

Bounding Fertility Elasticities

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Abstract

I propose a new methodology to bound the magnitude of fertility responses to financial incentives. Under mild assumptions, I show that raising the fertility rate by 0.1 children per woman in the United States in 2010 requires the cost of children to fall between \$7,514 and \$35,966. This bound is tighter than that in meta-studies of past estimates and is simple to compute for a different country or year.

1 Introduction

How much does fertility respond to financial incentives? The answer to this question is important for both policymakers and economists. For governments that want to use financial transfers to raise fertility, it is critical to understand how cost-effective these policies are. For economists, fertility elasticity provides a benchmark to discipline models with endogenous fertility, just as the marginal propensity to consume (MPC) informs modern macroeconomic models. In addition, estimates of the fertility elasticity allow one to evaluate how much the exogenous fertility assumption matters for the quantitative results in structural models on child-related policies, such as the Child Tax Credit (CTC) and the Earned Income Tax Credit (EITC).

There has been a large number of empirical studies estimating the fertility elasticity in different settings.¹ Pinning it down using historical policies has proven to be difficult for several reasons. First, there is a lack of large and persistent policies that change the cost of children drastically, making it unlikely for people to change their fertility behaviors, especially the total number of children, in a quantitatively significant way. Second, as most pro-natal policies are nationwide, finding a control group is not straightforward (Gauthier (2005)). This difficulty is especially acute given that most pro-natal policies are adopted to prevent future fertility crashes, while it is unclear whether

¹See Milligan (2005), Cohen et al. (2013), González (2013) among many others. For review studies, see McDonald (2006) and Stone (2020).

countries or regions in the control group also face a similar problem (Castles (2003)). In addition, many pro-natal policies come in bundles of incentives that go beyond lowering the financial cost of children, such as measures encouraging women’s labor force attachment. Therefore, estimating the fertility elasticity of the underlying policy is not equivalent to estimating the price elasticity of fertility. As a result of these difficulties, empirical estimates vary widely – “there is considerable disagreement across studies about the effectiveness of pro-natal policies” even though “the directional finding that pro-natal benefits boost fertility is nearly uniform” (Stone (2020)). Last but not least, it is unclear to what extent past estimates predict fertility elasticities in a different time or institutional context.

In this paper, I propose a new methodology to bound the fertility elasticities. Under mild assumptions, I show that a bound can be derived after knowing (1) the prevailing fertility rate and cost of children, (2) the maximum desired fertility, and (3) the cost of children such that parents would rather prefer not to have a child. Applying this method to the United States in 2010, I find that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman. I show that this bound is tighter than that in meta-studies of past estimates and is simple to compute for a different country or year under consideration.

The rest of the paper is organized as follows. Section 2 presents the theory that derives the bound. Section 3 applies the theory to the United States and compares the results with prior studies. Section 4 concludes.

2 Theory

Denote the Marshallian demand of fertility as $n(p; \mathbf{p}^{\text{other}}, y)$ where n is fertility, p is the cost of children, $\mathbf{p}^{\text{other}}$ is the price of other goods, and y is the household’s (lifetime) income.

Assumption 1 The Marshallian demand of fertility $n(p; \mathbf{p}^{\text{other}}, y)$ is downward sloping, continuously differentiable, and convex.

This assumption is satisfied by most models of endogenous fertility, either with warm glow (e.g., De La Croix and Doepke (2003)) or with dynastic altruism (e.g., Barro and Becker (1989) and Córdoba and Ripoll (2019)).

Assumption 2 There exists \bar{p} and \bar{n} such that $n(\bar{p}; \mathbf{p}^{\text{other}}, y) = 0$ and $n(0; \mathbf{p}^{\text{other}}, y) = \bar{n}$.

This is also a mild assumption. For example, set $\bar{p} = y$, the condition requires that if having a child costs the parents’ entire budget and leaves no resources for other goods, then the household would prefer not to have a child. The existence of \bar{n} reflects satiation in preferences or biological constraints of childbearing.

Proposition The fertility response to price around (n^0, p^0) is bounded by

$$\left. \frac{dn}{dp} \right|_{(n^0, p^0)} \in \left(\frac{n^0}{\bar{p} - p^0}, \frac{\bar{n} - n^0}{p^0} \right). \quad (1)$$

Proof Figure 1 illustrates the essence of the proof. The Marshallian demand of fertility is given by curve BAC where the coordinates of B and C are $(0, \bar{p})$ and $(\bar{n}, 0)$ correspondingly. Point A denotes the prevailing fertility and cost of children (n^0, p^0) . We can show that the slope of the demand curve at A is bounded by the slope of AB and the slope of AC using the Mean Value Theorem and the assumption that the curve is convex.

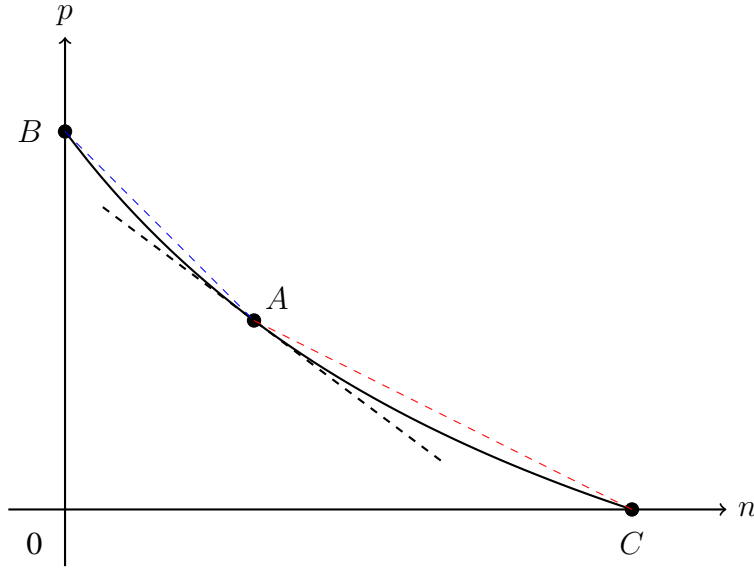


Figure 1: Illustration of the Proof

3 Quantification

I calculate the bound for the United States in 2010 where the prevailing fertility n^0 is 1.9 and the cost of one child p^0 is \$458,351 for a middle-income household (Córdoba and Ripoll (2019)). I make the following assumption on \bar{n} and \bar{p} :

Assumption 3 Set $\bar{n} = 8$. Choose \bar{p} such that an average household with one child lives in poverty for the rest of their lives, i.e.,

$$y - \bar{p} = y^{\text{poverty}}. \quad (2)$$

Following Córdoba and Ripoll (2019), I set $y = \$2,083,219$. Using the federal poverty guideline in 2010, I set $y^{\text{poverty}} = \$18,310 \times 18 + \$14,570 \times 42 = \$941,520$. This implies $\bar{p} = \$1,141,699$.

Applying the proposition, I find that to raise the fertility by 0.1 children per woman, the change in p needed is between \$7,514 and \$35,966. This change in p can be brought about by policies such as a baby bonus, a universal basic income, or a fully-refundable Child Tax Credit (CTC).

Next, I compare my results to meta-studies of past estimates. Stone (2020) summarizes 22 studies since 2000 using historical policies, mostly in low-fertility countries. He concludes that “an increase in the present value of child benefits equal to 10% of a household’s (annual) income can be expected to produce between 0.5% and 4.1% higher birth rates.” To make the measures comparable, I convert the bound in this paper into percentages using the median annual household income in 2010 (\$49,445). The bound predicts that an increase in the present value of child benefits equal to 10% of a household’s (annual) income can produce between 0.72% and 3.46% higher birth rates. As can be seen, the bound proposed here is tighter than existing estimates.

Another advantage of the bound is that it is simple to compute for a different country or year as long we know the prevailing (n^0, p^0) , y , and y^{poverty} , which are not difficult to obtain. Adopting an actual pro-natal policy and estimate the fertility elasticities afterwards will likely lead to a more precise estimate. But the bound could still be helpful as it provides a prediction of the program’s cost-effectiveness *ex ante* and a sanity check of the regression result *ex post*.

In addition, the bound provides a benchmark to discipline models with endogenous fertility. For instance, consider a simple model of fertility choice with quasi-linear utility where agents solve:

$$\max_{c,n} \quad c + \theta \cdot \frac{n^{1-\sigma}}{1-\sigma}$$

subject to

$$c + n \cdot p \leq y.$$

With interior solutions, optimal fertility is given by

$$n^* = \left(\frac{p}{\theta}\right)^{-1/\sigma}$$

which implies that when p falls by 1 percent, n^* rises by $1/\sigma$ percent. Thus, if one plans to calibrate this model to the U.S. economy in 2010, the bound suggests that the value of σ , which is typically chosen exogenously, should lie between 0.31 and 1.49. Similarly, one can use the bound to validate comparative static results in other, potentially more complicated, models of endogenous fertility.

Finally, the bound permits an evaluation of the exogenous fertility assumption in structural models that analyze child-related policies. For example, Guner et al. (2020) consider a policy counterfactual where the per-child tax credit rises about \$800 per child per year, amounting to about \$12,000 in net present value terms for eligible households. Another example is Daruich (2022) where the welfare-maximizing early-childhood development subsidy is around \$80,000 (Figure 3). The bound

suggests that these transfers might lead to non-negligible fertility responses and a dilution of family resources à la the quantity-quality trade-off mechanism (Becker and Lewis (1973)).

4 Conclusion

The magnitude of fertility responses to financial incentives is important to both policymakers and economists. In this paper, I propose a new methodology to bound the fertility elasticities. Under mild assumptions, I show that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman in the United States in 2010. This bound is tighter than that in meta-studies of past estimates and is simple to compute for a different country or year.

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