

# Intergenerational Altruism, Fertility, and Welfare Across Countries and Time

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## Abstract

In this paper, I incorporate intergenerational altruism into welfare comparisons across countries and time. I develop an expected utility framework where parents are altruistic towards the well-being of their children, and the degree of altruism is affected by fertility a la [Becker and Barro \(1988\)](#). Relative to the classic measures in [Jones and Klenow \(2016\)](#), welfare among many developed countries with high life expectancy, e.g., Japan, South Korea, Italy, is adjusted downward by up to 40 percentage points because of their low fertility. Furthermore, cross-country convergence of welfare in the past few decades might have been much slower than previously thought.

## 1 Introduction

A good measure of welfare is fundamental to understanding inequalities of living standards across countries, and how such inequalities change over time. As is well known, GDP per capita is a flawed measure, and economists have long been improving it using economic theories.<sup>1</sup> In this paper, I take one step further by incorporating intergenerational altruism toward future generations into welfare analysis where the degree of altruism depends on people's fertility choices a la [Becker and Barro \(1988\)](#).

Taking intergenerational altruism and fertility into account is important for several reasons. First, parents devote enormous amount time and resources towards their children in all societies.

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<sup>1</sup>See [Becker, Philipson, and Soares \(2005\)](#), [Stiglitz, Sen, Fitoussi, et al. \(2009\)](#), [Fleurbaey \(2009\)](#), and [Jones and Klenow \(2016\)](#) for notable examples

Such behavior can hardly be explained if children's well-being is not an integral part of parents' utility. There have been a large literature using intergenerational altruism to account for parents' decisions at the individual level,<sup>2</sup> yet there have not been much work integrating it into welfare analysis at the aggregate level. Second, fertility choices reveal costs and constraints that do not appear in traditional economic measures such as consumption, leisure, or life expectancy. For example, China's one-child policy might have accelerated its modernization and economic growth ([Liao, 2013](#)), but it also leads to first-order welfare losses through restrictions on people's choices ([Cordoba and Liu, 2016](#)). Last, variations in fertility, and hence the (effective) degree of altruism, is extremely large across countries and over time. For example, in 2019, an average women in Malawi have more than 4 children while an average women in South Korea have less than one child. The average total fertility rate across countries was more than 5 children in 1960s, but is less than 2.5 children today.

I build an expected utility model following [Jones and Klenow \(2016\)](#) that integrates consumption, leisure, mortality, and inequality. Different from their work, I incorporate intergenerational altruism a la [Becker and Barro \(1988\)](#) where parents are altruistic towards children's welfare, with the degree of altruism rising in fertility levels.

Using the calibrated model and a panel data set of countries, I show that relative to the results in [Becker et al. \(2005\)](#) and [Jones and Klenow \(2016\)](#), welfare among many developed countries with high life expectancy, e.g., Japan, South Korea, Italy, is adjusted downward by up to 40 percent because of their low fertility. Furthermore, cross-country convergence of welfare in the past few decades might have been slower than previously thought.

Before I proceed to the model, I address one concern about the approach in this paper. One may wonder how preferences over fertility differ across countries and time, and whether this potential preference heterogeneity affects welfare comparisons. First, [Manuelli and Seshadri \(2009\)](#) and [Córdoba, Liu, and Ripoll \(2019\)](#) show that one does not need preference heterogeneity to account for differences in fertility across countries. Moreover, this concern applies equally to consumption, leisure, and longevity. In this paper, I follow the approach in [Jones and Klenow \(2016\)](#) and calculate the consumption equivalent for a hypothetical person named Rawls under the veil of ignorance with fixed preferences.

<sup>2</sup>See [Cordoba, Liu, and Ripoll \(2016\)](#), [Daruich \(2018\)](#), and [Boar \(2020\)](#) for some recent examples.

## 2 Model

### 2.1 Welfare Measure Without Altruism

I first present the welfare measure without considering intergenerational altruism. The analysis closely follows [Jones and Klenow \(2016\)](#).

Denote  $a$  as an individual's age,  $C$  as annual consumption, and  $l$  as leisure plus time spent in home production. Expected lifetime utility from consumption and leisure is thus:

$$U = \mathbb{E} \sum_{a=1}^M \beta^a u(C_a, l_a) S(a) \quad (1)$$

For each country-year, I assume that individuals draw from the cross-section data on consumption, leisure, and morality by age. Assume that the flow utility of Rawls is given by:

$$u(C, l) = \bar{u} + \log C + v(l) \quad (2)$$

Under these assumptions, expected lifetime utility  $U$  is given by:

$$U = \left( \sum_{a=1}^M \beta^a S(a) \right) \cdot \left( \bar{u} + \log C + v(l) - \frac{1}{2} \cdot \sigma^2 \right)$$

where  $\sigma^2$  is the variance of log consumption.

In the special case where  $\beta = 1$ , the expression simplifies to:

$$U = \underbrace{e}_{\text{life expectancy}} \cdot \underbrace{\left( \bar{u} + \log C + v(l) - \frac{1}{2} \cdot \sigma^2 \right)}_{\text{flow utility}} \quad (3)$$

where  $e = \sum_{a=1}^M S(a)$  is the life expectancy at birth.

### 2.2 Adding Intergenerational Altruism

Following [Becker and Barro \(1988\)](#), I assume that parents derive value from children's lifetime utility  $U'$ . Rawl's expected welfare  $V$  is given by:

$$V = U + \Psi(n) \cdot \mathbb{E}U'$$

where  $\Psi(n)$  is the degree of altruism and  $n$  is the number of children. In the benchmark analysis where the value of children  $U'$  is positive, we assume  $\Psi(n) \in (0, 1)$ ,  $\Psi'(n) > 0$ , and  $\Psi''(n) < 0$ .

In the case where Rawls expect his children to have the same lifetime utility as himself, his welfare  $V$  is given by:

$$\underbrace{V}_{\text{welfare}} = \underbrace{\frac{1}{1 - \Psi(n)}}_{\text{altruism adjustment}} \cdot \underbrace{U}_{\text{lifetime utility}} \quad (4)$$

Combining Equations (3) and (4),

$$\begin{aligned} V &= \left( \frac{1}{1 - \Psi(n)} \cdot e \right) \cdot \left( \bar{u} + \log C + v(l) - \frac{1}{2} \cdot \sigma^2 \right) \\ &\equiv \tilde{e} \cdot \left( \bar{u} + \log C + v(l) - \frac{1}{2} \cdot \sigma^2 \right) \end{aligned} \quad (5)$$

where I define *altruism-adjusted life expectancy* ( $\tilde{e}$ ) as:

$$\tilde{e} \equiv \frac{1}{1 - \Psi(n)} \cdot e \quad (6)$$

It captures the idea that intergenerational altruism transforms a finite-lived agent into an infinitely-lived one where his discount rate depends on the number of children.

### 2.3 Welfare Comparisons

Let  $V_i(\lambda)$  denote expected welfare in country  $i$  if consumption is multiplied by a factor  $\lambda$  at each age *both for parents and all their descendants*:

$$\begin{aligned} V_i(\lambda) &= U_i(\lambda) + \Psi(n) \cdot \mathbb{E}U'_i(\lambda) \\ \text{where} \quad U_i(\lambda) &= \mathbb{E} \sum_{a=1}^M \beta^a u(\lambda C_{ai}, l_{ai}) S_i(a) \end{aligned} \quad (7)$$

The consumption equivalent  $\lambda_i$  is defined as the factor that makes Rawl's indifferent between living his life as a random person in the United States and in some other country  $i$ . Factor  $\lambda_i$  is found by solving:

$$V_{\text{U.S.}}(\lambda_i) = V_i(1)$$

Using Equation (5),  $\lambda_i$  across countries can be decomposed as:

$$\begin{aligned}
\log(\lambda_i) &= \frac{\tilde{e}_t}{\tilde{e}_{\text{U.S.}}} \cdot \left( \bar{u} + \log C_i + v(l_i) - \frac{1}{2} \cdot \sigma_i^2 \right) - \left( \bar{u} + \log C_{\text{U.S.}} + v(l_{\text{U.S.}}) - \frac{1}{2} \cdot \sigma_{\text{U.S.}}^2 \right) \\
&\stackrel{\text{decompose}}{=} \underbrace{\log(Y_i) - \log(Y_{\text{U.S.}})}_{\text{GDP per capita}} + \underbrace{\log(C_i/Y_i) - \log(C_{\text{U.S.}}/Y_{\text{U.S.}})}_{\text{consumption share}} \\
&\quad + \underbrace{v(l_i) - v(l_{\text{U.S.}})}_{\text{leisure}} + \underbrace{\frac{1}{2} \cdot (\sigma_{\text{U}}^2 - \sigma_{it}^2)}_{\text{inequality}} \\
&\quad + \underbrace{\frac{\tilde{e}_i - \tilde{e}_{\text{U.S.}}}{\tilde{e}_{\text{U.S.}}} \cdot \left( \bar{u} + \log C_i + v(l_i) - \frac{1}{2} \cdot \sigma_i^2 \right)}_{\text{altruism-adjusted life expectancy}}
\end{aligned} \tag{8}$$

Relative to GDP per capita, the welfare metric in Equation (8) integrates differences in consumption shares in output, leisure, levels of inequality, life expectancy, and fertility. Relative to the measure in Jones and Klenow (2016), the key difference after adding intergenerational altruism is the last term – instead of using life expectancy at birth  $e$ , this paper uses altruism-adjusted life expectancy  $\tilde{e}$  defined in Equation (6). Therefore, when I compare  $\lambda_i$  in this paper with those in Jones and Klenow (2016) in Section 3, I focus on comparing the last term in Equation (8).

Besides computing consumption equivalent across countries, I also use the decomposition in Equation (8) to calculate the growth rate of the consumption equivalent for each country over time. The growth rate for country  $i$  between year  $t_0$  and  $t_1$  is simply:

$$g_i \equiv -\frac{1}{t_1 - t_0} \log(\lambda_i) \tag{9}$$

where  $\lambda_i$  is computed by comparing  $V_{i,t_0}$  and  $V_{i,t_1}$ .

## 3 Results

### 3.1 Calibration

I follow Jones and Klenow (2016) in the parameterization of  $\bar{u}$  and  $v(\cdot)$ . In particular,  $C_{\text{U.S.},2007}$  is normalized to be 1;  $\bar{U} = 5.23$  is chosen so that a 40-year-old, facing the consumption and leisure uncertainty in the 2006 U.S. distribution, has a value of remaining life equal to \$6 million in 2007 prices. Utility from leisure is given by:

$$v(l) = -\frac{\theta\epsilon}{1+\epsilon} (1-l)^{\frac{1+\epsilon}{\epsilon}}$$

where  $\theta$  governs the disutility of working and  $\epsilon$  is the Frisch elasticity of labor supply. I choose  $\theta = 14.17$  to match the ratio of earnings to consumption and average leisure in the US Consumer Expenditure Survey (CE) in 2006. I consider a Frisch elasticity of 1.0 for the benchmark calibration.

Following [Cordoba et al. \(2016\)](#), I parameterize the degree of altruism  $\Psi(n)$  as:

$$\Psi(n) = 0.62 \cdot \exp(-1.85 \cdot n)$$

### 3.2 Welfare Across Countries in 2007 with Altruism

In this section, I compare the altruism-adjusted welfare metric with that in [Jones and Klenow \(2016\)](#). The main difference lies in the last term of the decomposition in Equation (8), i.e. life expectancy term. [Jones and Klenow \(2016\)](#) calculates it using raw life expectancy at birth while this paper uses altruism-adjusted life expectancy. Hereafter, I denote the life expectancy term in [Jones and Klenow \(2016\)](#) as LE, and the altruism-adjusted life expectancy term in this paper as  $\widetilde{\text{LE}}$ .

Figure 1 plots the LE and  $\widetilde{\text{LE}}$  for all countries in 2007. As the reference country,  $\text{LE}_{\text{U.S.}} = \widetilde{\text{LE}}_{\text{U.S.}} = 0$ . As can be seen, the two measures are positively correlated with many falling on the 45-degree line. For countries with higher (lower) fertility than the U.S.,  $\widetilde{\text{LE}}$  is larger (smaller) than LE. Upward adjustments of welfare when altruism is incorporated, however, are usually small while downward adjustments are quite large in many cases.<sup>3</sup> For example, while [Jones and Klenow \(2016\)](#) find that Japan's relative welfare should be 24% higher than the GDP differences because people in Japan have higher longer life expectancy, I find that Japan's welfare should be 16% lower than the GDP differences because in addition to long life expectancy, Japan has a low fertility of around 1.3 children per women, leading to a low altruism-adjusted life expectancy.

Figure 2 shows that the downward adjustments when intergenerational altruism is incorporated are sizable among OECD countries. Only 5 out of 34 OECD countries have a higher  $\widetilde{\text{LE}}$  than LE. There are 19 out of 34 OECD countries (in the lower-right quadrant of Figure 2) which are considered to have higher relative welfare than GDP differences according to [Jones and Klenow \(2016\)](#), but should have lower relative welfare when intergenerational altruism is considered. The average gap between  $\widetilde{\text{LE}}$  and LE is -30% among these 19 countries.

<sup>3</sup>This is due to the assumption that the degree of altruism  $\Psi(n)$  is a concave function of fertility  $n$ . As far as I know, this assumption holds in almost all models with altruism and fertility.

Figure 1: Life Expectancy Term - All countries

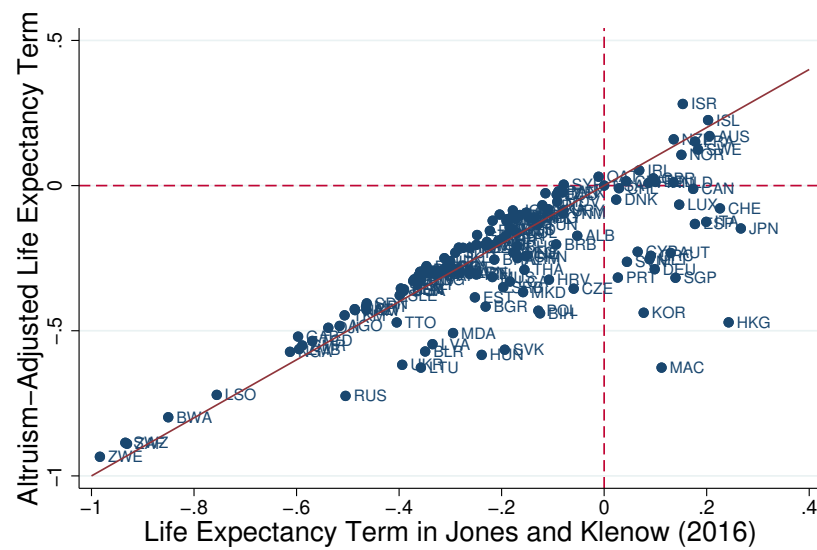
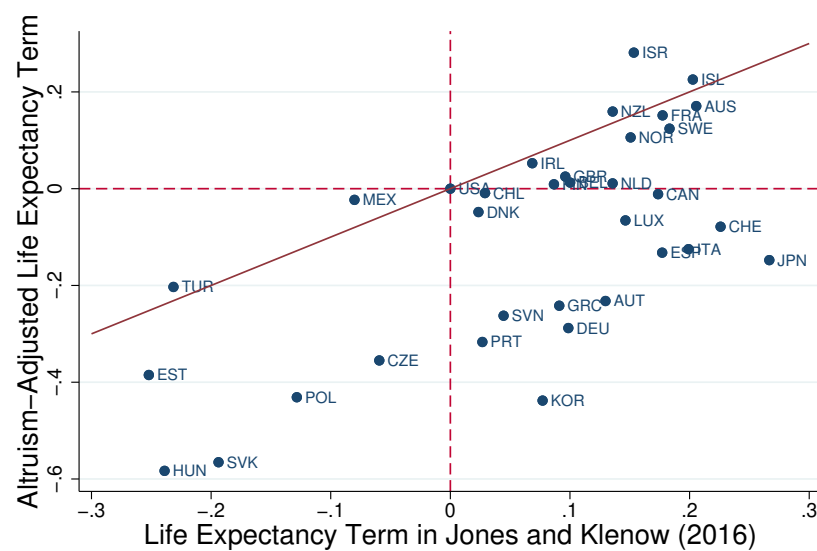


Figure 2: Life Expectancy Term - OECD countries



### 3.3 Welfare Growth from 1980 to 2007 with Altruism

In this section, I compare the altruism-adjusted welfare growth from 1980 to 2007, as defined in (9), with that in Jones and Klenow (2016). Similar to the previous results, the main difference lies in the use of fertility data. As a result, the measure in this paper not only considers changes in life expectancy over time, but also integrates the dynamics of fertility. Hereafter, I denote  $g$  the measure of annual welfare growth in Jones and Klenow (2016) as  $\tilde{g}$ , and the altruism-adjusted growth in this paper as  $\tilde{g}$ .

Figure 3 displays  $g$  and  $\tilde{g}$  for all countries where the dashed lines cross at  $(g_{\text{U.S.}}, \tilde{g}_{\text{U.S.}})$ . There are 112 out of 125 countries that have  $\tilde{g} < g$ . This reflects the fact that as demographic transition takes place in the process of development, life expectancy grows but fertility falls. The previous literature takes the former into account but ignores the latter. As a result, welfare growth is slower when intergenerational altruism is taken into account. If we regard the U.S. as the world frontier of welfare, 69 countries have higher growth than the U.S. in Jones and Klenow (2016), implying considerable amount of convergence, largely due to their fast rise in life expectancy. When intergenerational altruism is taken into account, only 38 countries grow faster than the U.S. In other words, cross-country convergence of welfare might have been slower than previously thought. This point can be further illustrated using the sample of OECD countries in Figure 4. Only 4 out of 34 OECD countries are considered to have lower welfare growth than the U.S. in Jones and Klenow (2016), whereas 13 out of 34 fall in this category when intergenerational altruism is considered.

Figure 3: Life Expectancy Term - All countries

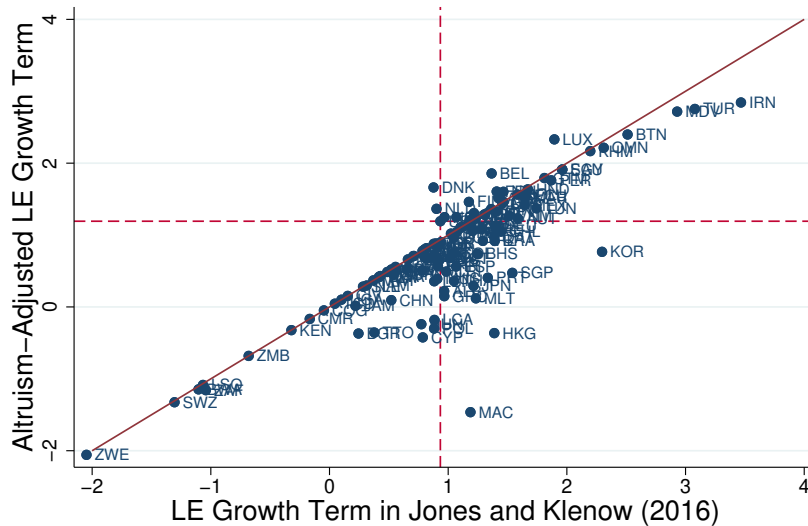
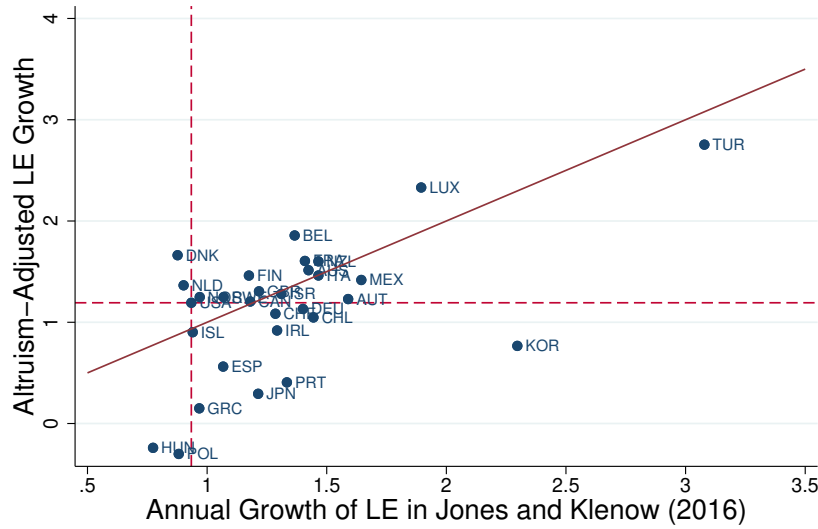




Figure 4: Life Expectancy Term - OECD countries



## 4 Conclusion

A good measure of welfare is fundamental to understanding inequalities of living standards across countries, and how such inequalities change over time. In this paper, I develop an expected utility framework where parents are altruistic towards the well-being of their children, and the degree of altruism is affected by fertility à la [Becker and Barro \(1988\)](#). Relative to the metric in [Jones and Klenow \(2016\)](#), welfare among many developed countries with high life expectancy, e.g., Japan, South Korea, Italy, is adjusted downward by up to 40 percentage points because of their low fertility. Furthermore, cross-country convergence of welfare in the past few decades might have been much slower than previously thought.

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