The Autumn of Patriarchy

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Abstract

This paper develops a unified model that explains the transition from patriarchal societies to egalitarian ones, featuring joint declines in fertility, marriage, and gender income gaps. I propose and empirically verify a novel Impossible Trinity hypothesis in family economics: high fertility, dual parenthood, and gender income equality cannot coexist. I also show that factor-neutral technological changes sow seeds of the inevitable demise of patriarchy by raising the opportunity cost of having children. The pace of the ultimate transition could vary across countries due to factors such as the social norm.

JEL classification: D13, J11, J12, J13, J16

Keywords: patriarchy, fertility, gender equality, family structure

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"...and the bells of glory that announced to the world the good news that the uncountable time of eternity had come to an end."

Gabriel García Márquez, *The Autumn of the Patriarch*

1. Introduction

After dominating human society over millennia, patriarchy has been tailing off in recent decades. Amidst the multifaceted transition towards a more egalitarian society (Doepke and Tertilt 2009, Folbre 2021), three key trends stand out: fertility rates have been falling (Greenwood et al. 2005a, Guinnane 2011), marriage and dual parenthood have been declining (Stevenson and Wolfers 2007), and gender gaps in wage, income, and wealth have been converging (Goldin 2014, Olivetti and Petrongolo 2016).[1]

While past papers propose distinct theories for each phenomenon or study at most two of them at the same time, there has not been any framework that explains the joint transitions of fertility, dual parenthood, and gender gaps, and more importantly, how they interact with each other. This paper fills the gap.

I first present a static model where males and females with different levels of human capital interact in the marriage market. Men can either remain single and childless, or they can get married and transfer a share of their income to their wives. On the other side of the market, women can be single mothers or get married and then have children. Childbirth within marriage is subject to veto (Doepke and Kindermann 2019).

In the model, the decisions on marriage, fertility, and labor supply are tightly connected via two simple yet intuitive assumptions. First, marriage is linked with fertility because a primary function of marriage is to share the costs of raising children. Second, fertility is linked with relative labor supply across genders because women shoulder a greater share of childcare responsibilities historically.

I characterize the model by showing that (1) individual optimization and marriage market clearing conditions uniquely pin down the equilibrium fertility rate and within-

[1] See Figures A.4, A.5, and A.6.

marriage income transfers, (2) the share of dual parenthood is a function of the within-marriage income transfers and the gender gap in human capital, and (3) the gender income gap is governed by the gender gap in human capital and endogenous female labor supply.

Based on the static model, I propose a novel Impossible Trinity hypothesis in family economics: high fertility, high share of dual parenthood, and gender income equality cannot coexist in the same economy. In particular, I establish that achieving any pair implies the opposite of the third. Even when policies can alter the economic fundamentals in the model, internal tensions are still present due to the equilibrium conditions. Therefore, while each outcome could be a desirable policy goal on its own, the Impossible Trinity implies that it would be difficult to achieve them all and policymakers need to make trade-offs. As a result, while the paper is silent on the normative concerns underlying the trade-offs, it points out an important boundary for policymakers.

I empirically test the Impossible Trinity hypothesis using data from a panel of countries between 1970 and 2014 where all three outcomes can be measured. I divide countries into high fertility, dual parenthood, and gender income equality groups using sample medians of each aspect so that the trinity is not ruled out by design. Then, I plot the Venn diagram to inspect their intersections. I find that a negligible share of the observations achieved high fertility, dual parenthood, and gender income equality jointly, far less than the random benchmark. This finding is consistent with the hypothesis, underscoring the inherent conflicts among the three outcomes.

Then, I study the demise of patriarchy by extending the static model into a dynamic framework. In modeling the evolution of gender-specific human capital across generations, I incorporate a new fact established by the recent empirical literature: dual parenthood has differential impacts on the human capital of boys relative to girls.[2] This fact implies that changes in family structures have profound implications for future gender gaps in human capital, and hence marriage, fertility, and female labor supply decisions.

Based on the dynamic model, I show that the demise of patriarchy is driven by two

[2]Quoting Wasserman (2020), “The evidence supports an emerging consensus that growing up in a family without biological married parents produces more adverse consequences for boys than for girls.”

key channels. First, factor-neutral technological progress raises the opportunity cost of having children and thus triggers declining fertility, falling within-marriage transfers, declining marriage rates, and increasing female labor supply. Second, rising single parenthood and the narrowing of gender human capital gaps form a powerful dynamic feedback mechanism that propagates the impacts of the first channel across generations.

Additionally, while the first channel applies uniformly across economies, the timing and magnitude of the second channel vary across countries due to differences in the mapping from within-marriage transfers to the share of dual parenthood. For example, in some economies with declining fertility rates and gender income gaps, single parenthood remains scarce due to factors such as social norms. Therefore, institutional differences could result in distinct transition patterns from patriarchal to egalitarian societies. I illustrate this argument using the United Kingdom and Japan as examples.

Lastly, I discuss whether gender equality in childcare responsibilities could resolve the Impossible Trinity. I propose several arguments against that possibility.

Related Literature

This paper is closely related to the literature on family economics and gender economics, especially the large body of papers that study historical changes in fertility, marriage, and gender gaps.[3] This paper makes four contributions.

First, while past papers propose distinct theories for each trend or study at most two trends at the same time (e.g., Galor and Weil 1996, Regalia and Rios-Rull 2001, Santos and Weiss 2016, Greenwood et al. 2016, Gayle et al. 2022, Greenwood et al. 2023), I propose a simple but unified model that knits all three facts together and highlight the tension in the trio.

Second, by taking a holistic approach, I propose and empirically test the Impossible Trinity hypothesis, a novel and overarching conjecture that links the scattered fields in the literature. The hypothesis underpins an important boundary for policymakers: jointly achieving high fertility, dual parenthood, and gender income equality is unlikely

[3] Also see Greenwood et al. (2017) and Greenwood (2019) for excellent reviews.

to be feasible.

Third, I show that factor-neutral technological growth can simultaneously generate falling fertility, marriage, and gender income gaps. This mechanism complements existing theories that rely on factor-biased technological changes, such as the skill-biased technical change that favors child quality over child quantity (Galor and Weil 2000, Fernandez-Villaverde 2001), the household appliance revolution that favors single household over married ones and encourages female labor supply (Greenwood et al. 2005b, Greenwood et al. 2023), or structural transformation that favors the labor demand of women over men (Galor and Weil 1996, Ngai and Petrongolo 2017, Cao et al. 2024).

Fourth, relative to the structural literature on demographic transition (e.g., Greenwood et al. 2023), I introduce a new mechanism that links marriage rates to gender gaps in human capital. While the differential effects of family structure on the outcomes of boys relative to girls are well documented in the empirical literature (e.g., see Bertrand and Pan 2013, Autor et al. 2019, Wasserman 2020, Reeves 2022, and Frimmel et al. 2024), this paper is the first to incorporate it into a dynamic macro model as a propagation channel.

The rest of the paper is organized as follows: Section 2 presents the static model; Section 3 proposes and tests the Impossible Trinity hypothesis; Section 4 studies the demise of patriarchy in the dynamic model; Section 5 discusses whether equal sharing of childcare responsibilities could resolve the Impossible Trinity; and Section 6 concludes.

2. The Static Model

I first present a static economy. I keep the time subscript t so that the model can be readily extended to a dynamic setting in Section 4.

Individuals are indexed by gender $g \in \{♂, ♀\}$ of equal mass. For each individual, the utility from consumption c and fertility n is given by

$$u(c, n) = \left((1-\beta) \cdot c^{\frac{\rho-1}{\rho}} + \beta \cdot n^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{1}$$

where $\rho > 1$ following Jones and Schoonbroodt (2010) and Carlos Córdoba and Ripoll (2019) so that the utility for childless individual $u(c, 0)$ is well-defined.

Within each gender, I assume that individuals have the same amount of human capital within each generation denoted by $h_t^{♂}$ and $h_t^{♀}$ respectively. Labor is the only productive factor in the economy. Therefore, $h_t^{♂}$ and $h_t^{♀}$ also determine wages. In the static model, $h_t^{♂}$ and $h_t^{♀}$ are exogenously given. The gender gap in human capital at time t is defined as

$$\Gamma_t^h = \frac{h_t^{♂}}{h_t^{♀}} \tag{2}$$

I use A_t to denote total factor productivity (TFP) at time t. In the baseline analysis, I assume that A_t is exogenously given.[4]

2.1 Single Individuals

Single males consume their labor income but have no children. They supply one unit of labor inelastically. Hence, their utility is given by

$$V_t^{♂,s} = u(A_t h_t^{♂}, 0) \tag{3}$$

where s in the superscript denotes "single."

Single females, on the other hand, can have children but do not receive any transfers or support from the absentee fathers.[5] They choose consumption $c_t^{♀,s}$, fertility n_t^s, and labor supply n_t^s to solve

$$V_t^{♀,s} = \max_{c_t^{♀,s}, l_t^s, n_t^s} \quad u(c_t^{♀,s}, n_t^s) \tag{4}$$

[4] Allowing for endogenous A_t may lead to additional channels. For example, besides enhancing aggregate productivity A_t à la Hsieh et al. (2019), rising female labor supply could stimulate innovation and hence economic growth (Chiplunkar and Goldberg 2021). Another example is Galor and Weil (1996) which discusses the feedback mechanism between fertility decline, which stimulates capital accumulation, and rising demand for female labor, which is more complementary to capital than male labor.

[5] This is a model simplification. If the biological father can be identified and located, it is possible to sue him for child support. Nevertheless, according to the calculation by the Annie E. Casey Foundation using data from the Current Population Survey wave 2020-2022, just 23% of U.S. female-headed families living with one or more children under age 18 reported receiving any amount of child support during the previous year.

subject to budget and time constraints

$$c_t^{\female,s} = A_t h_t^{\female} l_t^s, \quad \text{and} \quad l_t^s = 1 - \chi n_t^s$$

where χ is the time cost of raising each child. I follow the literature and assume that the fertility choice is continuous, i.e., $n_t^s \in \mathbb{R}_+$.

In other words, the decisions of single females involve a simple consumption-fertility trade-off via endogenous labor supply – a margin that is familiar to the female labor supply literature (e.g., Rosenzweig and Wolpin 1980).

2.2 Married Individuals

I assume that once married, husbands supply one unit of labor inelastically and are required to transfer α_t share of their income to their wives. This assumption captures the traditional role of marriage where husbands are the main breadwinners and provide income for the family. While individuals take α_t as given, it is an equilibrium object to be characterized in Section 2.5.

Husbands derive utility from their remaining income and fertility – a public good shared with their wives. Therefore, the value of marriage for males is

$$V_t^{\male,m} = u(\underbrace{(1-\alpha_t)A_t h_t^{\male}}_{\text{remaining income}}, \underbrace{n_t^m}_{\text{fertility}}). \tag{5}$$

Wives trade-off fertility and consumption via endogenous labor supply. They solve

$$V_t^{\female,m} = \max_{c_t^{\female,m}, l_t^m, n_t^m} \quad u(c_t^{\female,m}, n_t^m) \tag{6}$$

subject to budget and time constraints

$$c_t^{\female,m} = \underbrace{\alpha_t A_t h_t^{\male}}_{\text{transfer from husband}} + \underbrace{A_t h_t^{\female} l_t^m}_{\text{own labor income}}, \quad \text{and} \quad l_t^m = 1 - \chi n_t^m$$

where n_t^m and l_t^m are the fertility and labor supply of married women. From the wives' perspective, the transfers from husbands generate an income effect which leads to higher consumption and fertility given that both are normal goods in preferences. Rising wages, however, will also generate a substitution effect, leading to a lower demand for children.

Because husbands do not directly bear the costs of children after transferring α_t share of income, they prefer as much fertility n_t^m as possible.[6] Wives, however, prefer to have fewer children because they directly shoulder the burden of childcare. This observation is supported by the empirical findings in Doepke and Tertilt (2018).

Motivated by Doepke and Kindermann (2019), I assume that childbirth within marriage is subject to veto. Therefore, wives are the key decision-makers regarding fertility within marriage in this model.

2.3 Marriage Market

At the beginning of the period, I assume that each woman receives an idiosyncratic shock τ on the taste of marriage which follows a distribution $J(\tau)$.[7] For a woman with taste shock τ, her utility from marriage becomes $\tau \cdot V_t^{♀,m}$. After receiving the shock, individuals decide whether or not to get married and the marriage market clears. The distribution $J(\tau)$ is a reduced-form way to capture other considerations of marriage that are not explicitly included in the model, such as mutual affection, tax benefits, or risk-sharing.

For women, it is straightforward that there exists a threshold τ_t^* above which they would prefer marriage over staying single. The value of τ_t^* can be implicitly defined using the indifference condition

$$V_t^{♀,m} \cdot \tau^* = V_t^{♀,s}. \tag{7}$$

[6]The assumption that males do not share the cost of children is not crucial. In fact, all the results go through as long as wives shoulder *more* childcare responsibilities than their husbands–a pattern that holds widely across countries and over time (Kleven et al. 2019, Doepke et al. 2023). In Section 5, I discuss how gender equality in childcare would affect the results.

[7]Similar assumptions can be found in marriage models such as Greenwood et al. (2017).

Therefore, the share of men or women that are married in the equilibrium is given by

$$\mathcal{M}_t = 1 - J(\tau_t^*) \tag{8}$$

On the other hand, because males are homogeneous and are on the long side of the marriage market, the equilibrium imposes an indifference condition for men

$$V_t^{♂,m} = u((1-\alpha_t)A_t h_t^{♂}, n_t^m) = u(A_t h_t^{♂}, 0) = V_t^{♂,s} \tag{9}$$

where the share of transfers α_t acts as "prices" to clear the marriage market.

2.4 Aggregate Variables

With marriage rate $\mathcal{M}_t$ defined, the model gives expressions of other aggregate variables of interest. For example, aggregate fertility rate n_t is a weighted average of marital and non-marital fertility:

$$n_t = \mathcal{M}_t \cdot n_t^m + (1-\mathcal{M}_t) \cdot n_t^s \tag{10}$$

The share of children born with both parents, i.e., dual parenthood, is given by

$$\mathcal{D}_t = \frac{\mathcal{M}_t \cdot n_t^m}{n_t} \tag{11}$$

Average hours worked per female is

$$l_t^{♀} = \mathcal{M}_t \cdot l_t^m + (1-\mathcal{M}_t) \cdot l_t^s = 1 - \chi n_t \tag{12}$$

The average labor income of male and females are

$$y_t^{♂} = A_t \cdot h_t^{♂}, \qquad y_t^{♀} = A_t \cdot h_t^{♀} \cdot l_t^{♀}$$

which leads to a simple expression of the gender income gap

$$\Gamma_t^y = \frac{y_t^{♂}}{y_t^{♀}} = \frac{\Gamma_t^h}{l_t^{♀}} \tag{13}$$

2.5 Model Solution

In this section, I characterize the properties of the static model.

First, the indifference condition of males in the marriage market (9) implicitly defines α_t as a function of n_t^m:

$$(1-\beta) \cdot (A_t h_t^{♂})^{\frac{\rho-1}{\rho}} \left[1 - (1-\alpha_t)^{\frac{\rho-1}{\rho}}\right] = \beta \cdot (n_t^m)^{\frac{\rho-1}{\rho}} \tag{14}$$

When $\rho > 1$, using the implicit function theorem on Equation (14) reveals that the function $\alpha_t(n_t^m)$ is strictly increasing and convex. It takes the value of 0 when $n_t^m = 0$, and shifts up when A_t rises.

On the other hand, the first-order condition of married women gives the optimality condition where n_t^m is a function of α_t:

$$n_t^m \cdot \left[\left(\frac{(1-\beta)A_t h_t^{♀}\chi}{\beta}\right)^{\rho} + A_t h_t^{♀}\chi\right] = (1 + \alpha_t \Gamma_t^h) A_t h_t^{♀} \tag{15}$$

Equation (15) indicates that $n_t^m(\alpha_t)$ is an increasing and linear function. It takes a strictly positive value when $\alpha_t = 0$ and shifts down when A_t rises.

Taking the properties of $\alpha_t(n_t^m)$ and $n_t^m(\alpha_t)$ together generates the first lemma.

Lemma 1: For given A_t, there is a unique fixed point of (α_t, n_t^m).

Proof: See Appendix.

Second, by comparing $V_t^{♀,s}$ and $V_t^{♀,m}$, Lemma 2 provides a condition for the cutoff τ_t^* above which women choose to get married.

Lemma 2: The marriage threshold $\tau_t^* = 1/(1 + \alpha_t \Gamma_t^h)$.

Proof: See Appendix.

Lemma 2 indicates that the marriage threshold, and hence the marriage rate $\mathcal{M}_t$, is determined by the economic gains from marriage from the women's perspective. The "transfer potential" of males is a product of the gender gap in human capital Γ_t^h and men's willingness to transfer α_t.

Together with Equation (26) in the Appendix, Lemma 2 also indicates that the fraction of dual parenthood $\mathcal{D}_t$, defined in (11), is monotonically increasing in the marriage rate $\mathcal{M}_t$. Therefore, I will use the $\mathcal{M}_t$ and $\mathcal{D}_t$ interchangeably when I analyze the Impossible Trinity next.

3. The Impossible Trinity

In this section, I propose and empirically test the Impossible Trinity hypothesis.

3.1 Theory

Collecting the equilibrium conditions and results from Lemma 2, the relationship between fertility n_t, marriage $\mathcal{M}_t$, female labor supply $l_t^{♀}$, and gender income gap Γ_t^y can be summarized in the following three equations:

$$\mathcal{M}_t = 1 - J\left(\frac{1}{1+\alpha_t \Gamma_t^h}\right) \tag{16}$$

$$\Gamma_t^y = \frac{\Gamma_t^h}{l_t^{♀}} \tag{17}$$

$$l_t^{♀} = 1 - \chi n_t \tag{18}$$

Equations (16)-(18) illustrate the key tensions in the model. In particular, (16) shows that marriage rates are higher when there are larger gender gaps in human capital. But (17) implies that large gender gaps in human capital make it difficult to achieve gender income inequality unless the female labor supply is high. However, the direct implication of a high female labor supply is low fertility from (18).

The Impossible Trinity hypothesis: high fertility, dual parenthood (or equivalently high marriage rate), and gender income inequality cannot coexist.
Proof: I prove the hypothesis by discussing three possible cases.

1. *High fertility and dual parenthood.* With high fertility n_t, female labor supply $l_t^{\female}$ is low from (18). To achieve a high marriage rate $\mathcal{M}_t$, gender human capital gap Γ_t^h cannot be too low from (16). Therefore, the gender income gap Γ_t^y is necessarily high from (17).

2. *High fertility and gender income equality.* With high fertility n_t, female labor supply $l_t^{\female}$ is low from (18). To achieve a low gender income gap Γ_t^y, it must be the case that Γ_t^h is very low from (17). But a very low gender gap in human capital Γ_t^h leads to a low marriage rate $\mathcal{M}_t$ from (16).

3. *Dual parenthood and gender income equality.* To achieve a high marriage rate $\mathcal{M}_t$, (16) implies that the gender gap in human capital Γ_t^h needs to be high. With high Γ_t^h, the only way to achieve a low gender income gap Γ_t^y is to have a high female labor supply $l_t^{\female}$. needs to be very high from (18). Therefore, fertility n_t is very low from (18).

3.2 Empirical Results

To test the Impossible Trinity hypothesis empirically, I collect data on (1) total fertility rates (TFR) from the United Nations, (2) the share of children born outside of marriage from the OECD database, and (3) gender gaps in median earnings from the OECD database. The resulting dataset is an unbalanced panel of 37 countries from 1970 to 2014 with 721 country-year observations in total.

I categorize observations based on sample medians of each variable. I use medians to define "high" or "low" to guarantee that each group has a sizable mass of observations so that there is a fair chance to achieve the trinity. If the cutoffs can be set arbitrarily strict, then achieving the trinity becomes impossible by design.[8]

Using this definition, observations are labeled as

- "high fertility" if $\text{TFR}_{it} > 1.69$,

[8] I have also experimented with alternative cutoffs: (1) defining "high fertility" (e.g., $\text{TFR}_{it} > 2$) or (2) defining each category using upper quartiles of each variable. The results are plotted in Figures A.7 and A.8. As can be seen, the main finding remains robust.

- "dual parenthood" if out of marriage$_{it} < 31.4\%$, and
- "gender income equality" if gap$_{it} < 17.2\%$.

After labeling each observation, I plot the Venn diagram to inspect the intersections. The results are shown in Figure 1. Because I am defining each group using sample median, the share of observations that could achieve all three jointly would be 12.5% had these three outcomes been independent of each other. In the data, I find that less than 3% of the observations achieved high fertility, dual parenthood, and gender income equality jointly – much less than the random benchmark. This finding supports the Impossible Trinity hypothesis, highlighting the inherent conflicts among the three outcomes. Furthermore, it may well be possible that many countries only achieve one, or even none of the three outcomes.

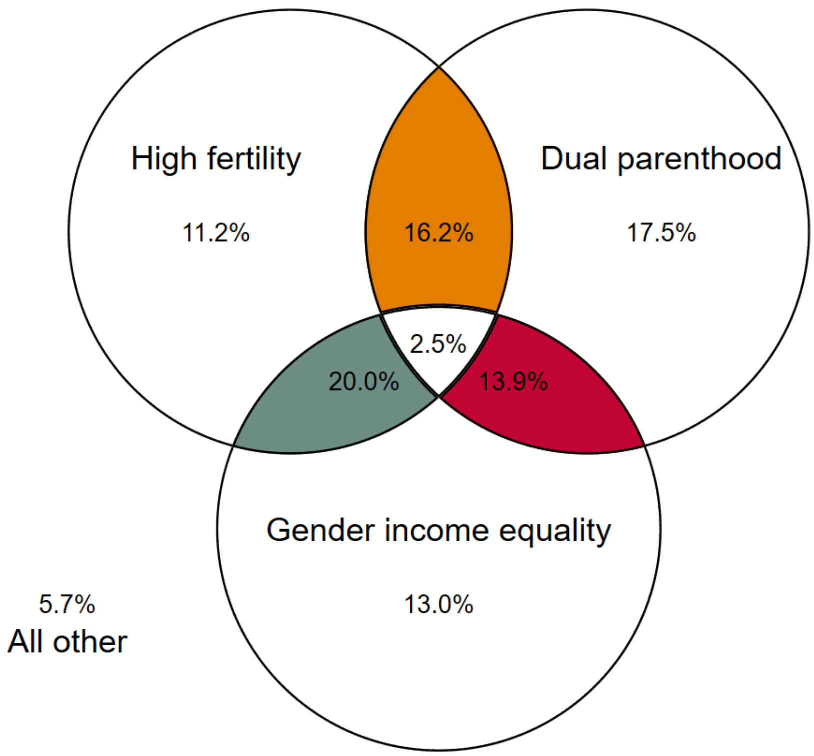


Figure 1: The Impossible Trinity in the Data

Table 1 gives some examples for each area of the Venn diagram. The main country that consistently achieved the trinity according to the definition is Australia between

D – dual parenthood, G – gender income equality, F – high fertility	
Category	Countries
None	Austria, United Kingdom 1995-2003
Only D	Canada, Switzerland, Germany 1992-2006, Japan, South Korea
Only G	Germany 2009-2014, Hungary, Portugal
Only F	United States 1994-2013, Finland
$D+G$	Greece, Italy, Poland
$G+F$	Belgium, Norway, New Zealand, Sweden
$D+F$	United Kingdom 1970-1994, Israel, USA 1973-1993
$D+G+F$	Australia 1992-2002 ($G+F$ afterwards)

Table 1: Examples of Countries

1992 and 2002.[9] After 2003, the share of single parenthood rose sharply in Australia so it lost the "dual parenthood" status. Figure A.9 presents a complete picture of the total number of outcomes achieved by each country over time.

3.3 Implications

The main takeaway from the Impossible Trinity hypothesis is that although each of the three outcomes could be a desirable policy goal,[10] it is difficult for policymakers to achieve them all due to the inherent incompatibility.

To understand the intuitions behind this result, it is noteworthy to point out that there are two key tensions in the static model. The first tension is between fertility and gender income equality through endogenous female labor supply. For example, consider family policies that change the cost of children χ in the model (e.g., baby bonuses and child tax credits). If the policymaker raises χ, then it can achieve more gender

[9] In fact, the Australian fertility rate is less than 1.9 in this era, so it would fall out of the trinity had we used the TFR¿2 definition.

[10] For instance, there have been many policies promoting childbirth and gender equality. Furthermore, even though the model does not explicitly consider cross-sectional inequality, studies like Kearney (2023) established a close link between the prevalence of dual parenthood and inequalities of children's outcomes.

equality at the cost of lower fertility because the female labor supply rises. On the other hand, if the policymaker lowers χ, then fertility is higher, but the female labor supply falls and hence the gender income gap widens.

The second tension is between dual parenthood and gender income gaps. For example, if anti-discrimination policies reduce Γ^h and lead to shrinking gender wage gaps, then marriage rates will decline because there is less "transfer potential" from males. If Γ^h rises, then the marriage rate rises but the economy fares worse in gender income equality.

The two tensions in the static model may not be fully satisfactory for several reasons. First, one might argue that due to the model setup, it will not capture the margins on leisure, and hence the possibility that some government policies, in particular subsidized childcare, could raise both fertility and female labor supply (Baker et al. 2008) – resolving the first tension. Second, governments may directly change the benefits of marriage and hence $J(\cdot)$ through – achieving higher marriage rates without sacrificing gender income equality – resolving the second tension.

The result in Figure 1, however, suggests that some other mechanism should also be present. Because if not, why don't we observe a larger share of countries achieving the trinity given that subsidized childcare or marriage tax benefits are available to governments and have already been widely adopted in many developed economies?

To answer this question, in Section 4, I extend the model into a dynamic setting and uncover another intrinsic tension between dual parenthood and the gender income gap through the formation of gender-specific human capital. Besides strengthening the argument for the Impossible Trinity hypothesis, the dynamic model also provides a roadmap on the demise of patriarchy.

4. The Autumn of Patriarchy

This section studies the transition from patriarchal societies to egalitarian societies in a dynamic model.

4.1 Human Capital Dynamics

I assume that the gender-specific human capital follows the law of motion[11] specified as

$$h_{t+1}^{♀} = (h_t^{♀})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{♀}} \tag{19}$$

$$h_{t+1}^{♂} = Z \cdot (h_t^{♂})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{♂}} \tag{20}$$

where $Z > 1, \theta \in (0,1)$ and more importantly, $\psi^{♂} > \psi^{♀} > 0$.

The production functions (19) and (20) are motivated by a large empirical literature that has documented that growing up in a family without biological married parents leads to more adverse consequences for boys than for girls (e.g., see Bertrand and Pan 2013, Autor et al. 2019, Wasserman 2020, Reeves 2022, and Frimmel et al. 2024). In practice, this result could be due to (1) role model effects operating within genders, (2) differential sensitivity to parental inputs across genders, or (3) differential exposure or sensitivity to inputs from other social institutions such as neighborhoods or schools.

The difference between between $\psi^{♂}$ and $\psi^{♀}$ is economically sizable. For example, Autor et al. (2019) show that the racial differences in the ratio of single motherhood could explain the bulk of the black-white differences in gender gaps. Autor et al. (2023) find that a substantial fraction of the gender gap in high school outcomes can potentially be explained by the differential effect of family socioeconomic status, in particular family structure, on boys' medium-run outcomes.

Taking the results on differential sensitivity as given, the model implies that the prevailing marriage rates determine gender gaps in human capital in the next generation and hence the evolution of Γ^h. To see this, note that dividing (19) by (20) yields

$$\Gamma_{t+1}^{h} = Z \cdot (\Gamma_t^{h})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{♂} - \psi^{♀}}$$

[11] I adopt Galton's approach to the intergenerational transmission of human capital for analytical and aggregation simplicity. As pointed out by Mulligan (1999), explicit modeling of parental human capital investment decisions, e.g., following Becker and Tomes (1979), often yields similar predictions.

which implies in steady-state

$$\Gamma^h = Z^{\frac{1}{1-\theta}} \cdot (\mathcal{M})^{\frac{\psi^{♂} - \psi^{♀}}{1-\theta}} \implies \frac{d\Gamma^h}{d\mathcal{M}} > 0 \tag{21}$$

Therefore, higher marriage rates generates larger gender human capital gaps through the channel of human capital formation.

4.2 Mechanism

With all elements in the dynamic system defined, this section discusses the mechanisms that result in the demise of patriarchy.

Lemma 3: The levels of α_t and n_t^m are decreasing in A_t.

Proof: See Appendix.

The intuition behind Lemma 3 is simple: because consumption and fertility are substitutes in the utility function, a higher total factor productivity A raises the opportunity costs of having children and the substitution effect dominates the income effect. Therefore, n_t^m is decreasing in A. Because the amount of transfers males are willing to pay their wives depends positively on marital fertility n^m, transfer share α also falls as A rises. This channel is presented as the red arrows in Figure 2

The second channel that leads to the demise of patriarchy is a chain reaction between single parenthood and gender human capital gaps presented as the blue arrows in Figure 2. When α falls, there is a decline in the economic gains from marriage for women ($\alpha\Gamma^h$). As a result, the marriage rate $\mathcal{M}$ drops. Because the decline in marriage hurts boys relatively more than girls, the gender gap in human capital Γ^h falls in the next generation, further dragging down the economic gains from marriage. The second channel propagates the effects of rising A_t over time, generating dynamic falls in marriage rates and human capital gaps.

More rigorously, the impact of the second channel is given by Lemma 4.

Lemma 4: Declining α_t reduces long-run $\mathcal{M}$ and Γ^h.

Proof: See Appendix.

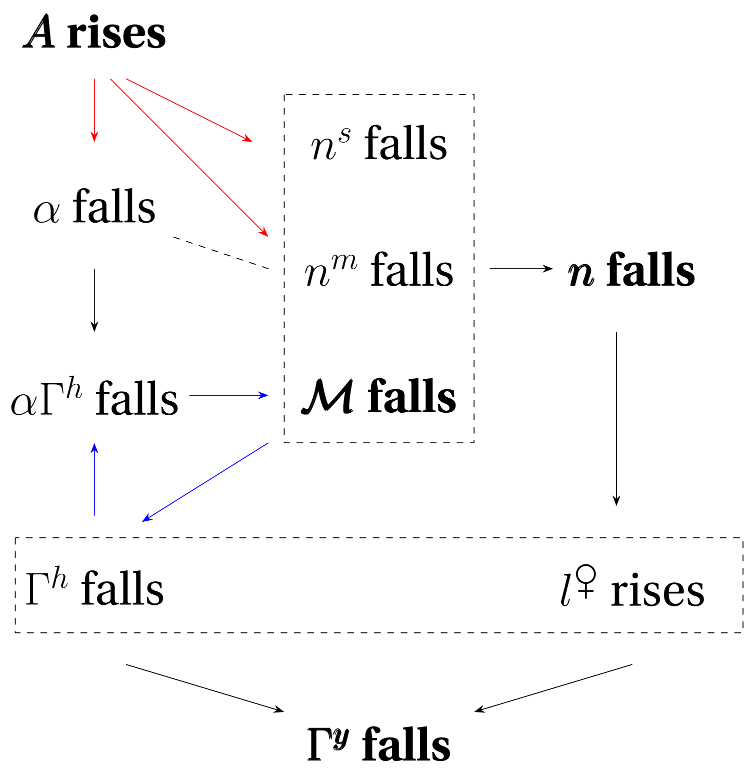


Figure 2: The Autopsy of Patriarchy

To take stock, Figure 2 indicates that the joint declines in fertility, marriage, and gender income can be explained by a unified framework that solely relies on rising total factor productivity A_t:

1. Fertility n falls because (1) rising A_t reduces both marital fertility n^m and single fertility n^s, and (2) a composition effect where marriage rate $\mathcal{M}$ falls and $n^m > n^s$.

2. Marriage $\mathcal{M}$ falls because there is less "transfer potential" from males $\alpha\Gamma^h$. The decline in marriage triggers a chain reaction, leading to lower gender gaps in human capital Γ^h.

3. Gender income gap Γ^y converges because (1) gender gaps in human capital Γ^h decreases, and (2) falling fertility n implies higher female labor supply $l^{♀}$.

4.3 The Role of Social Norms

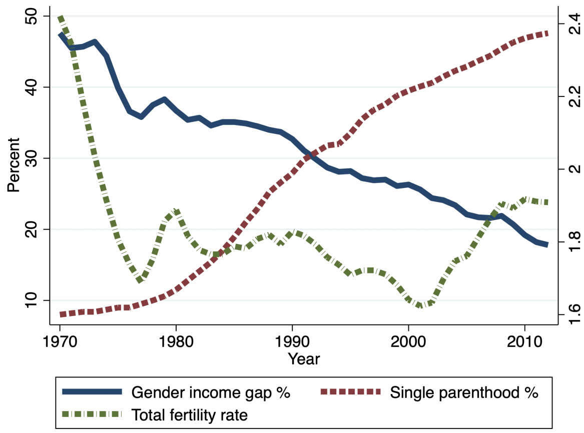
Another message from Figure 2 is that while the first channel, i.e., effects of A_t on n_t^m, n_t^s, and α_t, is the same across countries, the final impacts on marriage and gender income gaps could be different across countries depending on the quantitative magnitude of the second channel.

To be more specific, the mapping from the "transfer potential" $\alpha\Gamma^h$ to marriage rates $\mathcal{M}$ depends on the distribution of idiosyncratic shocks $J(\tau)$. This distribution could vary across countries due to factors such as culture, religion, and social norms (e.g., shotgun marriage). Depending on the mass of individuals around the cutoff τ^*, responses in the marriage rate $\mathcal{M}$ could be either large or small. As a result, the timing and magnitude of the feedback mechanism between $\mathcal{M}$ and Γ^h could vary dramatically across countries.

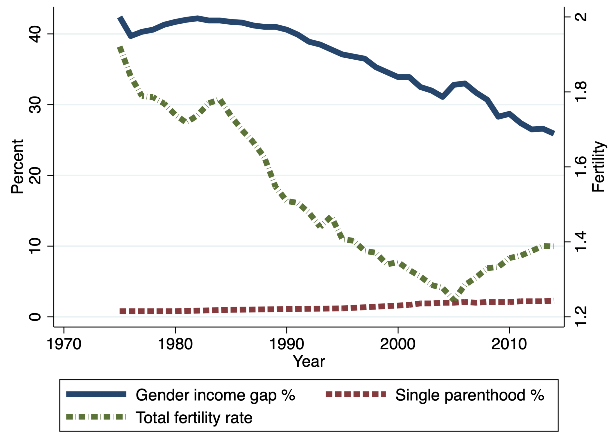
To give some concrete examples, Figure 3a displays the case for the United Kingdom. As its fertility fell after the Baby Boom, single parenthood surged after the 1980s. Through the lens of the model, rising female labor supply and converging gender human capital gaps jointly contributed to the converging gender income gaps.

In contrast, Figure 3b displays the case of Japan. While fertility also fell during the rapid economic growth era in the 1980s, single parenthood barely rose, owing to the strong influence of the Confucian tradition that stigmatizes out-of-marriage births (Myong et al. 2021). Through the lens of the model, only the rising female labor supply contributed to the converging gender income gaps. As a result, the speed of gender gap convergence in Japan is much slower than that in the United Kingdom. Such differences can be attributed, at least partly, to the heterogeneous $J(\tau)$ distribution between Japan and the United Kingdom.

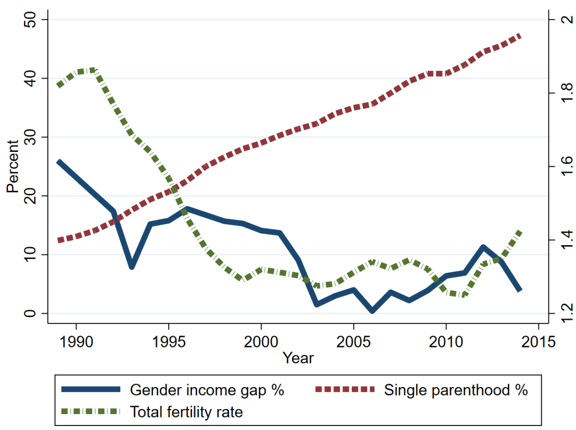
Similar juxtaposition and analyses can be drawn when we inspect other cases, such as Hungary (Figure 3c), South Korea (Figure 3d), Australia (Figure 3e), and Poland (3f).



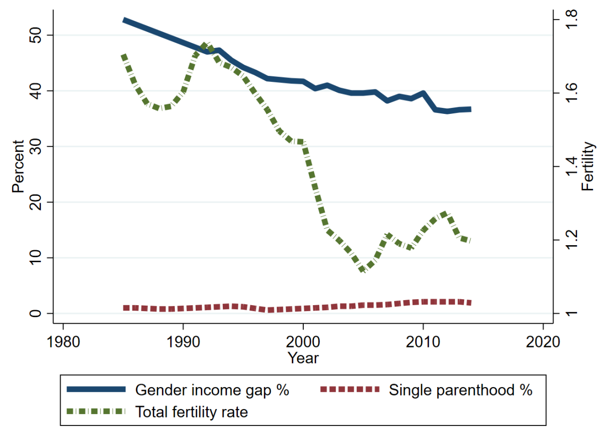
(a) The Case of the U.K.



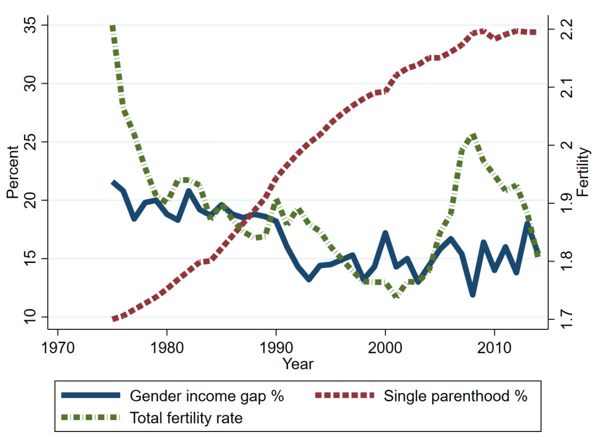
(b) The Case of Japan



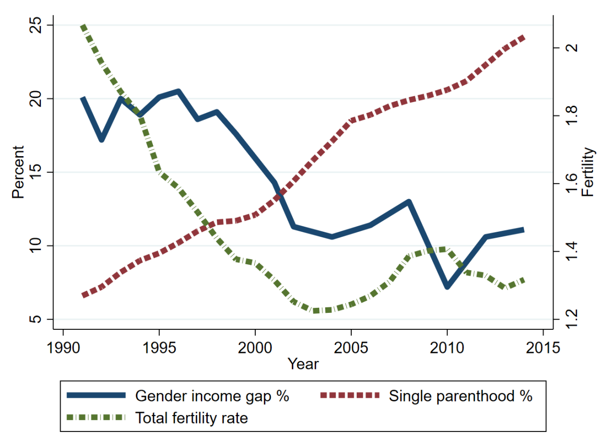
(c) The Case of Hungary



(d) The Case of South Korea



(e) The Case of Australia



(f) The Case of Poland

Figure 3: The Demise of Patriarchy: Some Examples

5. Discussions

An interesting and challenging question is whether gender equality in childcare responsibilities, which has been studied by many recent papers such as Doepke and Kindermann (2019), could resolve the Impossible Trinity. In particular, if both men and women participate in childcare, could countries achieve high fertility while preserving dual parenthood and gender income equality?

Through the lens of the model, if both genders share the same childcare burden, then the labor supply is the same across genders irrespective of the prevailing fertility. As a result, the gender income gap Γ^y entirely depends on the gender human capital gap Γ^h. But with high marriage rates $\mathcal{M}$, the gender human capital gap Γ^h is also high due to the differential sensitivity assumption $\psi^{♂} > \psi^{♀}$. Therefore, to achieve both dual parenthood and gender income equality, men need to take *more* childcare responsibilities than women. This requirement has three potential issues.

First, how large would the efficiency cost be for men to work less than women when their human capital is relatively higher? The efficiency cost could be even larger if women have an absolute advantage in childcare.

Second, because men have the outside option of staying single and having no children, the amount of transfer α needs to be very low for them to agree to take on more childcare responsibilities within marriage. But when α, and hence the economic gains from marriage, is small, more women would prefer to stay single, making high marriage rates an unlikely outcome.

Lastly, from an empirical point of view, even though there has been a lot of progress towards an equal sharing of childcare responsibilities, especially in many European countries, Figure 1 indicates that there hasn't been much evidence supporting it as a way out from the Impossible Trinity.

Due to the reasons mentioned above, I argue that it is unlikely that gender equality in childcare responsibilities will resolve the Impossible Trinity.

6. Conclusion

Human society is undergoing an unprecedented transition in which patriarchy is withering away. In this paper, I present a unified framework on the interactions between fertility, dual parenthood, and gender income gaps in this epoch.

The model offers three main insights. First, high fertility, dual parenthood, and gender income equality cannot coexist – a novel and overarching Impossible Trinity hypothesis in family economics. I also show that the hypothesis is supported by the data. Second, rising total factor productivity is sufficient to cause the demise of patriarchy – one does not need to assume factor-biased technological changes. Lastly, while the demise of patriarchy is inevitable, the pace of the transition could differ across countries due to social norms.

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A. Proofs

Proof of Lemma 1

Define function

$$f_1(\alpha_t) = A_t h_t^{♂} \cdot \left(\frac{1-\beta}{\beta} \cdot [1-(1-\alpha_t)^{\frac{\rho-1}{\rho}}]\right)^{\frac{\rho}{\rho-1}}, \qquad \alpha \in [0,1]$$

For $\rho > 1$, $f_1(\alpha_t)$ is strictly increasing, convex, and $f_1(0) = 0$. Moreover, $n_t^m = f_1(\alpha_t)$ satisfies men's indifference condition (9).

Define function

$$f_2(\alpha_t) = \frac{(1+\alpha_t \Gamma_t^h) A_t h_t^{♀}}{\left(\frac{(1-\beta)A_t h_t^{♀}\chi}{\beta}\right)^{\rho} + A_t h_t^{♀}\chi}, \qquad \alpha_t \in [0,1]$$

For $\rho > 1$, $f_2(\alpha_t)$ is strictly increasing, linear, and $f_2(0) > 0$. Moreover, $n_t^m = f_2(\alpha_t)$ satisfies women's optimality condition (15).

Thus, $f_3(\alpha_t) = f_1(\alpha_t) - f_2(\alpha_2)$ is strictly increasing, convex, and $f_3(0) < 0$. Therefore, there are two possibilities. If $f_3(\alpha_t)$ obtains the value of zero in the domain $\alpha \in [0,1]$, i.e., interior solution, then this solution is unique. Otherwise, there is a corner solution $\alpha_t = 1$, i.e., men strictly prefer marriage over being single and are willing to transfer the entirety of their income – a theoretically possible but empirically irrelevant case.

Figure A.1 provides a graphical illustration of the proof.

Proof of Lemma 2

For married women, the first-order condition is

$$(1-\beta)\cdot(c_t^{♀,m})^{-\frac{1}{\rho}} = \frac{\beta\cdot(n_t^m)^{-\frac{1}{\rho}}}{A_t h_t^{♀}\chi} \Longrightarrow c_t^{♀,m} = n_t^m \cdot \left(\frac{(1-\beta)A_t h_t^{♀}\chi}{\beta}\right)^{\rho} \tag{22}$$

Substituting (22) into the budget constraint, n_t^m satisfies

$$n_t^m \cdot \left(\frac{(1-\beta)A_t h_t^{♀}\chi}{\beta}\right)^{\rho} = \alpha_t \Gamma_t^h A_t h_t^{♀} + A_t h_t^{♀}(1-\chi n_t^m)$$

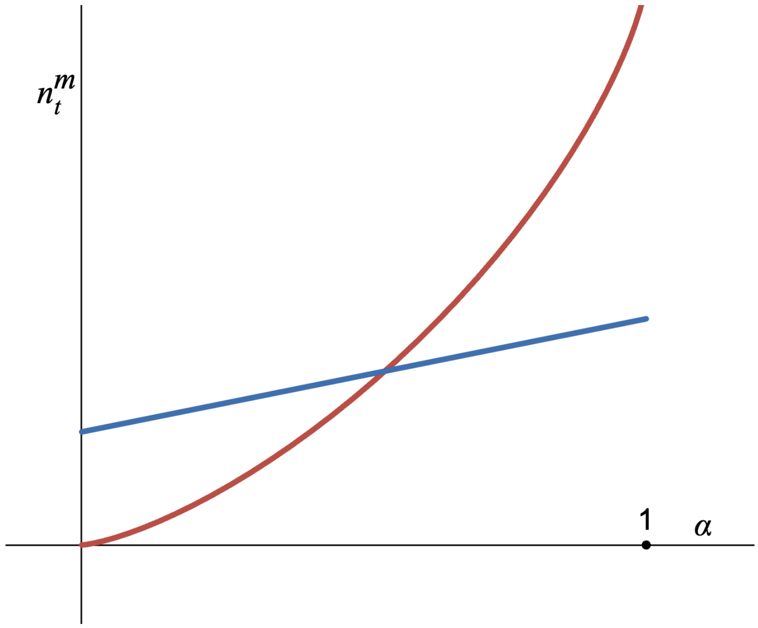

Figure A.1: $n_t^m(\alpha_t)$ (blue) and $\alpha_t(n_t^m)$ (red)

which is equivalent to

$$n_t^m \cdot \left[\left(\frac{(1-\beta)A_t h_t^{\female} \chi}{\beta} \right)^{\rho} + A_t h_t^{\female} \chi \right] = (1 + \alpha_t \Gamma_t^h) A_t h_t^{\female} \tag{23}$$

For single women, the first-order condition is

$$(1-\beta) \cdot (c_t^{\female,s})^{-\frac{1}{\rho}} = \frac{\beta \cdot (n_t^s)^{-\frac{1}{\rho}}}{A_t h_t^{\female} \chi} \Longrightarrow c_t^{\female,s} = n_t^s \cdot \left(\frac{(1-\beta)A_t h_t^{\female} \chi}{\beta} \right)^{\rho} \tag{24}$$

Substituting (24) into the budget constraint, $c_t^{\female,s}$ satisfies

$$n_t^s \cdot \left(\frac{(1-\beta)A_t h_t^{\female} \chi}{\beta} \right)^{\rho} = A_t h_t^{\female} (1 - \chi n_t^s)$$

which is equivalent to

$$n_t^s \cdot \left[\left(\frac{(1-\beta)A_t h_t^{\female} \chi}{\beta} \right)^{\rho} + A_t h_t^{\female} \chi \right] = A_t h_t^{\female} \tag{25}$$

Take the ratio between (23) and (25) gives

$$\frac{n_t^m}{n_t^s} = 1 + \alpha_t \Gamma_t^h \tag{26}$$

which is independent of A_t.

On the other hand,

$$V_t^{♀,m}(\tau) = \tau \cdot n_t^m \cdot \left((1-\beta) \cdot \left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho - 1} + \beta \right)^{\frac{\rho}{\rho-1}} \tag{27}$$

$$V_t^{♀,s} = n_t^s \cdot \left((1-\beta) \cdot \left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho - 1} + \beta \right)^{\frac{\rho}{\rho-1}} \tag{28}$$

Combining (27), (28), and (26),

$$\tau^* = \frac{V_t^{♀,s}}{V_t^{♀,m}} = \frac{n_t^s}{n_t^m} = \frac{1}{1 + \alpha_t \Gamma_t^h} \tag{29}$$

Proof of Lemma 3

When A_t increases, $f_1(\alpha_t)$ shifts up while $f_2(\alpha_t)$ shifts down. Therefore, $f_3(\alpha_t) = f_1(\alpha_t) - f_2(\alpha_2)$ shifts up. As a result, the interior solution, i.e., the value of α_t such that $f_3(\alpha_t) = 0$, necessarily decreases.

Figure A.2 provides a graphical illustration of the proof.

Proof of Lemma 4

When α_t falls, $\mathcal{M}(\Gamma^h; \alpha)$ shifts down while $\Gamma^h(\mathcal{M})$ is unaffected. As a result, the intersection (α, Γ^h) falls.

Figure A.3 provides a graphical illustration of the proof.

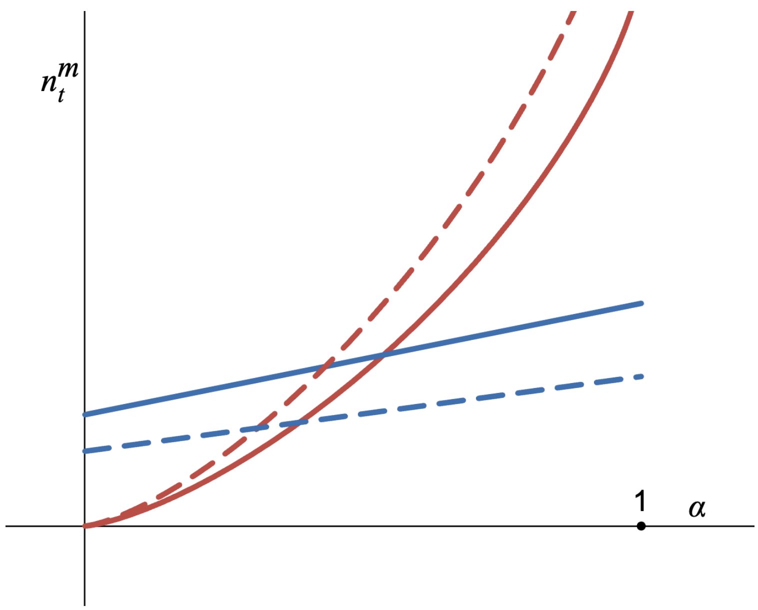

Figure A.2: $f_1(\alpha_t)$ (red) and $f_2(\alpha_t)$ (blue). Solid (before) and dashed (after)

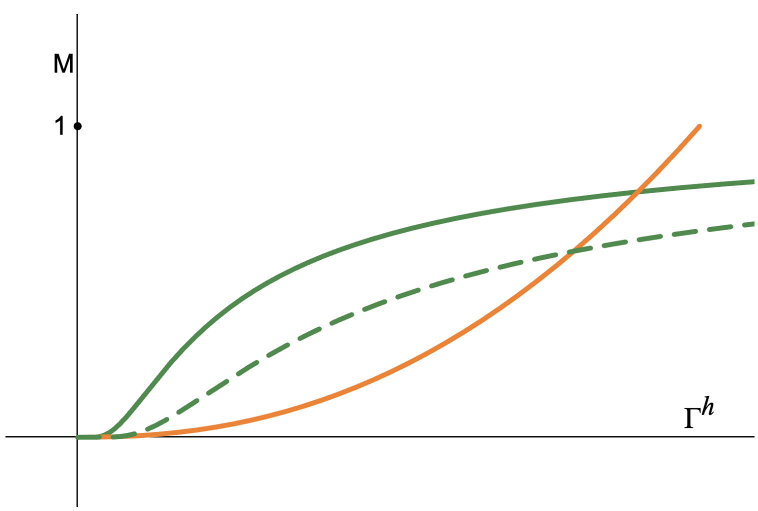

Figure A.3: $\mathcal{M}(\Gamma^h; \alpha)$ (green) and $\Gamma^h(\mathcal{M})$ (orange)

B. Figures

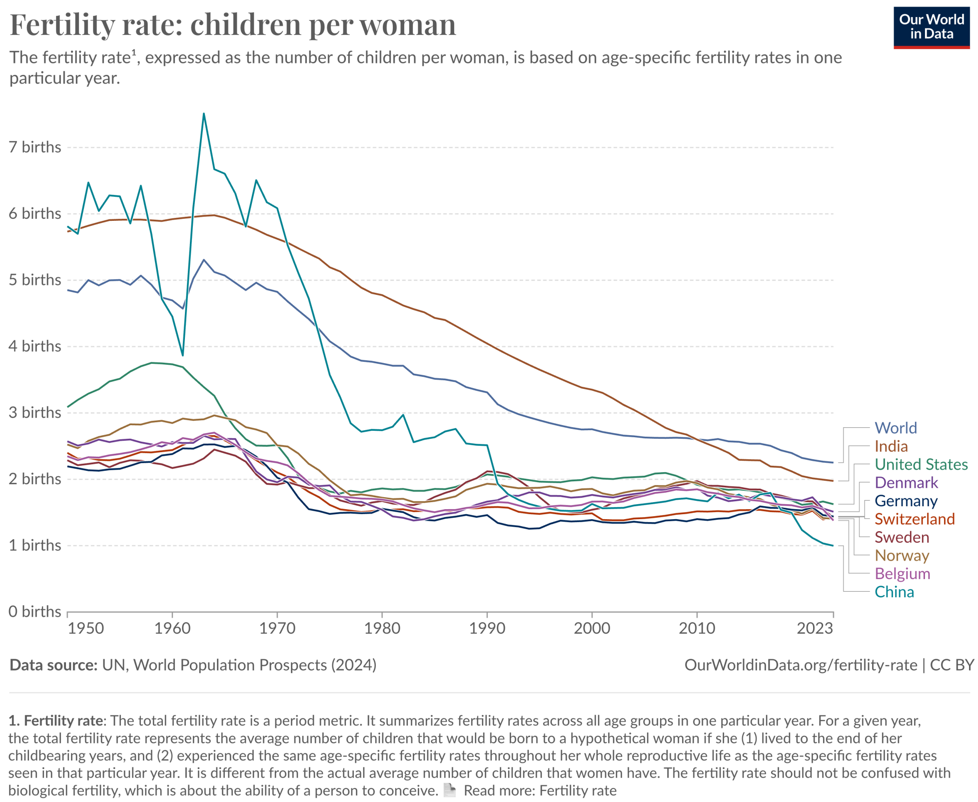

Figure A.4: Declining Fertility

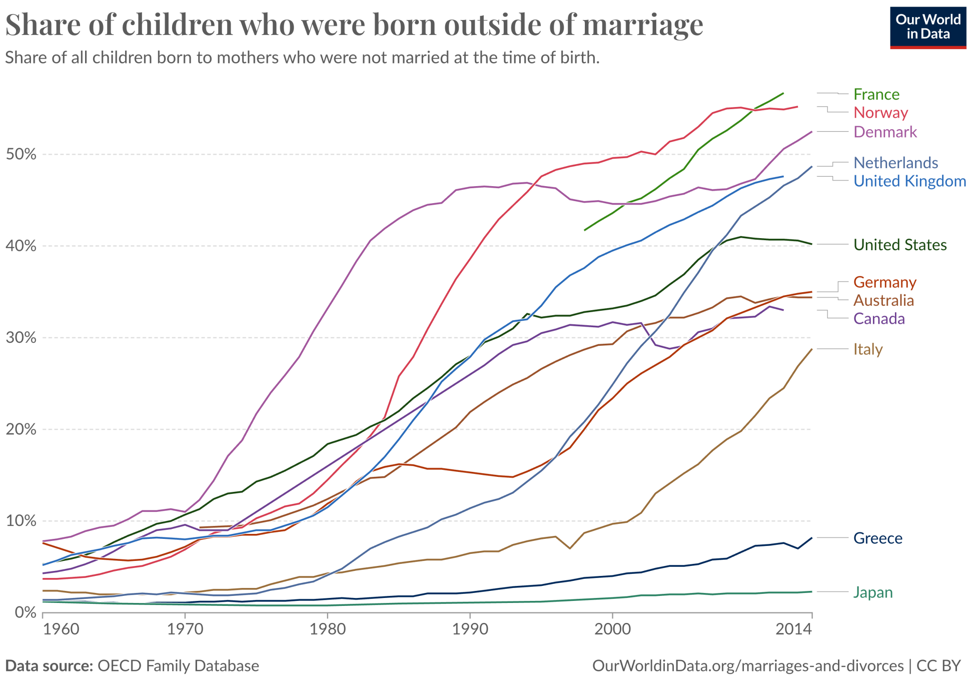

Figure A.5: Rising Single Parenthood

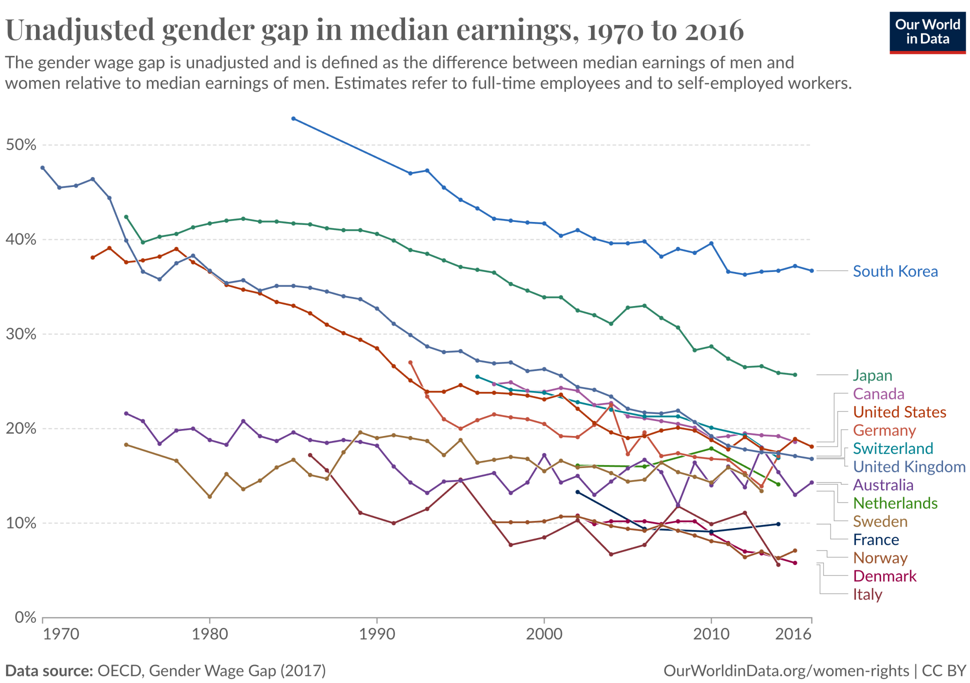

Figure A.6: Converging Gender Gaps

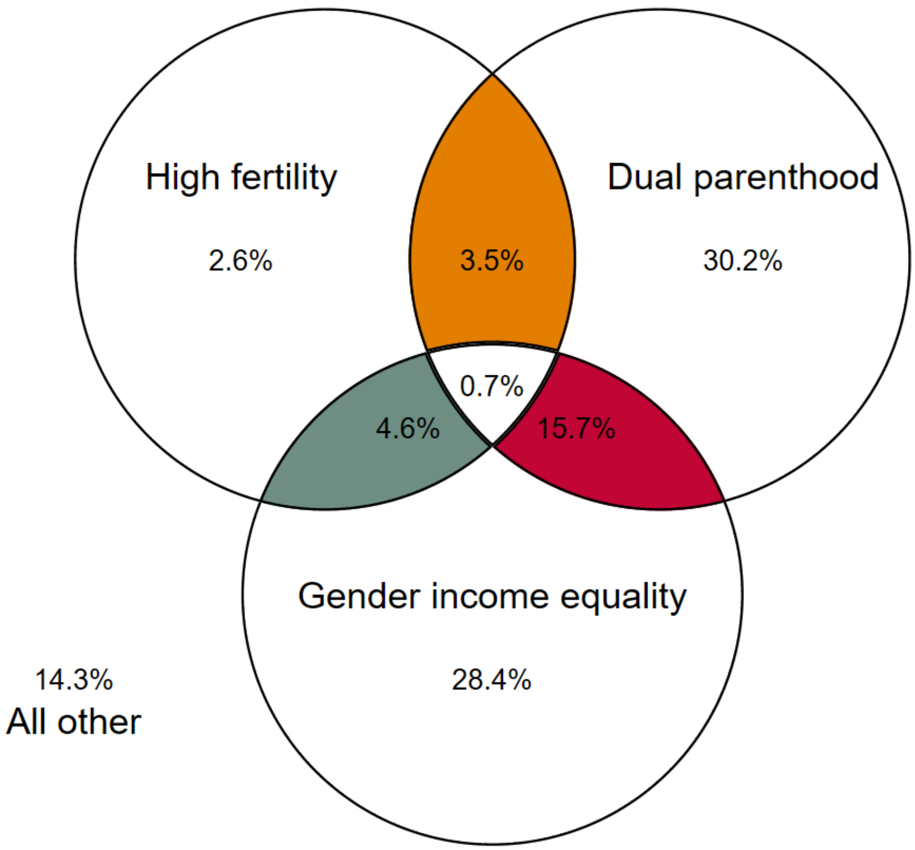

Figure A.7: Impossible Trinity: “High fertility” if TFR¿2

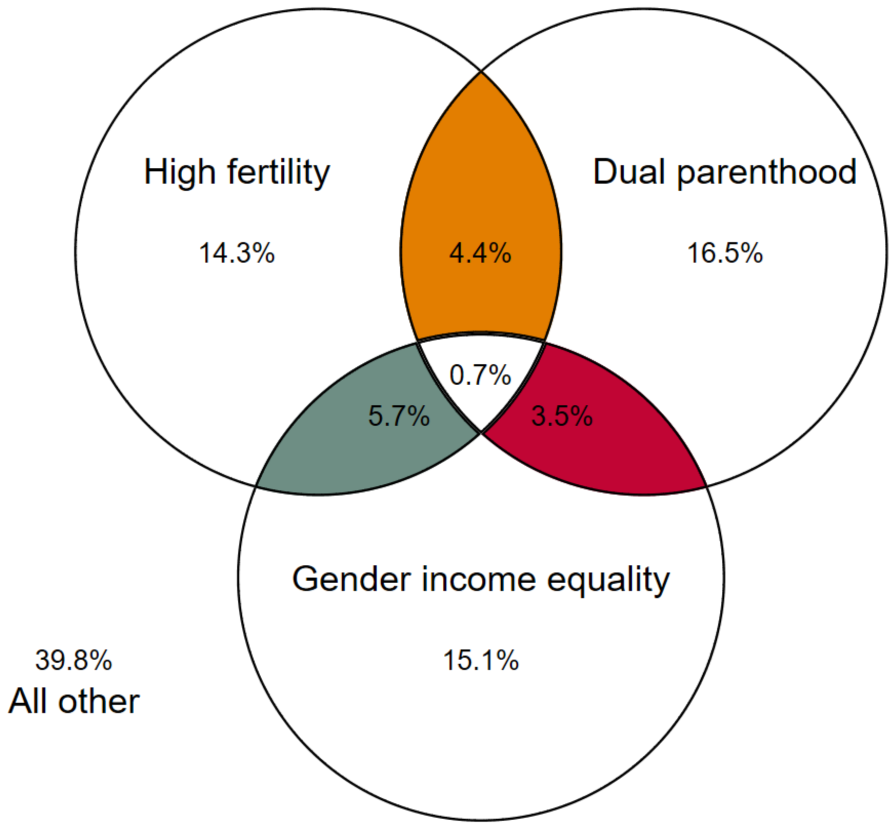

Figure A.8: Impossible Trinity: Define Categories using Upper Quartiles

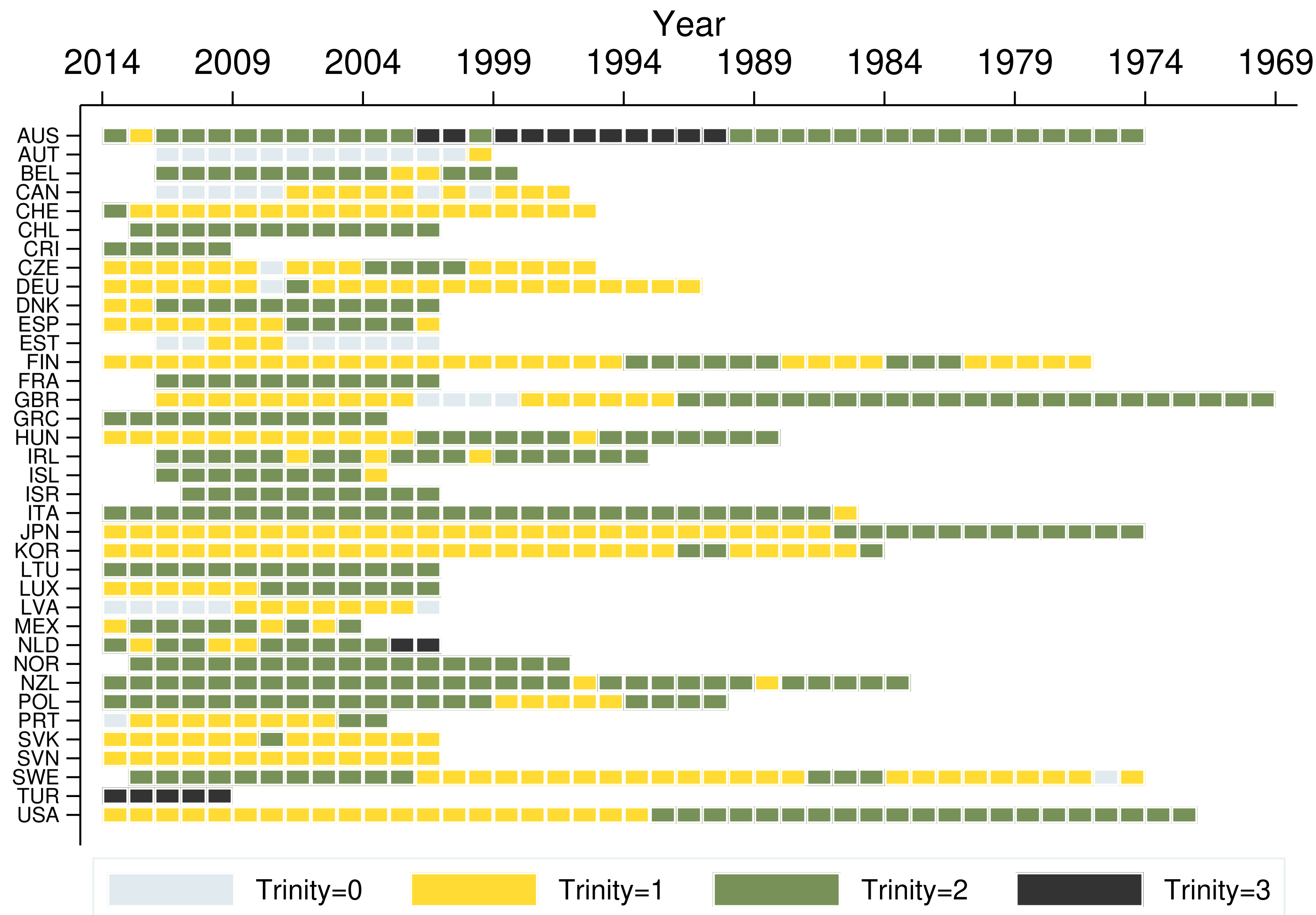

Figure A.9: Number of Outcomes Achieved by Country and Time