Bounding Fertility Elasticities

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Preliminary and Incomplete

June, 2022

Abstract

I propose a new methodology to bound the magnitude of fertility responses to financial incentives. Under mild assumptions, I show that raising the fertility rate by 0.1 children per woman in the United States in 2010 requires the cost of children to fall between \$7,514 and \$35,966. This bound is tighter than the range provided by meta-studies of past estimates and is simple to compute for a different country or year.

1 Introduction

How much does fertility respond to financial incentives? The answer to this question is important for both policymakers and economists. For governments that want to use financial transfers to raise fertility, it is critical to understand how cost-effective these policies are. For economists, fertility elasticity provides a benchmark to discipline models with endogenous fertility, just as the marginal propensity to consumer (MPC) informs modern macroeconomic models. In addition, estimates of the fertility elasticity allow one to evaluate how much the exogenous fertility assumption matters for the quantitative results in structural models on child-related policies, such as the Child Tax Credit (CTC) and the Earned Income Tax Credit (EITC).

There has been a large number of empirical studies estimating the fertility elasticity in different settings.[1] Pinning it down using historical policies has proven to be difficult for several reasons.

[1] See Milligan (2005), Cohen et al. (2013), González (2013) among many others. For review studies, see McDonald (2006) and Stone (2020).

First, there is a lack of large and persistent policies that change the cost of children drastically, making it unlikely for people to change their fertility behaviors, especially the total number of children, in a quantitatively significant way. Second, as most pro-natal policies are nationwide, finding a control group is not straightforward (Gauthier (2005)). This difficulty is especially acute given that most pro-natal policies are adopted to prevent future fertility crashes, while it is unclear whether countries or regions in the control group also face a similar problem (Castles (2003)). In addition, many pro-natal policies come in bundles of incentives that go beyond lowering the cost of children, such as measures encouraging women's labor force participation and attachment. Therefore, estimating the fertility elasticity of the underlying policy is not equivalent to estimating the price elasticity of fertility. As a result of these difficulties, empirical estimates vary widely – "there is considerable disagreement across studies about the effectiveness of pro-natal policies" even though "the directional finding that pro-natal benefits boost fertility is nearly uniform" (Stone (2020)). Last but not least, it is unclear to what extent past estimates predict fertility elasticities in a different time or institutional context.

In this paper, I propose a new methodology to bound the fertility elasticities. Under mild assumptions, I show that a bound can be computed after knowing (1) the prevailing fertility rate and cost of children, (2) the maximum desired fertility, and (3) the cost of children such that parents would rather prefer not to have a child. Applying this method to the United States in 2010, I find that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman. I show that this bound is tighter than that in meta-studies of past estimates and is simple to compute for a different country or year under consideration.

The rest of the paper is organized as follows. Section 2 presents the theory that derives the bound. Section 3 applies the the bound to the United States, compares the results with prior studies, and discusses further implications of the bound. Section 4 concludes.

2 Theory

Denote the Marshallian demand of fertility as $n(p; \boldsymbol{p}^{\text{other}}, y)$ where n is fertility, p is the cost of children, $\boldsymbol{p}^{\text{other}}$ is the price (vector) of other goods, and y is the household's lifetime income.

Assumption 1 The Marshallian demand of fertility $n(p; \boldsymbol{p}^{\text{other}}, y)$ is downward sloping, continuously differentiable, and convex.

This assumption is satisfied by most models of endogenous fertility, either with warm glow (e.g., De La Croix and Doepke (2003)) or with dynastic altruism (e.g., Barro and Becker (1989) and Córdoba and Ripoll (2019)).

Assumption 2 There exists $\overline{p}$ and $\overline{n}$ such that $n(\overline{p}; \boldsymbol{p}^{\text{other}}, y) = 0$ and $n(0; \boldsymbol{p}^{\text{other}}, y) = \overline{n}$.

This is also a mild and realistic assumption. For example, set $\overline{p} = y$, the assumption requires that if having a child costs the parents' entire income and leaves no resources for other goods, then the household would prefer not to have a child. The existence of $\overline{n}$ reflects satiation in preferences or biological constraints of childbearing.

Proposition The fertility response to price around (n^0, p^0) is bounded by

$$\left.\frac{dn}{dp}\right|_{(n^0,p^0)} \in \left(\frac{n^0}{\overline{p} - p^0}, \frac{\overline{n} - n^0}{p^0}\right). \tag{1}$$

Proof Figure 1 illustrates the essence of the proof. The Marshallian demand of fertility is given by curve BAC where the coordinates of B and C are $(0, \overline{p})$ and $(\overline{n}, 0)$ correspondingly. Point A denotes the prevailing fertility and cost of children (n^0, p^0). We can show that the slope of the demand curve at A is bounded by the slope of AC and the slope of AB using the Mean Value Theorem and the assumption that the curve is convex.

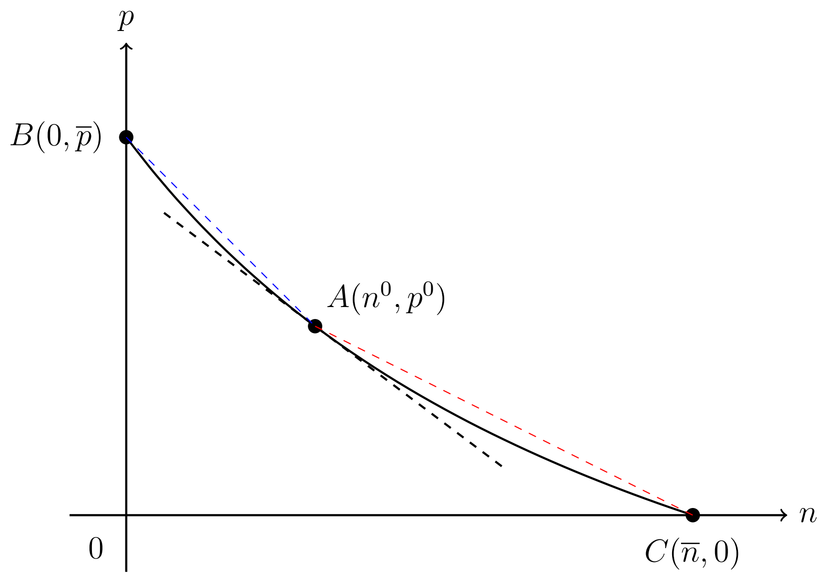

Figure 1: Illustration of the Proof

3 Quantification

In this section, I calculate bound for the United States, compare the results with prior studies, and discuss further implications of the bound.

3.1 Calculating the Bound

I calculate the bound for the United States in 2010 where the prevailing fertility n^0 is 1.9 and the cost of one child p^0 is \$458,351 for a middle-income household (Córdoba and Ripoll (2019)). I make the following assumption on $\overline{n}$ and $\overline{p}$:

Assumption 3 Set $\overline{n} = 8$. Choose $\overline{p}$ such that an average household with one child lives in poverty for the rest of their lives, i.e.,

$$y - \overline{p} = y^{\text{poverty}}. \tag{2}$$

Proposition 1 makes it clear that the choice of $\overline{n}$ and $\overline{p}$ affects the tightness of the bound – as $\overline{n}$ or $\overline{p}$ decreases, the bound becomes tighter. The assumption that $\overline{n} = 8$ is probably conservative towards the high end as the prevailing ideal number of children is around 2.5 children per woman (Stone (2018)). The assumption that $\overline{p}$ is the cost of children that makes an average married household fall into poverty status after one childbirth is also a likely upper bound for $\overline{p}$. Thus, there are reasons to believe that the bound could be made tighter under alternative reasonable assumptions.

Following Córdoba and Ripoll (2019), I set $y = \$2{,}083{,}219$. Using the federal poverty guideline in 2010, I set $y^{\text{poverty}} = \$18{,}310 \times 18 + \$14{,}570 \times 42 = \$941{,}520$. This implies $\overline{p} = \$1{,}141{,}699$.

Applying the proposition, I find that to raise the fertility by 0.1 children per woman, the change in p needed is between \$7,514 and \$35,966. This change in p can be brought about by policies such as a baby bonus, a universal basic income, or a fully-refundable Child Tax Credit (CTC).

Next, I compare my results to meta-studies of past estimates. Stone (2020) summarizes 22 studies since 2000 using historical policies, mostly in low-fertility countries. He concludes that "an increase in the present value of child benefits equal to 10% of a household's (annual) income can be expected to produce between 0.5% and 4.1% higher birth rates." To make the measures comparable, I convert the bound in this paper into percentages using the median annual household income in 2010 (\$49,445). The bound predicts that an increase in the present value of child benefits equal to 10% of a household's (annual) income can produce between 0.72% and 3.46% higher birth rates. As can be seen, the bound proposed here is tighter than existing estimates.[2]

Another advantage of the bound is that it is simple to compute for a different country or year as long we know the prevailing (n^0, p^0), y, and y^{poverty}, which are not difficult to obtain. Adopting an actual pro-natal policy and estimate the fertility elasticities afterwards will likely lead to a more precise estimate. But the bound could still be helpful as it provides a prediction of the program's cost-effectiveness *ex ante* and a check of the regression result *ex post*.

3.2 Implications

Beyond informing policymakers how costly it is to raise fertility, the bound provides a benchmark to discipline models with endogenous fertility and evaluate the exogenous fertility assumption in models that analyze child-related policies.

[2] Another difference worth noting is that the bound proposed in this paper is a *theoretical* bound under assumptions while the range in Stone (2020) is a *statistical* bound that summarizes past estimates.

3.2.1 Endogenous Fertility w/ Dynastic Altruism

Consider a model of fertility choice with dynastic altruism following Barro and Becker (1989). Agents solve

$$U_0 = \max_{c_t, n_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\prod_{i=0}^{t-1} n_i \right)^{1-\varepsilon} \cdot \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$y_t + (1 + r_t)k_t = c_t + n_t(\chi_t + k_{t+1}) \quad \forall t$$

where it is assumed that $\beta, \varepsilon, \sigma \in (0, 1)$ and $\varepsilon + \sigma < 1$ to ensure that children are goods.

Using first-order conditions, the Marshallian demand of fertility in this economy is given by

$$n_t = (\beta(1 + r_{t+1}))^{\frac{1}{\varepsilon}} \left[\frac{\chi_{t-1}(1 + r_t) - y_t}{\chi_t(1 + r_{t+1}) - y_{t+1}} \right]^{\frac{1-\sigma}{\varepsilon}} \tag{3}$$

where the net cost of creating a descendant at time t is $p_t \equiv \chi_t(1 + r_{t+1}) - y_{t+1}$. Therefore, as the net cost of children p_t falls by 1 percent, fertility n_t increases by $\frac{1-\sigma}{\varepsilon}$ percent *ceteris paribus*. Relating this elasticity to the numerical bound for the U.S. in 2010, the value of $\frac{1-\sigma}{\varepsilon}$ should lie between 0.67 and 3.23. Combining this condition with the prior conditions on σ and ε, we have

$$\varepsilon + \sigma < 1 \quad \text{and} \quad 3.23 \cdot \varepsilon + \sigma > 1.$$

The latter condition puts further restrictions on the choice of σ and ε. For example, Cordoba (2015) satisfies these restrictions with $\sigma = 0.3$ and $\varepsilon = 0.288$.

3.2.2 Endogenous Fertility w/ Warm Glow Utility

Consider another model of fertility choice with warm glow utility. Agents solve

$$\max_{c,n} \quad c + \beta \cdot \frac{n^{1-\sigma}}{1-\sigma}$$

subject to

$$c + n \cdot p \leq y.$$

As before, it is assumed that $\sigma \in (0, 1)$ to ensure that children delivers positive utility.

With interior solutions, optimal fertility is given by

$$n^* = \left(\frac{p}{\beta}\right)^{-1/\sigma}.$$

This implies that when p falls by 1 percent, n^* rises by $1/\sigma$ percent. Thus, if one plans to calibrate this model to the U.S. economy in 2010, the bound suggests that the value of σ, which is typically chosen exogenously, should lie between 0.31 and 1. Similarly, one can use the bound to validate comparative static results in other models of endogenous fertility, such as Manuelli and Seshadri (2009), Cordoba et al. (2016), and Daruich and Kozlowski (2020).

3.2.3 Models w/ Exogenous Fertility Assumption

The bound also permits an evaluation of the exogenous fertility assumption in structural models that analyze child-related policies. For example, Guner et al. (2020) consider a policy counterfactual where the per-child tax credit rises about \$800 per child per year, amounting to about \$12,000 in net present value terms for eligible households. The optimal policy in Mullins (2019) is a Negative Income Tax on mothers that is equivalent to an additional \$82 per week over 17 years, i.e., a transfer greater than \$50,000 in net present value. Daruich (2022) shows that the welfare-maximizing early-childhood development subsidy is around \$80,000.

A real-world policy example is the proposed renewal of the (fully-refundable) Child Tax Credit expansion that increases the annual transfer from \$2,000 dollars to to \$3,600 per child under age 6 and \$3,000 per child ages 6 through 17. The net present value of this expansion is above \$30,000 per child for eligible families.

With prior calculations, we see that a \$10,000 reduction in the cost of children increases fertility by 0.03 to 0.13 from the baseline level of 1.9 children per woman. Thus, these proposed policies might lead to non-negligible fertility responses and a dilution of family resources, triggering the quantity-quality trade-off mechanism à la Becker and Lewis (1973). As a result, the policies might not achieve the goal of raising children's human capital, but could nevertheless benefit the society in the long-run through mechanisms such as changing the dependency ratio (Zhou (2022)).

4 Conclusion

The magnitude of fertility responses to financial incentives is important to both policymakers and economists. In this paper, I propose a new methodology to bound the fertility elasticities. Under mild assumptions, I show that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman in the United States in 2010. This bound is tighter than the range provided by meta-studies and is simple to compute for a different country or year.

References

Barro, R. J. and Becker, G. S. (1989). Fertility choice in a model of economic growth. *Econometrica*, pages 481–501.

Becker, G. S. and Lewis, H. G. (1973). On the interaction between the quantity and quality of children. *Journal of Political Economy*, 81(2, Part 2):S279–S288.

Castles, F. G. (2003). The world turned upside down: below replacement fertility, changing preferences and family-friendly public policy in 21 oecd countries. *Journal of European Social Policy*, 13(3):209–227.

Cohen, A., Dehejia, R., and Romanov, D. (2013). Financial incentives and fertility. *Review of Economics and Statistics*, 95(1):1–20.

Córdoba, J. and Ripoll, M. (2019). The elasticity of intergenerational substitution, parental altruism, and fertility choice. *The Review of Economic Studies*, 86(5):1935–1972.

Cordoba, J. C. (2015). Children, dynastic altruism and the wealth of nations. *Review of Economic Dynamics*, 18(4):774–791.

Cordoba, J. C., Liu, X., and Ripoll, M. (2016). Fertility, social mobility and long run inequality. *Journal of Monetary Economics*, 77:103–124.

Daruich, D. (2022). The macroeconomic consequences of early childhood development policies. *Available at SSRN 3265081*.

Daruich, D. and Kozlowski, J. (2020). Explaining intergenerational mobility: The role of fertility and family transfers. *Review of Economic Dynamics*, 36:220–245.

De La Croix, D. and Doepke, M. (2003). Inequality and growth: why differential fertility matters. *American Economic Review*, 93(4):1091–1113.

Gauthier, A. H. (2005). Trends in policies for family-friendly societies. *The New Demographic Regime: Population Challenges and Policy Responses*, pages 95–110.

González, L. (2013). The effect of a universal child benefit on conceptions, abortions, and early maternal labor supply. *American Economic Journal: Economic Policy*, 5(3):160–88.

Guner, N., Kaygusuz, R., and Ventura, G. (2020). Child-related transfers, household labour supply, and welfare. *The Review of Economic Studies*, 87(5):2290–2321.

Manuelli, R. E. and Seshadri, A. (2009). Explaining international fertility differences. *The Quarterly Journal of Economics*, 124(2):771–807.

McDonald, P. (2006). Low fertility and the state: The efficacy of policy. *Population and Development Review*, pages 485–510.

Milligan, K. (2005). Subsidizing the stork: New evidence on tax incentives and fertility. *Review of Economics and Statistics*, 87(3):539–555.

Mullins, J. (2019). Designing cash transfers in the presence of children's human capital formation. *Job Market Paper.[235]*.

Stone, L. (2018). How many kids do women want? Link.

Stone, L. (2020). Pro-natal policies work, but they come with a hefty price tag. Link.

Zhou, A. (2022). The macroeconomic consequences of family policies. *Available at SSRN 3931927*.