Teacher Quality and the Dynamics of Inequality: An Analytical Framework *

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Abstract

How does teacher quality affect the dynamics of inequality? In a model with occupation choice and child investments, I propose a feedback mechanism between teacher quality and human capital dispersion where (1) heterogeneities in human capital amplify the selection effects on teacher quality, and (2) teacher quality affects the degree of heterogeneity itself through endogenous human capital formation. I use evidence on how duty-to-bargain laws affect teacher labor market and children's long-run outcomes to show that this dynamic feedback mechanism is quantitatively important. In counterfactuals, I find that lowering the returns to human capital among teachers has large spillover effects, raising income inequality among non-teachers and reducing intergenerational mobility.

JEL classification: I24, J24, J31, J45
Keywords: teacher quality, endogenous human capital, inequality

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1 Introduction

Teacher quality is one of the most important determinants of student achievement and path to upward mobility (Rivkin et al. (2005), Chetty et al. (2014b)). Education policies that attract the best talent into the teaching profession have profound implications on future generations (Hanushek (2011)). The scope of such policies, however, depends on the dispersion of talent in the population. After all, selection is redundant in an economy where everyone is the same. This paper argues that in an environment with human capital formation so that the degree of heterogeneity is endogenous, changes in teacher quality have large dynamic effects and lead to a rich set of policy trade-offs across occupations and over time.

I develop a model à la Benabou (2002) and consider occupation selection with teaching and non-teaching professions. The model uncovers a new mechanism that amplifies static changes in teacher quality. Consider a case where decreasing returns to human capital in the teaching profession pushes the most talented individuals into non-teaching occupations and thus reduces teacher quality. Deteriorating teacher quality lowers human capital in the next generation, with the effects being disproportionately larger among low-income families when teacher quality and parental investments are substitutes.[1] As a result, the economy has lower intergenerational mobility and a greater dispersion of human capital to start with in the next generation. Importantly, the increased human capital dispersion reinforces the teacher selection channel and leads to further reductions in teacher quality - a vicious cycle.

To quantify this mechanism, I show that the model provides analytical characterizations of optimal individual decisions and the transition path of the equilibrium human capital distribution. I also prove that the key parameters, in particular the human capital production function, are identified using cross-sectional moments and estimates on how teacher pay rigidities affects teacher labor market and children's long-run outcomes. I utilize variations in duty-to-bargain laws across states and over time to measure the effects of teacher pay compression on teacher labor markets. I also borrow the corresponding estimates on children's long-run outcomes from Lovenheim and Willén (2019).

In counterfactual analysis, I show that even though teachers account for less than 5% of the workforce, they have disproportionate impacts on the dynamics of inequality due to their vital roles in human capital formation. Moreover, the feedback mechanism proposed in this paper results in large amplifications along the transition path.

In particular, I first reduce the returns to human capital by 5% in the teaching profession to mimic the initial impacts that duty-to-bargain laws have on the income dispersion among

[1] See Lee and Seshadri (2019) and Yum (2023). I will also verify in this condition in the calibration results using empirical evidence from duty-to-bargain laws.

teachers found in the empirical section. I find that only one-fifth of the initial impacts on teachers' income inequality, measured in the coefficient of variations, survive in the long-run steady state. Income inequality among non-teachers, however, increases drastically by 4.1%, reflecting changes in the human capital dispersion. The degree of intergenerational mobility, measured in the reciprocal of the intergenerational elasticity of earnings, fall by 20%. In other words, changing returns to human capital in the teaching profession pushes the economy downward along the Great Gatsby curve. I also show that the magnitude of these effects grow along the transition path. Therefore, static or one-generation analyses overstate the effects of teacher pay compression on income inequality within teachers, and understate the spillover effects on intergenerational mobility and the income inequality among non-teachers.

I also find that endogenous teacher quality and human capital distribution greatly amplify the effects that the "convexification" of non-teaching occupations (Autor et al. (2020)) have on income inequalities. Absent changes in teacher quality, the increase in income inequality among non-teachers would be 50% smaller in the long run. A model-based decomposition also shows that to neutralize the effects of the SBTC, the returns to human capital in the teaching profession needs to rise *relatively more* than that in non-teaching occupations. Taken together, these findings shed new light on the spillover effects that teacher labor market and collective bargaining have in the aggregate economy.

Related Literature

This paper builds on the literature that investigates the determinants of inequality and intergenerational mobility. The literature has traditionally focused on factors that affect the demand for education, such as credit constraints (Lee and Seshadri (2019), Caucutt and Lochner (2020)), neighborhood effects (Chetty et al. (2014b), Durlauf and Seshadri (2018), Fogli and Guerrieri (2019)), and information frictions (Hoxby and Turner (2015)). This paper contributes to the literature by studying the supply side of education and show that teacher market reforms could be powerful instruments moving the economy along the Great Gatsby curve toward lower inequalities and greater intergenerational mobility.

This paper is also related to the large body of literature that studies teacher quality and its implications on students. The most related papers are Bacolod (2007), Rothstein (2015), Lovenheim and Willén (2019), Tincani (2021), and Biasi et al. (2021). The contribution of this paper is to introduce the endogenous human capital formation and hence uncover an important amplification mechanism to static changes in teacher quality. Complementary to the design-based approach in evaluating how the structure of teacher market, such as collective bargaining, affects children's outcomes, the structural model also allows me to analyze trade-off across occupations and generations. Such evaluations uncover the important spillover effects that teacher labor market policies have on the economy, and provide new angles for a more comprehensive cost-benefit analysis.

2 Model

I study an overlapping-generations economy populated by agents that live for two periods - children and adults. Children do not make any decisions. Their human capital is formed through a production function that takes teacher quality and parental investments as inputs. Adults with different levels of human capital supply labor inelastically and select into two occupations: teachers and workers (non-teachers). After occupation selection, adult parents choose child investments that maximizes their utility from consumption and preferences on their children's human capital.

2.1 Occupation Selection

In the beginning of each period, adults with human capital h make occupation choice after observing idiosyncratic preference shock ν that follows a Gumbel distribution with scale parameter θ. They solve the following occupation choice problem:

$$\max_{j\in\{1,2\}} \mathbb{1}_{j=1}(\underbrace{\log(wh^{\psi_1}\cdot\kappa)+\nu}_{\text{teachers}}) + \mathbb{1}_{j=2}\underbrace{\log(h^{\psi_2})}_{\text{workers}}$$

where w is the relative wage across occupations, κ captures the systematic cost or taste for becoming teachers, and ψ_j is the occupation-specific returns to human capital.

Assume that the underlying human capital distribution of adults $F(h)$ follows a log-normal distribution

$$\log(h) \sim F(h) = \mathcal{N}(\mu, \sigma^2)$$

The log-normality of the human capital distribution is a condition that I will verify in the equilibrium. Because human capital is endogenously formed, the values of μ and σ^2 are both equilibrium objects in the steady state.

Let $\overline{h}_1$ denote the average human capital of teachers. I define teacher quality ξ as the z-score of $\log(\overline{h}_1)$ in the population, i.e.,

$$\xi \equiv \frac{\log(\overline{h}_1)-\mu}{\sigma} \tag{1}$$

I further discuss this assumption in Section 2.3.

2.2 Human Capital Formation

After the occupation selection stage, adult workers have children[2] and solve the following maximization problem

$$\max_{c,e\geq 0} \ \log(c) + \beta \log(\mathbb{E}_\epsilon h')$$

subject to budget constraint

$$c + e = y = h^{\psi_2}$$

and human capital production function

$$\log(h') = A + \underbrace{\log(\epsilon)}_{\text{normal dist.}} + \underbrace{\delta_0 \xi + \delta_1 \log(e) + \delta_2 \log(e)\xi}_{\text{translog}} + \underbrace{\rho \log(h)}_{\text{residual persistence}} \tag{2}$$

where c is consumption, e is education investments, h' is children's human capital, and ϵ is an idiosyncratic shock that follows log-normal distribution

$$\log(\epsilon) \sim \mathcal{N}(-\sigma_\epsilon^2/2, \sigma_\epsilon^2)$$

so that $\mathbb{E}(\epsilon) = 1$.

I assume that teacher quality ξ and parental investments e interact in the human capital production function (2) in a translog fashion.[3] Parameter δ_2 governs whether they are gross complements or substitutes. I also allow for a direct spillover effect of parents' human capital to the next generation governed by ρ.

2.3 Discussions

There are several points worth noting here regarding the modeling choices on occupation selection and human capital formation.

First, I have assumed that individuals differ by a one-dimensional human capital h and an idiosyncratic taste shock ν. Human capital here summarizes a broad set of personal traits that affect productivities, including but not limited to cognitive ability, non-cognitive ability, experience, and education. An alternative modeling choice is to use a multi-dimensional human capital model à la Roy (1952) where individuals have comparative advantage in one of the occupations (Borjas (2002), Bacolod (2007)). With strongly positive correlation

[2] I assume that only workers have children for analytical tractability of the model. I do not think that including teachers' children in this stage will change the results significantly because teachers account for less than 5% of the adult population.

[3] One can also view the translog production function as the Kmenta (1967) approximation to the more commonly-used CES production function.

between different abilities, the Roy-style model offers the same intuition on the selection effects. Such a model, however, requires us to model separate production functions for each dimension of human capital and the corresponding mappings to productivities across occupations. In our analysis, I restrict to the one-dimensional case for data availability reasons, but I acknowledge that the quantitative results will be smaller if *relative to* overall earnings, skills that are more important in the teaching profession (e.g., communication and interpersonal relation skills) are less affected by changes in human capital production inputs.

To investigate this concern further, I collect data on the importance of five cross-functional skills by occupation from O*NET, including complex problem solving skills, resource management skills, social skills, system skills, and technical skills. I find that for each skill, the value of teachers are all within one standard deviation of the values across other non-teaching occupations. Thus, I do not expect including multi-dimensional human capital will change the results significantly.

Second, in our definition teacher quality ξ reflects the *relative* position of teachers' human capital in the population. In other words, parallel shifts in the population-wide human capital distribution, i.e., changes in μ, does not affect teacher quality ξ. I use this definition to emphasize the selection channel and its amplification mechanisms. Our results will be stronger if teacher quality also depends on the absolute level of teachers' human capital because there will be an additional feedback loop where teacher quality feeds back on average human capital in the population and so on.

Third, I have adopted a rather stylized model of occupation selection. To focus on the propagation mechanism through endogenous human capital formation, I have abstracted away from some important margins in the context of teacher selection and quality such as the role of risk preferences (Cortes and Pan (2018)), dynamic learning (Rothstein (2015)), the allocation of teachers across schools or districts (Biasi et al. (2021)), and teacher efforts (Bold et al. (2017)). I view these possibilities as intriguing avenues for future research.

Last, as teachers are heterogeneous, the human capital production function (2) should be interpreted as a reduced-form way of modeling how parental inputs and teacher quality jointly determine children's outcome without explicitly micro-founding the assignment problem (Sattinger (1975), Seshadri (2000)). Beyond purchasing books and computers, the role of $\delta_1 \log(e)$ also reflects parents' efforts in matching their own children to better teachers. Likewise, the interaction term $\delta_2 \log(e)\xi$ captures the idea that need to compete for better teachers varies as teacher quality changes.

2.4 Model Solution

The share of adults that becomes teachers by human capital h is given by

$$l(h) = \frac{(w\kappa h^{\psi_1-\psi_2})^{\theta}}{1+(w\kappa h^{\psi_1-\psi_2})^{\theta}} \approx (w\kappa h^{\psi_1-\psi_2})^{\theta}$$

Integrating $l(h)$ across the human capital distribution $F(h)$, the aggregate share of teachers π is

$$\pi = \int l(h)\, dF(h) = (w\kappa)^{\theta} \cdot \exp\left(\mu(\psi_1-\psi_2)\theta + \frac{((\psi_1-\psi_2)\sigma\theta)^2}{2}\right)$$

Using the definition in Equation (1), teacher quality ξ can be solved as

$$\xi = (\psi_1 - \psi_2) \cdot \sigma\theta \tag{3}$$

Equation (3) is important as it shows that teacher quality depends on three objects. First, the relative skill bias across sectors $\psi_1 - \psi_2$ represents the selection mechanism. If the relative return to human capital between teaching and non-teaching occupation rises, teacher quality ξ improves. Second, teacher quality is proportional to the dispersion of human capital among potential teachers σ. This captures the idea that selection is powerful when agents become more heterogeneous. Last, teacher quality also depends on θ which reflects the elasticity that individuals move across occupations in response to changes in valuations. The novel part of the model is to recognize that in a dynamic environment, σ is an endogenous object. In light of this, Equation (3) also provides a decomposition formula that will be used in later analysis:

$$\underbrace{d\log(\xi)}_{\text{change in teacher quality}} = \underbrace{d\log(\psi_1-\psi_2)}_{\text{change in selection}} + \underbrace{d\log(\sigma)}_{\text{change in h.c. dispersion}} \tag{4}$$

Now I turn to optimal individual decisions. For interior solutions of e, the optimal investment to income ratio is

$$\frac{e}{y} = \frac{\beta(\delta_1+\delta_2\xi)}{1+\beta(\delta_1+\delta_2\xi)} \approx \beta(\delta_1+\delta_2\xi)$$

Substitute e back into the human capital production function, we have

$$\log(h') = A + \log(\epsilon) + f(\xi;\vec{\delta}) + (\rho + \psi_2(\delta_1+\delta_2\xi))\log(h) \tag{5}$$

where $\vec{\delta} = \{\delta_0, \delta_1, \delta_2\}$ and

$$f(\xi; \vec{\delta}) = \delta_0 \xi + (\delta_1 + \delta_2 \xi) \cdot \log(\beta(\delta_1 + \delta_2 \xi))$$

The dynamics of Equation (5) indicate that when $\log(h)$ is normal, $\log(h')$ stays normal. The transition path of human capital distribution is therefore analytically characterized:

$$\begin{cases} \mu' = A + f(\xi; \vec{\delta}) + (\rho + \psi_2(\delta_1 + \delta_2 \xi)) \cdot \mu - \sigma_\epsilon^2/2 \\ (\sigma')^2 = (\rho + \psi_2(\delta_1 + \delta_2 \xi))^2 \cdot \sigma^2 + \sigma_\epsilon^2 \\ \xi = (\psi_1 - \psi_2) \cdot \sigma\theta \end{cases} \tag{6}$$

The system in (6) has a close relationship to Benabou (2002) which exploits the properties of the normal distribution to study the relationship between tax and education policies. The key difference here is the third equation in (6) where teacher quality ξ is an endogenous object and enters the evolution of human capital. As a result, changing ψ_1 or ψ_2 has dynamic implications through ξ and endogenous human capital distribution.

Equation (6) also highlights the key role that δ_2 plays in determining whether teacher quality is amplified or dampened when the dispersion of human capital σ is endogenous. Consider a small decrease of ξ and focus on how σ' changes. If $\delta_2 < 0$, then the intergenerational persistence $(\rho + \psi_2(\delta_1 + \delta_2 \xi))$ is higher and we have higher σ'. This feeds back to the determination of teacher quality in the next period and leads to further declines in ξ'. On the other hand, σ' will be smaller when teacher quality decreases when $\delta_2 > 0$. Therefore, relating back to the human capital production function in Equation (2), endogenous human capital amplifies (dampens) changes in selection if and only if teacher quality is gross substitute (complement) to parental investments.

3 Identification and Calibration

In this section, I carry out the calibration of the model by proving identification and finding empirical counterparts to the model's predictions.

3.1 Model Identification

The objects that need to be identified include

$$\underbrace{\delta_0, \delta_1, \delta_2, A, \rho, \sigma_\epsilon^2}_{\text{h.c. technologies}}, \underbrace{\kappa, \theta, \beta}_{\text{preferences}}, \underbrace{\psi_1, \psi_2}_{\text{labor market}}, \underbrace{\xi, w, \mu, \sigma^2}_{\text{endogenous objects}}.$$

The steady-state of the model can be summarized in the following equations:

$$\frac{\mathbb{E}(y_1)}{\mathbb{E}(y_2)} = w \cdot \exp\left(\mu(\psi_1 - \psi_2) + \frac{\sigma^2}{2}(\psi_1 - \psi_2)(\psi_1 + \psi_2 + 2\psi_1\theta)\right) \tag{7}$$

$$\mathrm{CV}(y_1) = \sigma\psi_1 \tag{8}$$

$$\mathrm{CV}(y_2) = \sigma\psi_2 \tag{9}$$

$$\pi = (w\kappa)^\theta \cdot \exp\left(\frac{\mu\xi}{\sigma} + \frac{\xi^2}{2}\right) \tag{10}$$

$$\xi = (\psi_1 - \psi_2) \cdot \theta\sigma \tag{11}$$

$$\mathrm{IGE} = \rho + \psi_2(\delta_1 + \delta_2\xi) \tag{12}$$

$$\sigma^2 = \frac{\sigma_\epsilon^2}{1 - \mathrm{IGE}^2} \tag{13}$$

$$\mu = \frac{A + \delta_0\xi + (\delta_1 + \delta_2\xi) \cdot \log\left(\frac{e}{y}\right) - \sigma_\epsilon^2/2}{1 - \mathrm{IGE}} \tag{14}$$

$$\frac{e}{y} = \beta(\delta_1 + \delta_2\xi) \tag{15}$$

where IGE denotes intergenerational elasticity of earnings when both parent and children work as non-teachers.

We can measure $\mathbb{E}(y_1), \mathbb{E}(y_2), \mathrm{CV}(y_1), \mathrm{CV}(y_2), \pi, \mathrm{IGE}, e/y$ in the data - more details below. Because the unit of human capital is undetermined, we have the freedom to make one normalization. After normalizing $\psi_2 = 1$, we need more information because there are 14 unknowns with 9 equations.[4] The additional information comes from the model's comparative statics regarding a small change to teacher market rigidity ψ_1:

$$\frac{\partial\overline{\log(y')}}{\partial\psi_1} = \psi_2 \cdot \underbrace{\sigma\theta}_{\frac{\partial\xi}{\partial\psi_1}} \left[\underbrace{\delta_0 + \delta_2\log(e/y) + \psi_2\delta_2\mu}_{\text{direct}} + \underbrace{\beta\delta_2 \cdot \frac{\delta_1 + \delta_2\xi}{e/y}}_{\text{indirect}}\right] \tag{16}$$

$$\frac{\partial^2 \log y'}{\partial\psi_1\partial\log y} = \sigma\theta \cdot \psi_2 \cdot \delta_2 \tag{17}$$

$$\frac{\partial\mathbb{E}(y_1)}{\partial\psi_1} = \mathbb{E}(y_1) \cdot \big(\mu + \sigma^2(\psi_1 + \theta(2\psi_1 - \psi_2))\big) \tag{18}$$

$$\frac{\partial\pi}{\partial\psi_1} = \pi \cdot (\theta\mu + \sigma\theta\xi) \tag{19}$$

[4]Normalizing $\psi_2 = 1$ is without loss of generality. It is equivalent to normalizing ψ_1, σ_ϵ, or A.

where y' denotes the average earnings of the next generation.

Equation (16) and (17) quantify how changes in ψ_1 affect income in the next generation. In (16), the effect on next generation's average income combines a direct effect coming from changes in teacher quality ξ and an indirect effect capturing parents' responses through endogenous investments e. Equation (17) highlights that the key parameter δ_2 is identified from the differential impacts across parents with different income. On the other hand, Equations (18) and (19) quantifies how changes in ψ_1 affect teacher employment and outcome.

With these additional information, the identification argument is presented next.

Proposition 1: The model is identified up to the calibration of ρ if we observe $\mathbb{E}(y_1)$, $\mathbb{E}(y_2)$, $\mathrm{CV}(y_1)$, $\mathrm{CV}(y_2)$, π, IGE, e/y and measure the left-hand-sides of Equations (16)-(19).

Proof. Given that $\psi_2 = 1$, Equation (9) identifies σ; then Equation (8) identifies ψ_1; Equation (13) identifies σ_ϵ. Combining Equations (18) and (19) by substituting out μ identifies θ. Then, Equation (11) identifies ξ; Equation (19) identifies μ; Equation (7) identifies w; Equation (10) identifies κ; Equation (17) identifies δ_2; Equation (12) identifies δ_1 given that I calibrated ρ; Equation (15) identifies β; and lastly Equation (14) identifies A. $\square$

3.2 Empirical Evidence

I measure the left-hand-sides of Equations (16)-(19) using the variations coming from the passage of duty-to-bargain (DTB) laws across states between 1960 and 1996.

To give some institutional background, I quote the excellent summary from Lovenheim and Willén (2019). "Prior to 1960, teachers' unions in the United States were predominantly professional organizations that had little role in the negotiation of contracts between teachers and school districts. Collective bargaining occurred in only a handful of large, urban school districts. Beginning with Wisconsin in 1960, states began passing pubic sector duty-to-bargain (DTB) laws, which mandated that districts have to negotiate in good faith with a union that has been elected for the purposes of collective bargaining. These laws gave considerable power to teachers' unions in the collective bargaining process. As a result, duty-to-bargain laws led to a sharp rise in teacher unionization and in the prevalence of collectively bargained contracts (Lovenheim (2009))."

Regarding the (differential) effects of DTB laws on the next generation's human capital and income, Lovenheim and Willén (2019) run the regression

$$Y_i = \alpha_1 \cdot \text{DTB exposure}_i + \zeta X_i + \varepsilon \tag{20}$$

using individual-level data from the American Community Survey (ACS). They find that a

10-year exposure to teachers under DTB laws reduces annual earnings by 2.36% (averaging men and women). Moreover, compared with Black and Hispanic children, the negative effects are 4.9 percentage points smaller among White and Asian children, whose parents had 60% higher income after adjusting for the number of children (calculated using data from the Current Population Survey (CPS) March Supplement).

Complementing their findings on children's earnings, I run additional regressions to measure Equations (18) and (19). In particular, I collect data on full-time workers from the CPS-ASEC and examine the effects of duty-to-bargain laws on employment share and teacher earnings statistics

$$Y_{\text{state,year}} = \alpha_2 \cdot \text{DTB}_{\text{state,year}} + \text{State FE} + \text{Year FE} + \varepsilon \tag{21}$$

where observations are at state-year level. The regression results are shown in Table 1.

Table 1. Regression Results

	(1) Teacher share	(2) CV(teacher)	(3) Average teacher earnings
DTB	-0.351**	-0.0292*	-591.3
	(-2.96)	(-2.11)	(-1.39)
# Observations	1378	1364	1378

Notes: This table displays the results of regression (21). t statistics in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. Average teacher earnings is measured in year 2000 dollars.

I find that DTB laws have negative but statistically insignificant effects on average earnings of teachers $\mathbb{E}(y_1)$ while reducing the coefficient of variations of teacher earnings by 0.03 from a base of 0.56. In addition, DTB laws reduce teacher employment share by 0.35 percentage points from a base of 4.6%. These results are consistent with the interpretation that these laws mainly affect teacher market by changing rigidity parameter ψ_1. One potential worry is that variations in employment and wages could also reflect other effects of DTB laws, such as changes in relative wages w or entry barriers κ. I provide a detailed discussion about how this possibility affects the identification and our results in Section 5.

To utilize these estimates, I revisit Equations (16)-(19) and rewrite them as:

$$\frac{\Delta \overline{\log(y')}}{\Delta \psi_1} = \text{CV}(y_2) \cdot \theta \left[\delta_0 + \delta_2 \log\left(e/y\right) + \psi_2 \delta_2 \mu + \beta \delta_2 \cdot \frac{\delta_1 + \delta_2 \xi}{e/y} \right] \tag{16'}$$

$$\frac{\Delta^2 \log y'}{\Delta \psi_1 \Delta \log y} = \theta \delta_2 \cdot \text{CV}(y_2) \tag{17'}$$

$$\frac{\Delta \log(y_1)}{\Delta\psi_1} = \mu + \sigma^2(\psi_1 + \theta(2\psi_1 - \psi_2)) \tag{18'}$$

$$\frac{\Delta\pi}{\pi\Delta\psi_1} = \theta\mu + \sigma\theta\xi \tag{19'}$$

The regression results in Table 1, together with the estimates in Lovenheim and Willén (2019), inform the left-hand-sides of these equations. In addition, the magnitude of changes in ψ_1 can be measured using changes in the coefficient of variations in teachers' earnings:

$$\Delta\psi_1 = \psi_1 \cdot \frac{\Delta \mathrm{CV}(y_1)}{\mathrm{CV}(y_1)} \tag{22}$$

3.3 Calibration Results

I calibrate $\rho = 0.6 \times \mathrm{IGE}$ following the results in Lefgren et al. (2012). It represents a "measure of ignorance" in our understandings of the determinants of intergenerational mobility. Then, I implement the calibration procedures following the proof of Proposition 1.

Other target moments, including $\mathbb{E}(y_1)$, $\mathbb{E}(y_2)$, $\mathrm{CV}(y_1)$, $\mathrm{CV}(y_2)$, π, IGE, and e/y, are derived from the Current Population Survey (CPS), the Panel Study of Income Dynamics (PSID), and the past literature on parental investments in children's education (Lee and Seshadri (2019), Daruich (2018)).

Table 2. Calibration Results

Human capital formation parameters		Value	Preference parameters		Value
δ_0	teacher effect (level)	-0.48	κ	teacher cost	0.21
δ_1	investment effect	-0.42	θ	taste shock dispersion	3.07
δ_2	teacher effect (gradient)	-0.88	β	weight on children's h.c.	0.35
A	TFP of h.c. production	1.12			
ρ	residual persistence	0.22			
σ_ϵ	ability shock dispersion	0.72	**Equilibrium objects**		**Value**
			ξ	teacher quality	-0.64
Labor market parameters		**Value**	w	relative wage	2.18
ψ_1	skill bias (teachers)	0.73	μ	average h.c.	1.15
ψ_2	skill bias (non-teachers)	1	σ	h.c. dispersion	0.77

Notes: This table displays the calibration results. See text for the identification and target moments.

The parameters of particular interest are those in the human capital production function. Combining the results on $\{\delta_0, \delta_1, \delta_2\}$ with the observed level of e/y, I find that a one standard deviation change of teacher quality throughout ones' education leads to a rise in human capital by 1.6 percent. The implied result from the structural model here is slightly larger but broadly consistent with Chetty et al. (2014a), who conclude that "a one standard deviation increase in teacher quality in a single grade raises annual earnings by 1.3 percent." The result that $\delta_2 < 0$ is of particular importance because as discussed before, it implies that the endogenous human capital formation serves as an amplifying mechanism of the teacher selection channel. The result that $\delta_2 < 0$ is also consistent with existing estimates on the substitutability of public and private investments (Kotera and Seshadri (2017), Yum (2023)) and the idea that public schooling serves as the "great equalizer" in the economy (Agostinelli et al. (2022)).

4 Counterfactual Analyses

In this section, I analyze two counterfactual to better understand the role of teacher quality and human capital formation in shaping the dynamics of inequality and intergenerational mobility.

4.1 Teacher Wage Rigidities

In the first counterfactual, I permanently reduce the returns to human capital by 5% in the teaching profession to mimic the initial impacts that duty-to-bargain laws have on the income dispersion among teachers in the empirical findings. Figures 1 to 4 shows the transition path of CV(teacher), CV(non-teacher), teacher quality ξ, and IGE from the original economy to the new steady state.

I find that in the long-run steady-state, only one-fifth of the initial impacts on CV(teacher) survive. In contrast, CV(non-teacher) increases drastically by 4.1%, reflecting changes in the dispersion of human capital σ. Utilizing the decomposition formula in Equation (4), I find that changes in σ also account for 24% of the overall reduction in teacher quality ξ (z-score of $\log(h_1)$). Intergenerational elasticity of earnings increase from 0.36 to 0.46, reflecting the fact that reduced teacher quality raises the role of parental investments and hence intergenerational persistence. These results also indicate that the magnitude of these effects vary the transition path. Therefore, empirical analyses that focus on short-run effects, i.e., $t = 0$ or $t = 1$, could overstate the effects of teacher pay compression on CV(teacher) and understate the dynamic effects on teacher quality ξ, non-teacher inequality CV(non-teacher), and intergenerational mobility IGE.

4.2 Skill-Biased Technical Change

In the second counterfactual, instead of changing ψ_1, I let ψ_2 rise by 1% permanently. Such changes mimic the skill-biased technical change (SBTC), i.e., the "convexification" of the

Figure 1. CV(teacher) Transition

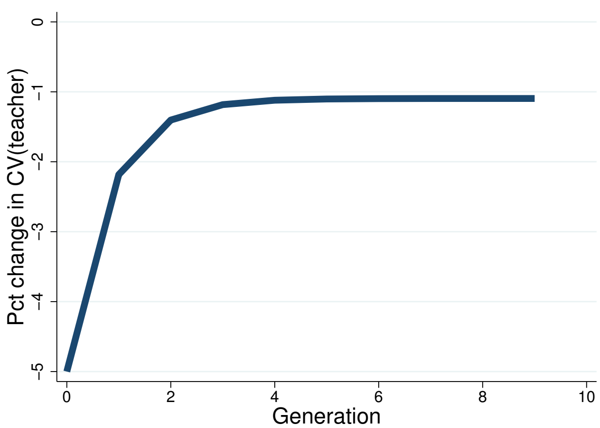


Figure 2. CV(non-teacher) Transition

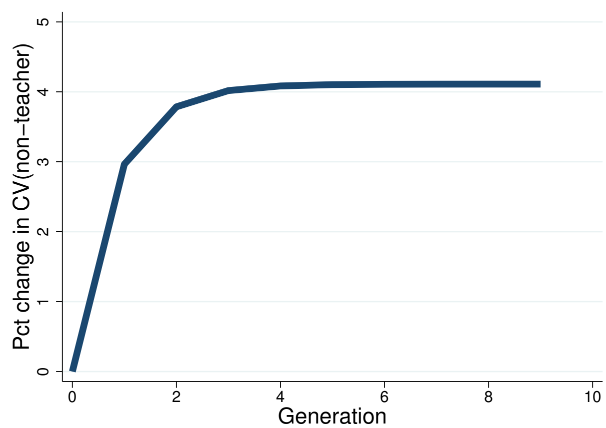


Figure 3. Teacher Quality Transition

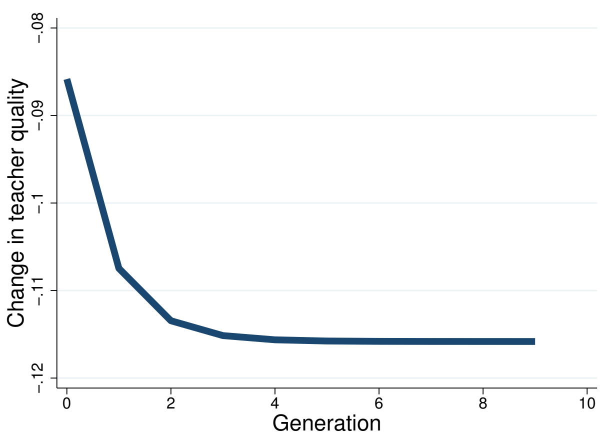
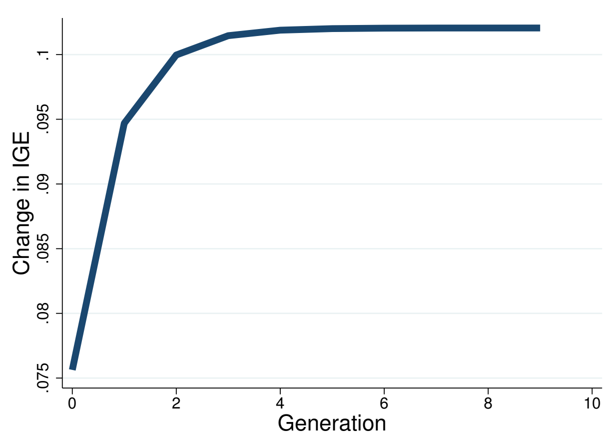


Figure 4. IGE Transition



Notes: These figures plot the transition path of CV(teacher), CV(non-teacher), teacher quality ξ, and IGE following a 5% permanent reduction of ψ_1 in the steady-state economy.

non-teaching labor market (Autor et al. (2020)). Figures 5 to 8 shows the transition path of CV(teacher), CV(non-teacher), teacher quality ξ, and IGE from the original economy to the new steady state.

I find that when ψ_2 increases, it not only raises inequalities within non-teachers but also those within teachers, reflecting changes in σ. Higher returns to human capital in non-teaching occupations attract the most talented workers and reduce teacher quality, with the effects being much larger in the long run (see Figure 5). Interestingly, intergenerational elasticity of earnings increases for two reasons. The first reason is the same as that in the previous counterfactual - reduced teacher quality raises the role of parental investments and hence intergenerational persistence. The second reason is that the dispersion of parental income is also greater due to higher ψ_2, magnifying disparities in parental investments.

Figure 5. CV(teacher) Transition

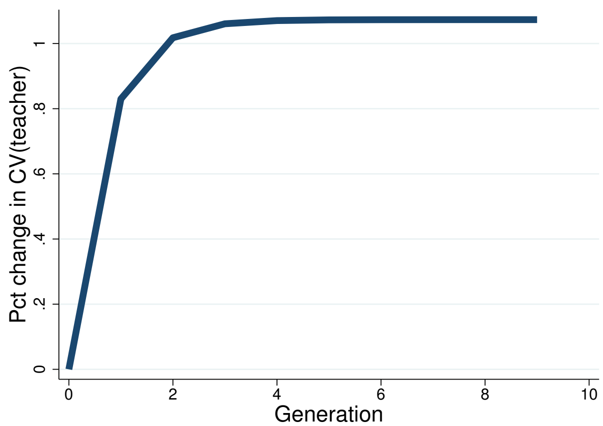


Figure 6. CV(non-teacher) Transition

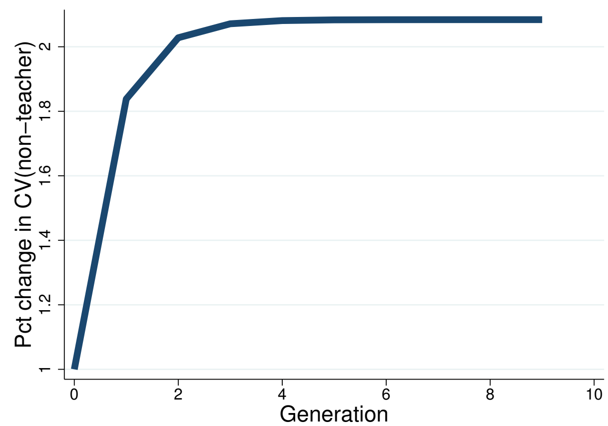


Figure 7. Teacher Quality Transition

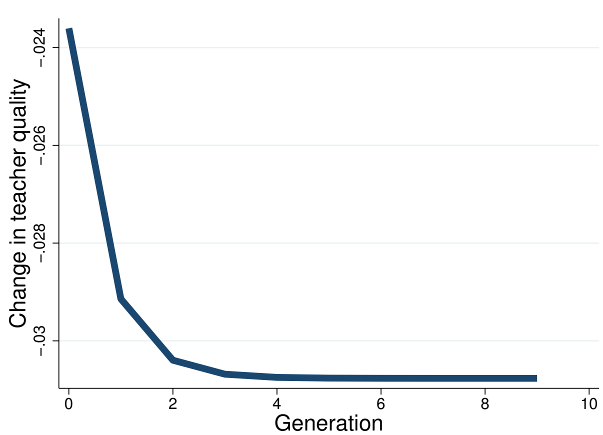
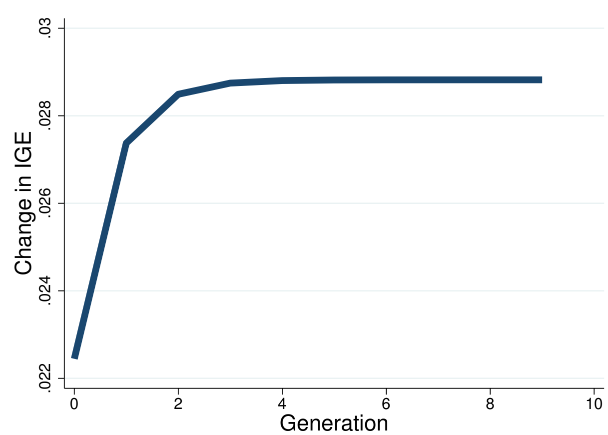


Figure 8. IGE Transition



Notes: These figures plot the transition path of CV(teacher), CV(non-teacher), teacher quality ξ, and IGE following a 1% permanent increase in ψ_2 in the steady-state economy.

To better understand the relative importance of these different channel, I conduct a model-based decomposition of $\mathrm{CV}(y_2)$ in the following steps:

1. First, because $\mathrm{CV}(y_2) \equiv \sigma\psi_2$, I decompose $\mathrm{CV}(y_2)$:

$$\underbrace{\mathrm{CV}(y_2)' - \mathrm{CV}(y_2)}_{\text{changes in inequality}} = \underbrace{\sigma'(\psi_2' - \psi_2)}_{\text{direct effect}} + \underbrace{(\sigma' - \sigma)\psi_2}_{\text{indirect h.c. dist.}}$$

   I find that the direct effect of changing ψ_2 accounts for 48% of total changes in $\mathrm{CV}(y_2)$ in the long run, with the rest being explained by changing σ.

2. To understand what drives changes in σ, I conduct a decomposition of IGE because in the steady-state σ is proportional to $\sqrt{1 - \mathrm{IGE}^2}$. The formula $\mathrm{IGE} = \rho + \psi_2(\delta_1 + \delta_2\xi)$ motivates the following decomposition:

$$\underbrace{\psi_2'(\delta_1 + \delta_2\xi') - \psi_2(\delta_1 + \delta_2\xi)}_{\text{changes in IGE}} = \underbrace{\psi_2\delta_2(\xi' - \xi)}_{\text{teacher quality}} + \underbrace{(\delta_1 + \delta_2\xi')(\psi_2' - \psi_2)}_{\text{parental income}}$$

   I find that teacher quality channel accounts for 94% of changes in IGE with the rest being explained by changes in parental income dispersion.

These decomposition results lead to an intriguing policy implication. Suppose ψ_2 increases by 1%, one can counteract its contemporaneous effects on teacher quality by also raising ψ_1 by 1%. But with the additional channel through parental income, this 1% increase is not enough to maintain teacher quality in later periods. Therefore, to neutralize the effects of the SBTC, the returns to human capital in the teaching profession needs to rise *relatively more* than that in the non-teaching profession. If instead, policymakers or teacher unions shield teachers from rising inequality elsewhere by reducing ψ_1, income inequalities among non-teachers will become even more severe in the future.

5 Robustness

As discussed before, one potential worry is that variations in employment and wages following DTB laws could also reflect changes in relative wages w or entry barriers κ. In that case, instead of Equations (18') and (19'), we have

$$\begin{cases} \Delta \log(y_1) = \Delta\psi_1[\mu + \sigma^2(\psi_1 + \theta(2\psi_1 - \psi_2))] + \Delta w \\ \Delta\pi = \Delta\psi_1 \cdot (\theta\mu + \sigma\theta\xi) + \theta(\Delta w + \Delta\kappa) \end{cases} \tag{23}$$

Note that potential changes in w and κ does not affect the interpretation of other equations including (16'), (17'), and (22) because they do not alter teacher quality ξ in the model.

They, however, do affect the calibration of other parameters. I divide the analysis into three cases.

First, suppose DTB laws affect entry barriers κ but not relative wage w. Then Equation (18') survives but equation (19') is contaminated because I do not have a direct measure of $\Delta\kappa$ in the data. In this case, I need to calibrate θ outside of the model and choose a value from the literature. An important insight here is that different value of θ does not affect the calibration of other variables except $\{\delta_0, \delta_2, \xi, w, \kappa\}$, and the results in all counterfactuals remain unchanged.

To see this, suppose θ is doubled. To make Equation (17') hold, δ_2 needs to halve. Similarly, δ_0 needs to halve to make Equation (16') hold. ξ needs to double because $\xi = (\mathrm{CV}(y_1) - \mathrm{CV}(y_2)) \cdot \theta$. Lastly, w and κ needs to adjust to maintain Equations (7) and (10). Beyond that, no other parameters need to change because δ_0, δ_2, and ξ enter the rest of the model, especially human capital dynamics, in a multiplicative fashion and these changes exactly cancel out.

Second, suppose DTB laws affect w but not κ. I can multiply the first equation in (23) by θ and subtract the second equation to cancel out Δw. Then, I can proceed like the previous case by calibrating θ exogenously. The results in counterfactuals are not affected.

Last, suppose DTB laws affect w as well as κ. Then the empirical results on how DTB laws affect teacher labor market are not informative about parameters and I need to find additional calibration targets. One option is to equate the effects of a one standard deviation increase in teacher quality on children's income, which is $(\delta_0 + \delta_2 \log(e/y)) \cdot \psi_1$ in the model, to existing empirical estimates (e.g., Chetty et al. (2014a)). Then, I can calibrate θ exogenously, and the counterfactual results are again not affected. As discussed before, when I compare teacher value added in the model to Chetty et al. (2014a) after calibration, the two values are quite similar.

To sum up, while potential changes in w and κ threaten the identification of θ, I can choose θ from the literature and counterfactual results will not be affected.

6 Conclusion

In this paper, I study how teacher quality affects inequality and intergenerational mobility. In a model with occupation choice and child investments, I uncover a feedback mechanism between teacher quality and human capital distribution. Heterogeneities in human capital amplify the selection channel in determining teacher quality, and teacher quality affects the degree of heterogeneity itself through human capital formation. The model is identified using empirical evidence on how duty-to-bargain laws affect teacher labor market and children's long-run outcomes. In counterfactuals, I find that reducing the returns to human capital by 5% in the teaching profession reduces income inequality, measured in the coefficient of variation, by 1.1% but raises inequality among non-teachers by 4.1% and reduces intergenerational mobility by 20%. Endogenous human capital distribution explains 24% of the overall decline in teacher quality in the long run. I also show that reducing teacher pay rigidities could alleviate the effects of skill-biased technical change on inequalities.

Lastly, I want to suggest that while this paper studies teacher quality, the same argument holds for other occupations, especially those employed in the public sector, as long as worker quality spills over to the productivity of future generations. These effects can be quantified in the same way through combining structural modeling and empirical analysis. I leave this as interesting future avenues of research.

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