The Macroeconomic Consequences of Family Policies

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Abstract

Many governments in developed countries adopt family policies (e.g. baby bonus and childcare subsidies) to combat population aging, improve children's outcomes, and boost social mobility. To study the macroeconomic consequences of family policies, I build a quantitative general equilibrium overlapping-generations model with heterogeneous agents, endogenous fertility, child human capital formation, and inter-vivos transfers. In contrast to conventional wisdom from structural models with exogenous fertility or design-based studies, I find that in the calibrated model (1) parents could optimally reduce child human capital investments in response to child-related cash transfers due to quality/quantity trade-off, which dampens social mobility since this reduction is larger among low-income households; (2) a $31,000-dollar baby bonus raises the aggregate fertility rate to replacement level but reduces average human capital by 1.5% in light of lower human capital investments and composition effects; (3) such baby bonus increases long-run welfare by 2.2% because higher population growth rate reduces the share of retired households, allowing the government to lowers tax rates; (4) such long-run improvements require a prolonged transition path with higher tax rates to finance larger child-related expenditures; and (5) among different policy options, subsidized childcare encourages childbearing and labor force participation; while education expenditures are the most effective in boosting child outcomes and social mobility despite having mild effects on fertility.

JEL classification: E62, H31, H52, J11, J13

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1 Introduction

Family policies are social programs, laws and public directives designed to promote and enhance marriage, reproduction and raising children. Examples of family policies include but not limited to baby bonus, subsidized childcare, and parental leaves. These policies are widely used in developed countries and large in scale. Public expenditures on family benefits as a percentage of GDP rises from 1.5% in 1980 to 2.1% in 2015 among OECD countries.

Family policy is a key instrument for governments to encourage childbirth and address population aging.[1] For example, parents of every child born in Australia after July 1st 2004 are eligible to receive a lump-sum payment of A$3,000.[2] Peter Costello, the Treasurer of Australia at that time, said that the baby bonus could help families to have “one (baby) for the Mum, one for the Dad, and one for the country". Similar benefits that are linked to the event of childbirth or designed to reduce the burden of child-rearing is widespread in the developed world and countries that are struggling with low fertility such as Russia, Spain, France, Italy, Canada, Singapore, Korea and so on. The concern of low-fertility is also becoming increasingly relevant for the United States as recent Census data shows record-low level of birthrate.

Governments also use family policies to alleviate child poverty, foster children’s human capital formation, and raise social mobility. In the United States, proposals to expand child tax credit (CTC) has received supports from bipartisan members of the Congress such as Senator Bennet (D-CO) and Senator Romney (R-UT) while the maximum amount awarded to each child per year has risen to more than $3,000 under the recent policy change. The recently-proposed $1.8 trillion American Families Plan also aims at reinforcing the change in CTC by providing more support to parents through childcare subsidies, early childhood education, and more generous parental leaves. Guided by empirical evidence suggesting that transfers to families with children could raise children’s human capital (see Heckman and Mosso (2014) for recent review), the administration views family policies as “an investment in our kids, our families, and our economic future".

With growing popularity and magnitude of family policies, it is important to understand their macroeconomic consequences. Are they cost-effective in raising fertility, reduce old-age dependency ratio, and improve welfare? Are generous child benefits effective in improving children’s human capital and boost social mobility? Moreover, will parents’ decisions help achieve or go against stated policy goals? This paper seek to understand these questions.

I study the macroeconomic consequences of family policies in an overlapping-generations model with heterogeneous agents and distortionary taxes in the general equilibrium. The model has two key components. First, households make endogenous fertility choices where parents weigh the ben-

[1] Alternative policy instruments include immigration policies and pension reforms. I keep these policies unchanged in this paper and focus on impacts of family policies.

[2] Equivalent to 4 times weekly average earnings and $2,800 in 2010 U.S. dollars.

efits of children against the costs of child-rearing. Second, parents make choices that affects child quality, a termed coined by a series of seminal papers by Gary Becker, via investments in child human capital formation and inter-vivos transfers. Family policies affect households' decisions along both quantity and quality margins.

Compared with previous literature that mainly focus on how transfers would affect the quality margin for existing children, considering the quantity margin, i.e. endogenous fertility, is crucial in understanding the impacts of family policies for three reasons. First, if parents raise fertility to take advantage of more generous child benefits, larger number of children would increase the marginal costs of child quality. Therefore, policies that aim at boosting fertility could actually worsen the outcome of children due to quality/quantity trade-off a lá Becker and Lewis (1973). Second, family policies induce a composition effect on aggregate variables. Due to intergenerational persistence of traits (e.g. human capital), aggregate variables gravitate towards the traits of those families that respond most strongly in fertility since these children account for larger share of the population in future generations. Third, higher fertility induces general equilibrium effects on government budget and taxes due to changes in demographic structure. As population growth rate rises, the burden of pension payments in countries with pay-as-you-go (PAYG) system could be relieved while public education expenditures on children will rise. These changes in expenditures are of first-order importance for government budget and taxes, which in turn affects people's welfare.

All three channels mentioned above depend on the magnitude of fertility response to government policies, hereafter I define as fertility elasticities. If child quantity is exogenous or fixed, transfers to parents would simply induce an income effect on children's quality and the model predictions would be similar to that in Daruich (2018) and Mullins (2019). I discipline the magnitude of fertility elasticities using estimated parameters in the calibration, including costs of children, child human capital production function, and fertility-income profile informed by data from Current Population Survey (2010), Panel Study of Income Dynamics (PSID), Survey of Income and Program Participation (SIPP), Consumer Expenditure Survey (CEX), mobility estimates from Chetty et al. (2014) as well as RCT evidence from García et al. (2020). I also provide external validation of fertility elasticities in using policy variation from Alaska Permanent Fund Dividend (APFD) and show that the quantitative predictions of the model is consistent with existing empirical evidence.

The calibrated model generates five key findings. For the first four findings, consider a baby bonus, i.e. one-time lump-sum transfer to parents of newborn child, of \$31,000 in 2010 dollars,[3]. First, the model predicts that parents would increase the number of children by 10% and average fertility rate would rise from 1.9 children per women to 2.1 children per women. But in contrast to

[3]To put this number in perspective, this baby bonus is similar to $1.1\times$ the size of expansions in Child Tax Credit (CTC) from 2010 to 2021 in net present value. It offsets the average cost of raising on child by 19% using estimates from U.S. Department of Agriculture (USDA).

conventional wisdom that more generous child benefits improve child outcomes, parents optimally reduce private educational investments by 8% due to quality/quantity trade-off and average human capital falls by 1.5% following the baby bonus.

Second, I find that intergenerational income mobility falls by 1.6% since parents with lower human capital have stronger responses in fertility and reduces private educational investments by a larger amount. While the effect is derived under a baby bonus that is uniform across households, the result of falling mobility will be even stronger when child-related transfers is designed to target sub-population with lower income.

Third, I find that despite reductions in human capital and mobility, long-run welfare rise by 2.2% in consumption equivalence units. As family policies raise population growth rate and lower the old-age dependency ratio[4], the government is able to reduce tax rates while still keeping fiscal budget at balance. Assuming exogenous fertility in the cost-benefit analysis of family policies misses a key channel that welfare gains are obtained.

Fourth, the transition towards such long-run welfare improvements with lower taxes features a prolonged path where the government actually needs to raise taxes to finance higher child-related expenditures. Total dependency ratio[5] spikes before converging to the long-run level. Welfare changes of future households along the transition path are positive but smaller than the long-run gain of 2.2% while most households in the current economy loses due to higher taxes.

Lastly, I use the calibrated model to study the macroeconomic impacts of alternative policies such as subsidized childcare and expansions in education expenditures. Among different policy options, I find that (1) subsidized childcare encourages childbearing and labor force participation at the same time, and (2) education expenditures are the most effective policy in improving child outcomes and boosting social mobility despite having mild effects on fertility.

Related Literature

This paper is related to four strands of literature. The first set of papers study fertility in a macroeconomic context and analyze how it is related to the fiscal stability, economic growth, intergenerational mobility and inequality. Prominent examples include Doepke (2004), Greenwood et al. (2005), Manuelli and Seshadri (2009), Cordoba et al. (2016), De Silva and Tenreyro (2020) and Jones (2020) among many others. The closest work to our is the paper by De La Croix and Doepke (2003) where they study how higher inequality reduces growth through the channel of fertility differential. While I adopt their basic framework featuring general equilibrium, heterogeneous agents, endogenous fertility and human capital formation, I extend the model in various dimensions and

[4]Old-age dependency ratio is defined as the number of retired households divided by the number of working population.

[5]Total dependency ratio is defined as the number of retired people plus the number of children, divided by the number of working population.

use the model to study aggregate impacts of family policies.

Second, the paper relates to the literature on both the positive and normative aspects of fiscal policies regarding fertility, redistribution and education. On the positive side, Daruich (2018) and Abbott et al. (2019) are the closest papers to ours. They both build heterogeneous agent overlapping generations model to study large-scale education policies. I differ from these work by modeling endogenous fertility choice and considering the interaction of family and education policies. On the normative side, Guner et al. (2020) considers optimal design child-related transfers with heterogeneous households making both intensive and extensive labor force participation choices. This paper add to the literature by studying the design family policies with endogenous fertility responses.

A number of empirical papers have studied the fertility effects of family policies (e.g. Milligan (2005), Drago et al. (2011), González (2013), Laroque and Salanié (2014) among many others). Meta-analyses of these studies have concluded that “the directional finding that pronatal benefits boost fertility is nearly uniform", and an increase in present value of child benefits equal to 10% of household annual income would lead to 0.5% to 4.1% increase in total fertility rate (McDonald (2006), Stone (2020)).[6] These papers, however, are not tailored to address the broader consequences of family policies beyond fertility, such as the effects on output, human capital and welfare. The design-based approach is also not able to predict the consequences of family policies in the long-run when children being affected become parents themselves. This paper takes a structural approach to answer these questions.

Lastly, the paper is related to the growing literature that evaluates the impacts of transfers or subsidies to families with children. Prominent examples include Akee et al. (2010), Dahl and Lochner (2012), Mullins (2019) and García et al. (2020) among many others (see Heckman and Mosso (2014) for recent summary). With both design-based methods and structural models, the conventional wisdom in this literature is that income transfers or education subsidies to families with children have positive impacts on children’s outcomes including but not limited to health, education attainment and criminal records. This papers uses experimental evidence from this literature to inform parameter choices in Section 3. But since this literature exclusively evaluates policy effects of transfers to families with existing children, I argue that the effects could not be directly used to extrapolate the aggregate impacts of family policies when fertility is endogenous.

The rest of the paper is organized as follows. In Section 2, I present the quantitative model. The calibration of the model is discussed in detail in Section 3. I conduct the validation exercise in Section 4. Section 5 presents our policy counterfactuals results. Section 6 discusses the robustness of our results to alternative assumptions. Section 7 concludes.

[6] The distinction of quantum versus tempo effects will be discussed with more details in Section 4.

2 Model

The model is based on the framework in De La Croix and Doepke (2003) where heterogeneous households make endogenous fertility and child human capital investment decisions in an overlapping-generations, general equilibrium economy. I extend it in multiple directions. First, I model realistic lifespan with working and retirement stages. Households make both life-cycle and precautionary savings under idiosyncratic uninsurable labor market shocks. By extending individual lifespan to more than two periods, the model captures changes in age structure of the population induced by family policies. Second, I incorporate government policies including taxation, pension payments, public education and expenditures on family benefits. In particular, I explicitly model the role of public education in children's formation of skills. These features would allow us to evaluate the comprehensive effects of family policies on the government budget balance and the potential interactions with education policies. Lastly, I consider parents' choices in childcare arrangements. They can either choose to take care of children at home by themselves or utilize marketable childcare services. This feature of the model allows us to evaluate the effects of childcare subsidies on parents' labor supply which could in turn affect their own human capital through learning by doing.

2.1 Household

Consider overlapping generations of households that can live up to 90 years old. I split the life-cycle into 9 periods where where each period stand for 10 years and is indexed by $j \in \{0, 1, 2, \dots, 8\}$. Period j stands for age $j \times 10$ to age $(j + 1) \times 10$.

The timeline of the model is presented in Figure 1. Children live with their parents from age 0 to 20 where they receive human capital investments from both public and private sources. Children also receive inter-vivos transfers from parents when they become independent and form their own household at age 20. Households make fertility choice, childcare arrangements, and private child human capital investments decisions from age 20 to 30. Parents make inter-vivos transfers choice at age 30 to 40 before the child becomes independent. Households work from age 20 to 70 where their wage depends on human capital and idiosyncratic uninsurable shocks. They retire at age 70 and receive income from savings and pension payments. Households make consumption-savings choices every period in their lives and face age-specific survival rate δ_j.[7]

For working adults, I assume that their human capital evolve over time following a learning-by-doing process:

$$h_{j+1} = L(h_j, t_w, z_{j+1}) \tag{1}$$

[7] Assets from deceased households are collected by the government and rebated to all living households uniformly as a lump-sum transfer.

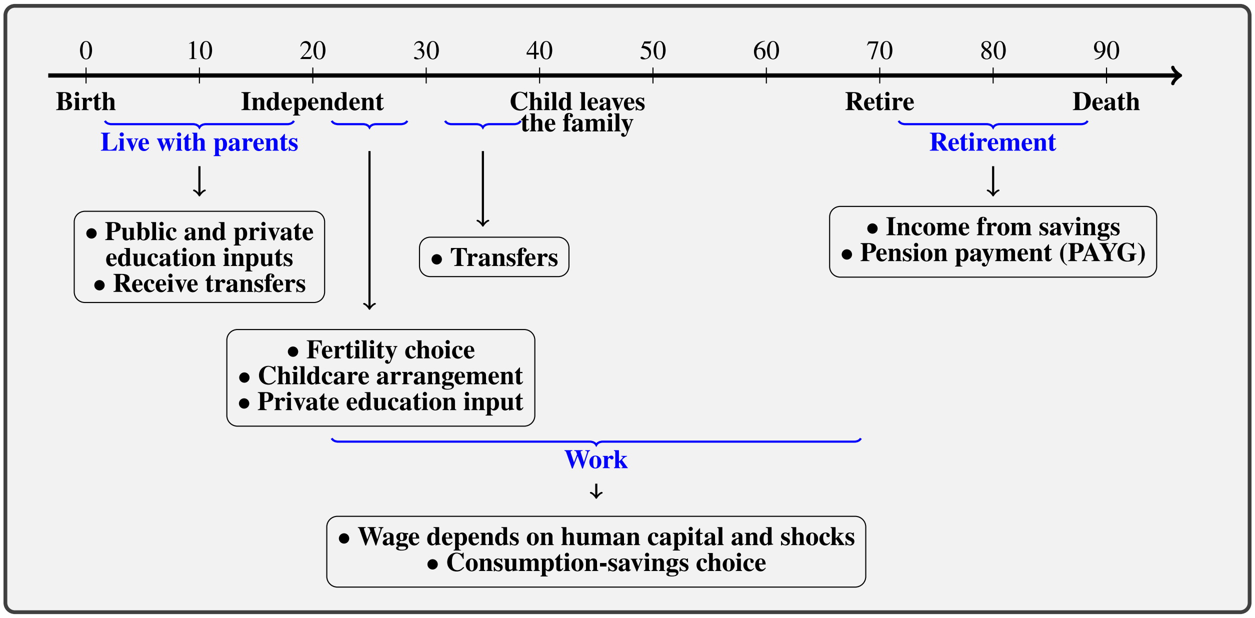


Figure 1: Model Timeline

where z_{j+1} is the idiosyncratic uninsurable shock to human capital that occurs at the beginning of period $j+1$; h_j is the amount of human capital at period j; and t_w is the time worked in period j. I assume that agents have unit time endowment and supply labor inelastically except in the period where they need to make childcare arrangement choices. This assumption captures the key trade-off between taking care of the child and working for households with children.[8]

Young adults at the beginning of period 2 (age 20-30) are heterogeneous in human capital h and assets a which are in turn determined by their own parents. Their maximization problem is given by:

$$V_2(h,a) = \max_{c,a',n,t_h,m,e \geq 0} u(c/q(n)) + \beta \mathbb{E} V_3(h', a', n, \mathbb{E}h_k)$$

subject to

$$n \cdot \chi = \left(t_h^{v/\iota} + (n \cdot m)^v \right)^{1/v} \qquad \text{[time cost]}$$

$$y = wh \cdot (1 - t_h) \qquad \text{[labor income]}$$

$$(1+\tau_c)(c + mp_m(1-\mathcal{S})n + e \cdot n) + a' = (1+r)a + y - \mathcal{T}(y,a,n) + \mathcal{B} \cdot n \qquad \text{[BC]}$$

$$h' = L(h, 1-t_h, z') \qquad \text{[learning OTJ]}$$

$$h_k = G(h, \mathcal{E}, e, \epsilon) \qquad \text{[skill formation]}$$

[8]In the class of models with endogenous fertility, it is common to abstract away from labor/leisure split and focus on the time costs of children, e.g. De La Croix and Doepke (2003) and Daruich and Kozlowski (2020). Incorporating leisure choice into the benchmark model increases computation time, especially in transition path, without adding more insights.

The state variables in $V_2(h,a)$ are household human capital h and starting asset a. Households choose consumption c, savings a', fertility n, total time spent at home taking care of children t_h, amount of market childcare purchased for each child m and private education investment for each child e. Parents' value is composed of flow utility of consumption $u(c/q(n))$ where consumption expenditure c is divided by equivalence scale $q(n)$ and the discounted continuation value $V_3(\cdot)$.

To raise n children, parents need to meet $n \cdot \chi$ amount of childcare needs in time units. Parents could satisfy this need either by spending their own time at home t_h or spend money purchasing market childcare services of m per child. I assume that these two forms of childcare is combined in a constant-elasticity-of-substitution (CES) production function where υ governs the elasticity of substitution. Home production of childcare enjoys an economy of scale with parameter $\iota \in (0,1)$.

The opportunity cost of home production of childcare t_h is hours spent in the labor market. Household labor income y is the product of market wage w, human capital h, and time worked $(1-t_h)$. Total resource available to the household is composed of asset multiplied by interest rate $(1+r)a$, labor income y, net taxes paid $\mathcal{T}(y,a,n)$, and the total amount of baby bonus received $\mathcal{B} \cdot n$. Household divide resources into savings a' and three kinds of expenditures which are multiplied by consumption tax τ_c. These expenditures are composed of consumption c, total spending on market childcare $mp_m n$ and private educational expenditure $e \cdot n$. I use p_m to denote the price of market childcare relative to consumption goods[9] and I assume that for each dollar spent on childcare, the government could choose to refund $\mathcal{S} \in [0,1]$ dollars back to parents.

Children's human capital production function $G(h,\mathcal{E},e,\epsilon)$ combines parents' human capital h, public education $\mathcal{E}$, private investment e and idiosyncratic ability shock ϵ that I assume is unknown to parents. I make the simplifying assumption that parental investment in children's human capital takes place when children's age is between 0 to 10 while public education $\mathcal{E}$ takes the major role from age 6 to 20. The function $G(\cdot)$ captures the overall human capital production function that spans age 0 to 20 without explicitly modeling several stages of production with dynamic complementarity. I also assume that arrangement of non-educational childcare does not affect children's human capital directly. This assumption holds when substituting home care by market care mainly shifts the burden of routine tasks without affecting intergenerational interactions that could affect children's human capital. Since the government also chooses public education expenditure $\mathcal{E}$, the model could replicate public provision of high-quality childcare as a policy that combines higher $\mathcal{S}$ with an expansion in $\mathcal{E}$.

I assume that households face inter-temporal borrowing constraint $a' \geq 0$ that is standard in the class of Aiyagari-Huggett models. With idiosyncratic uninsurable shocks, this induces households

[9] In this paper, I will be assuming that the supply of childcare workers is perfect elastic, at least in the long-run, at baseline price p_m. In the short-run, relative prices of childcare may rise in response to family policies, such as a large childcare subsidy.

to save for precautionary motives beyond life-cycle savings. Parents are also not allowed to invest in negative amounts of resources in children's education, $e \geq 0$. Therefore, public investment $\mathcal{E}$ guarantees a lower-bound for total educational investments received by any child.

Parents' choice problem at age 30-40 is given by:

$$V_3(h, a, n, \mathbb{E}h_k) = \max_{c, a', a_k \geq 0} u(c/q(n)) + \beta \mathbb{E} V_4(h', a') + v(n, \mathbb{E}h_k, a_k)$$

subject to

$$y = wh$$

$$(1 + \tau_c)c + a' + n \cdot a_k = (1 + r)a + y - \mathcal{T}(y, a, n)$$

$$h' = L(h, 1, z')$$

where parents choose consumption, savings and the amount of transfer to be received by each child at the beginning of next period a_k. Following the literature, it is assumed that parents face intergenerational borrowing constraints so that they are not allowed to make negative transfers to children, i.e. $a_k \geq 0$. I use $v(n, \mathbb{E}h_k, a_k)$ to denote parents' preferences on child quantity, child quality and inter-vivos transfers. The parametric form of $v(\cdot)$ will be discussed and calibrated in Section 3.3.

The first-order conditions for child quality choices e and a_k are:

$$\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial \mathbb{E}h_k} \cdot \frac{\partial \mathbb{E}h_k}{\partial e} = \lambda_2 \cdot (1 + \tau_c) \cdot n \qquad \text{FOC } [e]$$

$$\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial a_k} = \lambda_3 \cdot n \qquad \text{FOC } [a_k]$$

where the left-hand-sides denote marginal benefits of higher child quality and the right-hand-sides denote marginal costs[10]. Note that higher child quantity n raises the marginal costs of child quality due to the simple fact that quality and quantity interact in parents' budget constraint. Becker and Lewis (1973) gave this relationship between child quality and quantity the name "quality/quantity trade-off".

For $j \in \{4, 5, 6\}$, households solve a simple consumption-savings problem with idiosyncratic shocks to human capital. For computational simplicity, I assume that parents and children no longer interact in these periods.[11] The maximization problem is given by:

$$V_j(h, a) = \max_{c, a' \geq 0} u(c) + \beta \mathbb{E} V_{j+1}(h', a') \qquad j \in \{4, 5, 6\}$$

[10] I use λ_i to denote the Lagrange multiplier on budget constraint for period i.

[11] Otherwise, parents need to keep track of all children's state variables. This will make the problem computationally infeasible.

subject to

$$(1+\tau_c)c + a' = (1+r)a + y - T(y, a, 0)$$

$$h' = L(h, 1, z)$$

Finally, retired agents solve:

$$V_j(h, a) = \max_{c, a' \geq 0} u(c) + \beta V_{j+1}(h, a') \qquad j \in \{7, 8\}$$

$$(1+\tau_c)c + a' = (1 + r - \tau_a r)a + \pi \cdot wh$$

$$V_9(\cdot) \equiv 0$$

where household income is composed of assets and pension payment $\pi \cdot wh$. I use π to denote the replacement rate of pension and assume pension payments are not subject to income taxes. Capital income ra, on the other hand, is subject to a flat-rate tax rate τ_a.[12]

2.2 Firms

There is a representative firm in the economy that hires labor (in efficiency units) and borrows capital from households to produce final goods with Cobb-Douglas technology:

$$Y = A \cdot K^{\alpha} L^{1-\alpha} \tag{2}$$

In (2), K is aggregate capital used in production and H is total efficiency units employed. Total factor productivity A is normalized to be one.[13]

I assume physical capital depreciates at rate δ_K after use. With competitive factor markets, wage and risk-free interest rate in equilibrium are given by:

$$r = \alpha \left(\frac{K}{H}\right)^{\alpha-1} - \delta_K, \quad w = (1-\alpha)\left(\frac{K}{H}\right)^{\alpha}$$

2.3 Government

The government collects tax revenues from labor income, capital income and consumption. The expenditure side includes public education, family policies, pension and other exogenous expendi-

[12]Households in other periods also need to pay the same flat-rate capital income tax. It is embedded in the tax function $\mathcal{T}(y, a, n)$.

[13]I abstract away from population externalities that could affect aggregate production such as pollution (Bohn and Stuart (2015)) or ideas creation (Jones (2020)). This choice is made since (1) the literature on measurement of population externalities is still developing (2) our results will move in expected ways once positive/negative externalities are incorporated.

tures to balance the budget. I assume that the government balances budget from period to period without issuing debts.

I use $\{\mu_j\}_{j=0}^{8}$ to denote the distribution of households across state space and use $\{\omega_j\}_{j=0}^{8}$ to denote the fraction of each age group in the population with total mass normalized to one. The government balance constraint at each time is given by:

$$\begin{aligned}
&\underbrace{\left(\sum_{j=2}^{6}\omega_j\int\mathcal{T}(y_j^*,a_j^*,n_j^*)\,d\mu_j\right)}_{\text{labor and capital income taxes}}+\underbrace{\left(\sum_{j=2}^{8}\omega_j\int\tau_c c_j^*\,d\mu_j\right)}_{\text{consumption tax}}=\underbrace{\left(\sum_{j=7}^{8}\omega_j\int w\pi h\,d\mu_j\right)}_{\text{pension payments}}\\
&+\underbrace{(\omega_0+\omega_1)\cdot\mathcal{E}}_{\text{public education}}+\omega_2\left(\underbrace{\int n^*\cdot\mathcal{B}\,d\mu_2}_{\text{baby bonus}}+\underbrace{\int(1+\tau_c)m^*n^*p_m\cdot\mathcal{S}\,d\mu_2}_{\text{subsidized childcare}}\right)+\underbrace{\mathcal{X}}_{\textbf{other spendings}}
\end{aligned}\tag{3}$$

In (3), government balances budget by equalizing tax revenues from labor income, capital income and consumption to expenditures consisting of pension payments, public education, baby bonus, subsidized childcare and some other fixed spending per capita $\mathcal{X}$. I will fix $\mathcal{X}$ across designs and adjust tax rates to balance government budget when family policies induce general equilibrium changes on both revenues and expenditures.

I define social welfare in the long-run $\mathcal{W}$ as the average value of households at the beginning of their life-cycle, i.e.

$$\mathcal{W}=\int V_2\,d\mu_2 \tag{4}$$

This measures the expected utility of a newborn under the veil of ignorance. When making welfare comparisons, I will convert changes in $\mathcal{W}$ into percentage changes in consumption equivalence using utility function $u(\cdot)$. This assumption on welfare is a pragmatic approach that sidesteps theoretical and philosophical discussions on the definition of welfare with endogenous fertility (see Golosov et al. (2007) and De La Croix and Doepke (2021)). Here, I simply evaluate whether higher fertility, i.e. additional agents, is going to improve the average well-being of all households who are actually alive in the long-run economy.

This measure of long-run welfare $\mathcal{W}$ does not capture the well-being of existing agents or households along the transition path. I will discuss how they are affected by family policies in Section 5.2.

Government policies could be welfare-improving in the economy for two reasons. First, childbearing and child-rearing carries fiscal externalities since parents do not internalize the effects of having an additional child or investing in children's human capital on the tax base and government revenue. In other words, infinitesimal parents take the demographic structure $\{\omega_j\}_{j=0}^{8}$ and tax rates as given. But these objects will change when a mass of parents adjust their decisions on fertility and educational investments. This is due to the lack of property rights of parents on children's future output (Schoonbroodt and Tertilt (2014)). Second, parents face both inter-temporal and intergenerational borrowing constraints due to imperfections in capital markets. Parents will not be able to use children's future income to finance children's education. Parents are also constrained in borrowing from their own future income to finance current expenditures. Government could overcome these inefficiencies by providing resources to parents or expanding public education (Daruich (2018), Abbott et al. (2019)).

2.4 Equilibrium

I use t to denote time. The equilibrium of the economy is defined as a tuple composed of:

- Decision rules $\{c_t^*, a_t'^*, n_t^*, m_t^*, t_{h,t}^*, e_t^*, a_{k,t}^*\}_{t=0}^{\infty}$
- Prices $\{w_t^*, r_t^*\}_{t=0}^{\infty}$
- Government policies $\{T_t(\cdot), \tau_{c,t}, \mathcal{B}_t, \mathcal{S}_t, \mathcal{E}_t, \pi_t\}_{t=0}^{\infty}$
- Distribution of agents $\{\mu_t\}_{t=0}^{\infty}$

such that households maximize utility, prices clear labor and capital markets, government balances budget in each period, and the distribution of agents evolve following the dynamics shaped by decision rules and exogenous labor market shocks z and ability shock for children ϵ.

For given set of government policies, there exists a stationary distribution of agents and that distribution is unique. In particular, if I use $F(h)$ to denote the human capital distribution for parents at $j = 2$, the human capital of their children $F'(h)$ is given by:

$$F'(h) = \frac{1}{N} \iint n^*(x) \mathbb{1}_{h_k^*(x,\epsilon)<h} \, dG(\epsilon) \, dF(x) \tag{5}$$

where G denotes the lognormal distribution of child ability shock ϵ; $h_k^*(x, \epsilon)$ is the human capital of children born into family with parents human capital being x and receive ϵ as ability shock, and N is the aggregate fertility rate in the economy:

$$N = \int n^*(x) \, dF(x) \tag{6}$$

The stationary distribution $F(h)$ is the fixed point that solves (5). The process defined in (5) is a multi-type branching (or Galton-Watson) process. The existence and uniqueness result of $F(h)$ follows the proof presented in Mode (1971) and Chu (1990).

2.5 Discussion of Mechanisms

In this section, I discuss why adding endogenous fertility makes the model predictions on policy effects distinct from structural models with exogenous fertility or design-based studies with existing parent-child pairs.

2.5.1 Effects on Child Quality

Consider the effect of an increase in baby bonus $\mathcal{B}$ on transfers to each child a_k holding education investment e constant. Recall the first-order condition FOC $[a_k]$:

$$\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial a_k} = \lambda_3 \cdot n$$

When fertility is exogenous, i.e. n is fixed, the increase in $\mathcal{B}$ is an *income transfer*, which implies a_k rise unambiguously due to the income effect:

$$\underbrace{\lambda_3 \downarrow}_{\text{income effect on } MU_c} \quad \cdot n \quad \Rightarrow \quad \frac{v(n, \mathbb{E}h_k, a_k \uparrow)}{\partial a_k}$$

When fertility is endogenous, however, the increase in $\mathcal{B}$ is a *price change*. The direction in which a_k changes is ambiguous:

$$\underbrace{\lambda_3 ?}_{\text{change in } MU_c \text{ as } n \uparrow} \quad \cdot \quad \underbrace{n \uparrow}_{\textbf{fertility response}} \quad = \quad \underbrace{\frac{\partial v(n \uparrow, \mathbb{E}h_k, a_k \textbf{?})}{\partial a_k}}_{\text{interaction in preferences}} \tag{7}$$

As fertility rises due to more generous child benefits[14], it affects the first-order condition of child quality in three ways. First, it interacts will child quality in parents preferences $v(\cdot)$. Second, it could offset in income effect since a rise in family size raises in the marginal utility of consumption via change in equivalence scale $q(n)$. Lastly, it directly raises the marginal costs of child quality due to quality/quantity trade-off discussed above.

Considering endogenous fertility allows the possibility that child quality could fall when child benefits become more generous. This stands in contrast with previous literature on family policies that only considers the income effects of transfers.

[14]This will be true as long as child quantity is not an inferior good. See Bleakley and Ferrie (2016) for supporting evidence.

2.5.2 Composition Effects

Fertility responses to family policies generate composition effects on aggregate variables. Families or dynasties with stronger fertility responses will gain representation over time since their children will take a larger fraction of the future population. Since traits, e.g. human capital, are persistent across generations, aggregate variables will gravitate towards the traits of families that increase fertility the most due to policies. For example, Vogl and Freese (2020) argues that these composition effects could shape public opinions on policies regarding conservative family values. De La Croix and Doepke (2003) and Vogl (2016) show that changes in fertility differentials in the economy affect long-run growth via composition effects. This effect will not be present if fertility is exogenous.

2.5.3 General Equilibrium Effect

When fertility is endogenous, family policies change the population growth rate and hence the demographic structure $\{\omega_j\}_{j=0}^{8}$. This change has profound implications on the government budget constraint (3) since it shapes both the revenue side and expenditure side. In particular, a higher fertility affects the old-age dependency ratio $\left(\sum_{j=7}^{8} \omega_j\right) / \left(\sum_{j=2}^{6} \omega_j\right)$ as well as total dependency ratio $\left(\sum_{j=7}^{8} \omega_j + \sum_{j=0}^{1} \omega_j\right) / \left(\sum_{j=2}^{6} \omega_j\right)$. These changes affect the government budget (3), which in turn affects taxation and people's welfare. In fact, allowing for endogenous fertility is a necessary condition for a model to have the ability to study the impacts of family policies on population aging.

2.5.4 Summary

As discussed above, relative to design-based analysis or structural models with exogenous child quantity, explicitly modeling fertility choice of households incorporates quality/quantity trade-off, composition effects, and general equilibrium effects. The key moment in the model that governs the strength of these mechanisms is the magnitude of fertility responses to financial incentives. If households do not respond to family policies by adjusting fertility, than the model predictions would be similar to that in Daruich (2018) and Mullins (2019). A large part of the next two sections of the paper would be devoted to disciplining this elasticity using data from various sources and validating its magnitude using policy variations.

3 Calibration

In this section, I describe the calibration procedure of the model. There are a number of parameters that are picked exogenously. I calibrate 13 parameters inside the model using simulated method of moments by matching the steady-state of baseline model to the United States in 2010.

When the government provides more generous child benefits, households weigh these benefits against the costs of having an additional children which includes trade-offs in child quality. Hence, the magnitude of fertility elasticities is largely determined by (1) costs of children and childcare, (2) child's skill production function, and (3) parents' preferences on child quantity and quality.

3.1 Costs of Children and Childcare

The household equivalence scale $q(n)$ is taken from the OECD standard:

$$q(n) = 1.7 + 0.5 \cdot n$$

where n is the number of children residing with the family.

Recall the childcare requirement constraint, i.e. time costs of child, is given by:

$$n \cdot \chi = \left(t_h^{\upsilon/\iota} + (n \cdot m)^{\upsilon}\right)^{1/\upsilon}$$

I choose $\chi = 0.18$ following estimated by Folbre (2008) (Table 6.2) calculated using American Time Use Survey (ATUS) data. I calibrate the economies of scale in providing childcare at home $\iota = 0.7$ to match the estimates by Folbre (2008) (Table 6.4). The parameter governing the elasticity of substitution between home care and market care υ is calibrate to be 0.5 to match the average expenditure on childcare as a fraction of total family income using data from SIPP. Lastly, I choose the price of full-time market care for child aged 0 to 10, p_m, to be \$6,860 in 2010 following the statistic reported by the National Association of Child Care Resource & Referral Agencies.

3.2 Child's Human Capital Production

I parameterize the child human capital production function as

$$h_k = G(h, \mathcal{E}, e, \epsilon) = \underbrace{Z}_{\text{scalar}} \cdot \underbrace{\epsilon}_{\text{shock}} \cdot \underbrace{h^{\rho}}_{\text{spillover}} \cdot \left(\underbrace{\mathcal{E}^{\xi}}_{\text{public education}} + \underbrace{e^{\xi}}_{\text{private input}} \right)^{\kappa/\xi} \tag{8}$$

$$\log(\epsilon) \sim \mathcal{N}(-\frac{\sigma_\epsilon^2}{2}, \sigma_\epsilon^2)$$

Parameter Z is a scaling parameter that governs the overall scale of the economy. It also enables us to normalize the total factor productivity A to be one. I choose $Z = 2.57$ so that the median income in the model is one. Parameter σ_ϵ governs the dispersion of idiosyncratic shock to children's ability. I pick $\sigma_\epsilon = 0.45$ to match the initial dispersion of earnings for households in period 2 estimated by Huggett et al. (2011) using PSID data. I calibrate $\rho = 0.28$ to match the rank-rank intergenerational mobility estimated by Chetty et al. (2014) (see Figure 2).

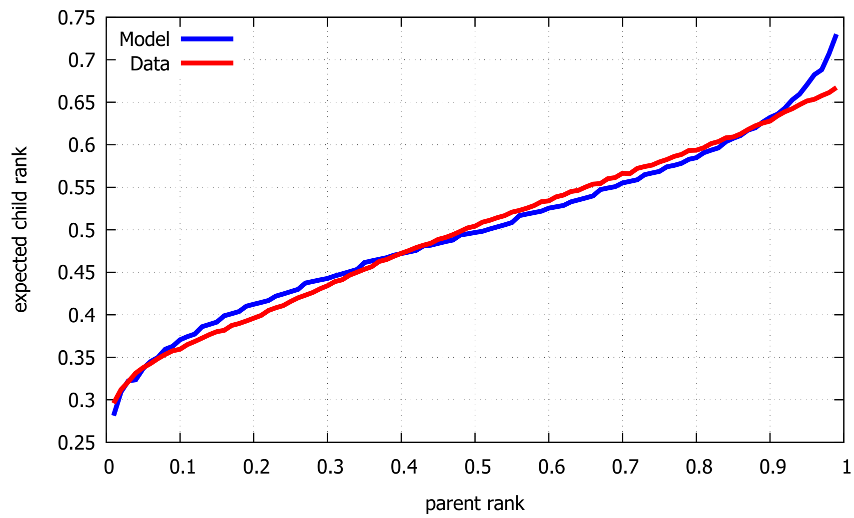


Figure 2: Mobility: Model vs Data

I use ξ to parameterize the elasticity of substitution between public and private education inputs and calibrate $\xi = 0.9$ to match the relationship between educational spending and household income using data from Consumer Expenditure Survey (CEX). The magnitude of public education $\mathcal{E}$ is chosen to match \$12,000 annual expenditure per student reported by National Center of Education Statistics (NCES).

The last parameter in child human capital production function that needs to be calibrated is κ which governs the productivity of educational investments. I use RCT evidence by García et al. (2020) to inform the value of κ. García et al. (2020) evaluates two early childhood programs (ABC and CARE) in the 1970s. They costs about \$13,500 per year for give years, which means the total program cost is around \$67,500 per child. The project follows the treated children into adulthood and observes their education and (part of life-cycle) incomes. The papers estimates that for every dollar invested, children's lifetime labor income increases by 1.3 dollars in net present value. I apply a similar policy in the model by expanding existing public education $\mathcal{E}$ by \$67,500. I apply it at a small scale so that prices and taxes remain unchanged. Like the original program, I target it at children with parents at 10th percentile of earnings. By comparing treatment and controls groups in the model, I arrive at the estimate of $\kappa = 0.17$.

3.3 Preferences over Quantity and Quality

I assume that parents' preference over child quantity and quality is given by:

$$v(n, \mathbb{E}h_k, a_k) = \underbrace{\Psi(n)}_{\text{child discounting}} \cdot (\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k)) \tag{9}$$

$$\Psi(n) = 1 - \exp(-\psi n) \tag{10}$$

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0,1), \quad x \in \{\mathbb{E}h_k, a_k, c\} \tag{11}$$

where parents value child quality weighted by child discounting $\Psi(n)$. The utility function of child quality and consumption is governed by γ which also known as the elasticity of intergenerational substitution (EGS) (c.f. Cordoba et al. (2016) and Carlos Córdoba and Ripoll (2019)). I use $\theta \cdot u(\mathbb{E}(h_k)) + \nu \cdot u(a_k)$ as a first-order approximation to the general class of preferences over child quality and transfers.[15]

Parameters $\theta = 2.1$ and $\nu = 0.36$ are calibrated to match the average human capital investment as a share of income and average inter-vivos transfers of \$60,000 in 2010 dollars calculated by Daruich (2018) using PSID data. I calibrate $\psi = 1.93$ to match the average fertility calculated using CPS June Fertility Supplement data from 2008 to 2014.[16]

Conditional on $\{\theta, \nu, \psi\}$ and other parameters that govern costs of child and children's human capital production function, parameter γ is an important parameter governing the fertility elasticity where higher γ implies smaller fertility responses to financial incentives. The illustrative model in Appendix C.2 shows that in comparative statics, I have the following relationship:

$$\frac{\Delta \Psi'(n)}{\Delta \chi} \propto 1 - \gamma$$

On the other hand, parameter γ is identified by the fertility-income profile in the data. The logic of this identification follows Cordoba et al. (2016): controlling for other parameters, higher γ implies a larger marginal rate of substitution of quantity for quality and a flatter fertility-income profile. Numerical illustration of the identification is displayed in Figure 3. I choose $\gamma = 0.61$ such that the model best fits the observed relationship between fertility and income.

[15]In Section 6, I show that our results are robust to (1) separable preferences (De La Croix and Doepke (2003)), (2) quality and quantity being substitutes (Jones and Schoonbroodt (2010)), and (3) dynastic altruism (Daruich and Kozlowski (2020)).

[16]I measure the completed fertility rate using the number of live births ever had for women aged 40 to 55.

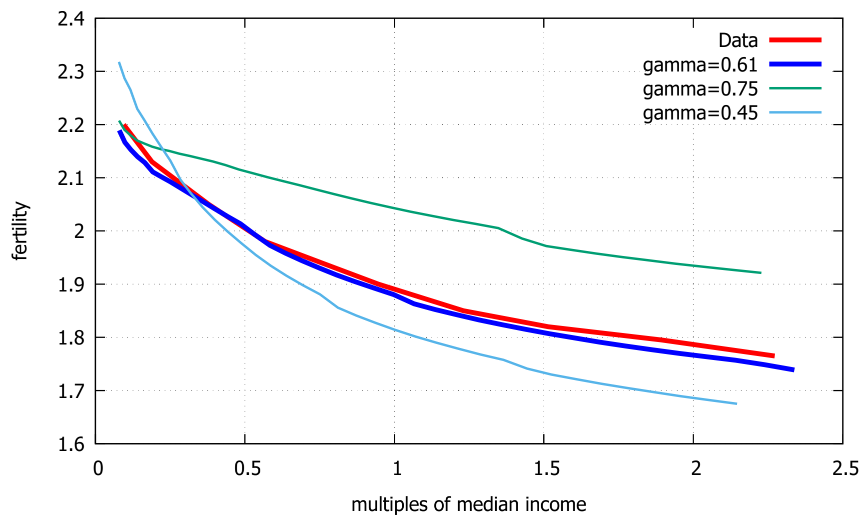


Figure 3: Identification of γ

3.4 Other Parameters

I assume the human capital of adults evolve with age according to:

$$h' = L(h, t_w, z') = \exp(z')(h + \zeta(h \cdot t_w))^{\eta} \tag{12}$$

$$\log(z) \sim \mathcal{N}(\mu_z, \sigma_z)$$

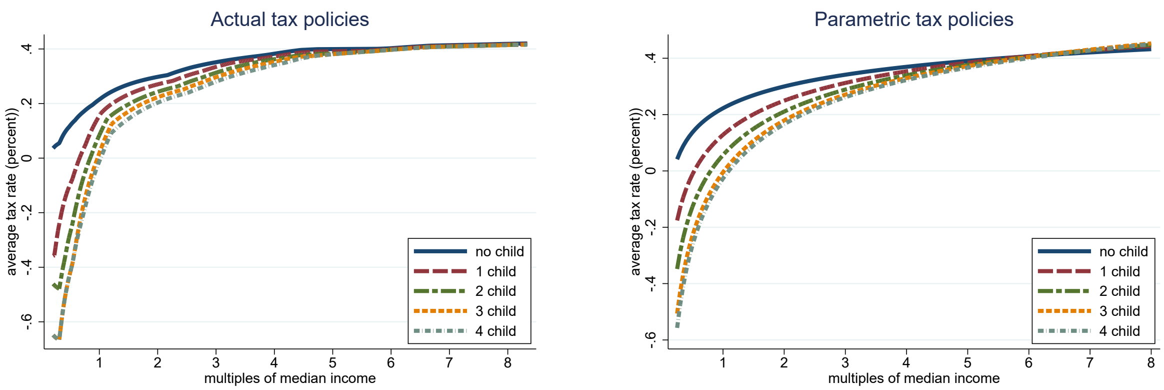
I calibrate $\eta = 0.61$, $\zeta = 0.72$ and $\sigma_z = 0.42$ to match life-cycle profile of earnings and gini coefficient of earnings calculated by Huggett et al. (2011). I choose $\mu_z = -0.23$ so that skill depreciation is 2% per year.

Following Heathcote et al. (2017), I parameterize income taxes as:

$$\mathcal{T}(y, a, n) = y \cdot (1 - \tau_y^n y^{-\lambda_y^n}) + \tau_a r a \tag{13}$$

where $\{\tau_y^n, \lambda_y^n\}$ parameterize the level and progressitivity of taxes depending on the number of children residing in the household while τ_a denotes the linear capital income taxes. I obtain $\{\tau_y^n, \lambda_y^n\}$ using simulated data from TAXSIM provided by the NBER (see Figure 4). Having an additional child significantly reduces the tax burden faced by households through various programs such as Earned Income Tax Credit (EITC) and Child Tax Credits (CTC). The tax system is progressive, with lower-income households receiving subsidies rather than paying taxes. As household income grows, labor income taxes converge to 40%.

Following McDaniel (2007) and Daruich and Fernández (2020), I choose capital income tax $\tau_a = 0.27$ and consumption taxes $\tau_c = 0.07$. Pension replacement rate π is set to be 40%. In



(a) Actual Taxes

(b) Parametric Taxes

Figure 4: Labor Income Taxes

the production function of the firms, I choose the capital share α to be 0.33 following the standard literature and set the capital depreciation rate δ_k at 4% per year. Lastly, I choose the annual discount rate $\beta = 0.98$ and calculate age-specific mortality rate $\{\delta_j\}_{j=0}^{8}$ using the actuarial life table from Social Security.

To summarize, the calibrated parameters are presented in Table 1.

4 Validation

In this section, I perform an external validation of the fertility responses by households to financial incentives in the model. This is an important exercise before I use the model to compute policy counterfactuals since it lends extra credibility to the quantitative magnitude of fertility elasticities, which is the crucial moment governing quality/quantity trade-off, composition effects and general equilibrium effects due to endogenous fertility.

I exploit empirical evidence from Alaska Permanent Fund Dividends (APFD). Briefly speaking, the dividend is officially established in 1982 after discovery of the petroleum and increased state revenues. It gives equal transfers to all residents regardless of income, employment or age ever since. In particular, the program allows a parent, guardian, or other authorized representative to claim a dividend on behalf of a child while Alaska law currently imposes no requirements whatsoever on how parents use a child's dividend. As a result, the policy has pronatal effects even though it is not explicitly labeled as a family policy.

I argue that APFD is an ideal policy variation to validate fertility elasticities for three reasons. First, compared with other family policies which mostly took place in European countries, the cultural and institutional background of Alaska is much more similar to the overall U.S. system that I calibrate the model to match. Second, compared with other family policies which is usually

	Interpretation	Value	Source		Interpretation	Value	Source
	Preferences				Child human capital production		
β	discount rate	0.98^{10}	standard	Z	normalizing scalar	2.57	median income =1
γ	elasticity of substitution	0.61	CPS	σ_ϵ	ability shock dispersion	0.45	PSID
ψ	fertility preference	1.93	CPS	ρ	intergenerational spillover	0.28	Chetty et al. (2014)
θ	quality preference	2.1	PSID	ξ	substitution of education	0.9	ATUS
ν	transfer preference	0.36	PSID	$\mathcal{E}$	public education	0.15	NCES
				κ	input productivity	0.17	García et al. (2020)
	Childcare arrangement				Adult human capital evolution		
χ	childcare cost	0.18	Folbre (2008)	η	learning curvature	0.61	PSID
ι	economies of scale at home	0.7	Folbre (2008)	ζ	learning level	0.72	PSID
υ	substitutability of care	0.5	SIPP	μ_z	skill depreciation	-0.23	PSID
p_m	price of full-time care	0.13	NACCRRA	σ_z	shock dispersion	0.42	PSID
	Taxes and pension				Firm production function		
τ_y^n, λ_y^n	tax levels and progressitivity	misc.	TAXSIM	A	total factor productivity	1	normalization
τ_c	consumption tax	0.07	McDaniel (2007)	α	capital share	0.33	standard
τ_a	capital income tax	0.27	McDaniel (2007)	δ_k	capital depreciation	0.04^{10}	standard
π	pension replacement rate	0.40	OECD Database				

Table 1: Calibrated Parameters

less than a few thousand dollars in net present value per child, the net present value that parents could receive and use with an additional child under APFD is almost $20,000.[17] With such a large scale, it is more likely that policies would have meaningful and observable impacts on people's behavior especially when it comes to the important and irreversible decision of having a child. Lastly, compared with other policies which are typically means-tested or depends on birth order, the APFD has a clearer implementation as a small-scale universal basic income (UBI).

I implement the exact same policy as APFD by transferring $1,500 to every household member including both parents and children. Parents will receive this transfer every period until they die and they are also entitled to receive additional transfers when children reside with them prior to becoming independent at age 20. I normalize the amount of transfer by median income in Alaska relative to that in the U.S. and conduct the policy experiment in partial equilibrium without changing prices.

The model predicts that completed fertility rate, i.e. the total number of children that parents end up having, increase 4.2% compared with the scenario without AFPD. Using synthetic control methods, Yonzan et al. (2020) concludes that total fertility rate increases by 13.1% due to AFPD. Note that effects on total fertility rate is typically larger than that on completed fertility rate since the former included both quantum effect (i.e. effects on completed fertility rate) and tempo effect (i.e. change in timing of birth). After reviewing a large set of empirical papers, Stone (2017) arrives at a crude estimate that roughly one-third of the effects on total fertility rate could be attributed to changes in completed effect. Hence if I multiply 13.1% by 1/3, the fertility elasticities in the model is similar to that found in the empirical evaluation of AFPD.

Since APFD was enacted almost 40 years ago, it is possible to explore direct effects on completed fertility rate without relying on changes in total fertility rate. Figure 5 plots the time series of completed fertility rate (i.e. total number of live births ever had by women aged 40-55) for Alaska and the rest of U.S. from 1982 to 2018 using CPS June Fertility Supplement data. As can be seen, the trend of Alaska is indistinguishable from the rest of the U.S. prior to 1996. After 16 years of the APFD implementation, however, completed fertility rate in Alaska is persistently higher than that in the rest of the U.S. [18] The gap is around 0.15 children per women after 1998, which makes the model prediction of 0.1 children per women slightly conservative.

The model also predicts heterogeneous responses across households. Since the parents are

[17] I calculated the average payment to be around $1,500 per year. Since the amount of the dividend is tied to the performance of the fund, which in turn depends largely on overall stock market performance, one might worry about how would the uncertainty about future payment amount affect people's responses in fertility. Given that given birth to a child is an irreversible decision, I argue that larger a mean-preserving spread of the dividend payment would reduce people's responses in fertility. Therefore, since the model generates fertility elasticities that are consistent with people's choices under uncertainty, our results on child benefits (without uncertainty) is likely going to be conservative.

[18] Unfortunately, the gap is only statistically significant at 5% for some of the years because the CPS sample of Alaskan women is small, leading to wide confidence intervals.

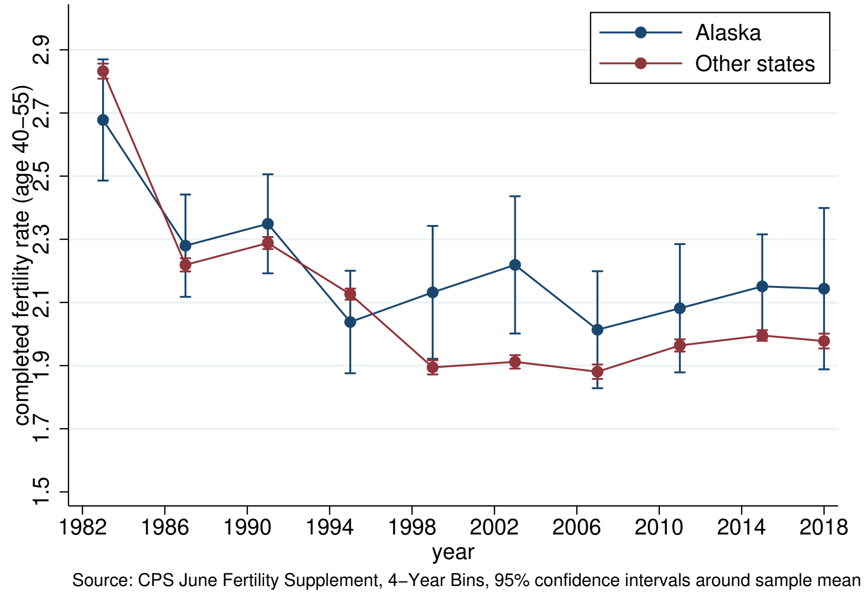


Figure 5: AFPD Effects on Completed Fertility Rate

entitled to claim children's dividends, and these dividends are the uniform in size, the model predicts that households with lower human capital have larger increase in fertility (e.g. see results on baby bonus in Section 5.1). This is in line with the finding by Cowan and Douds (2021) where they show larger fertility increases among Alaska natives and women without high school degree. This difference is also visible when inspecting the time series of completed fertility rate by education in Alaska versus the rest of the U.S. in Figure 6.

5 Counterfactuals

In this section, I use the model to evaluate policies of different sizes in the general equilibrium where prices and distribution of the population adjust. I focus on baby bonus since it has been widely adopted in the developed world (see Appendix A for more details) and is similar in structure to an expansion of refundable child tax credits (CTC).[19]. I will first discuss its long-run implications in Section 5.1 where I assume that the government is balancing the budget by adjusting consumption taxes. The outcome variables of interest include but not limited to fertility, human capital, average income, intergenerational mobility (measured by IGE^{-1}) and social welfare. In Section 5.2, I study

[19]There are two small differences between baby bonus and CTC. First, unlike CTC which has a phase-out criteria, baby bonus is usually not means-tested. I focus on the uniform case for simplicity and better exposition of heterogeneous responses. Second, baby bonus is a lump-sum transfer when the child is born while CTC is an annual transfer to parents when the child is below age 18. With borrowing constraints, low-income parents should strictly prefer baby bonus to CTC of the same net present value since they can replicate the latter through saving.

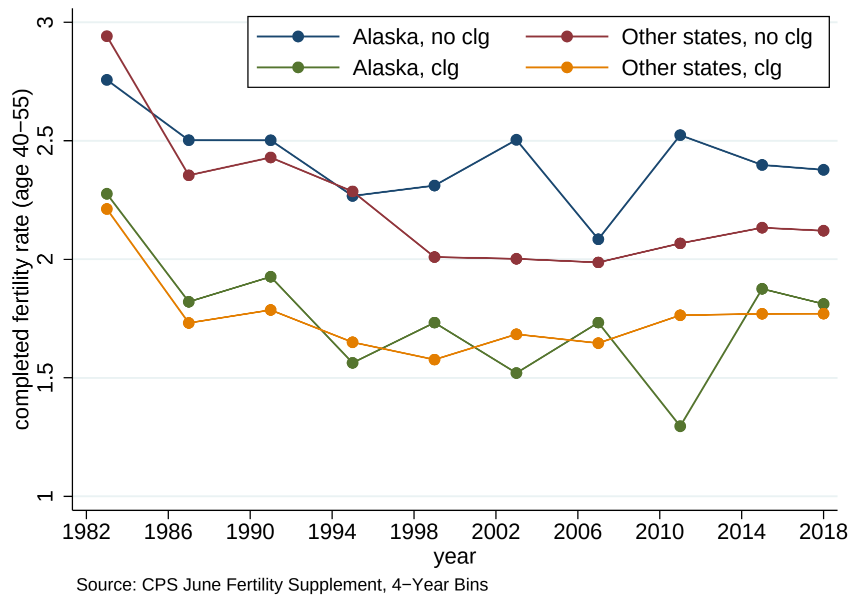


Figure 6: AFPD Effects on Completed Fertility Rate by Education

the transition path leading to that long-run outcome. Lastly in Section 5.3, I compare baby bonus with other policies including subsidized childcare $\mathcal{S}$ and public education expenditures $\mathcal{E}$.

5.1 Baby Bonus: Long-run Implications

In this section, I evaluate baby bonus of different size ranging from \$0 to \$50,000. I compute the long-run macroeconomic implications of these policies by comparing the long-run steady-state of the economy with the benchmark economy where baby bonus is zero.

Figure 7 and 8 shows the fertility effects of baby bonus. To reach replacement fertility level (2.1 children per family on average), the model predicts that it would require a baby bonus of \$31,000 which will be around 1.7% of GDP in the new steady-state. This amounts is similar to the increase in CTC from 2010 to 2021 in net present value and offsets 19% of the average cost of raising one child (USDA estimate). These results confirm the common perception by demographers that using financial incentives to raise fertility is not “cheap". Under realistic magnitude of fertility elasticities and reasonable scale of policies, the effects of financial incentives provided by the government are small relative to historical changes in preferences, social norms, contraceptive technologies, and perhaps most important of all, changes in returns to human capital. These results, however, suggest that using transfers to prevent large crashes in fertility, or even to raise it, is feasible in practice.

Figure 9 shows the quality/quantity trade-off channel discussed in Section 2.5.1. Baby bonus reduces the price of child quantity and parents respond by having more children. The increase quantity, in turn, raises the marginal cost of education as well as marginal utility of consumption

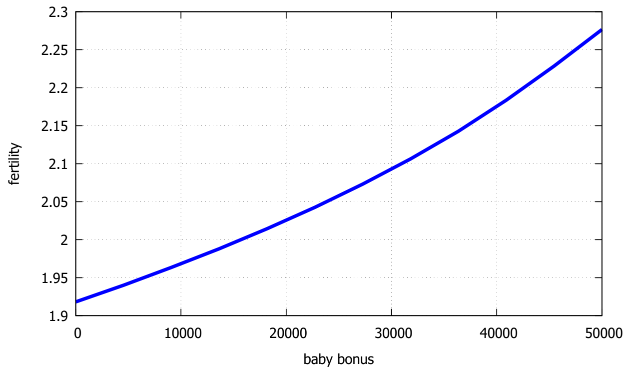


Figure 7: Effects on Fertility

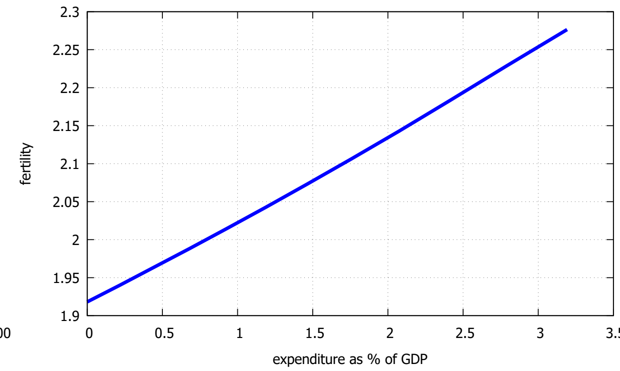


Figure 8: Expenditure Share as % of GDP

due to larger family size. As a result, even when quality and quantity are complements in parents' preferences (see Section 3.3), parents optimally reduce the amount invested in children's education when fertility changes are large.[20] When baby bonus is \$31,000, private investment per child is reduced by 8% on average.

Figure 9 and 10 shows that agents with lower human capital have stronger fertility responses and larger drops in private education investments in response to baby bonus. This is intuitive since the baby bonus of a given size accounts for a larger fraction of household income among low-income families. Due to intergenerational transmission of human capital in skill formation, heterogeneous response in fertility results in composition effects on aggregate variables. Coupled with reductions in private investment, Figure 11 shows that average human capital falls by 1.5% under a \$31,000 baby bonus. Since the reduction of education investment is larger among low-income households, baby bonus reduces social mobility by 1.6% rather than boosting it.

Figure 13 shows that despite lower average human capital and social mobility, per capita output rises initially before peaking at $\mathcal{B} = \$23,000$ dollars. On one hand, human capital distribution worsens as baby bonus increases. On the other hand, rising fertility shifts the composition of adults away from retired households, who are less productive.

As the size of baby bonus increases in size, Figure 15 shows that the government is able to reduce consumption tax for smaller baby bonus but needs to raise taxes for larger ones. This is a result of several forces shaping the government budget constraint in (3). First of all, tax revenues is affected by the change in human capital distribution. Second, as the share of retired households decrease, the government has less burden from pension payment. Third, as the share of children increases, the government needs to spend more on public education. When the government uses \$31,000 to raise the aggregate fertility to replacement level, consumption tax could be slashed by 0.8%.

[20]The initial mild increase in investment, especially among parents with high human capital, are due to complementarity between quantity and quality and income effects on child human capital.

Figure 16 plots the changes in welfare from the baseline economy expressed in change in consumption equivalence. Due to increased fertility and reduced consumption taxes, social welfare increases by 2.2% in the long-run economy.

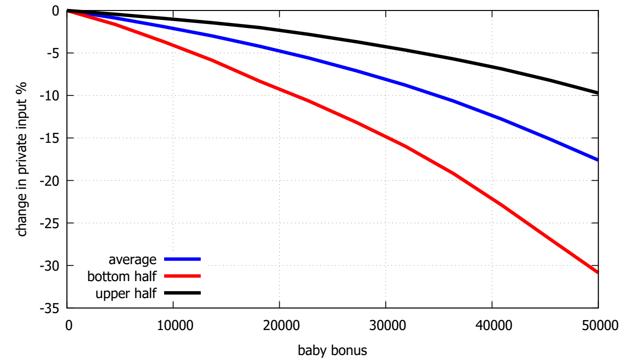


Figure 9: Average Private Investment

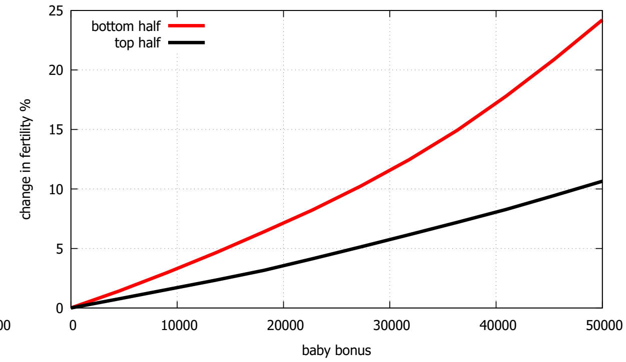


Figure 10: Heterogeneous Fertility Response

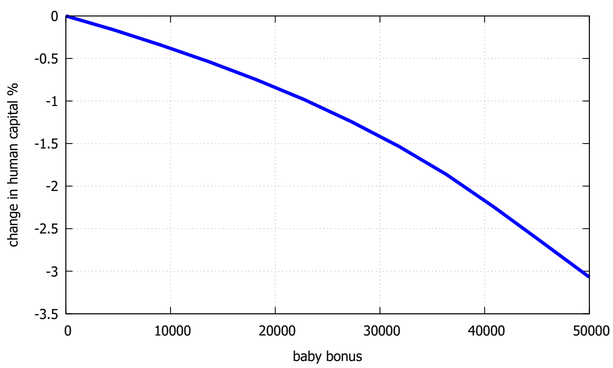


Figure 11: Average Human Capital

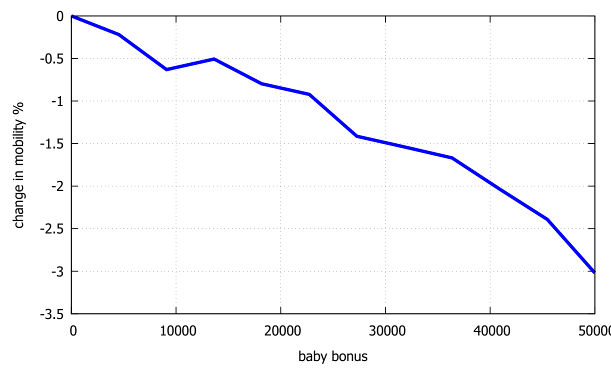


Figure 12: Intergenerational Mobility

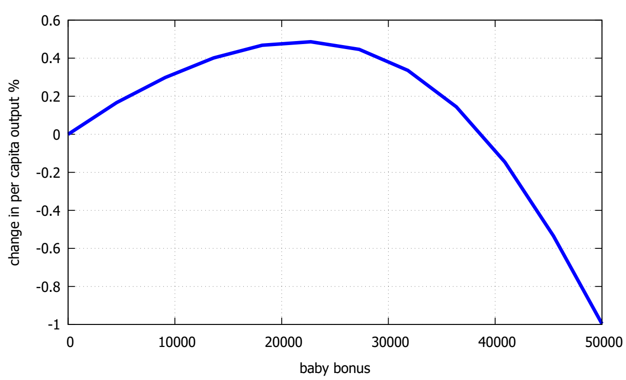


Figure 13: Per Capita Output

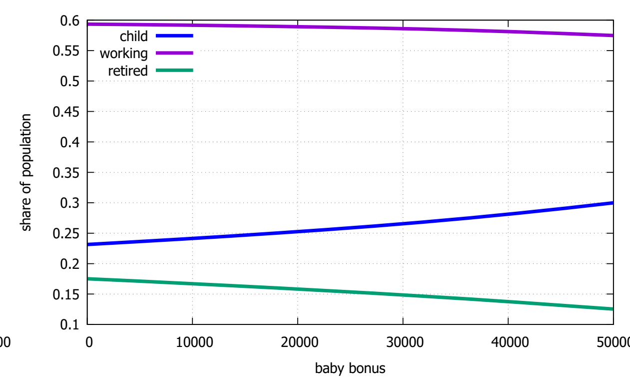


Figure 14: Change in Demographic Structure

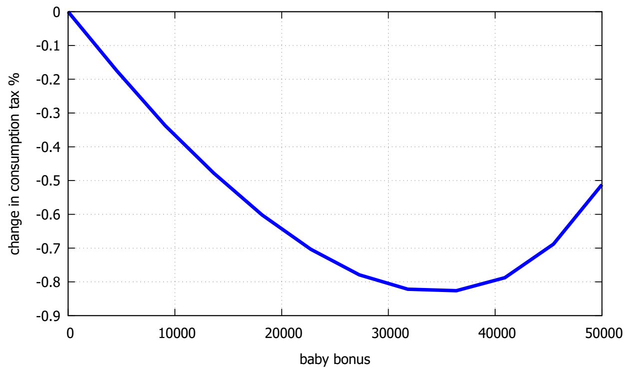


Figure 15: Change in Consumption Tax

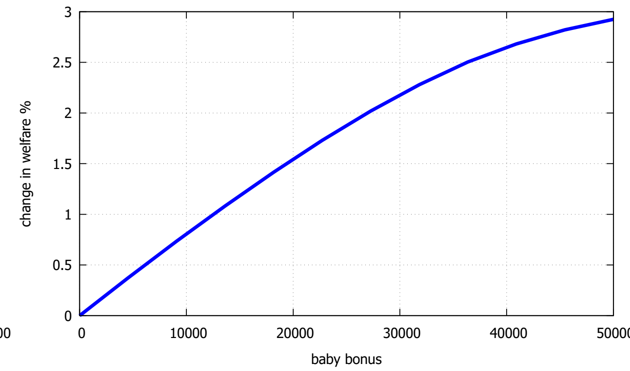


Figure 16: Change in Welfare

5.2 Baby Bonus: Transition Path

In this section, I show the transition path of aggregate variables under a \$31,000 baby bonus where the policy is enacted unexpectedly at period $t = 1$ and persists afterwards. It is assumed that the government changes consumption taxes along the transition path to balance government budget period by period where each period in the model represents a decade.

Figure 18 shows that the policy effects on fertility is immediate and persistent with a small overshooting from period 4 to 10. Figure 18 indicates that the old-age dependency ratio starts to have visible decline in period 6 since the share of working-age population and retired households are predetermined by past fertility rates. The total dependency ratio, however, increases immediately since there is a large increase in the number of children due to the baby bonus.

As a result of this increase in total dependency ratio, the government needs to raise consumption taxes in period 1 to 9 before being able to reduce it in the long-run. Hence, welfare changes of newborn agents are positive, but significantly smaller than that in the long-run. Lastly, since most agents in the current economy do not benefit from the baby bonus but are required to pay higher consumption taxes, they are worse off under the policy.

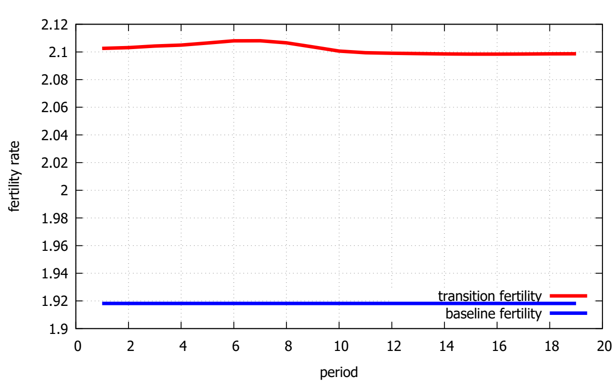


Figure 17: Fertility Rate

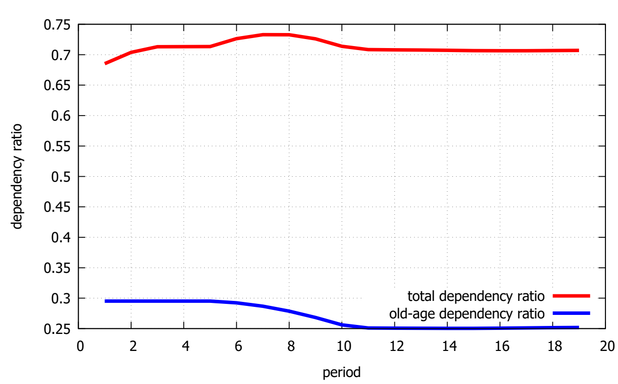


Figure 18: Change in Dependency Ratio

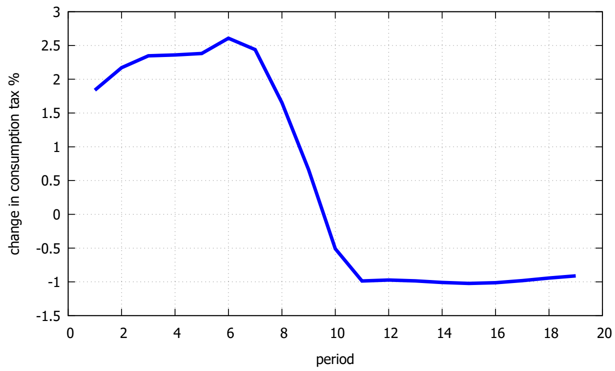


Figure 19: Chang in Consumption Tax

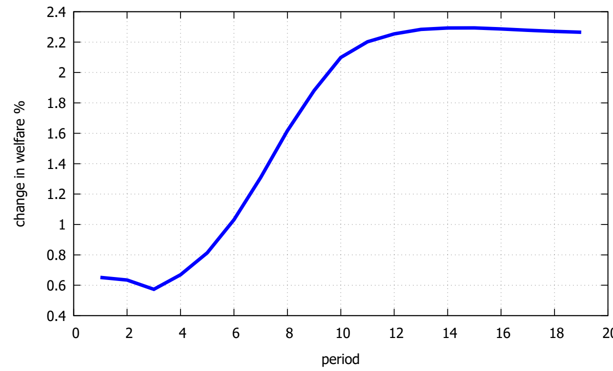


Figure 20: Change in Welfare

5.3 Comparing Policies

In this section, I briefly discuss the policy comparison results. For more detailed results of subsidized childcare and public education expenditures, see Appendix C.3 and C.4.

Compared with baby bonus, subsidized childcare is 50% less cost-effective in raising fertility. It encourages the combination of childbearing and labor force participation by substituting at-home care by market care. This in turn, fosters adults human capital accumulation via on-the-job learning. As a result, it leads to higher gains in per capita output and welfare in the long-run. The responses in fertility and education investments are similar across education levels of the parents. Intergenerational mobility is not significantly changed under different levels of childcare subsidies.

Compared with baby bonus, the fertility effects of public education expenditures is smaller by an order of the magnitude. The government should not expect to use education expenditures as pronatal policy instruments. Education expenditures also has strong crowding-out effects of private investments since they are substitutes in the child human capital production function. Expanding public education, however, is most effective in raising social mobility and improving children's outcomes. A $60,000 expansion of public education (in net present value) raises intergenerational mobility by 4.6% and long-run welfare by 2.4%.

6 Robustness and Discussions

6.1 Alternative Preferences

There are alternative ways to model parents' preferences on child quantity and quality. I argue that the modeling choice in Section 3.3 makes our results conservative.

A commonly seen assumption is separable preferences between quality and quantity (see De La Croix and Doepke (2003), Bar et al. (2018),Vogl (2016)) where

$$v(n, \mathbb{E}h_k, a_k) = \log(n) + \theta \log(\mathbb{E}h_k) + \nu \log(a_k)$$

Compared with our benchmark case where quality and quantity are complements, parents will raise quantity and reduce quality more strongly when costs of child is lower since the mechanism shown in the left-hand-side of Equation 7 is not present under separable preferences. I show this argument with closed-form solutions in Appendix C.1.

Another way of modeling parents preferences are dynastic altruism (see Cordoba et al. (2016), Daruich and Kozlowski (2020)) where

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n) V_2(\mathbb{E}h_k, a_k)$$

This formulation is quite similar to the one used in this paper except that utilities from child quality is pinned down recursively. Dynastic altruism is appealing aesthetically but face unresolved challenges in fitting observed transfers between parents and children (see Altonji et al. (1997) and Barczyk and Kredler (2020)). Paternalistic preferences, i.e. direct preference over children's outcomes, are often added to improve data fit (e.g. Abbott et al. (2019)). I argue that our results will be stronger under dynastic altruism since parents endogenize the fact that returns to child quality investments become lower as the government adopts policies such as their children would receive more generous family benefits. As a result, parents would further increase child quantity and reduce quality investments. This mechanism is based on the assumption that parents only care about children's utility, but place no weights on child quality directly. Similar arguments are found in Daruich and Fernández (2020) where they argue universal basic income reduces human capital of future generations.

6.2 Endogenous Timing of Childbirth

In the baseline model, I abstract away from birth timing by assuming that parents only make fertility choices from age 20-30. In reality, however, parents make decisions on when to give birth to children and family policies could affect that decision, i.e. tempo effects.[21]

I argue that adding endogenous timing will make our baseline results larger. As the model matches effects on completed fertility rate (quantum effects) in the validation exercise, policy effects

[21] An unresolved question in the literature is whether changes in timing is due to the relaxation of parents' constrains (e.g. down payments to buy a larger house) or the lack of commitment to policies by the government. In reality, governments often renege on family policies or reduces the scale of payments as fiscal condition changes. One example is the Australian baby bonus which got significantly downsized in 2014. This uncertainty creates an incentive for parents to shift timing when benefits are still in effect.

on total fertility rate (quantum+tempo) are going to be larger than just the quantum effect. Suppose that in addition to increasing the number of children, some parents shift birth from 30s to their 20s in response to a baby bonus. This shift in birth timing is likely going to be detrimental to children's human capital for two reasons. First, parents' human capital grows over time in the model, so early birth reduces the spillover that the children could receive. Second, family policies of realistic size are not be enough to compensate for the income differences between early and late birth, hence on average these children are induced to be born into households with less resources. This would also reduce children's human capital. As a result, family policies will have a larger observed fertility impact on the economy, but the outcome of children will be worse.

6.3 Alternative Policies

Even though I only use the model to evaluate baby bonus, subsidized childcare and education which are financed through consumption taxes in the paper, the framework can be applied to analyzing a range of family and redistributive policies. For instance, instead of changing consumption tax to balance the budget, it is straightforward to calculate scenarios where additional expenditures are financed via labor taxes τ_y^n or capital income taxes τ_a. The government could also increase income tax progressitivity by raising λ_y^n.

There are also alternative policies that would manifest themselves in the model as changes in parameters. For instance, government propagandas that promotes having child for the country, i.e. in the case of Australian baby bonus, or policies that promote gender equality in childcare, i.e. Doepke and Kindermann (2019), could lead to changes in preferences for child quantity ψ. Education policies that equalize regional education resources could lead to changes in intergenerational spillovers ρ. It is interesting and important for future studies to endogeneize these channels, compare their cost-effectiveness with more traditional fiscal policies studied in this paper, and investigate how different policies interact.

6.4 Optimal Policy Design

It is natural to ask what the optimal policy is in the model. The result from Figure 16 points to an "optimal baby bonus" under given policy instrument and a pragmatic definition of welfare - average utility of new-born under the veil of ignorance in the long-run. But there are deeper questions that consider the planners' problem, i.e. what is the optimal number of children that the social planner would like to have, and what is the optimal distribution of children across heterogeneous parents?

The biggest challenge to these questions is the definition of welfare criteria under endogenous fertility. Unlike standard comparisons between allocations where the set of agents are fixed, there will be agents who are born in one economy but not the other when fertility is endogenous. As a result, standard Pareto principle can not be used to conduct welfare analysis in this context. The field of population ethics is devoted to understanding and resolving this question. For instance, Parfit (1984) derives the famous “repugnant conclusion" under a set of axioms. Golosov et al. (2007) proposed two criteria called $\mathcal{A}-$efficiency and $\mathcal{P}-$efficiency. De La Croix and Doepke (2021) considers the optimal welfare from a soul’s perspective. The debate on what is the most appropriate welfare measure to use is far from being settled.

Even though this paper does not explore optimal policies, it can still contribute to the discussion in two ways. First, once a certain welfare metric and the set of policy instrument are chosen, the model could be applied to find the optimal policy combination numerically. Second, the paper highlights some major trade-offs in policy design. On one hand, subsidizing fertility among parents with low human capital leads to larger fertility changes per dollar spent on family policies, which could result in larger welfare improvement due to general equilibrium effects on age structure. On the other hand, subsidizing fertility among parents with high human capital could utilize the intergenerational transmission of human capital, which could raise welfare due to the composition effects. At the aggregate level, the government balances population growth rate and average human capital due to quality-quantity trade-off. The model provides a unified framework where all three effects are present.

7 Conclusion

Facing the “global fertility crash" (Tartar et al. (2019)), family policies are at the forefront of public attention and policy debates. Evidence on the effects of transfers to parents on existing children’s outcomes also lead policy makers and economists alike to conclude that family policies are excellent instruments to “lift children out of poverty today and help them tomorrow" (Schanzenbach et al. (2021)).

In this paper, I study the aggregate impacts of family policies in a quantitative general equilibrium overlapping-generations model with heterogeneous agents, endogenous fertility, child human capital formation, and inter-vivos transfers. Relative to previous studies with exogenous number of children, I argue that explicitly modeling endogenous fertility choice is crucial for analyzing the macroeconomic implications of family policies because it speaks to (1) quality/quantity trade-off a lá Becker and Lewis (1973), (2) composition effects due to changing representation, and (3) general equilibrium effects through population growth and demographic structure.

I calibrate the model to match salient features of the data including fertility-income relationship, human capital investments, childcare usage, and institutional details on child benefits through the tax system. In the external validation exercise, I show that the model-generated fertility elasticities are consistent with empirical evidence from Alaska Permanent Fund Dividend (APFD).

I find that when family policies are designed such that parents are awarded based on child quantity, they respond by having more children but optimally reduce child quality investments due to quality/quantity trade-off. Compared with education subsidies, family policies are not ideal instruments if the policy goal is to raise children's human capital or to boost social mobility. In fact, average human capital and intergenerational mobility both fall in the long-run after implementation of large-scale family policies due to (heterogeneous) reductions in child quality investments and composition effects. The pronatal effect of family policies, however, could lead to long-run welfare gains because population growth reduces old-age dependency ratio. This allows the government to reduce tax rates without violating the fiscal constraint in the general equilibrium.

I also show that long-run gains in welfare require a prolonged transition path where the government needs to raise consumption taxes to finance higher child-related expenditures. This finding highlights the implicit costs of family policies in transition. They are typically forgotten since most attention in policy discussions are paid to reductions in the pension liabilities in the long-run.

I conclude by discussing several avenues for future research. In this paper, I focus on the positive analysis of family policies since the normative principles with endogenous fertility is still hotly debated. It would be interesting for future research to adopt some leading criteria, e.g. Golosov et al. (2007), and conduct optimal policy design. Another fruitful area of research is to link family policies with population externalities such as pollution (e.g. Bohn and Stuart (2015)) and ideas creation (Jones (2020)) to study long-run impacts of family policies on the dynamics of fertility, environment and economic growth.

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A Empirical Evaluations of Family Policies

To be completed

B Calibration Details

C Illustrative Models

C.1 Simple Model of Quality/Quantity Trade-off

In this section, I show the quality/quantity trade-off mechanism with closed-form solution in a model that is adapted from De La Croix and Doepke (2003).

Consider parents solving the problem:

$$\max \quad \log(c) + \theta \log(n \cdot h_k)$$

subject to

$$c + n \cdot e = 1 - n \cdot \chi$$

$$h_k = e^{\gamma}, \quad \gamma \in (0, 1)$$

where h_k is human capital of children, e is private investment,[22] χ is fixed costs per child, and n is the number of children.

When fertility is exogenous, i.e. n is given, parents maximize over c and e. The optimal investment is given by:

$$e^* = \frac{\theta\gamma}{1+\theta\gamma} \cdot \frac{1 - n\chi}{n}$$

When the cost of child χ decreases, e^* increases due to an income effect.

When fertility is endogenous, parents maximize over c, e as well as n. The optimal fertility and investment are given by:

$$n^* = \frac{1}{\chi} \cdot \frac{\theta(1-\gamma)}{1+\theta}, \qquad e^* = \frac{\gamma\chi}{1-\gamma}$$

When the cost of child χ decreases, n^* increases while e^* decreases. The intuition for this result is simple. Parents increase fertility due to a price effect. This increase in n^*, in turn, raises the

[22] In De La Croix and Doepke (2003), children receives human capital endowments. This generates non-homotheticity over child quality for parents with different human capital and leads to a negative fertility-income relationship. I abstract away from steady-state heterogeneity across households in this simple model for clearer exposition of intuition in comparative statics, but all arguments carry through when human capital endowments are allowed.

shadow price of investment e^* due to interaction of n and e in the budget constraint. As a result, optimal investment e^* falls.

C.2 Simple Model of Fertility Elasticity

In this section, I build a simple model to illustrate the relationship between parameter γ, also known as elasticity of intergenerational substitution (EGS), and the magnitude of fertility response to family policies.

Consider a simplified problem for parents with very low income so that child quality is generated by public investments alone:

$$\max_{c,n} \quad u(c) + \Psi(n)u(\mathcal{E})$$

$$c + n \cdot \chi = 1$$

First-order condition for n:

$$\underbrace{\Psi'(n) \cdot u(\mathcal{E})}_{\text{MB of } n} = \underbrace{\lambda \cdot \chi}_{\text{MC of } n}$$

Plug in $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I have

$$\Psi'(n) = (1-\gamma) \cdot \chi \cdot \frac{\lambda}{\mathcal{E}^{1-\gamma}} \Longrightarrow \Delta\Psi'(n) \propto (1-\gamma) \cdot \Delta\chi$$

As can be seen, conditional on other parameters, higher γ implies smaller response in n for given changes in costs of child χ.

C.3 Public Childcare

In the second set of policy counterfactuals, I evaluate childcare subsidies where the government refunds $\mathcal{S} \in [0, 0.9]$ fraction of expenditures on market childcare services for all families.[23]

I find that subsidized childcare raises fertility, but at a much higher cost than baby bonus. The government needs to refund more than 80% of childcare expenses to raise the aggregate fertility rate to 2.1. This would cost almost 3.2% of aggregate GDP in the long-run steady-state economy, which means the policy is 50% less cost-effective than baby bonus.

[23] Another common proposal on childcare is to provide certain amount (days) of public childcare services that is free to all parents with children. For households with middle-to-high income, this policy would be equivalent to a baby bonus since their private spending on market childcare exceeds the subsidized quantity absent of the policy. For lower-income households, they would strictly prefer to have a baby bonus as cash subsidies can be used in alternatives with higher marginal benefits. Overall, the implications on aggregate variables are quantitatively similar to that of a baby bonus.

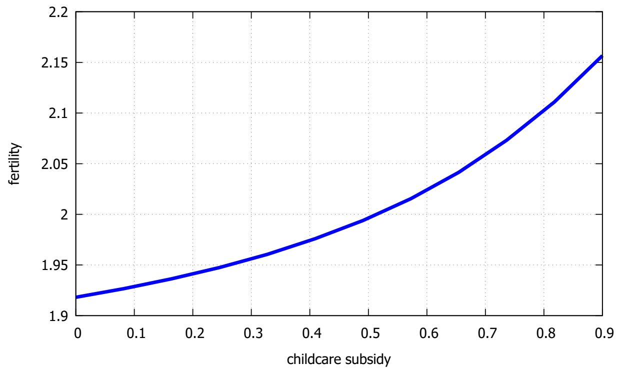

Figure 21: Effects on Fertility

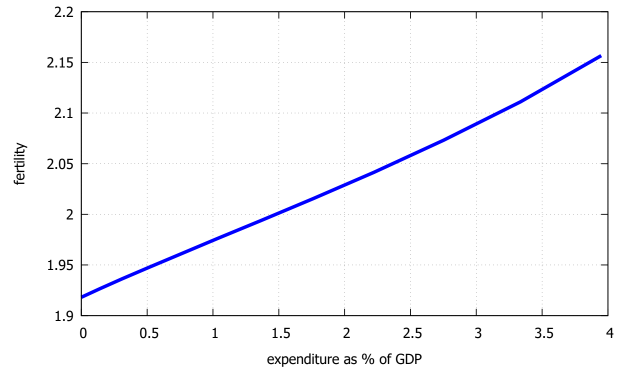

Figure 22: Expenditure Share as % of GDP

Figure 23 shows that similar to the case in baby bonus, parents reduces private education investments due to quality-quantity trade-off. Figure 24 shows that fertility responses are similar across households with different human capital levels. As subsidy raises, the fertility responses among low-income households exceed that among high-income ones. As a result, average human capital in the economy begins to decrease (see Figure 25). The overall effects on intergenerational mobility is small (see Figure 26) .

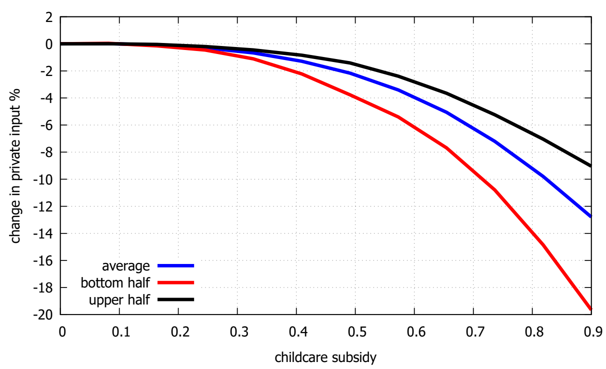

Figure 23: Average Private Investment

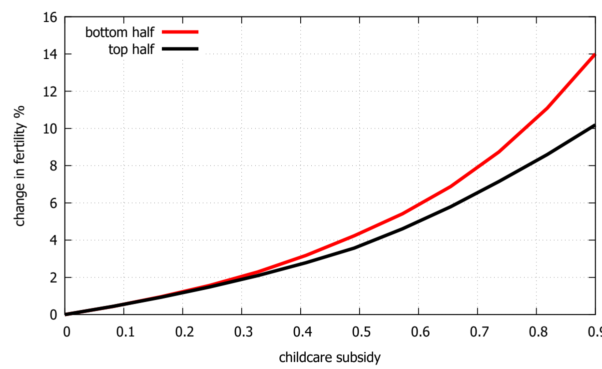

Figure 24: Heterogeneous Fertility Response

Figure 27 shows that output per capita can be increased by up to 3.5% when the government refunds 80 cents per dollar spent on market childcare. Compared with baby bonus, the magnitude of change in output is larger for three reasons. First, the composition effects through intergenerational transmission channel is smaller because changes in fertility is not very different across human capital (see Figure 24). Second, childcare subsidy raises labor supply by reducing the effective price of market care. This induces parents to reduce time spent at home taking care of children t_h. Lastly, the increased working time at age 20-30 translates to a high human capital for adults in later periods due to learning-by-doing in the labor market.

Figure 29 shows that the government is able to reduce consumption tax by 0.55% when $\mathcal{S} = 0.45$. Long-run welfare could be raised by 2.8% at replacement fertility rate with $\mathcal{S} = 0.8$.

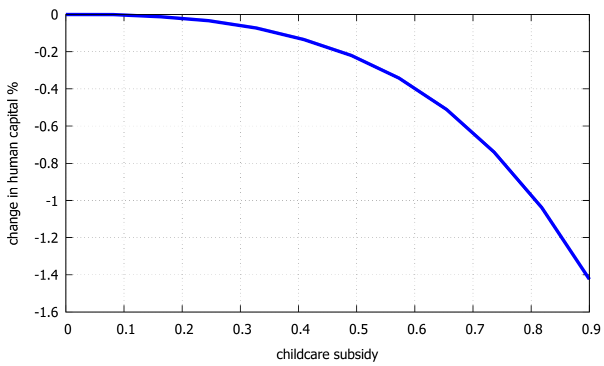

Figure 25: Average Human Capital

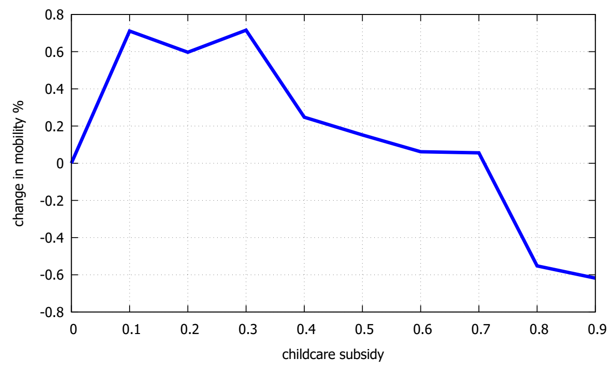

Figure 26: Intergenerational Mobility

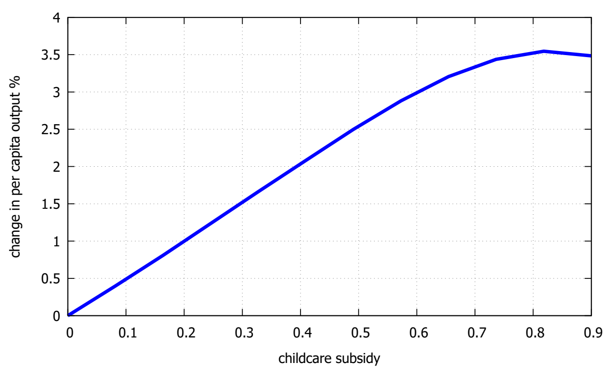

Figure 27: Per Capita Output

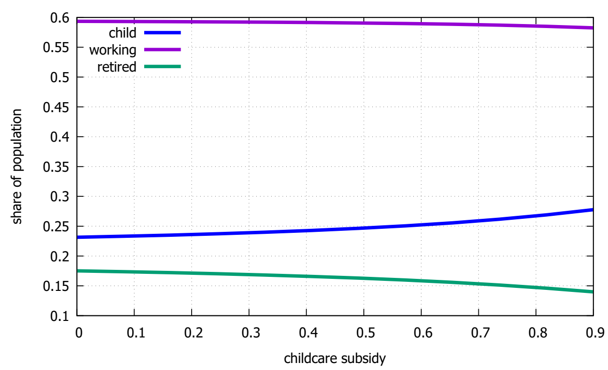

Figure 28: Change in Demographic Structure

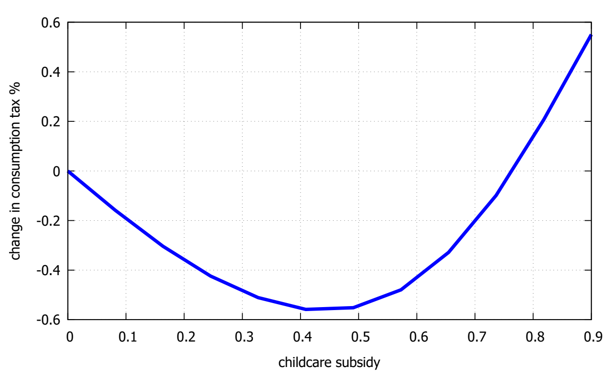

Figure 29: Change in Consumption Tax

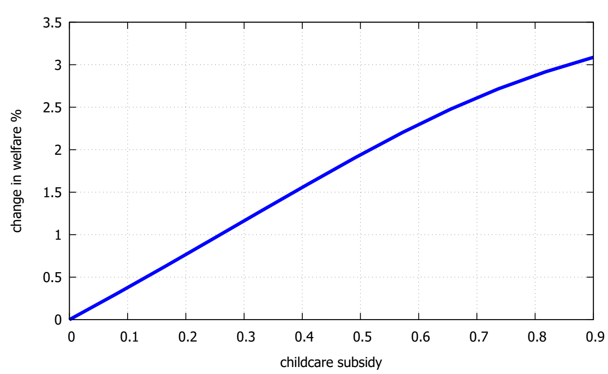

Figure 30: Change in Welfare

C.4 Public Education

Lastly, I evaluate an expansion of public education expenditures from the current level of \$12,000 per student per year by \$0 to \$100,000 in net present value. Figure 31 shows that the effect on fertility is positive because children become more desirable as public education raises children's quality. The magnitude of fertility effects here are much smaller than that of family policies. Compared with a \$30,000 baby bonus that raises fertility by 0.2, an education expansion of the same amount only raises fertility by 0.03. The government should not expect to use education policies to raise aggregate fertility to replacement level.

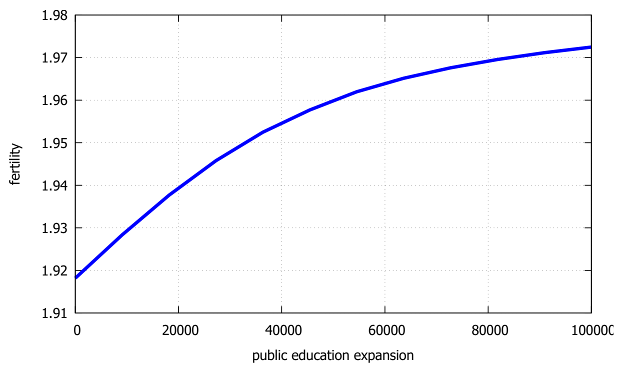

Figure 31: Effects on Fertility

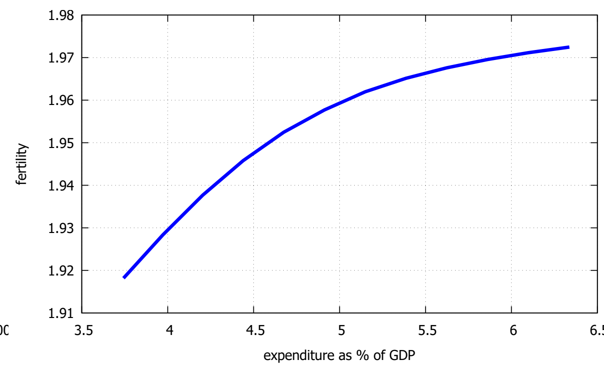

Figure 32: Expenditure Share as % of GDP

Figure 33 shows that fertility response is slightly higher for households with higher human capital. An increase in public education expenditure also crowds out private input (see Figure 35). Due to strong crowding-out effects, average human capital only increases by 2.3% when the education expansion exceeds \$80,000 in net present value. It is quite costly to raise average human capital in the economy using uniform policies in general equilibrium since (1) crowding out effect is strong, and (2) investments face decreasing marginal return as $\mathcal{E}$ is already \$12,000 per year in the baseline economy.

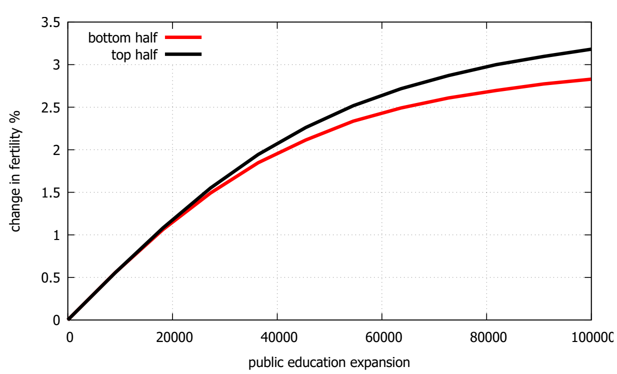

Figure 33: Heterogeneous Fertility Response

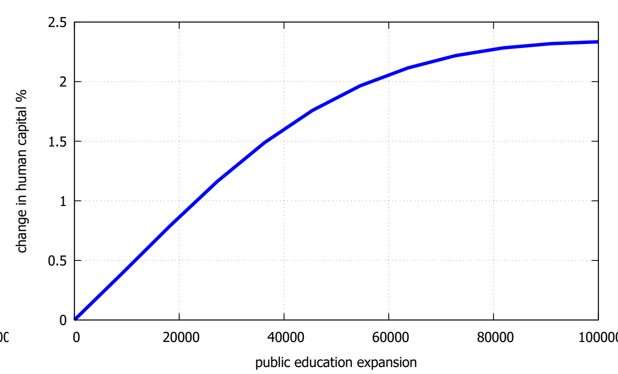

Figure 34: Average Human Capital

Expanding public education, however, is the most effective policy that could raise social mobility. As the expansion exceeds $60,000 in net present value, intergenerational mobility could be raised by almost 4.6%. If the policy is designed to target lower-income households, we can expect larger effects on mobility. Due to increased human capital, public education expansion of $60,000 raises output by 2.2%. The government needs to increase consumption tax by 0.5% to finance this expenditure. Figure 40 shows that welfare could be increased by up to 2.4%.

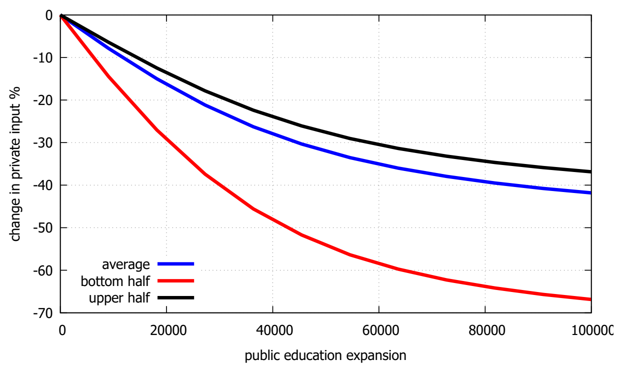

Figure 35: Average Private Investment

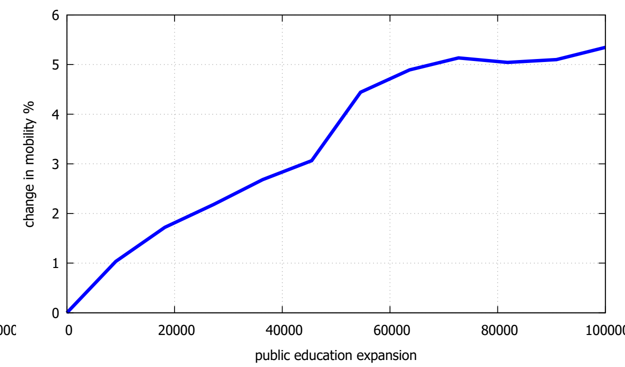

Figure 36: Intergenerational Mobility

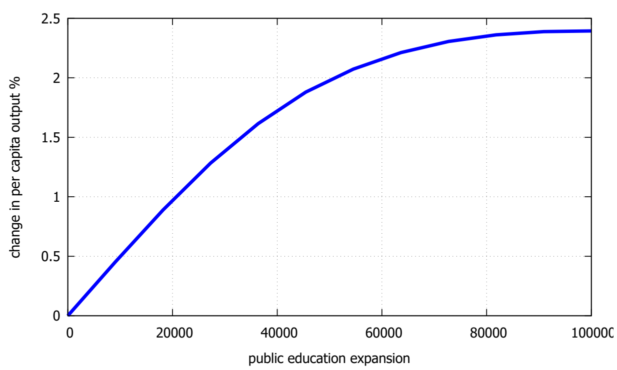

Figure 37: Per Capita Output

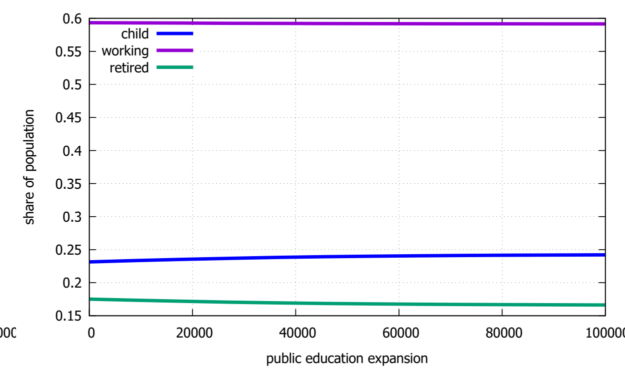

Figure 38: Change in Demographic Structure

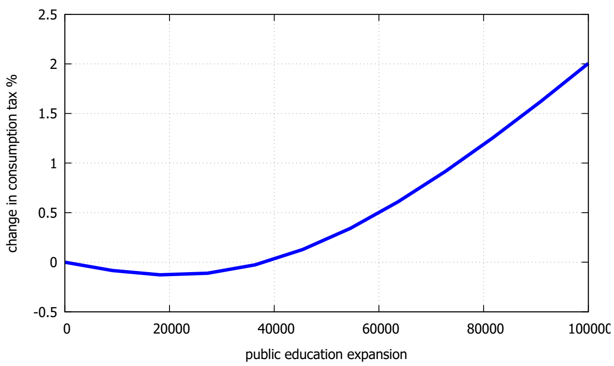

Figure 39: Change in Consumption Tax

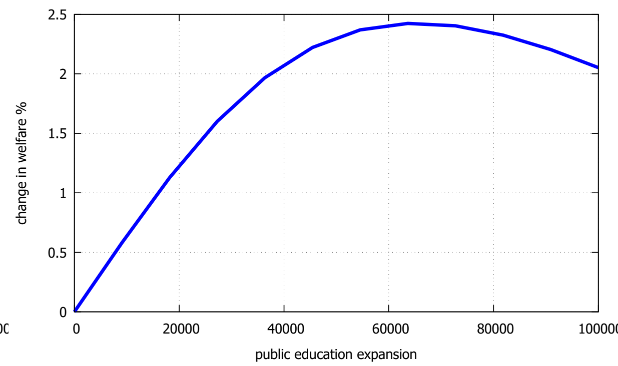

Figure 40: Change in Welfare