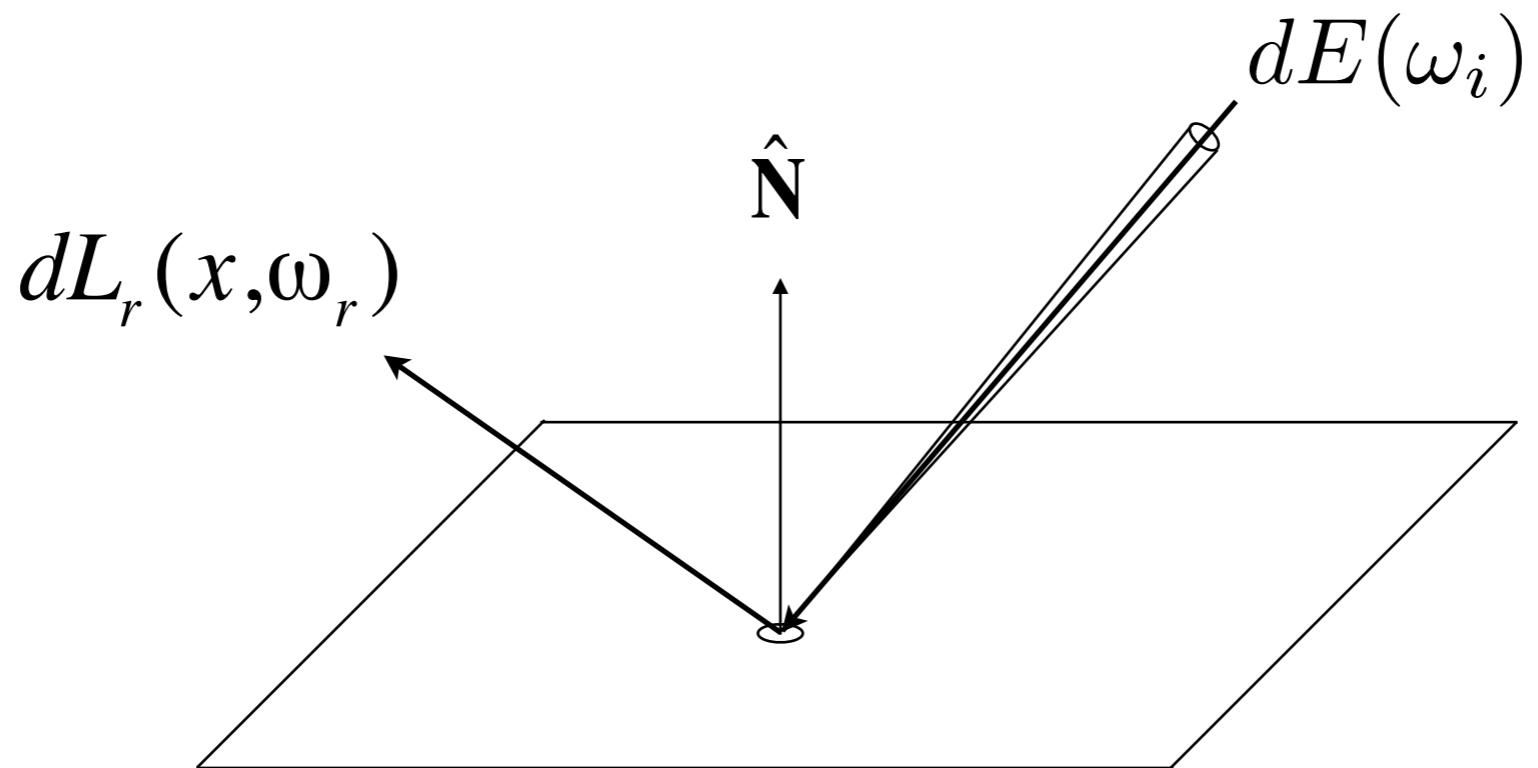
# Bidirectional Reflectance Distribution Function (BRDF)

双向反射分布函数

# Reflection at a Point

Radiance from direction  $\omega_i$  turns into the power  $E$  that  $dA$  receives

Then power  $E$  will become the radiance to **any other** direction  $\omega_o$ .



Differential irradiance incoming:

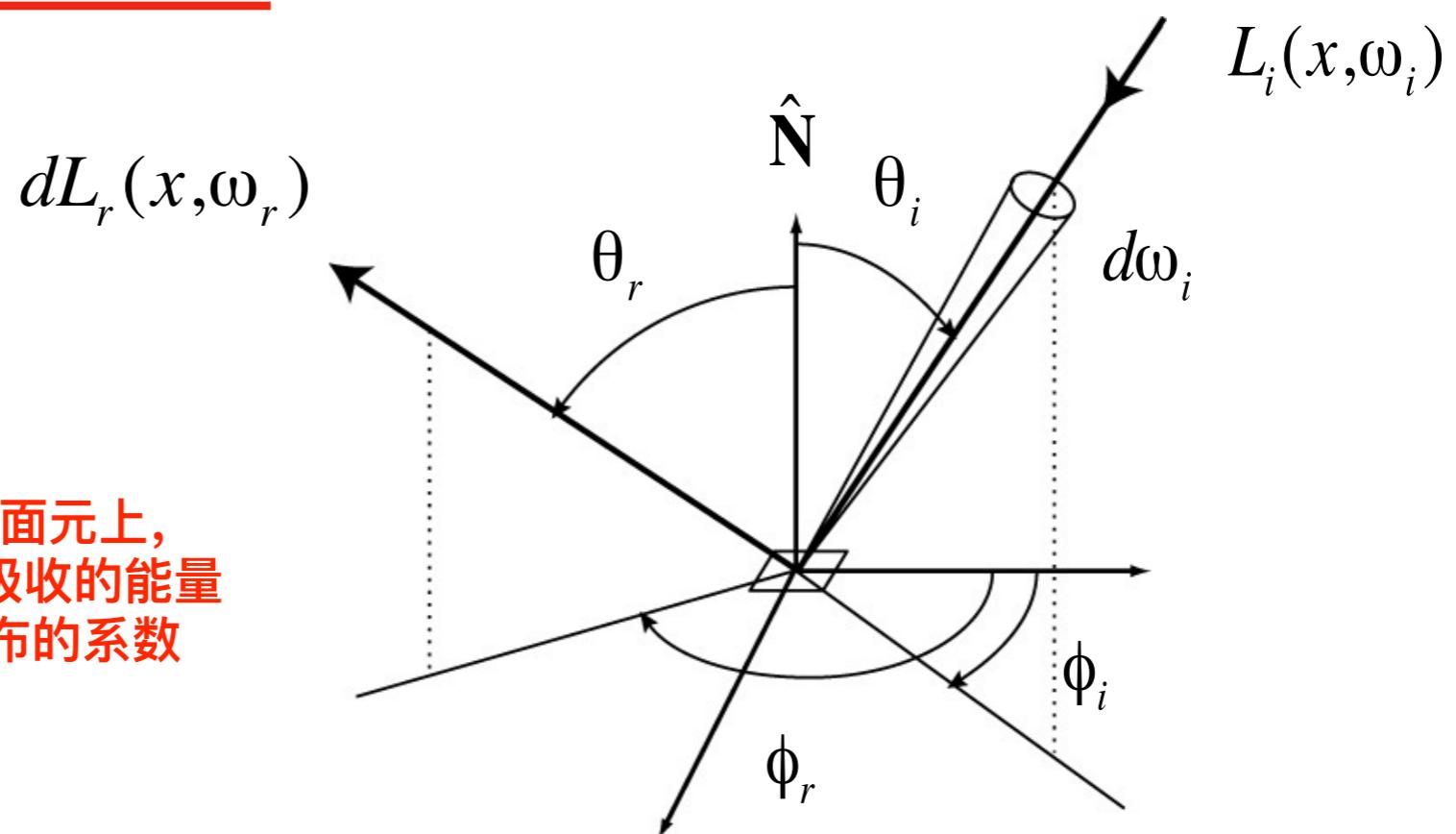
$$dE(\omega_i) = L(\omega_i) \cos \theta_i d\omega_i$$

Differential radiance exiting (due to  $dE(\omega_i)$ ):         $dL_r(\omega_r)$

系数: **dA是如何把吸收到的能量分布到空间中的**

# BRDF 描述了光线和物体如何作用：定义了不同物体的材质

The Bidirectional Reflectance Distribution Function (BRDF) represents how much light is reflected into each outgoing direction  $\omega_r$  from each incoming direction

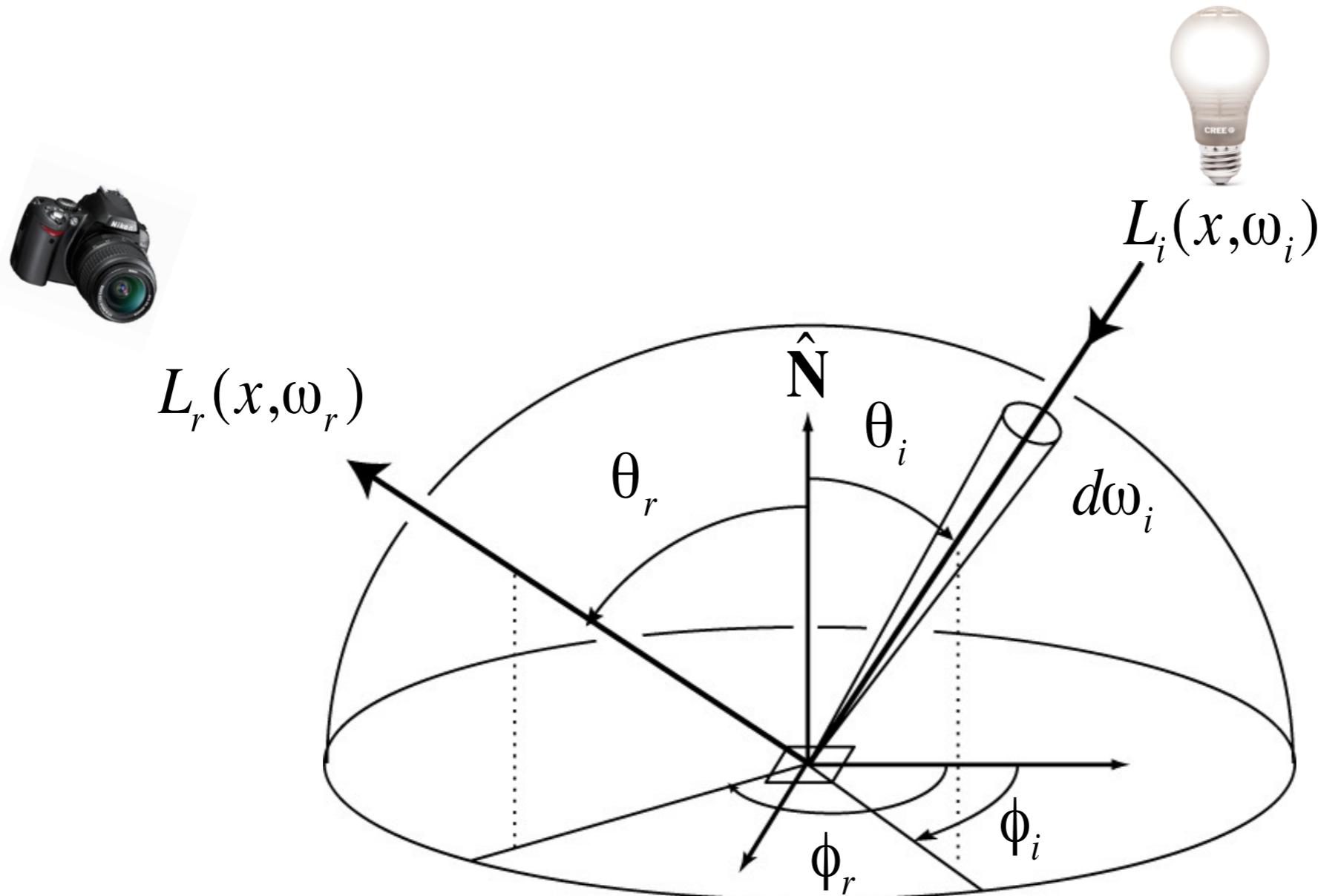


直观理解：某束光打到一个微分面元上，  
面元吸收光的能量；然后需要将吸收的能量  
继续向外辐射，f就代表辐射分布的系数

$$f_r(\omega_i \rightarrow \omega_r) = \frac{dL_r(\omega_r)}{dE_i(\omega_i)} = \frac{dL_r(\omega_r)}{L_i(\omega_i) \cos \theta_i d\omega_i}$$

$$\left[ \frac{1}{sr} \right]$$

# The Reflection Equation



反射方程

$$L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i d\omega_i$$

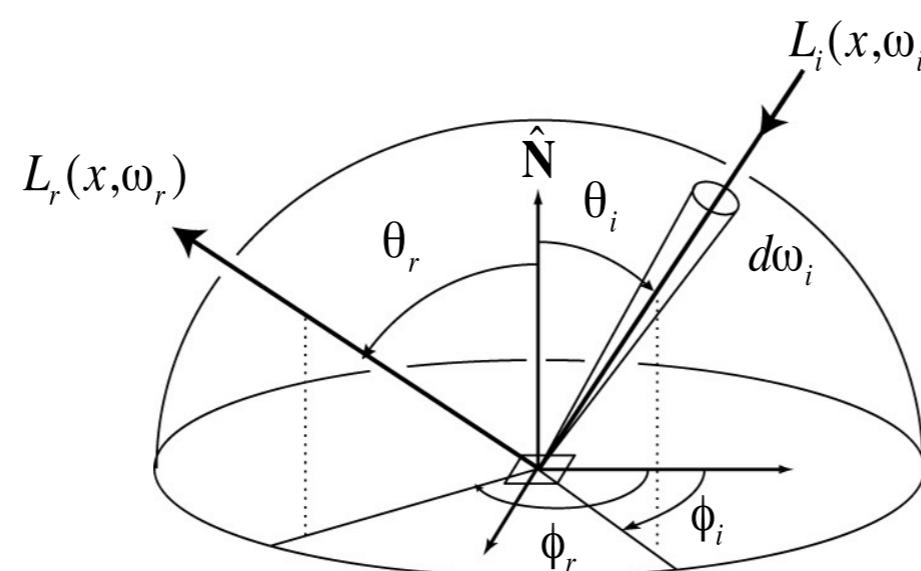
对所有入射光方向进行积分，最后反射到某一方向上的结果

# Challenge: Recursive Equation

光线在场景中不止弹射一次

Reflected radiance depends on incoming radiance

$$L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i d\omega_i$$



不仅仅是光源可以带来入射irradiance,  
其他物体表面反射的光  
也可以作为这个着色点的irradiance来源

But incoming radiance depends on reflected radiance (at another point in the scene)

# The Rendering Equation

渲染方程 (绘制方程)

Re-write the reflection equation:

$$L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i d\omega_i$$

by adding an Emission term to make it general!

The Rendering Equation

$$L_o(p, \omega_o) = L_e(p, \omega_o) + \int_{\Omega^+} L_i(p, \omega_i) f_r(p, \omega_i, \omega_o) (n \cdot \omega_i) d\omega_i$$

考虑物体本身发光

认为下半球的贡献一定为0

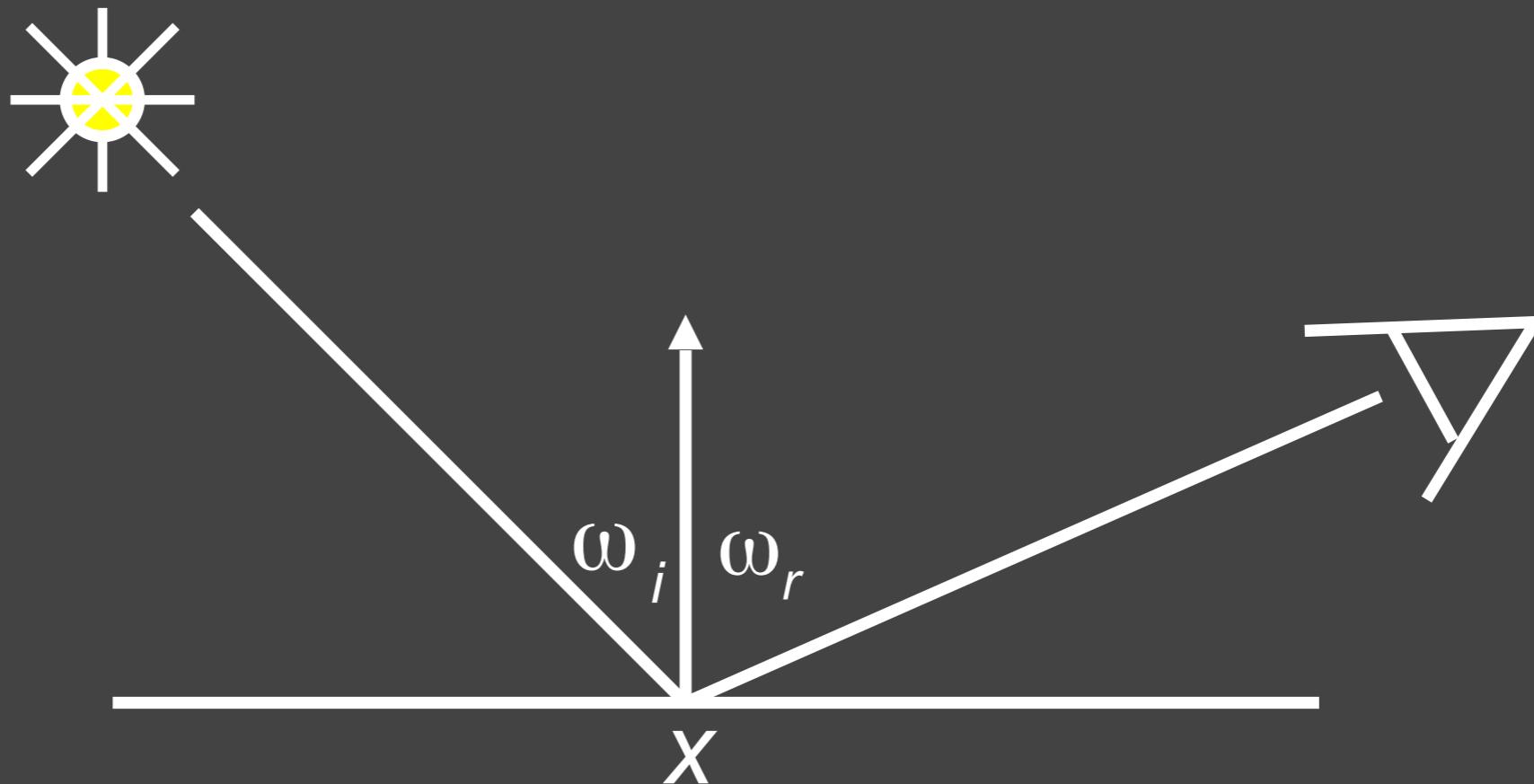
How to solve? Next lecture!

Note: now, we assume that all directions are pointing **outwards**!

$\omega_i$ 也是向外的

# Understanding the rendering equation

# Reflection Equation



$$L_r(x, \omega_r) = L_e(x, \omega_r) + L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i, n)$$

Reflected Light  
(Output Image)

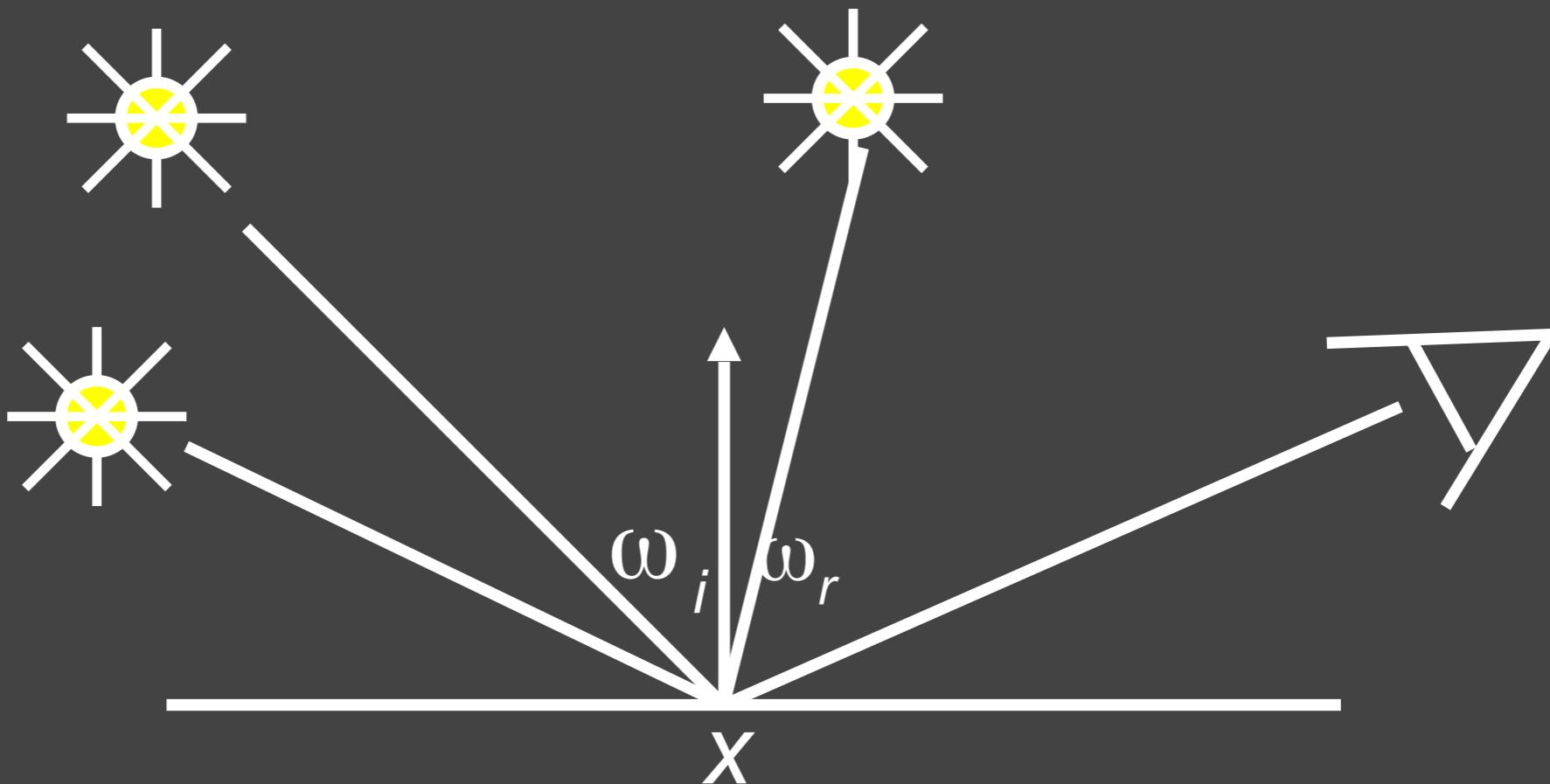
Emission

Incident  
Light (from  
light source)

BRDF

Cosine of  
Incident angle

# Reflection Equation



Sum over all light sources

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \sum L_i(x, \omega_i) f(x, \omega_i, \omega_r) (\omega_i, n)$$

Reflected Light  
(Output Image)

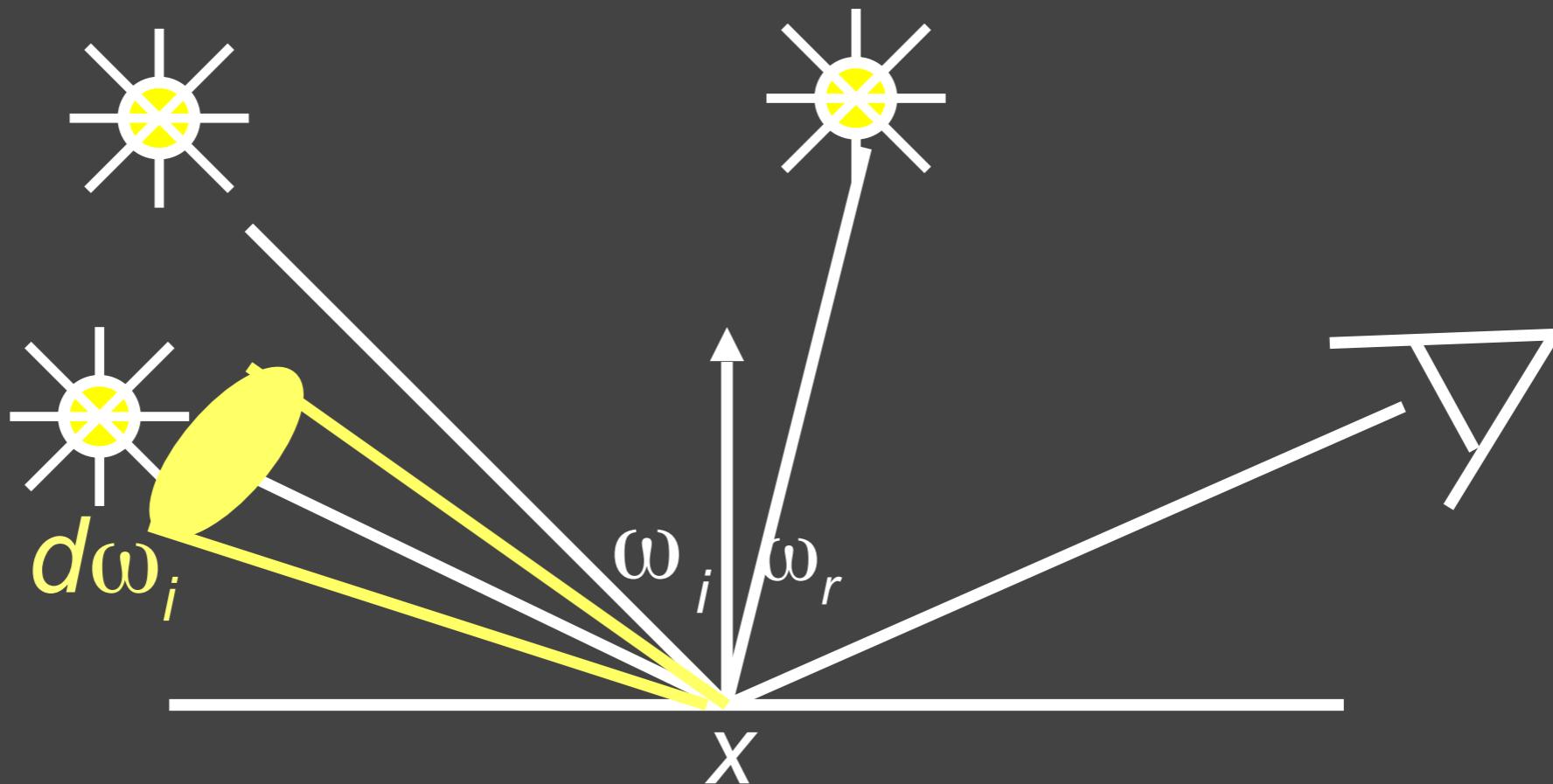
Emission

Incident  
Light (from  
light source)

BRDF

Cosine of  
Incident angle

# Reflection Equation



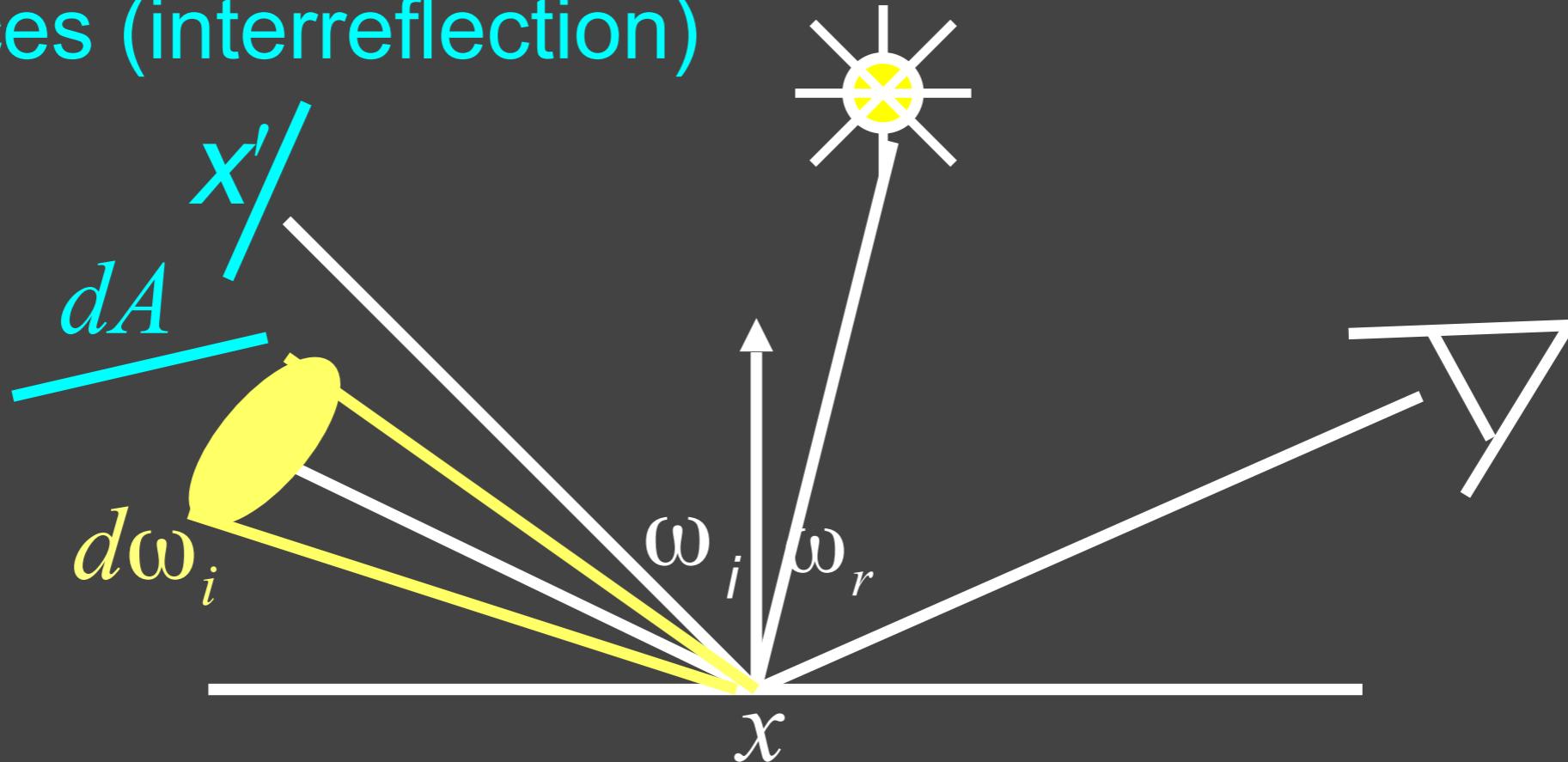
Replace sum with integral

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_i(x, \omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light (Output Image)      Emission      Incident Light (from light source)      BRDF      Cosine of Incident angle

# Rendering Equation

## Surfaces (interreflection)



$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos\theta_i d\omega_i$$

Reflected Light  
(Output Image)

UNKNOWN

Emission

KNOWN

Reflected Light

UNKNOWN

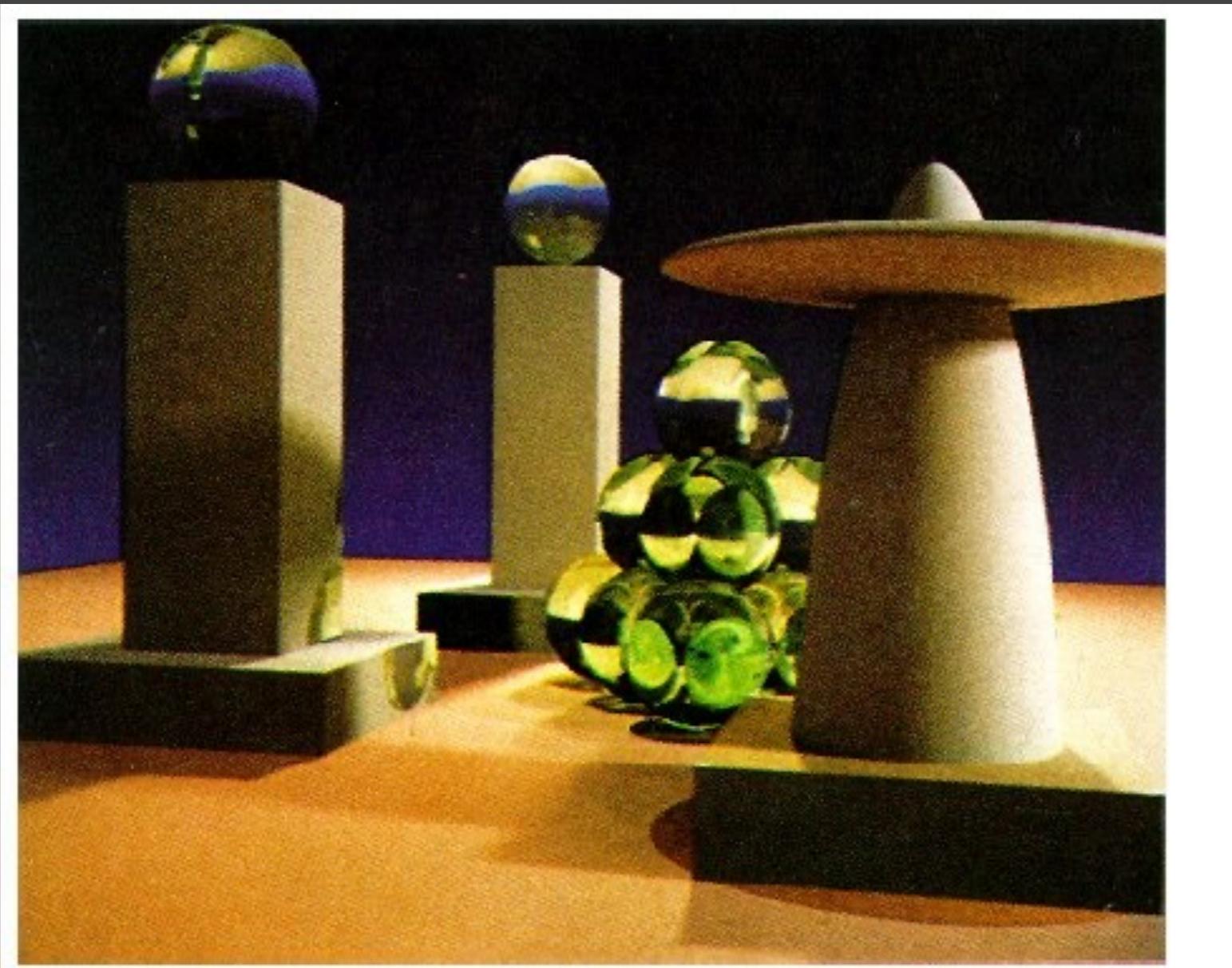
BRDF

KNOWN

Cosine of  
Incident angle

KNOWN

# Rendering Equation (Kajiya 86)



**Figure 6.** A sample image. All objects are neutral grey. Color on the objects is due to caustics from the green glass balls and color bleeding from the base polygon.

# Rendering Equation as Integral Equation

$$L_r(x, \omega_r) = L_e(x, \omega_r) + \int_{\Omega} L_r(x', -\omega_i) f(x, \omega_i, \omega_r) \cos \theta_i d\omega_i$$

Reflected Light  
(Output Image)

UNKNOWN

Emission

KNOWN

Reflected  
Light

UNKNOWN

BRDF

KNOWN

Cosine of  
Incident angle

KNOWN

Is a Fredholm Integral Equation of second kind  
[extensively studied numerically] with canonical form

$$I(u) = \Theta(u) + \int I(v) K(u, v) dv$$

Kernel of equation

# Linear Operator Equation

$$l(u) = e(u) + \int l(v) K(u, v) dv$$

Kernel of equation  
Light Transport Operator

$$L = E + KL$$

Can be discretized to a simple matrix equation  
[or system of simultaneous linear equations]  
(L, E are vectors, K is the light transport matrix)

# Ray Tracing and extensions

- General class numerical Monte Carlo methods
- Approximate set of all paths of light in scene

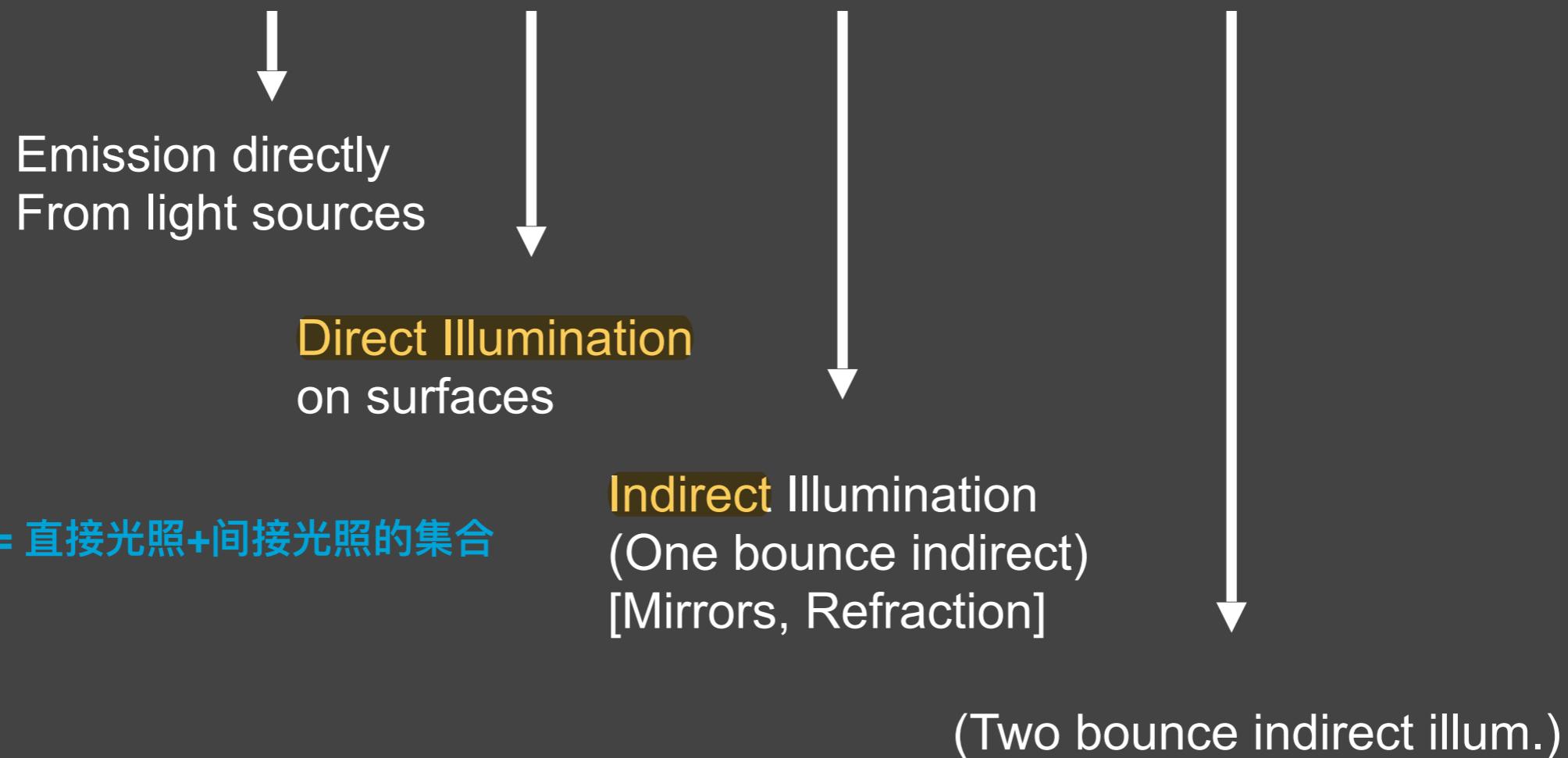
$$\begin{aligned} L &= E + KL \\ IL - KL &= E \\ (I - K)L &= E \\ L &= (I - K)^{-1}E \end{aligned}$$

Binomial Theorem

$$\begin{aligned} L &= (I + K + K^2 + K^3 + \dots)E \\ L &= E + KE + K^2E + K^3E + \dots \end{aligned}$$

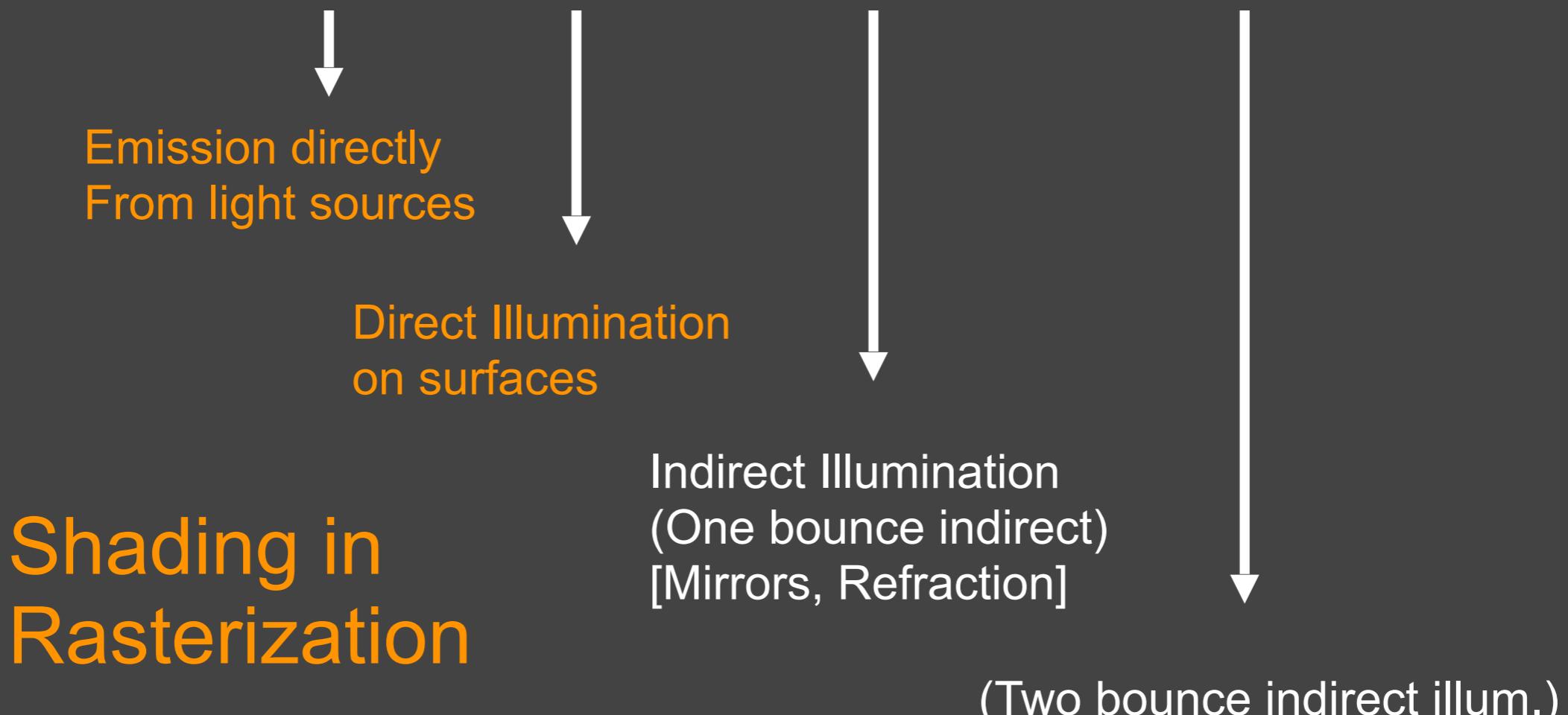
# Ray Tracing

$$L = E + KE + K^2E + K^3E + \dots$$



# Ray Tracing

$$L = E + KE + K^2E + K^3E + \dots$$



Shading in  
Rasterization

光栅化不能做间接光照的部分

对比：光栅化和光线追踪

**Direct illumination**

•*p*

•  
*p*

One-bounce global illumination (dir+indir)

Two-bounce global illumination

•  
*p*

$\bullet p$

Four-bounce global illumination

$\bullet p$

Eight-bounce global illumination

结果会收敛

•  
*p*

Sixteen-bounce global illumination

# Probability Review

# Random Variables

$X$

random variable. Represents a distribution of potential values

$X \sim p(x)$

probability density function (PDF). Describes relative probability of a random process choosing value

$x$

Example: uniform PDF: all values over a domain are equally likely

e.g. A six-sided die

$X$  takes on values 1, 2, 3, 4, 5, 6

$$p(1) = p(2) = p(3) = p(4) = p(5) = p(6)$$



# Probabilities

$n$  discrete values     $x_i$

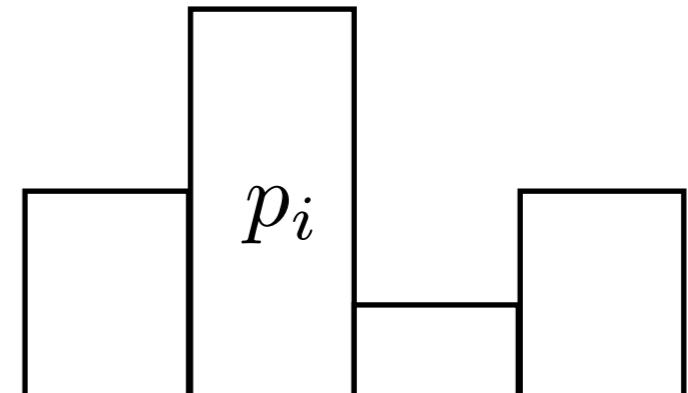
With probability     $p_i$

Requirements of a probability distribution:

$$p_i \geq 0$$

$$\sum_{i=1}^n p_i = 1$$

Six-sided die example:     $p_i = \frac{1}{6}$



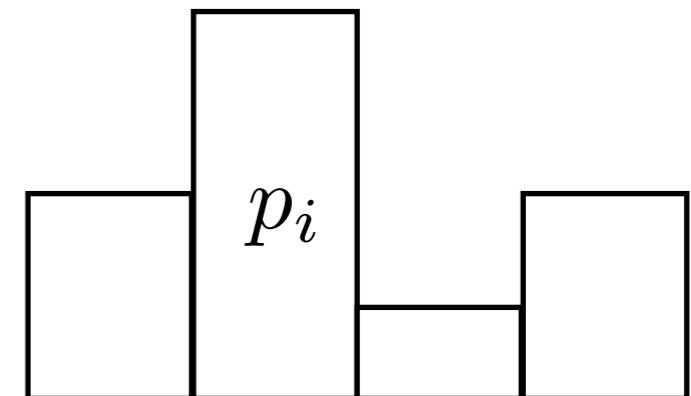
# Expected Value of a Random Variable

The average value that one obtains if repeatedly drawing samples from the random distribution.

$X$  drawn from distribution with

$n$  discrete values  $x_i$

with probabilities  $p_i$



Expected value of  $X$ :

$$E[X] = \sum_{i=1}^n x_i p_i$$

Die example:

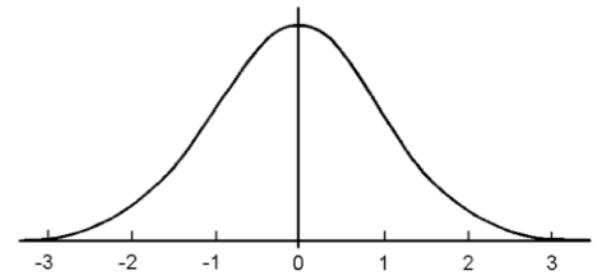
$$E[X] = \sum_{i=1}^n \frac{i}{6}$$

$$= (1 + 2 + 3 + 4 + 5 + 6)/6 = 3.5$$



## Continuous Case: **Probability Distribution Function (PDF)**

$$X \sim p(x)$$



A random variable  $X$  that can take any of a continuous set of values, where the relative probability of a particular value is given by a continuous probability density function  $p(x)$ .

Conditions on  $p(x)$ :

$$p(x) \geq 0 \text{ and } \int p(x) dx = 1$$

Expected value of  $X$ :

$$E[X] = \int x p(x) dx$$

# Function of a Random Variable

A function  $Y$  of a random variable  $X$  is also a random variable:

$$X \sim p(x)$$

$$Y = f(X)$$

Expected value of a function of a random variable:

$$E[Y] = E[f(X)] = \int f(x) p(x) dx$$

# Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)