The Macroeconomic Consequences of Family Policies*

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Abstract

I build a quantitative general equilibrium overlapping-generations model with heterogeneous agents, endogenous fertility, child human capital formation, and inter-vivos transfers to study the macroeconomic consequences of large-scale family policies (e.g. baby bonus, childcare subsidies). In the calibrated model that matches the U.S. data, I find that: (1) in contrast to conventional wisdom from studies with exogenous fertility, parents optimally reduce child human capital investments in response to child-related cash transfers, which dampens social mobility as this reduction is larger among low-income households; (2) a $31,000-dollar baby bonus raises the aggregate fertility rate to replacement level but reduces average human capital by 1.5%; (3) this baby bonus increases long-run welfare by 2.2% as old-age dependency ratio drops and taxes fall, but it requires heightened taxes in transition phase to finance larger child-related expenditures, hurting most households in the current economy; and (4) among different policies, subsidized childcare combines childbearing and labor force participation; while public education expenditure is the most effective policy in boosting child outcomes and social mobility despite having mild effects on fertility.

JEL classification: E62, H31, H52, J11, J13

Keywords: Family policies, quality/quantity trade-off, demographic structure

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1 Introduction

Family policies are social programs and laws designed to promote and enhance family formation, reproduction, and raising children (Bogenschneider, 2011). Examples of family policies include but not limited to baby bonus[1], parental leaves, and subsidized childcare. These policies are prevalent in developed countries and are often large in scale. By 2015, every OECD country has sizable public spendings exclusively on family and children. These expenditures, in-cash and in-kind, account for 2.1% of national GDP on average.[2]

In most cases, family policies are pursued by governments to achieve two goals. First, they are key instruments for governments to address population aging which poses great threat to economic growth (Aksoy et al., 2019) and the sustainability of public pension system (Bongaarts, 2004). For example, the Australian government made it eligible for parents of every child born after July 1st 2004 to receive a lump-sum payment that is equivalent to $2,800 so as to mitigate the effects of aging population. Peter Costello, the Treasurer of Australia at that time, set the ambitious goal of such baby bonus as to encourage each family to have "one (baby) for the Mum, one for the Dad, and one for the country". Similarly in Russia, Spain, Singapore, many countries struggling with low fertility are using family policies to incentivize childbirth. Concerns about low-fertility are also becoming increasingly relevant for the United States as it recently experiences record-low birthrate.[3]

Second, family policies are also adopted to reduce child poverty, improve children's outcomes, and raise social mobility. Stating such goals, proposals to expand (refundable) Child Tax Credit (CTC) have received bipartisan support in the Congress,[4] with the maximum amount of CTC awarded per child annually increasing from $2,000 to over $3,000 in 2021. In addition, the Biden Administration has also proposed the $1.8 trillion American Families Plan advertised as "an investment in our kids, our families, and our economic future" (The White House, 2021). If implemented, the Plan will provide further child-related tax benefits, more generous parental leaves, and greater support for public education. Guided by empirical evidence suggesting that transfers to families with children could improve children's future outcomes, economists predict that such policies will "lift children out of poverty today and help them tomorrow" (Schanzenbach et al., 2021) and boost social mobility (Pulliam and Reeves, 2021).

With growing popularity of large-scale family policies, it is important to understand their macroeconomic consequences. Are family policies cost-effective in raising fertility, reducing old-age de-

[1] Baby bonus is a one-time, lump-sum cash transfer to parents for each newborn or adopted child. Notably, baby bonus is different from "baby bond" proposed by William Darity and Darrick Hamilton to reduce wealth gaps. Under baby bonds, children themselves, but not the parents, gain access to the trust account at the age of 18.

[2] See https://data.oecd.org/socialexp/family-benefits-public-spending.htm for more details.

[3] For example, see https://www.wsj.com/articles/births-in-u-s-drop-to-levels-not-seen-since-1979-11620187260.

[4] See proposals from Senator Bennett (D-CO). Senator Romney (R-UT) proposed the Family Security Act which is similar to a fully refundable CTC but without minimum income requirement and other limitations..

pendency ratio, and improving social welfare? Will generous child benefits improve children's outcomes and raise social mobility? In this paper, I provide comprehensive answers to these questions by evaluating several family policies in a quantitative general equilibrium overlapping-generations model with heterogeneous agents and distortionary taxes.

To capture the key trade-offs in family policies, the model has two components regarding parents' choices. First, households make endogenous child quantity choices, internalizing the potential benefits of additional children provided by family policies. Second, parents make choices on child quality through investments in child human capital formation and inter-vivos transfers.

In contrast to design-based researches that focuses on the policy effects on qualities of existing children, considering the quantity margin is crucial to understanding the overall impacts of family policies for three reasons. First, when parents raise fertility to take advantage of more generous child benefits, the marginal cost of providing the same level of child quality increases. Therefore, policies that aim at boosting fertility may worsen the outcome of children due to *quality/quantity trade-off* a lá Becker and Lewis (1973). Second, family policies induce *composition effects* on aggregate variables. With intergenerational persistence of traits such as human capital, aggregate variables gravitate over time towards the traits of those families that respond most strongly in fertility because their children account for a larger share of the population in future generations. Third, higher fertility induces *demographic structure effects* on government budget and taxes due to changes in the share of population in each age group. As population growth rate rises, the burden of pension payments under pay-as-you-go (PAYG) system is relieved but public education expenditures on children rise.

All three channels mentioned above depend on the magnitude of fertility response to financial incentives, hereafter denoted as *fertility elasticities*. In the extreme case where fertility is exogenous or fixed, i.e. fertility elasticity is zero, child-related benefits would only have income effects on children's quality and the model predictions on child human capital and social mobility are similar to that in Daruich (2018) and Mullins (2019). I discipline the magnitude of fertility elasticities using a set of calibrated parameters that matches key empirical regularities in the United States in 2010. I also provide external validation using Alaska Permanent Fund Dividend (APFD) to show that the quantitative predictions of the model is consistent with design-based evidence.[5]

The calibrated model generates four key findings. First, a universal baby bonus of $31,000 in 2010 dollars[6] boosts the average fertility rate rises from 1.9 children per women to 2.1 children per women, the replacement level. Parents with lower human capital have stronger responses to the

[5] I conduct additional validation exercises using Australian baby bonus, Spanish child benefits, and Georgia's Cherokee Land Lottery of 1832 presented in Appendix B.

[6] To put this number in perspective, this baby bonus is similar to the size of expansions in Child Tax Credit (CTC) from 2010 to 2021 in net present value (author's own calculation). It offsets the average cost of raising on child by 19% using estimates from U.S. Department of Agriculture (USDA).

policy by having more additional children.

Second, this baby bonus is not effective in raising children's human capital and social mobility. Parents optimally reduce child educational investments by 8% on average due to quality/quantity trade-off, and this reduction is larger among low-income parents. Such (heterogeneous) reductions in educational investments, coupled with the composition effects due to heterogeneous fertility responses, lower the average human capital by 1.5% and intergenerational income mobility by 1.5% in the long-run economy.

Third, I find that despite reductions in average human capital and social mobility, long-run social welfare[7] rises by 2.2% in consumption equivalence units. As the baby bonus boosts population growth rate and lowers the old-age dependency ratio,[8] the government is able to reduce tax rates while still balancing fiscal budget. The policy is also highly progressive. It improves the welfare of parents with low human capital by more than 6%.

The transition towards such long-run welfare improvements, however, requires a prolonged path where the government temporarily raises taxes to finance higher child-related expenditures. While old-age dependency ratio falls gradually, total dependency ratio[9] spikes before converging to its long-run level. As a result of heightened taxes, welfare impacts on generations along the transition path are positive but smaller than that in the long-run steady-state while most households in the current economy experience welfare losses. In the study of optimal policies, I elaborate on this point by showing that the optimal baby bonus that maximizes the long-run welfare is significantly different from the one that prioritizes the well-being of existing parents in the economy.

Lastly, I use the calibrated model to study the macroeconomic impacts of alternative policies at the forefront of public debate such as subsidized childcare and expansions in public education expenditures. Among different policy options, I find that (1) subsidized childcare encourages child-bearing and labor force participation at the same time because it substitutes home production of childcare by marketable childcare services, and (2) among all policy instruments studied in this paper, public education expenditure, known as "the great equalizer", is the most effective one in improving child outcomes and boosting social mobility despite having mild effects on fertility.

Related Literature

This paper relates to four strands of literature. The first set of papers study fertility in a macroeconomic context and analyze how it is related to the fiscal stability, economic growth, intergenerational mobility, and inequality. Notable examples include Barro and Becker (1989), Doepke (2004), Greenwood et al. (2005), Manuelli and Seshadri (2009), De Silva and Tenreyro (2020), Cavalcanti et al. (2021), and Jones (2020) among many others. The closest works are de La Croix and Doepke

[7] Average utility of new-born under the veil of ignorance. See Section 6 for more discussions on welfare criteria.
[8] The number of retired households divided by the number of working population.
[9] The number of retired people plus the number of children, divided by the number of working population.

(2003) and Kim et al. (2021). de La Croix and Doepke (2003) study how higher inequality reduces growth through the channel of fertility differentials. While I adopt their framework featuring general equilibrium, heterogeneous agents, endogenous fertility, and human capital formation, I extend the basic model in various dimensions and use the model to study aggregate impacts of family policies. In a recent work, Kim et al. (2021) adopt the de La Croix and Doepke (2003) model and add status externality mechanism to explain the fertility-income relationship in Korea and study the impacts of pronatal transfers and education taxes. Different from their work, this paper allows for a richer life-cycle structure with savings and retirement so that demographic structure effects, an important motivation for most family policies, can be quantified.

Second, the paper is related to the literature on positive and normative aspects of government policies regarding fertility, redistribution and education. On the positive side, Daruich (2018) and Abbott et al. (2019) are the closest work to mine. Both papers use heterogeneous agent overlapping generations model to study large-scale education policies. I differ from these work by modeling endogenous fertility choice and considering the interaction of family and education policies. On the normative side, Guner et al. (2020) considers optimal design child-related transfers with heterogeneous households making both intensive and extensive labor force participation choices while keeping fertility exogenous. This paper add to the literature by studying the design of family policies with endogenous fertility responses.

This paper is also related to a large body of empirical work on the fertility effects of family policies (e.g. Milligan, 2005; Drago et al., 2011; González, 2013; Laroque and Salanié, 2014). Meta-analyses of these studies have concluded that "the directional finding that pronatal benefits boost fertility is nearly uniform". An increase in present value of child benefits equal to 10% of household annual income would lead to 0.5% to 4.1% increase in total fertility rate (McDonald, 2006; Stone, 2020).[10] These papers, however, are not tailored to address the broader consequences of family policies beyond fertility, such as the effects on output, human capital, social mobility, and welfare. The design-based approach is also not able to predict the consequences of family policies in the long-run when children being affected become parents themselves. This paper takes a structural approach to answer these questions.

Lastly, the paper is related to a growing literature that evaluates the impacts of transfers or subsidies to families with children. Prominent examples include Akee et al. (2010), Dahl and Lochner (2012), Mullins (2019) and García et al. (2020) among many others. With both design-based methods and structural models with exogenous fertility, the conventional wisdom in the literature is that transfers to families with children have positive impacts on children's outcomes including but not limited to health, education attainment and criminal records (Schanzenbach et al., 2021). This

[10]The distinction between quantum versus tempo effects on fertility, i.e. total fertility rate versus completed fertility rate, will be discussed with more details in Section 4.

papers utilizes experimental evidence from this literature to inform parameter choices in the calibration. But since this literature exclusively evaluates policy effects of transfers to families with existing children and abstracts away from fertility responses, I argue that their conclusions could not be directly used to extrapolate the aggregate impacts of family policies.

The rest of the paper is organized as follows. In Section 2, I present the quantitative model. The calibration of the model is discussed in detail in Section 3. I conduct validation exercises in Section 4. Section 5 presents the main policy counterfactuals results. Section 6 displays optimal policy results. Section 7 discusses the robustness of results to alternative assumptions. Section 8 concludes.

2 Model

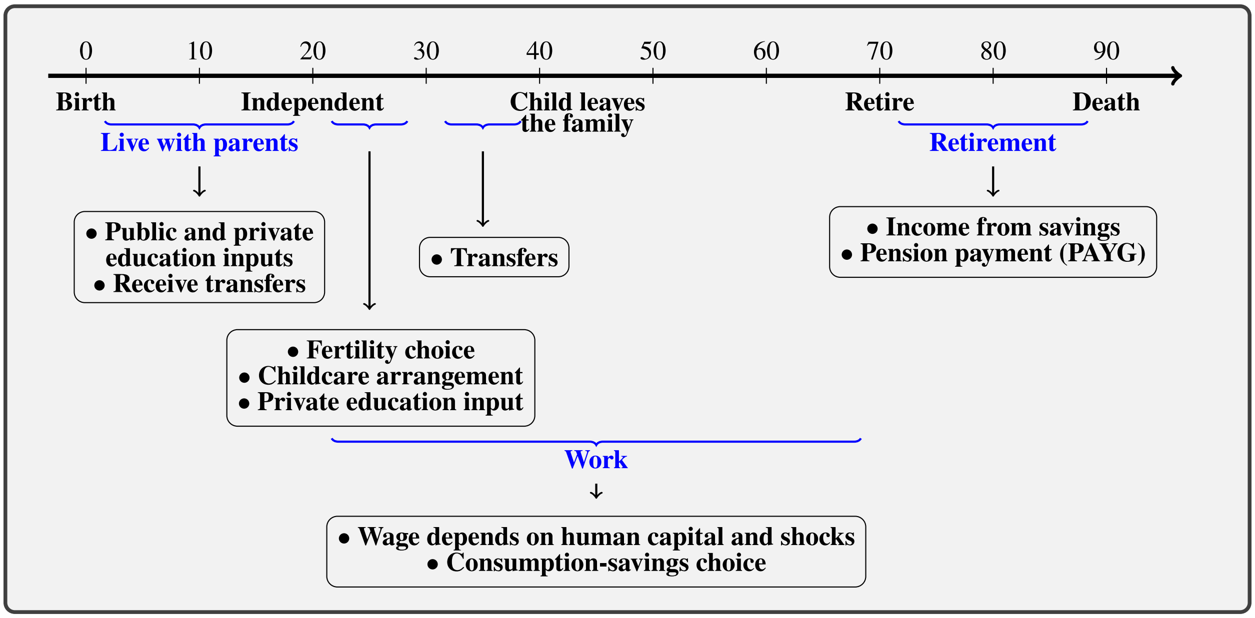
The model is based on the framework developed by de La Croix and Doepke (2003) where heterogeneous households make endogenous fertility and child human capital investment decisions in an overlapping-generations, general equilibrium economy. I extend it in multiple directions. First, I model realistic lifespan with working and retirement stages. Households make both life-cycle and precautionary savings under idiosyncratic uninsurable labor market shocks. By extending household lifespan to more than two periods, the model captures the changes in age structure of the population induced by family policies. Second, I incorporate a rich set of government policies including taxation, pension payments, public education and expenditures on family policies. In particular, I explicitly model the role of public education in children's formation of skills. These features allow me to evaluate the comprehensive effects of family policies on the government's budget. Lastly, I consider parents' choices in childcare arrangements. They can either choose to take care of children at home by themselves or utilize marketable childcare services. This feature enables the model to evaluate the effects of childcare subsidies on parents' labor supply which could in turn affect their own human capital accumulation through on-the-job learning.

2.1 Household

Consider overlapping generations of households that can live up to 90 years old. I split the life-cycle into 9 periods where where each period stand for 10 years and is indexed by $j \in \{0, 1, 2, \dots, 8\}$. Period j stands for age $j \times 10$ to age $(j + 1) \times 10$.

The timeline of the model is presented in Figure 1. Children live with their parents from age 0 to 20 where they make no choices and receive human capital investments from both public and private sources. Children also receive inter-vivos transfers (from parents) when they become independent and form their own families at age 20. Households make fertility, childcare arrangements, and

Figure 1: Model Timeline



private child human capital investments decisions from age 20 to 30.[11] Parents make inter-vivos transfers choice between age 30 and 40, one period before the child becomes independent. People work from age 20 to 70 where their wage depends on human capital and labor market shocks. They retire at age 70 and receive income from savings and pension payments. Households make consumption-savings choices every period in their lives and face age-specific survival rate δ_j.

For working adults, I assume that their human capital evolve over time following a learning-by-doing process:

$$h_{j+1} = L(h_j, t_w, z_{j+1}) \tag{1}$$

where z_{j+1} is an idiosyncratic uninsurable shock to human capital that occurs at the beginning of period $j+1$; h_j is the amount of human capital at period j; and t_w is the time worked in period j. I assume that agents have unit time endowment and supply labor inelastically except in period 2 where they trade off providing childcare at home against supplying labor in the market.[12]

The state variables of young adults at the beginning of period 2 (age 20-30) are human capital level h and assets a, which are both determined by their own parents. Households choose consumption c, savings a', fertility n, total time spent at home taking care of children t_h, amount of market

[11] In this paper, I model unitary/collective decision-making within families and abstract away from intra-household bargaining. Promoting gender equality in sharing childcare burden within household is an interesting alternative policy to raise fertility left for future research (e.g. Doepke and Kindermann, 2019).

[12] Abstraction away from labor/leisure choice is common in models with endogenous fertility, e.g. Barro and Becker (1989), de La Croix and Doepke (2003), Cordoba et al. (2016), and Daruich and Kozlowski (2020).

childcare purchased for each child m, and private education investment for each child e. Parents' value is composed of flow utility from consumption $u(c/\Lambda(n))$ where consumption expenditure c is divided by equivalence scale $\Lambda(n)$ and the discounted continuation value $V_3(\cdot)$. Their maximization problem is given by:

$$V_2(h,a) = \max_{c,a',n,t_h,m,e\geq 0} u(c/\Lambda(n)) + \beta\mathbb{E}V_3(h',a',n,\mathbb{E}h_k)$$

subject to

$$n\cdot\chi = \left(t_h^{\upsilon/\iota} + (n\cdot m)^{\upsilon}\right)^{1/\upsilon} \qquad \text{[time cost]}$$

$$y = wh\cdot(1-t_h) \qquad \text{[labor income]}$$

$$(1+\tau_c)(c + mp_m(1-\mathcal{S})n + e\cdot n) + a' = (1+r)a + y - \mathcal{T}(y,a,n) + \mathcal{B}\cdot n \qquad \text{[BC]}$$

$$h' = L(h, 1-t_h, z') \qquad \text{[learning OTJ]}$$

$$h_k = G(h, \mathcal{E}, e, \epsilon) \qquad \text{[skill formation]}$$

To raise n children, parents need to meet $n\cdot\chi$ amount of childcare needs in time units. Parents could satisfy this need either by spending their own time at home (t_h) or by spending money purchasing market childcare services m. I assume that these two forms of childcare is combined in a constant-elasticity-of-substitution (CES) production function where υ governs the elasticity of substitution. Home production of childcare enjoys an economy of scale with parameter $\iota\in(0,1)$. This captures the fact that taking care of two children at home takes less than two times the amount of hours needed to take care of one child (Folbre, 2008).

The opportunity cost of home production of childcare t_h is hours spent in the labor market. Household's labor income y is the product of market wage w, human capital h, and time worked $(1-t_h)$. Total resources available to the household consist of risk-free asset a multiplied by gross interest rate $(1+r)$, labor income y, net taxes paid $\mathcal{T}(y,a,n)$, and the total amount of baby bonus received $\mathcal{B}\cdot n$ where the amount $\mathcal{B}$ is chosen by the government. Household divide resources into savings a' and expenditures which are multiplied by consumption tax τ_c. Expenditures are composed of consumption c, total spending on market childcare $m\cdot p_m\cdot n$ and private educational expenditure $e\cdot n$. I use p_m to denote the price of market childcare relative to consumption goods.[13] For each dollar spent on childcare, the government could choose to refund $\mathcal{S}\in[0,1]$ dollars back to parents.

[13] In this paper, I will be assuming that the supply of childcare services is perfectly elastic at baseline price p_m. Since each period represents ten years, adjustments of labor in and out of the childcare industry equates prices of childcare to its long-run marginal costs if the industry is competitive. In the short-run, of course, relative prices of childcare may change in response to family policies.

Children's human capital production function $G(h, \mathcal{E}, e, \epsilon)$ combines parents' human capital h, public education $\mathcal{E}$, private investment e, and idiosyncratic ability shock ϵ that is unknown to parents. To keep the tractability of the model, I make a simplifying assumption that parental investment in children's human capital takes place when children's age is between 0 to 10 while public education $\mathcal{E}$ plays the major role from age 6 to 20. The function $G(\cdot)$ captures the overall human capital production function that spans age 0 to 20 without explicitly modeling several stages of production. To the extent that the bulk of childcare needs $n \cdot \chi$ as non-educational, i.e. preparing food or changing diapers, the arrangement of childcare does not affect children's human capital directly in the model.[14] Parents can invest in their children's human capital through monetary investments e. Since the government chooses public education expenditure $\mathcal{E}$, the model replicates public provision of high-quality childcare as a policy that combines higher $\mathcal{S}$ with an expansion in $\mathcal{E}$.

In the economy, households face inter-temporal borrowing constraint $a' \geq 0$ that is standard in the class of Aiyagari-Huggett models. With idiosyncratic uninsurable shocks, this induces households to save for precautionary motives besides life-cycle asset accumulation. Parents are not allowed to invest in negative amounts of resources in children's education, $e \geq 0$. Therefore, public investment $\mathcal{E}$ provides a lower-bound of total educational investments received by every child.

Parents' choice problem at age 30-40 is given by:

$$V_3(h, a, n, \mathbb{E}h_k) = \max_{c, a', a_k \geq 0} u(c/\Lambda(n)) + \beta \mathbb{E} V_4(h', a') + v(n, \mathbb{E}h_k, a_k)$$

subject to

$$y = wh$$

$$(1 + \tau_c)c + a' + n \cdot a_k = (1 + r)a + y - \mathcal{T}(y, a, n)$$

$$h' = L(h, 1, z')$$

where parents choose consumption, savings, and the amount of transfers to be received by each child at the beginning of next period (a_k). Following the literature, it is assumed that parents face intergenerational borrowing constraints so that they are not allowed to make negative transfers to children, i.e. $a_k \geq 0$. I use $v(n, \mathbb{E}h_k, a_k)$ to denote parents' preferences over child quantity, child human capital, and inter-vivos transfers. The parametric form of $v(\cdot)$ will be discussed in detail in Section 3.1.

For $j \in \{4, 5, 6\}$, households solve a simple consumption-savings problem with idiosyncratic

[14]In a recent work, Chaparro et al. (2020) develop a model of childcare with both quality and quantity aspects of maternal and non-maternal care. In their model, effects on child human capital is proportional to both care quality and time "exposed" to each type of care, but they do not explicitly consider time with children that are non-educational. The model in this paper separates the essence of childcare into non-education time χ and education time which is lumped into e and $\mathcal{E}$.

shocks to human capital. For tractability, I assume that parents and children no longer interact in these periods.[15] The maximization problem is given by:

$$V_j(h,a) = \max_{c,a' \geq 0} u(c/\Lambda(0)) + \beta \mathbb{E} V_{j+1}(h',a') \qquad j \in \{4,5,6\}$$

subject to

$$(1+\tau_c)c + a' = (1+r)a + y - \mathcal{T}(y,a,0)$$

$$h' = L(h,1,z)$$

Finally, retired agents solve:

$$V_j(h,a) = \max_{c,a' \geq 0} u(c/\Lambda(0)) + \beta V_{j+1}(h,a') \qquad j \in \{7,8\}$$

$$(1+\tau_c)c + a' = (1+r-\tau_a r)a + \pi \cdot wh$$

$$V_9(\cdot) \equiv 0$$

where household income is composed of risk-free assets and pension payment $\pi \cdot wh$. I use π to denote the replacement rate of pension and assume pension payments are not subject to income taxes. Capital income $r \cdot a$, on the other hand, is subject to a flat-rate tax rate τ_a.[16]

2.2 Firms

There is a representative firm in the economy that hires labor (in efficiency units) and borrows capital from households to produce final goods with Cobb-Douglas technology:

$$Y = A \cdot K^{\alpha} L^{1-\alpha} \tag{2}$$

In (2), K is aggregate capital used in production and H is total efficiency units employed. Total factor productivity A is normalized to be one.[17]

I assume physical capital depreciates at rate δ_K after use. With competitive factor markets, wage

[15] Otherwise, parents need to keep track of all children's state variables, including that of the grandchildren. This will make the problem computationally infeasible.

[16] Flat-rate capital income taxes for households in other periods are embedded in the tax function $\mathcal{T}(y,a,n)$.

[17] I abstract away from population externalities that could affect aggregate production such as pollution (Bohn and Stuart, 2015) and ideas creation (Jones, 2020). This choice is made since (1) the literature on measurement of population externalities is still developing, and (2) paper results will change in expected ways once positive/negative externalities are incorporated.

and risk-free interest rate in equilibrium are given by:

$$r = \alpha \left(\frac{K}{H}\right)^{\alpha-1} - \delta_K, \quad w = (1-\alpha)\left(\frac{K}{H}\right)^{\alpha}$$

2.3 Government

The government collects revenues from taxing labor income, capital income, and consumption. The expenditure side includes public education, family policies, pension and other per-capita exogenous expenditures. I assume that exogenous expenditures are invariant acorss policies, and the government balances budget from period to period.

I use $\{\mu_j\}_{j=0}^{8}$ to denote the distribution of households across state space and use $\{\omega_j\}_{j=0}^{8}$ to denote the fraction of each age group in the population with total mass normalized to one at each period. The government balance constraint is given by:

$$\begin{aligned} &\underbrace{\left(\sum_{j=2}^{6}\omega_j \int \mathcal{T}(y_j^*, a_j^*, n_j^*)\, d\mu_j\right)}_{\text{labor and capital income taxes}} + \underbrace{\left(\sum_{j=2}^{8}\omega_j \int \tau_c c_j^*\, d\mu_j\right)}_{\text{consumption tax}} = \underbrace{\left(\sum_{j=7}^{8}\omega_j \int w\pi h\, d\mu_j\right)}_{\text{pension payments}} \\ &+ \underbrace{(\omega_0+\omega_1)\cdot\mathcal{E}}_{\text{public education}} + \omega_2\left(\underbrace{\int n^*\cdot\mathcal{B}\, d\mu_2}_{\text{baby bonus}} + \underbrace{\int (1+\tau_c)m^*n^*p_m\cdot\mathcal{S}\, d\mu_2}_{\text{subsidized childcare}}\right) + \underbrace{\mathcal{X}}_{\textbf{other spendings}} \end{aligned} \tag{3}$$

2.4 Equilibrium

I use t to denote time. The equilibrium of the economy is defined as a tuple composed of:

- Decision rules $\{c_t^*, a_t'^*, n_t^*, m_t^*, t_{h,t}^*, e_t^*, a_{k,t}^*\}_{t=0}^{\infty}$
- Prices $\{w_t^*, r_t^*\}_{t=0}^{\infty}$
- Government policies $\{\mathcal{T}_t(\cdot), \tau_{c,t}, \mathcal{B}_t, \mathcal{S}_t, \mathcal{E}_t\}_{t=0}^{\infty}$
- Distribution of agents $\{\{\mu_{j,t}\}_{j=0}^{8}, \{\omega_{j,t}\}_{j=0}^{8}\}_{t=0}^{\infty}$

such that households maximize utility, prices clear labor and capital markets, government balances budget in each period, and the distribution of agents evolve following the dynamics shaped by household decision rules, exogenous labor market shocks z, and ability shock for children ϵ.

The evolution of the human capital distribution[18] from parents to children is given by:

$$\mu_2'(h) = \frac{1}{N} \iint n^*(x) \mathbb{1}_{h_k^*(x,\epsilon)<h} \, d\mathcal{Q}(\epsilon) \, d\mu_2(x) \tag{4}$$

where $\mathcal{Q}$ denotes the distribution of child ability shock ϵ; $h_k^*(x, \epsilon)$ is the human capital of children born into family with parents human capital being x and receive ϵ as ability shock, and N is the aggregate fertility rate in the economy:

$$N = \int n^*(x) \, d\mu_2(x) \tag{5}$$

In a stationary equilibrium of the economy, decision rule, prices, and distributions are unchanged over time.[19] The process defined in (4) is a multi-type branching (or Galton-Watson) process. Under mild conditions that are satisfied by the model, it can be shown that a stationary distribution of agents exists and is unique (see Mode, 1971; Chu, 1990).

2.5 Welfare

To facilitate comparisons between different government policies, I define social welfare in the long-run ($\mathcal{W}$) as the *average value* of households at the beginning of their life-cycle, i.e.

$$\mathcal{W} = \int V_2 \, d\mu_2 \tag{6}$$

$\mathcal{W}$ measures the expected utility of a newborn child under the veil of ignorance by summarizing the discounted utility flow from later life periods. When making welfare comparisons, I will convert changes in $\mathcal{W}$ into percentage changes in consumption equivalence using utility function $u(\cdot)$.[20]

Government policies could be welfare-improving in the economy for two reasons. First, child-bearing and child-rearing carry fiscal externalities as parents do not internalize the effects of having an additional child or investing in children's human capital on future tax base and government revenues. In other words, infinitesimal parents take the age structure $\{\omega_j\}_{j=0}^{8}$, distribution $\{\mu_j\}_{j=0}^{8}$, and tax rates as given, but these objects will change when a mass of parents adjust their decisions on fertility and educational investments. Due to the lack of property rights of parents on children's future output, equilibrium fertility and child investments are too low relative to the planner's solu-

[18] For simplicity of exposition, here I omit the notation of distribution over initial assets.

[19] The size of the population could change over time as aggregate fertility rate is not necessarily at replacement level.

[20] This benchmark definition of welfare evaluates the average well-being of households who are alive in the long-run economy. The effects of family policies on current households will be discussed in Section 5.2. Also see Section 6 for more discussions about different welfare criteria in models with endogenous fertility and corresponding optimal policies.

tion (Schoonbroodt and Tertilt, 2014). Second, parents face both inter-temporal and intergenerational borrowing constraints due to imperfections in capital markets. Parents will not be able to use children's future income to finance children's education. Parents are also constrained in borrowing from their own future income to finance current expenditures. Government policies could overcome these inefficiencies by providing resources to parents or expanding public education (Daruich, 2018, Abbott et al., 2019).

2.6 Discussion of Mechanisms

In this section, I discuss why adding endogenous fertility makes the model predictions on the overall impacts of family policies distinct from that using structural models with exogenous fertility or design-based studies with existing parent-child pairs.

2.6.1 Quality/Quantity Trade-off

Consider the effect of an increase in baby bonus $\mathcal{B}$ on transfers to each child a_k holding education investment e unchanged. The first-order condition for a_k is given by:

$$\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial a_k} = \lambda_3 \cdot n$$

where λ_3 denotes the Lagrangian multiplier on parents' budget in period 3.

When fertility is exogenous, i.e. n is fixed, the increase in $\mathcal{B}$ is an *income transfer*, which implies that a_k rises unambiguously due to income effect:

$$\underbrace{\lambda_3 \downarrow}_{\text{income effect on } MU_c} \cdot n \quad \Rightarrow \quad \frac{v(n, \mathbb{E}h_k, a_k \uparrow)}{\partial a_k}$$

When fertility is endogenous, however, the increase in $\mathcal{B}$ is a *price change*. The direction in which a_k changes is ambiguous:

$$\underbrace{\frac{\partial v(n\uparrow, \mathbb{E}h_k, a_k\mathbf{?})}{\partial a_k}}_{\text{interaction in preferences}} = \underbrace{\lambda_3?}_{\text{change in } MU_c \text{ as } n\uparrow} \cdot \underbrace{n\uparrow}_{\textbf{fertility response}} \tag{7}$$

As fertility rises due to more generous child benefits[21], it affects the first-order condition of child quality a_k in three ways. First, it interacts with a_k in parents preferences $v(\cdot)$. If quality and quantity are complements (substitutes), then parents would demand higher (lower) a_k ceteris paribus.

[21]This will be true as long as child quantity is not a Giffen good. See McDonald (2006) and Stone (2020) for supporting evidence.

Second, rising quantity could offset the income effect since higher n raises in the marginal utility of consumption via change in equivalence scale $\Lambda(n)$. Lastly, marginal costs of child quality a_k rises because it is proportional to fertility n due to their interaction in parents' budget constraint.[22]

To summarize, considering endogenous fertility allows for the possibility that child quality could fall when child benefits become more generous via family policies. This stands in contrast with previous literature on family policies that only considers the income effects of transfers.

2.6.2 Composition Effects

When fertility is endogenous, parents' responses to family policies generate composition effects on aggregate variables. Families or dynasties with stronger fertility responses will gain representation over time since their children will take a larger fraction of future population. Since individual traits, such as human capital, are persistent across generations, aggregate variables will gravitate over time towards the traits of those families that increase fertility the most due to policies. For example, Vogl and Freese (2020) argues that composition effects could shape public opinions towards policies regarding conservative family values. de La Croix and Doepke (2003) and Vogl (2016) show that changes in fertility differentials in the economy affect long-run growth via composition effects.

2.6.3 Demographic Structure Effects

When fertility is endogenous, family policies change the population growth rate and hence the demographic structure $\{\omega_j\}_{j=0}^{8}$. This change has profound implications on the government budget constraint (3) since it determines how each source of revenue or expenditure is weighted. In particular, higher fertility rate reduces the old-age dependency ratio $\left(\sum_{j=7}^{8}\omega_j\right) / \left(\sum_{j=2}^{6}\omega_j\right)$ but may raise the total dependency ratio $\left(\sum_{j=7}^{8}\omega_j + \sum_{j=0}^{1}\omega_j\right) / \left(\sum_{j=2}^{6}\omega_j\right)$. Since the government needs to balance its budget, family policies induce further adjustments to taxes in the general equilibrium. Allowing for endogenous fertility is a necessary condition for a model to have the ability to study the impacts of large-scale family policies on population aging.

[22] Becker and Lewis (1973) named this last effect quality/quantity trade-off. In this paper, I abuse this notation slightly to denote the overall effects of an increase in fertility on child quality. Using twin births as instruments, some recent studies (e.g. Black et al., 2005; Angrist et al., 2010) find little evidence of the trade-off. A recent study by Mogstad and Wiswall (2016), however, overturns the conclusion by relaxing the linear specification. They find evidence of a trade-off of quality and quantity for larger families and complementarities in small families.

2.6.4 Summary of Mechanisms

As discussed above, relative to design-based analysis or structural models with exogenous child quantity, explicitly modeling fertility choice of households incorporates quality/quantity trade-off, composition effects, and demographic structure effects through which aggregate variables are affected. The key moment in the model that governs the strength of these mechanisms is the magnitude of fertility responses to financial incentives because if households do not respond to family policies by adjusting fertility, the model simply predicts an income effect on child quality. Next two sections are devoted to disciplining this elasticity using calibrated parameters and validating it using existing policies.

3 Calibration

In this section, I discuss the calibration of the model. Following the standard procedures in the literature, I pick some parameters exogenously, and 13 other parameters are calibrated inside the model by matching steady-state moments to the United States in 2010.

3.1 Preferences over Quantity and Quality

I assume that parents' preference over child quantity and quality is given by:

$$v(n, \mathbb{E}h_k, a_k) = \underbrace{\Psi(n)}_{\text{child discounting}} \cdot \underbrace{(\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k))}_{\text{utility from quality}} \tag{8}$$

$$\Psi(n) = 1 - \exp(-\psi n) \tag{9}$$

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0,1), \quad x \in \{\mathbb{E}h_k, a_k, c\} \tag{10}$$

where parents value child quality weighted by child discounting $\Psi(n)$ (c.f. Barro and Becker, 1989; Scholz and Seshadri, 2007; Kim et al., 2021). The utility function of child quality and consumption is governed by γ which is known as the elasticity of intergenerational substitution (EGS) (c.f. Cordoba et al., 2016).[23] I use $\theta \cdot u(\mathbb{E}(h_k)) + \nu \cdot u(a_k)$ as a first-order approximation to general preferences over child quality and transfers. In Section 7, I argue that results in this paper are robust to other common specifications used in the literature, including separable preferences (de La Croix

[23] Since EGS is designed to match interactions across generations, its magnitude and interpretations are different from that of the elasticity of intertemporal substitution (EIS) used to capture risk-aversion in business cycle models. More specifically, when utility from consumption and child quality are separable, it is common to assume that EIS and EGS are the same to ensure the existence of long-run steady states (see Barro and Becker, 1989; Soares, 2005). For recent work that considers EGS and EIS jointly with non-separable utilities, see Carlos Córdoba and Ripoll (2019).

and Doepke, 2003), quality and quantity being substitutes (Jones and Schoonbroodt, 2010), and dynastic altruism (Daruich and Kozlowski, 2020).

Parameters $\theta = 2.1$ and $\nu = 0.36$ are calibrated to match the average human capital investment as a share of income and average inter-vivos transfers of \$60,000 in 2010 dollars measured by Daruich (2018) using PSID data. I calibrate $\psi = 1.93$ to match the average fertility calculated using CPS June Fertility Supplement data from 2008 to 2014.[24]

Conditional on $\{\theta, \nu, \psi\}$ and other parameters of the model, γ is an important parameter governing the fertility elasticity where higher γ results in smaller fertility responses to financial incentives.[25] Smaller fertility responses, in turn, implies a larger responses in child quality due to income effects discussed in Section 2.6.

I identify γ by matching the fertility-income profile in the data. The identification argument follows Cordoba et al. (2016): controlling for other parameters, higher γ implies a larger marginal rate of substitution of quantity for quality and a flatter fertility-income profile. Numerical illustration of the identification is displayed in Figure 2. I choose $\gamma = 0.61$ such that the model best fits the observed relationship between fertility and income.[26]

3.2 Child's Human Capital Production

I parameterize the child human capital production function as:

$$h_k = G(h, \mathcal{E}, e, \epsilon) = \underbrace{Z}_{\text{scalar}} \cdot \underbrace{\epsilon}_{\text{shock}} \cdot \underbrace{h^{\rho}}_{\text{spillover}} \cdot \left(\underbrace{\mathcal{E}^{\xi}}_{\text{public education}} + \underbrace{e^{\xi}}_{\text{private input}} \right)^{\kappa/\xi} \tag{11}$$

$$\log(\epsilon) \sim \mathcal{N}(-\frac{\sigma_{\epsilon}^2}{2}, \sigma_{\epsilon}^2)$$

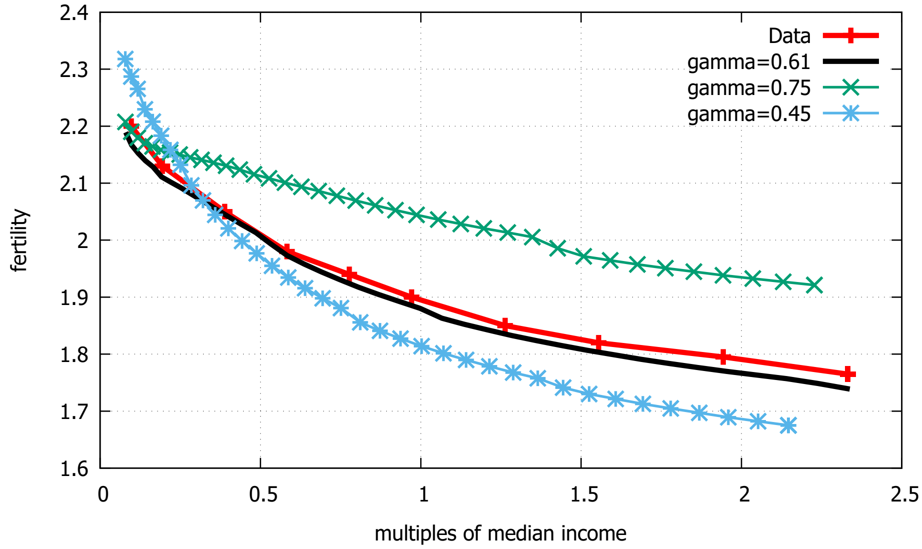
Parameter Z is a scaling parameter that governs the overall scale of the economy. It also enables us to normalize the total factor productivity A to be one. I choose $Z = 2.57$ so that the median income of families in the model is one, corresponding to \$49,445 estimated using Census data. Parameter σ_{ϵ} governs the dispersion of idiosyncratic shock to children's ability. I pick $\sigma_{\epsilon} = 0.45$ to match the dispersion of earnings for young households estimated by Huggett et al. (2011) calculated using Panel Study of Income Dynamics (PSID). I calibrate $\rho = 0.28$ to match the rank-rank intergenerational mobility estimated by Chetty et al. (2014). The model fit can be seen in Figure 3.

[24] I measure the completed fertility rate using the number of live births ever had for women aged 40 to 55.

[25] The illustrative model in Appendix C.2 shows that responses in fertility is proportional to $1 - \gamma$. Soares (2005) uses a similar argument in discussing the responses in fertility to changes in adult longevity and child mortality.

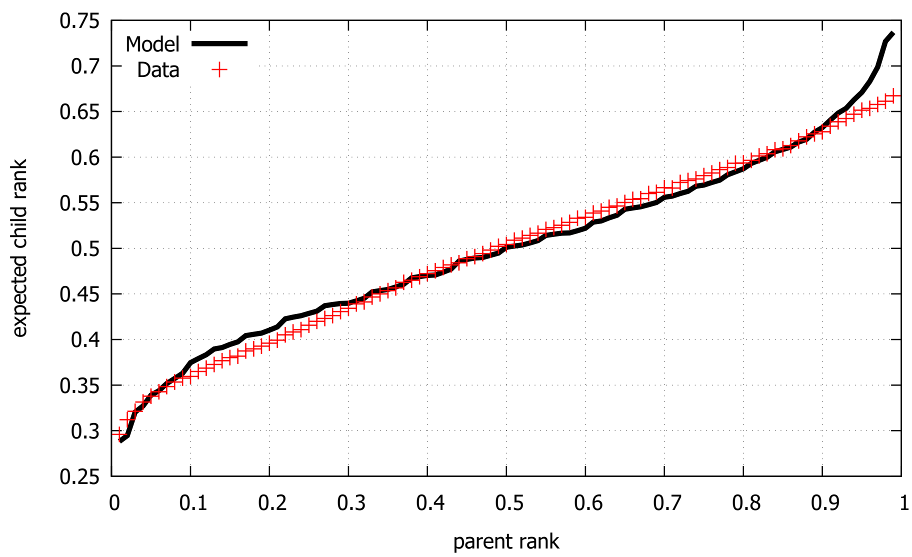
[26] In recent samples of CPS, completed fertility rate is mildly increasing in family income for families with very high income. Bar et al. (2018) relates this phenomenon to the marketization of time. Kim et al. (2021) discuss the possibility of status competition concerns for middle-income families as an explanation.

Figure 2: Identification of Elasticity of Intergenerational Substitution γ



Notes: This figure plots the relationship between parents' income and completed fertility in the model under different levels of γ. The data estimate uses CPS June Fertility Supplement data from 2008 to 2014. Completed fertility rate is calculated using *frever* variable for women between 40 and 50 years old.

Figure 3: Intergenerational Income Mobility: Model vs Data



Notes: This figure plots the relationship between parents' income rank and children's expected income rank in the calibrated model and that estimated by Chetty et al. (2014).

I use ξ to parameterize the elasticity of substitution between public and private education inputs and calibrate $\xi = 0.9$ to match the correlation between educational spending and household income using data from Consumer Expenditure Survey (CEX) which includes tuition, test preparation, tutoring, books, and supplies. The magnitude of public education $\mathcal{E}$ is chosen to match $12,000 annual expenditure per student reported by National Center of Education Statistics (NCES).

The last parameter in child human capital production function that needs to be calibrated is κ which governs the productivity of educational investments. I use RCT evidence from García et al. (2020) to inform the value of κ. García et al. (2020) evaluates two early childhood programs (ABC and CARE) in the 1970s. These two programs cost about $13,500 per year for five years, which implies that the total program cost is around $67,500 per child. Treated children are followed into adulthood with education and (part of life-cycle) incomes observed by researchers. García et al. (2020) estimate that for every dollar invested, children's lifetime labor income increases by 1.3 dollars in net present value. I apply a similar policy in the model by expanding existing public education $\mathcal{E}$ by $67,500 targeted at children with parents at 10th percentile of earnings. The counterfactual is evaluated at a small scale so that prices and taxes remain unchanged. By comparing the discounted life-time earnings of children in the treatment group versus those in the baseline economy, κ is calibrated to be 0.17.

3.3 Costs of Children and Childcare

The household equivalence scale $\Lambda(n)$ is taken from the OECD standard:

$$\Lambda(n) = 1.7 + 0.5 \cdot n$$

where n is the number of children residing with the family.

Recall the childcare requirement constraint, i.e. time costs of child, is given by:

$$n \cdot \chi = \left(t_h^{\upsilon/\iota} + (n \cdot m)^{\upsilon} \right)^{1/\upsilon}$$

I choose $\chi = 0.18$ following estimated by Folbre (2008) (Table 6.2) calculated using American Time Use Survey (ATUS) data. I calibrate the economies of scale in providing childcare at home $\iota = 0.7$ to match the estimates by Folbre (2008) (Table 6.4). The parameter governing the elasticity of substitution between home care and market care υ is calibrate to be 0.5 to match the average expenditure on childcare as a fraction of total family income using estimates from Herbst (2018). Lastly, I choose the price of full-time market care for child aged 0 to 10, p_m, to be $6,860 in 2010 following the statistic reported by the National Association of Child Care Resource & Referral

Agencies (NACCRRA, 2011).[27]

3.4 Other Parameters

I assume the human capital of adults evolve with age according to:

$$h' = L(h, t_w, z') = \exp(z')(h + \zeta(h \cdot t_w))^{\eta} \tag{12}$$

$$\log(z) \sim \mathcal{N}(\mu_z, \sigma_z)$$

I calibrate $\eta = 0.61$, $\zeta = 0.72$ and $\sigma_z = 0.42$ to match the life-cycle profile of earnings and gini coefficient of earnings calculated by Huggett et al. (2011). I choose $\mu_z = -0.23$ so that skill depreciation is 2% per year.

Following Heathcote et al. (2017), I parameterize income taxes as:

$$\mathcal{T}(y, a, n) = y \cdot (1 - \tau_y^n y^{-\lambda_y^n}) + \tau_a r a \tag{13}$$

where $\{\tau_y^n, \lambda_y^n\}$ denote the level and progressitivity of taxes depending on the number of children residing in the household[28] while τ_a denotes the linear capital income taxes. I obtain $\{\tau_y^n, \lambda_y^n\}$ using simulated data from TAXSIM provided by the NBER (see Figure 4). Having an additional child significantly reduces the tax burden faced by households through various programs such as Earned Income Tax Credits (EITC), Dependent Care Tax Credits (DCTC), and Child Tax Credits (CTC). The tax system is progressive, with lower-income households receiving subsidies on net. As household income grows, labor income taxes converge to 40%. Following McDaniel (2007) and Daruich and Fernández (2020), I choose capital income tax $\tau_a = 0.27$ and consumption taxes $\tau_c = 0.07$. Pension replacement rate π is set to be 40%.

In the production function of the firms, I choose the capital share α to be 0.33 following the standard literature and set the capital depreciation rate δ_k at 4% per year. Lastly, I choose the annual discount rate $\beta = 0.98$ and calculate age-specific mortality rate $\{\delta_j\}_{j=0}^{8}$ using the actuarial life table from Social Security.

Table 1 provides a summary of the model parametrization.

[27] To be more specific, the average costs of full-time childcare across states are \$9,303 (infant), \$7,377 (4-year-old), and \$4,753 (school-age) for child care centers; \$6,926 (infant), \$6,131 (4-year-old), and \$4,405 (school-age) for family child care (NACCRRA, 2011 Appendix 1). I take an age-weighted average of these costs to calculate the full-time childcare costs for children age 0 to 10.

[28] I use linear interpolation to calculate $\{\tau_y^n, \lambda_y^n\}$ in the model when n is not an integer.

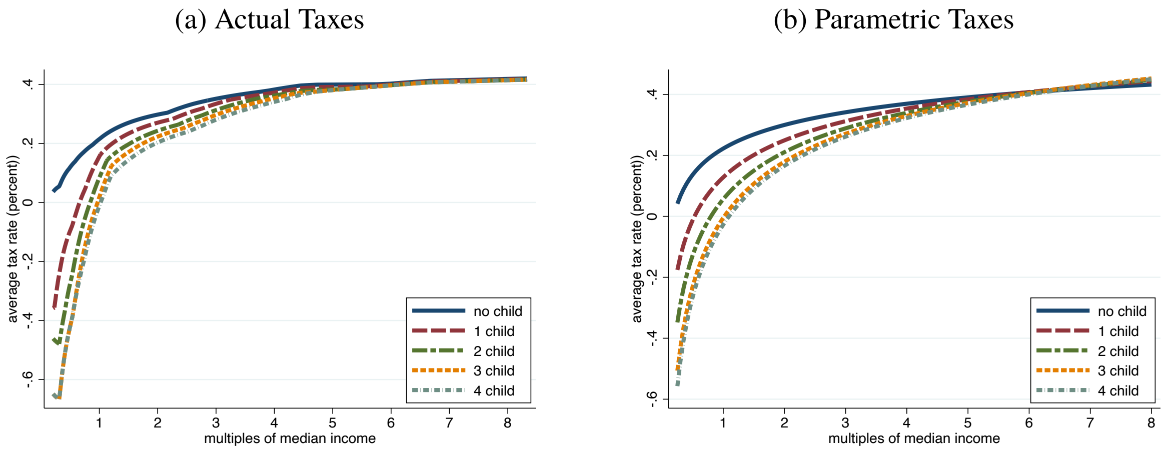
Table 1: Calibrated Parameters

	Interpretation	Value	Source
	Preferences		
β	discount rate	0.98^{10}	standard
γ	elasticity of substitution	0.61	CPS
ψ	fertility preference	1.93	CPS
θ	quality preference	2.1	PSID
ν	transfer preference	0.36	PSID
	Childcare arrangement		
χ	childcare cost	0.18	Folbre (2008)
ι	economies of scale at home	0.7	Folbre (2008)
υ	substitutability of care	0.5	SIPP
p_m	price of full-time care	0.13	NACCRRA (2011)
	Taxes and pension		
τ_y^n, λ_y^n	tax levels and progressitivity	misc.	TAXSIM
τ_c	consumption tax	0.07	McDaniel (2007)
τ_a	capital income tax	0.27	McDaniel (2007)
π	pension replacement rate	0.40	OECD Database

	Interpretation	Value	Source
	Child human capital production		
Z	normalizing scalar	2.57	median income =1
σ_ϵ	ability shock dispersion	0.45	PSID
ρ	intergenerational spillover	0.28	Chetty et al. (2014)
ξ	substitution of education	0.9	ATUS
$\mathcal{E}$	public education	0.15	NCES
κ	input productivity	0.17	García et al. (2020)
	Adult human capital evolution		
η	learning curvature	0.61	PSID
ζ	learning level	0.72	PSID
μ_z	skill depreciation	-0.23	PSID
σ_z	shock dispersion	0.42	PSID
	Firm production function		
A	total factor productivity	1	normalization
α	capital share	0.33	standard
δ_k	capital depreciation	0.04^{10}	standard

Notes: This table displays the list of parameters used in the model. Parameters in red are calibrated within the model while those in black are chosen exogenously.

Figure 4: Labor Income Taxes



Notes: These figures plot average tax rates depending on household income and number of dependent children residing with the family. The left panel shows calculations using TAXSIM. The right panel displays fits using parametric tax formulas.

4 Validation

In this section, I perform an external validation of the fertility responses to financial incentives. This is an important exercise before I use the model to conduct policy counterfactuals since it lends extra credibility to the quantitative magnitude of fertility elasticities, which is the crucial to quality/quantity trade-off, composition effects, and demographic structure effects. The model predictions on fertility, child quality, and childcare arrangements are also validated by additional evidence from Australian baby bonus, Spanish child benefits, and Georgia's Cherokee Land Lottery (see Appendix B).

The main validation exercise exploits empirical evidence from Alaska Permanent Fund Dividends (APFD). Briefly speaking, the dividend was officially established in 1982 after discovery of the petroleum and increased state revenues. Every year, it gives uniform transfers to all residents regardless of income, employment or age. In particular, the program allows a parent, guardian, or other authorized representative to claim a dividend on behalf of a child while Alaska law imposes no requirements whatsoever on how parents use a child's dividend. As a result, the policy has pronatal effects even though it is not explicitly advertised as a family policy that encourages fertility.

APFD is an ideal policy variation to validate fertility elasticities in the model for four reasons. First, compared with other family policies which mostly took place in European countries, the cultural and institutional background of Alaska is much more similar to the overall U.S. system that I calibrate the model to match. Second, compared with other family policies which is usually less than a few thousand dollars in net present value per child (McDonald, 2006; Drago et al., 2011; González, 2013), the net present value that parents could receive and use with an additional child under APFD is almost $20,000.[29] With sizable benefits, it is more likely that APFD would have meaningful and observable impacts on people's behavior especially when it comes to the important and irreversible decision of having a child. Third, compared with other family policies which are typically means-tested or depends on birth order, the APFD has a clearer implementation with more than 90% of state population filing for the application historically.[30] Lastly, APFD mimics a universal basic income for parents plus a refundable Child Tax Credit (CTC) for children. Given that it is not advertised as a policy that aims to encourage childbirth, its effects on fertility provides a conservative benchmark for family policies that are explicitly pronatal which may change parents' behavior through preferences.

In the model, I implement APFD by transferring $1,500 (annual amount) to every household member including both parents and children. Parents will receive this transfer every period until they die. They are also entitled to receive additional transfers when children reside with them. I normalize the amount of transfer by median income in Alaska relative to that in the U.S. and conduct the policy experiment in partial equilibrium without changing prices and distribution.

The model predicts that completed fertility rate, i.e. the total number of children that parents end up having, increase 4.2% compared with the scenario without AFPD. Using synthetic control methods, Yonzan et al. (2020) concludes that total fertility rate increases by 13.1% due to AFPD using Natality files. Since policy effects on total fertility rate is typically larger than that on completed fertility rate[31], I multiply it by a common adjustment of 1/3 (Stone, 2017) to get a crude estimate of the effects on completed fertility rate of 4.5% which is similar to the model estimate.

Since APFD was enacted decades ago, it is possible to explore the effects on completed fertility rate directly without extrapolating based on changes in total fertility rate. Figure 5 plots the time series of completed fertility rate (i.e. total number of live births ever had by women aged 40-55) for Alaska and the rest of U.S. from 1982 to 2018 using CPS June Fertility Supplement data. As can be seen, the trend of Alaska is indistinguishable from the rest of the U.S. prior to 1996. After 16 years of the APFD implementation, however, completed fertility rate in Alaska is persistently higher than that in the rest of the U.S.[32] by around or above 0.15 children per women. This makes

[29] I calculated the average payment to be around $1,500 per year. Since the amount of the dividend is tied to the performance of the fund, which in turn depends largely on overall stock market performance, one might worry about how would the uncertainty about future payment amount affect people's responses in fertility. Given that childbirth is an irreversible decision, I argue that a mean-preserving spread of the dividend payment would reduce people's responses in fertility. Therefore, since the model generates fertility elasticities that are consistent with people's choices under uncertainty, our results on child benefits without uncertainty is likely going to be conservative.

[30] Cowan and Douds (2021) argues that the migration effects, also known as "population magnets effect", of APFD was not large empirically, with net migration rates around one-tenth of a percent in the sample period.

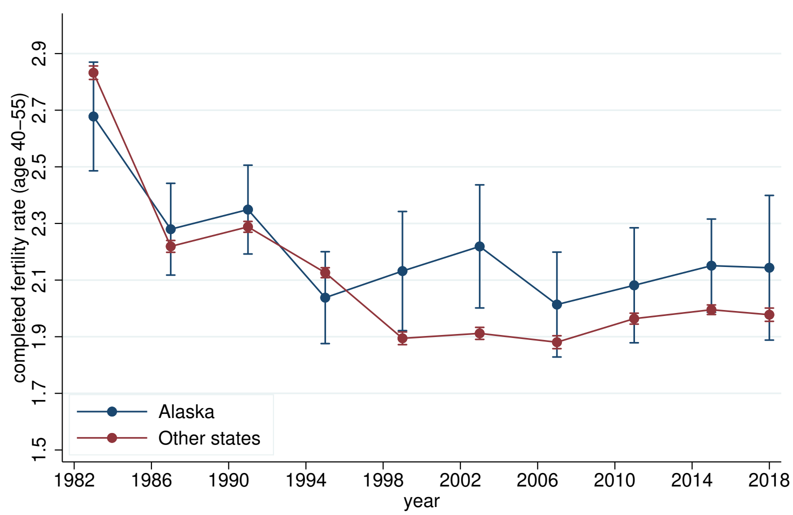
[31] Changes in total fertility rate includes both quantum effect (i.e. effects on completed fertility rate) and tempo effect (i.e. change in timing of birth, also known as compression effect).

[32] The gap is only statistically significant at 10% for some of the years because the CPS sample of Alaskan women is small, leading to wide confidence intervals.

the model prediction of 0.1 children per women (4.2% increase) slightly conservative.

The model also predicts heterogeneous fertility responses across households. Since the parents are entitled to claim children's dividends, and these dividends are the uniform in size, the model predicts that households with lower human capital have larger increase in fertility (see Section 5.1). This is in line with the finding by Cowan and Douds (2021) where they show larger fertility increases among Alaska natives and women without high school degree. This difference is also visible when inspecting the time series of completed fertility rate by education in Alaska versus the rest of the U.S. in Figure 6. Completed fertility rates among Alaskan women with a college degree is similar to that in the other states after APFD was adopted, while Alaskan women without a college degree increased childbirth relative to the other states.

Figure 5: AFPD Effects on Completed Fertility Rate



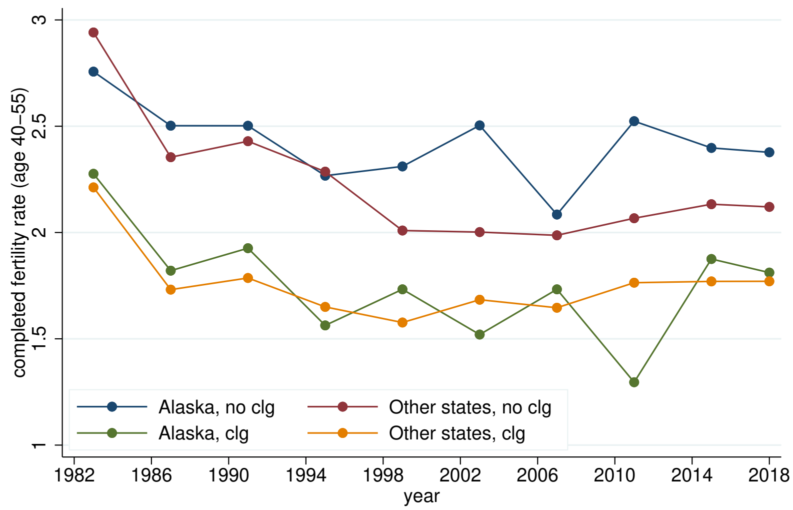
Notes: This figure plots the average completed fertility rates for women aged 40-55 by state of residence from 1982 to 2018 using data from CPS June Fertility Supplement combined into 4-year bins. Bars around sample means show 90% confidence intervals.

5 Counterfactuals

In this section, I use the model to evaluate policies of different sizes in the general equilibrium where prices and distribution of the population adjust. I focus on baby bonus since it has been widely adopted in the developed world (e.g. Drago et al., 2011; González, 2013) and is similar in structure to an expansion of fully refundable child tax credits (CTC).[33]. I discuss its long-run implications in

[33]There are two main differences between baby bonus and CTC. First, unlike CTC which has income requirement and phase-out region, baby bonus is usually not means-tested. I focus on the uniform case for simplicity and better

Figure 6: AFPD Effects on Completed Fertility Rate by Education



Notes: This figure plots the average completed fertility rates for women aged 40-55 by state of residence and education from 1982 to 2018 using data from CPS June Fertility Supplement combined into 4-year bins. I define college-educated women as those who have obtained a bachelor's degree or above.

Section 5.1 where I assume that the government is balancing the budget by adjusting consumption taxes. The outcome variables of interest include but not limited to fertility, human capital, average income, intergenerational mobility (measured by IGE^{-1}) and social welfare. In Section 5.2, I study the transition path leading to that long-run outcome. Lastly in Section 5.3, I compares baby bonus with other policies including subsidized childcare $\mathcal{S}$ and education expenditures $\mathcal{E}$.

5.1 Baby Bonus: Long-run Implications

In this section, I evaluate baby bonus of different sizes ranging from \$0 to \$50,000. I compute the long-run macroeconomic implications of these policies by comparing the long-run steady-state of the economy with the benchmark economy where baby bonus is zero.

Figure 7a and 7b shows the fertility effects of baby bonus. To reach replacement fertility level (2.1 children per family on average), the model predicts that it would require a baby bonus of \$31,000 which will be around 1.7% of GDP in the new steady-state economy. This amounts is similar to the increase in CTC from 2010 to 2021 in net present value and offsets 19% of the average cost of raising one child (USDA estimate). These results confirm the common perception

exposition of heterogeneous responses. As will be discussed, the main results are stronger when family policies target low-income households. Second, baby bonus is a lump-sum transfer when the child is born while CTC is an annual transfer to parents when the child is below age 18. With borrowing constraints, low-income parents prefer baby bonus to CTC of the same net present value since they can replicate the latter through savings.

by demographers that using financial incentives to raise fertility is not “cheap". Under realistic magnitude of fertility elasticities and reasonable scale of policies, the effects of financial incentives provided by the government are small relative to historical changes in preferences, social norms, contraceptive technologies, and perhaps most important of all, changes in skill premium. The results, however, suggest that using cash transfers to prevent further crashes in fertility, or even to raise it, is feasible in practice.

Figure 7c shows the quality/quantity trade-off channel discussed in Section 2.6.1. Baby bonus reduces the price of child quantity and parents respond by having more children. The increase quantity, in turn, raises the marginal cost of education as well as marginal utility of consumption due to larger family size. As a result, even when quality and quantity are complements in parents' preferences (see Section 3.1), parents optimally reduce the amount invested in children's education when fertility increases. When baby bonus is \$31,000, private investment per child is reduced by 8% on average.

Figure 7c and 7d shows that agents with lower human capital have stronger fertility responses and larger drops in private education investments in response to baby bonus. This is intuitive since the baby bonus of a given size accounts for a larger fraction of household income among low-income families. Due to intergenerational transmission of human capital in skill formation, heterogeneous responses in fertility induces gravitation of aggregate variables towards that of low-income families. Coupled with reductions in private investment, Figure 7e shows that average human capital falls by 1.5% under a \$31,000 baby bonus.[34] Since the reduction of education investment is larger among low-income households, baby bonus reduces social mobility by 1.5% rather than boosting it as policy makers might have hoped.

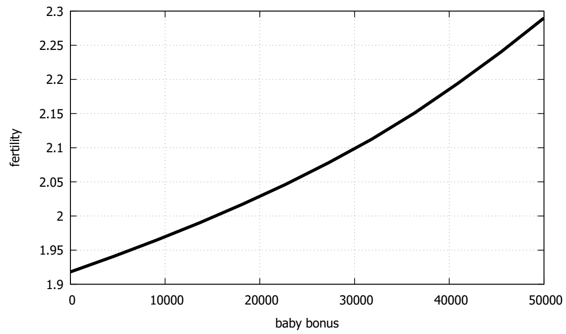
Figure 7g shows that despite lower average human capital and dampened social mobility, per capita output rises initially before falling around $\mathcal{B} = \$23,000$. Changes in per capita output is not monotonic due to three reasons. First, as parents reduce investments and transfers to children due to quality/quantity trade-off, they optimally save part of the baby bonus for their future consumption. This raises the supply of capital in the economy. Second, human capital distribution worsens as baby bonus increases. This reduces the supply of capital and labor due to composition effects. Lastly, demographic structure effect reduces the mass of retired households and boosts the mass of children (see Figure 7h). As baby bonus gets larger, per capita output falls below the steady-state level as composition effects dominate.

As the size of baby bonus increases, Figure 7i shows that in the long-run steady-state, the government is able to reduce consumption tax for small baby bonus but needs to raise taxes for larger ones. This is a result of several forces shaping the government budget constraint in (3). First, tax

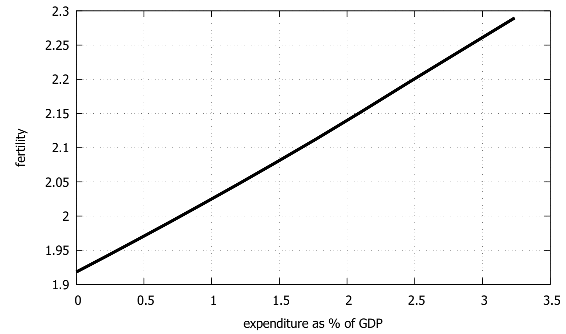
[34] Reductions in average human capital (-1.5%) is much smaller than that of the private investment (-8%) since public education investment, i.e. $\mathcal{E}$, is unchanged.

Figure 7: Effects of Baby Bonus of Different Sizes

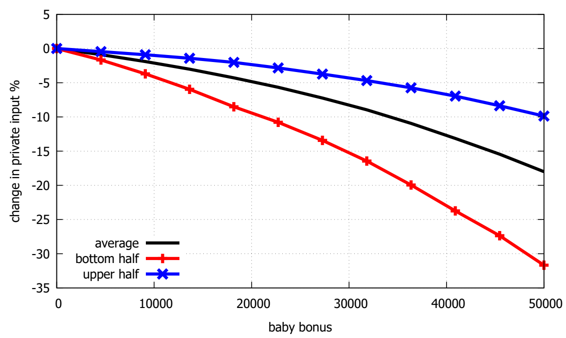
(a) Effects on Fertility



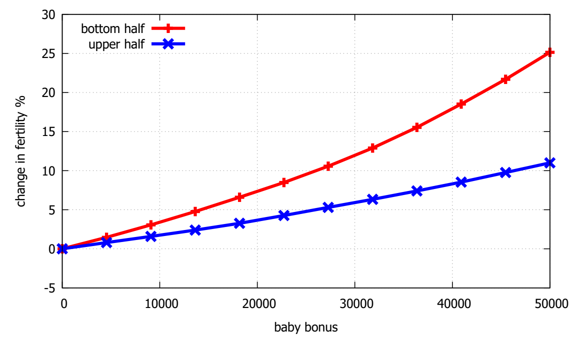
(b) Expenditure Share as % of GDP



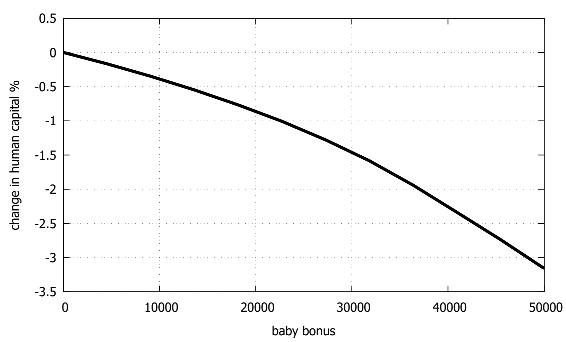
(c) Average Private Investment



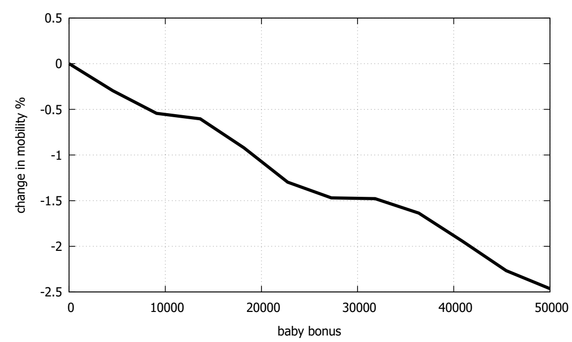
(d) Heterogeneous Fertility Response



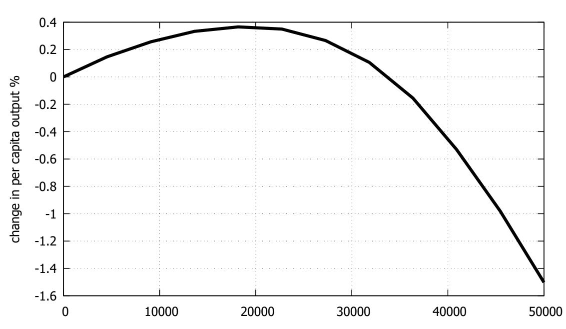
(e) Average Human Capital



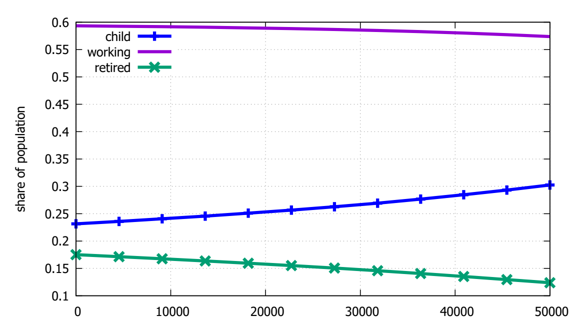
(f) Intergenerational Mobility



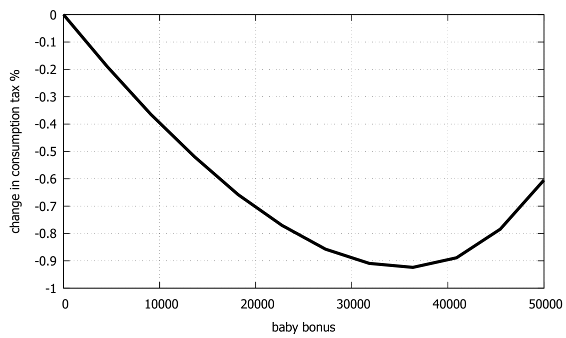
(g) Per Capita Output



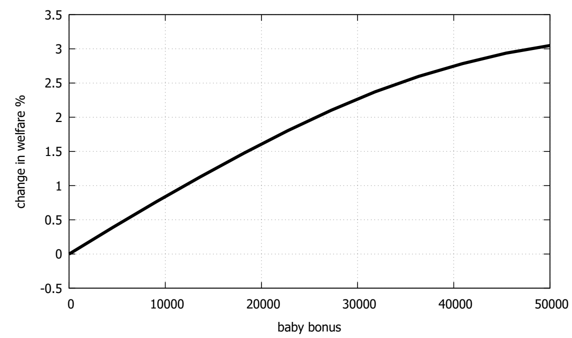
(h) Demographic Structure



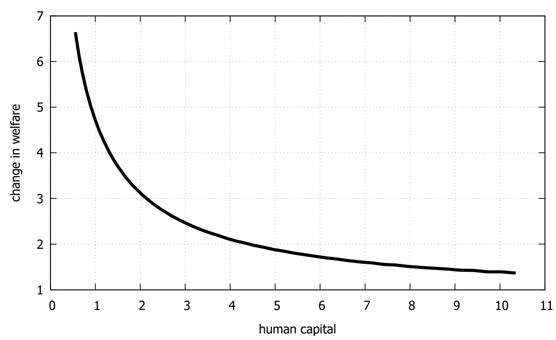
(i) Change in Consumption Tax



(j) Change in Welfare



(k) Change in Welfare by Human Capital of Parents



Notes: These figures plot macroeconomic impacts of baby bonus of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. See text for discussions.

revenues is affected by the change in human capital distribution. Second, as the share of retired households decrease, the government has less burden from pension payment. Lastly, as the share of children increases, the government needs to allocate more resources to public education. When the $\mathcal{B} = \$31,000$, consumption taxes could be slashed by 0.9% in the long-run.

Figure 7j plots the welfare effects in consumption equivalence. Due to increased fertility and reduced consumption taxes, average utility of new-born under the veil of ignorance increases by 2.2% in the long-run economy when aggregate fertility reaches replacement level. Besides higher fertility and lower consumption tax, welfare rises because baby bonus provides a social safety net for low-income households. This generates insurance benefits for risk-averse agents ex ante.

Lastly, Figure 7k plots the change in welfare by parents' human capital at $j = 2$ under a \$31,000 baby bonus.[35] Welfare improvements for parents with low human capital are much higher because the magnitude of each bonus is larger relative to their labor income. They also receive more bonuses as they have more children (see Figure 2 and 7d).

[35] I plot welfare changes against parents human capital rather than multiples of median income because the level of median income changes with policies.

5.2 Baby Bonus: Transition Path

In this section, I show the transition path results of aggregate variables under a \$31,000 baby bonus that leads to a replacement level fertility rate in the long-run. The policy is enacted unexpectedly at period $t = 1$ and stays afterwards. It is assumed that the government changes consumption taxes along the transition path to balance budget period by period.

Figure 8b shows that the policy effects on fertility is immediate and persistent with a small overshooting from period 4 to 10.[36] Figure 8b indicates that the old-age dependency ratio starts to have visible decline in period 6 since the share of working-age population and retired households in the short-run are predetermined by past fertility rates. The total dependency ratio, however, increases immediately since there is a large increase in the number of children due to the baby bonus.

As a result of this increase in total dependency ratio, the government needs to raise consumption taxes in period 1 to 9 before being able to reduce it in the long-run. Hence, welfare changes of newborn agents in transition are positive, but significantly smaller than that in the long-run. Since most existing agents in the economy at $t = 1$ do not benefit from the baby bonus but are required to pay higher consumption taxes, they are worse off under the baby bonus.

5.3 Comparing Policies: Subsidized Childcare and Education Expenditures

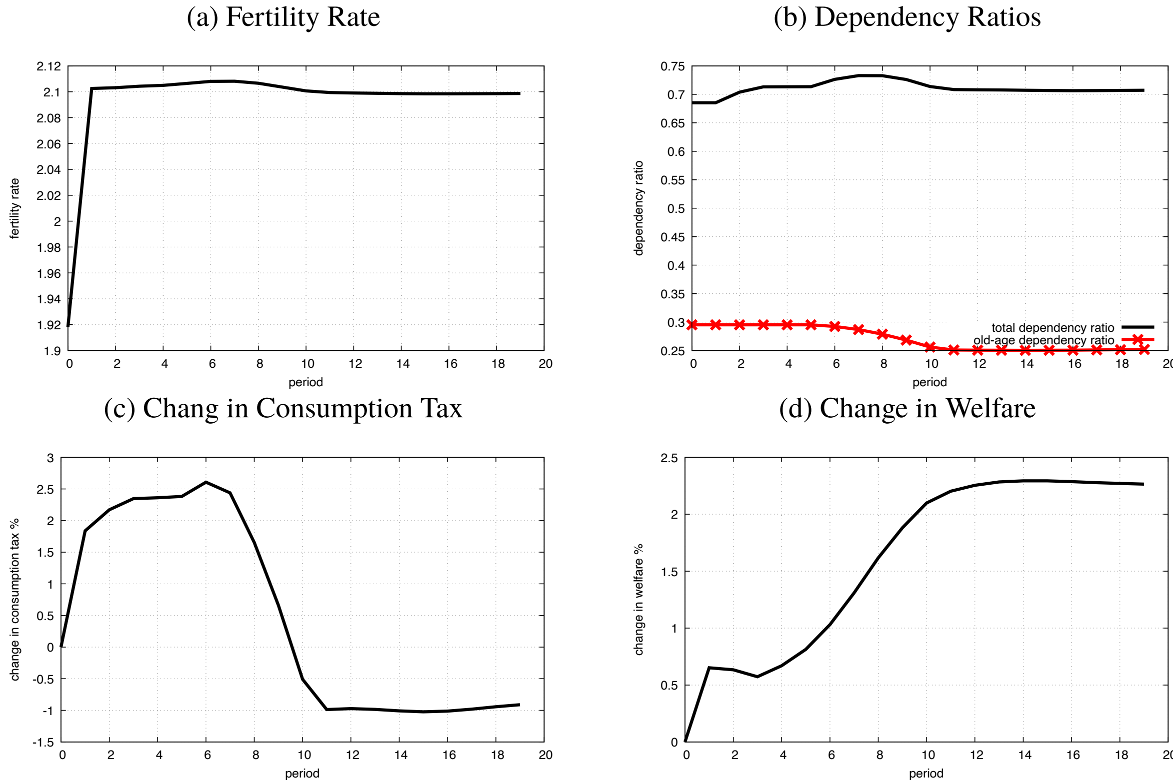
In this section, I briefly discuss the policy comparison results. For more detailed results regarding subsidized childcare and public education expenditures, see Appendix A.1 and A.2.

Compared with baby bonus, subsidized childcare is 50% less cost-effective in raising fertility because low-income parents, who are more price-sensitive, use less market childcare to start with. Since subsidized childcare changes the relative price between home care and market care, it encourages the combination of childbearing and labor force participation. This in turn, fosters adults human capital accumulation via on-the-job learning. As a result, subsidized childcare leads to higher gains in per capita output and welfare in the long-run than baby bonus. The responses in fertility and education investments to subsidized childcare are similar across education levels of the parents. Intergenerational mobility is not significantly changed under different levels of childcare subsidies.

Compared with baby bonus, the fertility effects of public education expenditures is smaller by an order of the magnitude for two reasons. First, it only indirectly affects quantity choices through changes in child quality. Second, education increases the share of parents with higher human capital who have smaller number of children. Therefore, the government should not expect to use education expenditures as an effective pronatal policy instrument. Education expenditures

[36]Recall that each period in the model is 10 years.

Figure 8: Aggregate Variables in Transition under $31,000 Baby Bonus



Notes: These figures plot evolution of aggregate variables under a baby bonus of $31,000. Each period represents 10 years. The policy is enacted at period 1. See text for discussions.

have strong crowding-out effects of private investments since public and private investments are substitutes in the child human capital production function. Expanding public education, however, is most effective in raising social mobility and improving children's outcomes. A $60,000 expansion of public education (in net present value) raises intergenerational mobility by 5% and long-run welfare by 2.5%.[37]

6 Optimal Policy

In this section, I study the optimal family policy to address externalities of childbearing and inefficiencies caused by borrowing constraints. I begin with a discussion of welfare criteria under heterogeneous-agent and endogenous fertility. Then, I show optimal policy results and conclude by

[37]These values are considerably smaller than the projected policy effects by Daruich (2018) for two reasons. First, Daruich (2018) does not consider the existence of public education expenditures in the benchmark economy. As a result, additional public education expenditures have significantly higher marginal effects on children's human capital. Second, Daruich (2018) does not consider endogenous fertility which dampens the policy effects on child quality through quality/quantity trade-off.

proposing some principles for designs of family policies in general.

6.1 Welfare Criteria

Welfare criteria in models with endogenous fertility is complicated both conceptually and philosophically. Unlike standard comparisons between allocations where the set of agents are fixed, there will be agents who are born in one economy but not in the other. As a result, standard Pareto principle can not be used to conduct welfare analysis in this context. The field of population ethics is devoted to understanding and resolving this question.[38]

Since the debate on welfare criteria is far from being settled, I adopt two definitions in studying the optimal policy. The first criterion is the long-run welfare $\mathcal{W}$ used in previous sections. It evaluates the expected utility of a newborn child in the long-run stationary equilibrium under the veil of ignorance. The second criterion evaluates the average utility of current parents at age $j = 2$ when the policy is adopted.[39] This measure has two features. First, it is forward looking as it incorporates tax changes in later periods that affects these parents' utility. Second, with this criterion, the unique solution to the planning problem corresponds to the notion of $\mathcal{A}$-efficiency defined in Golosov et al. (2007) which focuses on the welfare of those who are already alive.

Rather than computing the unconstrained optimum, I follow the Ramsey tradition and allow the government/planner to use only a certain policy instrument, namely baby bonus that is uniform across households and birth order. As in previous sections, I consider the scenario where the government adopts a baby bonus at the beginning of time $t = 1$. The adoption of baby bonus is a permanent policy change financed by consumption tax changes. I use $\mathcal{B}^*_{lr}$ to denote the optimal baby bonus that maximizes average utility in the long run, and use $\mathcal{B}^*_{sr}$ to denote the optimal baby bonus that maximizes the average utility of current parents at $j = 2$.

[38] For instance, Parfit (1984) derives the famous "repugnant conclusion" where he shows that under a set of intuitively appealing assumptions, one can prove "for any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better, other things being equal." Golosov et al. (2007) proposed two criteria called $\mathcal{A}-$efficiency and $\mathcal{P}-$efficiency which differs by whether the planner evaluates the welfare of those who are not born. de La Croix and Doepke (2021) considers the optimal welfare from a soul's perspective where one needs to consider both the utility of being born and the average "waiting time" for incarnation.

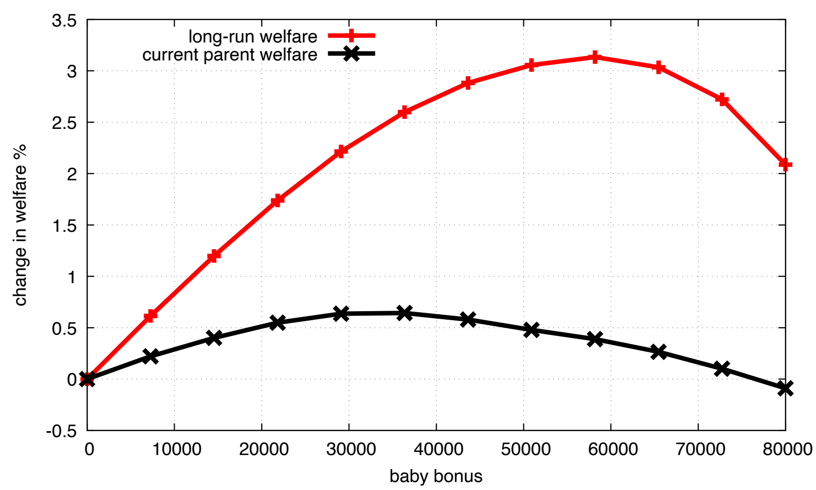
[39] There are two points worth noting. First, as discussed in Section 5.2, welfare effects on other agents in the current economy with age $j > 2$ simply depends on changes in consumption taxes since their choices related to children have already been made before the policy change. Second, following much of the quantitative macroeconomic literature, I use equal weights as a benchmark. Another approach is to use Negishi weights which puts a greater weight on households with higher human capital and initial assets. This would eliminate the redistributive benefits of baby bonus. See Kim et al. (2021) for an example.

6.2 Optimal Policy Results

Figure 9 shows the changes in welfare under baby bonus of different sizes for two welfare criteria. First, the baby bonus that maximizes welfare of current parents is $\mathcal{B}^*_{sr} = \$31,000$. Coincidentally, this is the baby bonus studied in Section 5.2 that leads to a long-run replacement fertility and a long-run welfare gain of 2.2%. As shown in Figure 8c, $\mathcal{B}^*_{sr}$ requires heightened taxes in the transition phase due to an increase in public expenditures on children's education. The gain in current parents' welfare, therefore, comes entirely from a redistribution of income from richer and elderly households.

On the other hand, the baby bonus that maximizes long-run welfare is $\mathcal{B}^*_{lr} = \$58,000$ which is significantly larger in size that $\mathcal{B}_{sr}$. It boosts aggregate fertility to 2.4, raises long-run welfare by 3.1%, and costs around 4.1% of GDP. The spike in taxes along the transition path, however, offsets much of its strong redistributive effects on current parents' welfare. The long-run increase in welfare also comes at a cost of all households in the current economy with age $j > 2$.

Figure 9: Optimal Baby Bonus by Welfare Measure



Notes: This figure plots the changes in welfare under baby bonus of different sizes for two welfare criteria.

6.3 Discussions

Even though this paper restricts the study of optimal policy design to baby bonus under two objective functions, it highlights the major trade-offs in policy design which could be used in a broader context with other instruments and goals.[40] On one hand, subsidizing fertility among parents with low human capital leads to larger fertility changes per dollar spent on family policies,

[40]For instance, it is straightforward to incorporate subsidized childcare and public education expenditures studied in Appendix A.1 and A.2.

which could result in larger welfare improvement due to demographic structure effects. On the other hand, subsidizing fertility among parents with high human capital could utilize the intergenerational transmission of human capital, which could raise welfare due to the composition effects. At the aggregate level, the government balances population growth rate and average human capital due to quality/quantity trade-off, composition effects, and demographic structure effects.

With all three countervailing forces present in the unified model, first-order stochastic dominance in equilibrium human capital distribution is neither necessary nor sufficient for choosing better policies. For instance, the equilibrium human capital distribution under a $31,000 baby bonus is first-order stochastic dominated by that in the baseline economy, but social welfare is improved by such baby bonus. Thus, the paper provides a novel counter-argument to the common conclusion in past literature which argues for restrictions on childbirth among parents with low human capital (e.g. Chu and Koo, 1990). These parents, with higher fertility elasticities, could be key to solving population aging problems.

Moreover, beyond the specific example considered in this paper, conflict of interests across generations is prevalent in the context of family policies for two reasons. First, due to quality/quantity trade-off, policies that encourage childbirth among existing parents usually come at a cost of lower human capital of future generations.[41] Second, desirable outcomes from reductions in old-age dependency ratio take decades to realize while costs from additional children appear right after policies are adopted. Existing households beyond fertile age do not share the benefits of family policies but could potentially bear some costs. The design of politically feasible policy reform, therefore, requires re-directing costs to future generations through long-term financing mechanisms such as international borrowing. I leave this possibility for future research.

7 Robustness

7.1 Alternative Preferences

There are alternative ways to model parents' preferences over child quantity and quality. I argue that the modeling choice in Section 3.1 makes our results a conservative benchmark.

One commonly-used assumption is separable preferences between quality and quantity (see de La Croix and Doepke, 2003; Bar et al., 2018; Vogl, 2016) where:

$$v(n, \mathbb{E}h_k, a_k) = \log(n) + \theta \log(\mathbb{E}h_k) + \nu \log(a_k)$$

Compared with our benchmark case where quality and quantity are complements, parents will raise

[41] One notable exception is expansions in education expenditures (see Appendix A.2).

quantity and reduce quality even more strongly when costs of child is lower since the interaction in preferences at the left-hand-side of Equation 7 is not present under separable preferences. I show this result with closed-form solutions in Appendix C.1. For the same reason, results in this paper will be conservative if quantity and quality are substitutes (Jones and Schoonbroodt, 2010).

Another way of modeling parents preferences is dynastic altruism (see Cordoba et al., 2016; Daruich and Kozlowski, 2020) where:

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n) V_2(\mathbb{E}h_k, a_k)$$

This formulation is quite similar to the one used in this paper except that utilities from child quality is pinned down recursively. Dynastic altruism is appealing aesthetically but face unresolved challenges in fitting observed transfers between parents and children (see Altonji et al., 1997 and Barczyk and Kredler, 2020). Paternalistic motives are often added to improve data fit (e.g. Abbott et al., 2019). I argue that our results will also be stronger under dynastic altruism since parents endogenize the fact that returns to child quality become lower as the government adopts policies such that children with low human capital are eligible to receive generous family benefits as a "social safety net". As a result, parents would further increase child quantity and reduce quality investments. For instance, Daruich and Fernández (2020) argues that universal basic income reduces human capital of future generations due to this mechanism.

7.2 Endogenous Timing of Childbirth

In the baseline model, I abstract away from birth timing by assuming that parents only make fertility choices from age 20 to 30. In reality, however, parents make decisions on when to give birth to children and family policies could affect that decision, also known as the tempo effects.[42]

I argue that adding endogenous timing will make our baseline results larger. As the model matches effects on completed fertility rate (quantum effects) in the validation exercise, policy effects on total fertility rate (quantum+tempo) are going to be larger. Suppose that in addition to increasing the number of children, some parents shift birth from 30s to their 20s in response to a baby bonus. This shift in birth timing is likely going to be detrimental to children's human capital for two reasons. First, early birth reduces the spillover that the children could receive because parents' human capital grows rapidly in from age 20 to 40. Second, family policies of realistic size fall short of offsetting the income differences of parents between early and late births. Hence, these children are induced to

[42] An unresolved question in the literature is whether changes in timing is due to the relaxation of parents' constrains (e.g. down payments to buy a larger house) or the lack of commitment to policies by the government. Historically, governments often renege on family policies or reduces the scale of payments as fiscal condition changes. One example is the Australian baby bonus which got significantly downsized in 2014. This uncertainty creates an incentive for parents to shift timing when benefits are still in effect.

be born into households with less resources on average, which would also reduce children's human capital as investment falls. As a result, family policies will have a larger observed fertility impact on the economy with endogenous timing of birth, but the outcome of children will be even worse.

7.3 Alternative Policies

Even though I only use the model to evaluate baby bonus, subsidized childcare and education which are financed through consumption taxes in the paper, the framework can be applied to analyzing a range of family and redistributive policies. For instance, instead of changing consumption tax to balance the budget, it is straightforward to calculate scenarios where additional expenditures are financed via labor taxes τ_y^n or capital income taxes τ_a. The government could also raise income tax progressitivity λ_y^n.

Some alternative policies will manifest themselves in the model as changes in parameters. For instance, government propaganda that encourages parents to have child for the country (Australian baby bonus), or policies that promote gender equality in childcare (see Doepke and Kindermann, 2019) could lead to changes in preferences for child quantity ψ. Education policies that equalize access to educational resources across income groups could lead to changes in intergenerational spillovers ρ.

8 Conclusion

Facing the "global fertility crash" (Tartar et al., 2019), family policies are at the forefront of public attention and policy debates. Evidence on how transfers to parents affect children's outcomes lead policy makers and economists alike to think that family policies are good instruments to "lift children out of poverty today and help them tomorrow" (Schanzenbach et al., 2021).

In this paper, I study the aggregate impacts of family policies in a quantitative general equilibrium overlapping-generations model with heterogeneous agents, endogenous fertility, child human capital formation, and inter-vivos transfers. Relative to previous studies with exogenous number of children, I argue that explicitly modeling endogenous fertility choice is crucial for analyzing the macroeconomic implications of family policies because it allows for (1) quality/quantity trade-off a lá Becker and Lewis (1973), (2) composition effects due to changing representation, and (3) demographic structure effects through population growth, government budget, and taxes.

I calibrate the model to match salient features of the data including fertility-income relationship, human capital investments, childcare usage, and institutional details on child benefits through the tax system. In external validation exercises, I show that the model-generated fertility elasticities are consistent with empirical evidence from Alaska Permanent Fund Dividend (APFD) and other family policies around the world.

I find that when family policies are designed such that parents are awarded based on child quantity, they respond by having more children but optimally reduce child quality due to quality/quantity trade-off. Compared with education subsidies, family policies are not ideal instruments if the policy goal is to raise children's human capital or to boost social mobility. In fact, average human capital and intergenerational mobility both fall in the long-run after implementation of a baby bonus due to (heterogeneous) reductions in child quality investments and composition effects. The pronatal effect of family policies, however, could lead to long-run welfare gains because population growth reduces old-age dependency ratio. This allows the government to reduce tax rates without violating the fiscal constraint in the general equilibrium.

I also show that long-run gains in welfare require a prolonged transition path where the government needs to raise consumption taxes temporarily to finance higher child-related expenditures. This finding highlights the implicit costs of family policies in transition. Those costs are important but rarely mentioned in policy discussions since most attentions are paid to the reductions in the pension liabilities in the long-run.

I conclude by discussing several avenues for future research. First, it would be interesting for future research to consider optimal policy design with additional policy instruments and different welfare criteria. Second, given that family policies such as baby bonus generate conflicts of interest across generations by increasing welfare of households in the long-run but reducing that of current agents (see Section 5.2), it is important to explore whether such policies could be financed by borrowings that are paid back by future generations. Lastly, another fruitful area of research is to link family policies to other population externalities such as pollution (Bohn and Stuart, 2015), ideas creation (Jones, 2020), and firm dynamics (Hopenhayn et al., 2018; Peters and Walsh, 2020).

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A Additional Results

A.1 Public Childcare

In the second set of policy counterfactuals, I evaluate childcare subsidies where the government refunds $\mathcal{S} \in [0, 0.9]$ fraction of expenditures on market childcare services for all families.[43]

I find that subsidized childcare raises fertility, but at a much higher cost than baby bonus. The government needs to refund more than 80% of childcare expenses to raise the aggregate fertility rate to 2.1. This would cost almost 3.2% of aggregate GDP in the long-run steady-state economy, which means the policy is 50% less cost-effective than baby bonus.

Figure 10c shows that similar to the case in baby bonus, parents reduces private education investments due to quality/quantity trade-off. Figure 10d shows that fertility responses are similar across households with different human capital levels. As subsidy raises, the fertility responses among low-income households exceed that among high-income ones. As a result, average human capital in the economy begins to decrease (see Figure 10e). The overall effects on intergenerational mobility is small (see Figure 10f) .

Figure 10g shows that output per capita can be increased by up to 3.5% when the government refunds 80 cents per dollar spent on market childcare. Compared with baby bonus, the magnitude of change in output is larger for three reasons. First, the composition effects through intergenerational transmission channel is smaller because changes in fertility is not very different across human capital (see Figure 10d). Second, childcare subsidy raises labor supply by reducing the effective price of market care. This induces parents to reduce time spent at home taking care of children t_h. Lastly, the increased working time at age 20-30 translates to a high human capital for adults in later periods due to learning-by-doing in the labor market.

Figure 10i shows that the government is able to reduce consumption tax by 0.6% when $\mathcal{S} = 0.45$. Long-run welfare could be raised by 3% at replacement fertility rate with $\mathcal{S} = 0.8$.

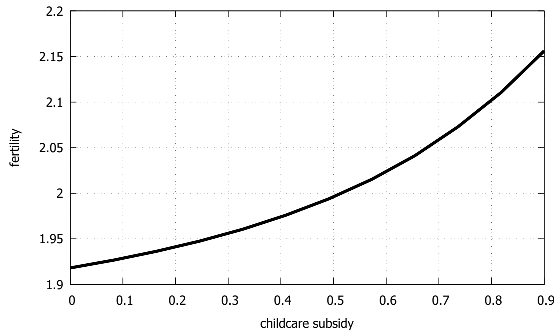
A.2 Public Education

Lastly, I evaluate an expansion of public education expenditures from the current level of $12,000 per student per year by $0 to $100,000 in net present value. Figure 11a shows that the effect on fertility is positive because children become more desirable as public education raises children's quality.

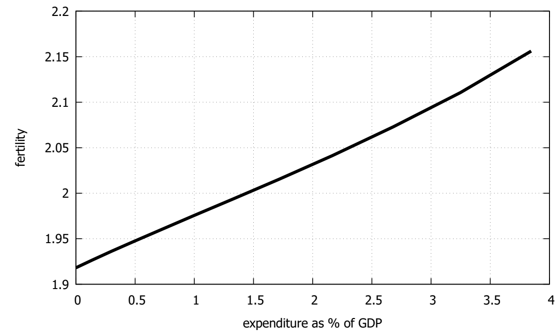
[43] Another common proposal on childcare is to provide certain amount (days) of public childcare services that is free to all parents with children. For households with middle-to-high income, this policy would be equivalent to a baby bonus since their private spending on market childcare exceeds the subsidized quantity absent of the policy. For lower-income households, they would strictly prefer to have a baby bonus as cash subsidies can be used in alternative expenditures with higher marginal benefits. Overall, the implications on aggregate variables are quantitatively similar to that of a baby bonus.

Figure 10: Effects of Childcare Subsidies of Different Sizes

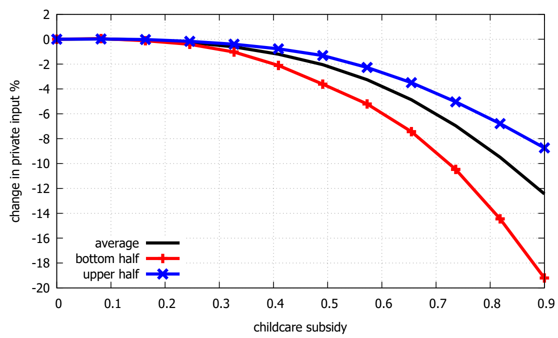
(a) Effects on Fertility



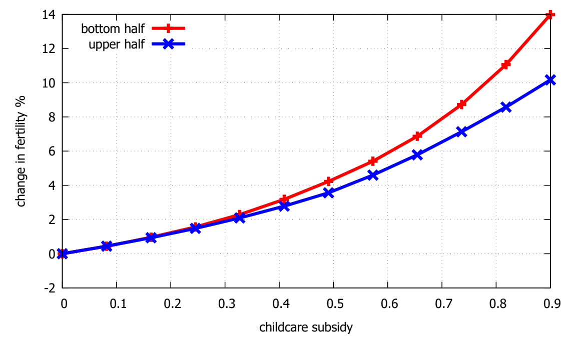
(b) Expenditure Share as % of GDP



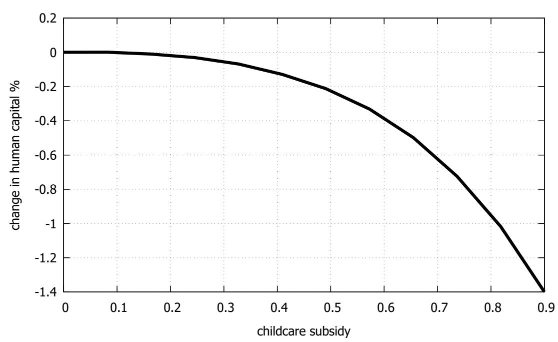
(c) Average Private Investment



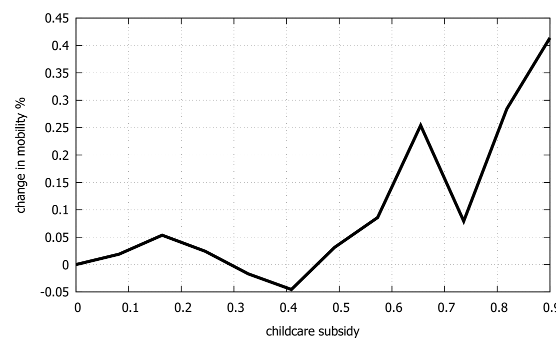
(d) Heterogeneous Fertility Response



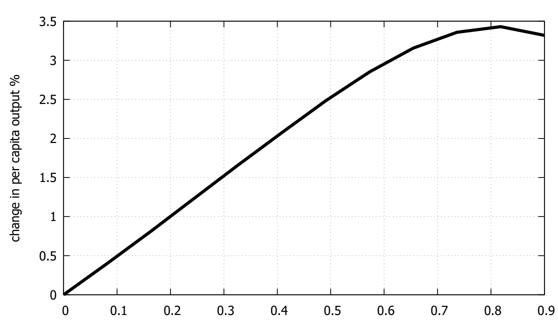
(e) Average Human Capital



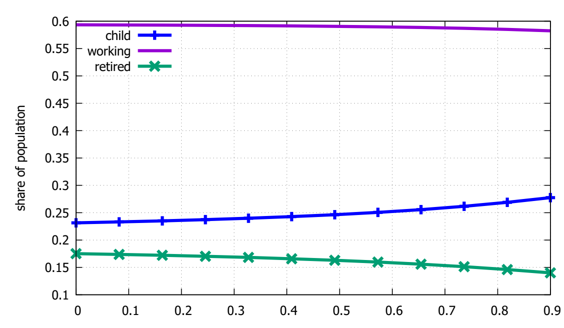
(f) Intergenerational Mobility



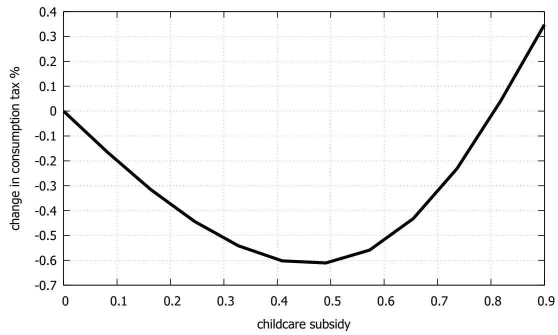
(g) Per Capita Output



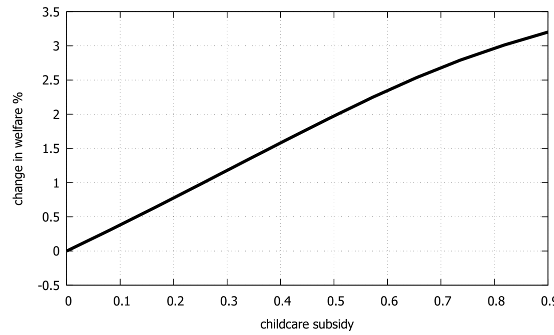
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of childcare subsidies of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. See text for discussions.

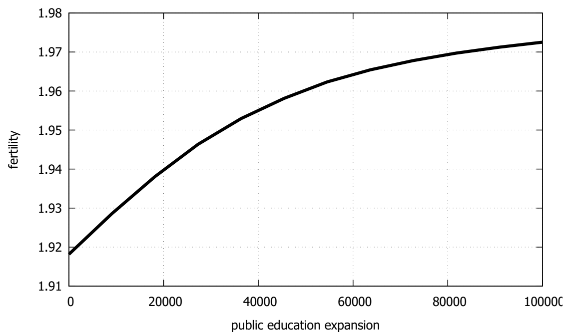
The magnitude of fertility effects here are much smaller than that of family policies. Compared with a \$30,000 baby bonus that raises fertility by 0.2, an education expansion of the same amount only raises fertility by 0.03. The government should not expect to use education policies to raise aggregate fertility to replacement level.

Figure 11d shows that fertility response is slightly higher for households with higher human capital. An increase in public education expenditure also crowds out private input (see Figure 11c). Due to strong crowding-out effects, average human capital only increases by 2.3% when the education expansion exceeds \$80,000 in net present value. It is quite costly to raise average human capital in the economy using uniform policies in general equilibrium since (1) crowding out effect is strong, and (2) investments face decreasing marginal return as $\mathcal{E}$ is already \$12,000 per year in the baseline economy.

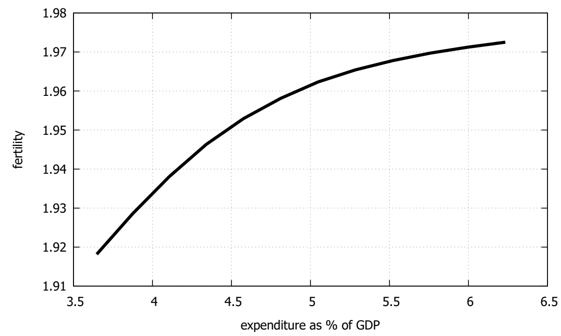
Expanding public education, however, is the most effective policy to raise social mobility. As the expansion exceeds \$60,000 in net present value, intergenerational mobility could be raised by almost 5%. If the policy is designed to target lower-income households, we can expect larger effects on mobility. Due to increased human capital, public education expansion of \$60,000 raises output by 2.3%. The government needs to increase consumption tax by 0.5% to finance this expenditure. Figure 11j shows that welfare could be increased by up to 2.5%.

Figure 11: Effects of Expansions in Public Education Expenditures of Different Sizes

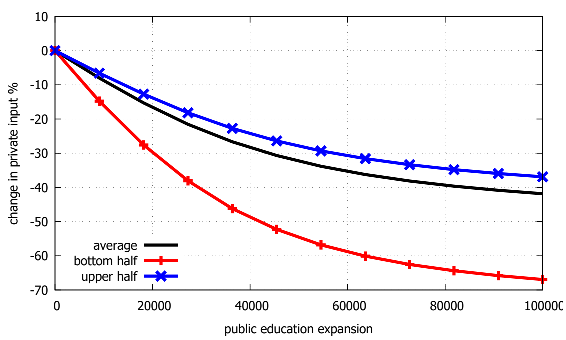
(a) Effects on Fertility



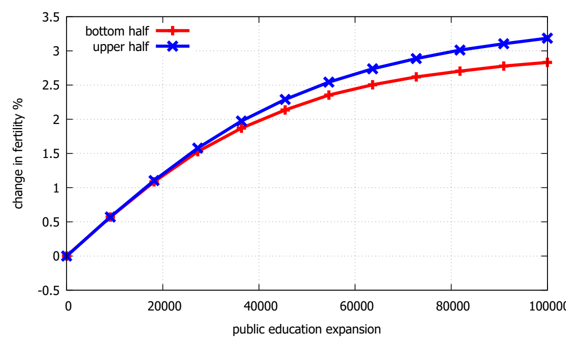
(b) Expenditure Share as % of GDP



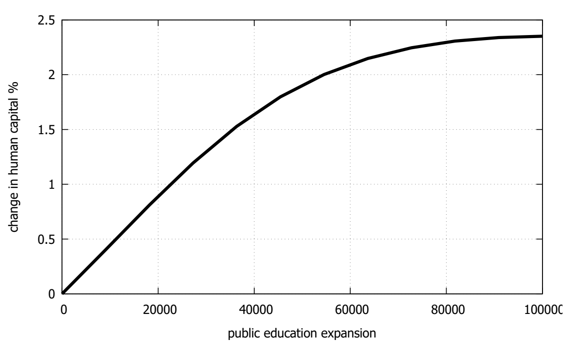
(c) Average Private Investment



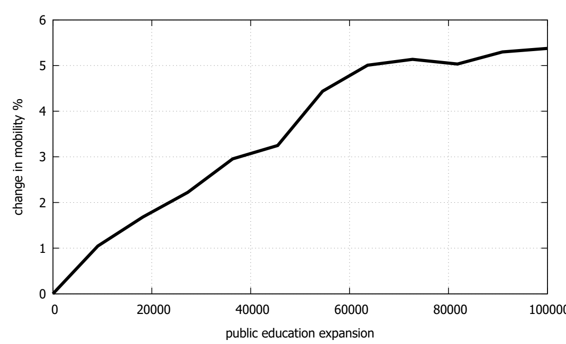
(d) Heterogeneous Fertility Response



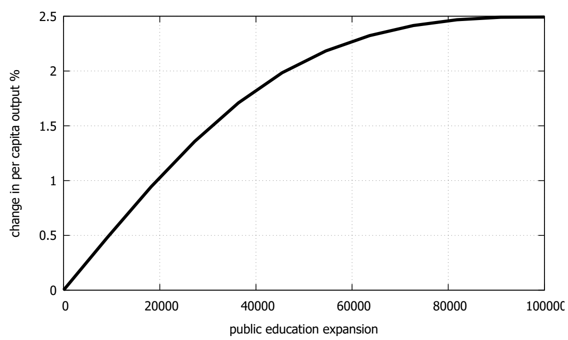
(e) Average Human Capital



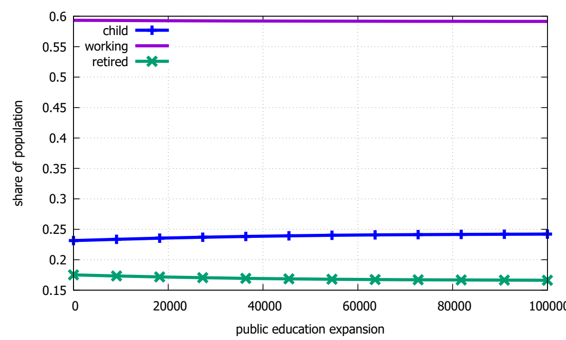
(f) Intergenerational Mobility



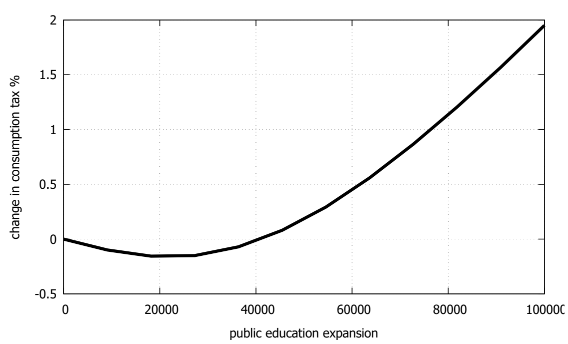
(g) Per Capita Output



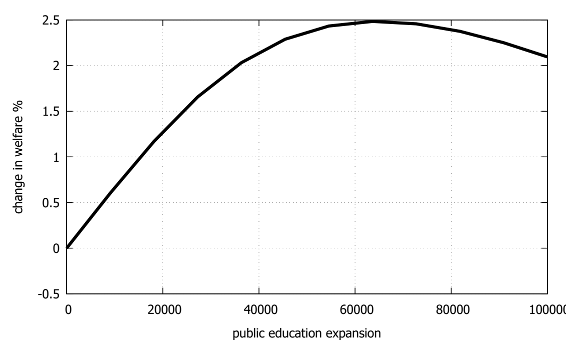
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of expansions in public education expenditures of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. See text for discussions.

B Additional Validating Evidence

B.1 Australian Baby Bonus

In this section, we compare the implications of the calibrated model with the empirical evidence from a recently adopted baby bonus in Australia. Even though I do not re-calibrate the model to match moments in Australia, this comparison still provides valuable insights into the mechanism through which child benefits affects parents decisions and estimates of fertility elasticities that is the core of this paper.

The Australian government announced the introduction of a universal cash payment, i.e. baby bonus, in its Federal budget on May 12$^{\text{th}}$ 2004. It was designed to be a non-means tested and non-taxable lump-sum payment of A$3,000 to encourage childbirth, paid to parents following a childbirth or adoption of a child up to 2 years of age after July 1$^{\text{st}}$ 2004. Such payment is independent of family income, maternal employment status, or the number of existing children in the household. The payment is equivalent to 4 times weekly average earnings or $2,800 in 2010 U.S. dollars.

Drago et al. (2011) conduct a comprehensive analysis of the fertility effects of the baby bonus using household panel data ($N = 14,932$) from the Household, Income and Labour Dynamics in Australia Survey (HILDA) using a simultaneous equations approach. They first estimate the effect of the baby bonus on households' birth intentions in a linear regression, which, in turn, predicts actual births in a binomial probit regression. They exploited the panel structure of the data to test and correct for announcement effects, compression/tempo effects, and delayed effects. Regression results show that the baby bonus have significant and positive impacts on fertility. Moreover, the fertility responses are concentrated among low-income women.[44] Risse (2010) reports similar findings on women's birth intentions.

When I compare these empirical studies to the model, both the finding on overall fertility effect and the heterogeneities across households are consistent with predictions in Section 5.1. Drago et al. (2011) estimates that the marginal costs to the government for an additional birth is at least A$126,000 which is roughly 4 times the GDP per capita in 2004 Australia. The calibrated model generates a quantitatively similar conclusion where the marginal costs for an additional birth using baby bonus in 2010 USA is around 3.5 times the GDP per capita.

Gaitz and Schurer (2017) evaluates the effect of the Australian baby bonus on children's human capital accumulation using high-quality panel data from Kindergarten cohort (K-cohort) of the Longitudinal Study of Australian Children (LSAC). They find that the baby bonus, despite be-

[44] This finding is different from that in Milligan (2005) which evaluates a baby bonus in Quebec and finds larger responses among parents with higher income. Drago et al. (2011) conjecture that the difference could be due to the fact that Quebec baby bonus was significantly more generous for high-parity births. Milligan (2005) discusses some other possible explanations such as unobserved heterogeneities that are systematically related to income.

ing significant in size, was not effective in boosting learning, socio-emotional or physical health outcomes of the average pre-school child. This finding is consistent with results in Section 5.1 where parents optimally reduce private educational investments due to quality/quantity trade-off.

B.2 Spanish Child Benefits

In this section, I provide additional evidence on the fertility effects of child benefits using a universal child benefits in Spain.

The Spanish government announced the new child benefit on July 3rd 2007. The benefit was a one-time payment of €2,500 to the mother immediately after birth on and after July 1st 2007. Since the cash benefit was universal and independent of recipient's characteristics, it was essentially a universal baby bonus (c.f. Australian baby bonus, and $\mathcal{B}$ in the model). The size of the payment is 4.5 times the monthly gross minimum wage for a full-time worker or $3,500 in 2010 U.S. dollars.

González (2013) studies the effect of the Spanish child benefits on fertility and mothers' labor supply using monthly vital statistic, monthly abortion statistics, and Household Budget Survey 2008 ($N = 958$). In the paper, the fertility effects are gauged by inspecting the time series of births and abortions while the effects on labor supply are estimated using a regression-discontinuity design by comparing households who had birth right before and right after the cutoff date. González (2013) finds that the fertility effects is positive and significant. The policy raises total fertility rate by 6%. Around 80% of this increase is due to higher number of conception while the remaining 20% is due to reductions in abortion. Applying the adjustment in Stone (2020) (see Section 4), the result suggests that in Spain, the marginal costs for an additional birth using baby bonus is 3.6 times the GDP per capita. In the model, this statistic is 3.5.

González (2013) also finds that the baby bonus reduced mothers' labor force participation. The calibrated model produces this effect via two channels. First, as the baby bonus raises fertility, parents need to generate more childcare services, which raises their time at home ceteris paribus. Second, higher fertility reduces the relative cost of home versus market care due to economies of scale in home production of childcare ($\iota < 1$). Therefore, the model predicts that parents optimally change the childcare arrangements so that their children spend less time in market care. González (2013) confirms this prediction by showing that Spanish parents reduces enrollment in formal child care after the baby bonus.

B.3 Georgia's Cherokee Land Lottery of 1832

In this section, I compare the model predictions with Georgia's Cherokee Land Lottery in Northwest Georgia, 1832.

Following the eviction of the Cherokee from northwest Georgia, the state allocated more than 18,000 parcels of land to the public in a large-scale lottery in 1832. Every man age 18 and older who had resided in Georgia for the three years prior to the 1832 drawing was entitled to one draw, and any man who had a wife or had children under 18 and met the three-year residency requirement was entitled to an additional draw. The lottery winners were able to immediately sell the winning draw. Therefore, winning the lottery can be viewed as a substantial shock to wealth. Bleakley and Ferrie (2013) calculate that 98% of eligible man participated in the lottery and winners were about $748 wealthier than losers by 1850, equivalent to 1,010 days of earnings for an unskilled laborer in the South at that time.

There are two notable things about the Cherokee Land Lottery. First, unlike baby bonus that provides an estimate of the *price elasticity* of fertility, the Cherokee Land Lottery reveals the *income elasticity* of fertility. This statistic, if used directly, is mostly informative about fertility effects of policies such as a universal basic income (UBI) rather than family policies that are pronatal by design. The model in this paper, nevertheless, could generate the counterpart to the income elasticity of fertility by varying the initial assets of parents at period 2. Second, numerous changes have taken place in the U.S. in the past two centuries. The average white women gave birth to around seven children in 1830s but less than two children in 2010. The biggest factor explaining the decline of fertility and the rise of schooling is the increase in skill premium (Greenwood and Seshadri, 2002). Therefore, the Cherokee Land Lottery is expected to generate responses that are *upper bounds in fertility* and *lower bounds in child human capital investment* to windfalls.

Bleakley and Ferrie (2016) identify the winners of this lottery using a list published by the State of Georgia , and link winners to their decedents using Census data. Results show that winners had slightly more children than nonwinners, but they did not send them to school more. "Sons of winners have no better adult outcomes (wealth, income, literacy) than the sons of nonwinners, and winners' grandchildren do not have higher literacy or school attendance than nonwinners' grandchildren" (Bleakley and Ferrie, 2016).

In the calibrated model, I compare $n^*(h, a), e^*(h, a)$ with $n^*(h, a'), e^*(h, a')$ for fixed h and $a < a'$. I find that if the parents are endowed with a higher level of initial assets, they choose to have a slightly smaller number of children, i.e. $n^*(h, a') \le n^*(h, a)$ but invest significantly more in children's education, i.e. $e^*(h, a') \gg e^*(h, a)$. Since the returns to education in 2010, to which the model targets using RCT results from García et al. (2020), is much higher than that in 1830s, these results are consistent with those found in Bleakley and Ferrie (2016).

C Illustrative Models

C.1 Simple Model of Quality/Quantity Trade-off

In this section, I show the quality/quantity trade-off mechanism with closed-form solutions in a model that is adapted from de La Croix and Doepke (2003).

Agents in the economy live for two periods, child and adult. Adult parents solve the problem:

$$\max_{c,n,e} \quad \log(c) + \theta \log(n \cdot h_k)$$

subject to

$$c + n \cdot e = 1 - n \cdot \chi$$

$$h_k = e^{\gamma}, \quad \gamma \in (0, 1)$$

where c is consumption, n is fertility, h_k is human capital of children, e is private investment,[45] and χ is fixed costs per child.

When fertility is exogenous, i.e. n is given, parents maximize over c and e. Optimal investment is given by:

$$e^* = \frac{\theta\gamma}{1 + \theta\gamma} \cdot \frac{1 - n\chi}{n}$$

When the cost of child χ decreases, e^* increases due to income effect.

When fertility is endogenous, parents maximize over c, e, and n. Optimal fertility and investment are given by:

$$n^* = \frac{1}{\chi} \cdot \frac{\theta(1-\gamma)}{1+\theta}, \qquad e^* = \frac{\gamma\chi}{1-\gamma}$$

When the cost of child χ decreases, n^* increases while e^* decreases. The intuition for this result is simple. Parents increase fertility due to substitution effect. The increase in n^*, in turn, raises the shadow price of investment e^* due to interaction of n and e in the budget constraint. As a result, optimal investment e^* falls.

Compared with the benchmark model where n and h_k are complements in parents' preferences, reduction in e^* in response to change in χ is higher in this simple model. This is because the marginal utility in child quality h_k is independent of fertility n.

[45] In de La Croix and Doepke (2003), children receive human capital endowments. This generates non-homotheticity over child quality for heterogeneous parents with different human capital and leads to a negative fertility-income relationship. I abstract away from steady-state heterogeneity across households in this simple model for clearer exposition of intuition in comparative statics, but all arguments carry through when human capital endowments are allowed.

C.2 Simple Model of Fertility Elasticity

In this section, I build a simple model to illustrate the relationship between parameter γ, also known as elasticity of intergenerational substitution (EGS), and the magnitude of fertility response to family policies.

Consider a simplified problem for parents with very low income so that child quality is generated by public investments alone:

$$\max_{c,n} \quad u(c) + \Psi(n)u(\mathcal{E})$$

$$c + n \cdot \chi = 1$$

First-order condition for n:

$$\underbrace{\Psi'(n) \cdot u(\mathcal{E})}_{\text{MB of } n} = \underbrace{\lambda \cdot \chi}_{\text{MC of } n}$$

Plug in $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I have

$$\Psi'(n) = (1-\gamma) \cdot \chi \cdot \frac{\lambda}{\mathcal{E}^{1-\gamma}} \Longrightarrow \Delta\Psi'(n) \propto (1-\gamma) \cdot \Delta\chi$$

As can be seen, conditional on other parameters, higher γ implies a smaller response in n for given changes in costs of child χ.