Bounding Fertility Elasticities

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Abstract

I propose a technique for bounding the fertility elasticity, i.e., the magnitude of fertility responses to changes in the cost of children. I show that a bound can be derived under mild assumptions for any country and year with minimal information required. Overall, the range is consistent with empirical estimates and is more precise than current meta-analyses. The bound imposes additional restrictions on parameters in models with endogenous fertility. Furthermore, it provides a transparent evaluation of the exogenous fertility assumption that is widely used in structural models studying child-related policies.

1 Introduction

Just like other elasticities in an economy, such as labor supply and consumer demand elasticities, fertility elasticity (the price elasticity of fertility demand) is an important moment for both economists in and outside the fields of labor and macroeconomics, as well as policymakers across the globe. For economists working with endogenous fertility models (e.g., Barro and Becker (1989)), fertility elasticity provides a fundamental discipline to the selection of parameters. Additionally, for any economist interested in child-related policies such as the Child Tax Credit (CTC), the Earned Income Tax Credit (EITC), and Universal Basic Income (UBI), fertility elasticity enables a transparent evaluation of the exogenous fertility assumption, a simplification widely used in a class of structural models. For many governments that attempt to use transfers to raise fertility, it is also crucial to understand how cost-effective these measures are.

Despite the importance of fertility elasticity and a large body of empirical literature estimating it, little consensus has been reached on its quantitative magnitude.[1] Estimation based on historical policies has proven to be difficult for several reasons. First, many historical policies are not sizable enough (relative to the cost of children) to induce measurable changes in fertility (Bergsvik et al.

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[1] "There is considerable disagreement across studies about the effectiveness of pro-natal policies" (Stone (2020)).

(2020)). Second, given that pro-natal policies are usually nationwide or income-dependent, finding a control group is not straightforward (Gauthier (2005)). This difficulty is especially acute because most pro-natal policies are adopted to address contemporaneous or future fertility decline, while it is unclear if countries in the control group also face a similar situation (Castles (2003)). Third, family policies often come in bundles of incentives in excess of lowering the cost of children, such as measures encouraging women's labor force participation. Therefore, estimating the fertility responses to a policy bundle is not the same as estimating the fertility elasticity. Lastly, even if past estimates were precise, a widely acknowledged issue with the design-based methodology is that changes in time and institutions limit the applicability of past estimates in a different context.

In this paper, I propose a simple theoretical approach to *bounding* fertility elasticities and discuss its implications. In Section 2, I show that under mild assumptions, a bound can be derived after knowing (1) the prevailing fertility rate and the prevailing cost of children, (2) the maximum desired fertility, and (3) the cost of children such that potential parents would rather prefer not to have a child. In Section 3, I apply this method to the United States in 2010 and find that a transfer of size between \$7,514 and \$35,966 raises the fertility rate by 0.1 children per woman, a range tighter than conclusions from meta-analyses of past estimates. Moreover, I demonstrate that the bound is simple to compute for any country and year of interest. In Section 4, I show that this bound imposes additional restrictions on parameters in endogenous fertility models. Lastly, after comparing the bound with the optimal child-related policies in a number of structural models assuming exogenous fertility, I argue that the fertility responses to the proposed policies are non-negligible.

This paper contributes to two strands of literature. The first one is a large body of empirical literature quantifying fertility elasticities using historical policies (e.g., Milligan (2005), Laroque and Salanié (2008), Cohen et al. (2013), and González (2013) among many others). This paper has the same goal in mind but approaches the question from a different perspective. Rather than exploiting the local perturbation of prices, this paper bounds the local elasticity using the global properties of the demand curve. With its flexibility and accuracy, this method is complementary to design-based inquiries and has practical implications for policymakers that want to pursue large-scale family policies. The second literature models fertility choices in the context of labor and macroeconomics (e.g., Becker and Lewis (1973), Barro and Becker (1989), Doepke (2005), Córdoba and Ripoll (2019) among others). Despite being a fundamental moment disciplining the quantitative predictions of these models, fertility elasticity is rarely a targeted moment, primarily due to the challenges in the empirical measurement. This paper makes progress by showing that under mild assumptions, the bound derived here could already inform the selection of parameters in this class of models.

2 Theory

Consider an economy populated by representative agents. I denote the Marshallian demand of fertility as $n(p; \boldsymbol{p}^{\text{other}}, y)$ where n is fertility, p is the cost of children, $\boldsymbol{p}^{\text{other}}$ is the price (vector) of other goods, and y is the household's lifetime income.

Assumption 1 The Marshallian demand of fertility $n(p; \boldsymbol{p}^{\text{other}}, y)$ is downward sloping, continuously differentiable, and convex in p.

This assumption is satisfied by most models of endogenous fertility, whether with warm glow (e.g., De La Croix and Doepke (2003)) or with dynastic altruism (e.g., Barro and Becker (1989) and Córdoba and Ripoll (2019)). See Section 4 for examples.

Assumption 2 There exists $\overline{p}$ and $\overline{n}$ such that $n(\overline{p}; \boldsymbol{p}^{\text{other}}, y) = 0$ and $n(0; \boldsymbol{p}^{\text{other}}, y) = \overline{n}$.

This is a mild and realistic assumption. For the first equation, an example is to let $\overline{p} = y$. The assumption then requires that if having a child costs the parents' entire income and leaves no resources for other goods, then the household would prefer not to have a child in the first place.[2] The existence of $\overline{n}$ reflects biological constraints of childbearing or satiation in preferences.

Proposition 1 The fertility response to price around (n^0, p^0) is bounded by

$$\left.\frac{dn}{dp}\right|_{(n^0,p^0)} \in \left(\frac{n^0}{\overline{p} - p^0}, \frac{\overline{n} - n^0}{p^0}\right). \tag{1}$$

Proof Figure 1 gives an illustration of the proof. The Marshallian demand of fertility is given by curve BAC where the coordinates of B and C are $(0, \overline{p})$ and $(\overline{n}, 0)$ correspondingly. Point A denotes the prevailing fertility and cost of children (n^0, p^0). The slope of the demand curve at A (black) is bounded by the slope of AC (red) and the slope of AB (blue) under the Mean Value Theorem and the assumption that curve BAC is decreasing, continuously differentiable, and convex.

In other words, whereas estimation using historical policies relies on local perturbations to p around (n^0, p^0), the method proposed here exploits the global properties of the demand curve under Assumptions 1 and 2 to bound the local slope. After knowing the slope, the local fertility elasticity $\left.\frac{dn}{dp}\frac{\log p}{\log n}\right|_{(n^0,p^0)}$ can be easily computed.

Given the practical difficulties in identifying and quantifying a local shock to p, the bounding approach provides a valuable alternative. Of course, adopting a large-scale pro-natal policy or a

[2] In general, one does not need fertility demand to be exactly zero at $\overline{p}$. The argument goes through as long as one can choose a small $\underline{n}$.

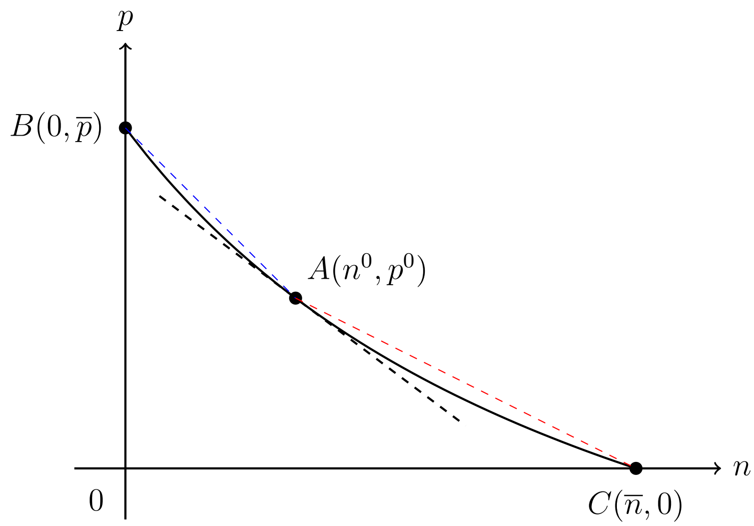

Figure 1: Essence of the Proof

Notes: This figure shows the essence of the proof. The fertility demand curve is given by BAC. The prevailing fertility and cost of children is at point A.

randomized control trial could lead to a more precise estimate for the specific country and year being studied (see Figure 2 for more details). But even in that case, the bound remains complementary to the design-based approach in that it provides a prediction of the program's cost-effectiveness *ex ante* and a first check of the results *ex post*.

3 Quantification

In this section, I calculate the bound for the United States, compare the results with prior studies, and show that the bound is simple to calculate for other combinations of country and year.

3.1 Calculating the Bound

I calculate the bound for the United States in 2010 where the prevailing fertility n^0 is 1.9 and the cost of one child p^0 is \$458,351 for a middle-income household (Córdoba and Ripoll (2019)). I make the following assumption on $\overline{n}$ and $\overline{p}$:

Assumption 3 Set $\overline{n} = 8$. Choose $\overline{p}$ such that an average household with one child lives in poverty for the rest of their lives, i.e.,

$$y - \overline{p} = y^{\text{poverty}}. \tag{2}$$

Proposition 1 suggests that the choice of $\overline{n}$ and $\overline{p}$ affects the tightness of the bound. When we express the bound as the change in p needed for a unit increase in n, the bound becomes tighter as

$\overline{n}$ or $\overline{p}$ decreases.

The assumption that $\overline{n} = 8$ is probably conservative towards the high end as the prevailing ideal number of children is around 2.5 children per woman (Stone (2018)). The assumption that $\overline{p}$ is the cost of children that makes an average married household fall into poverty status after one childbirth is also a likely upper bound for $\overline{p}$. Thus, there are reasons to believe that the bound could be tighter under reasonable alternative assumptions.

Following Córdoba and Ripoll (2019), I set $y =$ \$2,083,219. Using the federal poverty guideline in 2010, I set $y^{\text{poverty}} =$ \$941,520. This implies $\overline{p} =$ \$1,141,699.

Applying Proposition 1, I find that to raise the fertility by 0.1 children per woman, the change in p needed is between \$7,514 and \$35,966. This change in p can be brought about by policies such as a baby bonus, a universal basic income, a child allowance, or a fully-refundable Child Tax Credit (CTC) expansion.

3.2 Discussions

One advantage of the bound is that it is tighter than meta-analyses of past estimates. For example, Stone (2020) summarizes and harmonizes a large number of research estimating fertility elasticity from historical policy changes, mostly in low-fertility countries. He concludes that "an increase in the present value of child benefits equal to 10% of a household's (annual) income can be expected to produce between 0.5% and 4.1% higher birth rates" (see Figure 2).

To make the measures comparable, I convert the bound in this paper into percentages using the median annual household income in 2010 (\$49,445). The bound predicts that an increase in the present value of child benefits equal to 10% of a household's (annual) income can produce between 0.72% and 3.46% higher birth rates.[3] As can be seen, the bound proposed here provides a narrower range of predictions for the fertility effects of pro-natal policies.

Another advantage of the bound is that it is simple to compute for a different combination of country and year as long we know the prevailing (n^0, p^0), y, and y^{poverty}. In particular, p^0 acts as a *sufficient statistic* that captures differences in policies, markets, and social norms that affect the costs of childraising across time and space, while n^0 is the revealed preference for fertility demand under prevailing prices.

For example, in the United Kingdom in 2016, the fertility rate was 1.79 children per woman, and the cost of raising a child from birth to 21 years old is £231,843 (CEBR (2016)).[4] The lifetime

[3] In Figure 2, there are some estimates that lie outside of the bound derived in this paper. This could arise because these policies were implemented in a different time and institutional setting.

[4] Unlike Córdoba and Ripoll (2019), this cost does not include the opportunity costs of time in childraising. Thus, p^0 could be biased downward. As a result, the bound reported below will be wider than the case where the bias is eliminated.

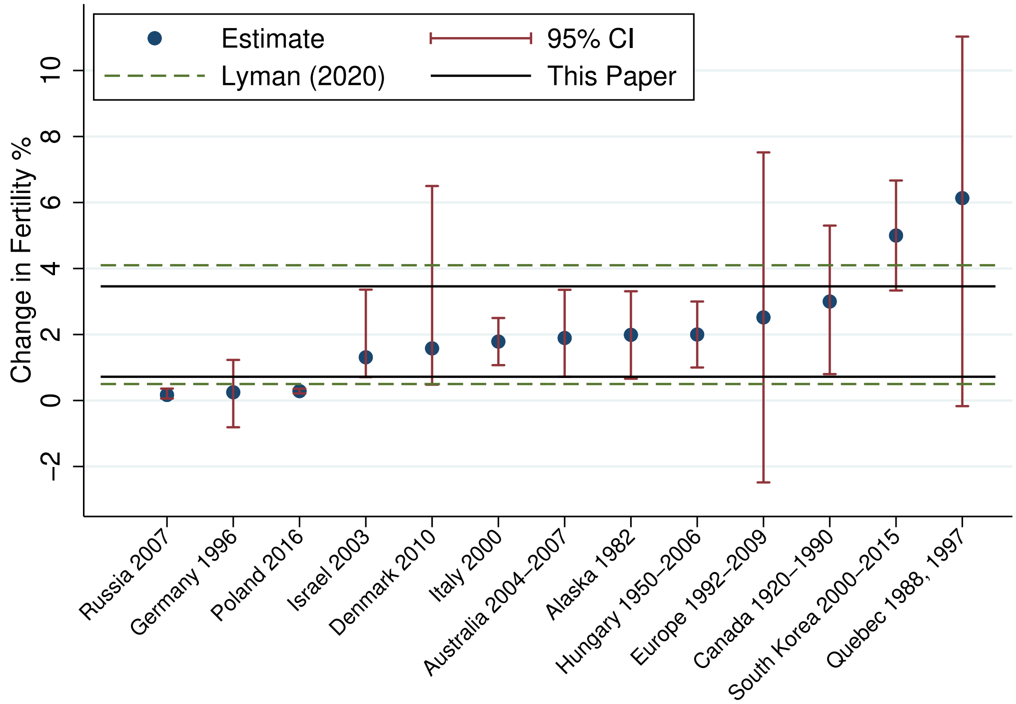

Figure 2: Past Estimates of Fertility Elasticities

Notes: This figure presents a summary of fertility elasticities estimated using historical policies and different bounds. As the goal of the paper is to pin down the price elasticity of fertility, I select policy changes that includs universal child benefits, baby bonuses, and universal basic income from the summary file compiled by Stone (2020). When there are multiple estimates exploiting the same policy change, I take the average across studies. The dots represent point estimates of fertility responses to a transfer with a net present value that is 10% of a household's annual income. The red intervals correspond to 95% confidence intervals. The two horizontal dashed lines represent the bound of fertility elasticity suggested by Stone (2020). The two horizontal solid lines represent the bound derived in this paper for the United States in 2010.

income y is approximately £1,400,000. p^{poverty} is chosen to be 60% of y (£840,000) following the definition used by the Department of Work and Pensions of the British government. Holding Assumption 3 unchanged and applying Proposition 1, it can be seen that to raise the fertility by 0.1 children per woman in the United Kingdom, the change in p required is between £3,733 and £18,333.

To summarize, the accuracy and flexibility of this method make it a useful benchmark for policymakers that are pursuing large-scale policies to raise fertility.

4 Further Implications

Beyond informing policymakers how costly it is to raise fertility, the bound disciplines models with endogenous fertility. It also allows one to evaluate the exogenous fertility assumption in models that analyze child-related policies. Below are some examples.

4.1 Endogenous Fertility w/ Dynastic Altruism

Consider a model of fertility choice with dynastic altruism following Barro and Becker (1989). Agents solve

$$U_0 = \max_{c_t, n_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\prod_{i=0}^{t-1} n_i \right)^{1-\varepsilon} \cdot \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$w_t + (1+r_t)k_t = c_t + n_t(\chi_t + k_{t+1}) \quad \forall t$$

and the initial k_0. It is assumed that $\beta, \varepsilon, \sigma \in (0,1)$ to ensure that children are goods and $\varepsilon \leq \sigma$ for the second-order condition to hold.[5]

Using first-order conditions, the Marshallian demand of fertility in this economy is given by

$$n_t = (\beta(1+r_{t+1}))^{\frac{1}{\varepsilon}} \left[\frac{\chi_{t-1}(1+r_t) - w_t}{\chi_t(1+r_{t+1}) - w_{t+1}} \right]^{\frac{\sigma}{\varepsilon}} \tag{3}$$

where the net cost of creating a descendant at time t is $p_t \equiv \chi_t(1+r_{t+1}) - w_{t+1}$. Therefore, as the net cost of children p_t falls by 1 percent, fertility n_t increases by $\frac{\sigma}{\varepsilon}$ percent *ceteris paribus* in this model. Relating this elasticity to the numerical bound for the U.S. in 2010 computed in the previous section, the value of $\frac{\sigma}{\varepsilon}$ should lie between 0.67 and 3.23. Combining with the prior assumptions on σ and ε, the full set of restrictions are

$$0 < \varepsilon \leq \sigma < 1 \quad \text{and} \quad \sigma < 3.23 \cdot \varepsilon. \tag{4}$$

As can be seen, the bound imposes *additional restrictions* on the choice of σ and ε. For example, Manuelli and Seshadri (2009) satisfy these restrictions with $\sigma = 0.62$ and $\varepsilon = 0.35$; Córdoba (2015) considers $\sigma = 0.3$ and $\varepsilon = 0.288$; and Daruich and Kozlowski (2020) use $\sigma = 0.5$ and $\varepsilon = 0.25$. Under different choices of $\overline{n}$ or $\overline{p}$, the restrictions from the bound could become more binding for this class of models.

A point worth noting here is that the model presented above generates a Marshallian demand that satisfies Assumption 1 in Section 2 but not Assumption 2. In light of Assumption 2, the isoelastic demand in Equation (3) can be interpreted as an approximation of the true underlying fertility demand around (n^0, p^0). The existence of $\overline{p}$ and $\overline{n}$, and hence the bound, provides restrictions

[5] Jones and Schoonbroodt (2010) discuss an alternative set of assumptions which implies that the quantity and quality are substitutes rather than complements as in standard Barro-Becker models.

on the *local properties* of $\frac{\sigma}{\varepsilon}$. The same interpretation applies to the endogenous fertility model with warm glow utilities presented below.

4.2 Endogenous Fertility w/ Warm Glow Utilities

Consider a model of fertility choice with warm glow utilities. Agents solve

$$\max_{c,n} \quad c + \beta \cdot \frac{n^{1-\sigma}}{1-\sigma}$$

subject to

$$c + n \cdot p \leq y.$$

As before, it is assumed that $\sigma \in (0, 1)$ to ensure that fertility delivers positive utility.

With interior solutions, the Marshallian demand of fertility is given by

$$n^* = \left(\frac{p}{\beta}\right)^{-1/\sigma}. \tag{5}$$

This implies that when p falls by 1 percent, n^* rises by $1/\sigma$ percent. Thus, if one calibrated this model to the U.S. economy in 2010, the bound suggests that the value of σ, which is often chosen exogenously, should lie between 0.31 and 1. Relative to the standard assumption on σ, the bound further requires that $\sigma > 0.31$.

In general, one can use the bound to validate comparative static results in other, potentially more complicated, models of endogenous fertility before using the model to conduct counterfactuals. For instance, Zhou (2022) shows that in a heterogeneous-agent model of quantity-quality trade-off calibrated to the U.S. economy in 2010, raising the aggregate fertility rate by 0.1 children per woman requires the cost of children to fall by $15,000 - a number that is within the bound.

4.3 Models w/ Exogenous Fertility Assumption

The bound also permits a transparent evaluation of the exogenous fertility assumption in structural models that analyze child-related policies.

For example, Guner et al. (2020) consider a policy counterfactual where the per-child tax credit rises about $800 per child per year, amounting to about $12,000 in net present value terms for eligible households.[6] The optimal policy in Mullins (2019) is a Negative Income Tax on mothers that is equivalent to an additional $82 per week over 17 years, i.e., a transfer greater than $50,000 in net present value. Daruich (2022) shows that the welfare-maximizing early-childhood development

[6]The net present values are calculated using a 3% annual discount rate.

subsidy is around \$80,000. A real-world policy example is the expansion of the Child Tax Credit in the American Rescue Plan which increases the annual transfer from \$2,000 dollars to \$3,600 per child under age 6 and \$3,000 per child ages 6 through 17. The net present value of this expansion is above \$30,000 per child for fully eligible families.

In these models, the exogenous fertility assumption is typically justified based on design-based studies that find little fertility responses to financial incentives. With prior calculations, however, it can be seen a \$10,000 reduction in the cost of children should increase fertility by 0.03 to 0.13 from the baseline level of 1.9 children per woman. Thus, the proposed policies are likely going to lead to non-negligible fertility responses and a dilution of family resources, triggering the quantity-quality trade-off mechanism à la Becker and Lewis (1973). This channel could be strong enough to overturn the direct effects of the transfer on children's outcomes. As a result, the implications of these child-related policies on children's human capital, intergenerational mobility, and social welfare could be affected and deserve further inspection.

5 Conclusion

Fertility elasticity is important to economists and policymakers, yet pinning it down using historical policies has been proven difficult. In this paper, I propose a technique for bounding fertility elasticities. Under mild assumptions, I show that a bound can be derived for any country and year with minimal information required. For example, in the United States in 2010, a transfer of size between \$7,514 and \$35,966 is expected to raise fertility by 0.1 children per woman. This range is tighter than the conclusion from past meta-analyses and is simple to compute for different country and year combinations. With its flexibility and accuracy, this method is complementary to design-based inquiries and has practical implications for policymakers that want to pursue large-scale family policies. For economists working with endogenous fertility models, the bound imposes additional restrictions on parameters. Additionally, it provides a transparent evaluation of the exogenous fertility assumption that is widely used in structural models studying child-related policies.

There are several interesting avenues for future research. First, one can make additional assumptions and derive the bound for different subgroups of the population. This can guide the policy targeting to achieve maximum cost-effectiveness. Second, one can use survey questions to inform the choices of $\overline{p}$ and $\overline{n}$. Lastly, one can compute the bound for different countries and years and compare them with design-based estimates in a systematic way.

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