Bounding Fertility Elasticities

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Abstract

I propose a technique for bounding the magnitude of fertility responses to financial incentives. Under mild assumptions, I show that raising the fertility rate by 0.1 children per woman in the United States in 2010 requires the cost of children to fall by $7,514 to $35,966. This bound is tighter than the range provided by meta-analyses of past estimates and is simple to compute for a different country and year. I also discuss the implications of this bound for models with and without endogenous fertility choices.

1 Introduction

How much do fertility choices respond to changes in the cost of children? This question has significant implications for policymakers and economists alike. For many governments that attempt to use transfers to raise fertility in order to combat population aging, it is vital to understand how cost-effective these measures are. For economists, the price elasticity of fertility demand (hereafter fertility elasticity) provides a fundamental discipline to models with endogenous fertility choices and hence our understandings of the decision process. Furthermore, fertility elasticity enables a transparent evaluation of the exogenous fertility assumption, an simplification widely used in structural models devoted to analyzing child-related policies.

Despite the great significance of fertility elasticity and a large body of empirical studies estimating it, little consensus has been reached regarding its quantitative magnitude. "There is considerable disagreement across studies about the effectiveness of pro-natal policies" (Stone (2020)). Estimation based on historical policies has proven to be difficult for several reasons. First, many historical policies are not sizable enough (relative to the cost of children) to induce measurable changes in fertility choices (Bergsvik et al. (2020)). Second, as pro-natal policies are typically nationwide or income-dependent, finding a control group is not straightforward (Gauthier (2005)). This difficulty

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is especially acute given that most pro-natal policies are adopted to address contemporaneous or future fertility decline, but it is unclear whether countries in the control group also face a similar situation (Castles (2003)). Third, family policies often come in bundles of incentives in excess of lowering the cost of children, such as measures encouraging women's labor force participation. Therefore, estimating the fertility responses to the policy bundle differs from estimating the price elasticity of fertility. Last but not least, it is unclear to what extent past estimates predict fertility elasticity in a vastly different time or institutional context.

In this paper, I propose a simple technique for *bounding* the fertility elasticities and discuss its implications. In Section 2, I show that under mild assumptions, a bound can be derived after knowing (1) the prevailing fertility rate and cost of children, (2) the maximum desired fertility, and (3) the cost of children such that potential parents would rather prefer not to have a child. In Section 3, I apply this method to the United States in 2010 and find that a transfer of size between $7,514 and $35,966 raises the fertility rate by 0.1 children per woman, a range tighter than conclusions from meta-analyses of past estimates. Moreover, I demonstrate that the bound is simple to compute for a different country and year of interest. In Section 4, I show that this bound puts additional restrictions on parameters in models with endogenous fertility choices. Last, after comparing the bound with proposed policies in past research, I argue that fertility responses to these policies are non-negligible and could affect the mechanisms being studied.

This paper contributes to two strands of literature. The first one is a large body of empirical studies quantifying fertility elasticity using historical policies (e.g., Milligan (2005), Laroque and Salanié (2008), Cohen et al. (2013), and González (2013) among many others). This paper has the same goal in mind but approaches the question from a different perspective. Rather than exploiting local perturbation of prices, this paper bounds the local elasticity using the global properties of the demand curve. With its flexibility and accuracy, this method is complementary to design-based inquiries and has practical implications for policymakers that want to pursue large-scale family policies. The second literature concerns endogenous determination of fertility (e.g., Becker and Lewis (1973), Barro and Becker (1989), Doepke (2005), Córdoba and Ripoll (2019) among others). Despite being a fundamental moment disciplining the quantitative predictions of these models, fertility elasticity is rarely a targeted moment, primarily due to the challenges in the empirical measurement. This paper makes progress in resolving this difficulty by showing that under mild assumptions, the bound derived here could already put additional restrictions on parameters in this class of models.

2 Theory

Consider an economy populated by representative agents. I denote the Marshallian demand of fertility as $n(p; \boldsymbol{p}^{\text{other}}, y)$ where n is fertility, p is the cost of children, $\boldsymbol{p}^{\text{other}}$ is the price (vector) of other goods, and y is the household's lifetime income.

Assumption 1 The Marshallian demand of fertility $n(p; \boldsymbol{p}^{\text{other}}, y)$ is downward sloping, continuously differentiable, and convex in p.

This assumption is satisfied by most models of endogenous fertility, either with warm glow (e.g., De La Croix and Doepke (2003)) or with dynastic altruism (e.g., Barro and Becker (1989) and Córdoba and Ripoll (2019)). See Section 4 for examples.

Assumption 2 There exists $\overline{p}$ and $\overline{n}$ such that $n(\overline{p}; \boldsymbol{p}^{\text{other}}, y) = 0$ and $n(0; \boldsymbol{p}^{\text{other}}, y) = \overline{n}$.

This is a mild and realistic assumption. For example, let $\overline{p} = y$, the assumption requires that if having a child costs the parents' entire income and leaves no resources for other goods, then the household would prefer not to have a child in the first place.[1] If The existence of $\overline{n}$ reflects biological constraints of childbearing or satiation in preferences.

Proposition 1 The fertility response to price around (n^0, p^0) is bounded by

$$\left.\frac{dn}{dp}\right|_{(n^0,p^0)} \in \left(\frac{n^0}{\overline{p} - p^0}, \frac{\overline{n} - n^0}{p^0}\right). \tag{1}$$

Proof Figure 1 shows the essence of the proof. The Marshallian demand of fertility is given by curve BAC where the coordinates of B and C are $(0, \overline{p})$ and $(\overline{n}, 0)$ correspondingly. Point A denotes the prevailing fertility and cost of children (n^0, p^0). The slope of the demand curve at A (black) is bounded by the slope of AC (red) and the slope of AB (blue) under the Mean Value Theorem and the assumption that curve BAC is decreasing, continuously differentiable, and convex.

In other words, whereas estimation using historical policies relies on local perturbations to p around (n^0, p^0), the method proposed here exploits the global properties of the demand curve under Assumptions 1 and 2 to bound the local slope. After knowing the slope, the local fertility elasticity $\left.\frac{dn}{dp}\frac{\log p}{\log n}\right|_{(n^0,p^0)}$ can be easily computed.

Given the practical difficulties in identifying and quantifying a local shock to p, the bounding approach provides a valuable alternative. Of course, adopting a large-scale pro-natal policy or a

[1] In general, one does not need fertility demand to be exactly zero at $\overline{p}$. The argument goes through as long as one can choose a small $\underline{n}$.

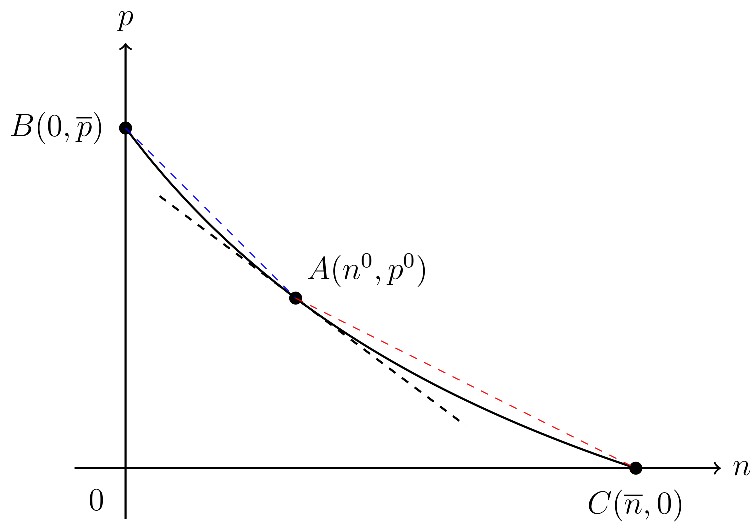

Figure 1: Essence of the Proof

Notes: This figure shows the essence of the proof. The fertility demand curve is given by BAC. The prevailing fertility and cost of children is at point A.

randomized control trial could lead to a more precise estimate for the specific country and year being studied (see Figure 2 for more details). But even in that case, the bound remains complementary to the design-based approach as it provides a prediction of the program's cost-effectiveness *ex ante* and a first check of the results *ex post*.

3 Quantification

In this section, I calculate bound for the United States, compare the results with prior studies, and show that the bound is simple to calculate for another country and year.

3.1 Calculating the Bound

I calculate the bound for the United States in 2010 where the prevailing fertility n^0 is 1.9 and the cost of one child p^0 is \$458,351 for a middle-income household (Córdoba and Ripoll (2019)). I make the following assumption on $\overline{n}$ and $\overline{p}$:

Assumption 3 Set $\overline{n} = 8$. Choose $\overline{p}$ such that an average household with one child lives in poverty for the rest of their lives, i.e.,

$$y - \overline{p} = y^{\text{poverty}}. \tag{2}$$

Proposition 1 makes it clear that the choice of $\overline{n}$ and $\overline{p}$ affects the tightness of the bound. When we express the bound as the change in p needed for a unit increase in n, the bound becomes tighter as

$\overline{n}$ or $\overline{p}$ decreases.

The assumption that $\overline{n} = 8$ is probably conservative towards the high end as the prevailing ideal number of children is around 2.5 children per woman (Stone (2018)). The assumption that $\overline{p}$ is the cost of children that makes an average married household fall into poverty status after one childbirth is also a likely upper bound for $\overline{p}$. Thus, there are reasons to believe that the bound could be tighter under alternative reasonable assumptions.

Following Córdoba and Ripoll (2019), I set $y =$ \$2,083,219. Using the federal poverty guideline in 2010, I set $y^{\text{poverty}} =$ \$941,520. This implies $\overline{p} =$ \$1,141,699.

Applying Proposition 1, I find that to raise the fertility by 0.1 children per woman, the change in p needed is between \$7,514 and \$35,966. This change in p can be brought about by policies such as a baby bonus, a universal basic income, a child allowance, or a fully-refundable Child Tax Credit (CTC) expansion.

3.2 Discussions

One advantage of the bound is that it tighter than to meta-analyses of past estimates. For example, Stone (2020) summarizes and harmonizes a large number of research estimating fertility elasticity from historical policy changes, mostly in low-fertility countries. He concludes that "an increase in the present value of child benefits equal to 10% of a household's (annual) income can be expected to produce between 0.5% and 4.1% higher birth rates" (see Figure 2).

To make the measures comparable, I convert the bound in this paper into percentages using the median annual household income in 2010 (\$49,445). The bound predicts that an increase in the present value of child benefits equal to 10% of a household's (annual) income can produce between 0.72% and 3.46% higher birth rates.[2] As can be seen, the bound proposed here provides a narrower range of predictions for the fertility effects of pro-natal policies.

Another advantage of the bound is that it is simple to compute for a different country and year as long we know the prevailing (n^0, p^0), y, and y^{poverty}. In particular, p^0 acts as a *sufficient statistic* that captures differences in policies and social norms that affect the costs of child-raising across time and space while n^0 is the revealed preference for fertility demand under prevailing prices.

For example, in the United Kingdom in 2016, the fertility rate is 1.79 children per woman and the cost of raising a child from birth to 21 years old is £231,843 (CEBR (2016)).[3] The lifetime income y is approximately £1,400,000. p^{poverty} is chosen to be 60% of y (£840,000) following the

[2]In Figure 2, there are some estimates that lie outside of the bound derived in this paper. This could arise because these policies were implemented in a different time and institutional setting.

[3]Unlike Córdoba and Ripoll (2019), this cost does not include the opportunity costs of time in childraising. Thus, p^0 could be biased downward. As a result, the bound reported below will be wider than the case where the bias is eliminated.

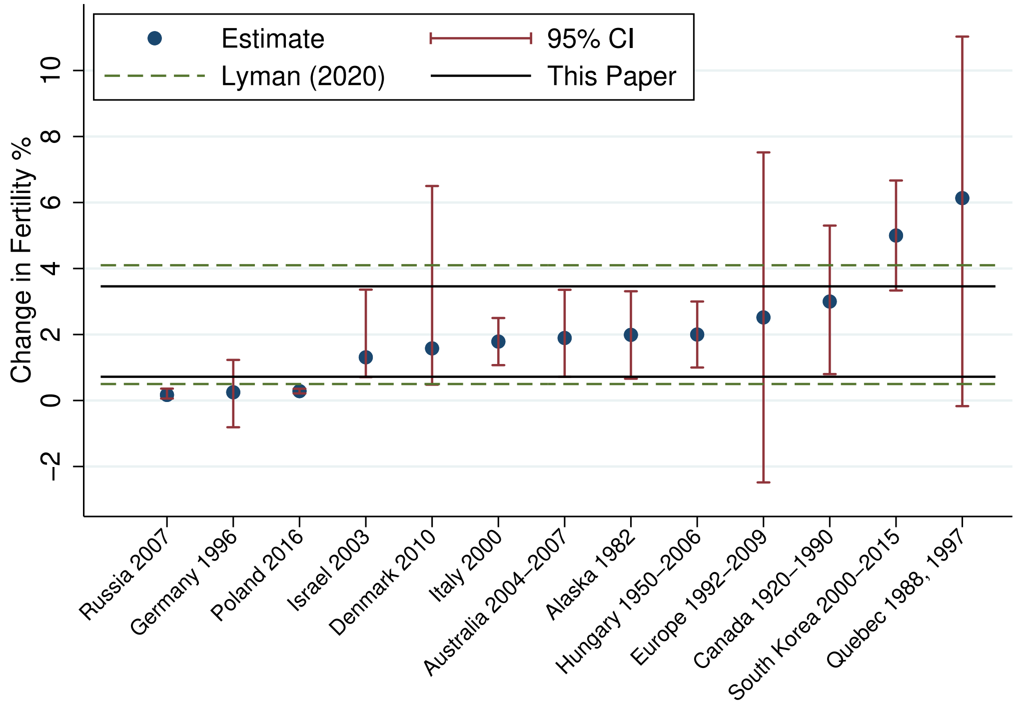

Figure 2: Past Estimates of Fertility Elasticities

Notes: This figure presents a summary of fertility elasticities estimated using historical policies and different bounds. As the goal of the paper is to pin down the price elasticity of fertility, I select policy changes including universal child benefits, baby bonuses, and universal basic income from the summary file compiled by Stone (2020). When there a multiple estimates exploiting the same policy change, I take the average across studies. The dots represent point estimates of fertility responses to a transfer with a net present value that is 10% of a household's annual income. The red intervals corresponds to 95% confidence intervals. The two horizontal dashed lines represent the bound of fertility elasticity suggested by Stone (2020). The two horizontal solid lines are the bound derived in this paper for the United States in 2010.

definition used by the British government's Department of Work and Pensions. Holding Assumption 3 unchanged and apply Proposition 1, it can be seen that to raise the fertility by 0.1 children per woman in the United Kingdom, the change in p required is between £3,733 and £18,333.

To summarize, the accuracy and flexibility of this method make it a useful benchmark for policymakers that are pursuing large-scale policies to raise fertility.

4 Further Implications

Beyond informing policymakers how costly it is to raise fertility, the bound disciplines models with endogenous fertility and allows one to evaluate the exogenous fertility assumption in models that analyze child-related policies. Below are some examples.

4.1 Endogenous Fertility w/ Dynastic Altruism

Consider a model of fertility choice with dynastic altruism following Barro and Becker (1989). Agents solve

$$U_0 = \max_{c_t, n_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\prod_{i=0}^{t-1} n_i \right)^{1-\varepsilon} \cdot \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$w_t + (1+r_t)k_t = c_t + n_t(\chi_t + k_{t+1}) \quad \forall t$$

and the initial k_0. It is assumed that $\beta, \varepsilon, \sigma \in (0,1)$ to ensure that children are goods and $\varepsilon \leq \sigma$ for the second-order condition to hold.[4]

Using first-order conditions, the Marshallian demand of fertility in this economy is given by

$$n_t = (\beta(1+r_{t+1}))^{\frac{1}{\varepsilon}} \left[\frac{\chi_{t-1}(1+r_t) - w_t}{\chi_t(1+r_{t+1}) - w_{t+1}} \right]^{\frac{\sigma}{\varepsilon}} \tag{3}$$

where the net cost of creating a descendant at time t is $p_t \equiv \chi_t(1+r_{t+1}) - w_{t+1}$. Therefore, as the net cost of children p_t falls by 1 percent, fertility n_t increases by $\frac{\sigma}{\varepsilon}$ percent *ceteris paribus* in this model. Relating this elasticity to the numerical bound for the U.S. in 2010 computed in the previous section, the value of $\frac{\sigma}{\varepsilon}$ should lie between 0.67 and 3.23. Combining with the prior assumptions on σ and ε, the full set of restrictions are

$$0 < \varepsilon \leq \sigma < 1 \quad \text{and} \quad \sigma < 3.23 \cdot \varepsilon. \tag{4}$$

As can be seen, the bound puts *additional restrictions* on the choice of σ and ε. For example, Manuelli and Seshadri (2009) satisfies these restrictions with $\sigma = 0.62$ and $\varepsilon = 0.35$; Córdoba (2015) considers $\sigma = 0.3$ and $\varepsilon = 0.288$; and Daruich and Kozlowski (2020) uses $\sigma = 0.5$ and $\varepsilon = 0.25$. Under different choices of $\overline{n}$ or $\overline{p}$, the restrictions from the bound could become more binding for this class of models.

A point worth noting here is that the model presented above generates a Marshallian demand that satisfies Assumption 1 in Section 2 but not Assumption 2. In light of Assumption 2, the isoelastic demand in Equation (3) can be interpreted as an approximation of the true underlying fertility demand around (n^0, p^0). The existence of $\overline{p}$ and $\overline{n}$, and hence the bound, puts restrictions on the *local properties* of $\frac{\sigma}{\varepsilon}$. The same interpretation applies to the endogenous fertility model with warm glow utility presented below.

[4]Jones and Schoonbroodt (2010) discuss an alternative set of assumptions which implies that the quantity and quality are substitutes rather than complements as in standard Barro-Becker models.

4.2 Endogenous Fertility w/ Warm Glow Utility

Consider a model of fertility choice with warm glow utility. Agents solve

$$\max_{c,n} \quad c + \beta \cdot \frac{n^{1-\sigma}}{1-\sigma}$$

subject to

$$c + n \cdot p \leq y.$$

As before, it is assumed that $\sigma \in (0, 1)$ to ensure that children delivers positive utility.

With interior solutions, the Marshallian demand of fertility is given by

$$n^* = \left(\frac{p}{\beta}\right)^{-1/\sigma}. \tag{5}$$

This implies that when p falls by 1 percent, n^* rises by $1/\sigma$ percent. Thus, if one calibrates this model to the U.S. economy in 2010, the bound suggests that the value of σ, which is often chosen exogenously, should lie between 0.31 and 1. Relative to the standard assumption on σ, the bound further requires that $\sigma > 0.31$.

In general, one can use the bound to validate comparative static results in other, potentially more complicated, models of endogenous fertility before using the model to conduct counterfactuals. For instance, Zhou (2022) shows that in a heterogeneous-agent model of quantity-quality trade-off calibrated to the U.S. economy in 2010, raising aggregate fertility rate by 0.1 children per woman requires the cost of children to fall by $15,000 - a number that is within the bound.

4.3 Models w/ Exogenous Fertility Assumption

The bound also permits a transparent evaluation of the exogenous fertility assumption in structural models that analyze child-related policies.

For example, Guner et al. (2020) consider a policy counterfactual where the per-child tax credit rises about $800 per child per year, amounting to about $12,000 in net present value terms for eligible households.[5] The optimal policy in Mullins (2019) is a Negative Income Tax on mothers that is equivalent to an additional $82 per week over 17 years, i.e., a transfer greater than $50,000 in net present value. Daruich (2022) shows that the welfare-maximizing early-childhood development subsidy is around $80,000. A real-world policy example is the expansion of the Child Tax Credit in the American Rescue Plan that increases the annual transfer from $2,000 dollars to $3,600 per child under age 6 and $3,000 per child ages 6 through 17. The net present value of this expansion

[5]The net present values are calculated using a 3% annual discount rate.

is above \$30,000 per child for fully eligible families.

In these models, the exogenous fertility assumption is typically justified based on design-based studies that find little fertility responses to financial incentives. With prior calculations, however, it can be seen a \$10,000 reduction in the cost of children should increase fertility by 0.03 to 0.13 from the baseline level of 1.9 children per woman. Thus, the proposed policies are likely going to lead to non-negligible fertility responses and a dilution of family resources, triggering the quantity-quality trade-off mechanism à la Becker and Lewis (1973). This channel could be strong enough to overturn the direct effects of the transfer on children's outcomes. As a result, the implications of these child-related policies on children's human capital, intergenerational mobility, and social welfare could be affected and deserves further inspection.

5 Conclusion

Fertility elasticity is important to both policymakers and economists, yet pinning it down using historical policies has been proven difficult. In this paper, I propose a technique for bounding the fertility elasticities. Under mild assumptions, I show that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman in the United States in 2010. This range is tighter than the conclusion from past meta-analyses and is simple to compute for a different country and year. The bound puts further restrictions on parameters in endogenous fertility models and allows one to assess the relevance of the exogenous fertility assumption in structural models that analyze child-related policies.

There are several interesting avenues for future research. First, one can make additional assumptions and derive the bound for different subgroups of the population. Second, one can use survey questions to guide the choices of $\overline{p}$ and $\overline{n}$. Last, one can compute the bound for different countries and years and compare them with design-based estimates in a systematic way.

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