The Macroeconomic Consequences of Family Policies*

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Abstract

Most developed countries have adopted large-scale family policies, but little is known about the macroeconomic consequences of such policies on output, social mobility, and welfare. Using a heterogeneous-agent overlapping-generations model, I argue that the child quality-quantity trade-off, an endogenous demographic structure, and the modeling of childcare arrangements are important for analyzing the effects of family policies. With the model calibrated to the U.S. data, I find that a $30,000 baby bonus boosts the aggregate fertility rate to the replacement level, but contrary to the conventional wisdom, average human capital and social mobility both fall. Despite these drawbacks, such baby bonus raises welfare in the long run by 1.6% as the old-age dependency ratio decreases. The long-run welfare gain, however, requires a transition period with higher child-related government spending, which may hurt the welfare of existing households depending on how those expenditures are funded. Other family policies are less cost-effective in raising fertility than the baby bonus but have their own advantages: subsidized childcare encourages parents' labor force participation and reduces inequalities in wage growth, while public education expansions improve child outcomes and social mobility.

JEL classification: E62, H31, H52, J11, J13

Keywords: Family policies, quality-quantity trade-off, demographic structure

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1 Introduction

Family policies are social programs and laws designed to promote and enhance family formation, reproduction, and raising children (Bogenschneider, 2011). Examples of family policies include baby bonuses[1], parental leaves, subsidized childcare, and so on. These policies are prevalent in developed countries and are often large in scale. As of the most recent tabulations in 2017, every OECD country has sizable public spending exclusively on family and children. These expenditures, in-cash and in-kind, account for 2.1% of national GDP on average.[2]

In most cases, governments pursue family policies to achieve two goals. First, family policies are among the key instruments[3] to address population aging that poses imminent threat to economic growth (Aksoy et al., 2019) and the sustainability of public pension system (Bongaarts, 2004). For example, the Australian government made parents eligible to receive a lump-sum payment equivalent to $2,800 for every child born after July 1, 2004 to mitigate the effects of its aging population. The Treasurer of Australia at that time stated that the goal for such policy was to encourage each family to have "one (baby) for the Mum, one for the Dad, and one for the country". Similarly in Russia, Spain, Singapore, many countries struggling with low fertility are using family policies to incentivize childbirth. Concerns over the low fertility have also been voiced in the United States as it recently experiences record-low birthrate.[4]

Second, governments also endorse family policies to reduce child poverty, improve children's outcomes, and raise social mobility. Stating such goals, expansions in (refundable) Child Tax Credit (CTC) have received bipartisan support in the U.S. Congress,[5] while the maximum amount of CTC awarded per child annually increased from $2,000 to over $3,000 in 2021. In addition, the Biden Administration has also proposed the $1.8 trillion American Families Plan advertised as "an investment in our kids, our families, and our economic future" (The White House, 2021). If implemented, the Plan would provide further child-related tax benefits, more generous parental leaves, and greater support for public education. Guided by empirical evidence suggesting that transfers to families with children could improve children's future outcomes, economists predict that such policies will "lift children out of poverty today and help them tomorrow" (Schanzenbach et al., 2021) and boost social mobility (Pulliam and Reeves, 2021).

With wide implementation of large-scale family policies, it is important to understand their

[1] A baby bonus is a one-time, lump-sum cash transfer to parents for each newborn or adopted child. Notably, a baby bonus is different from a "baby bond" proposed to reduce wealth gaps. Under baby bonds, children themselves, but not the parents, gain access to the trust account at the age of 18.

[2] See https://data.oecd.org/socialexp/family-benefits-public-spending.htm for more details.

[3] Other important instruments include immigration policies and pension system reform. In this paper, I abstract away from immigration and keep the pension system fixed in policy counterfactuals.

[4] See National Vital Statistics report on births at https://www.cdc.gov/nchs/data/vsrr/vsrr012-508.pdf.

[5] See past proposals from Senator Bennett (D-CO). Senator Romney (R-UT) also proposed the Family Security Act which is similar to a fully refundable CTC but without minimum income requirement and other limitations..

macroeconomic consequences. Are family policies cost-effective in raising fertility, reducing old-age dependency ratio, and improving social welfare? Will generous child benefits improve children's outcomes and raise social mobility? How should policy makers choose between in-cash benefits such as baby bonuses versus in-kind ones such as subsidized childcare? In this paper, I analyze these questions through the lens of a quantitative heterogeneous-agent model with overlapping-generations in the general equilibrium.

Compared with the existing work on family policies, the model has three novel features. First, unlike most design-based studies that focus on fertility or child outcomes in isolation, this model allows households to choose the number of children to have and simultaneously decide on child quality through child human capital investments and inter-vivos transfers. Both margins are important since they correspond to the two main goals of family policies, namely raising fertility and child outcomes. They are also closely linked in the model due to *quality-quantity trade-off* a lá Becker and Lewis (1973). When parents increase fertility to take advantage of child benefits, the marginal cost of providing the same level of child quality goes up. Therefore, as I show in this paper, policies that aim at boosting fertility may worsen children's outcomes.

The joint determination of child quality and quantity has further implications on aggregate variables through a mechanism called the *composition effects*. With heterogeneities in human capital and wealth, parents respond differently in fertility and child quality choices to family policies. Some parents may increase fertility more relative to others, which results in their children accounting for a larger share of the population in the future. Due to the intergenerational transmission of traits, e.g., human capital, aggregate variables gravitate over time towards the traits of those parents that respond most strongly in fertility.

Second, while most structural models with endogenous fertility consider two-period-lived agents, the framework in this paper explicitly models realistic life cyle with childhood, working age, and retirement totaling nine periods where each period represents 10 years. This allows the model to quantify the *demographic structure effects* of family policies where following a rise in the population growth rate, the burden of pension payments under pay-as-you-go (PAYG) system is relieved but public education expenditures on children rise. The rich demographic structure also highlights the distributive consequences of family policies across generations, especially along the transition path.

Third, unlike most models with endogenous fertility where each child costs a fixed amount of time, I consider endogenous *childcare arrangements* in the model where parents can either choose to take care of children at home by themselves or utilize market childcare services at a price. Besides allowing the model to better match childcare utilization patterns in the data, this feature facilitates the comparisons between family policies that are in-cash, e.g., baby bonuses or child allowances, and those that are in-kind, e.g., subsidized childcare (vouchers). Distinguishing in-cash and in-kind transfers are important for predicting the policy effects on parents' labor force participation, which in turn affects earnings inequality over the life cycle due to human capital accumulation on the job.

All mechanisms discussed above depend on the magnitude of fertility response to financial incentives, hereafter denoted as *fertility elasticities*. In the extreme case where fertility is exogenous, i.e., fertility elasticity is zero, child-related benefits simply have an income effect on children's quality and improves social mobility if the benefits are progressive (Daruich, 2018; Mullins, 2019). But if households increase fertility after family policies are adopted, i.e., fertility elasticity is bigger than zero, the above-mentioned mechanisms generate novel and sometimes counter-intuitive predictions. I discipline the magnitude of fertility elasticities using a set of calibrated parameters that matches key empirical regularities in the United States in 2010. I also provide external validations using the Alaska Permanent Fund Dividend (APFD) and several other programs to show that the quantitative predictions of the model is consistent with existing design-based evidence.[6]

The calibrated model generates three main findings. First, a baby bonus of $30,000 in 2010 dollars[7] boosts the average fertility rate from 1.9 children per women to 2.1 children per women, the replacement level. Parents with lower human capital have stronger responses to the policy by having more additional children. Surprisingly, I find that such baby bonus is not effective in raising children's human capital and social mobility. Parents optimally reduce child educational investments by 4% on average due to the quality-quantity trade-off, and this reduction is larger among low-income parents. Such heterogeneous reduction in educational investments, coupled with the composition effects due to heterogeneous fertility responses discussed above, will lower the average human capital by 2% and intergenerational income mobility by 1.8% in the long run.

Second, despite reductions in average human capital and social mobility, long-run social welfare[8] rises by 1.6% in consumption equivalence units due to the demographic structure effects of the $30,000 baby bonus. As the old-age dependency ratio[9] falls following an increase in fertility, the government is able to reduce tax rates while still balancing fiscal budget. Such baby bonus is also highly progressive. It improves the welfare of parents with low human capital by nearly 6%.

The transition towards such long-run welfare improvements, however, requires the government to finance higher child-related expenditures in the first few decades. While the old-age dependency ratio falls gradually, the total dependency ratio[10] spikes before converging to its long-run level. The welfare consequences on existing agents can be very different from those in transition or in the

[6] Additional validation exercises include Australian baby bonus, Spanish child benefits, and Georgia's Cherokee Land Lottery of 1832 presented in Appendix B.

[7] To put this number in perspective, this baby bonus is similar to the size of expansions in Child Tax Credit (CTC) from 2010 to 2021 in net present value (author's own calculation). It offsets the average cost of raising on child by 19% using estimates from U.S. Department of Agriculture (USDA).

[8] Average utility of new-born under the veil of ignorance. See Section 6 for more discussions on welfare criteria.

[9] The number of retired people divided by the number of working population.

[10] The number of retired people plus the number of children, divided by the number of working population.

long-run steady-state depending on how these additional expenditures are funded. For example, if the government chooses to balance the budget from period to period using changes in consumption tax, most households in the current economy experience welfare losses.

Lastly, I use the model to study the macroeconomic impacts of alternative policies at the forefront of public debate, such as subsidized childcare and expansions in public education expenditures. These policies are less cost-effective in raising fertility than the baby bonus but have other advantages. In particular, subsidized childcare encourages parents to work in the labor market since it substitutes home production of childcare with market childcare services. The increased labor force participation, especially among parents with low human capital, leads to a reduction in inequality in wage growth. Public education expenditure, on the other hand, has the smallest fertility effects. Nonetheless, among all policy instruments studies in this paper, it is the most effective one in improving child outcomes and boosting social mobility.

Related Literature

This paper relates to four strands of literature. The first set of papers study fertility in a macroeconomic context and analyze how it is related to the fiscal stability, economic growth, intergenerational mobility, and inequality. Notable examples include Barro and Becker (1989), Doepke (2004), Greenwood et al. (2005), Manuelli and Seshadri (2009), De Silva and Tenreyro (2020), Cavalcanti et al. (2021), and Jones (2020) among many others. The closest works are de La Croix and Doepke (2003) and Kim et al. (2021). de La Croix and Doepke (2003) study how higher inequality reduces growth through the channel of fertility differentials. While I adopt their framework featuring general equilibrium, heterogeneous agents, endogenous fertility, and human capital formation, I extend the canonical model in various dimensions and use the model to study aggregate impacts of family policies. In a recent work, Kim et al. (2021) adopt the de La Croix and Doepke (2003) model and add a status competition mechanism to explain the fertility-income relationship in Korea and study the impacts of pronatal transfers and education taxes. Different from their work which assumes two-period-lived agents, this paper allows for a richer life-cycle structure so that demographic structure effects, an important motivation for most family policies, can be quantified.

Second, this paper is related to the literature on positive and normative aspects of government policies concerning fertility, redistribution, and education. On the positive side, Daruich (2018) and Abbott et al. (2019) are the closest work to mine. Both papers use a heterogeneous-agent overlapping-generations model to study large-scale education policies. I differ from these papers by modeling endogenous fertility choice and considering the interaction of family and education policies. On the normative side, Guner et al. (2020) considers optimal design child-related transfers with heterogeneous households making both intensive and extensive labor force participation choices while keeping fertility exogenous. This paper adds to the literature by studying the design of family policies with endogenous fertility responses.

Third, this paper also relates to a large body of empirical work on the fertility effects of family policies (e.g., Milligan, 2005; Drago et al., 2011; González, 2013; Laroque and Salanié, 2014). Meta-analyses of these studies have concluded that "the directional finding that pronatal benefits boost fertility is nearly uniform." An increase in present value of child benefits equal to 10% of household annual income would lead to 0.5% to 4.1% increase in the total fertility rate (McDonald, 2006; Stone, 2020).[11] These papers, however, are not suited to address the broader consequences of family policies beyond fertility, such as the effects on output, human capital, social mobility, and welfare. The design-based approach is also not able to predict the consequences of family policies in the long-run when children being affected become parents, which changes the distribution of the underlying population of interest. This paper takes a structural approach to answer these questions.

Lastly, the paper connects a growing literature that evaluates the impacts of transfers or subsidies to families with children. Prominent examples include Akee et al. (2010), Dahl and Lochner (2012), Mullins (2019), and García et al. (2020), among many others. With both design-based methods and structural models with exogenous fertility, the conventional wisdom in the literature is that transfers to families with children have positive impacts on children's outcomes including health, education attainment, and criminal records (Schanzenbach et al., 2021). This paper utilizes experimental evidence from this literature to inform parameter choices in the calibration. Meanwhile, given that this literature exclusively evaluates policy effects of transfers to families with existing children and abstracts away from fertility responses, I argue that their conclusions could not be directly used to extrapolate the aggregate impacts of family policies.

The rest of the paper is organized as follows. In Section 2, I present the quantitative model. Section 3 describes the calibration of the model in detail. I conduct validation exercises in Section 4. Section 5 presents the main policy counterfactual results of the paper. Section 6 discusses the optimal policy design. Section 7 displays the robustness of the main results and Section 8 concludes.

2 Model

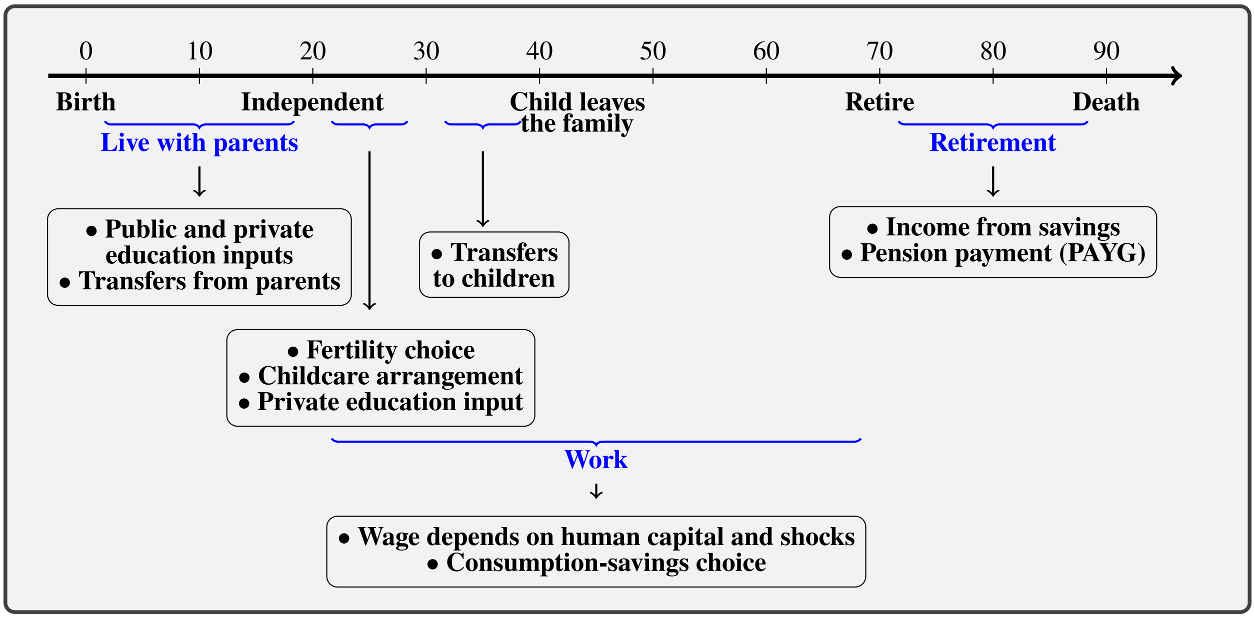
2.1 Household

Consider overlapping generations of households[12] that can live up to 90 years old. I split the life-cycle into 9 periods where where each period represents 10 years and is indexed by $j \in \{0, 1, 2, \dots, 8\}$. Period j stands for age $j \times 10$ to age $(j + 1) \times 10$.

[11] The distinction between quantum versus tempo effects on fertility, i.e., the completed fertility rate versus the total fertility rate, will be discussed with more details in Section 4.

[12] In this paper, I model unitary/collective decision-making within families and abstract away from intra-household bargaining. Hereafter I will be using agents, households, and adults interchangeably.

Figure 1: Model's Life Cycle



The life cycle of each agent in the model is presented in Figure 1. Children live with their parents from age 0 to 20 where they make no choices and receive human capital investments from both public and private sources. Children also receive inter-vivos transfers from parents when they become independent and form families in their 20s. Households choose fertility, childcare arrangements, and private child human capital investments from age 20 to 30.[13] Between age 30 to 40, parents determines the amount of inter-vivos transfers that their children would receive at the beginning of the next period.

People work from age 20 to 70 where their earnings depend on the amount of efficiency units, i.e., human capital, supplied to the market. For working adults, I assume that their human capital evolves over time following an age-dependent learning-by-doing process:

$$h_{j+1} = L_j(h_j, t_w, z_{j+1}) \tag{1}$$

where z_{j+1} is an idiosyncratic uninsurable shock to human capital that occurs at the beginning of period $j+1$; h_j is the amount of human capital at period j; t_w is the time worked in period j. I assume that agents have unit time endowment and supply labor inelastically except in period 2 where they trade off providing childcare at home against supplying labor in the market.[14]

[13] In Section 7.2, I argue that the main results of this paper are robust to allowing for endogenous timing for childbirth where parents can also give birth in their 30s.

[14] I have experimented with adding endogenous labor supply with leisure in the utility function. Quantitative results remain largely unchanged, but computation time is significantly increased, especially for the transition path. Therefore,

Agents retire at the end of period 6 (age 60 to 70). Retired adults receive income from savings and pension payments for the remaining of their lives. Households make consumption-savings choices in each period and face age-specific survival rate δ_j.

The state variables of young adults at the beginning of period 2 (age 20-30) are human capital level h and assets a. Both of them are endogenously determined in equilibrium by these young adults' parents in previous periods. Agents choose consumption c, savings a', fertility n, total time spent at home taking care of children t_h, amount of market childcare purchased for each child m, and private education investment for each child e.[15] Parents' value $V_2(\cdot)$ is composed of flow utility from consumption $u(c/\Lambda(n))$ where consumption expenditure c is divided by equivalence scale $\Lambda(n)$ and the discounted continuation value $V_3(\cdot)$. Their maximization problem is given by:

$$V_2(h,a) = \max_{c,a',n,t_h,m,e\geq 0} u(c/\Lambda(n)) + \beta \mathbb{E} V_3(h', a', n, \mathbb{E}h_k)$$

subject to

$$n \cdot \chi = \left(t_h^{\upsilon/\iota} + (n \cdot (m + \mathcal{S}))^{\upsilon}\right)^{1/\upsilon} \quad \text{[time cost]}$$

$$y = wh \cdot (1 - t_h) \quad \text{[labor income]}$$

$$(1+\tau_c)(c + p_m \cdot n \cdot m + e \cdot n) + a' = (1+r)a + y - \mathcal{T}(y,a,n) + \mathcal{B} \cdot n \quad \text{[BC]}$$

$$h' = L_2(h, 1 - t_h, z') \quad \text{[learning OTJ]}$$

$$h_k = G(h, \mathcal{E}, e, \epsilon) \quad \text{[skill formation]}$$

To raise n children, parents need to produce $n \cdot \chi$ amount of childcare (in time units). Parents could satisfy this need either by spending their own time at home (t_h) or by spending money purchasing market childcare services m. The government could choose to provide in-kind childcare $\mathcal{S} \in [0, \chi]$ free of charge to parents. I assume that these two forms of childcare are combined in a constant-elasticity-of-substitution (CES) production function where υ governs the elasticity of substitution. Household production of childcare enjoys an economy of scale with parameter $\iota \in (0,1)$. This captures the fact that taking care of two children at home simultaneously costs less than two times the hours needed to take care of a single child (Folbre, 2008).

The opportunity cost of home production of childcare t_h is hours spent in the labor market. The household's labor income y is the product of market wage w, human capital h, and time worked $(1 - t_h)$. Total resources available to the parents consist of risk-free asset a multiplied by gross interest rate $(1 + r)$, labor income y, net taxes payment $\mathcal{T}(y, a, n)$, and the total amount of baby bonuses received $\mathcal{B} \cdot n$ where the amount $\mathcal{B}$ is chosen by the government. The household divides

I omit labor supply in the baseline model.

[15] Following de La Croix and Doepke (2003), fertility choice n in the model is continuous.

the resources into savings a' and different expenditures including consumption c, total spending on market childcare $p_m \cdot n \cdot m$, and private educational expenditure $e \cdot n$. I use p_m to denote the price of market childcare relative to consumption goods.[16] All expenditures are subject to a proportional consumption tax τ_c.

Children's human capital production function $G(h, \mathcal{E}, e, \epsilon)$ combines parents' human capital h, public education $\mathcal{E}$, private investment e, and idiosyncratic ability shock ϵ that is unknown to the parents. To keep the model tractable, I make a simplifying assumption that parental investments in children's human capital take place when children's age is between 0 to 10 while public education $\mathcal{E}$ plays the major role from age 6 to 20. The function $G(\cdot)$ captures the overall human capital production function that spans age 0 to 20 without explicitly modeling several stages of production. Moreover, to the extent that the bulk of childcare needs $n \cdot \chi$ are non-educational, i.e., preparing food or changing diapers, the arrangement of childcare does not affect children's human capital directly in the model.[17] Parents can invest in their children's human capital through monetary investments e. Since the government chooses public education expenditure $\mathcal{E}$, the model could replicate the public provision of high-quality childcare or pre-K by adopting high $\mathcal{S}$ and $\mathcal{E}$ at the same time.

In the economy, households face inter-temporal borrowing constraint $a' \geq 0$ that is standard in the class of Aiyagari-Huggett models. Moreover, parents are not allowed to invest in negative amounts of resources in children's education, i.e., $e \geq 0$. As a result, public investment $\mathcal{E}$ gives a lower-bound of total educational investments received by every child.[18]

Parents' maximization problem at age 30-40 is given by:

$$V_3(h, a, n, \mathbb{E}h_k) = \max_{c, a', a_k \geq 0} u(c/\Lambda(n)) + \beta \mathbb{E} V_4(h', a') + v(n, \mathbb{E}h_k, a_k)$$

subject to

$$y = wh$$

$$(1 + \tau_c)c + a' + n \cdot a_k = (1 + r)a + y - \mathcal{T}(y, a, n) \tag{2}$$

[16]In this paper, I will be assuming that the supply of childcare services is perfectly elastic at baseline price p_m. Since each period represents ten years, adjustments of capitla and labor in and out of the childcare industry equates prices of childcare to its long-run marginal costs as long as the industry is perfectly competitive. In the short run, of course, relative prices of childcare may change in response to family policies.

[17]As Guryan et al. (2008) noted, educational childcare time accounts for at most a quarter of time that parents spend with children. In a recent work, Chaparro et al. (2020) develop a model of childcare with both quality and quantity aspects of maternal and non-maternal care. In their model, effects on child human capital is proportional to both care quality and time "exposed" to each type of care, but they do not explicitly consider time with children that are non-educational. The model in this paper separates the essence of childcare into non-education chores χ and endogenous educational investments, money or time, captured by e and $\mathcal{E}$.

[18]Since public education funding depends on local taxation (Kotera and Seshadri, 2017), one might worry that the amount of education resources received by each child are not uniformly $\mathcal{E}$. The child human capital production function $G(\cdot)$ allows for this possibility through its dependency on parents' human capital h. In other words, $\partial G(\cdot)/\partial h$ captures not only the genetic transmissions of ability but also other channels including residential segregation.

$$h' = L_3(h, 1, z')$$

where parents choose consumption, savings, and the amount of transfers to be received by each child at the beginning of next period (a_k). Following the literature, I assume that parents face intergenerational borrowing constraints so that they are not allowed to make negative transfers to children, i.e., $a_k \geq 0$. I use $v(n, \mathbb{E}h_k, a_k)$ to denote parents' preferences over child quantity, child human capital, and inter-vivos transfers. The parametric form of $v(\cdot)$ will be discussed in detail in Section 3.1.

For $j \in \{4, 5, 6\}$, households solve a simple consumption-savings problem with idiosyncratic shocks to human capital. For tractability, I assume that parents and children no longer interact in these periods.[19] The maximization problem is given by:

$$V_j(h, a) = \max_{c, a' \geq 0} u(c/\Lambda(0)) + \beta \mathbb{E} V_{j+1}(h', a') \qquad j \in \{4, 5, 6\}$$

subject to

$$(1 + \tau_c)c + a' = (1 + r)a + y - \mathcal{T}(y, a, 0)$$

$$h' = L_j(h, 1, z)$$

Finally, retired agents solve:

$$V_j(h, a) = \max_{c, a' \geq 0} u(c/\Lambda(0)) + \beta V_{j+1}(h, a') \qquad j \in \{7, 8\}$$

$$(1 + \tau_c)c + a' = (1 + r)a + \pi \cdot wh - \mathcal{T}(0, a, 0)$$

$$V_9(\cdot) \equiv 0$$

where household income is composed of risk-free assets and pension payment $\pi \cdot wh$ where π to denote the pension replacement rate.[20]

2.2 Firms

There is a representative firm in the economy that hires labor and borrows capital from households to produce final goods with Cobb-Douglas technology:

$$Y = A \cdot K^{\alpha} H^{1-\alpha} \tag{3}$$

[19] Otherwise, parents need to keep track of all children's state variables, including that of the grandchildren. This will make the problem computationally infeasible.

[20] For simplicity, I assume that pension payments are not subject to labor income taxes.

In (3), K is aggregate capital used in production and H is total efficiency units employed. Total factor productivity A is normalized to be one.[21]

I assume that physical capital depreciates at rate δ_K after use. With competitive factor markets, the equilibrium wage and risk-free interest rate are given by:

$$r = \alpha \left(\frac{K}{H}\right)^{\alpha-1} - \delta_K, \quad w = (1-\alpha)\left(\frac{K}{H}\right)^{\alpha}$$

2.3 Government

The government collects revenues from taxing labor income, capital income, and household expenditures (including spendings on market childcare and children's education). Government expenditures include public education, family policies, pension payments, and other per-capita exogenous expenditures denoted as $\mathcal{X}$. I assume that the exogenous expenditure $\mathcal{X}$ is invariant across policies. The government balances the budget from period to period.[22]

I use $\{\mu_j\}_{j=0}^{8}$ to denote the distribution of households across state space and use $\{\omega_j\}_{j=0}^{8}$ to denote the fraction of each age group in the population with total mass normalized to one at each date. The government budget is given by:

$$\underbrace{\left(\sum_{j=2}^{6} \omega_j \int \mathcal{T}(y_j^*, a_j^*, n_j^*)\, d\mu_j\right)}_{\text{labor and capital income taxes}} + \underbrace{\tau_c \left(\sum_{j=2}^{8} \omega_j \int c_j^*\, d\mu_j + \omega_2 \int n^* \cdot (p_m \cdot m^* + e^*)\, d\mu_2\right)}_{\text{consumption taxes}} = \underbrace{\underbrace{(\omega_0+\omega_1)}_{\text{mass of children}} \cdot \mathcal{E}}_{\text{public education expenditures}}$$
$$+ \underbrace{\pi \cdot \left(\sum_{j=7}^{8} \omega_j \int wh\, d\mu_j\right)}_{\text{pension payments}} + \underbrace{\omega_2 \left(\underbrace{\int n^* \cdot \mathcal{B}\, d\mu_2}_{\text{baby bonus}} + \underbrace{\int (1+\tau_c) \cdot n^* \cdot p_m \cdot \mathcal{S}\, d\mu_2}_{\text{subsidized childcare}}\right)}_{\text{family policy expenditures}} + \underbrace{\mathcal{X}}_{\textbf{other spendings}} \tag{4}$$

2.4 Equilibrium

I use t to denote time. The equilibrium of the economy is defined as a tuple composed of:

- Decision rules $\{c_t^*, a_t'^*, n_t^*, m_t^*, t_{h,t}^*, e_t^*, a_{k,t}^*\}_{t=0}^{\infty}$
- Prices $\{w_t^*, r_t^*\}_{t=0}^{\infty}$

[21] I abstract away from population externalities that could affect aggregate production such as pollution (Bohn and Stuart, 2015) and ideas creation (Jones, 2020). This choice is made since (1) the literature on measurement of population externalities is still developing, and (2) paper results will change in expected ways once positive/negative externalities are incorporated.

[22] I discuss the possibility of allowing for government borrowing in Section 7.4.

- Government policies $\{\mathcal{T}_t(\cdot), \tau_{c,t}, \mathcal{B}_t, \mathcal{S}_t, \mathcal{E}_t\}_{t=0}^{\infty}$
- Distribution of agents $\{\{\mu_{j,t}\}_{j=0}^{8}, \{\omega_{j,t}\}_{j=0}^{8}\}_{t=0}^{\infty}$

such that households maximize utility, prices clear labor and capital markets, the government balances budget in each period, and the distribution of agents evolve following the dynamics shaped by household decision rules, exogenous labor market shocks z, and the ability shock for children ϵ.

The evolution of the human capital distribution[23] from parents to children is given by:

$$\mu_2'(h) = \frac{1}{N} \iint n^*(x) \mathbb{1}_{h_k^*(x,\epsilon)<h} \, d\mathcal{Q}(\epsilon) \, d\mu_2(x) \tag{5}$$

where $\mathcal{Q}$ denotes the distribution of child ability shock ϵ; $h_k^*(x, \epsilon)$ is the human capital of a child whose ability shock is ϵ and parents' human capital is x; N is the aggregate fertility rate in the economy defined as:

$$N = \int n^*(x) \, d\mu_2(x) \tag{6}$$

In a stationary equilibrium of the economy, decision rules, prices, and distributions are unchanged over time. The size of the population, however, could change over time as aggregate fertility rate is not necessarily at the replacement level. The process defined in (5) is a multi-type branching (or Galton-Watson) process. Under mild conditions that are satisfied by the model, it can be shown that a stationary distribution exists and is unique (see Mode, 1971; Chu, 1990).

2.5 Welfare

To facilitate comparisons between different government policies, I define social welfare in the long-run ($\mathcal{W}$) as the *average value* of households at the beginning of their life-cycle, i.e.,:

$$\mathcal{W} = \int V_2 \, d\mu_2 \tag{7}$$

where both V_2 and μ_2 are endogenous equilibrium objects. Welfare metric $\mathcal{W}$ measures the expected utility of a newborn child under the veil of ignorance by summing up the discounted utility flow from later life periods. When making welfare comparisons, I will convert changes in $\mathcal{W}$ into percentage changes in consumption equivalence using utility function $u(\cdot)$.

Welfare metric $\mathcal{W}$, on the other hand, fails to capture the policy effects on households who are *already alive* when the policy is enacted, or on those who will be born in the *transition phase*. I will discuss how these people are affected by family policies in Section 5.3. Section 6 contains more discussions about different welfare criteria in models with endogenous fertility.

[23] For simplicity of exposition, here I omit the notation of distribution over initial assets.

No matter how welfare is defined, government policies have the potential to improve the well-beings of agents for two reasons. First, childbearing and child-rearing carry fiscal externalities as parents do not internalize the effects of having an additional child or investing in children's human capital on future tax base and government revenues. Infinitesimal parents take the age structure $\{\omega_j\}_{j=0}^{8}$, distribution $\{\mu_j\}_{j=0}^{8}$, and tax rates as given, but these objects will change when a mass of parents adjust their decisions on fertility and educational investments. Due to the lack of property rights of parents on children's future output, the equilibrium level of child human capital investments are too low relative to planner's solution a lá Schoonbroodt and Tertilt (2014). In contrast to their paper, with heterogeneous agents in the model, the equilibrium fertility level could be either too high or too low depending on parental characteristics.[24]

Second, parents face both inter-temporal and intergenerational borrowing constraints due to imperfections in capital markets. Parents cannot borrow against their children's future income or their own future income to finance current expenditures. Government policies can overcome these inefficiencies by providing in-cash or in-kind transfers (Daruich, 2018; Abbott et al., 2019).

2.6 Discussion of Mechanisms

In this section, I discuss the mechanisms through which family policies affect the economy. In particular, I highlight how the novel features of this model, namely fertility and child quality choice, endogenous demographic structure, and childcare arrangements make the model predictions on the overall impacts of family policies distinct from those in standard models.

2.6.1 Quality-quantity Trade-off

Consider the effect of an increase in the baby bonus $\mathcal{B}$ on inter-vivos transfer to each child (a_k).[25] The first-order condition of a_k is given by:

$$\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial a_k} = \lambda_3 \cdot n$$

where λ_3 denotes the Lagrangian multiplier on parents' budget in period 3.

When fertility is exogenous, i.e., n is fixed, the increase in $\mathcal{B}$ is an *income transfer*, which implies that a_k rises unambiguously due to income effect. As marginal utility of consumption MU_c

[24]The policy implications of this observation, however, is less straightforward than one might think. Due to intergenerational persistence of human capital, a tempting conclusion is that the policy maker should restrict fertility among the poor (Chu and Koo, 1990). I show that this argument is not valid in Section 5.1 and 6.3.

[25]The same argument applies to the child human capital investments e.

decreases, a_k need to increase to keep the first-order condition satisfied:

$$\underbrace{\lambda_3 \downarrow}_{\text{income effect on } MU_c} \cdot n \quad \Rightarrow \quad \frac{v(n, \mathbb{E}h_k, a_k \uparrow)}{\partial a_k}$$

When fertility is endogenous, however, the increase in $\mathcal{B}$ is a *price change*. The direction in which a_k changes is ambiguous. As fertility n rises due to more generous child benefits,[26] it affects the first-order condition of child quality a_k in three ways. First, it interacts with a_k in parents preferences $v(\cdot)$. If quality and quantity are complements (substitutes), then parents would demand higher (lower) a_k ceteris paribus. Second, rising quantity could potentially offset the income effect since higher n raises in the marginal utility of consumption via change in equivalence scale $\Lambda(n)$. Lastly, marginal cost of child quality a_k rises because it is proportional to fertility n due to their interaction in parents' budget constraint (see Equation (2)).[27]

$$\underbrace{\frac{\partial v(n \uparrow, \mathbb{E}h_k, a_k \mathbf{?})}{\partial a_k}}_{\text{interaction in preferences}} = \underbrace{\lambda_3 ?}_{\text{change in } MU_c \text{ as } n\uparrow} \cdot \underbrace{n \uparrow}_{\textbf{fertility response}} \tag{8}$$

While most design-based research find positive effects of child benefits on the outcomes of *existing* children being affected (Heckman and Mosso, 2014; Schanzenbach et al., 2021), considering endogenous fertility in the model allows for the possibility that child quality, either human capital or transfers received, could fall when family policies become more generous. This possibility is not only of theoretical interest but also of empirical relevance since (1) increasing fertility is one of the major goals of family policies, and (2) pronatal transfers have clear and significant impacts on fertility choices (McDonald, 2006; Stone, 2020; also see Section 4).

2.6.2 Composition Effects

Since parents differ by human capital h and asset holdings a, they respond differently to the same baby bonus $\mathcal{B}$ or subsidized childcare $\mathcal{S}$.[28] With endogenous quality-quantity trade-off, families with stronger fertility responses gain representation in the economy over time as their children account for a larger fraction of the future population. This change in the fertility differential across households, combined with intergenerational transmission of traits, lead to what I call the *compo-*

[26] This holds if child quantity is not a Giffen good. See McDonald (2006) and Stone (2020) for supporting evidence.

[27] Becker and Lewis (1973) named this last effect quality-quantity trade-off. In this paper, I abuse this notation slightly to denote the overall effects of an increase in fertility on child quality. Using twin births as instruments, some recent studies (e.g., Black et al., 2005; Angrist et al., 2010) find little evidence of the trade-off. A recent study by Mogstad and Wiswall (2016), however, overturns the conclusion by relaxing the linear specification. They find evidence of a trade-off of quality and quantity for larger families and complementarities in small families.

[28] See Drago et al., 2011 and Figure 9 for empirical evidence.

sition effects where aggregate variables, such as average human capital, gravitate towards those of the households with the highest fertility elasticities.

From a theoretical perspective, the composition effects generate an interesting scenario where aggregate human capital decreases despite positive policy effects on individual child's human capital. Quantitatively speaking, the composition effects are shown to be of empirically important for the evolution of aggregate variables such as economic growth (de La Croix and Doepke, 2003) and public opinions on family values (Vogl and Freese, 2020). Therefore, policy makers should consider the composition effects when evaluating and designing family policies.

2.6.3 Demographic Structure Effects

With endogenous fertility, family policies change the population growth rate and hence the demographic structure $\{\omega_j\}_{j=0}^{8}$. This change has profound implications on the government budget constraint (4) since it determines how each source of revenue or expenditure is weighted.

Most macroeconomic structural models with endogenous fertility assume two-period-lived agents (e.g., de La Croix and Doepke, 2003; Kim et al., 2021). From the government budget point of view, an increase in the population growth rate leads to an unambiguous rise of the fiscal burden in this class of models since there are fewer tax-paying adults to finance the public education expenditures on children. As a result, tax rates need to rise. Most developed countries that actually adopt family policies, however, have another story in mind where tax rates should be *lower* instead of higher in the long run. The key missing piece in the analysis is the presence of retired households that receives pension payments.

By considering a realistic life cycle with childhood, working-age, and retirement in this model, higher fertility rates reduce the old-age dependency ratio $\left(\sum_{j=7}^{8}\omega_j\right) / \left(\sum_{j=2}^{6}\omega_j\right)$. Therefore, the model allows for the possibility that the government may be able to *reduce tax rates* in the long-run after the adoption of family policies. Depending on the scale of the family policies, however, long-run tax rates may either rise or fall since higher fertility could boost the total-age dependency ratio $\left(\sum_{j=7}^{8}\omega_j + \sum_{j=0}^{1}\omega_j\right) / \left(\sum_{j=2}^{6}\omega_j\right)$. I denote the effects of family policies through changes in the mass of each age group as the *demographic structure effects*.

2.6.4 Childcare Arrangements

Consider a comparison between two family policies, a baby bonus $\mathcal{B}$ and subsidized childcare $\mathcal{S}$. In standard macroeconomic models with endogenous fertility, each child costs a fixed amount of time for parents.[29] Since subsidized childcare reduces the time costs, the total income in this class of models is given by:

$$y = wh \underbrace{(1 - (\chi - \mathcal{S})}_{\text{hours worked}} \cdot n) + n \cdot \mathcal{B}$$

For parents with human capital h, a baby bonus $\mathcal{B}$ is equivalent, or can be replicated by, the subsidized childcare with $\mathcal{S} = \frac{\mathcal{B}}{wh}$ in these models. Therefore, parents should be indifferent between in-cash and in-kind child benefits. They are also expected to react in the same way in labor supply and child-related decisions.

These predictions are nevertheless counter-intuitive since one can always purchase market childcare with in-cash transfers, but in most settings one cannot freely convert in-kind benefits, e.g., childcare vouchers, back to cash and spend on other uses. Therefore, in-cash transfers should be (weakly) preferred by parents to in-kind benefits of the same face value. Moreover, empirical evidence also indicate that parents react differently to in-cash versus in-kind benefits. Upon receiving a baby bonus, parents do not increase their labor supply but rather choose to stay at home and use less market childcare (González, 2013). In contrast, upon receiving subsidized childcare, parents increase their labor supply drastically and substitute home care by (subsidized) market care (Milligan, 2005). Lastly, the utilization of market childcare is increasing in maternal education,[30] but traditional models without childcare arrangements are unable to account for this pattern.

In this model, considering childcare arrangement choices of parents provides an explanation to the observed data facts and a natural way to compare family policies of different kinds. With market childcare, a substitute to home production of childcare, available for purchase at a uniform price, richer parents use more market childcare services than the poorer ones. When the government offers a childcare subsidy $\mathcal{S}$, it will be equivalent to a baby bonus $\mathcal{B}$ of the same face value for parents with high human capital who are already spending more than $\mathcal{S}$ on market childcare in the original economy. Subsidized childcare just reduces their out-of-pocket expenditures. On the other hand, when parents with low human capital receive a childcare subsidy $\mathcal{S}$, they would rather have an in-cash transfer $\mathcal{B}$ of the same face value since they prefer spending the money on consumption or education as the price of market care is higher than their wage. As a result, the subsidized childcare $\mathcal{S}$ is "binding" for low-human capital parents in the sense that the in-kind policy pushes them into the labor market if they would like to take up the benefits. The welfare improvements for them will be smaller than that under a baby bonus $\mathcal{B}$ of the same face value. As they work more, however, these parents accumulate more human capital through learning-by-doing, and the economy sees a

[29] Modeling child costs as time costs rather than goods costs is crucial for explaining a negative fertility-income relationship (Jones et al., 2008). Also see discussions in Doepke and Tertilt (2016). A notable exception that considers the use of market childcare is Bar et al. (2018). They show that the adoption of market childcare services and rising income inequality explains the recent rise in fertility among married women with very high income in the recent U.S.

[30] See Figure 5 for evidence. Also see Bar et al. (2018) and Chaparro et al. (2020).

reduction in the inequality of life-cycle wage growth across parents.

2.7 Summary of Mechanisms

As discussed above, the framework contains a unique mix of modeling ingredients so that it explains more data patterns (e.g., childcare usage) and speaks to several important mechanisms through which family policies affect the economy.

The quantitative magnitude of these mechanisms depends crucially on the fertility responses to family policies, i.e., the fertility elasticities. In particular, if child quantity is fixed, then family policies will have (1) a simple income effect on child outcomes, (2) no composition effects, and (3) no demographic structure effects. In the quantitative analysis of the model, the profile of fertility elasticities across households are disciplined by the calibrated model parameters. In Section 4, I show that the model predictions on fertility responses are consistent with empirical evidence.

3 Calibration

In this section, I discuss the parametrization, the calibration procedures, and the fit of the model. Following the literature, I pick some parameters exogenously, most of which either have standard values or have observable counterparts. The other 14 parameters are calibrated inside the model by matching steady-state moments to the United States in 2010.

3.1 Preferences over Quantity and Quality

I assume that parents' preference over child quantity and quality is given by:

$$v(n, \mathbb{E}h_k, a_k) = \underbrace{\Psi(n)}_{\text{child discounting}} \cdot \underbrace{(\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k))}_{\text{utility from quality}} \tag{9}$$

$$\Psi(n) = 1 - \exp(-\psi n) \tag{10}$$

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0,1), \quad x \in \{\mathbb{E}h_k, a_k, c\} \tag{11}$$

where parents value child quality weighted by the child discounting function $\Psi(n)$ (c.f., Barro and Becker, 1989; Scholz and Seshadri, 2007; Kim et al., 2021). The utility function of child quality and consumption is governed by γ which is known as the elasticity of intergenerational substitution (EGS) (c.f., Cordoba et al., 2016).[31] I use $\theta \cdot u(\mathbb{E}(h_k)) + \nu \cdot u(a_k)$ as a first-order approximation to

[31] Since EGS is designed to match interactions across generations, its magnitude and interpretations are different from that of the elasticity of intertemporal substitution (EIS) used to capture risk-aversion in business cycle models.

general preferences over child quality and transfers. In Section 7.1, I argue that results in this paper are robust to other common specifications used in the literature, including separable preferences (de La Croix and Doepke, 2003), quality and quantity being substitutes (Jones and Schoonbroodt, 2010), and dynastic altruism (Daruich and Kozlowski, 2020).

Parameters $\theta = 2.85$ and $\nu = 0.29$ are calibrated to match the average human capital investment as a share of income[32] and average inter-vivos transfers of \$48,381 in 2000 dollars (Daruich, 2018). I calibrate $\psi = 2.3$ to match the average fertility calculated using the CPS June Fertility Supplement data from 2008 to 2014.

Conditional on $\{\theta, \nu, \psi\}$ and other parameters of the model, γ is an important parameter governing the fertility elasticity where higher γ results in smaller fertility responses to financial incentives.[33] Smaller fertility responses, in turn, imply larger responses in child quality due to income effects discussed in Section 2.6. I identify γ by matching the fertility-income profile in the data. The identification argument follows Cordoba et al. (2016): controlling for other parameters, higher γ implies a larger marginal rate of substitution of quantity for quality and a flatter fertility-income profile. Numerical illustration of the identification is displayed in Figure 2. I choose $\gamma = 0.73$ such that the model best fits the observed relationship between fertility and income.[34]

Even though γ is restricted to be between $(0, 1)$, the model generates surprisingly good fit to the data in terms of life-cycle asset accumulation. Figure 3 shows the median net worth in the model and that in the data estimated using Survey of Consumer Finances (SCF). As non-targeted moments, the model generates a life-cycle profile of net worth that is similar to the data both in trend and in level.

3.2 Child's Human Capital Production

I parameterize the child human capital production function as:

$$h_k = G(h, \mathcal{E}, e, \epsilon) = \underbrace{Z}_{\text{scalar}} \cdot \underbrace{\epsilon}_{\text{shock}} \cdot \underbrace{h^{\rho}}_{\text{spillover}} \cdot \left(\underbrace{\mathcal{E}^{\xi}}_{\text{public education}} + \underbrace{e^{\xi}}_{\text{private input}} \right)^{\kappa/\xi} \tag{12}$$

More specifically, when utility from consumption and child quality are separable, it is common to assume that EIS and EGS are the same to ensure the existence of long-run steady states (see Barro and Becker, 1989; Soares, 2005). For recent work that considers EGS and EIS jointly with non-separable utilities, see Carlos Córdoba and Ripoll (2019).

[32] Daruich (2018) uses the PSID and the CEX to estimate that average expenditures on each child below age 12 include 18 hours of "quality" hours per week as well as \$1,966 (in 2000 dollars) spent on childcare and educational investments. These two expenditures combined amount to 13.4% of median family income. The calibration targets this moment by computing average education investments e plus expenditures on market childcare services m. These moments are also similar to the ones reported in Lee and Seshadri (2019).

[33] The illustrative model in Appendix C.2 shows that responses in fertility is proportional to $1 - \gamma$. Soares (2005) uses a similar argument in discussing the responses in fertility to changes in adult longevity and child mortality.

[34] See Section 7.3 for discussions and robustness to alternative measures of fertility.

Figure 2: Identification of γ

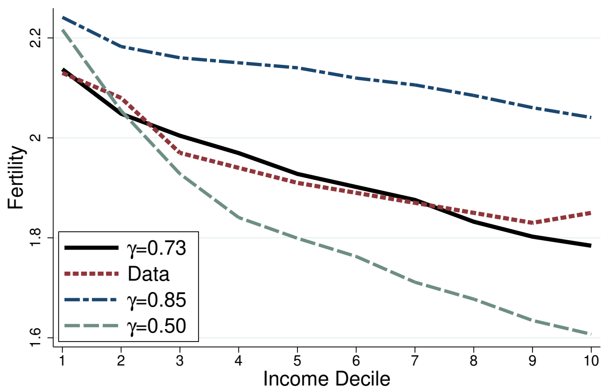
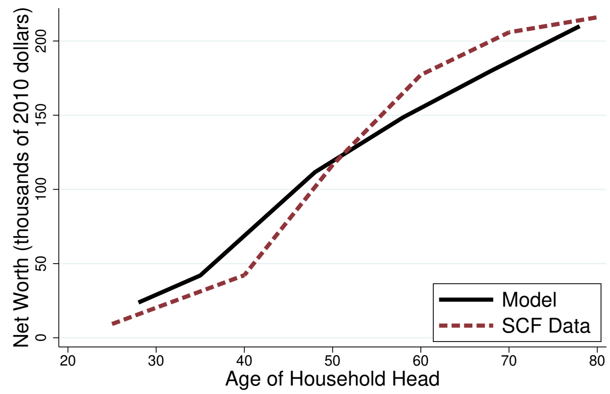


Figure 3: Non-Targeted: Net Worth by Age



Notes: Figure 2 plots the relationship between family income and fertility in the model under different γ. Fertility rate by income is calculated using the number of live births ever had for married women between 40 and 55 years old in the CPS June Fertility Supplement data from 2008 to 2014.
— Figure 3 plots the median net worth by age in the model and in the data. Median net worth in the model is calculated using net asset position a integrated over the equilibrium distribution of households by age. In the data, median net worth by age of the household head is collected from the Federal Reserve System estimated using Survey of Consumer Finances (SCF).

$$\log(\epsilon) \sim \mathcal{N}\left(-\frac{\sigma_\epsilon^2}{2}, \sigma_\epsilon^2\right)$$

Parameter Z is a scaling parameter that governs the overall scale of the economy. It also enables us to normalize the total factor productivity A to be one. I choose $Z = 2.5$ so that the median income of families in the model is one, corresponding to \$49,445 estimated using Census data. Parameter σ_ϵ governs the dispersion of idiosyncratic shock to children's ability. I pick $\sigma_\epsilon = 0.58$ to match the dispersion of earnings for young households.[35] I calibrate $\rho = 0.30$ to match the rank-rank intergenerational mobility estimated by Chetty et al. (2014). Figure 4 shows that the model generates a good fit both in absolute upward mobility and relative mobility.

I use ξ to parameterize the elasticity of substitution between public and private education inputs and calibrate $\xi = 0.9$ to match the relationship between educational spending and household income using data from the Consumer Expenditure Survey (CEX) which includes tuition, test preparation, tutoring, books, and supplies. The public education $\mathcal{E}$ in the baseline economy is chosen to match \$12,000 annual expenditure per student reported by National Center of Education Statistics (NCES).

The last parameter in child human capital production function that needs to be calibrated is κ which governs the productivity of educational investments. I use RCT evidence from García et al. (2020) to inform the value of κ. García et al. (2020) evaluates early childhood programs (ABC/CARE) in the 1970s. The yearly cost of the program was \$18,514 per participant (in 2014

[35] I calibrate σ_ϵ to match the Gini coefficient of income among married households aged 23 to 29 in the CPS-ASEC data (2008-2014).

dollars) for five years. Treated children are followed into adulthood with education and incomes observed by researchers. García et al. (2020) estimate that for every dollar invested, children's lifetime labor income increases by 1.3 dollars in net present value. I apply a similar policy in the model by expanding existing public education $\mathcal{E}$ by \$17,000 (in 2010 dollars) for five years. The policy is targeted at children with parents at 10$^{\text{th}}$ percentile of earnings. The counterfactual is evaluated at a small scale so that prices and taxes remain unchanged. By comparing the discounted life-time earnings of children in the treatment group versus those in the baseline economy, κ is calibrated to be 0.13.

3.3 Costs of Children and Childcare

The household equivalence scale $\Lambda(n)$ is taken from the OECD standard:

$$\Lambda(n) = 1.7 + 0.5 \cdot n$$

where n is the number of children residing with the family.

Recall the childcare requirement constraint, i.e., time costs of child, is given by:

$$n \cdot \chi = \left(t_h^{\upsilon/\iota} + (n \cdot (m + \mathcal{S}))^{\upsilon} \right)^{1/\upsilon}$$

I choose $\chi = 0.18$ following the estimates by Folbre (2008) (Table 6.2) calculated using the American Time Use Survey (ATUS) data.[36] I calibrate the economies of scale in providing childcare at home $\iota = 0.7$ to match the estimates by Folbre (2008) (Table 6.4).[37] The parameter governing the elasticity of substitution between home care and market care υ is calibrated to be 0.38 to match the average expenditure on childcare as a fraction of total family income using estimates from Herbst (2018).[38] Lastly, I choose the price of full-time market care for child aged 0 to 10, p_m, to be \$6,860 in 2010 following the statistic reported by the National Association of Child Care Resource & Referral Agencies (NACCRRA, 2011).[39] Figure 5 shows that as non-targeted moments, the model

[36] Folbre (2008) documents that an average child in two-parent households spends about 22 hours in active non-parental care, and 23 hours in active parental care (adjusting for presence of both parents). Therefore, I calculate χ, total active care required by one child as a fraction of parents' total time endowment, as $\chi = (22+23)/((24-6.5)\cdot 7 \cdot 2) \approx 0.18$ assuming 6.5 hours of sleep for each parent.

[37] Folbre (2008) reports that for two-parent families, the active parental time per child is 1.5 times higher for one-child families than that for two-child families.

[38] Herbst (2018) uses Wave 8 of 2008 panel in SIPP which covers Winter and Spring of 2011. The results show that average childcare expenditures in the whole sample are \$134.44 in 2013 dollars, which translates to 16% of median income. The calibration targets this moment by computing average share of income spent on market childcare services, which is measured by $n \cdot m \cdot (1 + \tau_c)$ integrated over the equilibrium distribution.

[39] The average costs of full-time childcare across states are \$9,303 (infant), \$7,377 (4-year-old), and \$4,753 (school-age) for child care centers; \$6,926 (infant), \$6,131 (4-year-old), and \$4,405 (school-age) for family child care (NAC-

Figure 4: Model Matches Mobility

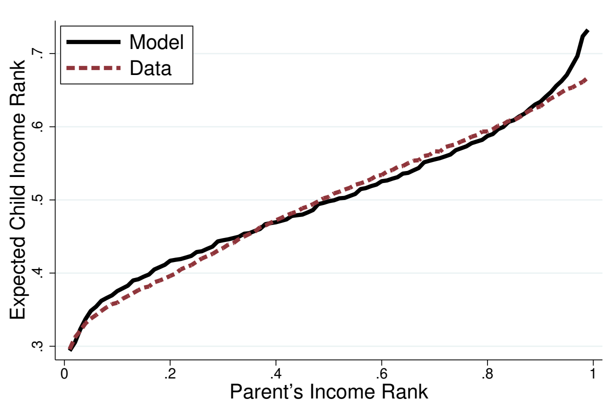
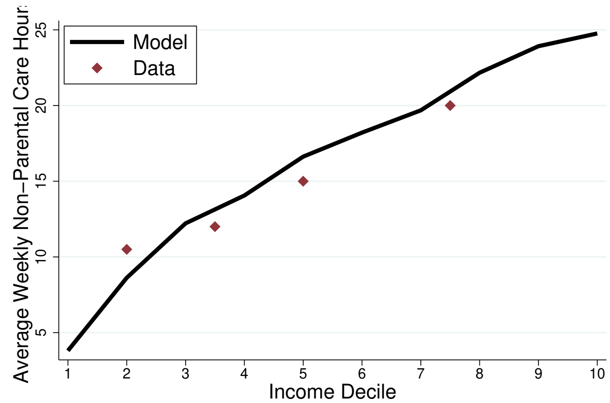


Figure 5: Non-Targeted: Childcare Usage



Notes: Figure 4 plots the relationship between parents' income rank and children's expected income rank in the calibrated model and that estimated by Chetty et al. (2014).
— Figure 5 plots the average weekly non-parental cares received by children aged 0-2 conditional on parents' income decile. The data estimate is obtained using public-use file of National Survey of Early Care and Education (NSECE) wave 2012. I calculate the average weekly non-parental care hours in the data by maternal education categorized into "less than high school", "high school", "some college", and "bachelor's degree and above". Education category is then mapped into income deciles using median income by education and income distribution estimates from the Current Population Survey (CPS).

generates the pattern of childcare usage by parents' income decile that fits the data from National Survey of Early Care and Education (NSECE).

3.4 Other Parameters

I assume the human capital of adults evolve with age according to:

$$h_{j+1} = L_j(h_j, t_w, z') = \exp(z') \left[h_j + \zeta_j (h_j \cdot t_w)^\eta\right] \tag{13}$$

$$\log(z) \sim \mathcal{N}(\mu_z, \sigma_z)$$

I calibrate $\eta = 1.22$ to match the heterogeneous growth rate of income by initial income decile (see Figure 6).[40] Parameters $\{\zeta_j\}_{j=2}^{5}$ and $\sigma_z = 0.38$ are calibrated to match the life-cycle profile of average and dispersion of household income.[41]. I choose $\mu_z = -0.23$ exogenously so that human capital depreciates at an annual rate of 2%.

CRRA, 2011 Appendix 1). I take an age-weighted average of these costs to calculate the full-time childcare costs for children aged 0 to 10.

[40]This model reproduces the well-documented hump-shaped life-cycle earnings profile as well as a Gini coefficient of human capital that is increasing in age (Huggett et al., 2006). These two facts are reconciled by considering a profile of learning ability $\{\zeta_j\}_{j=2}^{5}$ that is decreasing in age and the self-production of human capital with $\eta > 1$.

[41]I calculate average income and Gini coefficient of income by age using data from married households in the CPS-ASEC data (2008-2014). I have experimented with panel data methods a lá Huggett et al. (2011) and the results are quantitatively similar.

Figure 6: Income Growth By Initial Decile

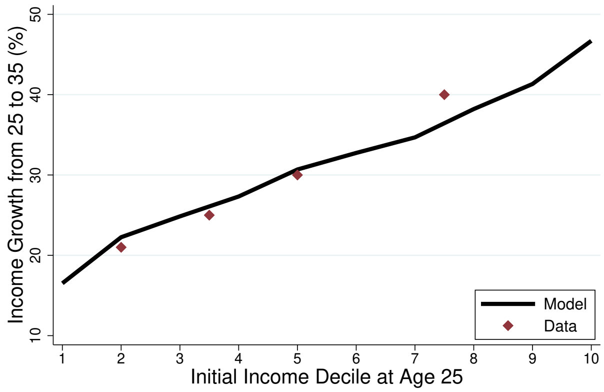
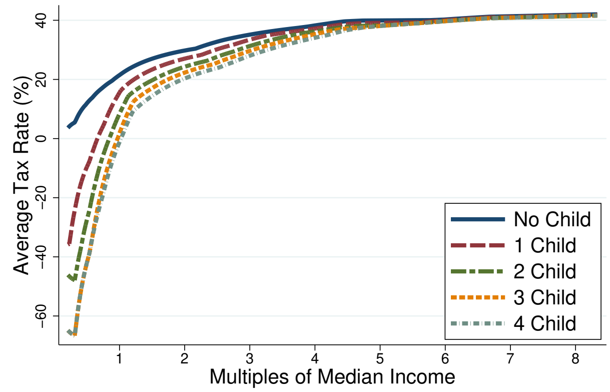


Figure 7: Labor Income Taxes



Notes: Figure 6 plots the growth rate of average income from age 25 to 35, depending on the initial income decile at 25. For married households in the CPS-ASEC data (2008-2014), I calculate the growth rate of average household income by four education categories: "less than high school", "high school", "some college", and "bachelor's degree and above". Initial income decile of each education category is calculated using the overall income distribution at age 25.
— Figure 7 plots average tax rates depending on household income and number of dependent children residing with the family calculated TAXSIM.

Following Heathcote et al. (2017), I parameterize income taxes as:

$$\mathcal{T}(y, a, n) = y \cdot (1 - \tau_y^n y^{-\lambda_y^n}) + \tau_a r a \tag{14}$$

where $\{\tau_y^n, \lambda_y^n\}$ denote the level and progressitivity of taxes depending on the number of children residing in the household while τ_a denotes the linear capital income taxes. I obtain $\{\tau_y^n, \lambda_y^n\}$ using simulated data from TAXSIM provided by the NBER (see Figure 7) and use linear interpolation to calculate $\{\tau_y^n, \lambda_y^n\}$ in the model when n is not an integer. Having an additional child significantly reduces the tax burden faced by households through various programs such as Earned Income Tax Credits (EITC), Dependent Care Tax Credits (DCTC), and Child Tax Credits (CTC). The tax system is progressive, with lower-income households receiving subsidies on net. As the household income grows, labor income taxes converge to 40%. Following McDaniel (2007) and Daruich and Fernández (2020), I choose capital income tax $\tau_a = 0.27$ and consumption taxes $\tau_c = 0.07$. Pension replacement rate π is set to be 40%.

In the production function of the firms, I choose the capital share α to be 0.33 following the standard literature and set the capital depreciation rate δ_k at 4% per year. Lastly, I choose the annual discount rate $\beta = 0.98$ and calculate age-specific mortality rate $\{\delta_j\}_{j=0}^{8}$ using the actuarial life table from Social Security.

Table 1 provides a summary of the model parametrization.

Table 1: Calibrated Parameters

	Interpretation	Value	Source		Interpretation	Value	Source
	Preferences				**Child human capital production**		
β	discount rate	0.98^{10}	standard	Z	normalizing scalar	2.50	median income =1
γ	elasticity of substitution	0.73	CPS	σ_ϵ	ability shock dispersion	0.58	PSID
ψ	fertility preference	2.30	CPS	ρ	intergenerational spillover	0.30	Chetty et al. (2014)
θ	quality preference	2.85	PSID	ξ	substitution of education	0.9	ATUS
ν	transfer preference	0.29	PSID	$\mathcal{E}$	public education	0.16	NCES
				κ	input productivity	0.13	García et al. (2020)
	Childcare arrangement				**Adults' human capital evolution**		
χ	childcare cost	0.18	ATUS	η	learning curvature	1.22	PSID
ι	economies of scale at home	0.7	ATUS	$\{\zeta\}_{j=2}^{5}$	learning level	misc.	PSID
υ	substitutability of care	0.38	SIPP	μ_z	skill depreciation	-0.23	PSID
p_m	price of full-time care	0.15	NACCRRA (2011)	σ_z	shock dispersion	0.38	PSID
	Taxes and pension				**Firm production function**		
τ_y^n, λ_y^n	tax levels and progressitivity	misc.	TAXSIM	A	total factor productivity	1	normalization
τ_c	consumption tax	0.07	McDaniel (2007)	α	capital share	0.33	standard
τ_a	capital income tax	0.27	McDaniel (2007)	δ_k	capital depreciation	0.04^{10}	standard
π	pension replacement rate	0.40	OECD Database				

Notes: This table displays the list of parameters used in the model. Parameters in red are calibrated within the model while those in black are chosen exogenously.

4 Validation

In this section, I discuss the main external validation of the fertility responses to financial incentives. The purpose of validation exercises is to lend extra credibility to the quantitative predictions of the model before evaluating policy counterfactuals. In Appendix B, I provide further confirmation of the model's predictions using evidence from the Australian baby bonus, the Spanish child benefits, and Georgia's Cherokee Land Lottery.

The main validation exercise exploits empirical evidence from the Alaska Permanent Fund Dividends (APFD). The dividend was officially established in 1982 after the discovery of petroleum and increased state revenues. Every year, it gives uniform transfers to all residents regardless of income, employment or age. In particular, the program allows a parent, guardian, or other authorized representative to claim a dividend on behalf of a child while Alaska law imposes no requirements whatsoever on how parents use a child's dividend. As a result, the policy has pronatal effects even though it is not explicitly advertised as a family policy that encourages fertility.

The APFD is an ideal policy variation to validate fertility elasticities in the model for four reasons. First, compared with other family policies which are usually less than a few thousand dollars in net present value per child (McDonald, 2006; Luci-Greulich and Thévenon (2013)), the net present value that parents could receive and use with an additional child under the APFD is almost $20,000.[42] With sizable benefits, it is more likely that the APFD would have meaningful and observable impacts on people's behavior especially when it comes to the important and irreversible decision of having a child. Second, compared with other family policies which are typically means-tested or depends on birth order, the APFD has a clearer implementation with more than 90% of state population filing for the application historically.[43] Third, the APFD mimics a universal basic income for parents plus a refundable Child Tax Credit (CTC) for children. Given that it is not advertised as a policy that aims to encourage childbirth, its effects on fertility provides a conservative benchmark for family policies that are explicitly pronatal which may change parents' behaviors through preferences or information.[44] Lastly, compared with other family policies which mostly took place in European countries, the cultural and institutional background of Alaska is much more similar to the overall U.S. system that I calibrate the model to match.

In the model, I implement the APFD by transferring $1,500 (annual amount) to every household

[42] I calculated the average payment to be around $1,500 per year. Since the amount of the dividend is tied to the performance of the overall stock market, future payment uncertainties might affect people's responses in fertility. Given that childbirth is an irreversible decision, I argue that a mean-preserving spread of the dividend payment would reduce people's responses in fertility. Therefore, since the model generates fertility elasticities that are consistent with people's choices under uncertainty, our results on child benefits without uncertainty is likely going to be conservative.

[43] Cowan and Douds (2021) argues that the migration effects, also known as "population magnets effect," of the APFD was not large empirically, with net migration rates around one-tenth of a percent in the sample period.

[44] One example is Kearney and Levine (2015) where the authors show that the introduction of the widely viewed MTV reality show 16 and Pregnant reduces childbearing among teens.

member including both parents and children. Parents will receive this transfer (for themselves) every period until they die. They are also entitled to receive the children's share of dividends before their children becomes independent. I normalize the amount of transfer by median income in Alaska relative to that in the U.S. and conduct the policy experiment in the partial equilibrium without changing prices and distribution. The model predicts that the completed fertility rate, i.e., the total number of children that parents end up having, increases by 5% compared with the scenario without AFPD.

As noted by McDonald (2006) and Stone (2020), most empirical research on family policies focus on total fertility rates since it simply sums up age-specific fertility rates in a given year which could be derived right after the adoption of policies. Measuring the effects on the total fertility rate, however, is not satisfactory from the policy evaluation point of view since the government ultimately cares about how many additional children are born due to the policies. A better measure is the changes in the completed fertility rate, i.e., number of children ever born for women who have past their fertile years. Compared with the total fertility rate, effects on the completed fertility rate is much harder to measure because researchers need to wait 15 to 20 years for women at childbearing age to finish their fertility choices. Since the APFD was enacted decades ago, it provides a rare opportunity to explore the effects on the completed fertility rate directly without extrapolating based on changes in the total fertility rate.[45] Figure 8 plots the time series of the completed fertility rate (i.e., the total number of live births ever had by women aged 40-55) for Alaska and the rest of U.S. from 1982 to 2018 using the CPS June Fertility Supplement data. As the figure shows, the trend of Alaska is indistinguishable from the rest of the U.S. prior to 1996. After 16 years of the APFD implementation, however, the completed fertility rate in Alaska is persistently higher than that in the rest of the U.S. by about 0.14 children per women. The model prediction of 0.1 additional children per women in Alaska (5% increase) is within the 90% confidence intervals of the observed differences.

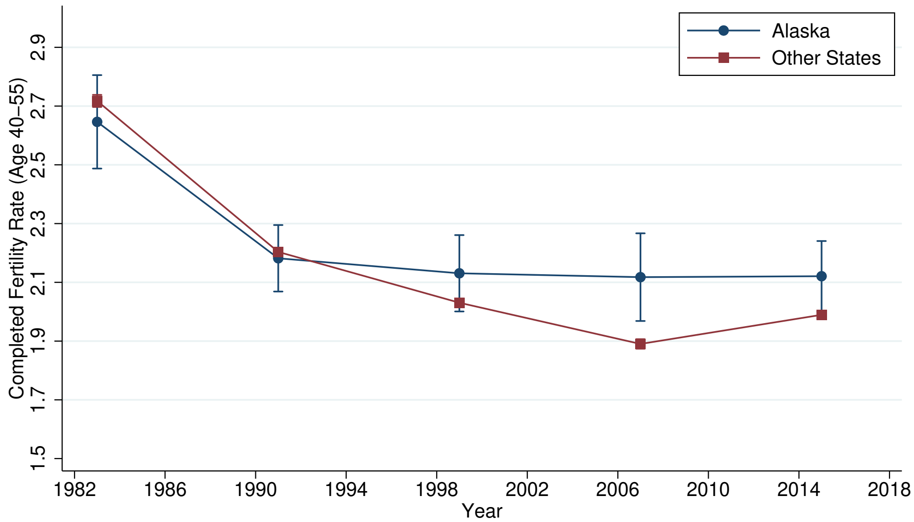
The model also predicts heterogeneous fertility responses across households. Since parents are entitled to claim children's dividends, and these dividends are uniform in size, the model predicts that households with lower human capital will have a larger increase in fertility. I find empirical support for this prediction by calculating the time series of the completed fertility rate by education in Alaska versus the rest of the U.S. in Figure 9.[47] The completed fertility rates among Alaskan women with high education (at least an associate degree) is similar to that in the other states after

[45]Using synthetic control methods, Yonzan et al. (2020) concludes that the total fertility rate increases by 13.1% due to AFPD using Natality files. The increase is immediate and significant. Since policy effects on the total fertility rate is typically larger than that on the completed fertility rate[46], I multiply it by a common adjustment of 1/3 (Stone, 2017) to get a crude estimate of the effects on the completed fertility rate of 4.5% which is quantitatively close to the model estimate.

[47]Using data from the total fertility rates after the policy was implemented, Cowan and Douds (2021) also find larger fertility increases among Alaska natives and women without high school degree.

the APFD was adopted, while Alaskan women with low education increased childbirth significantly relative to the other states.

Figure 8: AFPD Effects on the Completed Fertility Rate



Notes: This figure plots the average completed fertility rates for women aged 40-55 by state of residence from 1982 to 2018 using data from the CPS June Fertility Supplement combined into 8-year bins. Bars around sample means show 90% confidence intervals. Two-sample t tests rejects the null hypothesis of equal mean at 0.1 significance level for year 1999, 2007, and 2015.

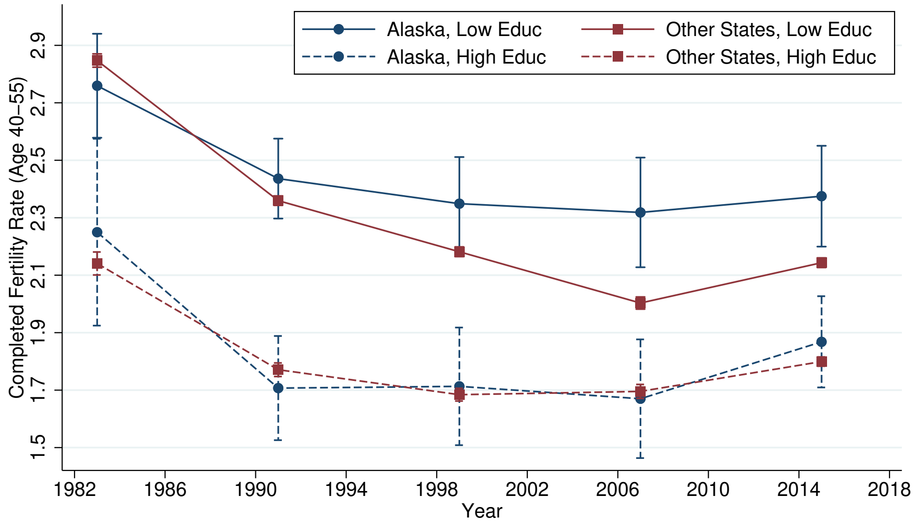
5 Counterfactuals

In this section, I use the model to evaluate family policies of different size in the general equilibrium where prices and distribution of the population adjust. I focus on baby bonuses $\mathcal{B}$ since the model mechanisms can be most clearly displayed under the simple structure of the baby bonus which is similar to an expansion of the fully refundable child tax credit (CTC).[48] Moreover, baby bonuses are empirically relevant as it has been widely adopted in the developed world (e.g., Drago et al., 2011; González, 2013). I discuss its long-run implications in Section 5.1 where I assume that the government is balancing the budget by adjusting consumption taxes.[49] The outcome variables

[48]There are two main differences between baby bonuses and CTC. First, unlike CTC which has income requirement and phase-out region, a baby bonus is usually not means-tested. As will be discussed later, the main results are stronger when family policies target low-income households. Second, a baby bonus is a lump-sum transfer when the child is born while CTC is an annual transfer to parents when the child is below age 18. With borrowing constraints, low-income parents prefer baby bonuses to CTC of the same net present value since they can replicate the latter through savings.

[49]I will discuss the robustness of the results to alternative funding methods in Section 7.4.

Figure 9: AFPD Effects on the Completed Fertility Rate by Education



Notes: This figure plots the average completed fertility rates for women aged 40-55 by state of residence and education from 1982 to 2018 using data from the CPS June Fertility Supplement combined into 6-year bins. I define women with high education as those who have obtained an associate degree or above. Bars around sample means show 90% confidence intervals. Two-sample t tests rejects the null hypothesis of equal mean for women without college education at 0.1 significance level for year 1999, 2007, and 2015.

of interest include but not limited to fertility, human capital, average income, intergenerational mobility (measured by IGE^{-1})[50] and social welfare. In Section 5.2, I compare the baby bonus with other policies including subsidized childcare and education expenditures. Lastly in Section 5.3, I study the transition path the baby bonus and its distributional effects across generations.

5.1 Baby Bonus: Long-run Implications

This section evaluates baby bonuses of different sizes ranging from \$0 to \$50,000. I compute the long-run macroeconomic implications of these policies by comparing the long-run steady state of the economy with the benchmark economy where the baby bonus is zero.

Figures 10a and 10b show the fertility effects and the direct fiscal costs of baby bonuses. To reach replacement fertility level (2.1 children per family on average), the model predicts that it would require a baby bonus of \$30,000 which will be around 1.7% of GDP in the new steady-state economy. This amount is similar to the increase in CTC from 2010 to 2021 in net present value, and it offsets 19% of the average cost of raising one child estimated by the USDA. These results confirm the common perception by demographers that using financial incentives to raise fertility is

[50]Intergenerational Elasticity of Earnings (IGE) is calculated by simulating parent-child pairs and calculate the correlation between their earnings at $j = 2$.

not "cheap." Under realistic magnitude of fertility elasticities and reasonable scale of policies, the effects of financial incentives provided by the government are small relative to historical changes in preferences, social norms, contraceptive technologies, and perhaps most important of all, changes in the skill premium. The results, however, suggest that using cash transfers to prevent further crashes in fertility, or even to raise it, is feasible in practice.

Figure 10c shows the quality-quantity trade-off channel discussed in Section 2.6.1. Baby bonuses reduces the price of child quantity and parents respond by having more children. The increase quantity, in turn, raises the marginal cost of education as well as marginal utility of consumption due to larger family size. As a result, even when quality and quantity are complements in parents' preferences (see Section 3.1), parents may optimally reduce the amount invested in children's education when fertility increases. When the baby bonus is \$30,000, private investment per child e falls by 4% on average. This result is in sharp contrast with design-based research, in which parents with a fixed number of children receive transfers and spend more on children's human capital formation.

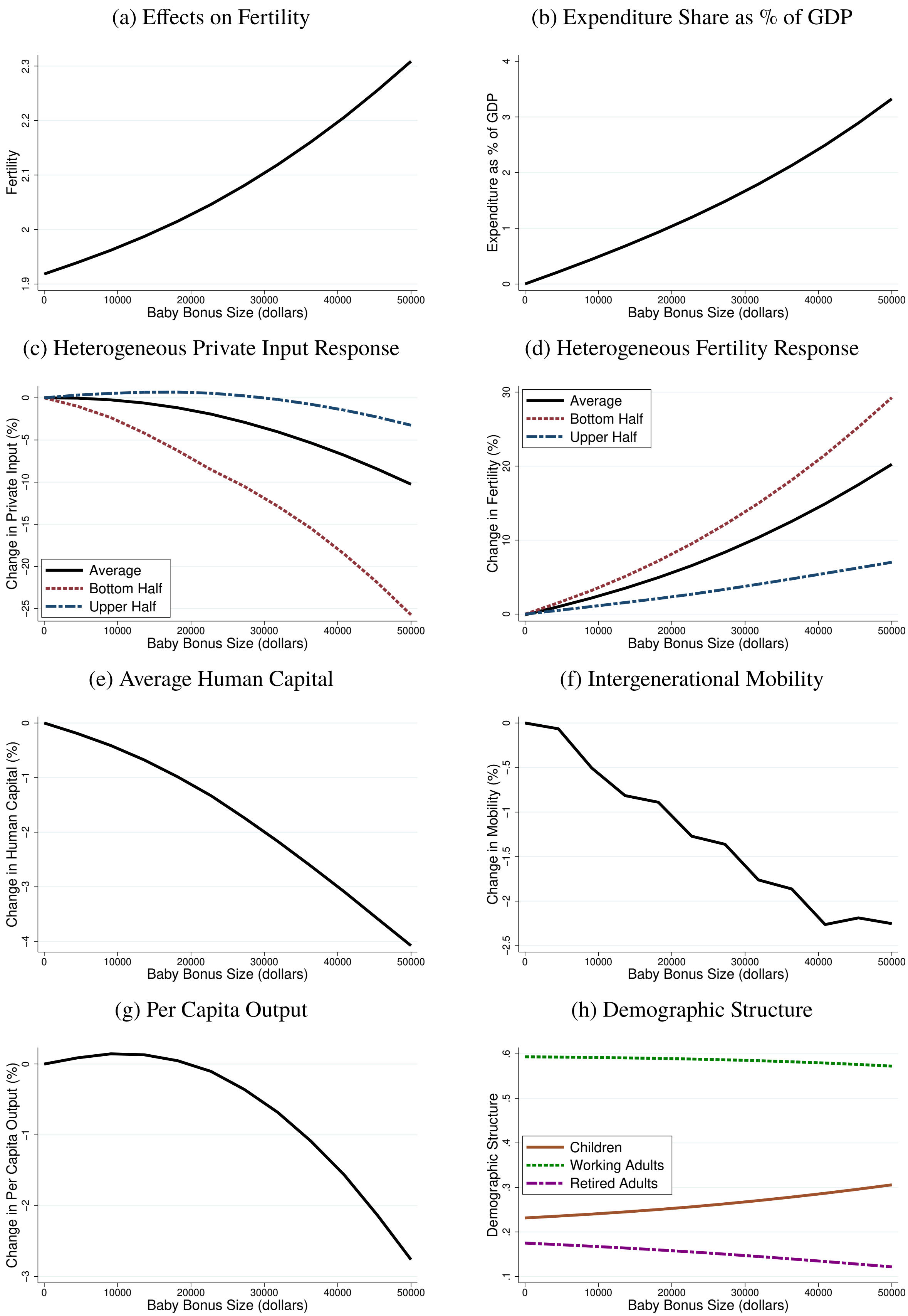
Figures 10c and 10d show that agents with lower human capital have stronger fertility responses and larger drops in private education investments in response to baby bonuses. This is intuitive since the baby bonus of a given size accounts for a larger fraction of household income among low-income families. Due to intergenerational transmission of human capital in skill formation, heterogeneous responses in fertility induces gravitation of aggregate variables towards that of low-income families. Composition effects, in addition to the reductions in private investment per child, lead to a reduction of the average human capital by 2% under a \$30,000 baby bonus (see Figure 10e).[51] Since the reduction of education investment is larger among low-income households, the baby bonus also reduces social mobility, measured by IGE^{-1}, by 1.8% rather than boosting it as policy makers have hoped.[52]

Figure 10g shows that per capita output rises initially before falling around $\mathcal{B} = \$14,000$ despite lower average human capital and dampened social mobility in the economy. Changes in per capita output is not monotonic due to three forces that shape the aggregate supply of capital and labor. First, as parents reduce investments and transfers to children due to quality-quantity trade-off, they optimally save part of the baby bonus for their future consumption, which leads to an increase in the supply of capital in the economy. Second, the human capital distribution worsens as the baby bonus increases, which reduces the supply of capital and labor due to composition effects. Lastly, demographic structure effects alter the mass of agents in each cohort, which in turn affects the aggregation of capital and labor across age groups (see Figure 10h). As the baby bonus gets larger, per capita output falls below the steady-state level as the second effect dominates.

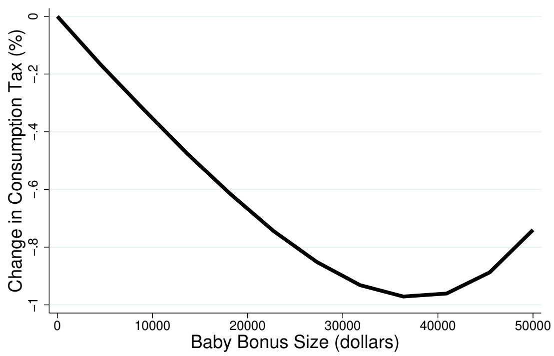
[51] Reductions in average human capital (-2%) is smaller than that of the private investment (-4%) since public education investment, i.e., $\mathcal{E}$, is unchanged.

[52] Due to the quality-quantity trade-off mechanism, the policy effects on social mobility will be even stronger if the baby bonus is exclusively given to low-income households.

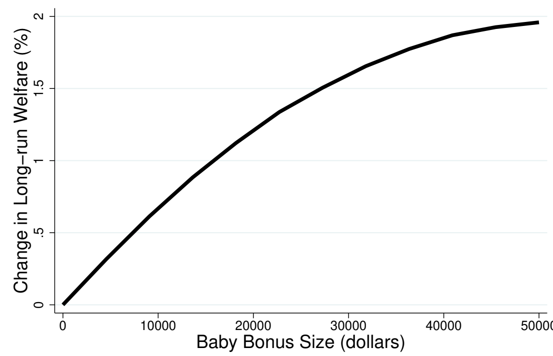
Figure 10: Long-Run Effects of Baby Bonuses



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of baby bonuses of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. In Figure 10c and 10d, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

Despite the potential reductions in per capita output, Figure 10i shows that in the long-run steady-state, the government is able to reduce the consumption tax under a small baby bonus. This is a result of several forces shaping the government budget constraint in (4). First, the change in human capital distribution affects tax revenues. Second, as the share of retired households decrease, the government has less burden from pension payment. Lastly, as the share of children increases, the government needs to allocate more resources to public education. Contrary to structural models with two-period-lived agents where the government needs to raise taxes to support more children, this model predicts that when the $\mathcal{B} = \$30,000$, consumption taxes could be lower by 0.9% in the long-run.

Figure 10j plots the welfare effects in consumption equivalence. Due to increased fertility and reduced consumption taxes, the average utility of new-born under the veil of ignorance increases by 1.6% in the long-run economy when aggregate fertility reaches replacement level. Besides these two benefits, welfare rises because the baby bonus provides a social safety net for low-income households. This generates insurance benefits for risk-averse agents ex ante.

5.2 Comparing Policies: Subsidized Childcare and Education Expenditures

In this section, I highlight some similarities and differences between various family policies. For more detailed results, see Appendix A.1 and A.2.

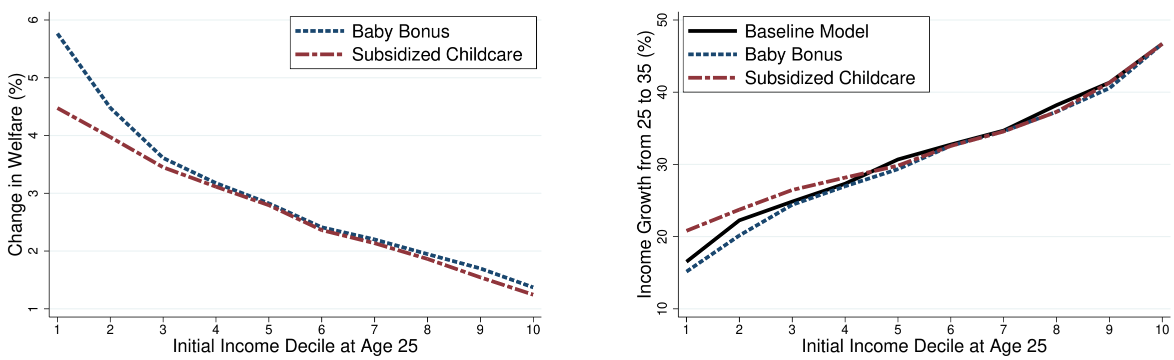
Compared with baby bonuses, subsidized childcare is less cost-effective in raising fertility. While it takes a baby bonus equal to 1.7% of GDP to boost aggregate fertility to the replacement level, the government needs to spend 2.5% of GDP if it wants to use subsidized childcare to achieve the same goal. This result is consistent with evidence from historical policies (Luci-Greulich and Thévenon, 2013; Stone, 2020). It is also intuitive since compared with cash benefits, subsidized childcare offers less benefits per child to parents with low human capital given that their wages are lower than the price of market childcare services (see Figure 11a).

Different from baby bonuses, subsidized childcare encourages the combination of childbearing and labor force participation, especially among parents with low human capital.[53] This in turn, fosters adults' human capital accumulation via on-the-job learning. As a result, subsidized childcare leads to a more equal income growth across agents with different initial human capital and reduces inequality in life-cycle income growth (see Figure 11b). The quantitative effects on income-growth inequality are nevertheless mild since most of the gap is driven by differences in initial human capital rather than hours worked (see Section 3).

Figure 11: Baby Bonuses and Subsidized Childcare: Effects on Welfare and Income Growth

(a) Effects on Welfare

(b) Effects on Income Growth



Compared with baby bonuses, the fertility effects of public education expenditures are smaller by an order of the magnitude for two reasons. First, it only indirectly affects quantity choices through changes in child quality. Second, in the long run, education expenditures increase the share of parents with higher human capital who have smaller number of children. Therefore, the government should not expect to use education expenditures as an effective pronatal policy instrument. While a $30,000 baby bonus boosts aggregate fertility from 1.92 to 2.1 (the replacement level), an education expansion of the same size only raises aggregate fertility by 0.011.

Similar to baby bonuses, private investments into children's education also falls under public education expansion, but for a different reason. While private investments drop due to quality-quantity trade-off when the government subsidizes childbirth, parents spend less on children's education under an expansion in public education expenditure due to crowding-out effects since public

[53] Hours worked among parents with low human capital fall after the implementation of a baby bonus for two reasons. First, these parents are most responsive in fertility, which drives up the total childcare time. Second, due to increasing economies of scale in home production of childcare, the increase in fertility decreases the relative price of home versus market childcare. See González (2013) for empirical evidence of the predictions. As discussed in Section 2.6.4, hours worked among parents with low human capital increases under subsidized childcare as the policy "pushes" them from home production of childcare to market work if they want to benefit from the in-kind transfers.

and private monetary inputs into children's human capital formation are substitutes.

Despite mild fertility effects, expanding public education is the most effective policy among the ones evaluated in raising social mobility and improving children's outcomes. A $60,000 expansion of public education (in net present value) raises intergenerational mobility by 5.2% and average human capital by 2.3%.[54]

5.3 Baby Bonus: Transition Path

In this section, I discuss the transition path results under a $30,000 baby bonus that leads to the replacement level fertility rate in the long run. The policy is enacted unexpectedly at period $t = 1$ and stays afterwards. It is assumed that the government changes consumption taxes along the transition path to balance budget period by period.[55] Each period stands for 10 years.

Figure 12a shows that the policy effects on fertility are immediate and persistent. Figure 12b indicates that the old-age dependency ratio starts to decline in period 6, given that past fertility rates predetermine the ratio between the retired and the working-age population in the short-run. The total dependency ratio, however, increases immediately after the policy is adopted since there is a large increase in the number of children due to the baby bonus.

As a result of this increase in the total dependency ratio, the government needs to finance additional expenditures in the first few periods. Such expenditures include education and transfers to parents with children through the tax system. These "induced" expenditures could make the overall costs of family policies much larger than the direct expenditures.[56]

In the baseline model, the government is required to balance the budget period by period using consumption taxes. As a result, Figure 12c shows that consumption taxes rise at first before falling in the long-run. Hence, welfare changes of new-born agents in transition are positive, but significantly smaller than that in the long run. Since most existing agents in the economy at $t = 1$ do not benefit from the baby bonus but are required to pay higher consumption taxes, they are worse off with the baby bonus.

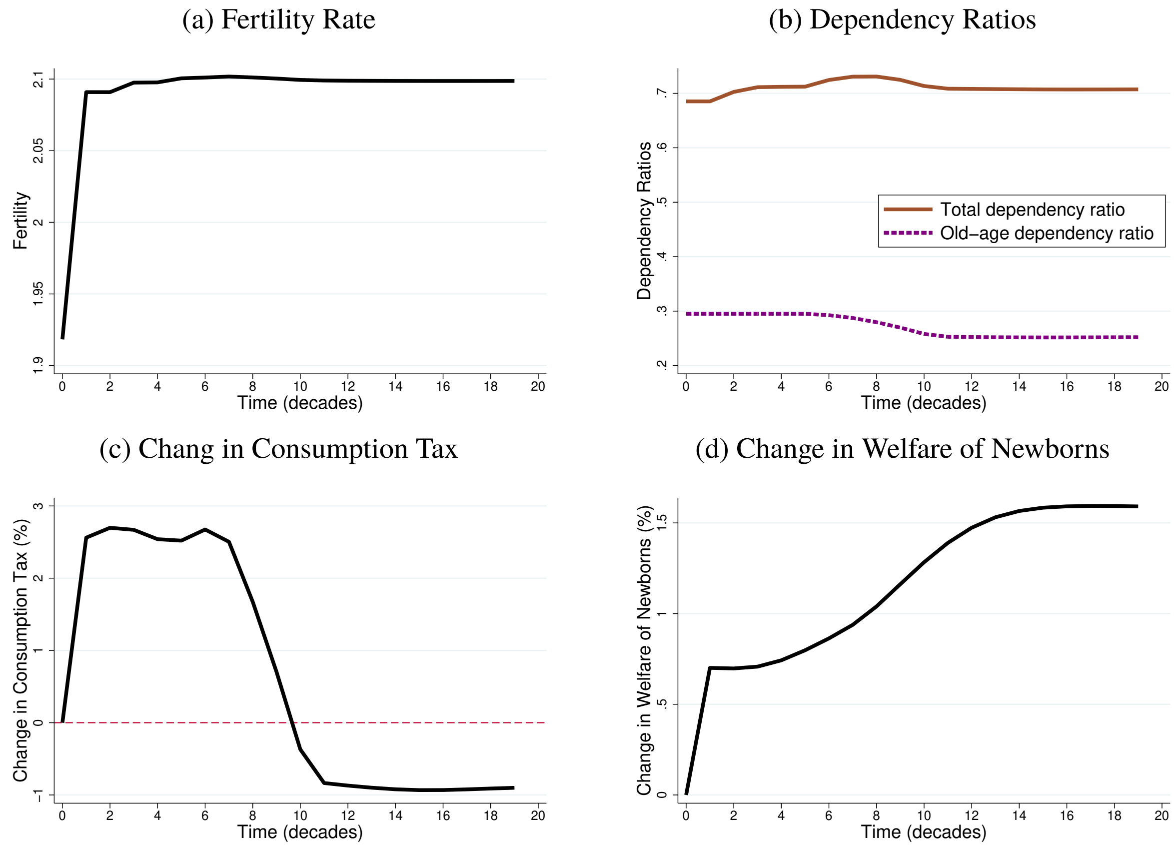
The transition path results shed light on two insights that hold more generally beyond the case of a baby bonus. First, the overall fiscal burden induced by family policies are usually much higher than the policy expenditure itself, especially in the first few decades in the transition path. When

[54] These values are considerably smaller than the projected policy effects by Daruich (2018) for two reasons. First, Daruich (2018) does not consider the existence of public education expenditures in the benchmark economy. As a result, additional public education expenditures have significantly higher marginal effects on children's human capital. Second, Daruich (2018) does not consider endogenous fertility which dampens the policy effects on child quality through quality-quantity trade-off.

[55] I discuss an alternative way of using government borrowings to fund fiscal expenditures in Section 7.4.

[56] For example, while the direct cost of a $30,000 baby bonus is 1.7% of GDP, the government needs to raise consumption taxes by 2.6% for the first seven decades in the transition.

Figure 12: Transition Path of a $31,000 Baby Bonus



Notes: These figures plot evolution of aggregate variables under a baby bonus of $31,000. Each period represents 10 years. The policy is enacted at period 1.

additional children are born, the government needs to finance additional expenditures on existing child-related policies. The potential fiscal benefits via the reductions in the old-age dependency ratio, on the other hand, will realize much later than the upfront costs. Second, the amount of political support of family policies depend on how the fiscal costs are distributed across generations. The government will find a hard time in gathering enough agents to support family policies that benefits the economy in the long run at the cost of existing households. In that sense, family policies are very similar to climate policies where countries wait "too long" to act, and parents' preferences towards their children's welfare alone are not sufficient to restore efficient outcome, however defined, if they need to make huge sacrifices to their living standards.

6 Optimal Policy

In this section, I study the optimal baby bonus to address externalities of childbearing and inefficiencies caused by borrowing constraints. I begin with a discussion of welfare criteria under heterogeneous-agents and endogenous fertility. Then, I show optimal policy results and conclude this section by proposing some principles for the design of family policies in general.

6.1 Welfare Criteria

Welfare criteria in models with endogenous fertility are complicated both conceptually and philosophically. Unlike standard comparisons between allocations where the set of agents are fixed, there will be agents who are born in one economy but not in the other. As a result, standard Pareto principle cannot be used to conduct welfare analysis in this context. The field of population ethics is devoted to understanding and resolving this question.[57]

Since the debate on welfare criteria is far from being settled, I adopt two definitions in studying the optimal policy. The first criterion is the *long-run welfare* $\mathcal{W}$ used in previous sections. It evaluates the expected utility of a newborn child in the long-run stationary equilibrium under the veil of ignorance. The second criterion evaluates the *average utility of existing agents* when the policy is adopted.[58] I decompose it into welfare changes for existing parents ($j = 2$) who receive the baby bonus and for other households ($j > 2$) who do not receive it. The latter criterion has two features. First, it is forward looking as it incorporates tax changes in later periods that affects these households' utility. Second, with this criterion, the unique solution to the planning problem corresponds to the notion of $\mathcal{A}$-efficiency defined in Golosov et al. (2007) which focuses on the welfare of those who are already alive.

Rather than computing the unconstrained optimum, I follow the Ramsey tradition and allow the government/planner to use only a certain policy instrument, namely a baby bonus that is uniform across households and birth order. As in previous sections, I consider the scenario where the government adopts the baby bonus at the beginning of time $t = 1$. The policy change is permanent and financed by consumption tax changes. I use $\mathcal{B}^*_{lr}$ to denote the optimal baby bonus that maximizes average utility in the long run, and use $\mathcal{B}^*_{sr}$ to denote the optimal baby bonus that maximizes the average utility of existing households when the policy is adopted.

[57] For instance, Parfit (1984) derives the famous "repugnant conclusion" where he shows that under a set of intuitively appealing assumptions, one can prove "for any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better, other things being equal." Golosov et al. (2007) proposed two criteria called $\mathcal{A}-$efficiency and $\mathcal{P}-$efficiency which differs by whether the planner evaluates the welfare of those who are not born. de La Croix and Doepke (2021) considers the optimal welfare from a soul's perspective where one needs to consider both the utility of being born and the average "waiting time" for incarnation.

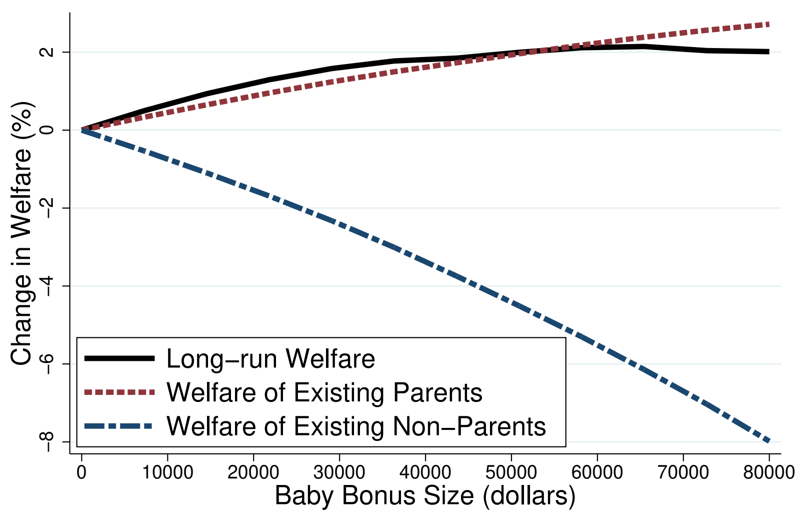
[58] Following the standard practice in quantitative macroeconomic literature, I use equal weights as a benchmark. Another approach is to use Negishi weights which puts a greater weight on households with higher human capital and initial assets. This would eliminate the redistributive benefits of baby bonuses. See Kim et al. (2021) for an example.

6.2 Optimal Policy Results

Figure 13 shows the changes in welfare under baby bonuses of different sizes for two welfare criteria. From a long-run perspective, the optimal baby bonus is $\mathcal{B}^*_{lr} = \$60,000$. It boosts aggregate fertility to 2.4, raises long-run welfare by 2%, and costs around 4.1% of GDP.

The baby bonus that maximizes welfare of existing households, however, is $\mathcal{B}^*_{sr} = \$0$. Current parents who receive the baby bonus directly prefers larger $\mathcal{B}$ for two reasons. First, despite the higher taxes in transition, they are subsidized by elderly households ($j > 2$) on net. Second, the baby bonus redistributes towards poorer parents which improves the average welfare. Elderly households in the economy, however, strongly opposes the baby bonus since they do not gain from it but need to pay more taxes in their remaining lives.[59] If each household has the same voting power, then $\mathcal{B}^*_{sr} = 0$ is the most likely outcome from a political perspective. As discussed in Section 5.3, this observation provides an explanation to the puzzle where many countries with extremely low fertility rates fail to implement large-scale family policies despite knowing the dire consequences in the long run, and even when they do, countries often renege on those policies due to fiscal pressures. For example, the Australian baby bonus got significantly downsized in 2014.

Figure 13: Optimal Baby Bonus by Welfare Measure



Notes: This figure plots the changes in welfare under baby bonuses of different sizes. I consider two welfare criteria: (1) average utility of a new-born child in the long run and (2) average utility of existing households when the policy is enacted, decomposed into agents who receive the baby bonus and agents who do not receive it.

[59] Note that in the case of the Child Tax Credit (CTC), parents at $j = 3$ are also beneficiaries since they are eligible to receive the transfers.

6.3 Discussions

Even though this paper restricts the study of optimal policy design to choosing the optimal level of baby bonus financed by consumption tax changes to balance the government budget period by period, it highlights the major trade-offs in policy design which could be used in a broader context with other instruments and goals.[60] On one hand, subsidizing childbirth among parents with low human capital leads to larger fertility changes per dollar spent on family policies, which could result in larger welfare improvement due to the demographic structure effects. On the other hand, subsidizing childbirth among parents with high human capital improves the equilibrium human capital distribution through the intergenerational transmission of skills, which could raise welfare due to the composition effects. At the aggregate level, the government balances population growth rate, average human capital, and distortions through the tax system by weighing the quality-quantity trade-off, composition effects, demographic structure effects, and changes in childcare arrangements.

With countervailing forces present in the unified model, the first-order stochastic dominance in equilibrium human capital distribution is neither necessary nor sufficient for choosing better policies. For instance, the equilibrium human capital distribution under a $30,000 baby bonus is first-order stochastic dominated by that in the baseline economy, but social welfare is improved in the long run. The key insight is that besides comparing human capital distributions across economies, one also needs to pay attention to differences in the *age distribution*, i.e., demographic structures. Thus, the paper provides a novel counterargument to the common conclusion in past literature on family policies which argue for restrictions on childbirth among parents with low human capital (e.g., Chu and Koo, 1990).[61] These parents, with higher fertility elasticities, could be key to solving population aging problems.

7 Robustness

7.1 Alternative Preferences

There are alternative ways to model parents' preferences over child quantity and quality. I argue that the modeling choice in Section 3.1 makes the results in this paper a conservative benchmark.

One commonly-used assumption is separable preferences between quality and quantity (see

[60] For instance, it is straightforward to incorporate subsidized childcare and public education expenditures studied in Appendix A.1 and A.2. Instead of choosing a permanent policy change in the baby bonus, one could also conduct dynamic optimal policy design where the size of the bonus varies with time.

[61] Cordoba and Liu (2016) make a complementary argument invoking the Lucas' Critique by showing that family policies have direct effects on households' utility.

de La Croix and Doepke, 2003; Bar et al., 2018; Vogl, 2016) where:

$$v(n, \mathbb{E}h_k, a_k) = \log(n) + \theta \log(\mathbb{E}h_k) + \nu \log(a_k)$$

Compared with our benchmark case where quality and quantity are complements, parents will raise quantity and reduce quality even more strongly when the government rewards childbirth. This is because the interaction in preferences at the left-hand-side of Equation 8 is not present under separable preferences. I show this result with closed-form solutions in Appendix C.1. For the same reason, the results in this paper will be conservative if quantity and quality are substitutes (Jones and Schoonbroodt, 2010).

Another way of modeling parents preferences is dynastic altruism (see Cordoba et al., 2016; Daruich and Kozlowski, 2020) where:

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n) V_2(\mathbb{E}h_k, a_k)$$

This formulation is quite similar to the one used in this paper except that utilities from child quality is pinned down recursively. Dynastic altruism is appealing aesthetically but face unresolved challenges in fitting observed transfers between parents and children (see Altonji et al., 1997; Barczyk and Kredler, 2020). Paternalistic motives are often added to improve data fit (e.g., Abbott et al., 2019). I argue that the results will also be stronger under dynastic altruism: parents endogenize the fact that returns to child quality become lower when the government creates a "social safety net" for children with low human capital by providing generous family benefits. As a result, parents would further increase child quantity and reduce quality investments. For instance, Daruich and Fernández (2020) argue that universal basic income reduces the human capital of future generations due to this mechanism.

7.2 Endogenous Timing of Childbirth

In the baseline model, I abstract away from birth timing by assuming that parents only make fertility choices from age 20 to 30. In reality, however, parents make decisions on when to give birth to children and family policies could affect that decision, also known as the tempo effects.[62]

I argue that adding endogenous timing will make my baseline results stronger. As the model matches effects on the completed fertility rate (quantum effects) in the validation exercise, policy

[62] An unresolved question is whether changes in timing is due to the relaxation of parents' constrains (e.g., down payments to buy a larger house) or the lack of commitment to policies by the government. Historically, governments often renege on family policies or reduces the scale of payments as fiscal condition changes. One example is the Australian baby bonus which got significantly downsized in 2014. This uncertainty creates an incentive for parents to shift timing when benefits are still in effect.

effects on the total fertility rate (quantum plus tempo) are going to be larger. Suppose that in addition to increasing the number of children, some parents shift birth from 30s to their 20s in response to a baby bonus. This shift in birth timing is likely going to be detrimental to children's human capital for two reasons. First, early birth reduces the spillover that the children could receive because parents' human capital grows rapidly in from age 20 to 40. Second, family policies of realistic size fall short of offsetting the income differences of parents between early and late births. Hence, these children are induced to be born into households with less resources on average, which would also reduce children's human capital as investment falls. As a result, family policies will have a larger observed fertility impact on the economy with endogenous timing of birth, but the outcomes of children will be even worse.

7.3 Alternative Measure of Fertility

Since the baseline model focuses on the total number of children that parents decide to have, the most appropriate measure of fertility in this context is completed fertility rate (CFR) which is measured using either "child ever born" in Census data (dropped after 1990) or "live births ever had" in the CPS June Fertility Supplement. As discussed in Section 4, CFR also has the advantage of being invariant to shifts in birth timing due to policy changes. For these reasons, In Figure 5.2 and the calibration, I use CFR by household income decile, calculated using the sample of married women age 40-55 in 2008-2014 CPS June Fertility Supplement.[63]

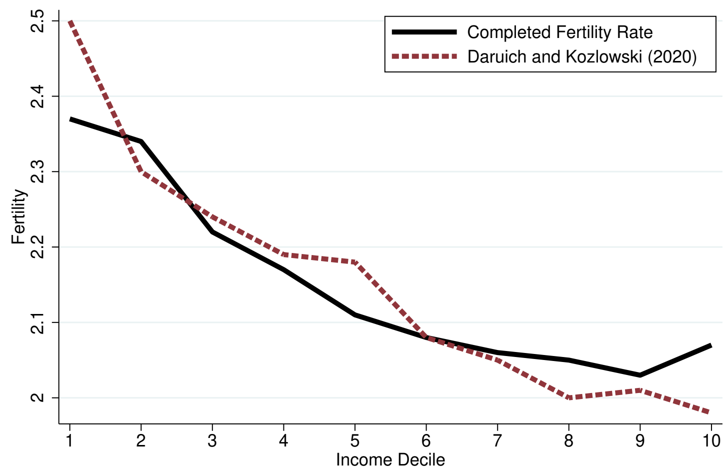
One disadvantage of using the completed fertility rate is that these women made their fertility decisions prior to 2010 where they might have faced different trade-offs. The Total fertility rate (TFR) is an alternative measure that is more responsive to contemporaneous situations by summing the age-specific birth rates over all women alive in a given year. This measure is simple to compute and is widely used in the literature (Kremer and Chen, 2002; Manuelli and Seshadri, 2009).

TFR *by income decile*, however, is less straightforward to compute than CFR by income decile. First, one do not want to assign income ranks that mixes young women with old women due to life-cycle changes in the earnings profile. Second, given that the appropriate measure of income is total family income, one do not want to mix married women with single women. Daruich and Kozlowski (2020) provides a recent estimate of TFR by income decile using the Census data in year 2000. They restrict the sample to married women between age 15-49 who are either heads of households or spouses of heads of households. Income deciles are assigned by comparing the

[63] In a steady-state economy, fertility choices across cohorts are stationary and hence CFR coincides with the total fertility rate (TFR). For this reason, I normalize the CFR target in the calibration of ψ to match the total fertility rate in 2010 which is 1.92 children per women. Model results are quantitatively similar if I omit this step. I have also experimented with a version of the model where a fraction of adults is single and (exogenously) childless, so that the overall CFR in the economy is a weighted average between single adults without children and married households whose within-group CFR is higher than 1.92. The overall results are again similar to the benchmark ones.

total family income of each women with other observations in the same age group. To alleviate selection bias into marriage for younger age groups, they also drop age-income groups with small number of observations. For each income decile, they calculate the total fertility rate by adding up the age-specific fertility rates in that decile. Figure 14 shows that their result is quantitatively similar to the CFR estimates used in the calibration.

Figure 14: Fertility Rate by Income Decile



Notes: This figure plots the total fertility rate by income decile measured in Daruich and Kozlowski (2020) and the completed fertility rate used in model calibration.

7.4 Government Borrowing

In the baseline policy counterfactual and the optimal policy analysis, I assume that the government changes consumption taxes τ_c to balance the budget in every period. This is an abstraction from the reality mainly for computational reasons.

I argue that many results are unlikely to change when the government is allowed to borrow from domestic or international lenders. In particular, all main policy mechanisms, i.e., quality-quantity trade-off, composition effects, changes in demographic structure, and adjustments in childcare arrangements, are independent from whether the government is allowed to borrow or not. As a result, I expect the model predictions on fertility, human capital, output, and mobility to be unchanged in a model with government borrowing.

Results on welfare implications, however, will likely change depending on how government expenditures are funded. For example, since the \$30,000 baby bonus generates welfare improvements for all agents in the long run, it is natural for the government to shift the fiscal burden to these beneficiaries via borrowing instead asking existing elderly households to fund the additional expenditures from which they derive little benefits (see Figure 13). If the government chooses to do so, then the welfare effect in the long-run economy will be smaller while that on existing agents will be less negative. Such policy change is also more likely to gain enough political support to be implemented since it does not directly hurt existing agents who have voting power.[64]

Despite the intuitive appeal of using government deficits to finance large-scale family policies that benefit future generations, an interesting observation is that in current policy proposals, family policies are commonly financed by adjusting fiscal revenues and expenditures. For example, the American Families Plan vows to increase the tax rates among the very rich, while the Family Security Act proposed by Senator Romney plans on reforming and consolidating outmoded federal programs. I leave the normative implications of such financing choices for future research.

8 Conclusion

Facing the "global fertility crash" (Tartar et al., 2019), family policies are at the forefront of public attention and policy debates. Evidence on how transfers to parents affect children's outcomes lead policy makers and economists alike to think that family policies are good instruments to "lift children out of poverty today and help them tomorrow" (Schanzenbach et al., 2021).

In this paper, I study the aggregate impacts of family policies in a quantitative general equilibrium overlapping-generations model with heterogeneous agents, endogenous fertility, childcare arrangements, child human capital formation, and inter-vivos transfers. Relative to previous studies on family policies, I show the importance of considering the quality-quantity trade-off, composition effects, an endogenous demographic structure, and changes in child arrangements.

I calibrate the model to match salient features of the data including fertility-income relationship, human capital investments, childcare usage, and institutional details on child benefits through the tax system. In external validation exercises, I show that the model-generated fertility elasticities are consistent with empirical evidence from the Alaska Permanent Fund Dividend (APFD) and other family policies around the world.

I find that when governments design family policies that reward having more children, parents respond by raising child quantity but optimally reduce child quality. Compared with education subsidies, family policies are not ideal instruments if the policy goal is to raise children's human capital or to boost social mobility. The pronatal effect of family policies, however, could lead to long-run welfare gains because population growth reduces old-age dependency ratio. This allows

[64] Note that if the government issues debt to finance large-scale family policies, the overall borrowing cost of the government could increase in the general equilibrium if lenders require higher compensations for a larger debt-to-GDP ratio. This might crowd out other government borrowings that benefit existing households. As a result, whether current agents prefer government borrowing or changes in taxes also depends on the opportunity costs of family policies.

the government to reduce tax rates.

I also show that the long-run gains in welfare require a transition path where the government needs to raise finance higher child-related expenditures in the first few decades. Those costs are important, but policy discussions often overlook them due to a greater focus on pension liabilities in the long run. Depending on how the government decide to finance those costs, the welfare of existing households could be hurt.

I conclude by discussing avenues for future research. First, it would be interesting to consider the optimal policy design with additional instruments and different welfare criteria. Second, the framework can be calibrated to match the institutional details of other countries and conduct policy counterfactuals on a case-by-case basis. Lastly, another fruitful area of research is to link family policies to other population externalities such as pollution (Bohn and Stuart, 2015), ideas creation (Jones, 2020), and firm dynamics (Hopenhayn et al., 2018; Peters and Walsh, 2020).

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A Additional Results

A.1 Public Childcare

In this section, I evaluate childcare subsidies where the government offers $\mathcal{S} \in [0, 0.9 \cdot \chi]$ market childcare services for all families, which is equivalent to 0 to 36 hours of public childcare per week.

I find that subsidized childcare raises fertility, but at a higher cost than baby bonuses. The government needs to provide 25 hours of subsidized childcare to raise the aggregate fertility rate to 2.1. This would cost almost 2.5% of aggregate GDP in the long-run steady-state economy, which is 0.8% higher than the baby bonus that achieves the same fertility goal.

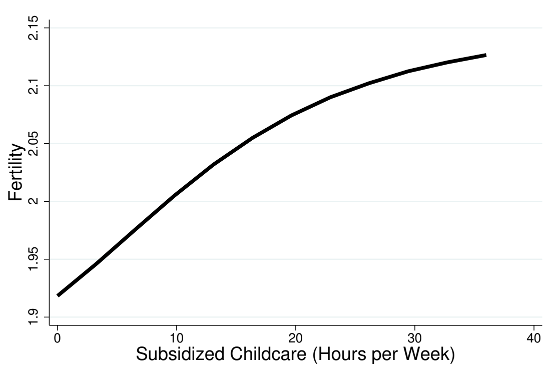
Figure 15c shows that similar to the baby bonus counterfactual, parents reduce private education investments due to quality-quantity trade-off. Figure 15d shows that fertility responses are higher among low-income parents. Due to reductions in private investment and the composition effects, average human capital in the economy decreases (see Figure 15e). The overall effects on intergenerational mobility are small (see Figure 15f).[65]

Figure 15g shows that output per capita increases with subsidized childcare. Compared with the baby bonus, the output changes are more positive for three reasons. First, the composition effects is milder because the fertility differentials between parents with low and high human capital are smaller than that under the baby bonus (see Figure 15d). Second, childcare subsidy raises labor supply by reducing the effective price of market care. This induces parents to reduce time spent at home taking care of children t_h. Lastly, the increased working time at age 20-30 translates to a high human capital for adults in later periods due to learning-by-doing in the labor market.

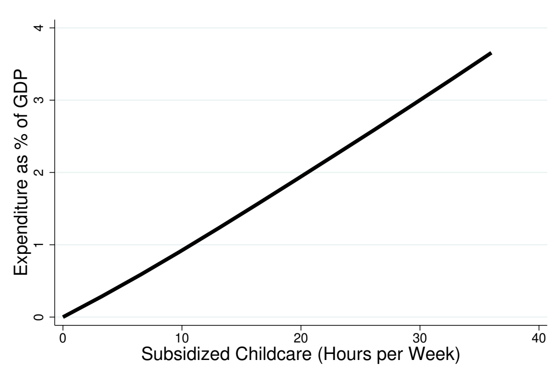
Lastly, Figure 15i shows that the government can reduce consumption taxes by 0.6%. Long-run welfare could be raised by 2.5% when aggregate fertility is around the replacement level.

Figure 15: Effects of Childcare Subsidies of Different Sizes

(a) Effects on Fertility

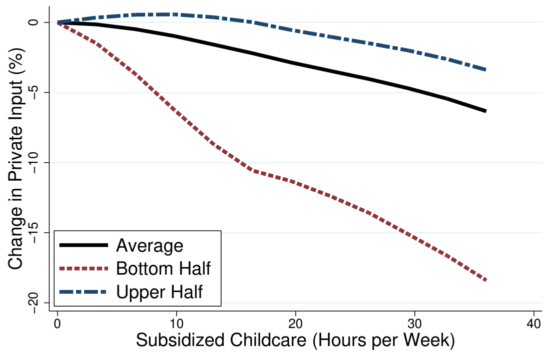


(b) Expenditure Share as % of GDP

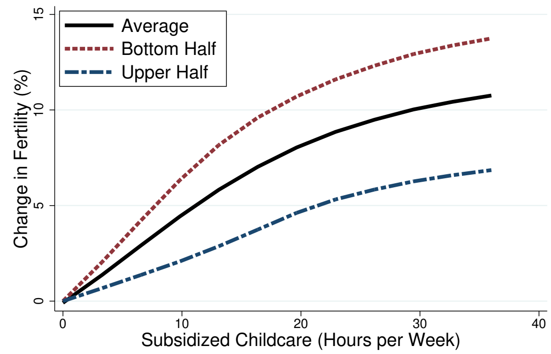


[65]The effects on mobility are not monotonic since besides changing children's human capital, subsidized childcare also changes hours worked which affect earnings.

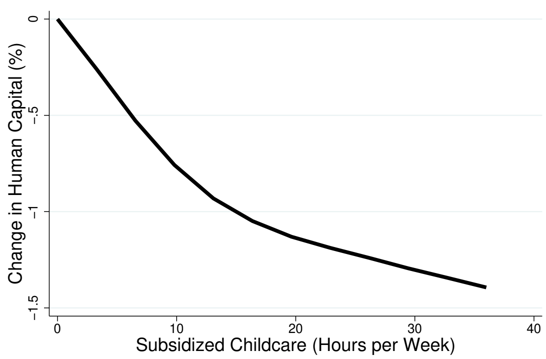
(c) Heterogeneous Private Input Response



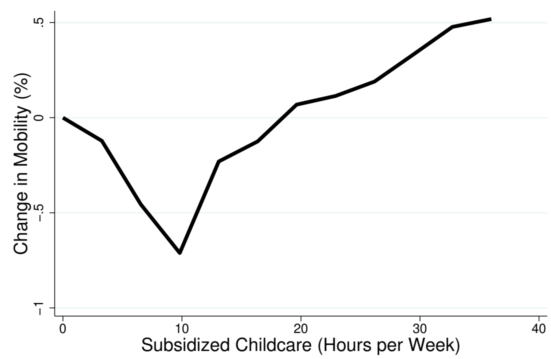
(d) Heterogeneous Fertility Response



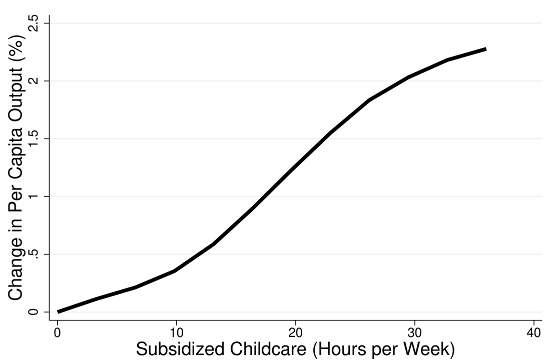
(e) Average Human Capital



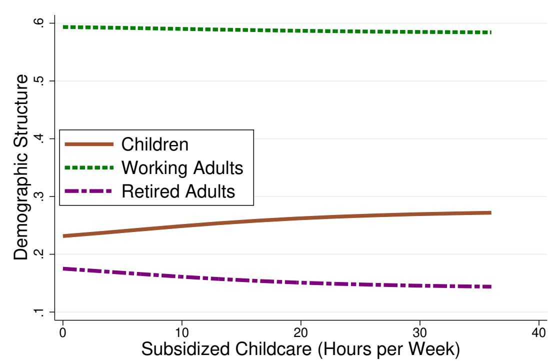
(f) Intergenerational Mobility



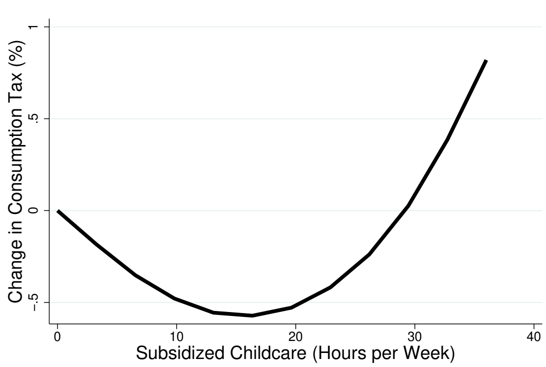
(g) Per Capita Output



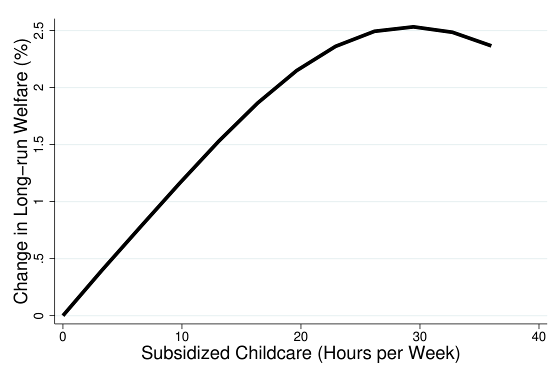
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of childcare subsidies of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. In Figure 15c and 15d, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

A.2 Public Education

In this section, I evaluate expansions of public education expenditures from the current level of \$12,000 per student per year by \$0 to \$100,000 in net present value. Figure 16a shows that the effect on fertility is positive, because children become more desirable as public education raises children's quality. The magnitude of fertility effects here are much smaller than that of family policies. Compared with a \$30,000 baby bonus that raises fertility by 0.2, an education expansion of the same amount only raises fertility by 0.011. The government should not expect to use education policies to raise aggregate fertility to replacement level.

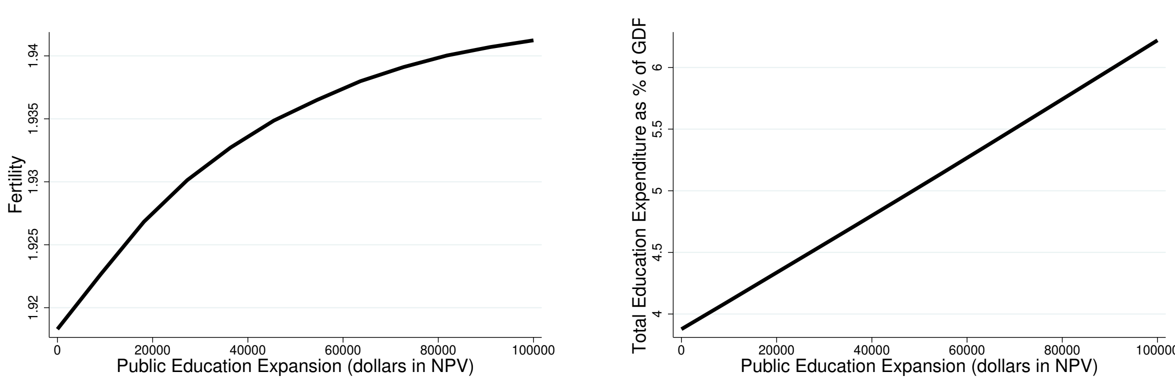
Figure 16d shows that the fertility response is larger for households with higher human capital. An increase in public education expenditure also crowds out private input (see Figure 16c). Due to strong crowding-out effects, average human capital only increases by 2.5% when the education expansion exceeds \$80,000 in net present value. It is quite costly to raise average human capital in the economy using uniform policies in the general equilibrium since (1) the crowding out effects are strong and (2) investments face decreasing marginal returns as $\mathcal{E}$ is already \$12,000 per year in the baseline economy.

Expanding public education, however, is the most effective policy to raise social mobility. As the expansion exceeds \$60,000 in net present value, intergenerational mobility could be raised by almost 5.2%. If the policy is designed to target lower-income households, we can expect larger effects on mobility. Due to increased human capital, public education expansion of \$60,000 raises output by 2.3%. The government needs to increase consumption tax by 0.75% to finance such expenditure. Figure 16j shows that welfare could be increased by up to 2.5%.

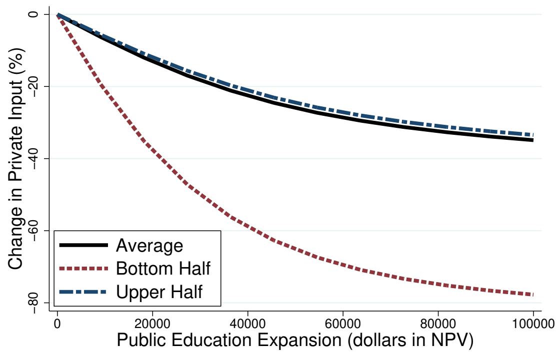
Figure 16: Effects of Expansions in Public Education Expenditures of Different Sizes

(a) Effects on Fertility

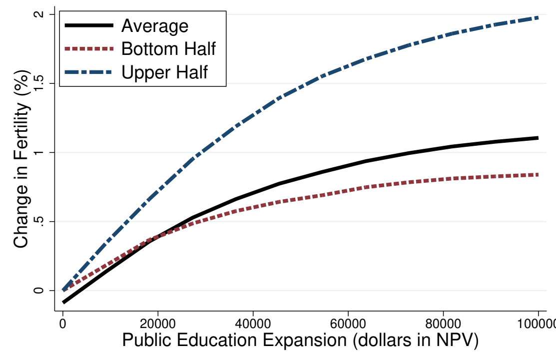
(b) Total Education Expenditure Share as % of GDP



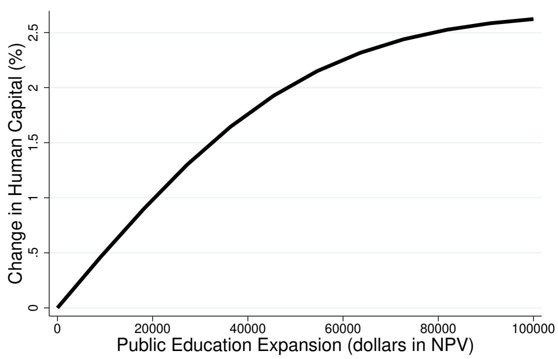
(c) Heterogeneous Private Input Response



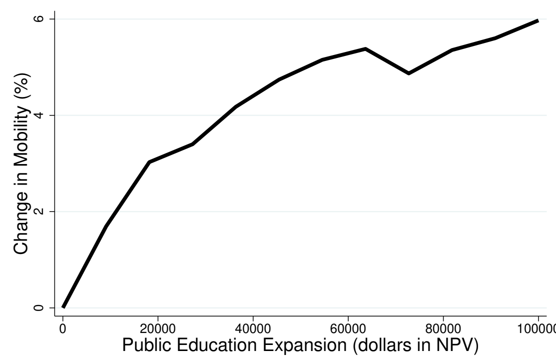
(d) Heterogeneous Fertility Response



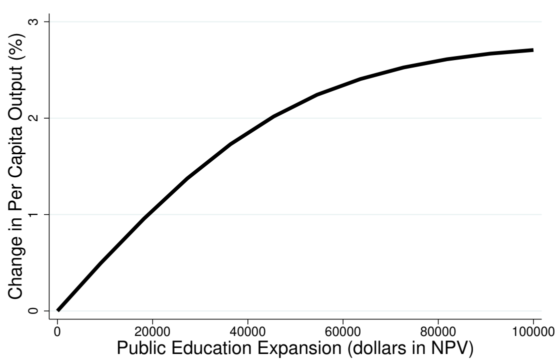
(e) Average Human Capital



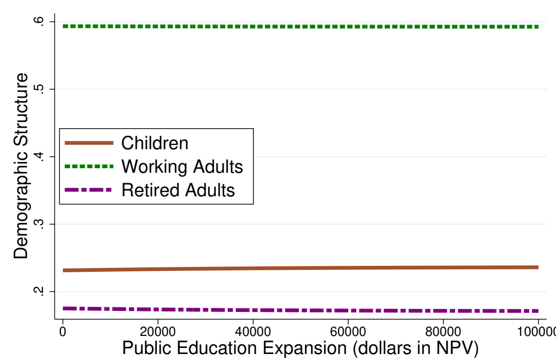
(f) Intergenerational Mobility



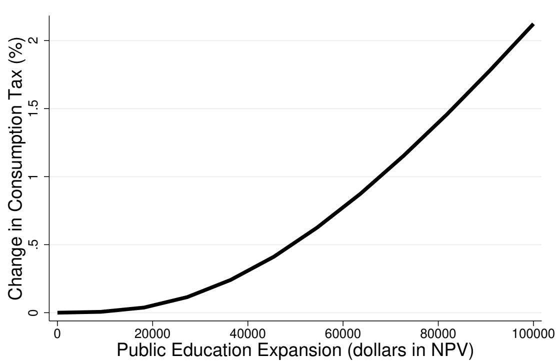
(g) Per Capita Output



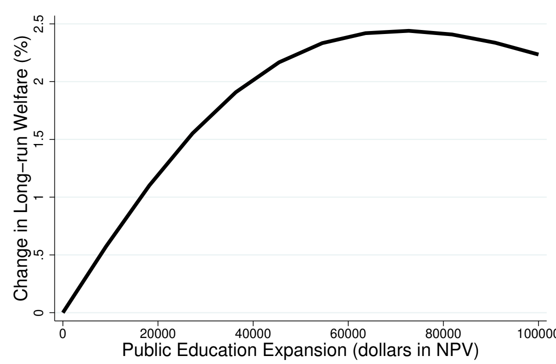
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of expansions in public education expenditures of different sizes by comparing long-run steady-states under policies with baseline steady-state economy. In Figure 16c and 16d, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

B Additional Validating Evidence

B.1 Australian Baby Bonus

In this section, we compare the implications of the calibrated model with the empirical evidence from a recently adopted baby bonus in Australia. Even though I do not re-calibrate the model to match moments in Australia, this comparison still provides valuable insights into the mechanism through which child benefits affects parents' decisions and estimates of fertility elasticities that are the core of this paper.

The Australian government announced the introduction of a universal cash payment, i.e., a baby bonus, in its Federal budget on May 12, 2004. It was designed to be a non-means tested and non-taxable lump-sum payment of A\$3,000 to encourage childbirth, paid to parents following a childbirth or adoption of a child up to 2 years of age after July 1, 2004. Such payment is independent of family income, maternal employment status, or the number of existing children in the household. The payment is equivalent to 4 times weekly average earnings or \$2,800 in 2010 U.S. dollars.

Drago et al. (2011) conduct a comprehensive analysis of the fertility effects of the baby bonus using household panel data ($N = 14,932$) from the Household, Income and Labour Dynamics in Australia Survey (HILDA) using a simultaneous equations approach. They first estimate the effect of the baby bonus on households' birth intentions in a linear regression, which, in turn, predicts actual births in a binomial probit regression. They exploited the panel structure of the data to test and correct for announcement effects, compression/tempo effects, and delayed effects. Regression results show that the baby bonus have significant and positive impacts on fertility. Moreover, the fertility responses are concentrated among low-income women.[66] Risse (2010) reports similar findings on women's birth intentions.

When I compare these empirical studies to the model, both the finding on overall fertility effect and the heterogeneities across households are consistent with predictions in Section 5.1. Drago et al. (2011) estimates that the marginal costs to the government for an additional birth is at least A\$126,000 which is roughly 4 times the GDP per capita in 2004 Australia dollars. The calibrated model generates a quantitatively similar conclusion where the marginal costs for an additional birth using baby bonuses in 2010 USA is around 3.5 times the GDP per capita.

Gaitz and Schurer (2017) evaluates the effect of the Australian baby bonus on children's human capital accumulation using high-quality panel data from Kindergarten cohort (K-cohort) of the Longitudinal Study of Australian Children (LSAC). They find that the baby bonus, despite be-

[66] This finding is different from that in Milligan (2005) which evaluates a baby bonus in Quebec and finds larger responses among parents with higher income. Drago et al. (2011) conjecture that the difference could be due to the fact that the Quebec baby bonus was significantly more generous for high-parity births. Milligan (2005) discusses some other possible explanations such as unobserved heterogeneities that are systematically related to income.

ing significant in size, was not effective in boosting learning, socio-emotional or physical health outcomes of the average pre-school child. This finding is consistent with results in Section 5.1 where parents optimally reduce private educational investments due to quality-quantity trade-off.

B.2 Spanish Child Benefits

In this section, I provide additional evidence on the fertility effects of child benefits using a universal child benefits in Spain.

The Spanish government announced the new child benefit on July 3, 2007. The benefit was a one-time payment of €2,500 to the mother immediately after birth on and after July 1, 2007. Since the cash benefit was universal and independent of recipient's characteristics, it was essentially a universal baby bonus (c.f., the Australian baby bonus, and $\mathcal{B}$ in the model). The size of the payment is 4.5 times the monthly gross minimum wage for a full-time worker or \$3,500 in 2010 U.S. dollars.

González (2013) studies the effect of the Spanish child benefits on fertility and mothers' labor supply using monthly vital statistic, monthly abortion statistics, and Household Budget Survey 2008 ($N = 958$). In the paper, the fertility effects are gauged by inspecting the time series of births and abortions while the effects on labor supply are estimated using a regression-discontinuity design by comparing households who had birth right before and right after the cutoff date. González (2013) finds that the fertility effects is positive and significant. The policy raises the total fertility rate by 6%. Around 80% of this increase is due to higher number of conception while the remaining 20% is due to reductions in abortion. Applying the adjustment in Stone (2020) (see Section 4), the result suggests that in Spain, the marginal costs for an additional birth using baby bonuses is 3.6 times the GDP per capita. In the model, this statistic is 3.5.

González (2013) also finds that the baby bonus reduced mothers' labor force participation. The calibrated model produces this effect via two channels. First, as the baby bonus raises fertility, parents need to generate more childcare services, which raises their time at home ceteris paribus. Second, higher fertility reduces the relative cost of home versus market care due to economies of scale in home production of childcare ($\iota < 1$). Therefore, the model predicts that parents optimally change the childcare arrangements so that their children spend less time in market care. González (2013) confirms this prediction by showing that Spanish parents reduces enrollment in formal childcare after the baby bonus.

B.3 Georgia's Cherokee Land Lottery of 1832

In this section, I compare the model predictions with Georgia's Cherokee Land Lottery in Northwest Georgia, 1832.

Following the eviction of the Cherokee from northwest Georgia, the state allocated more than 18,000 parcels of land to the public in a large-scale lottery in 1832. Every man aged 18 and older who had resided in Georgia for the three years prior to the 1832 drawing was entitled to one draw, and any man who had a wife or had children under 18 and met the three-year residency requirement was entitled to an additional draw. The lottery winners were able to immediately sell the winning draw. Therefore, winning the lottery can be viewed as a substantial shock to wealth. Bleakley and Ferrie (2013) calculate that 98% of eligible man participated in the lottery and winners were about $748 wealthier than losers by 1850, equivalent to 1,010 days of earnings for an unskilled laborer in the South at that time.

There are two notable things about the Cherokee Land Lottery. First, unlike a baby bonus that provides an estimate of the *price elasticity* of fertility, the Cherokee Land Lottery reveals the *income elasticity* of fertility. This statistic, if used directly, is mostly informative about fertility effects of policies such as a universal basic income (UBI) rather than family policies that are pronatal by design. The model in this paper, nevertheless, could generate the counterpart to the income elasticity of fertility by varying the initial assets of parents at period 2. Second, numerous changes have taken place in the U.S. in the past two centuries. The average white women gave birth to around seven children in 1830s but less than two children in 2010. The biggest factor explaining the decline of fertility and the rise of schooling is the increase in skill premium (Greenwood and Seshadri, 2002). Therefore, the Cherokee Land Lottery is expected to generate responses that are *upper bounds in fertility* and *lower bounds in child human capital investment* to windfalls.

Bleakley and Ferrie (2016) identify the winners of this lottery using a list published by the State of Georgia , and link winners to their decedents using Census data. Results show that winners had slightly more children than nonwinners, but they did not send them to school more. "Sons of winners have no better adult outcomes (wealth, income, literacy) than the sons of nonwinners, and winners' grandchildren do not have higher literacy or school attendance than nonwinners' grandchildren" (Bleakley and Ferrie, 2016).

In the calibrated model, I compare $n^*(h,a), e^*(h,a)$ with $n^*(h,a'), e^*(h,a')$ for fixed h and $a < a'$. I find that if the parents are endowed with a higher level of initial assets, they choose to have a slightly smaller number of children, i.e., $n^*(h,a') \leq n^*(h,a)$ but invest significantly more in children's education, i.e., $e^*(h,a') \gg e^*(h,a)$. Since the returns to education in 2010, to which the model targets using RCT results from García et al. (2020), is much higher than that in 1830s, these results are consistent with those found in Bleakley and Ferrie (2016).

C Illustrative Models

C.1 Simple Model of quality-quantity Trade-off

In this section, I show the quality-quantity trade-off mechanism with closed-form solutions in a model that is adapted from de La Croix and Doepke (2003).

Agents in the economy live for two periods, child and adult. Adult parents solve the problem:

$$\max_{c,n,e} \quad \log(c) + \theta \log(n \cdot h_k)$$

subject to

$$c + n \cdot e = 1 - n \cdot \chi$$

$$h_k = e^{\gamma}, \quad \gamma \in (0,1)$$

where c is consumption, n is fertility, h_k is human capital of children, e is private investment,[67] and χ is fixed costs per child.

When fertility is exogenous, i.e., n is given, parents maximize over c and e. Optimal investment is given by:

$$e^* = \frac{\theta\gamma}{1+\theta\gamma} \cdot \frac{1-n\chi}{n}$$

When the cost of child χ decreases, e^* increases due to income effect.

When fertility is endogenous, parents maximize over c, e, and n. Optimal fertility and investment are given by:

$$n^* = \frac{1}{\chi} \cdot \frac{\theta(1-\gamma)}{1+\theta}, \qquad e^* = \frac{\gamma\chi}{1-\gamma}$$

When the cost of child χ decreases, n^* increases while e^* decreases. The intuition for this result is simple. Parents increase fertility due to the substitution effect. The increase in n^*, in turn, raises the shadow price of investment e^* due to interaction of n and e in the budget constraint. As a result, optimal investment e^* falls.

Compared with the benchmark model where n and h_k are complements in parents' preferences, reduction in e^* in response to change in χ is higher in this simple model. This is because the marginal utility in child quality h_k is independent of fertility n.

[67] In de La Croix and Doepke (2003), children receive human capital endowments. This generates non-homotheticity over child quality for heterogeneous parents with different human capital and leads to a negative fertility-income relationship. I abstract away from steady-state heterogeneity across households in this simple model for clearer exposition of intuition in comparative statics, but all arguments carry through when human capital endowments are allowed.

C.2 Simple Model of Fertility Elasticity

In this section, I build a simple model to illustrate the relationship between parameter γ, also known as elasticity of intergenerational substitution (EGS), and the magnitude of fertility response to family policies.

Consider a simplified problem for parents with very low income so that child quality is generated by public investments alone:

$$\max_{c,n} \quad u(c) + \Psi(n)u(\mathcal{E})$$

$$c + n \cdot \chi = 1$$

First-order condition for n:

$$\underbrace{\Psi'(n) \cdot u(\mathcal{E})}_{\text{MB of } n} = \underbrace{\lambda \cdot \chi}_{\text{MC of } n}$$

Plug in $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I have

$$\Psi'(n) = (1-\gamma) \cdot \chi \cdot \frac{\lambda}{\mathcal{E}^{1-\gamma}} \Longrightarrow \Delta\Psi'(n) \propto (1-\gamma) \cdot \Delta\chi$$

As can be seen, conditional on other parameters, higher γ implies a smaller response in n for given changes in costs of child χ.