The Macroeconomic Consequences of Family Policies

(Work in Progress)

Anson Linshuo Zhou University of Wisconsin-Madison April 24, 2021

Motivation: Recent Trends in Family Policies

Developed countries are increasingly relying on family policies to raise fertility

- As population aging problem looms near, the share of countries with explicit pronatal policies increases from 10% in 1976 to 28% in 2015
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Recent calls for transfers to low-income households with children in the U.S.

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- Policy initiatives on providing universal childcare and expanding parental leaves
- Expected effects: Child-related transfers improve children's outcome (Dahl and Lochner 2012, Mullins 2019) and boost social mobility (Pulliam and Reeves 2021)

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Particularly on: average income, intergenerational mobility and welfare. short-run vs long-run

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Children's human capital is shaped by both public and private investment;

• Government: taxation, pension, baby bonus, childcare subsidy, public education

- + inter-vivos transfers
- Potential role for government policies to improve welfare because of:
 - 1 Fiscal externalities of fertility and child human capital investments
 - 2 Imperfect capital and insurance markets (Abbott, Gallipoli, Meghir and Violante 2019)

Key Channels of Family Policies

- Fertility responses
 - Original goal of family policies is to raise fertility
 - Capture findings that **family policies boost fertility** (McDonald 2006, Stone 2020)
 - Financial transfers have the biggest influence on fertility among women with lower income (Drago et al. 2011, Sobotka, Matysiak and Brzozowska 2019)
- Skill formation
 - When family policies affects childbearing decisions, intergenerational linkage is important in predicting changes in children's human capital
 - Endogenous educational choices by parents
- General equilibrium effects
 - Changes in human capital distribution and demographic structure (i.e. age distribution) affect fiscal balances and taxes in general equilibrium

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Preview of Key Results

- In contrast to the predictions by models with exogenous fertility, parents could optimally reduce private education investments in response to a baby bonus. With heterogeneous responses across households, baby bonus could reduce intergenerational mobility
- 2 Child-related cash transfers need to be \$28k+ (NPV) to raise fertility from baseline U.S. level (cfr=1.9) to replacement level (cfr=2.1). It improves long-run welfare¹by 3.2% despite a reduction in average human capital by 1.5% because changes in demographic structure lead to reduced tax rates in equilibrium
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¹Average utility of new-born under the veil of ignorance converted to consumption equivalence

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Related Literature

Fiscal Policies, Education, Income transfers, and Mobility

- Benabou (2002), de la Croix and Doepke (2003), Heckman and Mosso (2014), Daruich (2019), Abbott, Gallipoli, Meghir and Violante (2019), Mullins (2019), Guner, Kaygusuz and Ventura (2020)...
- <u>Contribution</u>: Endogenous fertility could reverse policy effects on child human capital and mobility. General equilibrium effects on welfare

Family Policies

- Empirical: Milligan (2005), Laroque and Salanié (2008), Drago et al. (2011), Luci-Greulich and Thévenon (2013), González (2013), Raute (2019)...
- Structural: Liao (2013), Bairoliya and Miller (2020), Moschini (2020)
- <u>Contribution</u>: Evaluate large-scale policy counterfactuals with both endogenous fertility and endogenous child skill formation

Outline

- Model
- **2 Calibration** (2010 USA)
 - Key moments on fertility profile (CPS), childcare arrangements (SIPP), parental investments and transfers (PSID) and lifecycle earnings profile (PSID)
 - Skill formation based on RCT evidence (Garcia, Heckman, Leaf and Prados 2020)
 and intergenerational mobility estimates (Chetty, Hendren, Kline and Saez 2014)

Validation

 Model matches fertility effects estimated for Alaska's Permanent Fund Dividend (Kelly, Timilsina and Yonzan 2020, Cowan and Douds 2020)

4 Policy:

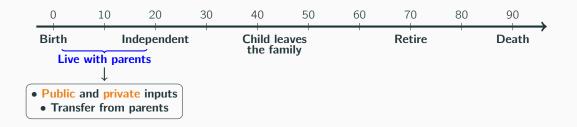
- Evaluate aggregate impacts of large-scale family and education policies
- Next Steps

Model



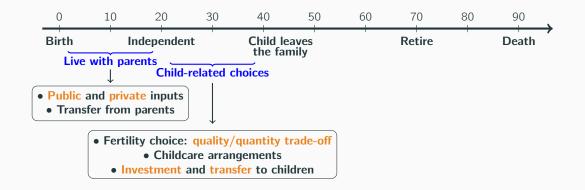
Key Elements

- Life-cycle Aiyagari
- Endogenous Fertility
- Endogenous Child Link



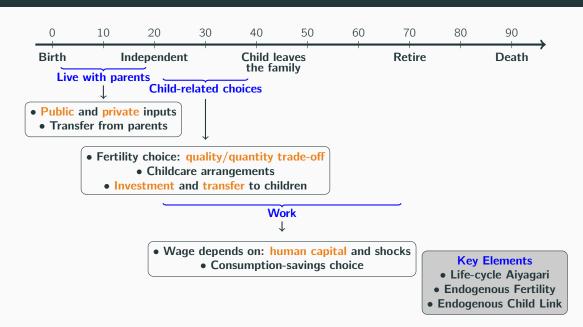
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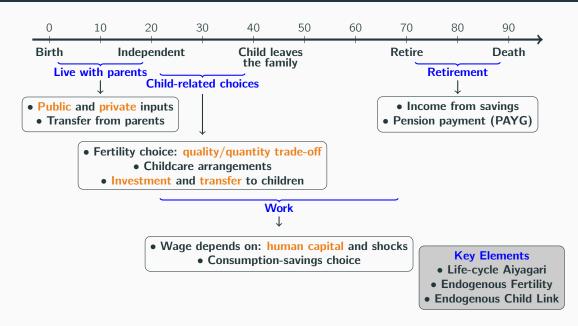
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Birth Independent
$$V_2(h, a) = \max_{c,a',n,t_h,m,e>0} u(c/q_n) + \beta \mathbb{E} V_3(h', a', n, \mathbb{E} h_k)$$

h: parents' skills *n* : fertility a: assets t_h : home care m: market care e: private educ. input χ : childcare needs p_m : market care price q_n : equivalence scale \mathcal{S} : childcare subsidy \mathcal{B} : baby bonus \mathcal{E} : public education

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$$V_2(h, a) = \max_{c,a',n,t_h,m,e \geq 0} u(c/q_n) + \beta \mathbb{E} V_3(h', a', n, \mathbb{E} h_k)$$

$$n \cdot \chi = \left(t_h^{v/\iota} + (n \cdot m)^v\right)^{1/v}$$
 [childcare]

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Birth Independent Child leaves the family Retire Death
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$$y = wh \cdot (1 - t_h) \qquad \text{[labor income]}$$

$$(1 + \tau_c)(c + mp_m(1 - \mathcal{S})n + e \cdot n) + a' = (1 + r)a + y - \mathcal{T}(y,a,n) + \mathcal{B} \cdot n \qquad \text{[BC]}$$

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$$h' = L(h,1-t_h,z') \qquad h_k = G(h,\mathcal{E},e,e) \qquad \text{[technology]}$$

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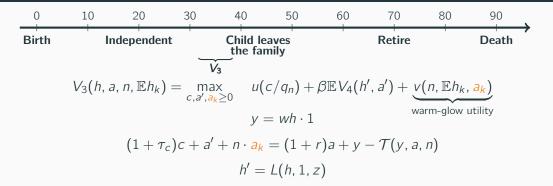
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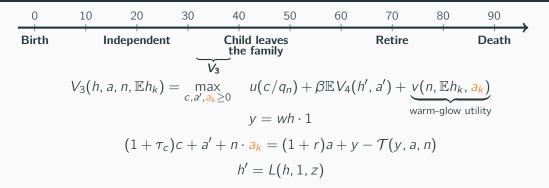
- $G(h, \mathcal{E}, e, \epsilon)$ captures children's skill formation from age 0 to 20 with:
 - Private investment *e* from age 0 to 10
 - Public education E from age 0 to 18
- Assumption: childcare arrangement does not affect children's skill formation

Parent-to-Child Transfer



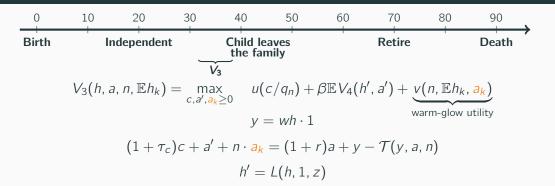
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 of child "quality" e and ak, and vice versa (Becker and Lewis 1972)
- ullet To parents, baby bonus ${\cal B}$ and childcare subsidy ${\cal S}$ reduce the shadow price of child quantity while public education ${\cal E}$ raises child quality

Firms and the Government

• Representative firm with **Cobb-Douglas production function**²:

$$Y = AK^{\alpha}H^{1-\alpha}$$

²We abstract away from aggregate population externalities such as pollution (Bohn and Stuart 2015) and knowledge creation (Jones 2020).

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- Government uses predetermined policy instruments $\{\mathcal{T}(\cdot), \mathcal{B}, \mathcal{S}, \mathcal{E}\}$
- Denote demographic structure as $\{\Omega_i\}_{i=0}^8$ (with $\sum_{i=0}^8 \Omega_i = 1$) and distribution of households across state space as μ . Government fiscal budget:

$$\underbrace{\left(\sum_{j=2}^{6}\Omega_{j}\int\mathcal{T}(y_{j}^{*},a_{j}^{*},n_{j}^{*})\,d\mu\right)}_{\text{labor and capital income taxes}} + \underbrace{\left(\sum_{j=2}^{8}\Omega_{j}\tau_{c}c_{j}^{*}\,d\mu\right)}_{\text{public education}} = \underbrace{\left(\sum_{j=7}^{8}\Omega_{j}wh\cdot\pi\,d\mu\right)}_{\text{public education}} + \underbrace{\left(\Omega_{0}+\Omega_{1}\right)\mathcal{E}}_{\text{baby bonus}} + \underbrace{\int\Omega_{2}(1+\tau_{c})m^{*}n^{*}p_{m}\cdot\mathcal{S}\,d\mu}_{\text{subsidized childcare}} + \underbrace{\int\Omega_{2}(1+\tau_{c})m^{*}n^{*}p_{m}\cdot\mathcal{S}\,d\mu}_{\text{other spendings}} + \underbrace{\int\Omega_{2}(1+\tau_{c})m^{*}$$

stationary equilibrium

[►] results ²We abstract away from aggregate population externalities such as pollution (Bohn and Stuart 2015) and knowledge creation (Jones 2020).

Role for Government Policies

Why may government policies improve welfare?

Long-run welfare³: average value of newborn under the veil of ignorance:

$$\mathcal{W} = \int V_2 \, d\mu$$

Pragmatic approach that evaluates whether higher fertility, i.e. additional agents, improves the **average well-being** in the economy⁴

⁴Distributional effects on **existing agents** will be evaluated in transition dynamics

⁴See Golosov, Jones and Tertilt (2007) and de la Croix and Doepke (2021) for more discussions of welfare with endogenous fertility

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Externalities/incompleteness that government could address:

- 1 Fiscal externalities of childbearing and childrearing
 - Private returns \neq social returns (i.e. $\{\Omega_j\}_{j=0}^8$ and μ)
- Parents cannot borow against children's future income
 - Lack of compensation mechanism
- **3** Life-cycle borrowing constraints

► policy

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Consider an increase in \mathcal{B} (baby bonus) or \mathcal{S} (childcare subsidy)

On child quality h_k for a given parent with human capital h:

- With exogenous fertility, this is a **lump-sum transfer** $h_k(h)$ rises unambiguously
- With endogenous fertility, $n^*(h)$ increases⁵, additional channels on $h_k(h)$:
 - **1** Complementarity/substitutability via $v(n, \mathbb{E}h_k, a_k)$
 - **2** Higher costs of childrearing $n\chi$ and q_r
 - 3 Interactions in budget constraint higher shadow price of e

- With exogenous fertility, N is fixed while \overline{h} increases
- With endogenous fertility:
 - **1** Aggregate fertility rises \Rightarrow lowers **old-age dependency ratio** $(\Omega_7 + \Omega_8)/(\sum_{j=2}^6 \Omega_j)$
 - **a** Besides change in $h_k(h)$, effects on \overline{h} rely on relative changes in $n^*(h)$ across h

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 $^{^5}$ A sufficient condition is that n is a normal good. See Bleakley and Ferrie (2016) for evidence

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On aggregate fertility N and average human capital \overline{h} in the economy:

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Policy effects of transfers to families with existing children are qualitatively different from the aggregate effects of family policies with endogenous fertility

Calibration

Parametrization: Preferences

• Utility from consumption:

$$u(c) = \frac{c^{1-\gamma_c}}{1-\gamma_c} \qquad \gamma_c \in (0,1)$$

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• Utility from child quantity and quality:

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n) \cdot (\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k))$$

$$\Psi(n) = 1 - \exp(-\psi n) \qquad \psi > 0$$

where parents value child quality via $u(\cdot)$ (Barro and Becker 1989) with exponential child discounting (Córdoba, Ripoll and Liu 2016)

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• Utility from child quantity and quality:

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$$\Psi(n) = 1 - \exp(-\psi n) \qquad \psi > 0$$

where parents value child quality via $u(\cdot)$ (Barro and Becker 1989) with exponential child discounting (Córdoba, Ripoll and Liu 2016)

• v_{12} , $v_{13} > 0$ \Longrightarrow main results are driven by interactions in **budget constraint** (Becker and Lewis 1972) instead of assumptions on preferences⁶

fit $\triangleright V_2$ $\triangleright V_3$ identification \triangleright results ⁶Robust to (1) separable preferences (de la Croix and Doepke 2003, Moav 2005), (2) quality and quantity being substitutes (Jones and Schoonbroodt 2010), or (3) dynastic altruism (Daruich and Kozlowski 2020)

Child's Skill Production Function

• Children's skill production function:

$$h_k = \underbrace{Z} \cdot \underbrace{\epsilon}_{\text{scalar shock spillover}} \cdot \underbrace{h^{\rho}}_{\text{public education}} + \underbrace{e^{\omega}}_{\text{private input}}$$

$$\log(\epsilon) \sim \mathcal{N}\left(-\frac{\sigma_{\epsilon}^2}{2}, \sigma_{\epsilon}^2\right)$$

- $\rho = 0.28$ rank-rank mobility (Chetty, Hendren, Kline and Saez 2014)
- $\mathcal{E} = 0.165$ \$12k per pupil per year (NCES)



Child's Skill Production Function Cont'd

Use **RCT** evidence to inform the productivity of inputs γ :

$$h_k = Z \cdot \epsilon \cdot h^{\rho} \left(\mathcal{E}^{\omega} + e^{\omega} \right)^{\gamma/\omega}$$

Child's Skill Production Function Cont'd

Use **RCT** evidence to inform the productivity of inputs γ :

$$h_k = Z \cdot \epsilon \cdot h^{\rho} \left(\mathcal{E}^{\omega} + e^{\omega} \right)^{\gamma/\omega}$$

- Garcia, Heckman, Leaf and Prados (2020)
 - Two US early childhood programs (ABC, CARE) in 1970s
 - Cost \approx \$13.5k per year for five years total \$67.5k per child
 - Followed up into adulthood and observe education/income
 - For every dollar invested, children's lifetime labor income increases by 1.3 dollars

Child's Skill Production Function Cont'd

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 - Followed up into adulthood and observe education/income
 - For every dollar invested, children's lifetime labor income increases by 1.3 dollars
- ullet Apply similar policy in the model: expand existing ${\mathcal E}$ by \$67.5k
 - Small scale: prices and taxes remain unchanged
 - Target: children of parents at 10th percentile of earnings
- Comparing labor income change with program costs gives $\gamma = 0.16$



Model Parameters

Table 1: Model Parameters

	Interpretation	Value	Source		Interpretation	Value	Source
Preferences				Child human capital production			
β	discount rate	0.98 ¹⁰	standard	Z	normalizing scalar	2.45	
γ_c	elasticity of substitution	0.52	CPS	σ_{ϵ}	ability shock dispersion	0.45	PSID
ψ	fertility preference	1.72	CPS	ρ	intergenerational spillover	0.28	Chetty et al. (2014)
θ	quality preference	1.70	PSID	ω	substitution of education	0.9	ATUS
ν	transfer preference	0.67	PSID	$\mathcal E$	public education	0.165	NCES
				γ	input productivity	0.16	Garcia et al. (2020)
Childcare arrangement							
χ	childcare cost	0.18	Folbre (2008)	Adult human capital evolution			
L	economies of scale at home	0.7	Folbre (2008)	η	learning curvature	0.61	PSID
υ	substitutability of care	0.5	SIPP	ζ	learning level	0.72	PSID
p_m	price of full-time care	0.13	NACCRRA	μ_Z	skill depreciation	-0.23	PSID
				σ_z	shock dispersion	0.42	PSID
Taxes and pension							
$\tau_{V}^{n}, \lambda_{V}^{n}$	tax levels and progressitivity	misc.	TAXSIM	Firm production function			
τ_c	consumption tax	0.07	McDaniel (2007)	A	total factor productivity	1	normalization
τ_a	capital income tax	0.27	McDaniel (2007)	α	capital share	0.33	standard
π	pension replacement rate	0.40	OECD Database	δ_k	capital depreciation	0.04^{10}	standard

- 13 parameters are calibrated within the model using SMM
- Model matches salient features of fertility profile, childcare, parental investment, inter-vivos transfers, intergenerational mobility and lifecycle earnings profile

Validation

Fertility Response to Financial Incentives

Question: The model matches data well. But does the model generate **quantitative** response to policies that match empirical estimates?

Answer: Validation using Alaska Permanent Fund Dividends (APFD)

Fertility Response to Financial Incentives

Answer: Validation using Alaska Permanent Fund Dividends (APFD)

- Established in 1982 after discovery of the petroleum. Equal transfer to **all residents** regardless of income, employment or age
- Allows parent to claim dividend on behalf of a child with no requirements on how parents use a child's dividend.
- **Ideal policy variation** to test fertility responses:
 - 1 Similar institution and cultural background
 - 2 Large in scale (≈ \$1.5k per year) relative to other family policies
 - 3 Clear implementation: not tied to labor market status or policy bundles

Fertility Response to Financial Incentives

Answer: Validation using Alaska Permanent Fund Dividends (APFD)

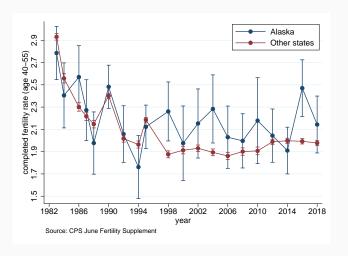
- **Ideal policy variation** to test fertility responses:
 - Similar institution and cultural background
 - **2** Large in scale (\approx \$1.5k per year) relative to other family policies
 - **3 Clear implementation**: not tied to labor market status or policy bundles
- Apply same policy in the model: UBI to all household members by \$1.5k (normalized by median income in Alaska relative to the rest of U.S. + partial equilibrium)
 - Completed fertility rate increases by 4.3% in the model:

$$\underbrace{4.3\%}_{\text{quantum effect (CFR)}} \approx \underbrace{13.1\%}_{\text{quantum+tempo (TFR)}} \times \underbrace{1/3 \text{ to } 1/2}_{\text{quantum/(quantum+tempo)}}$$

$$\underbrace{\text{quantum-tempo (TFR)}}_{\text{(Kelly, Timilsina and Yonzan 2020)}} \times \underbrace{1/3 \text{ to } 1/2}_{\text{quantum/(quantum+tempo)}}$$

 Heterogeneous responses: Larger responses from households with lower opportunity costs of time (model) - larger fertility increases among Alaska Natives and women without high school degree (Cowan and Douds 2020)

Evidence from Completed Fertility Rates



• 95% confidence intervals of sample mean

Counterfactuals

Large Scale and Permanent Policy

Evaluate universal policies of different scale (NPV for \mathcal{B} , \mathcal{E} and % for \mathcal{S})

- **Budget balance**: consumption tax adjusts
- General equilibrium: prices and distributions adjust

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Outcomes of Interest

- Aggregate fertility, average human capital, per capita income and intergenerational mobility (¹/_{IGE})
- Social welfare in consumption equivalence

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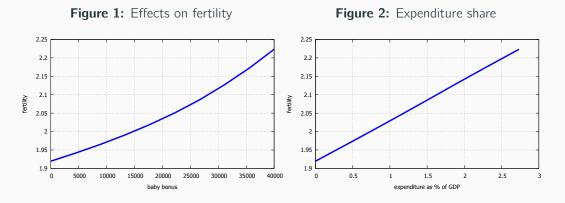
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Outline

- Long-run effects comparing new steady-state economy to baseline economy
- Synergies among policies (in progress)
- **Transition** (in progress)

Fertility Effects of Baby Bonus \mathcal{B}

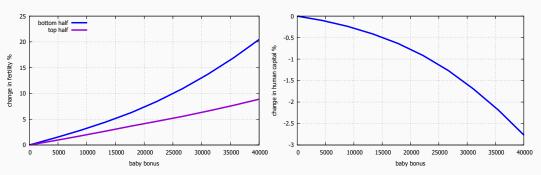


- Fertility increases as shadow price of child quantity falls
- Baby bonus needs to be greater than \$28k (NPV) to raise aggregate fertility rate to replacement level (2.1, commonly stated long-run policy goal). The policy costs around 1.6% of GDP in the new equilibrium

Heterogeneous Response and Effects on Human Capital

Figure 3: Heterogeneous fertility response

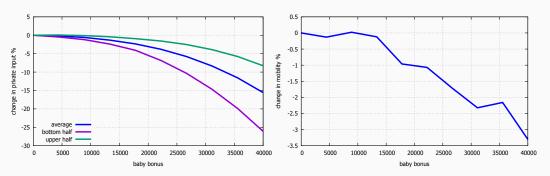
Figure 4: Average human capital



- Households with lower human capital respond more to baby bonus
- ullet Recall intergenerational transmission of human capital $h^{
 ho}$
- Average human capital decreases by 1.5% at replacement rate fertility

Figure 5: Average private investment

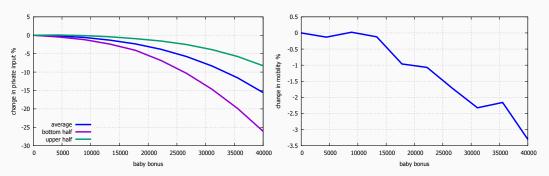
Figure 6: Intergenerational mobility



- Parents reduce private investments per child
- Dominant force: higher *n* increases the **shadow price of** *e* and **costs of childrearing**
- Predictions consistent with empirical evidence from Australian baby bonus
- Intergenerational mobility decreases by 2.5% at replacement fertility
- Results are **stronger** when cash transfers are targeted at low-income households



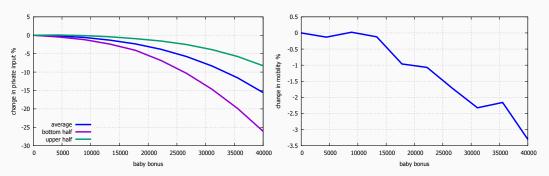
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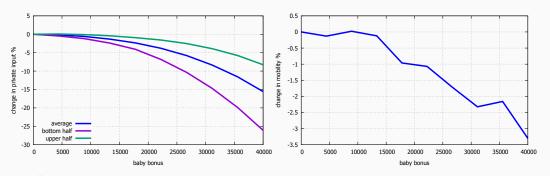
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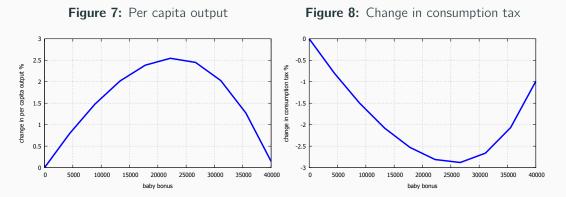
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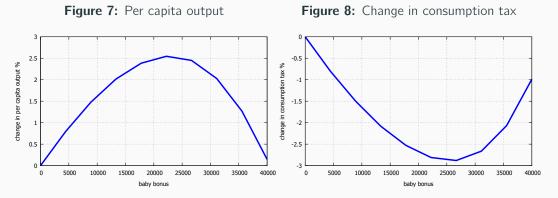
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Output Response and Change in Consumption Tax



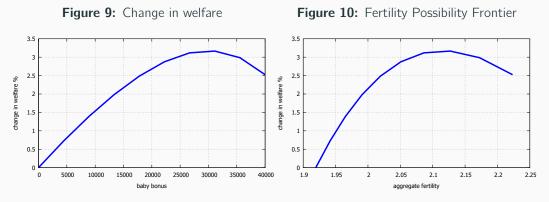
• Per capita output increases initially due to changes in demographic structure

Output Response and Change in Consumption Tax



- Per capita output increases initially due to changes in demographic structure
- Consumption tax could be **reduced** while keeping government budget satisfied
- Larger baby bonus is **not always beneficial** as average human capital worsens

Social Welfare under Baby Bonus \mathcal{B}

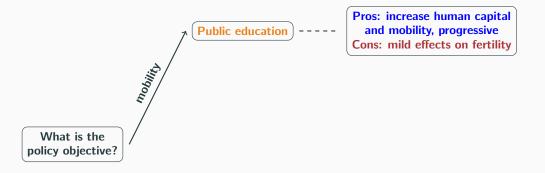


- Policy-induced relationship between fertility and welfare is **not monotonic**
- Welfare maximized around replacement fertility. Consumption equivalence increases by 3.2%. Sources: (1) higher n, (2) lower τ_c , (3) change in μ
- Progressive policy with large welfare improvement for low-income households

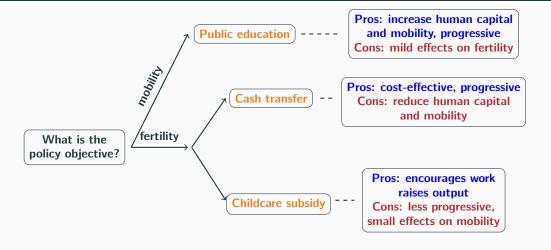
Taking Stock

What is the policy objective?

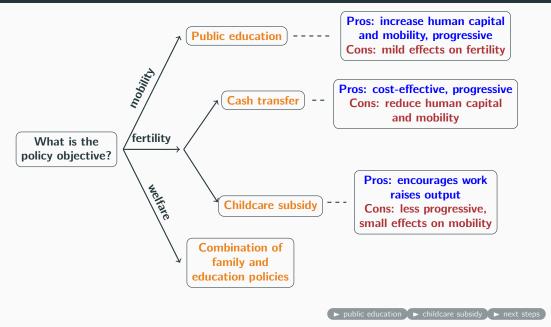
Taking Stock



Taking Stock



Taking Stock



Conclusion

Consequences of large-scale family policies depend on:

Fertility responses, endogenous skill formation and GE effects

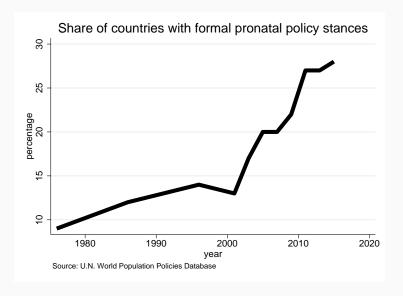
Model

• Introduce **endogenous fertility** a lá **quality/quantity trade-off** into a GE-OLG model with incomplete market and distortionary taxes

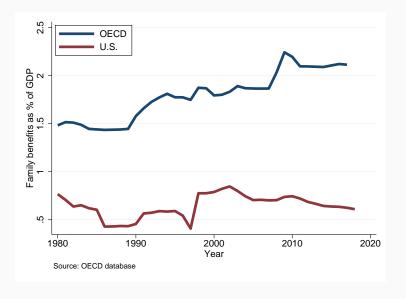
Results:

- Parents could reduce private investments in response to child-related cash transfers when fertility is endogenous. Cash transfers (e.g. baby bonus) could reduce social mobility if there is no restrictions on how the money could be spent
- Despite having negative effects on average human capital and social mobility, a \$28k baby bonus raises fertility to replacement level and improves welfare by 3.2% via effects on demographic structure and taxes in general equilibrium
- 3 Public education raises output, mobility and welfare, but has mild effects on fertility

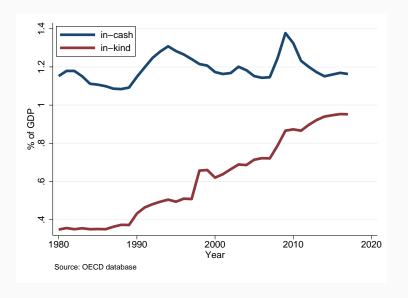
Time Trend of Pronatal Policy Stances



Time Trend of Public Expenditures on Family Benefits



Expenditure Breakdown



Working Without Children and Retirement

• For households working without children:

$$V_{j}(h, a) = \max_{c, a' \ge 0} u(c) + \beta \mathbb{E} V_{j+1}(h', a')$$
$$(1 + \tau_{c})c + a' = (1 + r)a + y - \mathcal{T}(y, a, 0)$$
$$h' = L(h, 1, z)$$

For retired households:

$$V_j(h, a) = \max_{c, a' \ge 0} u(c) + \beta V_{j+1}(h, a')$$
$$(1 + \tau_c)c + a' = (1 + r - r\tau_a)a + \pi \cdot wh$$
$$V_9(\cdot) \equiv 0$$

where π is pension replacement rate

Stationary Equilibrium

- Distributions:
 - Demographic structure $\{\Omega_j\}_{j=0}^8$ and distribution of agents over states μ are invariant over time periods
 - Distribution of initial states is determined by older generations and shock processes
- Households Optimize: Households choose consumption, savings, fertility, childcare arrangements, child investments and inter-vivos transfers such that utility is maximized
- Firms maximize profits
- Prices clear markets
- Government balances budget in period to period

$$u(c) = \frac{c^{1-\gamma_c}}{1-\gamma_c} \quad \gamma_c \in (0,1)$$

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n) \cdot (\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k))$$

$$\Psi(n) = 1 - \exp(-\psi n) \qquad \psi > 0$$

• Recall parametric assumptions:

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- Córdoba, Ripoll and Liu (2016): γ_c (EGS) governs quality/quantity trade-off.
 Higher γ_c ⇒ higher MRS of quantity for quality ⇒ MB_n rises relatively faster with h than MC_n ⇒ flatter or even positive income-fertility profile
- Calibrate $\gamma_c=0.52$ to match income-fertility profile (CPS)

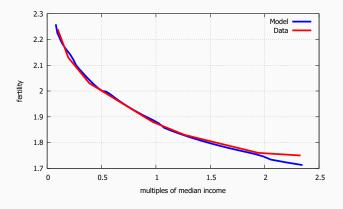
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- Calibrate $\gamma_c = 0.52$ to match income-fertility profile (CPS)
- Córdoba and Ripoll (2019) disentangles EIS from EGS. To fit data:
 - ullet EGS is significantly larger than one (c.f. $\gamma_c < 1$)
 - EIS is at most one (c.f. risk-aversion to fit wealth accumulation)

Model Fit: Fertility-Income Profile





Childcare Arrangement

• Childcare arrangements:

$$n \cdot \chi = \left(t_h^{\upsilon/\iota} + (n \cdot m)^{\upsilon}\right)^{1/\upsilon}$$

Set $\chi = 0.18$. Returns to scale within family $\iota = 0.7$ (Folbre 2008)

- Elasticity of substitution: v = 0.5 average share of income spent on childcare by education (SIPP) (Malik 2019)
- Price of full-time childcare: $p_m = \$6,860$ per year for child aged 0-10 (The National Association of Child Care Resource & Referral Agencies 2011)

Skill Evolution for Adults

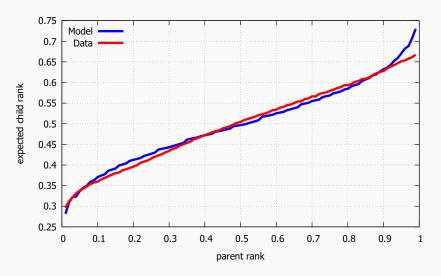
• Human capital of working adults evolves:

$$h_{j+1} = \exp(z) (h_j + \zeta(h_j t))^{\eta}$$

 $\log(z) \sim \mathcal{N}(\mu_z, \sigma_z)$

- $\eta = 0.61$, $\zeta = 0.72$ lifecycle earnings (PSID) (Huggett, Ventura and Yaron 2011)
- $\mu_Z = -0.23$, $\sigma_Z = 0.42$ 2% skill depreciation and lifecycle Gini coefficient of earnings (Huggett, Ventura and Yaron 2011)

Intergenerational Mobility: Model vs Data



• Rank-rank slope = 0.34 (Chetty, Hendren, Kline and Saez 2014)



Other Parameters

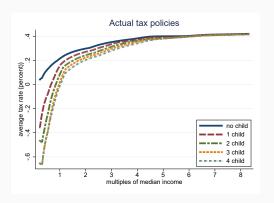
- Firms' production function: capital share $\alpha = 0.33$ and 4% capital depreciation
- Government taxes
 - Income taxes:

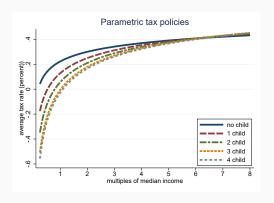
$$\mathcal{T}(y, a, n) = y \cdot (1 - \tau_y^n y^{-\lambda_y^n}) + \tau_a ra$$

where $\{\tau_{v}^{n}, \lambda_{v}^{n}\}_{n=0}^{6}$ estimated using TAXSIM

- Tax rates from McDaniel (2014): $\tau_c = 0.07$ and $\tau_a = 0.27$
- Pension replacement rate: $\pi = 40\%$

Income Taxes: Model vs Data

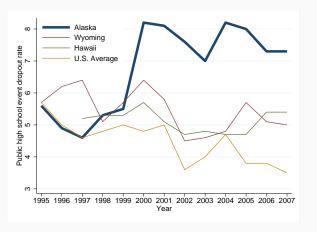




• Child tax benefits (reduction in tax rates) are larger for low-income households



High School Dropout - Alaska vs Other U.S. States



 Wyoming and Hawaii are states with largest synthetic control weights in Kelly, Timilsina and Yonzan (2020)



Baby Bonus in Australia

- **A\$3,000 baby bonus**⁷ to every child born on or after July 1st 2004
- Peter Costello (Treasurer of Australia): "One (baby) for the Mum, one for the Dad, and one for the country"

► other

• $tfr_{AUS,2004} = 1.77 > 1.73 = tfr_{USA,2018}$ before the Covid Baby Bust

⁸More details: (1) Announced on Mar.12th 2004, (2) universal coverage, lump-sum payment, (3) Equivalent to 4 times average weekly earnings, (4) Equivalent to \$2,800 in 2010 USD.

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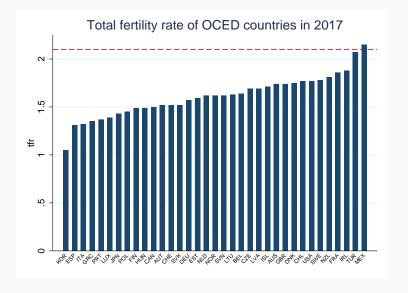
- Drago et al. (2011) finds:
 - Significant fertility responses and evidence for long-term/quantum effects
 - Marginal cost for an additional birth to be at least A\$126,000⁸
 - Larger fertility responses from low-income households
- Gaitz and Schurer (2017) finds that the baby bonus was **ineffective** in boosting learning, socio-emotional or physical health outcomes of pre-school children

► back to results

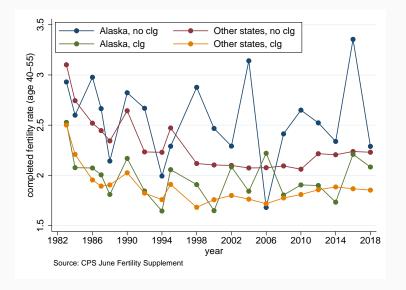
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⁸Equivalent to \$117,600 in 2010 USD. Quantitative model predicts \$130,000 is needed for an additional birth (for the U.S.).

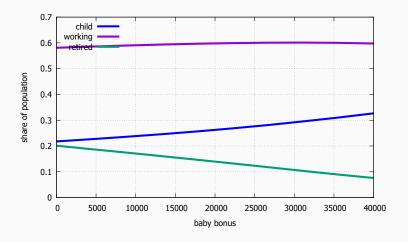
Total Fertility Rate Across Countries



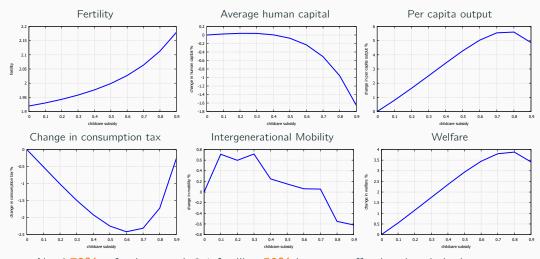
Evidence from Completed Fertility Rates



Change in Demographic Structure



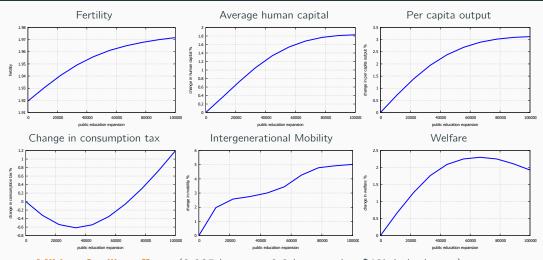
Highlights for Subsidized Childcare ${\mathcal S}$



- Need 78% refund to reach 2.1 fertility, 50% less cost-effective than baby bonus
- Impacts of subsidized childcare are qualitatively similar to baby bonus except that it encourages work (Guner, Kaygusuz and Ventura 2020) and mobility effect is small
- Less progressive because subsidy amount depends on childcare expenditures



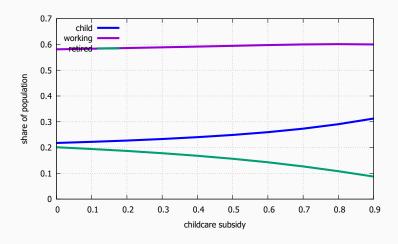
Highlights for Public Education Expansion ${\cal E}$



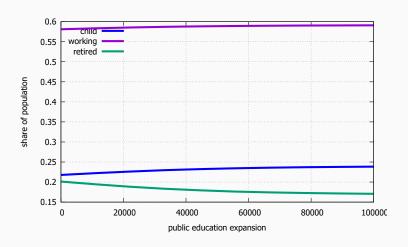
- Mildest fertility effects (0.035 boost vs 0.3 boost using \$40k baby bonus)
- Most effective in improving mobility (4% increase using \$65k)
- Progressive policy with large welfare improvement for low-income households



Change in Demographic Structure



Change in Demographic Structure



Next Steps

Transition path

- Question: How long does aggregate effects take place?
- **Question**: What are the distributional consequences for households in the original steady-state? Majority support for policy reform?
- **Observation**: If the government funds policies using change in τ_c , it is likely that the reforms will not be supported by older agents despite having large gains in the long-run. Private vs social discounting (c.f. Caplin and Leahy 2004)
- Conjecture: The government may need to use transfers within cohorts

Optimal policies

- Question: What is the optimal policy combination to maximize welfare?
- Next month: Explore complementarity or substitutability among policies
- Over the summer: Numerical search for (restricted) optimal policies

