Bounding Fertility Elasticities

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Abstract

I propose a new methodology to bound the magnitude of fertility responses to financial incentives. Under mild assumptions, I show that raising the fertility rate by 0.1 children per woman in the United States in 2010 requires the cost of children to fall between $7,514 and $35,966. This bound is tighter than the range provided by meta-studies of past estimates and is simple to compute for a different country or year. I also discuss further implications of this bound for models with and without endogenous fertility choice.

1 Introduction

How much does fertility respond to financial incentives? The answer to this question is important for both policymakers and economists. For governments that want to use financial transfers to raise fertility, it is critical to understand how cost-effective these policies are. For economists, fertility elasticity provides a benchmark to discipline models with endogenous fertility, just as the marginal propensity to consumer (MPC) informs modern macroeconomic models. In addition, estimates of the fertility elasticity allow one to evaluate how much the exogenous fertility assumption matters for the quantitative results in structural models on child-related policies, such as the Child Tax Credit (CTC) and the Earned Income Tax Credit (EITC).

There has been a large number of empirical studies estimating the fertility elasticity in different settings.[1] Pinning it down using historical policies has proven to be difficult for several reasons. First, there is a lack of large and persistent policies that change the cost of children drastically, making it unlikely for people to change their fertility behaviors, especially the total number of children, in a quantitatively significant way. Second, as pro-natal policies are typically nationwide or income-dependent, finding a control group is not straightforward (Gauthier (2005)). This difficulty is especially acute given that most pro-natal policies are adopted to address contemporaneous or future fertility decline, while it is unclear whether countries in the control group also face a similar problem (Castles (2003)). In addition, many pro-natal policies come in bundles of incentives that go beyond lowering the cost of children, such as measures encouraging women's labor force participation. Therefore, estimating the fertility elasticity of the policy bundle is not equivalent to estimating the price elasticity of fertility because the shadow price of other goods are affected simultaneously. As a result of these difficulties, empirical estimates vary widely – "there is considerable disagreement across studies about the effectiveness of pro-natal policies" even though "the directional finding that pro-natal benefits boost fertility is nearly uniform" (Stone (2020)). Last but not least, it is unclear to what extent past estimates predict fertility elasticities in a different time or institutional context.

In this paper, I propose a new methodology to *bound* the fertility elasticities. Under mild assumptions, I show that a bound can be computed after knowing (1) the prevailing fertility rate and cost of children, (2) the maximum desired fertility, and (3) the cost of children such that potential parents would rather prefer not to have a child. Applying this method to the United States in 2010, I find that a transfer of size between $7,514 and $35,966 is required to raise the fertility rate by 0.1 children per woman. I show that this bound is tighter than that in meta-studies of past estimates and is simple to compute for a different country or year under consideration. I also discuss further implications of this bound for models with and without endogenous fertility choice.

The rest of the paper is organized as follows. Section 2 presents the theory that derives the

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[1] See Milligan (2005), Cohen et al. (2013), González (2013) among many others. For review studies, see McDonald (2006) and Stone (2020).

bound. Section 3 applies the the bound to the United States, compares the results with prior studies, and discusses further implications of the bound. Section 4 concludes.

2 Theory

Denote the Marshallian demand of fertility as $n(p; \boldsymbol{p}^{\text{other}}, y)$ where n is fertility, p is the cost of children, $\boldsymbol{p}^{\text{other}}$ is the price (vector) of other goods, and y is the household's lifetime income.

Assumption 1 The Marshallian demand of fertility $n(p; \boldsymbol{p}^{\text{other}}, y)$ is downward sloping, continuously differentiable, and convex.

This assumption is satisfied by most models of endogenous fertility, either with warm glow (e.g., De La Croix and Doepke (2003)) or with dynastic altruism (e.g., Barro and Becker (1989) and Córdoba and Ripoll (2019)).

Assumption 2 There exists $\overline{p}$ and $\overline{n}$ such that $n(\overline{p}; \boldsymbol{p}^{\text{other}}, y) = 0$ and $n(0; \boldsymbol{p}^{\text{other}}, y) = \overline{n}$.

This is also a mild and realistic assumption. For example, set $\overline{p} = y$, the assumption requires that if having a child costs the parents' entire income and leaves no resources for other goods, then the household would prefer not to have a child. The existence of $\overline{n}$ reflects satiation in preferences or biological constraints of childbearing.

Proposition The fertility response to price around (n^0, p^0) is bounded by

$$\left. \frac{dn}{dp} \right|_{(n^0, p^0)} \in \left(\frac{n^0}{\overline{p} - p^0}, \frac{\overline{n} - n^0}{p^0} \right). \tag{1}$$

Proof Figure 1 shows the essence of the proof. The Marshallian demand of fertility is given by curve BAC where the coordinates of B and C are $(0, \overline{p})$ and $(\overline{n}, 0)$ correspondingly. Point A denotes the prevailing fertility and cost of children (n^0, p^0). We can show that the slope of the demand curve at A is bounded by the slope of AC and the slope of AB using the Mean

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Value Theorem and the assumption that the curve is decreasing, continuously differentiable, and convex.
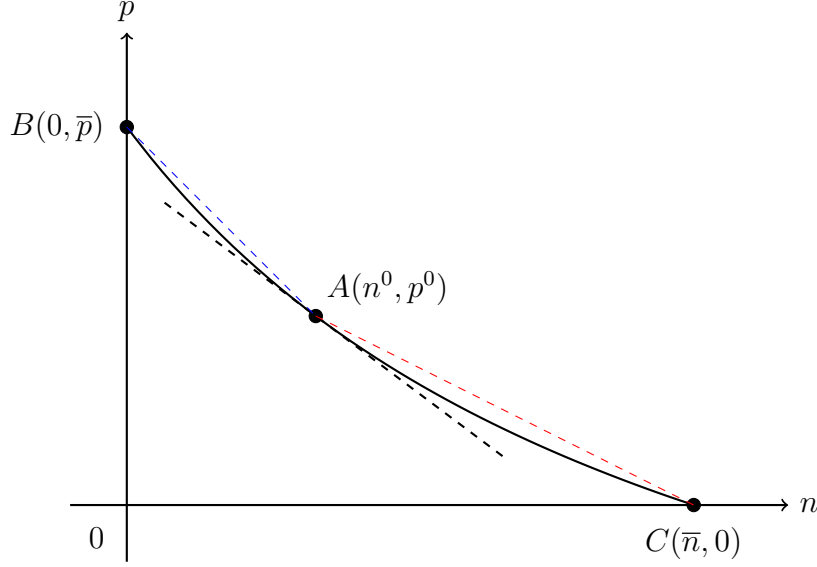

Figure 1: Illustration of the Proof

3   Quantification

In this section, I calculate bound for the United States, compare the results with prior studies, and discuss further implications of the bound.

3.1   Calculating the Bound

I calculate the bound for the United States in 2010 where the prevailing fertility n^0 is 1.9 and the cost of one child p^0 is \$458,351 for a middle-income household (Córdoba and Ripoll (2019)). I make the following assumption on $\overline{n}$ and $\overline{p}$:

Assumption 3 Set $\overline{n} = 8$. Choose $\overline{p}$ such that an average household with one child lives in poverty for the rest of their lives, i.e.,

$$y - \overline{p} = y^{\text{poverty}}. \tag{2}$$

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Proposition 1 makes it clear that the choice of $\overline{n}$ and $\overline{p}$ affects the tightness of the bound – as $\overline{n}$ or $\overline{p}$ decreases, the bound becomes tighter. The assumption that $\overline{n} = 8$ is probably conservative towards the high end as the prevailing ideal number of children is around 2.5 children per woman (Stone (2018)). The assumption that $\overline{p}$ is the cost of children that makes an average married household fall into poverty status after one childbirth is also a likely upper bound for $\overline{p}$. Thus, there are reasons to believe that the bound could be made tighter under alternative reasonable assumptions.

Following Córdoba and Ripoll (2019), I set $y = \$2,083,219$. Using the federal poverty guideline in 2010, I set $y^{\text{poverty}} = \$18,310 \times 18 + \$14,570 \times 42 = \$941,520$. This implies $\overline{p} = \$1,141,699$.

Applying the proposition, I find that to raise the fertility by 0.1 children per woman, the change in p needed is between \$7,514 and \$35,966. This change in p can be brought about by policies such as a baby bonus, a universal basic income, or a fully-refundable Child Tax Credit (CTC) expansion.

3.2 Comparing to Previous Estimates

I compare my results to meta-studies of past estimates. Stone (2020) summarizes 22 studies since 2000 using historical policies, mostly in low-fertility countries. He concludes that "an increase in the present value of child benefits equal to 10% of a household's (annual) income can be expected to produce between 0.5% and 4.1% higher birth rates." To make the measures comparable, I convert the bound in this paper into percentages using the median annual household income in 2010 (\$49,445). The bound predicts that an increase in the present value of child benefits equal to 10% of a household's (annual) income can produce between 0.72% and 3.46% higher birth rates. As can be seen, the bound proposed here is tighter than existing estimates.[2]

Another advantage of the bound is that it is simple to compute for a different country or year as long we know the prevailing (n^0, p^0), y, and y^{poverty}, which are not difficult to obtain. Adopting an actual pro-natal policy and estimate the fertility elasticities afterwards will likely lead to a more precise estimate. But the bound could still be helpful as it provides a prediction of the program's

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[2]Another difference worth noting is that the bound proposed in this paper is a *theoretical* bound under assumptions while the range in Stone (2020) is a *statistical* bound that summarizes past estimates.

cost-effectiveness *ex ante* and a check of the regression result *ex post*.

3.3 Further Implications

Beyond informing policymakers how costly it is to raise fertility, the bound provides a benchmark to discipline models with endogenous fertility and evaluate the exogenous fertility assumption in models that analyze child-related policies.

3.3.1 Endogenous Fertility w/ Dynastic Altruism

Consider a model of fertility choice with dynastic altruism following Barro and Becker (1989). Agents solve

$$U_0 = \max_{c_t, n_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\prod_{i=0}^{t-1} n_i \right)^{1-\varepsilon} \cdot \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$w_t + (1 + r_t)k_t = c_t + n_t(\chi_t + k_{t+1}) \quad \forall t$$

and the initial k_0. It is assumed that $\beta, \varepsilon, \sigma \in (0, 1)$ to ensure that children are goods and $\varepsilon \leq \sigma$ for the second-order condition to hold.[3]

Using first-order conditions, the Marshallian demand of fertility in this economy is given by

$$n_t = (\beta(1 + r_{t+1}))^{\frac{1}{\varepsilon}} \left[\frac{\chi_{t-1}(1 + r_t) - w_t}{\chi_t(1 + r_{t+1}) - w_{t+1}} \right]^{\frac{\sigma}{\varepsilon}} \tag{3}$$

where the net cost of creating a descendant at time t is $p_t \equiv \chi_t(1 + r_{t+1}) - w_{t+1}$. Therefore, as the net cost of children p_t falls by 1 percent, fertility n_t increases by $\frac{\sigma}{\varepsilon}$ percent *ceteris paribus*. Relating this elasticity to the numerical bound for the U.S. in 2010, the value of $\frac{\sigma}{\varepsilon}$ should lie between 0.67 and 3.23. Combining with the assumption on σ and ε, the full set of restrictions are

$$0 < \varepsilon \leq \sigma < 1 \quad \text{and} \quad \sigma < 3.23 \cdot \varepsilon \tag{4}$$

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[3]Jones and Schoonbroodt (2010) discuss an alternative set of assumptions which implies that the quantity and quality are substitutes rather than complements as in standard Barro-Becker models.

As can be seen, the bound puts further restrictions on the choice of σ and ε. For example, Doepke (2005) satisfies these restrictions with $\sigma = \varepsilon = 0.5$; Manuelli and Seshadri (2009) chooses $\sigma = 0.62$ and $\varepsilon = 0.35$; and Córdoba (2015) considers $\sigma = 0.3$ and $\varepsilon = 0.288$.

3.3.2 Endogenous Fertility w/ Warm Glow Utility

Consider another model of fertility choice with warm glow utility. Agents solve

$$\max_{c,n} \quad c + \beta \cdot \frac{n^{1-\sigma}}{1-\sigma}$$

subject to

$$c + n \cdot p \leq y.$$

As before, it is assumed that $\sigma \in (0,1)$ to ensure that children delivers positive utility.

With interior solutions, the Marshallian demand of fertility is given by

$$n^* = \left(\frac{p}{\beta}\right)^{-1/\sigma}.$$

which implies that when p falls by 1 percent, n^* rises by $1/\sigma$ percent. Thus, if one calibrates this model to the U.S. economy in 2010, the bound suggests that the value of σ, which is typically chosen exogenously, should lie between 0.31 and 1.

In general, one can use the bound to validate comparative static results in other models of endogenous fertility, such as Manuelli and Seshadri (2009), Córdoba et al. (2016), and Daruich and Kozlowski (2020). For instance, Zhou (2022) shows that in a Huggett-Aiyagari model of quantity-quality trade-off calibrated to the U.S. economy, raising fertility rate by 0.1 children per woman requires the cost of children to fall by $15,000 - a number that is within the bound. Moreover, as suggested by Córdoba and Ripoll (2019), the elasticity of intergenerational substitution (EGS) in the model plays a vital role in the magnitude of the fertility elasticity.

3.3.3 Models w/ Exogenous Fertility Assumption

The bound also permits an evaluation of the exogenous fertility assumption in structural models that analyze child-related policies. For example, Guner et al. (2020) consider a policy counterfactual where the per-child tax credit rises about $800 per child per year, amounting to about $12,000 in net present value terms for eligible households.[4] The optimal policy in Mullins (2019) is a Negative Income Tax on mothers that is equivalent to an additional $82 per week over 17 years, i.e., a transfer greater than $50,000 in net present value. Daruich (2022) shows that the welfare-maximizing early-childhood development subsidy is around $80,000.

A real-world policy example is the proposed renewal of the (fully-refundable) Child Tax Credit expansion that increases the annual transfer from $2,000 dollars to to $3,600 per child under age 6 and $3,000 per child ages 6 through 17. The net present value of this expansion is above $30,000 per child for eligible families.

With prior calculations, it can be seen a $10,000 reduction in the cost of children increases fertility by 0.03 to 0.13 from the baseline level of 1.9 children per woman. Thus, these proposed policies are likely going to lead to non-negligible fertility responses and a dilution of family resources, triggering the quantity-quality trade-off mechanism à la Becker and Lewis (1973). As a result, the policies might not achieve the goal of raising children's human capital or improving social mobility (due to heterogeneous fertility responses). Nevertheless, such policies could still benefit the society in the long-run through mechanisms such as changing the dependency ratio (Zhou (2022)).

4 Conclusion

The magnitude of fertility responses to financial incentives is important to both policymakers and economists. In this paper, I propose a new methodology to bound the fertility elasticities. Under mild assumptions, I show that a transfer of size between $7,514 and $35,966 is required to raise the fertility rate by 0.1 children per woman in the United States in 2010. This bound is tighter than the

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[4]The net present values are all calculated using a 4% annual discount rate.

range provided by meta-studies and is simple to compute for a different country or year. The bound puts further restrictions on parameters in endogenous fertility models and allows an evaluation of the exogenous fertility assumption in structural models that analyze child-related policies.

The bound is complementary to empirical inquiries into fertility elasticity using quasi-experimental variations and/or randomized control trials. It provides a prediction of the intervention's cost-effectiveness *ex ante* and a check of the regression result *ex post*.

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