The Macroeconomic Consequences of Family Policies*

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Abstract

Most developed countries have adopted family policies, but little is known about their macroeconomic consequences. I build a heterogeneous-agent overlapping-generations framework with the child quality-quantity trade-off, an endogenous demographic structure, and childcare arrangements choices to analyze family policies. With the model calibrated to U.S. data, my key findings indicate that a $30,000 cash reward to childbirth raises fertility to the replacement level but worsens human capital distribution and social mobility due to the quality-quantity trade-off. Nevertheless, social welfare improves by 1.6% in the long run as the demographic structure adjusts. Such long-run gains in welfare, however, require a transition path with higher child-related government spending, which may hurt the existing households' welfare depending on how those expenditures are funded. In contrast to cash rewards, in-kind family policies are less cost-effective in raising fertility but have their own advantages: subsidized childcare encourages parents' labor force participation and reduces inequalities in wage growth, while public education expansions improve children's outcomes and social mobility.

JEL classification: E62, H31, H52, J11, J13

Keywords: Family policies, quality-quantity trade-off, demographic structure

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1 Introduction

With growing concerns over population aging, family policies[1] are at the forefront of policy discussion in developed economies. Policymakers and economists also believe that family policies are effective instruments to "invest in our kids, our families, and our economic future" (The White House, 2021), and to improve social mobility (Pulliam and Reeves, 2021). Today, public expenditures on family policies have exceeded 2% of national GDP on average among OECD countries.[2]

Despite the great importance and wide implementation of large-scale family policies, we know little about their macroeconomic implications to date. Are family policies cost-effective in raising fertility, reducing fiscal burden, and improving social welfare? When fertility is endogenous, will generous child benefits still improve children's outcomes and raise social mobility? How do short-run policy effects differ from long-run ones, or those in transition, given that changes in demographic structure take decades to come about? Last but not least, how should policymakers choose between in-cash benefits such as baby bonuses versus in-kind ones like subsidized childcare?

In this paper, I analyze family policies through the lens of a quantitative heterogeneous-agent model with overlapping generations in the general equilibrium. Compared with the existing work on family policies, the model I develop has three novel features. First, unlike most design-based studies that focus on fertility or child outcomes in isolation, this model allows households to choose the number of children to have and simultaneously decide on child quality through child human capital investments and inter vivos transfers. Both margins are important since they correspond to the two main goals of family policies: raising fertility and improving children's outcomes. They are also closely linked in the model due to *quality-quantity trade-off* a lá Becker and Lewis (1973). When parents increase fertility to take advantage of child benefits, the marginal cost of providing the same level of child quality goes up. Therefore, as I show in this paper, policies that aim at boosting fertility may worsen children's outcomes.

The joint determination of child quality and quantity has further implications for aggregate variables via *composition effects*. With heterogeneities in human capital and wealth, parents respond differently to family policies when making fertility and child quality decisions. Some parents may increase fertility more than others, meaning that their children comprise a larger share of the population in the future. Due to the intergenerational transmission of traits (e.g., human capital), aggregate variables gravitate over time towards the traits of parents with strong fertility responses.

Second, while most structural models with endogenous fertility consider two-period-lived agents, the framework in this paper explicitly models a realistic life cycle with childhood, working-age, and

[1] Family policies are social programs and laws designed to promote and enhance family formation, reproduction, and child-rearing (Bogenschneider, 2011). Family policies include both in-cash benefits (e.g. baby bonuses and child tax benefits) and in-kind ones (e.g. subsidized childcare, public pre-K).

[2] See https://data.oecd.org/socialexp/family-benefits-public-spending.htm for more details.

retirement totaling nine periods each representing 10 years. This allows the model to quantify the *demographic structure effects* of family policies: following an increase in the population growth rate, the burden of pension payments under the pay-as-you-go (PAYG) system is relieved, but public education expenditures for children rise. The rich demographic structure also highlights the distributive consequences of family policies across generations, especially along the transition path.

Third, unlike most models with endogenous fertility where each child costs a fixed amount of time, I consider endogenous *childcare arrangements* where parents can choose to either take care of children at home by themselves or utilize market childcare services at a price. Besides allowing the model to better match childcare utilization patterns in the data, this feature facilitates comparisons between family policies that are in-cash (e.g., baby bonuses or child allowances) and those that are in-kind (e.g., subsidized childcare vouchers). Distinguishing between in-cash and in-kind transfers is important for predicting policy effects on parents' labor force participation, which in turn affects inequality in wage growth over the life cycle due to human capital accumulation on the job.

The quantitative strength of each of the mechanisms discussed above depends on the magnitude of the fertility response to financial incentives, hereafter denoted *fertility elasticities*. In the extreme case where fertility is exogenous, i.e., fertility elasticity is zero, child-related benefits simply have an income effect on children's quality and improve social mobility if the benefits are progressive (Daruich, 2018; Mullins, 2019). However, if households increase fertility after family policies are adopted (i.e., fertility elasticity is greater than zero), these mechanisms generate novel and sometimes counter-intuitive predictions. I discipline the magnitude of fertility elasticities using model parameters calibrated to match key empirical regularities in the United States in 2010. I also provide external validations using the Alaska Permanent Fund Dividend (APFD) and several other programs to show that the quantitative predictions of the model are consistent with existing design-based evidence.[3]

The calibrated model generates three main findings. First, a baby bonus (cash reward to childbirth) of $30,000 in 2010 dollars[4] boosts the average fertility rate from 1.9 children per woman to 2.1 children per woman, the replacement level. Parents with lower human capital respond more strongly to the policy by having more additional children. Surprisingly, I find that such a child benefit is not effective at raising child human capital or social mobility. Parents optimally reduce child educational investments by an average of 4% due to the quality-quantity trade-off, and this reduction is larger among low-income parents. The heterogeneous reductions in educational investments, coupled with composition effects due to differential fertility responses, will lower average human

[3]Additional validation exercises include comparisons with the Australian baby bonus, the Spanish child benefits, and Georgia's Cherokee Land Lottery of 1832. See Appendix C for details.

[4]To put this number in perspective, this baby bonus is similar to the size of expansions to the Child Tax Credit (CTC) from 2010 to 2021 in net present value (author's own calculation). A bonus of this size would offset the average cost of raising on child by 19% using estimates from U.S. Department of Agriculture (USDA).

capital by 2% and intergenerational income mobility by 1.8% in the long run.

Second, despite reductions in average human capital and social mobility, long-run social welfare[5] rises by 1.6% in consumption equivalents due to the demographic structure effects of the $30,000 baby bonus. As the old-age dependency ratio[6] falls following an increase in fertility, the government is able to reduce tax rates while still balancing the fiscal budget. By considering an endogenous demographic structure (age distribution), the model provides a novel counter-argument to the existing theoretical literature on family policies which argues that the first-order stochastic dominance (FSD) of human capital distribution is a sufficient condition for higher social welfare (Chu and Koo, 1990).

Third, the transition towards potential long-run welfare improvements, however, requires increased child-related government expenditures during the first few decades. While the old-age dependency ratio falls gradually, the total dependency ratio[7] spikes before converging to its long-run level. These additional expenditures are quantitatively significant, but policy discussion often overlook them due to a greater focus on pension liabilities in the long-run. The welfare consequences for existing agents can be very different from those in transition or in the long-run steady-state, depending on how these expenditures are funded. For example, if the government chooses to balance the budget from period to period using changes in the consumption tax, most households in the current economy will experience welfare losses.

Last, I use the model to study the macroeconomic impacts of alternative policies that are in-kind, such as subsidized childcare and expanded public education funding. These policies are less cost-effective in raising fertility than the cash rewards but offer other advantages. In particular, subsidized childcare encourages parents to work in the labor market since it substitutes home production of childcare with market childcare services. The resulting increase in labor force participation, especially among parents with low human capital, reduces inequality in wage growth. Public education expenditures, on the other hand, have the smallest fertility effects. Nonetheless, among all policy instruments studies in this paper, public education is the most effective at improving children's outcomes and boosting social mobility.

Related Literature

This paper relates to and builds on four strands of literature. The first studies fertility in a macroeconomic context and analyzes its relationship to fiscal stability, economic growth, intergenerational mobility, and inequality. Notable examples include Barro and Becker (1989), Doepke (2004), Greenwood et al. (2005), Manuelli and Seshadri (2009), De Silva and Tenreyro (2020), Cavalcanti et al. (2021), and Jones (2020) among many others. The closest papers to the work presented

[5] The average utility of a new-born under the veil of ignorance. See Section 6 for further discussion.

[6] The number of retired people divided by the size of the working population.

[7] The number of retired people plus the number of children, divided by the total working population.

in this paper are de La Croix and Doepke (2003) and Kim et al. (2021). de La Croix and Doepke (2003) study how higher inequality reduces growth through the channel of fertility differentials. While I adopt their framework featuring general equilibrium, heterogeneous agents, endogenous fertility, and human capital formation, I extend the canonical model in various dimensions and use the model to study aggregate impacts of family policies. In a recent paper, Kim et al. (2021) adopt the de La Croix and Doepke (2003) model and add a status competition mechanism to explain the fertility-income relationship in Korea. The authors then study the impacts of pro-natal transfers and education taxes. However, their work assumes two-period-lived agents, while this paper allows for a richer life-cycle structure. As a result, demographic structure effects, an important motivation for most family policies, can be quantified.

Second, this paper is related to the literature on positive and normative aspects of government policies concerning fertility, redistribution, and education. On the positive side, Daruich (2018) and Abbott et al. (2019) are the closest works to mine. Both papers use heterogeneous-agent overlapping-generations models to study large-scale education policies. I differ from these papers by modeling endogenous fertility choices and considering the interaction of family and education policies. On the normative side, Guner et al. (2020) study the optimal design of child-related transfers with heterogeneous households making both intensive and extensive labor force participation choices while keeping fertility exogenous. This paper contributes to the literature by studying the implications and optimal design of family policies with endogenous fertility responses.

Third, this paper relates to a large body of empirical work on the fertility effects of family policies (e.g., Milligan, 2005; Drago et al., 2011; González, 2013; Laroque and Salanié, 2014). Meta-analyses of these studies have concluded that "the directional finding that pro-natal benefits boost fertility is nearly uniform" (Stone, 2020). An increase in the present value of child benefits equal to 10% of annual household income would lead to a 0.5% to 4.1% increase in the total fertility rate (McDonald, 2006; Stone, 2020).[8] These papers, however, are not suited to address the broader consequences of family policies beyond fertility, such as the effects on output, human capital, social mobility, and welfare. The design-based approach is also unable to predict the consequences of family policies in the long-run when affected children become parents themselves, changing the distribution of the underlying population of interest. This paper takes a structural approach to answer these questions.

Finally, the paper contributes to a growing literature evaluating the impacts of transfers or subsidies to families with children. Prominent examples include Akee et al. (2010), Dahl and Lochner (2012), Mullins (2019), and García et al. (2020), among many others. Using both design-based methods and structural models with exogenous fertility, the conventional wisdom in the literature

[8]The distinction between quantum versus tempo effects on fertility (i.e., the completed fertility rate versus the total fertility rate) is discussed in more detail in Section 4.

is that transfers to families with children have positive impacts on children's outcomes, including improving health and education attainment and reducing criminal behavior (Schanzenbach et al., 2021). This paper utilizes experimental evidence from this literature to inform parameter choices in the calibration. Further, this literature exclusively evaluates policy effects of transfers to families with existing children and abstracts away from fertility responses, whereas I argue that their conclusions could not be directly applied to extrapolating the aggregate impacts of family policies.

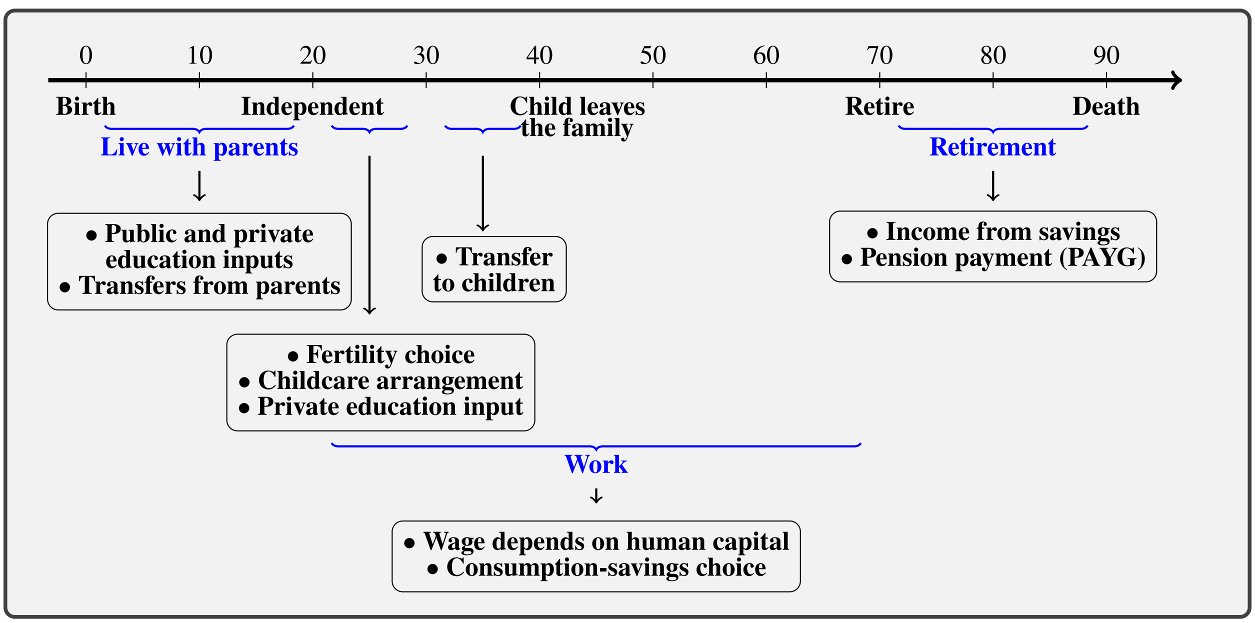
The rest of the paper is organized as follows. In Section 2, I present the quantitative model. Section 3 describes the calibration of the model in detail. I conduct validation exercises in Section 4. Section 5 presents the main policy counterfactual results of the paper. Section 6 discusses optimal policy design and Section 7 concludes.

2 Model

2.1 Household

Consider overlapping generations of households[9] that live up to 90 years old. I split the life cycle into 9 periods, each representing 10 years, and is indexed by $j \in \{0, 1, 2, \ldots, 8\}$. Period j therefore reflects age $j \times 10$ through $(j + 1) \times 10$.

Figure 1: Model's Life Cycle



[9]In this paper, I model unitary/collective decision-making within families and abstract away from intra-household bargaining. Hereafter, I use agents, households, and adults interchangeably.

The life cycle of each agent in the model is presented in Figure 1. Children live with their parents from age 0 to 20, during which time they make no choices and receive human capital investments from both public and private sources. Children also receive inter vivos transfers from parents when they become independent and form families in their 20s. Households choose fertility, childcare arrangements, and private child human capital investments from age 20 to 30.[10] Between ages 30 and 40, parents determine the amount of inter vivos transfers that their children would receive at the beginning of the next period.

People work from age 20 to 70, and their earnings depend on the amount of efficiency units, i.e., human capital, supplied to the market. Human capital for working adults evolves over time following an age-dependent learning-by-doing process:

$$h_{j+1} = L_j(h_j, t_w, z_{j+1}) \tag{1}$$

where z_{j+1} is an idiosyncratic, uninsurable shock to human capital that occurs at the beginning of period $j+1$; h_j is the amount of human capital at period j; and t_w is the time worked in period j. Agents have unit time endowment and supply labor inelastically, with the exception of period 2 where they trade off providing childcare at home against supplying labor in the market.[11]

Agents retire at the end of period 6 (age 60 to 70). Retired adults receive income from savings and pension payments for the remainder of their lives. Households make consumption-savings choices in each period and face an age-specific survival rate δ_j.

At the beginning of period 2 (age 20-30), the state variables for young adults are the human capital level h and assets a. Both h and a are endogenously determined in equilibrium by the young adults' parents during previous periods. Agents choose consumption c, savings a', fertility n, total time spent providing at-home childcare t_h, amount of market childcare purchased for each child m, and private education investment for each child e.[12] Parents' value $V_2(\cdot)$ is composed of flow utility from consumption $u(c/\Lambda(n))$ where consumption expenditure c is divided by equivalence scale $\Lambda(n)$ and the discounted continuation value $V_3(\cdot)$. Their maximization problem is given by:

$$V_2(h,a) = \max_{c,a',n,t_h,m,e\geq 0} u(c/\Lambda(n)) + \beta \mathbb{E} V_3(h', a', n, \mathbb{E}h_k)$$

subject to

$$n \cdot \chi = \left(t_h^{v/\iota} + (n \cdot (m + \mathcal{S}))^v \right)^{1/v} \qquad \text{[time cost]}$$

[10]In Appendix A.2, I argue that the main results of this paper are robust to allowing for endogenous timing of childbirth where parents can also give birth in their 30s.

[11]I have experimented with adding endogenous labor supply with leisure in the utility function. Quantitative results remain largely unchanged, but computation time is significantly increased, especially for the transition path. Therefore, I omit labor supply in the baseline model.

[12]Following de La Croix and Doepke (2003), fertility choice n is continuous in the model.

$$y = wh \cdot (1 - t_h) \qquad \text{[labor income]}$$

$$(1+\tau_c)(c + p_m \cdot n \cdot m + e \cdot n) + a' = (1+r)a + y - \mathcal{T}(y, a, n) + \mathcal{B} \cdot n \qquad \text{[BC]}$$

$$h' = L_2(h, 1 - t_h, z') \qquad \text{[learning OTJ]}$$

$$h_k = G(h, \mathcal{E}, e, \epsilon) \qquad \text{[skill formation]}$$

To raise n children, parents need to produce $n \cdot \chi$ amount of childcare (in time units). Parents could satisfy this need either by spending their own time at home (t_h) or by purchasing market childcare services m. The government could choose to provide in-kind childcare $\mathcal{S} \in [0, \chi]$ free of charge to parents. These two forms of childcare are combined in a constant-elasticity-of-substitution (CES) production function where υ governs the elasticity of substitution. Household production of childcare enjoys an economy of scale with parameter $\iota \in (0, 1)$. This captures the fact that taking care of two children at home simultaneously costs less than two times the hours needed to take care of a single child (Folbre, 2008).

The opportunity cost of home production of childcare t_h is hours spent in the labor market. The household's labor income y is the product of the market wage w, human capital h, and time worked $(1 - t_h)$. Total resources available to the parents consist of risk-free assets a multiplied by the gross interest rate $(1 + r)$, labor income y, net taxes payment $\mathcal{T}(y, a, n)$, and the total amount of baby bonuses received $\mathcal{B} \cdot n$ where $\mathcal{B}$ is chosen by the government. The household divides resources into savings a' and different expenditures, including consumption c, total spending on market childcare $p_m \cdot n \cdot m$, and private education expenditures $e \cdot n$. I use p_m to denote the price of market childcare relative to consumption goods.[13] All expenditures are subject to a proportional consumption tax τ_c.

Child human capital production function $G(h, \mathcal{E}, e, \epsilon)$ combines parents' human capital h, public education $\mathcal{E}$, private investment e, and idiosyncratic ability shocks ϵ which are unknown to the parents. To keep the model tractable, I make a simplifying assumption that parental investments in child human capital take place when children are between age 0 and 10 while public education $\mathcal{E}$ plays the major role from age 6 to 20. The function $G(\cdot)$ captures the overall human capital production function that spans age 0 to 20 without explicitly modeling several stages of production. Moreover, to the extent that the bulk of childcare needs $n \cdot \chi$ are non-educational, i.e., preparing food or changing diapers, childcare arrangements do not affect child human capital directly in the model.[14] Parents can invest in their children's human capital through monetary investments

[13] In the model, the supply of childcare services is perfectly elastic at baseline price p_m. Since each period represents ten years, adjustments of capital and labor in and out of the childcare industry equate childcare prices to long-run marginal costs so long as the industry is perfectly competitive. In the short run, of course, relative prices of childcare may change in response to family policies.

[14] As Guryan et al. (2008) have noted, educational childcare time accounts for at most one quarter of the time that parents spend with their children. In a recent paper, Chaparro et al. (2020) develop a model of childcare with both quality and quantity aspects of maternal and non-maternal care. In their model, effects on child human capital are

e. Since the government chooses public education expenditures $\mathcal{E}$, the model could replicate the public provision of high-quality childcare or pre-K by adopting high $\mathcal{S}$ and $\mathcal{E}$ at the same time.

In the economy, households face an inter-temporal borrowing constraint $a' \geq 0$ that is standard in the class of Aiyagari-Huggett models. Moreover, parents cannot invest negative amounts of resources in children's education, i.e., $e \geq 0$. As a result, public investment $\mathcal{E}$ serves as a lower-bound of total educational investments received by each child.[15]

Parents' maximization problem at age 30-40 is given by:

$$V_3(h, a, n, \mathbb{E}h_k) = \max_{c,a',a_k \geq 0} u(c/\Lambda(n)) + \beta \mathbb{E} V_4(h', a') + v(n, \mathbb{E}h_k, a_k)$$

subject to

$$\begin{aligned} y &= wh \\ (1+\tau_c)c + a' + n \cdot a_k &= (1+r)a + y - \mathcal{T}(y, a, n) \qquad (2) \\ h' &= L_3(h, 1, z') \end{aligned}$$

where parents choose consumption, savings, and the amount of transfers to be received by each child at the beginning of next period (a_k). Parents face intergenerational borrowing constraints so that they are not allowed to make negative transfers to children, i.e., $a_k \geq 0$. I use $v(n, \mathbb{E}h_k, a_k)$ to denote parents' preferences over child quantity, child human capital, and inter vivos transfers. The parametric form of $v(\cdot)$ is discussed in detail in Section 3.1.

For $j \in \{4, 5, 6\}$, households solve a simple consumption-savings problem with idiosyncratic shocks to human capital. For tractability, I assume that parents and children no longer interact in these periods.[16] The maximization problem is given by:

$$V_j(h, a) = \max_{c,a' \geq 0} u(c/\Lambda(0)) + \beta \mathbb{E} V_{j+1}(h', a') \qquad j \in \{4, 5, 6\}$$

subject to

$$\begin{aligned} (1+\tau_c)c + a' &= (1+r)a + y - \mathcal{T}(y, a, 0) \\ h' &= L_j(h, 1, z) \end{aligned}$$

proportional to both care quality and time "exposed" to each type of care, but they do not explicitly consider non-educational time spent with children. The model in this paper separates the essence of childcare into non-education chores χ and endogenous educational investments, money or time, captured by e and $\mathcal{E}$.

[15] Since public education funding depends on local taxation (Kotera and Seshadri, 2017), one might worry that the amount of education resources received by each child is not uniformly $\mathcal{E}$. The child human capital production function $G(\cdot)$ allows for this possibility through its dependency on parents' human capital h. In other words, $\partial G(\cdot)/\partial h$ captures not only the genetic transmission of ability but also ability transmitted via other channels such as residential segregation.

[16] Otherwise, parents need to keep track of all children's state variables, as well as those of their grandchildren. This will make the problem computationally infeasible.

Finally, retired agents solve

$$V_j(h,a) = \max_{c,a'\geq 0} u(c/\Lambda(0)) + \beta V_{j+1}(h,a') \qquad j \in \{7,8\}$$

$$(1+\tau_c)c + a' = (1+r)a + \pi \cdot wh - \mathcal{T}(0,a,0)$$

$$V_9(\cdot) \equiv 0$$

where household income is composed of risk-free assets and pension payments $\pi \cdot wh$, and where π denotes the pension replacement rate.[17]

2.2 Firms

There is a representative firm in the economy that hires labor and borrows capital from households to produce final goods with Cobb-Douglas technology

$$Y = A \cdot K^{\alpha} H^{1-\alpha} \tag{3}$$

In Equation (3), K is aggregate capital used in production and H is total efficiency units employed. Total factor productivity A is normalized to be one.[18]

Physical capital depreciates at rate δ_K after use. With competitive factor markets, the equilibrium wage and risk-free interest rate are given by:

$$r = \alpha \left(\frac{K}{H}\right)^{\alpha-1} - \delta_K, \quad w = (1-\alpha)\left(\frac{K}{H}\right)^{\alpha}$$

2.3 Government

The government collects revenues from taxing labor income, capital income, and household expenditures (including spending on market childcare and children's education). Government expenditures include public education, family policies, pension payments, and other per-capita policy-invariant expenditures denoted $\mathcal{X}$. The government balances the budget from period to period.[19]

I use $\{\mu_j\}_{j=0}^{8}$ to denote the distribution of households across state space and $\{\omega_j\}_{j=0}^{8}$ to denote the fraction of each age group in the population with total mass normalized to one at each date. The

[17] For simplicity, I assume that pension payments are not subject to labor income taxes.

[18] I abstract away from population externalities that could affect aggregate production such as pollution (Bohn and Stuart, 2015) and ideas creation (Jones, 2020). This choice is made since (1) the literature on measurement of population externalities is still developing, and (2) the results will change in expected ways once positive/negative externalities are incorporated.

[19] I discuss the possibility of allowing for government borrowing in Section A.4.

government budget is given by:

$$\underbrace{\left(\sum_{j=2}^{6}\omega_j\int\mathcal{T}(y_j^*,a_j^*,n_j^*)\,d\mu_j\right)}_{\text{labor and capital income taxes}}+\underbrace{\tau_c\left(\sum_{j=2}^{8}\omega_j\int c_j^*\,d\mu_j+\omega_2\int n^*\cdot(p_m\cdot m^*+e^*)\,d\mu_2\right)}_{\text{consumption taxes}}=\underbrace{\underbrace{(\omega_0+\omega_1)}_{\text{mass of children}}\cdot\mathcal{E}}_{\text{public education expenditures}}$$
$$+\underbrace{\pi\cdot\left(\sum_{j=7}^{8}\omega_j\int wh\,d\mu_j\right)}_{\text{pension payments}}+\underbrace{\omega_2\left(\underbrace{\int n^*\cdot\mathcal{B}\,d\mu_2}_{\text{baby bonus}}+\underbrace{\int(1+\tau_c)\cdot n^*\cdot p_m\cdot\mathcal{S}\,d\mu_2}_{\text{subsidized childcare}}\right)}_{\text{family policy expenditures}}+\underbrace{\mathcal{X}}_{\textbf{other spendings}} \tag{4}$$

2.4 Equilibrium

I use t to denote time. The equilibrium of the economy is defined as a tuple composed of:

- decision rules $\{c_t^*, a_t'^*, n_t^*, m_t^*, t_{h,t}^*, e_t^*, a_{k,t}^*\}_{t=0}^{\infty}$
- prices $\{w_t^*, r_t^*\}_{t=0}^{\infty}$
- government policies $\{\mathcal{T}_t(\cdot), \tau_{c,t}, \mathcal{B}_t, \mathcal{S}_t, \mathcal{E}_t\}_{t=0}^{\infty}$
- distribution of agents $\{\{\mu_{j,t}\}_{j=0}^{8}, \{\omega_{j,t}\}_{j=0}^{8}\}_{t=0}^{\infty}$

such that households maximize utility, prices clear labor and capital markets, the government balances the budget in each period, and the distribution of agents evolves following the dynamics shaped by household decision rules, exogenous labor market shocks z, and the ability shock for children ϵ.

The evolution of the human capital distribution[20] from parents to children is given by:

$$\mu_2'(h)=\frac{1}{N}\iint n^*(x)\mathbb{1}_{h_k^*(x,\epsilon)<h}\,d\mathcal{Q}(\epsilon)\,d\mu_2(x) \tag{5}$$

where $\mathcal{Q}$ denotes the distribution of the child ability shock ϵ; $h_k^*(x,\epsilon)$ is the human capital of a child whose ability shock is ϵ and parents' human capital is x; and N is the aggregate fertility rate in the economy defined as:

$$N=\int n^*(x)\,d\mu_2(x) \tag{6}$$

In a stationary equilibrium of the economy, decision rules, prices, and distributions are unchanged over time. The size of the population, however, could vary over time as the aggregate fertility rate is not necessarily at the replacement level. The process defined in (5) is a multi-type

[20]For simplicity of exposition, here I omit the notation of distribution over initial assets.

branching (or Galton-Watson) process. Under mild conditions that are satisfied by the model, it can be shown that a stationary distribution exists and is unique (see Mode, 1971; Chu, 1990).

2.5 Welfare

To facilitate comparisons between different government policies, I define social welfare in the long run ($\mathcal{W}$) as the *average value* of households at the beginning of their life cycle:

$$\mathcal{W} = \int V_2 \, d\mu_2 \tag{7}$$

where both V_2 and μ_2 are endogenous equilibrium objects. The welfare metric $\mathcal{W}$ measures the expected utility of a newborn child under the veil of ignorance by summing up the discounted utility flow from later life periods. When making welfare comparisons, I convert the changes in $\mathcal{W}$ into percentage changes in consumption equivalents using the utility function $u(\cdot)$.

The welfare metric $\mathcal{W}$, on the other hand, fails to capture the policy effects for households who are *already alive* when the policy is enacted, or for those who will be born in the *transition phase*. I discuss how these people are affected by family policies in Section 5.3. Section 6 contains more discussion of different welfare criteria in models with endogenous fertility.

No matter how welfare is defined, government policies have the potential to improve the well-being of agents for two reasons. First, childbearing and child-rearing carry fiscal externalities as parents do not internalize the effects of having an additional child or investing in child human capital on the future tax base and government revenues. Atomic parents take the age structure $\{\omega_j\}_{j=0}^{8}$, distribution $\{\mu_j\}_{j=0}^{8}$, and tax rates as given, but these objects will change when a mass of parents adjust their decisions on fertility and educational investments. Due to parents' lack of property rights over children's future output, the equilibrium level of child human capital investments is too low relative to the planner's solution (Schoonbroodt and Tertilt, 2014). In contrast to Schoonbroodt and Tertilt (2014), with heterogeneous agents in the model, the equilibrium fertility level could be either too high or too low depending on parental characteristics.[21]

Second, parents face both inter-temporal and intergenerational borrowing constraints due to imperfections in capital markets. Parents cannot borrow against their children's future income or their own future income to finance current expenditures. Government policies can overcome these inefficiencies by providing in-cash or in-kind transfers (Daruich, 2018; Abbott et al., 2019).

[21]The policy implications of this observation, however, are less straightforward than one might think. Due to intergenerational persistence of human capital, a tempting conclusion is that the policymaker should restrict fertility among the poor (Chu and Koo, 1990). Setting aside the rampant ethical concerns over such a policy, I show that this argument is not valid in Sections 5.1 and 6.3.

2.6 Discussion of Mechanisms

In this section, I discuss the mechanisms through which family policies affect the economy. In particular, I highlight how the novel features of this model (namely, fertility and child quality choice, endogenous demographic structure, and childcare arrangements) make the model's predictions on the overall impacts of family policies distinct from those in standard models.

2.6.1 Quality-Quantity Trade-off

Consider the effect of an increase in the baby bonus $\mathcal{B}$ on inter vivos transfers to each child (a_k).[22] The first-order condition of a_k is given by:

$$\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial a_k} = \lambda_3 \cdot n$$

where λ_3 denotes the Lagrangian multiplier on the parents' budget in period 3.

When fertility is exogenous, i.e., n is fixed, the increase in $\mathcal{B}$ is an *income transfer*, which implies that a_k rises unambiguously due to income effects. As the marginal utility of consumption MU_c decreases, a_k needs to increase to keep the first-order condition satisfied:

$$\underbrace{\lambda_3 \downarrow}_{\text{income effect on } MU_c} \cdot n \quad \Rightarrow \quad \frac{v(n, \mathbb{E}h_k, a_k \uparrow)}{\partial a_k}$$

When fertility is endogenous, however, the increase in $\mathcal{B}$ is a *price change*. The direction in which a_k changes is ambiguous. As fertility n rises due to more generous child benefits,[23] it affects the first-order condition of child quality a_k in three ways. First, it interacts with a_k in parents' preferences $v(\cdot)$. If quality and quantity are complements (substitutes), parents demand higher (lower) a_k *ceteris paribus*. Second, rising quantity could potentially offset the income effect since higher n raises the marginal utility of consumption via the change in the equivalence scale $\Lambda(n)$. Last, the marginal cost of child quality a_k rises because it is proportional to fertility n due to their

[22]The same argument applies to child human capital investments e.

[23]This holds if child quantity is not a Giffen good. See McDonald (2006) and Stone (2020) for supporting evidence.

interaction in parents' budget constraint (see Equation (2)).[24]

$$\underbrace{\frac{\partial v(n\uparrow, \mathbb{E}h_k, a_k\mathbf{?})}{\partial a_k}}_{\text{interaction in preferences}} = \underbrace{\lambda_3?}_{\text{change in } MU_c \text{ as } n\uparrow} \cdot \underbrace{n\uparrow}_{\textbf{fertility response}} \tag{8}$$

While most design-based research finds positive effects of child benefits on the outcomes of *existing* children (Heckman and Mosso, 2014; Schanzenbach et al., 2021), considering endogenous fertility in the model allows for the possibility that child quality, either as human capital or transfers received, could fall when family policies become more generous. This possibility is of both theoretical interest and empirical relevance since (1) increasing fertility is one of the major goals of family policies, and (2) pro-natal transfers have clear and significant impacts on fertility choices (McDonald, 2006; Stone, 2020; see also Section 4).

2.6.2 Composition Effects

Since parents differ by human capital h and asset holdings a, they respond differently to the same baby bonus $\mathcal{B}$ or subsidized childcare $\mathcal{S}$.[25] With an endogenous quality-quantity trade-off, families with stronger fertility responses gain representation in the economy over time as their children account for a larger fraction of the future population. This change in the fertility differential across households, combined with the intergenerational transmission of traits, leads to *composition effects* whereby aggregate variables, such as average human capital, gravitate towards those of the households with the highest fertility elasticities.

From a theoretical perspective, composition effects generate an interesting scenario where aggregate human capital decreases despite positive policy effects on individual children's human capital. Empirically speaking, composition effects are important for the evolution of aggregate variables such as economic growth (de La Croix and Doepke, 2003) and public opinion on family values (Vogl and Freese, 2020). Therefore, policymakers should consider composition effects when evaluating and designing family policies.

2.6.3 Demographic Structure Effects

With endogenous fertility, family policies change the population growth rate, and, as a result, the demographic structure $\{\omega_j\}_{j=0}^{8}$. This has profound implications for the government budget

[24]Becker and Lewis (1973) named this last effect the quality-quantity trade-off. In this paper, I abuse this term slightly to denote the overall effects of an increase in fertility on child quality. Using twin births as instruments, some recent studies (e.g., Black et al., 2005 and Angrist et al., 2010) find little evidence of such a trade-off. A recent study by Mogstad and Wiswall (2016), however, overturns that conclusion by relaxing the linear specification constraint. They find evidence of a trade-off between quality and quantity for larger families and complementarities in small families.

[25]See Drago et al., 2011 and Table 2 for empirical evidence.

constraint (4) since demographic structure determines how each source of revenue or expenditure is weighted.

Most macroeconomic structural models with endogenous fertility assume two-period-lived agents (see de La Croix and Doepke, 2003 and Kim et al., 2021, among others). From the government budget perspective, an increase in the population growth rate leads to an unambiguous increase in the fiscal burden in this class of models since there are fewer tax-paying adults to finance public education expenditures for children. As a result, tax rates need to rise. Most developed countries that actually adopt family policies, however, have another story in mind where tax rates should be *lower* instead of higher in the long run. The key missing piece in this analysis is the presence of retired households receiving pension payments.

By considering a realistic life cycle with childhood, working-age, and retirement in the model, higher fertility rates reduce the old-age dependency ratio $\left(\sum_{j=7}^{8}\omega_j\right) / \left(\sum_{j=2}^{6}\omega_j\right)$. Therefore, the model allows for the possibility that the government may *reduce tax rates* in the long run after the adoption of family policies. Depending on the scale of the family policies, long-run tax rates may either rise or fall since higher fertility could boost the total-age dependency ratio $\left(\sum_{j=7}^{8}\omega_j + \sum_{j=0}^{1}\omega_j\right) / \left(\sum_{j=2}^{6}\omega_j\right)$. I denote the effects of family policies through changes in the mass of each age group as *demographic structure effects*.

2.6.4 Childcare Arrangements

Consider a comparison between two family policies, a baby bonus $\mathcal{B}$ and subsidized childcare $\mathcal{S}$. In standard macroeconomic models with endogenous fertility, each child costs a fixed amount of time for parents.[26] Since subsidized childcare reduces time costs, total income in this class of models is given by:

$$y = wh\underbrace{(1-(\chi-\mathcal{S})}_{\text{hours worked}}\cdot n) + n\cdot\mathcal{B}$$

For parents with human capital h, a baby bonus $\mathcal{B}$ can be replicated by subsidized childcare with $\mathcal{S} = \frac{\mathcal{B}}{wh}$ in these models. Therefore, parents should be indifferent between in-cash and in-kind child benefits. They are also expected to react to each policy in the same way with respect to labor supply and child-related decisions.

These predictions are nevertheless counter-intuitive since parents can always purchase market childcare with in-cash transfers, but in most settings parents cannot freely convert in-kind benefits

[26]Modeling child costs as time costs rather than goods costs is crucial for explaining the negative fertility-income relationship (Jones et al., 2008). See the discussion in Doepke and Tertilt (2016). A notable exception that considers the use of market childcare is Bar et al. (2018). Those authors show that the adoption of market childcare services and rising income inequality explain the recent rise in fertility among high-income married women in the U.S.

(childcare vouchers) back to cash for spending on other uses. Therefore, in-cash transfers should be (weakly) preferred by parents to in-kind benefits of the same face value. Moreover, empirical evidence also indicates that parents react differently to in-cash versus in-kind benefits. Upon receiving a baby bonus, parents do not increase their labor supply but rather choose to stay at home and use less market childcare (González, 2013). In contrast, upon receiving subsidized childcare, parents increase their labor supply drastically and substitute home care for (subsidized) market care (Milligan, 2005). Finally, use of market childcare is increasing in maternal education,[27] but traditional models without childcare arrangements are unable to account for this pattern.

In this model, considering parents' childcare arrangement choices provides an explanation for patterns observed in the data and offers a natural way to compare family policies of different kinds. With market childcare, a substitute for home production of childcare, available at a uniform price, richer parents use more market childcare services than poorer ones. A government-offered childcare subsidy $\mathcal{S}$ will therefore be equivalent to a baby bonus $\mathcal{B}$ of the same face value for parents with high human capital who are already spending more than $\mathcal{S}$ on market childcare. Subsidized childcare simply reduces their out-of-pocket expenditures. On the other hand, parents with low human capital who receive a childcare subsidy $\mathcal{S}$ would rather have an in-cash transfer $\mathcal{B}$ of the same face value since they prefer spending the money on consumption or education. This is because the price of market care is higher than their own wage. As a result, subsidized childcare $\mathcal{S}$ is "binding" for low-human capital parents in the sense that the in-kind policy pushes them into the labor market if they want to take up the benefits. Their welfare improvements will be smaller than if they were offered a baby bonus $\mathcal{B}$ of the same face value. As they work more, however, these parents accumulate more human capital through learning-by-doing, and the economy sees a reduction in the inequality of life-cycle wage growth across parents.

2.7 Summary of Mechanisms

As discussed above, the framework contains a unique mix of modeling ingredients. Therefore, the model explains more observed data patterns (e.g., childcare usage) and speaks to several important mechanisms through which family policies affect the economy.

The quantitative magnitude of these mechanisms depends crucially on the fertility responses to family policies (the fertility elasticities). In particular, if child quantity is fixed, then family policies will have (1) a simple income effect on children's outcomes, (2) no composition effects, and (3) no demographic structure effects. In the quantitative analysis of the model, fertility elasticities across households are disciplined by the calibrated model parameters. In Section 4, I show that the model predictions on fertility responses are consistent with existing empirical evidence.

[27]See Figure 5 for evidence. Also see Bar et al. (2018) and Chaparro et al. (2020).

3 Calibration

In this section, I discuss the parameterization, calibration procedures, and model fit. Following the literature, I pick some parameters exogenously, most of which either have standard values or have observable counterparts. The other 14 parameters are calibrated inside the model by matching steady-state moments to the United States in 2010.

3.1 Preferences over Quantity and Quality

Parents' preferences over child quantity and quality is given by:

$$v(n, \mathbb{E}h_k, a_k) = \underbrace{\Psi(n)}_{\text{child discounting}} \cdot \underbrace{(\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k))}_{\text{utility from quality}} \tag{9}$$

$$\Psi(n) = 1 - \exp(-\psi n) \tag{10}$$

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0,1), \quad x \in \{\mathbb{E}h_k, a_k, c\} \tag{11}$$

where parents value child quality weighted by the child discounting function $\Psi(n)$ (c.f., Barro and Becker, 1989; Scholz and Seshadri, 2007; Kim et al., 2021). The utility function of child quality and consumption is governed by γ, known as the elasticity of intergenerational substitution (EGS) (c.f., Cordoba et al., 2016).[28] I use $\theta \cdot u(\mathbb{E}(h_k)) + \nu \cdot u(a_k)$ as a first-order approximation to general preferences over child quality and transfers. In Appendix A.1, I argue that results in this paper are robust to other common specifications used in the literature, including separable preferences (de La Croix and Doepke, 2003), quality and quantity being substitutes (Jones and Schoonbroodt, 2010), and dynastic altruism (Daruich and Kozlowski, 2020).

Parameters $\theta = 2.85$ and $\nu = 0.29$ are calibrated to match the average human capital investment as a share of income[29] and average inter vivos transfers of \$48,381 in 2000 dollars (Daruich, 2018). I calibrate $\psi = 2.3$ to match the average fertility calculated using the CPS June Fertility Supplement data from 2008 to 2014.

[28] Since EGS is designed to match interactions across generations, its magnitude and interpretations are different from that of the elasticity of intertemporal substitution (EIS) used to capture risk-aversion in business cycle models. More specifically, when utilities from consumption and child quality are separable, it is common to assume that EIS and EGS are the same to ensure the existence of long-run steady states (see Barro and Becker, 1989; Soares, 2005). For recent work modeling EGS and EIS jointly with non-separable utilities, see Carlos Córdoba and Ripoll (2019).

[29] Daruich (2018) uses the PSID and the CEX to estimate that average expenditures on each child below age 12 include 18 hours of "quality" hours per week and \$1,966 (in 2000 dollars) spent on childcare and educational investments. These two expenditures combined amount to 13.4% of median family income. The calibration targets this moment by computing average education investments e plus expenditures on market childcare services m. These moments are also similar to those reported in Lee and Seshadri (2019).

Figure 2: Identification of γ

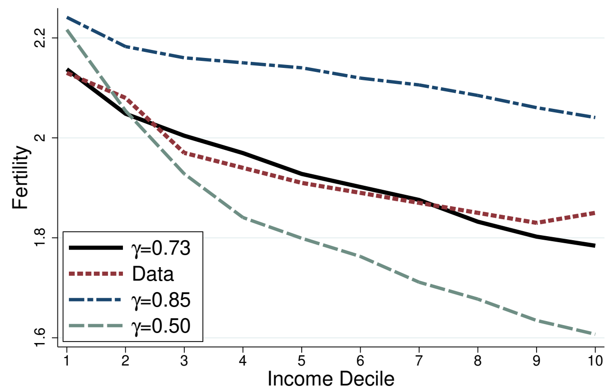
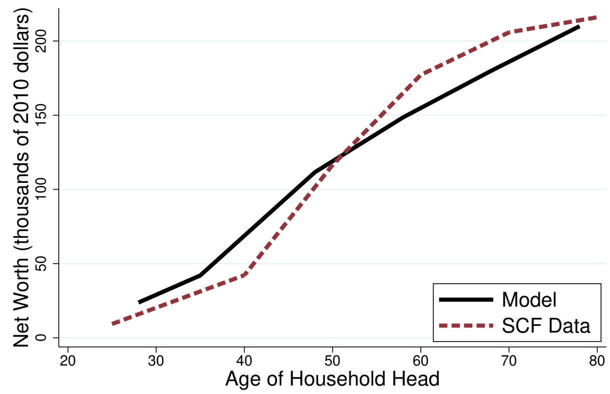


Figure 3: Non-Targeted: Net Worth by Age



Notes: Figure 2 plots the relationship between family income and fertility in the model under different γ. Fertility rates by income are calculated using the number of live births ever had for married women between 40 and 55 years old in the CPS June Fertility Supplement data from 2008 to 2014.
— Figure 3 plots the median net worth by age in the model and in the data. Median net worth in the model is calculated using net asset position a integrated over the equilibrium distribution of households by age. In the data, median net worth by age of the household head is collected from the Federal Reserve System estimated using Survey of Consumer Finances (SCF).

Conditional on $\{\theta, \nu, \psi\}$ and other parameters of the model, γ is an important parameter governing the fertility elasticity, with higher γ resulting in smaller fertility responses to financial incentives.[30] Smaller fertility responses, in turn, imply larger responses in child quality due to the income effects discussed in Section 2.6. I identify γ by matching the fertility-income profile in the data. The identification argument follows Cordoba et al. (2016): controlling for other parameters, higher γ implies a larger marginal rate of substitution of quantity for quality and a flatter fertility-income profile. Figure 2 shows a numerical illustration of the identification. I choose $\gamma = 0.73$ such that the model best fits the observed relationship between fertility and income.[31]

Even though γ is restricted to be between $(0, 1)$, the model generates a surprisingly good fit to the data in terms of life-cycle asset accumulation. Figure 3 shows the median net worth in the model and that in the data estimated using Survey of Consumer Finances (SCF). As non-targeted moments, the model generates a life-cycle profile of net worth that is similar to the data both in trend and in level.

[30]The illustrative model in Appendix D.2 shows that responses in fertility are proportional to $1 - \gamma$. Soares (2005) uses a similar argument in discussing fertility responses to changes in adult longevity and child mortality.

[31]See Appendix A.3 for discussion and tests of robustness to alternative measures of fertility.

3.2 Child's Human Capital Production

I parameterize the child human capital production function as

$$h_k = G(h, \mathcal{E}, e, \epsilon) = \underbrace{Z}_{\text{scalar}} \cdot \underbrace{\epsilon}_{\text{shock}} \cdot \underbrace{h^{\rho}}_{\text{spillover}} \cdot \left(\underbrace{\mathcal{E}^{\xi}}_{\text{public education}} + \underbrace{e^{\xi}}_{\text{private input}} \right)^{\kappa/\xi} \tag{12}$$

$$\log(\epsilon) \sim \mathcal{N}\left(-\frac{\sigma_{\epsilon}^2}{2}, \sigma_{\epsilon}^2\right)$$

Parameter Z is a scaling parameter that governs the overall scale of the economy. It also enables me to normalize the total factor productivity A to be one. I choose $Z = 2.5$ so that the median income of families in the model is one, corresponding to the Census-estimated household median income of \$49,445. Parameter σ_{ϵ} governs the dispersion of idiosyncratic shocks to children's ability. I pick $\sigma_{\epsilon} = 0.58$ to match the dispersion of earnings for young households.[32] I calibrate $\rho = 0.30$ to match the rank-rank intergenerational mobility estimated by Chetty et al. (2014). Figure 4 shows that the model generates a good fit both in absolute upward mobility and relative mobility.

I use ξ to parameterize the elasticity of substitution between public and private education inputs. $\xi = 0.9$ is calibrated to match the relationship between educational spending and household income using data from the Consumer Expenditure Survey (CEX) which includes tuition, test preparation, tutoring, books, and supplies. Public education $\mathcal{E}$ in the baseline economy is chosen to match the \$12,000 annual expenditure per student reported by the National Center of Education Statistics (NCES).

The last parameter to be calibrated in the child human capital production function is κ which governs the productivity of educational investments. I use RCT evidence from García et al. (2020) to inform the value of κ. García et al. (2020) evaluate early childhood programs (ABC/CARE) from the 1970s. The yearly cost of the program was \$18,514 per participant (in 2014 dollars) for five years. Treated children were followed into adulthood with education and incomes observed by researchers. García et al. (2020) estimate that children's lifetime labor income increases by 1.3 dollars in net present value for every dollar invested. I apply a similar policy in the model by expanding existing public education $\mathcal{E}$ by \$17,000 (in 2010 dollars) for five years. The policy targets children with parents at the 10^{th} percentile of earnings. The counterfactual is evaluated at a small scale so that prices and taxes remain unchanged. By comparing the discounted life-time earnings of children in the treatment group versus those in the baseline economy, κ is calibrated to be 0.13.

[32] I calibrate σ_{ϵ} to match the Gini coefficient of income among married households age 23 to 29 in the CPS-ASEC data (2008-2014).

3.3 Costs of Children and Childcare

The household equivalence scale $\Lambda(n)$ is taken from the OECD standard:

$$\Lambda(n) = 1.7 + 0.5 \cdot n$$

where n is the number of children residing with the family.

Recall that the childcare requirement constraint, i.e., time costs of children, is given by:

$$n \cdot \chi = \left(t_h^{\upsilon/\iota} + (n \cdot (m + \mathcal{S}))^{\upsilon} \right)^{1/\upsilon}$$

I choose $\chi = 0.18$ following estimates by Folbre (2008) (Table 6.2) calculated using data from the American Time Use Survey (ATUS).[33] I calibrate the economies of scale in providing childcare at home $\iota = 0.7$ to match the estimates by Folbre (2008) (Table 6.4).[34] The parameter governing the elasticity of substitution between home care and market care υ is calibrated to be 0.38 to match the average expenditure on childcare as a fraction of total family income using estimates from Herbst (2018).[35] Lastly, I choose the price of full-time market care for a child aged 0 to 10, p_m, to be \$6,860 per year in 2010 following the statistic reported by the National Association of Child Care Resource & Referral Agencies (NACCRRA, 2011).[36] Figure 5 shows that as non-targeted moments, the model generates the pattern of childcare usage by parents' income decile that fits the data from the National Survey of Early Care and Education (NSECE).

3.4 Other Parameters

Human capital of adults evolves with age according to

$$h_{j+1} = L_j(h_j, t_w, z') = \exp(z') \left[h_j + \zeta_j (h_j \cdot t_w)^{\eta} \right] \tag{13}$$

[33] Folbre (2008) documents that an average child in a two-parent household spends about 22 hours per week in active non-parental care, and 23 hours per week in active parental care (adjusting for presence of both parents). Therefore, I calculate χ, the total active care required by one child as a fraction of parents' total time endowment, as $\chi = (22 + 23)/((24 - 6.5) \cdot 7 \cdot 2) \approx 0.18$ assuming 6.5 hours of sleep for each parent.

[34] Folbre (2008) reports that for two-parent families, the active parental time per child is 1.5 times higher in one-child families than that of two-child families.

[35] Herbst (2018) uses Wave 8 of the 2008 SIPP panel, which covers winter and spring of 2011. The results show that average childcare expenditures in the whole sample are \$134.44 in 2013 dollars, which translates to 16% of the median household income. The calibration targets this moment by computing the average share of income spent on market childcare services, measured by $n \cdot m \cdot (1 + \tau_c)$ integrated over the equilibrium distribution.

[36] The average costs of full-time childcare across all U.S. states are \$9,303 (infant), \$7,377 (4-year-old), and \$4,753 (school-age) for child care centers; family child care, on the other hand, costs \$6,926 (infant), \$6,131 (4-year-old), and \$4,405 (school-age) (NACCRRA, 2011 Appendix 1). I take an age-weighted average of these costs to calculate full-time childcare costs for children aged 0 to 10.

Figure 4: Model Matches Mobility

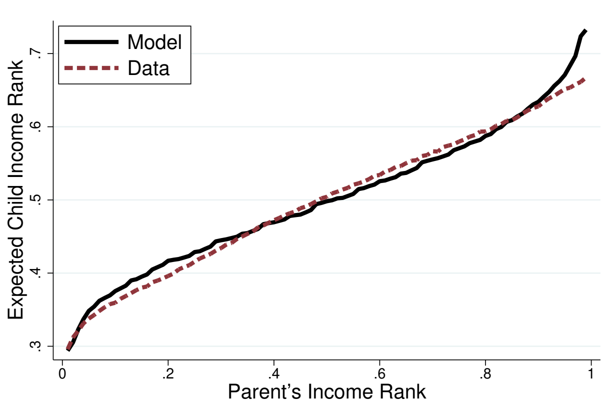
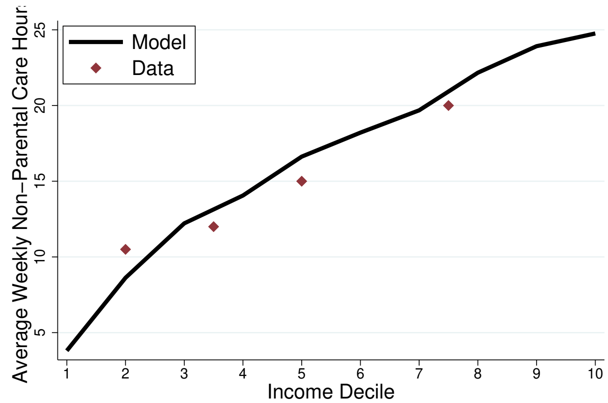


Figure 5: Non-Targeted: Childcare Usage



Notes: Figure 4 plots the relationship between parents' income rank and children's expected income rank in the calibrated model and that estimated by Chetty et al. (2014).
— Figure 5 plots the average weekly non-parental cares received by children aged 0-2 conditional on parents' income decile. The data estimate is obtained using the public-use file of National Survey of Early Care and Education (NSECE) wave 2012. I calculate the average weekly non-parental care hours in the data by maternal education categorized into "less than high school", "high school", "some college", and "bachelor's degree and above". The education category is then mapped into income deciles using median income by education and income distribution estimates from the Current Population Survey (CPS).

$$\log(z) \sim \mathcal{N}(\mu_z, \sigma_z)$$

I calibrate $\eta = 1.22$ to match the heterogeneous growth rate of income by initial income decile (see Figure 6).[37] Parameters $\{\zeta_j\}_{j=2}^{5}$ and $\sigma_z = 0.38$ are calibrated to match the life-cycle profile of average household income and its dispersion.[38]. I choose $\mu_z = -0.23$ exogenously so that human capital depreciates at an annual rate of 2%.

Following Heathcote et al. (2017), I parameterize income taxes as

$$\mathcal{T}(y, a, n) = y \cdot (1 - \tau_y^n y^{-\lambda_y^n}) + \tau_a r a \tag{14}$$

where $\{\tau_y^n, \lambda_y^n\}$ denote the level and progressivity of taxes depending on the number of children residing in the household while τ_a denotes the linear capital income taxes. I obtain $\{\tau_y^n, \lambda_y^n\}$ using simulated data from TAXSIM provided by the NBER (see Figure 7) and use linear interpolation to calculate $\{\tau_y^n, \lambda_y^n\}$ when n is not an integer. Having an additional child significantly reduces the tax burden faced by households through various programs such as the Earned Income Tax Credit (EITC), Dependent Care Tax Credit (DCTC), and Child Tax Credit (CTC). The tax system is pro-

[37]This model reproduces the well-documented hump-shaped life-cycle earnings profile as well as a Gini coefficient of human capital that is increasing in age (Huggett et al., 2006). These two facts are reconciled by considering a profile of learning ability $\{\zeta_j\}_{j=2}^{5}$ that is decreasing in age and the self-production of human capital with $\eta > 1$.

[38]I calculate average income and the Gini coefficient of income by age using data from married households in the CPS-ASEC data (2008-2014). I have experimented with panel data methods a lá Huggett et al. (2011) and the results are quantitatively similar.

Figure 6: Income Growth By Initial Decile

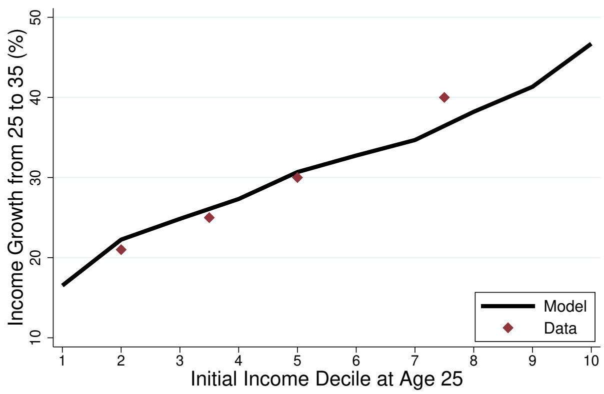
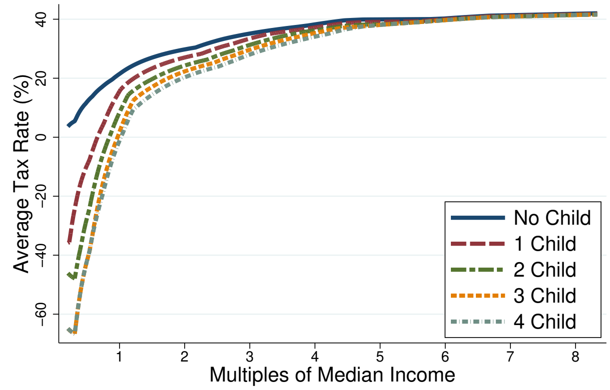


Figure 7: Labor Income Taxes



Notes: Figure 6 plots the growth rate of average income from age 25 to 35, depending on the initial income decile at age 25. For married households in the CPS-ASEC data (2008-2014), I calculate the growth rate of average household income by four education categories: "less than high school", "high school", "some college", and "bachelor's degree and above". The initial income decile of each education category is calculated using the income distribution at age 25. — Figure 7 plots average tax rates depending on household income and number of dependent children residing with the family calculated TAXSIM.

gressive, with lower-income households receiving subsidies. As household income grows, labor income taxes converge to 40%. Following McDaniel (2007) and Daruich and Fernández (2020), I choose capital income tax $\tau_a = 0.27$ and consumption taxes $\tau_c = 0.07$. The pension replacement rate π is set to be 40%.

In the production function of the representative goods-producing firms, I choose the capital share α to be 0.33 following the standard literature and set the capital depreciation rate δ_k at 4% per year. Lastly, I choose the annual discount rate $\beta = 0.98$ and calculate the age-specific mortality rate $\{\delta_j\}_{j=0}^{8}$ using the actuarial life table from Social Security.

Table 1 provides a summary of the model parametrization.

4 Validation

In this section, I discuss the main external validation of the fertility responses to financial incentives. The purpose of validation exercises is to lend extra credibility to the quantitative predictions of the model before evaluating policy counterfactuals. In Appendix C, I provide further confirmation of the model's predictions using evidence from the Australian baby bonus, the Spanish child benefits, and Georgia's Cherokee Land Lottery.

The main validation exercise exploits empirical evidence from the Alaska Permanent Fund Dividends (APFD). The dividend was officially established in 1982 after the discovery of petroleum and increased state revenues. Every year, it gives uniform transfers to all residents regardless of income, employment or age. In particular, the program allows a parent, guardian, or other authorized

Table 1: Calibrated Parameters

	Interpretation	Value	Source		Interpretation	Value	Source
	Preferences				Child human capital production		
β	discount rate	0.98^{10}	standard	Z	normalizing scalar	2.50	median income =1
γ	elasticity of substitution	0.73	CPS	σ_ϵ	ability shock dispersion	0.58	PSID
ψ	fertility preference	2.30	CPS	ρ	intergenerational spillover	0.30	Chetty et al. (2014)
θ	quality preference	2.85	PSID	ξ	substitution of education	0.9	ATUS
ν	transfer preference	0.29	PSID	$\mathcal{E}$	public education	0.16	NCES
				κ	input productivity	0.13	García et al. (2020)
	Childcare arrangement				Adults' human capital evolution		
χ	childcare cost	0.18	ATUS	η	learning curvature	1.22	PSID
ι	economies of scale at home	0.7	ATUS	$\{\zeta\}_{j=2}^{5}$	learning level	misc.	PSID
υ	substitutability of care	0.38	SIPP	μ_z	skill depreciation	-0.23	PSID
p_m	price of full-time care	0.15	NACCRRA (2011)	σ_z	shock dispersion	0.38	PSID
	Taxes and pension				Firm production function		
τ_y^n, λ_y^n	tax levels and progressivity	misc.	TAXSIM	A	total factor productivity	1	normalization
τ_c	consumption tax	0.07	McDaniel (2007)	α	capital share	0.33	standard
τ_a	capital income tax	0.27	McDaniel (2007)	δ_k	capital depreciation	0.04^{10}	standard
π	pension replacement rate	0.40	OECD Database				

Notes: This table displays the list of parameters used in the model. Parameters in red are calibrated within the model while those in black are chosen exogenously.

representative to claim a dividend on behalf of a child, while Alaska law imposes no requirements whatsoever on how parents use a child's dividend. As a result, the policy has pro-natal effects even though it is not explicitly advertised as a family policy that encourages fertility.

The APFD is an ideal policy environment to validate fertility elasticities in the model for four reasons. First, compared with other family policies which are usually less than a few thousand dollars in net present value per child (McDonald, 2006; Luci-Greulich and Thévenon (2013)), the net present value that parents could receive and use with an additional child under the APFD is almost \$20,000.[39] With sizable benefits, it is more likely that the APFD would have meaningful and observable impacts on people's behavior especially when it comes to the important and irreversible decision of having a child. Second, compared with other family policies which are typically means-tested or depend on birth order, the APFD has a clearer implementation with more than 90% of the state population filing for the application historically.[40] Third, the APFD mimics a universal basic income for parents plus a refundable Child Tax Credit (CTC) for children. Given that it is not advertised as a policy that aims to encourage childbirth, its effects on fertility provide a conservative benchmark for family policies that are explicitly pro-natal and which may change parents' behaviors through preferences or information.[41] Lastly, compared with other family policies that mostly took place in European countries, the cultural and institutional background of Alaska is much more similar to the overall U.S. system that I calibrate the model to match.

To implement the APFD in the model, I first recalibrate the model to match the aggregate data in 1980s.[42] Then, I implement the APFD by transferring \$1,500 (annually) to every household member, including both parents and children. Parents will receive this transfer (for themselves) every period until they die. They are also entitled to receive the children's share of dividends before their children become independent. I normalize the amount of the transfer by median household income in Alaska relative to the total U.S. I conduct the policy experiment in the partial equilibrium without changing prices and distribution. The model predicts that the completed fertility rate, i.e., the total number of children that parents end up having, increases by 0.16 compared with the scenario without the AFPD.[43] The model also predicts heterogeneous fertility responses across households.

[39] I calculated the average payment to be around \$1,500 per year. Since the amount of the dividend is tied to the performance of the overall stock market, future payment uncertainties might affect people's responses in fertility. Given that childbirth is an irreversible decision, I argue that a mean-preserving spread of the dividend payment would reduce households' fertility responses. Therefore, since the model generates fertility elasticities that are consistent with people's choices under uncertainty, the results on child benefits without uncertainty is likely conservative.

[40] Cowan and Douds (2021) argue that the migration effect, also known as "population magnets effect," of the APFD was not large empirically, with net migration rates around one-tenth of a percent in the sample period.

[41] As one example, Kearney and Levine (2015) show that the introduction of the widely viewed MTV reality show *16 and Pregnant* reduces childbearing among teens.

[42] In particular, I re-calibrate the tax function $\mathcal{T}(y, a, n)$ using the TAXSIM,and I re-estimate the fertility parameters $\{\psi, \gamma\}$ to match the aggregate fertility level and fertility differential across income groups. The identification argument is identical to that in Section 3.

[43] I derive this result by calculating the difference in the aggregate fertility rate between the new stationary equilib-

Since parents are entitled to claim children's dividends and these dividends are uniform in size, the model predicts that parents with lower human capital will have a larger increase in fertility.

As noted by McDonald (2006) and Stone (2020), most empirical research on family policies focuses on total fertility rates (TFR). This is because the TFR measure simply sums up age-specific fertility rates in a given year, and this could be derived immediately following policy adoption. Measuring the effects on the total fertility rate, however, is not satisfactory from the policy evaluation point of view since the government ultimately cares about how many additional children are born due to the policies.[44] A better measure is the completed fertility rate (CFR), i.e., the number of children ever born to women who have passed their childbearing years. Compared with the total fertility rate, effects on the completed fertility rate are much harder to measure because researchers need to wait 15 to 20 years for women of childbearing age to finish making their fertility choices. Since the APFD was enacted decades ago, it provides a rare opportunity to directly explore effects on the completed fertility rate without extrapolating based on changes in the total fertility rate.

Using the CPS June Fertility Supplement data from 1982 to 2018, I collect micro-level data on the completed fertility rate, i.e., the total number of live births ever had, among women aged 40 to 55. I divide the Alaskan sample into three groups based on the survey years. The "not treated" group contains the data prior to year 1987 since these women had already past their childbearing years when the APFD was enacted. The "partially treated" group contains the sample from 1987 to 2005 since the APFD affected some, but not all, of their childbearing years. Lastly, the "fully treated" group contains the observations from 2006 to 2018 since these women fully took the policy into account when making fertility choices. Women in the other states are in the control group.

To estimate the policy effects on the completed fertility rate, I use a difference-in-difference strategy by regressing the completed fertility rates on state fixed effects, year fixed effects, and treatment dummies:

$$\text{fertility} = \beta_0 + \beta_1 T_1 + \boldsymbol{\beta_2} T_2 + \text{State FE} + \text{Year FE} + \epsilon \tag{15}$$

where T_1 is a dummy variable for being in the "partially treated" group, and T_2 is a dummy variable for being in the "fully treated" group. The standard errors are clustered at the state level. Since the model prediction concerns the long-run impacts of policies, coefficient β_2 is the one of interest. To explore the heterogeneous treatment effects, I also estimate the specification (15) separately for women with or without a high education, which is defined as having at least one year of college experience. Table 2 reports the regression results.

rium and the baseline economy without the APFD. Results from the transition path (see Figure 10a) indicate that the policy effect on fertility is almost unchanged if I use short-run effects in the transition phase instead of long-run ones.

[44] Changes in the total fertility rate includes both quantum effects (i.e., effects on the completed fertility rate) and tempo effect (i.e., change in timing of birth, also known as compression effect).

Table 2: Effects of the APFD on the Completed Fertility Rates

	(1) Full Sample	(2) Low Education	(3) High Education
β_1 (partial treatment effects)	0.098***	0.216***	0.074***
	(0.027)	(0.036)	(0.021)
$\boldsymbol{\beta_2}$ (full treatment effects)	0.172***	0.296***	0.105***
	(0.032)	(0.041)	(0.025)
Observations	146,804	69,511	77,293

Notes: This table reports the regression results of specification (15). Standard errors are clustered at the state level. Column (1) shows the results with the full sample. Column (2) shows the estimated coefficients among women without any college experience. Column (3) shows the estimated coefficients among women with at least one year of college experience.

The first column of Table 2 shows that the estimated effects of the APFD on the completed fertility rates is $\hat{\beta}_2 = 0.172$ children per woman with the 95% confidence interval $(0.109, 0.235)$.[45] The model-predicted policy effects of 0.16 children per woman is within this range. Results on $\hat{\beta}_2$ from the second and third columns also confirms the model's prediction on heterogeneous treatment effects – women without a high education responds more strongly to the APFD.[46]

5 Counterfactuals

In this section, I use the model to evaluate family policies of different sizes in a general equilibrium where prices and distribution of the population adjust. I focus on a uniform cash reward to childbirth (i.e. baby bonuses $\mathcal{B}$) since the policies' structure is simple and similar to expanding the fully refundable child tax credit (CTC).[47] The simplicity in policy structure facilitates better exposition of the model mechanisms. In addition, such cash rewards have already been widely adopted in the developed world (e.g., Drago et al., 2011; González, 2013). I discuss the long-run implications of baby bonuses in Section 5.1 where the government is balancing the budget by adjusting consumption taxes.[48] The outcome variables of interest include fertility, human capital, average

[45]This finding echos the results in Yonzan et al. (2020). Using synthetic control methods on Natality files, they conclude that the total fertility rate in Alaska increases by 13.1% due to the AFPD.

[46]This finding echos the results in Cowan and Douds (2021). They find larger increases in fertility among Alaska natives and women without a high school degree.

[47]There are two main differences between baby bonuses and the CTC. First, the CTC has an income requirement and phase-out region, whereas a baby bonus is usually not means-tested. As is discussed later, the main results are stronger when family policies target low-income households. Second, a baby bonus is a lump-sum transfer when the child is born, while the CTC is an annual transfer to parents when the child is below age 18. With borrowing constraints, low-income parents prefer baby bonuses to a CTC of the same net present value since they can replicate the latter through savings.

[48]I discuss the robustness of the results to alternative funding methods in Appendix A.4.

income, intergenerational mobility (measured by IGE^{-1}),[49] and social welfare. In Section 5.2, I compare the baby bonus with other policies, including subsidized childcare and public education expenditures. Last, I study the transition path of the baby bonus and its distributional effects across generations in Section 5.3.

5.1 Baby Bonus: Long-run Implications

This section evaluates baby bonuses of different sizes ranging from \$0 to \$50,000. I compute the long-run macroeconomic implications of these policies by comparing the long-run steady state of the economy with the benchmark economy where the baby bonus is zero.

Figures 8a and 8b show the fertility effects and the direct fiscal costs of baby bonuses. To reach the replacement fertility level (2.1 children per family on average), the model predicts that it would require a baby bonus of \$30,000, or approximately 1.8% of GDP in the new steady-state economy. This amount is similar to the increase in the CTC from 2010 to 2021 in net present value, and it offsets 19% of the average cost of raising one child as estimated by the USDA. These results confirm the common perception by demographers that using financial incentives to raise fertility is not "cheap." Under realistic magnitudes of fertility elasticities and reasonable policy scales, the effects of financial incentives provided by the government are small relative to historical changes in preferences, social norms, contraceptive technologies, and perhaps most important of all, changes in the skill premium. These results, however, suggest that using cash transfers to prevent further crashes in fertility, or even to raise the fertility rate, is feasible in practice.

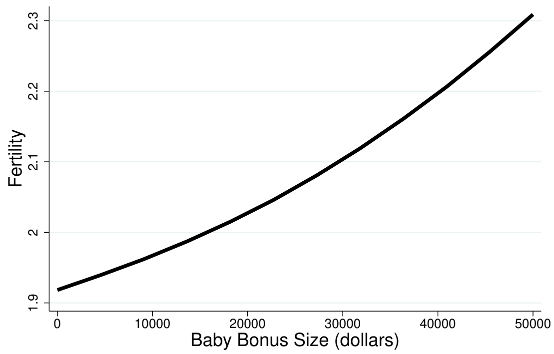
Figure 8c shows the quality-quantity trade-off channel discussed in Section 2.6.1. Baby bonuses reward parents for higher fertility, and parents respond by having more children. The increased quantity, in turn, raises the marginal cost of education as well as the marginal utility of consumption due to larger family sizes. As a result, even when quality and quantity are complements in parents' preferences (see Section 3.1), parents may optimally reduce the amount invested in children's education when fertility increases. When the baby bonus is \$30,000, private investment per child e falls by 4% on average. This result is in sharp contrast with design-based research, in which parents with a fixed number of children receive transfers and spend more on child human capital formation.

Figures 8c and 8d show that agents with lower human capital have stronger fertility responses and larger drops in private education investments in response to baby bonuses. This result is intuitive since a baby bonus of a given size accounts for a larger fraction of household income among low-income families. Due to intergenerational transmission of human capital in skill formation,

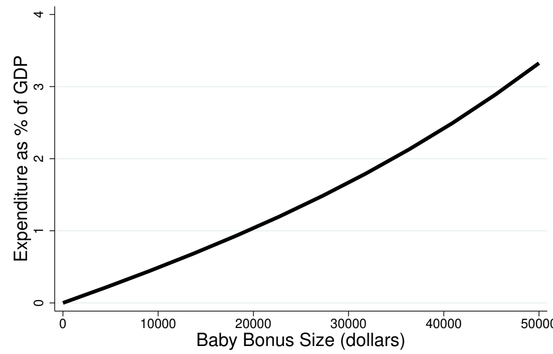
[49]Intergenerational Elasticity of Earnings (IGE) is calculated by simulating parent-child pairs and calculate the correlation between their earnings at $j = 2$.

Figure 8: Long-Run Effects of Baby Bonuses

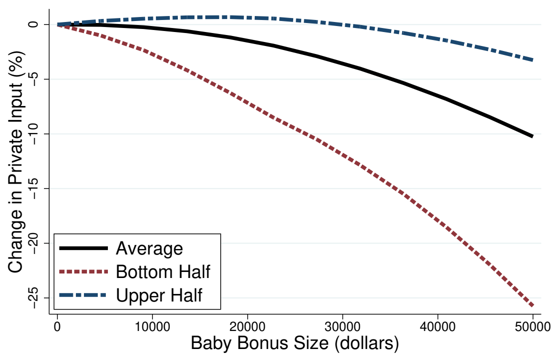
(a) Effects on Fertility



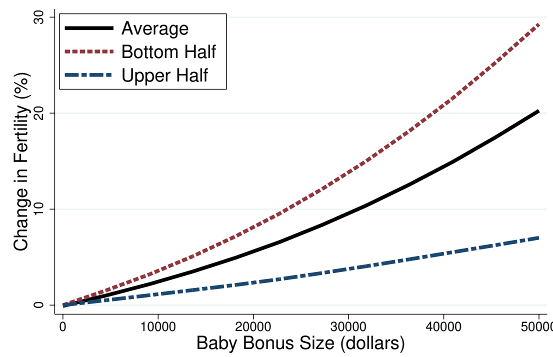
(b) Expenditure Share as % of GDP



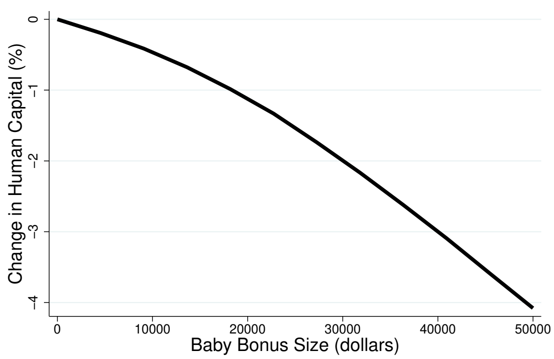
(c) Heterogeneous Private Input Response



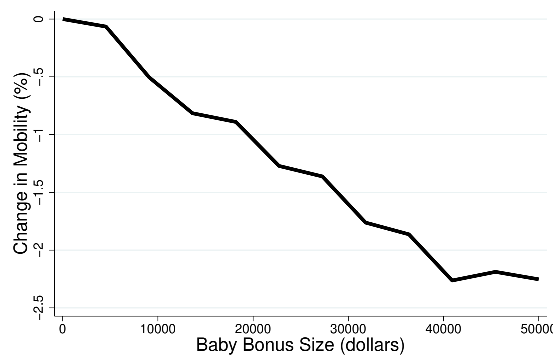
(d) Heterogeneous Fertility Response



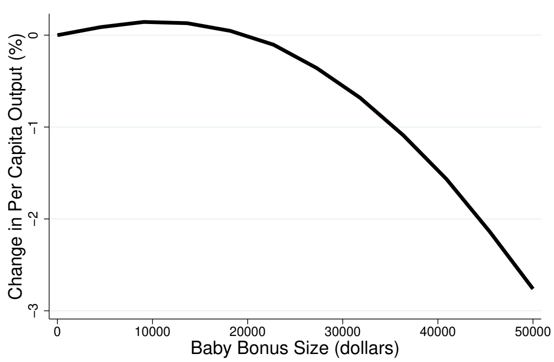
(e) Average Human Capital



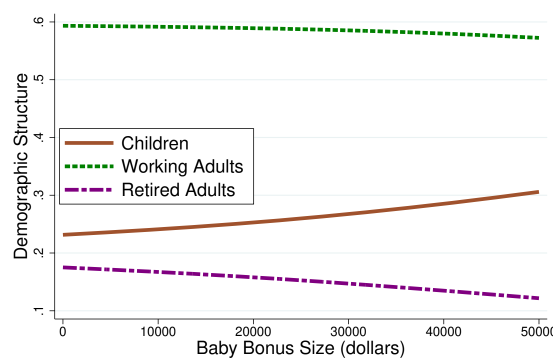
(f) Intergenerational Mobility

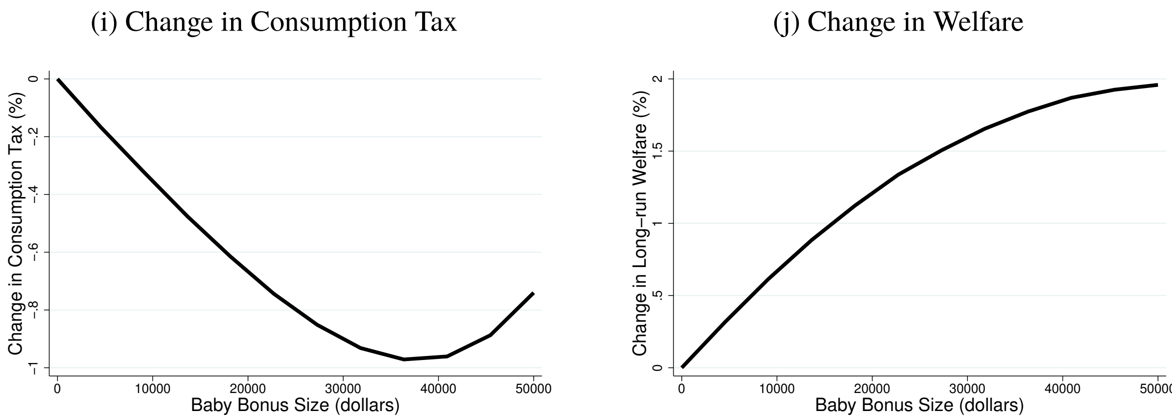


(g) Per Capita Output



(h) Demographic Structure





Notes: These figures plot macroeconomic impacts of baby bonuses of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. In Figure 8c and 8d, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

heterogeneous responses in fertility induce gravitation of aggregate variables towards low-income families. Composition effects, in addition to the reductions in private investment per child, lead to a reduction of the average human capital by 2% under a $30,000 baby bonus (see Figure 8e).[50] Since the decrease in education investments is larger among low-income households, the baby bonus also reduces social mobility, measured by IGE^{-1}, by 1.8% rather than boosting it as policymakers would have hoped.[51]

Figure 8g shows that per capita output rises initially before falling around $\mathcal{B} = \$14,000$ despite lower average human capital and dampened social mobility in the economy. Changes in per capita output are not monotonic due to three forces that shape the aggregate supply of capital and labor. First, as parents reduce investments and transfers to children due to the quality-quantity trade-off, they optimally save part of the baby bonus for their future consumption, which leads to an increase in the supply of capital in the economy. Second, the human capital distribution worsens as the baby bonus increases, which reduces the supply of capital and labor due to composition effects. Finally, demographic structure effects alter the mass of agents in each cohort, which in turn affects the aggregation of capital and labor across age groups (see Figure 8h). As the baby bonus gets larger, per capita output falls below the steady-state level because the second effect dominates.

Despite the potential reductions in per capita output, Figure 8i shows that in the long-run steady-state, the government is able to reduce the consumption tax under a small baby bonus. This is a result of several forces shaping the government budget constraint in (4). First, the change in human capital distribution affects tax revenues. Second, as the share of retired households decreases, the

[50] The 2% reduction in average human capital is smaller than that of the private investment (-4%) since public education investment $\mathcal{E}$ is unchanged.

[51] Due to the quality-quantity trade-off mechanism, the policy effects on social mobility will be even stronger if the baby bonus is exclusively given to low-income households.

government is less burdened by pension payments. Further, as the share of children increases, the government needs to allocate more resources to public education. Contrary to structural models with two-period-lived agents where the government needs to raise taxes to support more children, this model predicts that when $\mathcal{B} = \$30,000$, consumption taxes could be lower by 0.9% in the long-run.

Figure 8j plots the welfare effects in consumption equivalents. Due to increased fertility and reduced consumption taxes, the average utility of new-borns under the veil of ignorance increases by 1.6% in the long-run economy when aggregate fertility reaches the replacement level. Besides these two benefits, welfare rises because the baby bonus provides a social safety net for low-income households, generating insurance benefits to risk-averse agents *ex ante*.

5.2 Comparing Policies: Subsidized Childcare and Education Expenditures

In this section, I highlight some similarities and differences between various family policies. For more detailed results, see Appendices B.1 and B.2.

Compared with baby bonuses, subsidized childcare is less cost-effective in raising fertility. While it takes a baby bonus equal to 1.8% of GDP to boost aggregate fertility to the replacement level, the government needs to spend 2.5% of GDP if it uses subsidized childcare to achieve the same goal. This result is consistent with evidence from historical policies (Luci-Greulich and Thévenon, 2013; Stone, 2020). It is also intuitive since compared with cash benefits, subsidized childcare offers fewer benefits per child (in consumption equivalents) to parents with low human capital (see Figure 9a) given that they prefer to spend the money elsewhere.

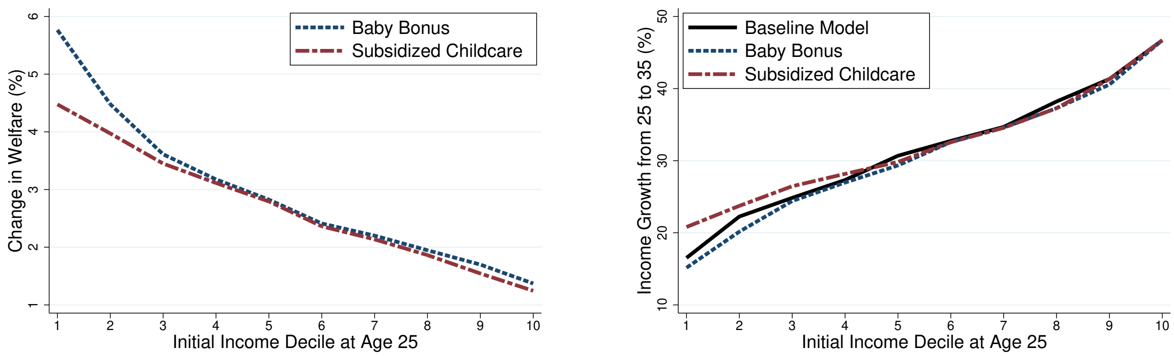
Different from baby bonuses, subsidized childcare encourages the combination of childbearing and labor force participation, especially among parents with low human capital.[52] This in turn, fosters adults' human capital accumulation via on-the-job learning. As a result, subsidized childcare leads to more equitable income growth across agents with different initial human capital and reduces inequality in life-cycle income growth (see Figure 9b). The quantitative effects on income-growth inequality are nevertheless mild since most of the gap is driven by differences in initial human capital rather than hours worked (see Section 3).

The fertility effects of public education expenditures are an order of magnitude smaller than those from baby bonuses for two reasons. First, public education expenditures only indirectly affect quantity choices through changes in child quality. Second, in the long run, education expenditures

[52] Hours worked among parents with low human capital fall after the implementation of a baby bonus for two reasons. First, these parents are most responsive in fertility, which drives up the total childcare time. Second, due to increasing economies of scale in home production of childcare, the increase in fertility decreases the relative price of home versus market childcare. See González (2013) for empirical evidence of these predictions. As discussed in Section 2.6.4, hours worked among parents with low human capital increase under subsidized childcare as the policy "pushes" them from home production of childcare to market work if they want to benefit from the in-kind transfers.

Figure 9: Baby Bonuses and Subsidized Childcare: Effects on Welfare and Income Growth

(a) Effects on Welfare (b) Effects on Income Growth



increase the share of parents with higher human capital who have a smaller number of children. Therefore, the government should not expect to use education expenditures as an effective pro-natal policy instrument. While a \$30,000 baby bonus boosts aggregate fertility from 1.92 to 2.1 (the replacement level), an education expansion of the same size only raises aggregate fertility by 0.011.

Similar to baby bonuses, private investments into children's education also fall under public education expansion, but for a different reason. While private investments drop due to the quality-quantity trade-off when the government subsidizes childbirth, parents spend less on children's education under an expansion in public education expenditure due to crowding-out effects. This is because public and private monetary inputs into child human capital formation are substitutes.

Despite mild fertility effects, expanding public education is the most effective policy among those evaluated at raising social mobility and improving children's outcomes. A \$60,000 expansion in public education (net present value) raises intergenerational mobility by 5.2% and average human capital by 2.3%.[53]

5.3 Baby Bonus: Transition Path

In this section, I discuss the transition path results under a \$30,000 baby bonus that leads to the replacement level fertility rate in the long run. The policy is enacted unexpectedly at period $t = 1$ and stays in place for all subsequent periods. The government changes consumption taxes along

[53]These values are considerably smaller than the projected policy effects by Daruich (2018) for two reasons. First, Daruich (2018) does not consider the existence of public education expenditures in the benchmark economy. As a result, additional public education expenditures have significantly higher marginal effects on child human capital. Second, Daruich (2018) does not consider endogenous fertility which dampens the policy effects on child quality through the quality-quantity trade-off.

the transition path to balance the budget period by period.[54] Each period stands for ten years.

Figure 10a shows that the policy effects on fertility are immediate and persistent. Figure 10b indicates that the old-age dependency ratio starts to decline in period 6, given that past fertility rates predetermine the ratio of the retired to working-age population in the short run. The total dependency ratio, however, increases immediately after the policy is adopted since there is a large increase in the number of children born due to the baby bonus.

As a result of this increase in the total dependency ratio, the government needs to finance additional expenditures in the first few periods. Such expenditures include education and transfers to parents with children through the tax system. These "induced" expenditures could make the overall costs of family policies much larger than the direct expenditures.[55]

In the baseline model, the government balances the budget period by period using consumption taxes. As a result, Figure 10c shows that consumption taxes rise at first before falling in the long run. Hence, welfare changes of new-born agents in transition are positive but significantly smaller than those in the long-run steady state. Since most existing agents in the economy at $t = 1$ do not benefit from the baby bonus but are still required to pay higher consumption taxes, they are worse off with the baby bonus.

The transition path results shed light on two insights that hold more generally beyond the case of a baby bonus. First, the overall fiscal burden induced by family policies is usually much higher than the policy expenditure itself, especially in the first few decades in the transition path. When additional children are born, the government needs to finance additional expenditures for existing child-related policies. The potential fiscal benefits of reductions in the old-age dependency ratio, on the other hand, will be realized much later than the upfront costs. Second, the amount of political support for family policies depends on how the fiscal costs are distributed across generations. The government will have a hard time gathering enough agents to support family policies that benefit the economy in the long run at the cost of existing households. In that sense, family policies are very similar to climate policies where countries wait "too long" to act.

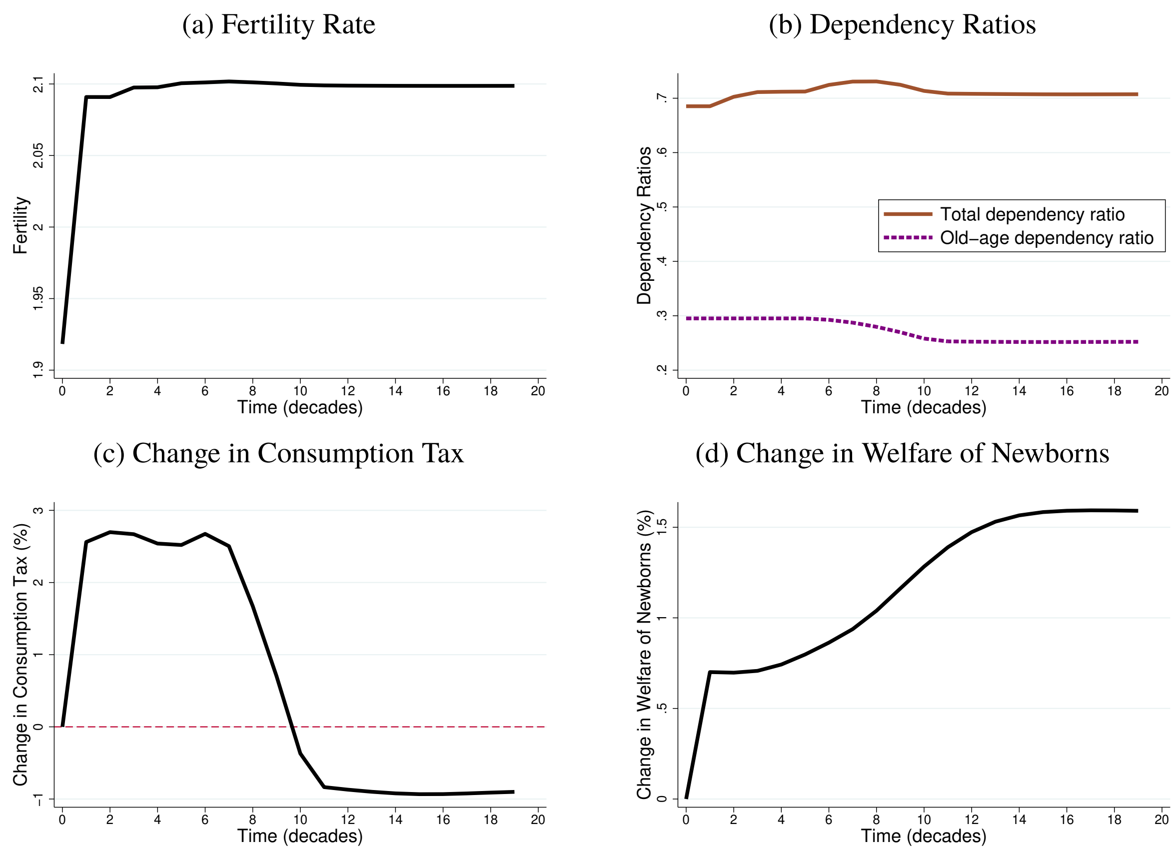
6 Optimal Policy

In this section, I study the optimal baby bonus to address the externalities of childbearing and inefficiencies caused by borrowing constraints. I begin with a discussion of welfare criteria under heterogeneous agents and endogenous fertility. Then, I show optimal policy results and propose principles for designing family policies in general.

[54] I discuss an alternative way of using government borrowings to fund fiscal expenditures in Section A.4.

[55] For example, while the direct cost of a \$30,000 baby bonus is 1.8% of GDP, the government needs to raise consumption taxes by 2.6% for the first seven decades of the transition.

Figure 10: Transition Path of a $31,000 Baby Bonus



Notes: These figures plot evolution of aggregate variables under a baby bonus of $31,000. Each period represents 10 years. The policy is enacted at period 1.

6.1 Welfare Criteria

Welfare criteria in models with endogenous fertility are complicated both conceptually and philosophically. Unlike standard comparisons between allocations where the set of agents is fixed, in this context there will be agents born in one economy but not in the other. As a result, the standard Pareto principle cannot be used to conduct welfare analysis in this context. The field of population ethics is devoted to understanding and resolving this question.[56]

Since the debate on welfare criteria is far from being settled, I adopt two definitions in studying the optimal policy. The first criterion is the *long-run welfare* $\mathcal{W}$ used in previous sections. It evaluates the expected utility of a newborn child in the long-run stationary equilibrium under the

[56]For instance, Parfit (1984) derives the famous "repugnant conclusion" showing that under a set of intuitively appealing assumptions, one can prove "for any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better, other things being equal." Golosov et al. (2007) propose two criteria called $\mathcal{A}-$efficiency and $\mathcal{P}-$efficiency which differ by whether the planner evaluates the welfare of those who are not born. de La Croix and Doepke (2021) consider the optimal welfare from a soul's perspective where one needs to consider both the utility of being born and the average "waiting time" for incarnation.

veil of ignorance. The second criterion assesses the *average utility of existing agents* when the policy is adopted.[57] I decompose this into welfare changes for existing parents ($j = 2$) who receive the baby bonus and for other households ($j > 2$) who are not direct beneficiaries of the policy. The latter criterion has two features. First, it is forward-looking as it incorporates tax changes in later periods that affect these households' utility. Second, with this criterion, the unique solution to the planning problem corresponds to the notion of $\mathcal{A}$-efficiency defined in Golosov et al. (2007) which focuses on the welfare of those already alive.

Rather than computing the unconstrained optimum, I follow the Ramsey tradition and allow the government/planner to use only a certain policy instrument (a baby bonus) that is uniform across households and birth order. As in previous sections, I consider the scenario where the government adopts the baby bonus at the beginning of time period $t = 1$. The policy change is permanent and financed by consumption tax changes. I use $\mathcal{B}^*_{lr}$ to denote the optimal baby bonus that maximizes average utility in the long run, and I use $\mathcal{B}^*_{sr}$ to denote the optimal baby bonus that maximizes the average utility of existing households when the policy is adopted.

6.2 Optimal Policy Results

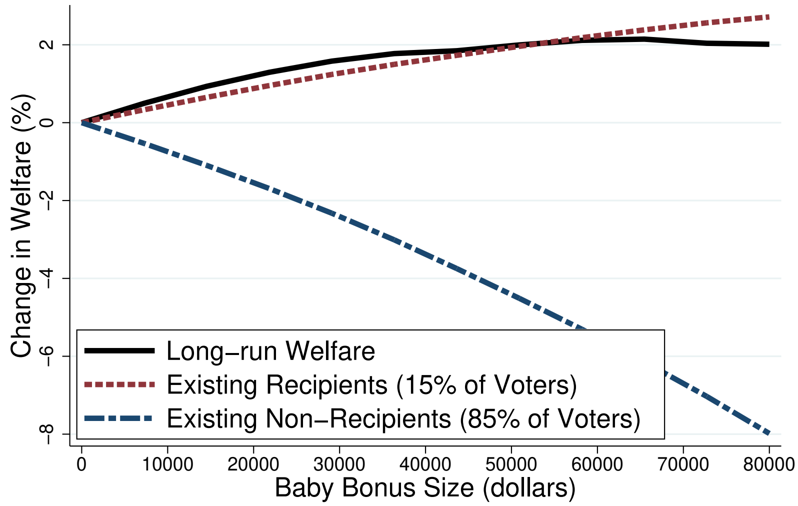
Figure 11 shows the changes in welfare under baby bonuses of different sizes for two welfare criteria. From a long-run perspective, the optimal baby bonus is $\mathcal{B}^*_{lr} = \$60,000$. $\mathcal{B}^*_{lr}$ boosts aggregate fertility to 2.4, raises long-run welfare by 2%, and costs around 4.1% of GDP.

The baby bonus that maximizes the welfare of existing households, however, is $\mathcal{B}^*_{sr} = \$0$. Current parents who receive the baby bonus directly prefer larger $\mathcal{B}$ for two reasons. First, despite the higher taxes in transition, these households are subsidized by elderly households ($j > 2$) on net. Second, the baby bonus redistributes towards poorer parents, which improves average welfare. Elderly households in the economy, however, strongly oppose the baby bonus since they do not benefit from the bonus but still pay higher taxes for the remainder of their lives.[58] If each household has the same voting power, then $\mathcal{B}^*_{sr} = 0$ is the most likely outcome from a political perspective. As discussed in Section 5.3, this observation explains the observed puzzle where many countries with extremely low fertility rates fail to implement large-scale family policies despite knowing the dire consequences in the long run. Even when they do implement these policies, governments often renege on the promises due to fiscal pressures. For example, the Australian baby bonus was significantly downsized in 2014.

[57]Following standard practice in quantitative macroeconomic literature, I use equal weights as a benchmark. Another approach is to use the Negishi weights which put a greater weight on households with higher human capital and initial assets. This would eliminate the redistributive benefits of baby bonuses. See Kim et al. (2021) for an example.

[58]Note that in the case of the Child Tax Credit (CTC), parents at $j = 3$ are also beneficiaries since they are eligible to receive the transfers.

Figure 11: Optimal Baby Bonus by Welfare Measure



Notes: This figure plots the changes in welfare under baby bonuses of different sizes. I consider two welfare criteria: (1) average utility of a new-born child in the long run and (2) average utility of existing households when the policy is enacted, decomposed into agents who receive the baby bonus and agents who do not receive it.

6.3 Discussion

Even though this paper restricts the study of optimal policy design to choosing the optimal level of a baby bonus financed by consumption tax changes (thus balancing the government budget period by period), it highlights the major trade-offs in policy design which are broadly applicable to other instruments and goals.[59] On one hand, subsidizing childbirth among parents with low human capital leads to larger fertility changes per dollar spent on family policies. This in turn leads to larger welfare improvements due to changes to the demographic structure. On the other hand, subsidizing childbirth among parents with high human capital improves the equilibrium human capital distribution via intergenerational transmission of skills, thereby raising the welfare due to composition effects. At the aggregate level, the government balances population growth, average human capital, and distortions through the taxation by weighing the quality-quantity trade-off, composition effects, demographic structure effects, and changes in childcare arrangements.

With countervailing forces present in the unified model, the first-order stochastic dominance (FSD) in equilibrium human capital distribution is neither necessary nor sufficient for choosing better policies. For instance, the equilibrium human capital distribution under a $30,000 baby bonus is first-order stochastic dominated by that in the baseline economy, but social welfare is im-

[59]For instance, it is straightforward to incorporate subsidized childcare and public education expenditures, shown in Appendices B.1 and B.2. Instead of choosing a permanent policy change to the baby bonus, one could also conduct dynamic optimal policy design where the size of the bonus varies over time.

proved in the long run. The key insight is that besides comparing human capital distributions across economies, one also needs to pay attention to differences in the *age distribution*, i.e., demographic structures. Thus, this paper provides a novel counterargument to the common conclusion in the existing literature on family policies arguing for childbirth restrictions among parents with low human capital (e.g., Chu and Koo, 1990).[60] These parents, with higher fertility elasticities, could be key to solving population aging problems.

7 Conclusion

Facing the "global fertility crash" (Tartar et al., 2019), family policies have been widely pursued to encourage childbirth. Evidence of the effects of transfers to parents on children's outcomes lead policymakers and economists alike to believe that family policies are good instruments to "lift children out of poverty today and help them tomorrow" (Schanzenbach et al., 2021).

In this paper, I study the aggregate impacts of family policies in a heterogeneous-agent general equilibrium overlapping-generations model. Relative to previous studies on family policies, I demonstrate the importance of considering the quality-quantity trade-off, an endogenous demographic structure, and changes in childcare arrangements.

I find that when governments design family policies that reward having more children, parents respond by increasing child quantity but optimally reduce child quality. Compared with education subsidies, child benefits are not ideal instruments if the government aims to raise child human capital or boost social mobility. However, the pro-natal effect of family policies could lead to long-run welfare gains. This is because population growth reduces the old-age dependency ratio, allowing the government to reduce tax rates. I also show that the long-run gains in welfare require a transition path where the government needs to finance higher child-related expenditures during the first few decades. Depending on how the government decides to finance these higher costs, the welfare of existing households could suffer.

I conclude by discussing avenues for future research. First, it would be interesting to consider the optimal policy design with additional instruments and different welfare criteria. Second, one can calibrate the model to match the institutional details of other countries and conduct policy counterfactuals on a case-by-case basis. Lastly, another fruitful area of research is to link family policies to other population externalities such as pollution (Bohn and Stuart, 2015), ideas creation (Jones, 2020), and firm dynamics (Hopenhayn et al., 2018; Peters and Walsh, 2020).

[60] Cordoba and Liu (2016) make a complementary argument invoking the Lucas' Critique by showing that family policies have direct effects on households' utility.

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A Robustness

A.1 Alternative Preferences

There are alternative ways to model parents' preferences over child quantity and quality. I argue that the modeling choice in Section 3.1 makes the results in this paper a conservative benchmark.

One commonly-used assumption is separable preferences between quality and quantity (see de La Croix and Doepke, 2003; Bar et al., 2018; Vogl, 2016) where:

$$v(n, \mathbb{E}h_k, a_k) = \log(n) + \theta \log(\mathbb{E}h_k) + \nu \log(a_k)$$

Compared with our benchmark case where quality and quantity are complements, parents will raise fertility and reduce child quality even more strongly when the government rewards childbirth. This is because the interaction in preferences on the left-hand side of Equation 8 is not present under separable preferences. I show this result with closed-form solutions in Appendix D.1. For the same reason, if quantity and quality are substitutes, the results in this paper will be conservative (Jones and Schoonbroodt, 2010).

Another way of modeling parents preferences is dynastic altruism (see Cordoba et al., 2016; Daruich and Kozlowski, 2020) where:

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n) V_2(\mathbb{E}h_k, a_k)$$

This formulation is similar to the one used in this paper with the exception that utilities from child quality are pinned down recursively. Dynastic altruism is appealing aesthetically but faces unresolved challenges in fitting observed transfers between parents and children (see Altonji et al., 1997; Barczyk and Kredler, 2020). Paternalistic motives are often added to improve data fit (e.g., Abbott et al., 2019). I argue that the results will also be stronger under dynastic altruism: parents endogenize the fact that returns to child quality become lower when the government creates a "social safety net" for children with low human capital by providing generous family benefits. As a result, parents would further increase child quantity and reduce quality investments. For instance, Daruich and Fernández (2020) argue that universal basic income reduces the human capital of future generations due to this mechanism.

A.2 Endogenous Timing of Childbirth

In the baseline model, I abstract away from birth timing by assuming that parents only make fertility choices from age 20 to 30. In reality, parents can decide when to give birth, and family policies could affect that decision. This is known as the tempo effects of family policies.[61]

I argue that adding endogenous timing will make my baseline results stronger. As the model matches effects on the completed fertility rate (quantum effects) in the validation exercise, policy effects on the total fertility rate (quantum plus tempo) will be larger. Suppose that in addition to increasing the number of children, some parents shift birth from 30s to their 20s in response to a baby bonus. This shift in birth timing will likely be detrimental to child human capital for two reasons. First, early birth reduces the spillover that the children could receive because parents' human capital grows rapidly from age 20 to 40. Second, family policies of realistic sizes fall short of offsetting the income differences of parents between early and late births. Hence, these children are born into households with less resources on average, which would also reduce child human capital as investment falls. As a result, family policies will have a larger observed fertility impact on the economy with endogenous timing of birth, but the outcomes of children will be even worse.

A.3 Alternative Measure of Fertility

Since the baseline model focuses on the total number of children that parents decide to have, the most appropriate measure of fertility in this context is the completed fertility rate (CFR), measured using data on either "children ever born" in Census data (dropped after 1990) or "live births ever had" in the CPS June Fertility Supplement. As discussed in Section 4, the CFR has the advantage of being invariant to shifts in birth timing due to policy changes. For these reasons, in Figure 5.2 and the calibration, I use the CFR by household income decile, calculated using the sample of married women age 40-55 in the 2008-2014 CPS June Fertility Supplement.[62]

One disadvantage of using the CFR is that these women made their fertility decisions prior to 2010 where they may have faced different trade-offs. The total fertility rate (TFR) is an alternative measure that is more responsive to contemporaneous situations by summing the age-specific birth rates of all women alive in a given year. This measure is simple to compute and widely used in the literature (Kremer and Chen, 2002; Manuelli and Seshadri, 2009).

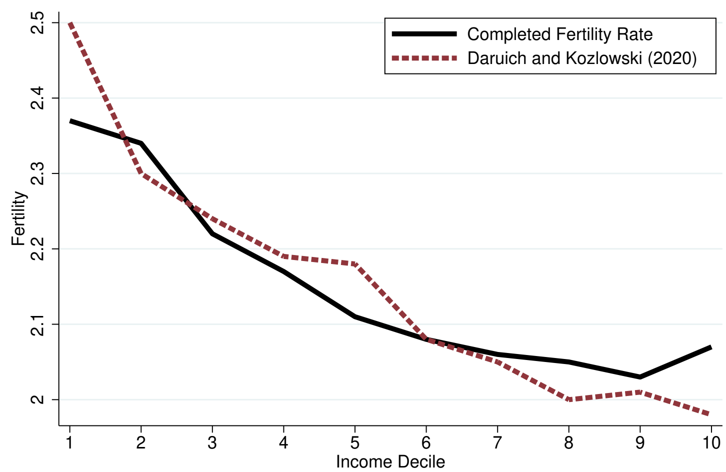
TFR *by income decile*, however, is less straightforward to compute than CFR by income decile.

[61] An unresolved question is whether changes in timing are due to the relaxation of parents' constraints (e.g., down payments to buy a larger house) or the lack of commitment to policies by the government. Historically, governments often renege on family policies as fiscal conditions change. One example is the Australian baby bonus which was significantly downsized in 2014. This uncertainty creates an incentive for parents to shift birth timing while benefits are still in effect.

[62] In a steady-state economy, fertility choices across cohorts are stationary, and hence the CFR coincides with the total fertility rate (TFR). For this reason, I normalize the CFR target in the calibration of ψ to match the total fertility rate in 2010, which was 1.92 children per woman. Model results are quantitatively similar if I omit this step. I have also experimented with a version of the model where a fraction of adults remain single and (exogenously) childless, meaning that the CFR in the economy is a weighted average between single adults without children and married households whose within-group CFR is higher than 1.92. The overall results are again similar..

First, one does not want to assign income ranks that mix young women with older women due to life-cycle changes in the earnings profile. Second, given that the appropriate measure of income is total family income, one would not want to mix married women with single women. Daruich and Kozlowski (2020) provide a recent estimate of TFR by income decile using data from the 2000 Census. They restrict the sample to married women between ages 15-49 who are either themselves household heads or spouses of the household head. Income deciles are assigned by comparing the total family income of each woman with other observations in the same age group. To alleviate selection bias into marriage for younger age groups, they also drop age-income groups with a small number of observations. For each income decile, they calculate the TFR by adding up the age-specific fertility rates in that decile. Figure A.1 shows that their result is quantitatively similar to the CFR estimates used in the calibration.

Figure A.1: Fertility Rate by Income Decile



Notes: This figure plots the total fertility rate by income decile measured in Daruich and Kozlowski (2020) and the completed fertility rate used in model calibration.

A.4 Government Borrowing

In the baseline policy counterfactual and the optimal policy analysis, the government changes consumption taxes τ_c to balance the budget in every period. This abstraction from reality is primarily for computational reasons.

I argue that many results are unlikely to change when the government can borrow from domestic or international lenders. In particular, all of the main policy mechanisms (i.e., quality-quantity trade-off, composition effects, changes in demographic structure, and adjustments in childcare arrangements) are independent of whether or not the government is allowed to borrow. As a result, I expect the model predictions on fertility, human capital, output, and mobility to be unchanged in a model with government borrowing.

Welfare implications, however, will likely change depending on how government expenditures are funded. For example, since the $30,000 baby bonus generates welfare improvements for all agents in the long run, it is natural for the government to shift the fiscal burden to these beneficiaries via borrowing instead of asking existing elderly households to fund additional expenditures from which they derive little benefit (see Figure 11). If the government chooses to do so, then the welfare effect in the long-run economy will be smaller, while the effect on existing agents will be less negative. Such a policy change is also more likely to gain sufficient political support for implementation as it does not directly hurt existing agents who have voting power.[63]

Despite the intuitive appeal of using government deficits to finance large-scale family policies that benefit future generations, family policies are often financed by adjusting fiscal revenues and expenditures in current policy proposals. For example, the American Families Plan vows to increase the tax rates among the very rich, while the Family Security Act proposed by Senator Romney would reform and consolidate outmoded federal programs. I leave the normative implications of such financing choices for future research.

[63]Note that if the government issues debt to finance large-scale family policies, the overall borrowing cost for the government could increase in the general equilibrium if lenders require higher compensation for a larger debt-to-GDP ratio. This might crowd out other government borrowing that benefits existing households. As a result, whether current agents prefer government borrowing or changes in taxation also depends on the opportunity costs of family policies.

B Additional Results

B.1 Public Childcare

In this section, I evaluate childcare subsidies where the government offers $\mathcal{S} \in [0, 0.9 \cdot \chi]$ market childcare services for all families, equivalent to 0 to 36 hours of public childcare per week.

I find that subsidized childcare raises fertility, but at a higher cost than baby bonuses. The government needs to provide 25 hours of subsidized childcare to raise the aggregate fertility rate to 2.1. This would cost almost 2.5% of aggregate GDP in the long-run steady-state economy, 0.8% higher than a baby bonus that achieves the same fertility goal.

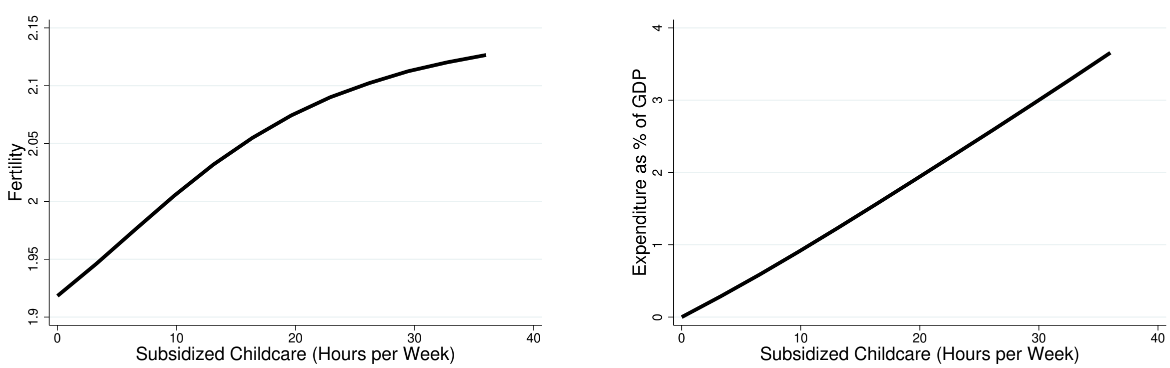
Figure B.2c shows that similar to the baby bonus counterfactual, parents reduce private education investments due to the quality-quantity trade-off. Figure B.2d shows that fertility responses are higher among low-income parents. Due to reductions in private investment and composition effects, average human capital in the economy decreases (see Figure B.2e). The overall effects on intergenerational mobility are small (see Figure B.2f).[64]

Figure B.2g shows that output per capita increases with subsidized childcare. Compared with the baby bonus, the output changes are more positive for three reasons. First, composition effects are milder because the fertility differentials between parents with low and high human capital are smaller than those under the baby bonus (see Figure B.2d). Second, the childcare subsidy raises the labor supply by reducing the effective price of market care. This induces parents to spend less time at home taking care of children t_h. Finally, the increased working time during age 20-30 translates to high human capital for adults in later periods due to learning-by-doing in the labor market.

Figure B.2i shows that the government can reduce consumption taxes by 0.6%. Long-run welfare could be raised by 2.5% when aggregate fertility is around the replacement level.

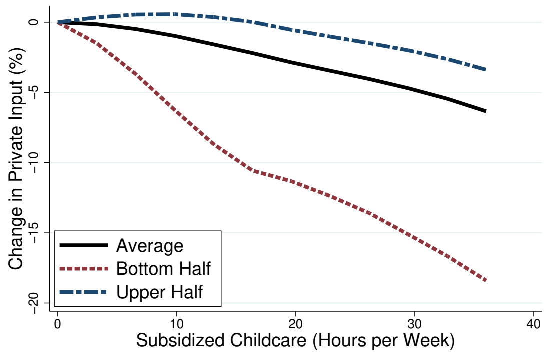
Figure B.2: Effects of Childcare Subsidies of Different Sizes

(a) Effects on Fertility (b) Expenditure Share as % of GDP

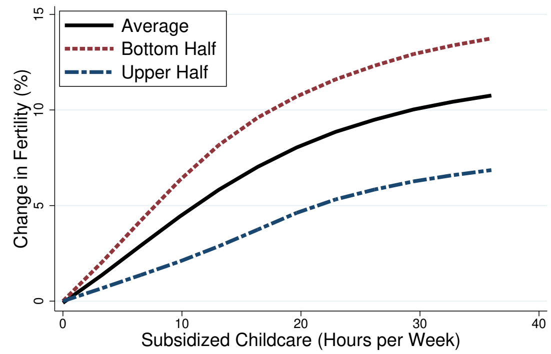


[64]The effects on mobility are not monotonic since besides changing child human capital, subsidized childcare also changes hours worked, affecting household earnings.

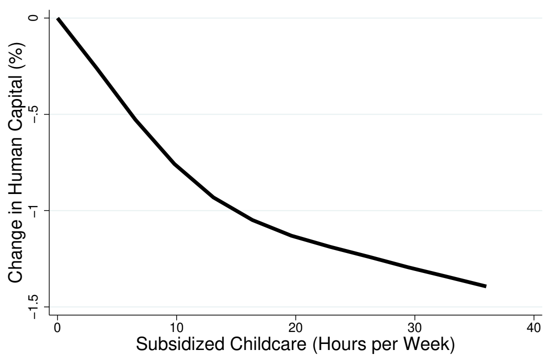
(c) Heterogeneous Private Input Response



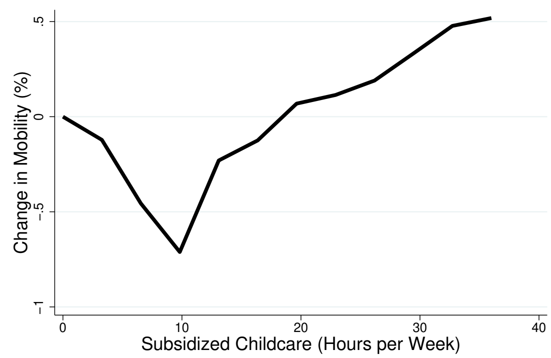
(d) Heterogeneous Fertility Response



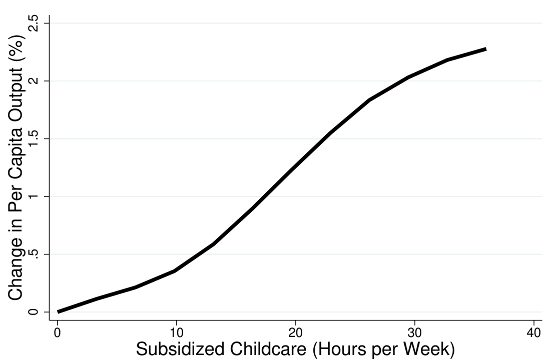
(e) Average Human Capital



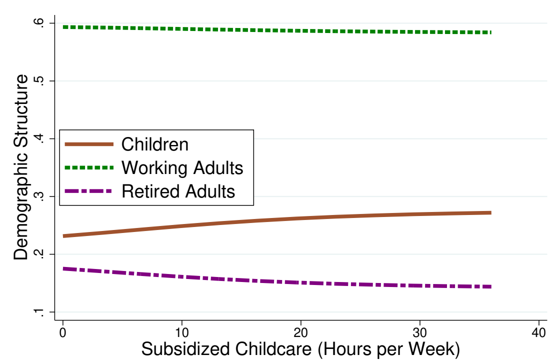
(f) Intergenerational Mobility



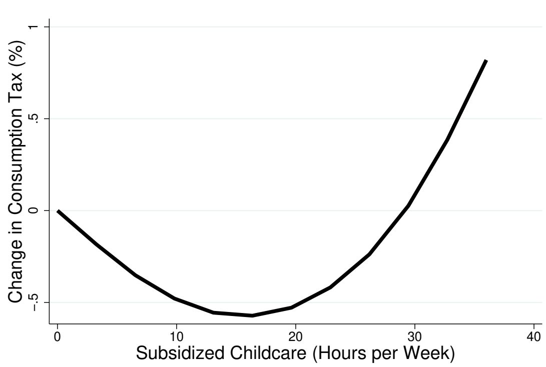
(g) Per Capita Output



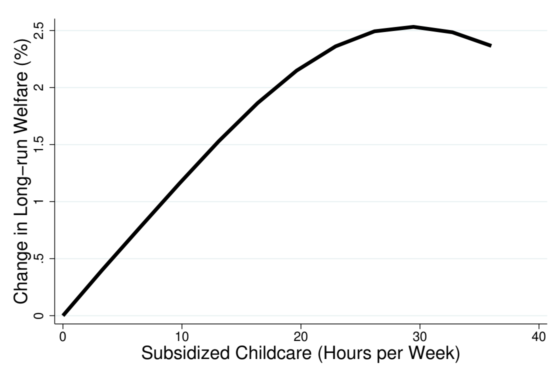
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of childcare subsidies of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. In Figure B.2c and B.2d, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

B.2 Public Education

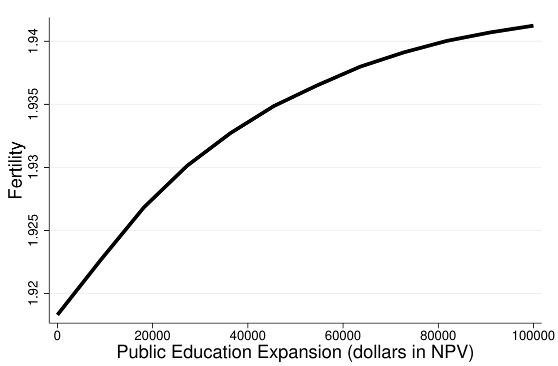
In this section, I evaluate expansions of public education expenditures from the current level of \$12,000 per student per year by between \$0 and \$100,000 in net present value. Figure B.3a shows that the effect on fertility is positive, because children become more desirable as public education raises children's quality. The magnitude of the fertility effect here is much smaller than that of family policies. Compared with a \$30,000 baby bonus that raises fertility by 0.2 children per woman, an education expansion of the same amount only raises fertility by 0.011. The government should therefore not expect to use education policies to raise aggregate fertility to the replacement level.

Figure B.3d shows that the fertility response is larger for households with higher human capital. An increase in public education expenditure also crowds out private input (see Figure B.3c). Due to strong crowding-out effects, average human capital only increases by 2.5% when the education expansion exceeds \$80,000 in net present value. It is costly to raise average human capital in the economy using uniform policies in the general equilibrium since (1) crowding-out effects are strong; (2) investments face decreasing marginal returns as $\mathcal{E}$ is already \$12,000 per year in the baseline economy.

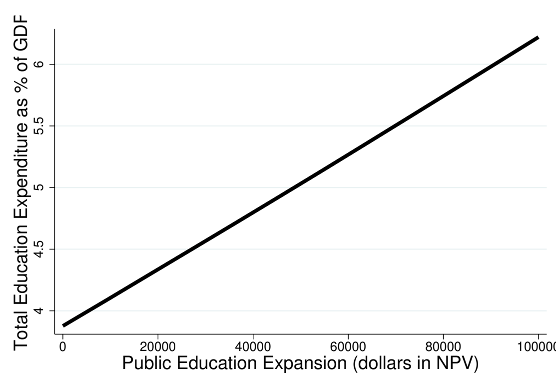
Expanding public education, however, is the most effective policy to raise social mobility. As the expansion exceeds \$60,000 in net present value, intergenerational mobility could be raised by almost 5.2%. If the policy targets lower-income households, one can expect larger effects on mobility. Due to increased human capital, public education expansion of \$60,000 raises output by 2.3%. The government needs to increase the consumption tax by 0.75% to finance such expenditures. Figure B.3j shows that welfare could be increased by up to 2.5%.

Figure B.3: Effects of Expansions in Public Education Expenditures of Different Sizes

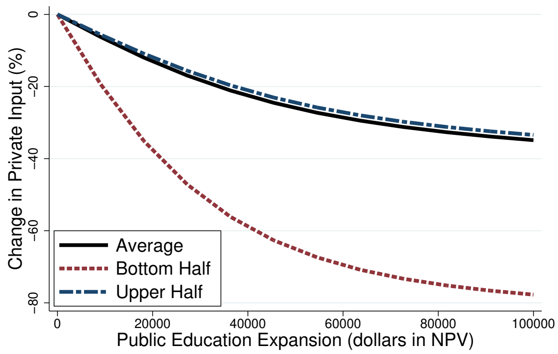
(a) Effects on Fertility



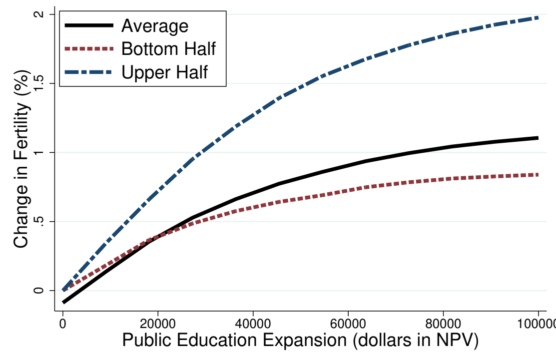
(b) Total Education Expenditure Share as % of GDP



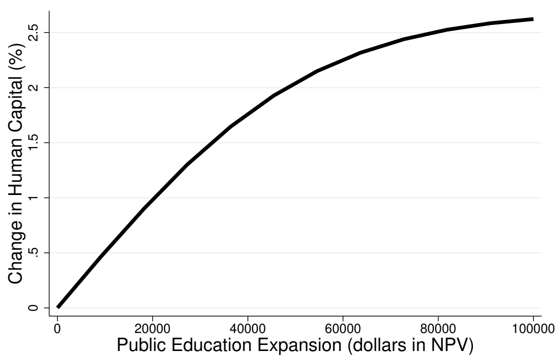
(c) Heterogeneous Private Input Response



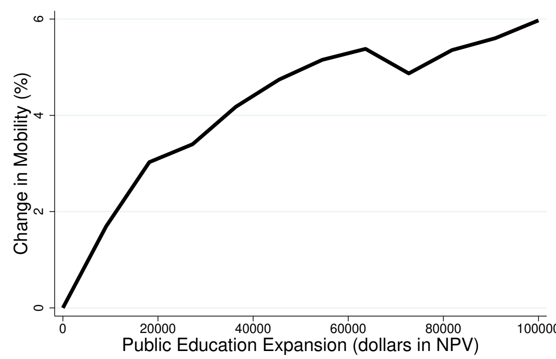
(d) Heterogeneous Fertility Response



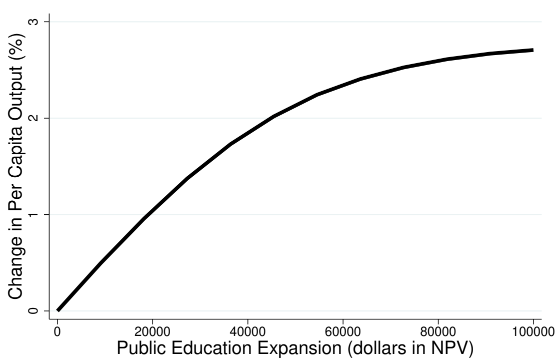
(e) Average Human Capital



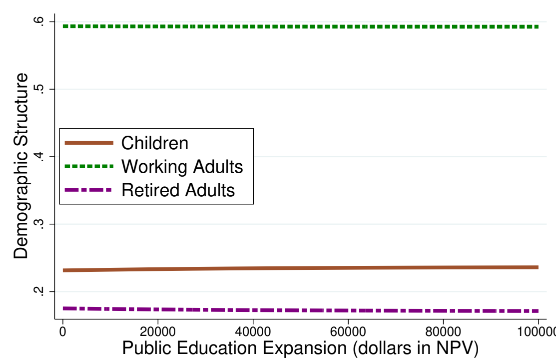
(f) Intergenerational Mobility



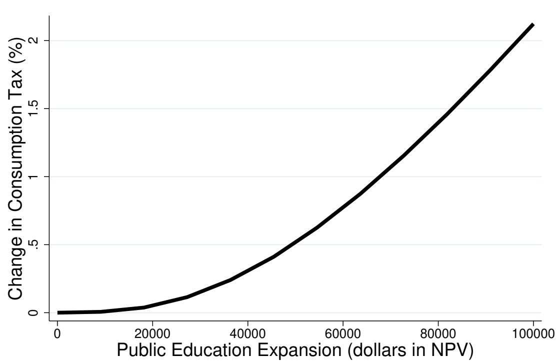
(g) Per Capita Output



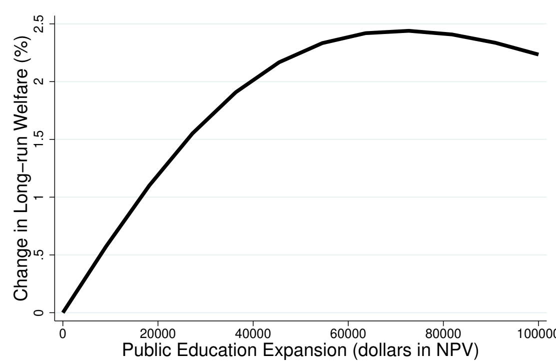
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of expansions in public education expenditures of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. In Figure B.3c and B.3d, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

C Additional Validating Evidence

C.1 Australian Baby Bonus

This section compares the implications of the calibrated model with the empirical evidence from a recently adopted baby bonus in Australia.

The Australian government announced the introduction of a universal cash payment, i.e., a baby bonus, in its federal budget on May 12, 2004. It was designed to be a non-means-tested and non-taxable lump-sum payment of AU$3,000 to encourage childbirth. The transfer would be paid to parents following the birth or the adoption of a child up to 2 years of age after July 1, 2004. Such payments are independent of family income, maternal employment status, or the number of existing children in the household. The payment is equivalent to 4 times the weekly average earnings or $2,800 in 2010 U.S. dollars.

Drago et al. (2011) conduct a comprehensive analysis of the fertility effects of the Australian baby bonus using household panel data ($N = 14,932$) from the Household, Income and Labour Dynamics in Australia Survey (HILDA) and a simultaneous equations approach. They first estimate the effect of the baby bonus on households' birth intentions in a linear regression, which, in turn, predicts actual births in a binomial probit regression. They exploit the panel structure of the data to test and correct for announcement effects, compression/tempo effects, and delayed effects. Regression results show that the baby bonus has significant and positive impacts on fertility. Moreover, the fertility responses are concentrated among low-income women.[65] Risse (2010) reports similar findings on women's birth intentions.

When I compare these empirical studies to the model, both the finding on the overall fertility effect and the heterogeneities across households are consistent with predictions in Section 5.1. Drago et al. (2011) estimate that the marginal cost to the government of an additional birth is at least A$126,000, roughly 4 times the GDP per capita in 2004 Australia dollars. The calibrated model generates a quantitatively similar conclusion where the marginal costs for an additional birth using a US baby bonuses in 2010 are around 3.5 times the GDP per capita.

Gaitz and Schurer (2017) evaluate the effect of the Australian baby bonus on child human capital accumulation using high-quality panel data from the Kindergarten cohort (K-cohort) of the Longitudinal Study of Australian Children (LSAC). They find that the baby bonus, despite being significant in size, is not effective in boosting learning, socio-emotional or physical health outcomes of the average pre-school child. This finding is consistent with results in Section 5.1 where parents

[65]This finding is different from that of Milligan (2005) where the author evaluates a child benefit in Quebec and finds larger responses among parents with higher incomes. Drago et al. (2011) propose that the difference could be due to the fact that the Quebec baby bonus was significantly more generous for high-parity births. Milligan (2005) discusses some other possible explanations such as unobserved heterogeneities that are systematically related to income.

optimally reduce private educational investments due to the quality-quantity trade-off.

C.2 Spanish Child Benefits

In this section, I provide additional evidence of the fertility effects of child benefits using a universal child benefits policy in Spain.

The Spanish government announced the new child benefit on July 3, 2007. The benefit was a one-time payment of €2,500 to the mother immediately following a child's birth on and after July 1, 2007. Since the cash benefit was universal and independent of the recipient's characteristics, it was essentially a universal baby bonus (c.f., the Australian baby bonus, and $\mathcal{B}$ in the model). The size of the payment is 4.5 times the monthly gross minimum wage for a full-time worker or $3,500 in 2010 U.S. dollars.

González (2013) studies the effect of the Spanish child benefit on fertility and mothers' labor supply using monthly vital statistics, monthly abortion statistics, and the 2008 Household Budget Survey ($N = 958$). In the paper, fertility effects are assessed by inspecting the time series of births and abortions, while effects on the labor supply are estimated using a regression-discontinuity design by comparing households who gave birth right before and right after the cutoff date. González (2013) finds that the fertility effects are positive and significant, with the policy increasing the total fertility rate by 6%. Around 80% of this increase is due to increased conception while the remaining 20% is due to reductions in abortion. The result suggests that (in Spain) the marginal cost of an additional birth[66] using baby bonuses is 3.6 times the GDP per capita. In the model, this statistic is 3.5.

González (2013) also finds that the baby bonus reduced mothers' labor force participation. The calibrated model produces this effect via two channels. First, as the baby bonus raises fertility, parents need to generate more childcare services, increasing their time at home ceteris paribus. Second, higher fertility reduces the relative cost of home versus market care due to economies of scale in home production of childcare ($\iota < 1$). Therefore, the model predicts that parents optimally change their childcare arrangements so that their children spend less time in market care. González (2013) confirms this prediction by showing that Spanish parents reduce enrollment in formal childcare after the baby bonus.

[66] Summarizing historical studies with both short-run and long-run effects on fertility, Stone (2020) show that one can multiply the policy effects on total fertility rates by 1/3 to derive a crude estimate of the effects on completed fertility rates.

C.3 Georgia's Cherokee Land Lottery of 1832

In this section, I compare the model predictions with Georgia's 1832 Cherokee Land Lottery in Northwest Georgia.

Following the eviction of the Cherokee from northwest Georgia, the state allocated more than 18,000 parcels of land to the public in a large-scale lottery in 1832. Every man aged 18 and older who had resided in Georgia for the three years prior to the 1832 drawing was entitled to one draw, and any man who had a wife or children under age 18 and met the three-year residency requirement was entitled to an additional draw. The lottery winners were able to sell the winning draw immediately. Therefore, winning the lottery can be viewed as a substantial shock to wealth. Bleakley and Ferrie (2013) calculate that 98% of eligible men participated in the lottery and winners were about $748 wealthier than losers by 1850, equivalent to 1,010 days of earnings for an unskilled laborer in the South at that time.

There are two notable things about the Cherokee Land Lottery. First, unlike a baby bonus that provides an estimate of the *price elasticity* of fertility, the Cherokee Land Lottery reveals the *income elasticity* of fertility. This statistic, if used directly, is mostly informative of the fertility effects of policies such as a universal basic income (UBI) rather than family policies that are pro-natal by design. Nevertheless, the model in this paper could generate the counterpart to the income elasticity of fertility by varying the initial assets of parents in period 2. Second, numerous changes have taken place in the U.S. in the past two centuries. The average white woman gave birth to around seven children in the 1830s but less than two children in 2010. The most significant factor explaining the decline of fertility and the rise of schooling is the increase in skill premium (Greenwood and Seshadri, 2002). Therefore, the Cherokee Land Lottery is expected to generate responses that are *upper bounds in fertility* and *lower bounds in child human capital investment*.

Bleakley and Ferrie (2016) identify the winners of this lottery using a list published by the State of Georgia, and they then link winners to their descendants using Census data. Results show that winners had slightly more children than non-winners, but they did not send them to school more. "Sons of winners have no better adult outcomes (wealth, income, literacy) than the sons of non-winners, and winners' grandchildren do not have higher literacy or school attendance than non-winners' grandchildren" (Bleakley and Ferrie, 2016).

In the calibrated model, I compare $n^*(h,a), e^*(h,a)$ with $n^*(h,a'), e^*(h,a')$ for fixed h and $a < a'$. I find that if parents are endowed with a higher level of initial assets, they choose to have a slightly smaller number of children, i.e., $n^*(h,a') \leq n^*(h,a)$ but invest significantly more in children's education, i.e., $e^*(h,a') \gg e^*(h,a)$. Since the return to education in 2010 is much higher than that in the 1830s, the model predictions are qualitatively consistent with those found in Bleakley and Ferrie (2016).

D Illustrative Models

D.1 Simple Model of Quality-quantity Trade-off

In this section, I show the quality-quantity trade-off mechanism with closed-form solutions in a model that is adapted from de La Croix and Doepke (2003).

Agents in the economy live for two periods, child and adult. Adult parents solve the problem:

$$\max_{c,n,e} \quad \log(c) + \theta \log(n \cdot h_k)$$

subject to

$$c + n \cdot e = 1 - n \cdot \chi$$

$$h_k = e^{\gamma}, \quad \gamma \in (0, 1)$$

where c is consumption, n is fertility, h_k is human capital of children, e is private investment,[67] and χ is fixed cost per child.

When fertility is exogenous, i.e., n is given, parents maximize over c and e. The optimal investment is given by

$$e^* = \frac{\theta\gamma}{1+\theta\gamma} \cdot \frac{1-n\chi}{n}$$

When the cost of child χ decreases, e^* increases due to the income effect.

When fertility is endogenous, parents maximize over c, e, and n. The optimal fertility and investment decisions are given by:

$$n^* = \frac{1}{\chi} \cdot \frac{\theta(1-\gamma)}{1+\theta}, \qquad e^* = \frac{\gamma\chi}{1-\gamma}$$

When the cost of a child χ decreases, n^* increases while e^* decreases. The intuition for this result is simple. Parents increase fertility due to the substitution effect. The increase in n^*, in turn, raises the shadow price of investment e^* due to the interaction between n and e in the budget constraint. As a result, the optimal investment e^* falls.

Compared with the benchmark model where n and h_k are complements in parents' preferences, reductions in e^* in response to a change in χ are higher in this simple model. This is because the marginal utility in child quality h_k is independent of fertility n.

[67] In de La Croix and Doepke (2003), children receive human capital endowments. This generates non-homotheticity over child quality for heterogeneous parents with different human capital and leads to a negative fertility-income relationship. I abstract away from steady-state heterogeneity across households in this simple model for clearer exposition of intuition in comparative statics, but all arguments carry through when human capital endowments are allowed.

D.2 Simple Model of Fertility Elasticity

In this section, I build a simple model to illustrate the relationship between parameter γ, also known as elasticity of intergenerational substitution (EGS), and the magnitude of fertility responses to family policies.

Consider a simplified problem for parents with very low income so that child quality is generated by public investments alone:

$$\max_{c,n} \quad u(c) + \Psi(n)u(\mathcal{E})$$

$$c + n \cdot \chi = 1$$

The first-order condition for n is therefore:

$$\underbrace{\Psi'(n) \cdot u(\mathcal{E})}_{\text{MB of } n} = \underbrace{\lambda \cdot \chi}_{\text{MC of } n}$$

By substituting in $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I have

$$\Psi'(n) = (1-\gamma) \cdot \chi \cdot \frac{\lambda}{\mathcal{E}^{1-\gamma}} \Longrightarrow \Delta\Psi'(n) \propto (1-\gamma) \cdot \Delta\chi$$

As can be seen, conditional on other parameters, higher γ implies a smaller response in n for given changes in costs of having a child χ.