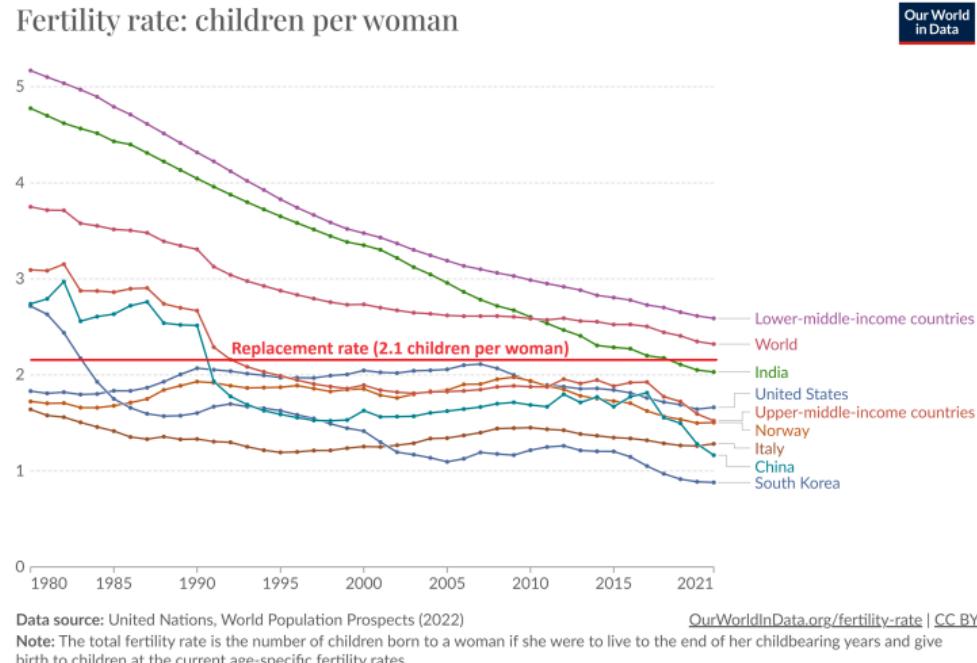


# Asymmetric Fertility Elasticities

Sam Engle    Chong Pang    Anson Zhou

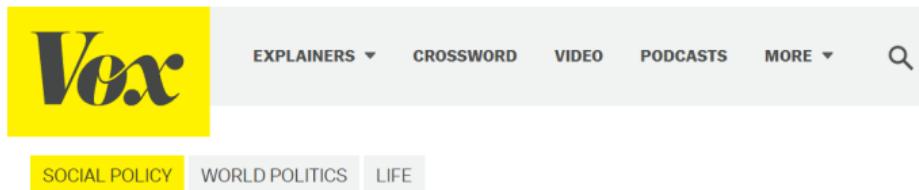
March 2024

# The emergence of below-replacement fertility



- Major implications for pension system, international relations, economic growth (Jones 2022), civilizational risk (Elon Musk)

# Raising fertility seems to be extremely difficult

The image shows the header of the Vox website. It features a yellow square logo with the word "Vox" in black, lowercase, sans-serif letters. To the right of the logo is a grey navigation bar with the following menu items: "EXPLAINERS ▾", "CROSSWORD", "VIDEO", "PODCASTS", "MORE ▾", and a search icon. Below the navigation bar, there are three categories: "SOCIAL POLICY" (highlighted in yellow), "WORLD POLITICS", and "LIFE".

SOCIAL POLICY WORLD POLITICS LIFE

## You can't even pay people to have more kids

These countries tried everything from cash to patriotic calls to duty to reverse drastically declining birth rates. It didn't work.

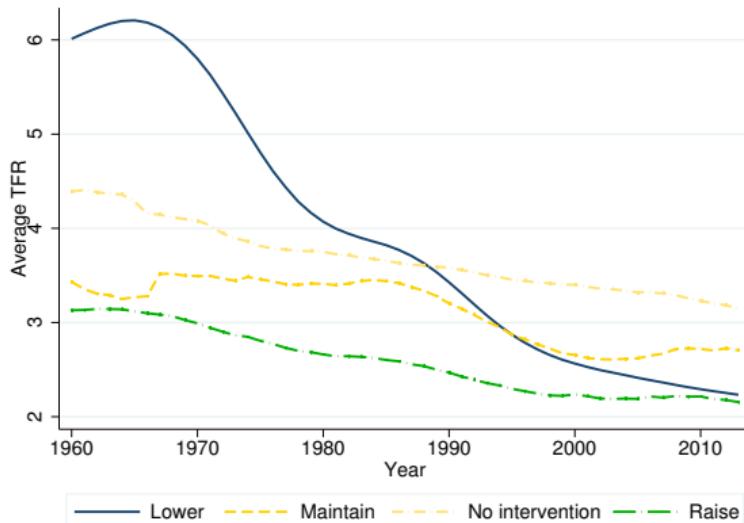
By Anna North | Nov 27, 2023, 8:00am EST

- “There are 27 countries in the Human Fertility Database where, since the 1950 birth cohort, ‘completed cohort fertility’ has ever fallen below a lifetime average of 1.9 children per woman. Never, in any one of these countries, has the average ever again risen above 2.” (Spears 2023)

# Reducing fertility feels easier

Many countries with low fertility problems now were reducing fertility (successfully) not so long ago (e.g., China, Thailand, Singapore, . . .)

[background](#)



Fertility trends by country groups based on policy stance in 1976

# Research question

- At first glance, the performance of pro- versus anti-fertility policies looks quite different
- Research questions:
  1. Is it systematically more difficult to raise fertility than to reduce it?
  2. If so, what are the macro implications and micro-foundations?
- This paper: new fact + new model + new policy implications

# This paper

## 1. Establish a new fact

- Collect historical data on fertility policy stance and funding
- Compare fertility responses at the aggregate and the individual levels
- Discuss robustness to a variety of checks

## 2. Why does the asymmetry matter?

- Develop a dynamic stochastic model of cost minimization by the government where the asymmetry enters in a reduced-form way
  - i Rethink the global campaign towards the replacement rate
  - ii Re-examine the cost-benefit analysis of fertility policies

## 3. What explains the asymmetry?

- Asymmetry challenges existing fertility theories
- Propose a micro-foundation using loss aversion

# Literature

- Empirical evaluations of fertility policies

McElroy and Yang (2000), Liu and Raftery (2020), Schultz (2007), Milligan (2005), Laroque and Salanié (2014), Raute (2019), González and Trommlerová (2023)

Contribution: first to systematically compare +ve and -ve policies

- Structural models of fertility

Barro and Becker (1989), de la Croix and Doepke (2004), Córdoba and Ripoll (2019), Kim, Tertilt, and Yum (2024)

Contribution: first to incorporate loss aversion into fertility choice

- Long-run fertility trajectory

Malthus (1872), Becker (1960), Easterlin (1968), Galor and Weil (2000), Feyrer et al. (2008), Lutz et al. (2007), Ibbotson (2019)

Contribution: a “slippery slope” perspective and new policy insights

# Outline

- Empirical results
- Simple model and policy implications
- Behavioral theory (if time permits)
- Conclusion

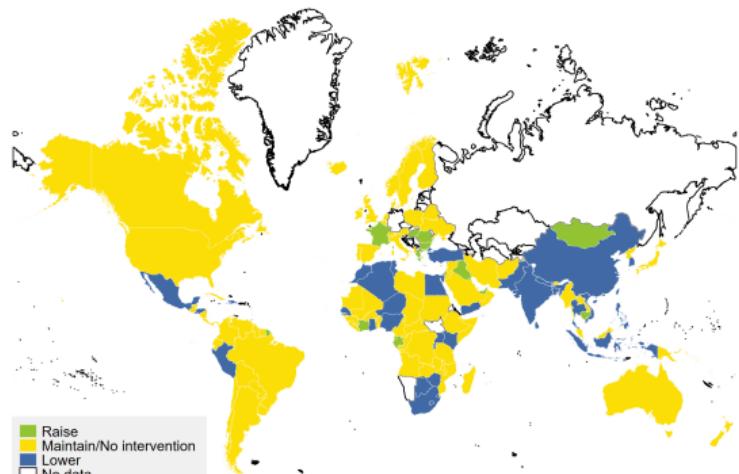
Establish a new fact

Empirical findings

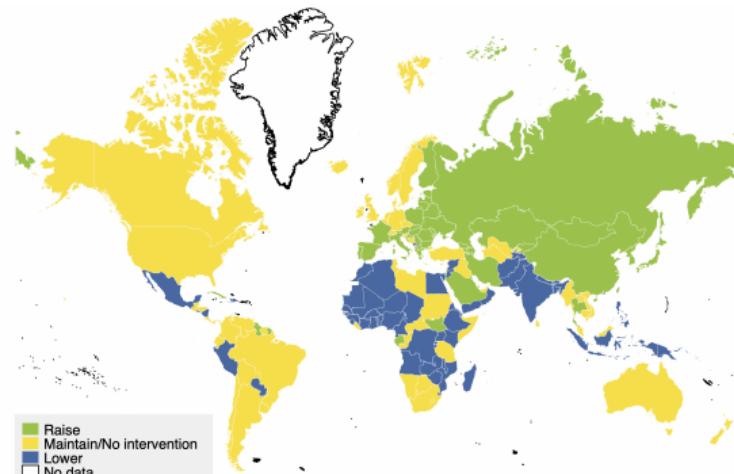
# Data

- Fertility level and policy data from the United Nations
  - Policy stance dummy assigned by the UN Population Division since 1976 - lower, raise, maintain, no intervention
- Aggregate variables from PWT and WDI: GDP per capita, urbanization, infant mortality, female labor force participation
- Family planning funding from de Silva and Tenreyro (2017)
- Individual-level data on fertility, education, and income from the World Value Survey (WVS) Database

# Fertility policy in 1986 and 2021



Source: United Nations Population Division



Source: United Nations Population Division

distribution

# 1. Panel regressions

- We estimate the following specification

$$\begin{aligned}\Delta \text{TFR}_{it}/\text{TFR}_{it-1} = & \alpha + \beta_1 \text{Policy\_Lower}_{it} + \beta_2 \text{Policy\_Raise}_{it} \\ & + \beta_3 \text{Control}_{it} + \sigma_i + \eta_t + \epsilon\end{aligned}\tag{1}$$

- $\text{Control}_{it}$  includes the level and growth rate of GDP per capita, urbanization, infant mortality, and female labor force participation
- Explanatory variables constructed by

$$\text{Policy\_Lower}_{it} = \frac{1}{N} \sum_{T=t-N}^{t-1} \mathbb{I}(\text{Policy}_{iT} = \text{Lower})$$

$$\text{Policy\_Raise}_{it} = \frac{1}{N} \sum_{T=t-N}^{t-1} \mathbb{I}(\text{Policy}_{iT} = \text{Raise})$$

# Results

Table 1: Population Policy and TFR

Policy Variables	ΔTotal Fertility Rate/Lagged Fertility Rate					
	Last Year		Average in the Last Five Years		Average in the Last Ten Years	
	(1)	(2)	(3)	(4)	(5)	(6)
Lower fertility	-0.0118*** (0.0013)	-0.0071*** (0.0055)	-0.0129*** (0.0015)	-0.0076*** (0.0016)	-0.0102*** (0.0020)	-0.0042* (0.0022)
Raise fertility	0.0013 (0.0034)	0.0016 (0.0030)	0.0034 (0.0039)	0.0013 (0.0034)	0.0023 (0.0040)	0.0002 (0.0039)
Country Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes	Yes	Yes
Control Variables	No	Yes	No	Yes	No	Yes
Observations	10726	9146	10726	9146	9937	8462
R <sup>2</sup>	0.133	0.174	0.133	0.173	0.123	0.170

comparison

## 2. Cohort exposure

- Using individual-level data, we estimate the following specification

$$\text{Child}_{icbt} = \alpha + \beta_1 \text{Policy\_Lower}_{cb} + \beta_2 \text{Policy\_Raise}_{cb} + \eta \text{Age}_i \times \text{Gender}_i + \gamma_{ct} + \delta_b + \epsilon \quad (2)$$

- Construct individual's exposure to policies in a 10-year fertile window around mean age of childbirth  $\text{MAC}_{cb}$ :

$$\text{Policy\_Lower}_{cb} = \frac{1}{11} \sum_{t \in [b + \text{MAC}_{cb} - 5, b + \text{MAC}_{cb} + 5]} \mathbb{I}(\text{Policy}_{ct} = \text{Lower})$$

$$\text{Policy\_Raise}_{cb} = \frac{1}{11} \sum_{t \in [b + \text{MAC}_{cb} - 5, b + \text{MAC}_{cb} + 5]} \mathbb{I}(\text{Policy}_{ct} = \text{Raise})$$

# Results

Table 2: Population Policy and the Number of Children

Interpolation of MAC	Number of Children								
	Country-Specific Year Polynomial				Nearest Neighbor			Socioeconomic Variables	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Target: Lower fertility	-0.776*** (0.220)	-0.762*** (0.210)	-0.624*** (0.185)	-0.844*** (0.201)	-0.655*** (0.188)	-0.875*** (0.208)	-0.831*** (0.243)	-0.821*** (0.232)	-0.631*** (0.215)
Target: Raise fertility	0.278 (0.181)	0.304* (0.162)	0.131 (0.186)	0.168 (0.167)	-0.007 (0.185)	0.141 (0.189)	0.259 (0.221)	0.262 (0.191)	0.046 (0.202)
Baseline Controls	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Income Level-Age-Gender FE	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Education Level-Age-Gender FE	No	Yes	Yes	No	Yes	Yes	No	Yes	Yes
Macroeconomic Controls	No	No	Yes	No	No	Yes	No	No	Yes
Observations	205324	183738	163768	231257	205288	182719	210785	186911	170841
R <sup>2</sup>	0.281	0.294	0.301	0.285	0.297	0.303	0.279	0.295	0.298

### 3. Intensive margin

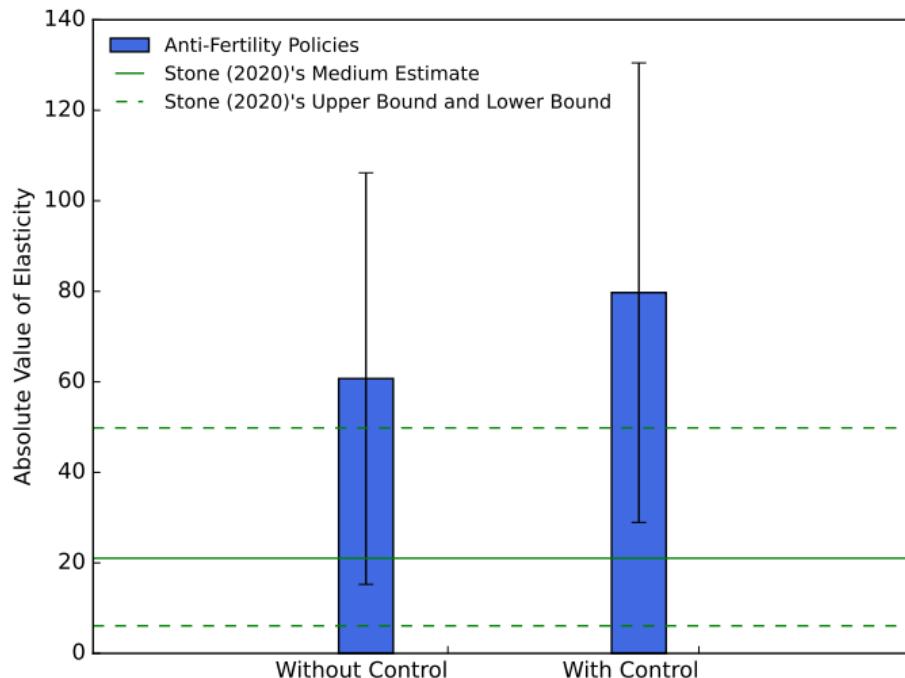
- Using data on family planning funding (de Silva and Tenreyro 2017)

*Table 3: Elasticity Estimation for Anti-Fertility Policy*

Dependent Variable	$\Delta$ Total Fertility Rate/ Lagged Total Fertility Rate	
Construction of Policy Variables	Average in the Last Five Years	
	(1)	(2)
Family planning funding-GDP Ratio	-60.72*** (22.65)	-79.71*** (25.29)
Country Fixed Effect	Yes	Yes
Year Fixed Effect	Yes	Yes
Control Variables	No	Yes
Observations	2754	2648
$R^2$	0.220	0.278

# Results

- Combine with harmonized estimates of pro-fertility policies (Stone (2020))



# Robustness

- Empirical finding is robust to
  1. Use levels instead of percentage changes in fertility
  2. Policy effects at different horizons
  3. Country-specific trends
  4. Controlling for past fertility to mitigate reverse causality
  5. Split sample by initial fertility and GDP per capita
  6. Evaluate the cumulative contributions of policies to fertility changes for specific countries and compare with existing studies (in progress)

The object we care about is the **ratio** between coefficients

Why does the fact matter?

A simple quantitative model

# Model setup

- Government takes the status quo fertility  $n_t^r$  as given and chooses realized fertility  $n_t$  to maximize the present value:

$$\underbrace{\mathcal{W}(n_t^r)}_{\text{positional value}} = \max_{n_t} - \underbrace{\mathcal{P}(n_t, n_t^r)}_{\text{policy expenditure}} - \underbrace{\mathcal{S}(n_t, \bar{n})}_{\text{social cost}} + \beta \cdot \mathbb{E}_\epsilon \mathcal{W}(n_{t+1}^r)$$

$$\mathcal{P}(n_t, n_t^r) = \begin{cases} \pi^+ \cdot (\log(n_t) - \log(n_t^r)) & \text{if } n_t \geq n_t^r \\ \pi^- \cdot (\log(n_t^r) - \log(n_t)) & \text{if } n_t < n_t^r \end{cases} \quad \underbrace{\pi^+ > \pi^- > 0}_{\text{asymmetry}}$$

$$\mathcal{S}(n_t, \bar{n}) = \lambda \cdot (\log(n_t) - \log(\bar{n}))^2$$

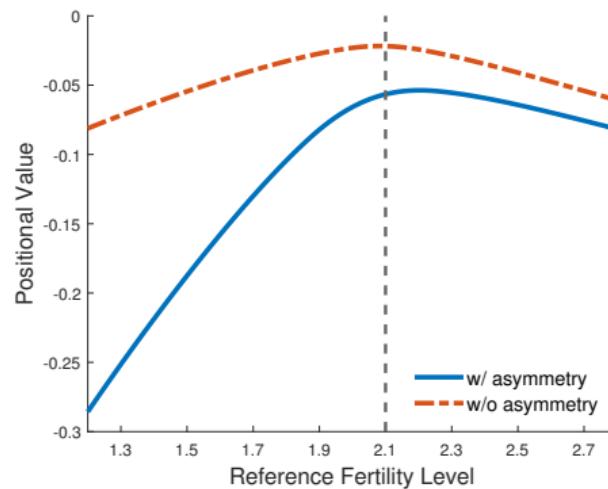
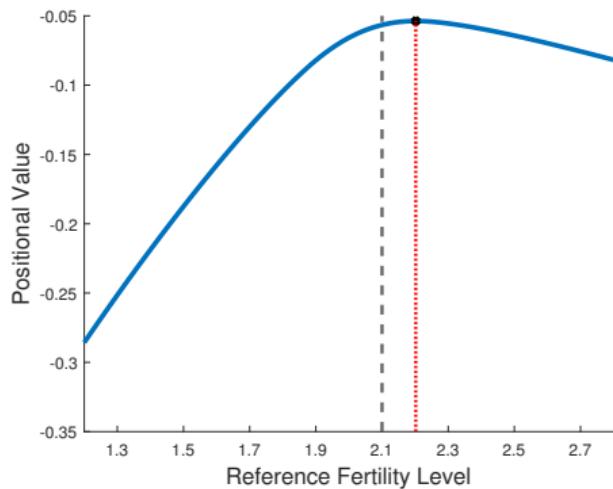
- Adaptive reference updating subject to shocks (Thakral and Tô 2021)

$$\log(n_{t+1}^r) = \phi \cdot \log(n_t) + (1 - \phi) \cdot \log(n_t^r) + \epsilon, \quad \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

# Calibration

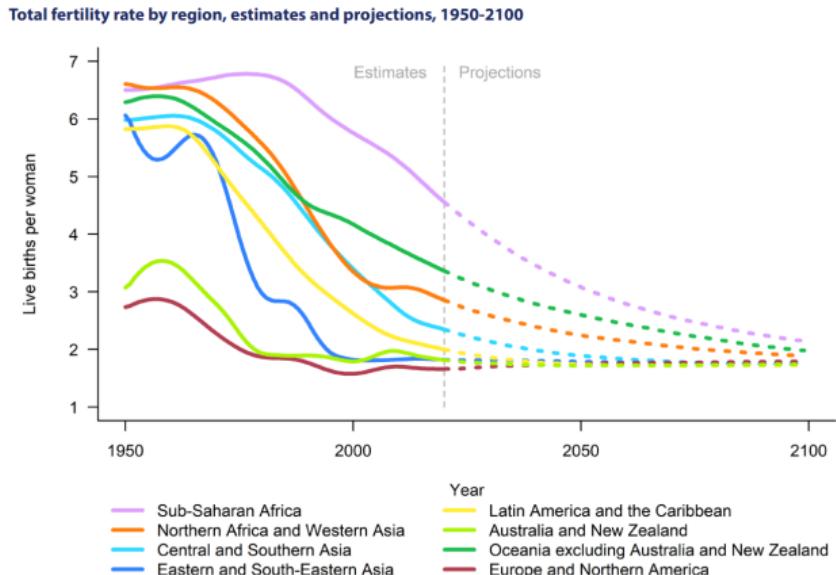
- $\beta = 0.96$  for an annual model
- Policy expenditures needed to change fertility  $\pi^+ = 0.05$  and  $\pi^- = 0.014$  (% of GDP) **from empirical estimates**
- Social costs of fertility  $\mathcal{S}(n_t, \bar{n}) = \lambda \cdot (\log(n_t) - \log(\bar{n}))^2$ 
  - $\bar{n} = 2.1$ : a commonly stated policy goal
  - $\lambda \in \{0.02, 0.2, 2\}$ : TFR=1.64 (USA 2022) results in a social cost of 0.65% of GDP annually
- Law of motion  $\log(n_{t+1}^r) = \phi \cdot \log(n_t) + (1 - \phi) \cdot \log(n_t^r) + \epsilon, \epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$ 
  - $\phi \in \{0.05, 0.13, 0.25\}$ : the expected half-life of the  $n_t^r$  is five years
  - $\sigma_\epsilon \in \{0.01, 0.05, 0.1\}$  - a one s.t.d. shock in fertility is 5%

# Positional value of fertility level



- Key observation:  $\pi^+ > \pi^- \implies \text{argmax}_n \mathcal{W}(n) > \bar{n}$
- Countries ignoring asymmetric fertility elasticities might go “too far” when they reduce fertility, landing in the steep part of  $\mathcal{W}(n)$

# 1. Rethink the global campaign towards $\bar{n} = 2.1$



Source: United Nations Department of Economic and Social Affairs, Population Division (2019a). *World Population Prospects 2019*.

- This paper:  $\bar{n}$  is not a good target in the presence of asymmetry

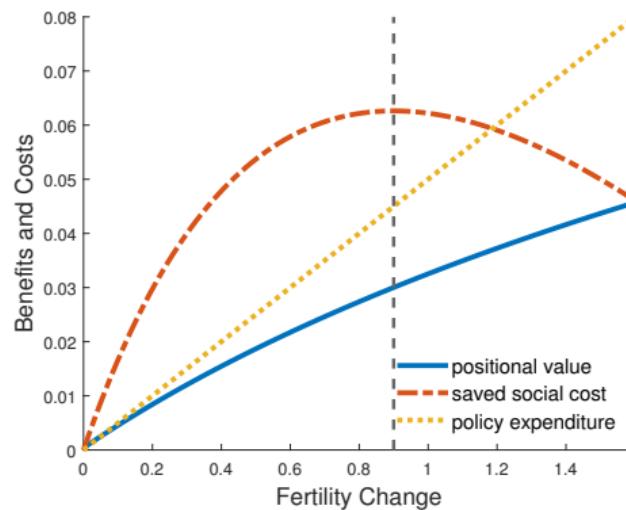
# If not $\bar{n}$ , then what?

- Following the **buffer-stock intuition**, we find that the cost-minimizing reference fertility  $n^* = \operatorname{argmax}_n \mathcal{W}(n)$ :
  - is greater than  $\bar{n}$  as long as  $\pi^+ > \pi^-$
  - increases with the social cost of fertility deviations from  $\bar{n}$
  - increases with the magnitude of reference level shocks
- $n^*$  does not depend much on the speed of reference updating

cost shock speed

## 2. A missing part in static cost-benefit analysis

$$\mathcal{W}(n_t^r) = \max_{n_t} -\mathcal{P}(n_t, n_t^r) - \mathcal{S}(n_t, \bar{n}) + \beta \cdot \mathbb{E}_\epsilon \mathcal{W}(n_{t+1}^r)$$



- Start with  $n_t^r = 1.2$  and simulate different pro-fertility policies
- Gains in positional value due to changing future state variable  $n_{t+1}^r$
- Such gains are particularly large in the presence of asymmetry

To recap, we have

- Established asymmetric fertility responses to policies
- Before micro-foundng why  $\pi^+ > \pi^-$ , we show that
  1. A cost-minimizing government should set  $n^* > \bar{n}$  for precaution
  2. Changes in positional value should be considered in the cost-benefit analysis of fertility policies

What explains the fact?

A behavioral model of fertility

# Asymmetry challenges existing models

- Existing models of fertility choice typically look like

$$\max_{c,n,(e,\dots)} U(c, n, e, \dots) \quad \text{subject to} \quad c + \chi n + \dots = I$$

⇒ smooth aggregate Marshallian demand  $n(\chi, \dots)$

- The smoothness result holds uniformly in this class of models

- Static and dynamic problems
- Altruistic and warm glow preferences
- Continuous and discrete fertility choices
- Representative and heterogeneous agents
- With and without quantity-quality trade-off or status competition

- Inconsistent with asymmetric elasticities  $\left. \frac{\partial n}{\partial \chi} \right|_+ > \left. \frac{\partial n}{\partial \chi} \right|_-$

# Why loss aversion?

- A large theoretical and empirical literature on loss aversion w/ applications to labor supply, portfolio choice, voting, tax filing, ...
- “The sweet, sweet life of America’s DINKs” (Business Insider)



- Loss aversion as a potential explanation for the asymmetry

alternatives

# A Behavioral Theory of Fertility Choice

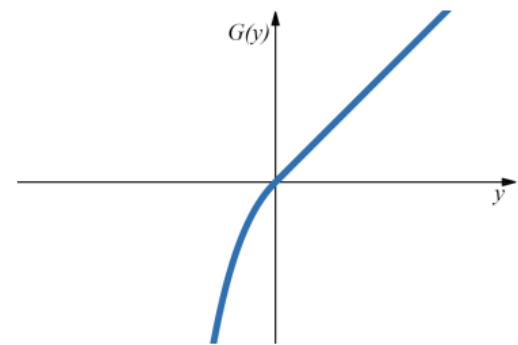
- Households solve

$$\max_{c,n} (1 - \alpha)(u(c) + v(n)) + \alpha G(u(c) - u(x))$$

$$c + \chi n = I \quad v(n) = \frac{n^{1-\gamma} - 1}{1 - \gamma} \quad \gamma > 1$$

- Loss aversion à la Santoro et al. (2014)

$$G(y) = \begin{cases} y & y \geq 0 \\ 1 - \exp(-y) & y < 0 \end{cases}$$



- Consistency:  $x = c$  in equilibrium with RA

# Result

- *Proposition:* In the comparative statics of this economy,

$$\left. \frac{\partial n}{\partial \chi} \right|_+ > \left. \frac{\partial n}{\partial \chi} \right|_-$$

- Intuition of the proof:
  - Loss aversion creates a kink in the marginal benefit of  $c$  around the reference level  $x$
  - When changes in the cost of fertility  $\chi$  affects the marginal cost of  $c$ 
    - consumption responses are different depending on  $\chi \uparrow$  or  $\chi \downarrow$
    - Due to the budget constraint, fertility responses are different
- Future: integrate the static problem into a dynamic quantitative model of demographic transition and study policy impacts

proof details

# Conclusion

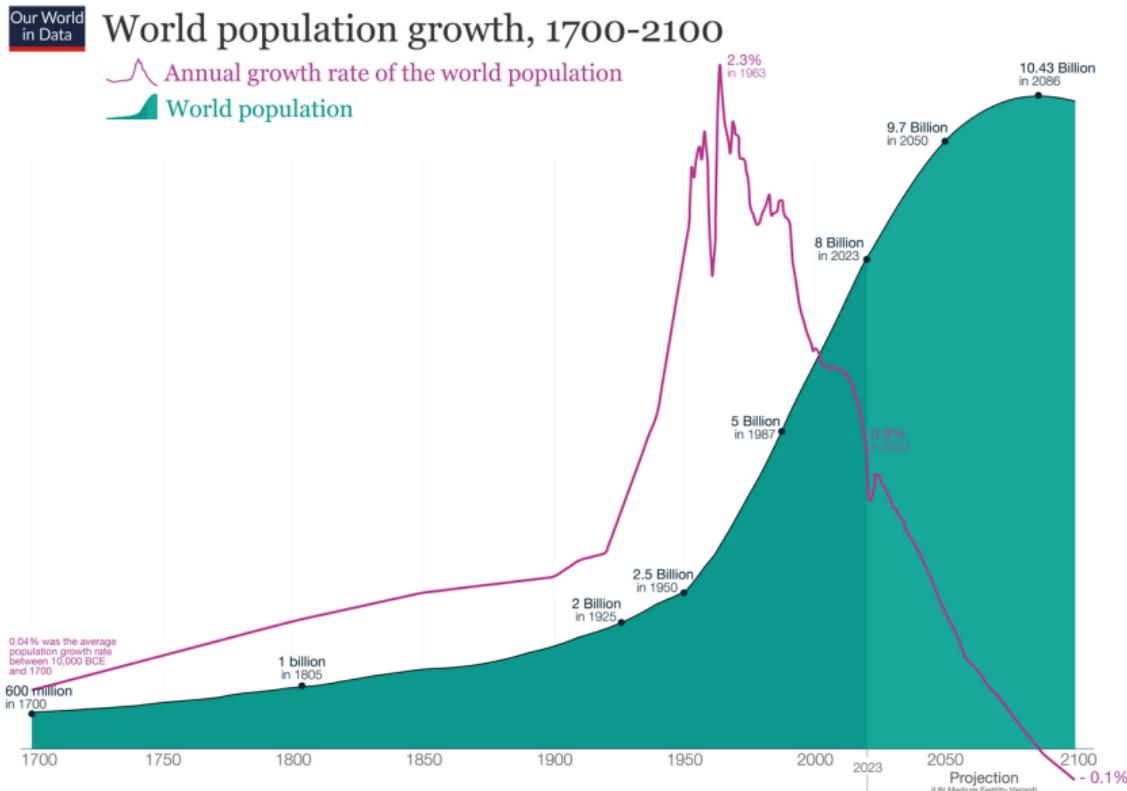
1. Document a new fact: asymmetric fertility elasticities
2. Build a dynamic cost-minimization model of the government
  - The cost-minimizing fertility is higher than the commonly-targeted replacement level
  - Fertility level has large **buffer-stock value**
3. Provide a micro-foundation using a behavioral theory of fertility choice under loss aversion

"Demographics determine the destiny of a people."

–Lee Kuan Yew

## Appendix

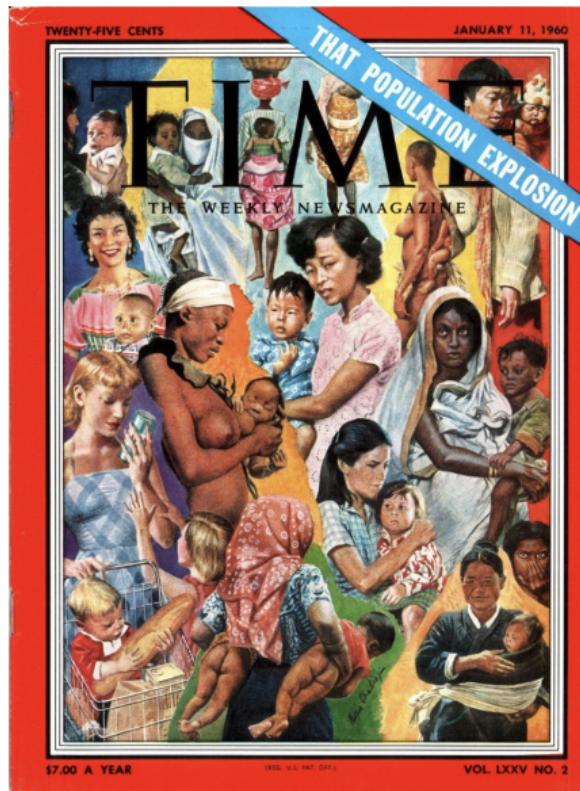
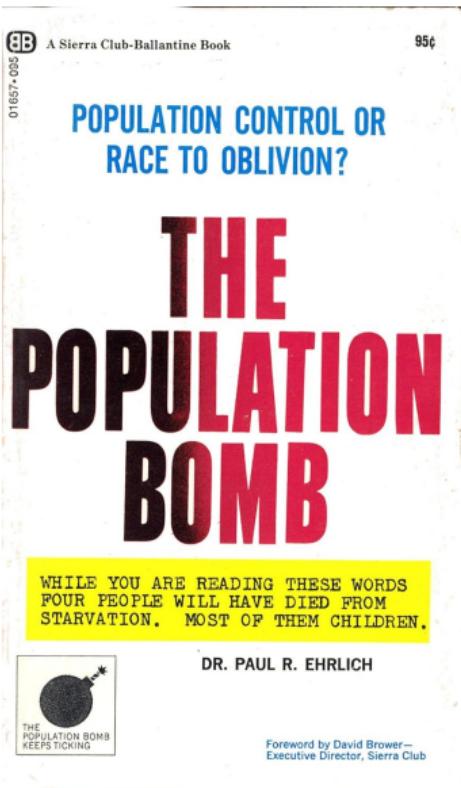
# The specter of Malthus in the 1960s



Data sources: Our World in Data based on HYDE, UN, and UN Population Division [2022 Revision].  
This is a visualization from [OurWorldinData.org](https://ourworldindata.org), where you find data and research on how the world is changing.

Licensed under CC-BY by the authors Max Roser and Hannah Ritchie.

# The population bomb

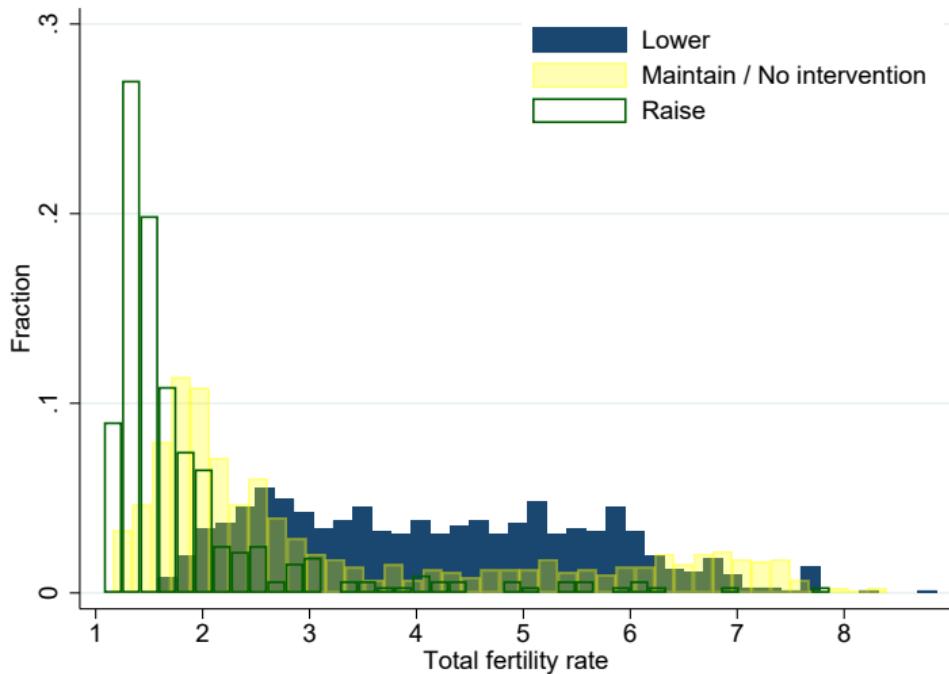


# The global family planning movement

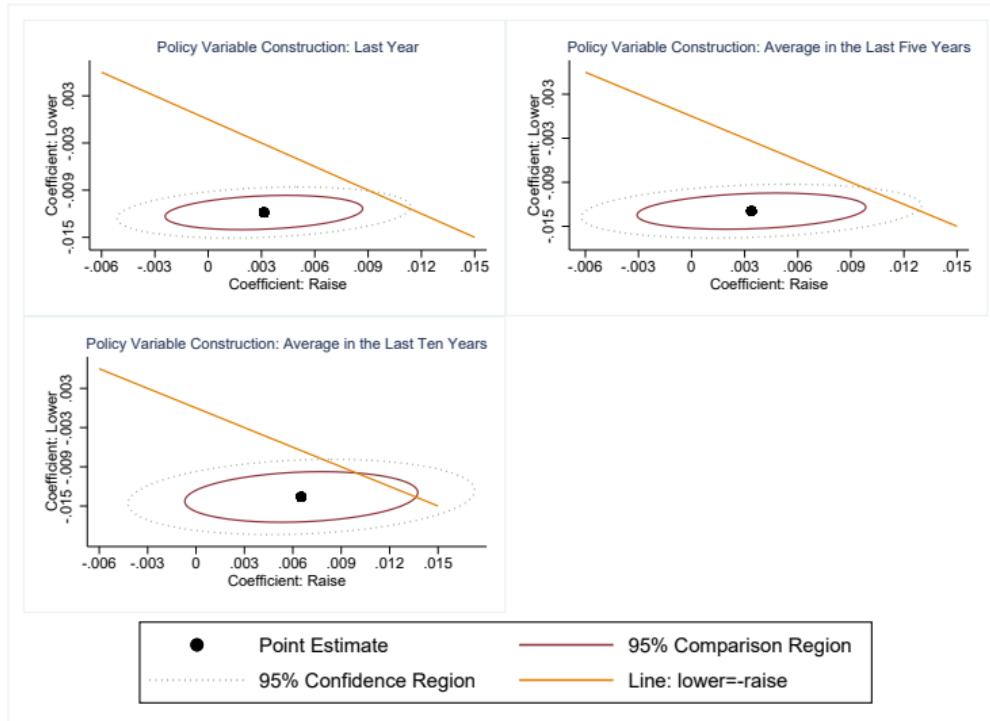
- Led by global organizations such as the United Nations, the World Bank, USAID, and Bill & Melinda Gates Foundation
- \$4.2 billion spent across low- & lower-middle-income countries in 2021
- Many country-specific policies (e.g., the one-child-policy in China)
- Gradually attaches more benefits to low fertility: economic development, health, gender equity, environment...
- Evidence that fertility policies played an important role in the rapid fertility decline (de Silva and Tenreyro 2020)

back

# Fertility policy distribution



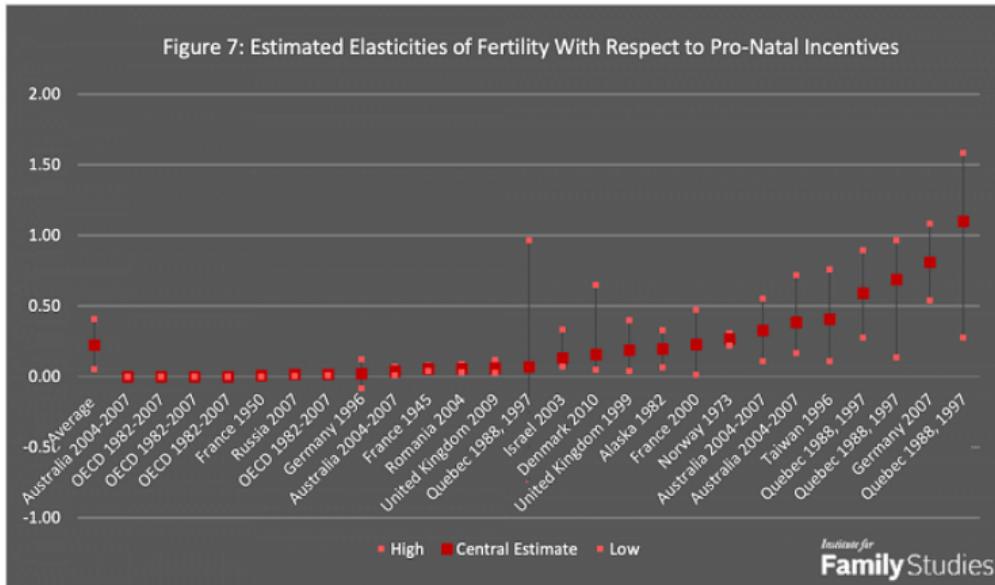
# Confidence region of coefficients



- Wald test-based comparison regions (Eckert and Vach 2020)

back

# Responses to pro-fertility policies



- “An increase in the present value of child benefits equal to 10% of a household’s income can be expected to produce between 0.5% and 4.1% higher birth rates.” (Stone 2020)

back

# Alternatives

- Propagating mechanisms?
  - E.g., peer pressure, human capital complementarities ...
  - They make elasticities larger, but not asymmetric
- Asymmetric technological feasibility?
  - **Insight:** The toolbox of policymakers is diverse but symmetric more
  - Within the toolbox, the observed choices could be different depending on policy direction. But what explains this pattern?

back

# Technological Reversibility

1. Propaganda: "It's better to make a family disappear than to make a second new birth appear" (China) & "have one for mum, one for dad and one for the country" (Australia) & "Do it for Denmark"
2. Family policies: childlessness tax (Soviet) & maternity capital (Russia)
3. Access to tech.: planned parenthood (global) & Decree 770 (Romania)
4. Reproductive coercion: forced sterilization (Bangladesh) & monthly gynecological exam w/ plant-level birth target (Romania)

Fertility policies have different combinations of cost-effectiveness and repugnancy. But each of them is **technologically feasible** in either direction

back

# Control for past fertility

Table 4: Population Policy and TFR: Control Average TFR in the Last Five Years

Dependent Variable Construction of Policy Variables	$\Delta$ Total Fertility Rate/Lagged Fertility Rate			
	Last Year		Average in the Last Five Years	
	(1)	(2)	(3)	(4)
Lower fertility	-0.0121*** (0.0014)	-0.0065*** (0.0015)	-0.0134*** (0.0016)	-0.0070*** (0.0017)
Raise fertility	0.0031 (0.0037)	0.0013 (0.0033)	0.0033 (0.0043)	0.0009 (0.0038)
Country Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Control Variables	No	Yes	No	Yes
Average TFR in the Last Five Years	Yes	Yes	Yes	Yes
Observations	9881	8446	9881	8446
$R^2$	0.134	0.182	0.133	0.182

back

# Country-specific trends

Table 5: Population Policy and TFR: Control Country-Specific Linear Trend

Construction of Policy Variables	Dependent Variable ΔTotal Fertility Rate/Lagged Fertility Rate			
	Last Year		Average in the Last Five Years	
	(1)	(2)	(3)	(4)
Lower fertility	-0.0040** (0.0018)	-0.0050** (0.0019)	-0.0038 (0.0026)	-0.0054** (0.0026)
Raise fertility	-0.0006 (0.0039)	-0.0001 (0.0037)	-0.0004 (0.0047)	0.0009 (0.0045)
Country Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Country-Specific Linear Trend	Yes	Yes	Yes	Yes
Control Variables	No	Yes	No	Yes
Observations	10726	9146	10726	9146
$R^2$	0.204	0.220	0.203	0.220

# Split samples

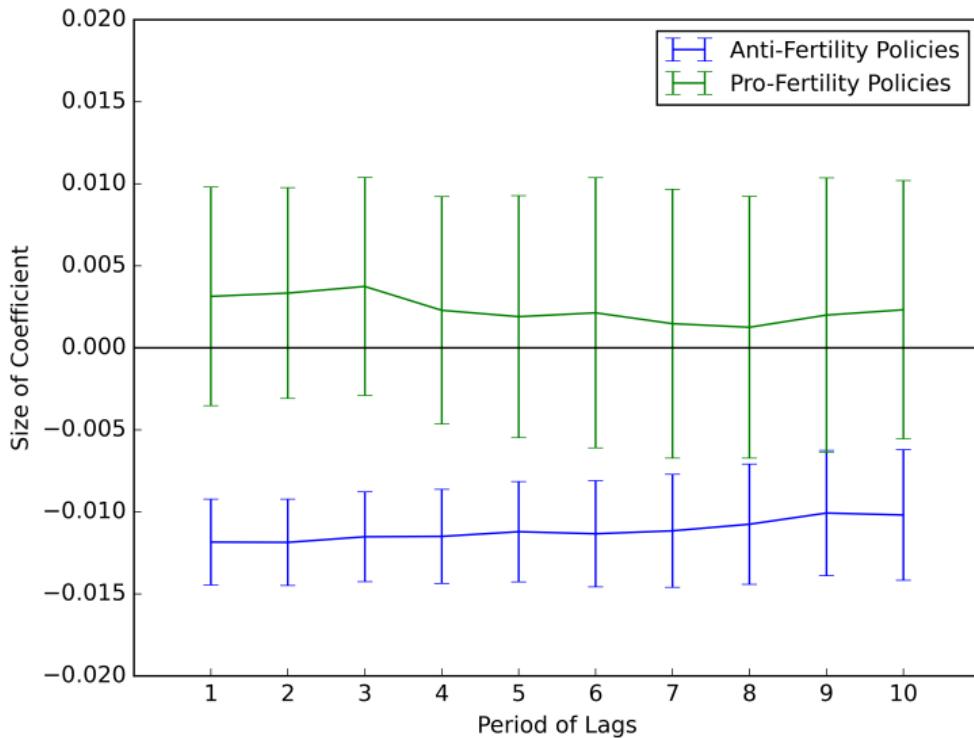
Panel A: Subsample with High TFR in 1960				
Dependent Variable	$\Delta$ Total Fertility Rate/Lagged Fertility Rate			
Construction of Policy Variables	Last Year		Average in the Last Five Years	
	(1)	(2)	(3)	(4)
Lower fertility	-0.0076*** (0.0014)	-0.0056*** (0.0014)	-0.0080*** (0.0018)	-0.0057*** (0.0018)
Raise fertility	0.0003 (0.0034)	0.0005 (0.0055)	0.0009 (0.0062)	0.0007 (0.0056)
Observations	5936	5247	5936	5247
$R^2$	0.339	0.390	0.337	0.388

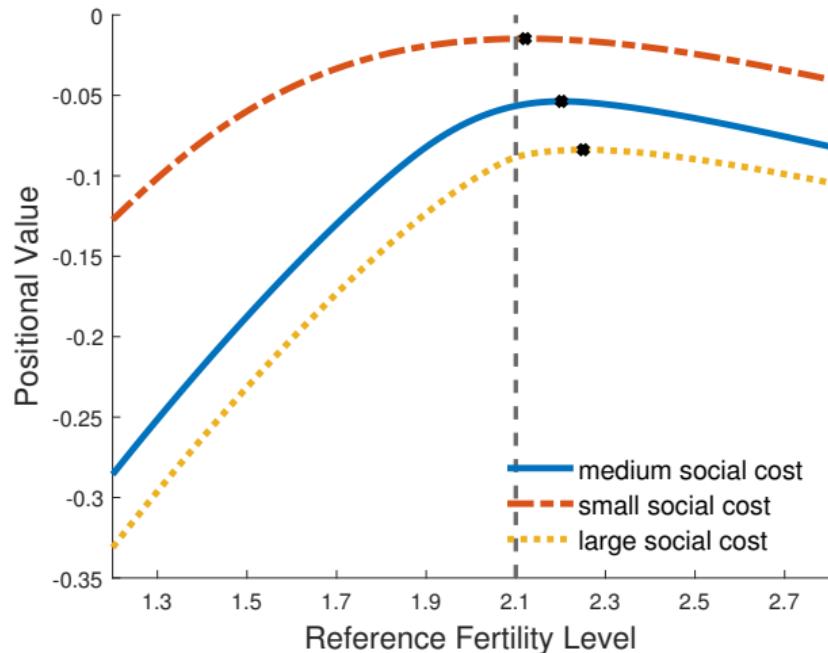
Panel B: Subsample with Low TFR in 1960				
Dependent Variable	$\Delta$ Total Fertility Rate/Lagged Fertility Rate			
Construction of Policy Variables	Last Year		Average in the Last Five Years	
	(1)	(2)	(3)	(4)
Lower fertility	-0.0150** (0.0028)	-0.0117** (0.0049)	-0.0151*** (0.0023)	-0.0117** (0.0047)
Raise fertility	0.0016 (0.0038)	0.0030 (0.0037)	0.0024 (0.0044)	0.0038 (0.0043)
Country Fixed Effect	Yes	Yes	Yes	Yes
Year Fixed Effect	Yes	Yes	Yes	Yes
Control Variables	No	Yes	No	Yes
Observations	4789	3899	4789	3899
$R^2$	0.128	0.147	0.128	0.147

back

# Different horizons

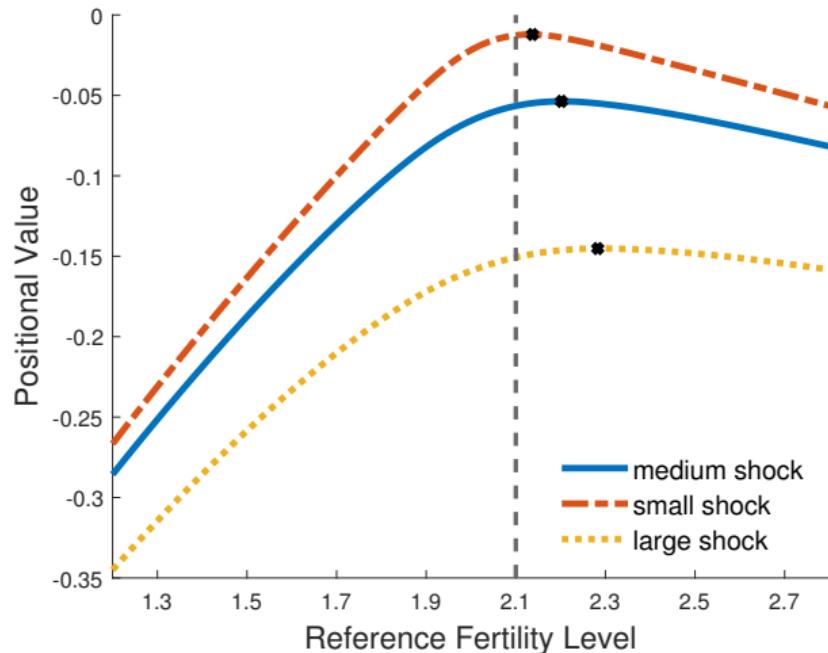


# The role of social cost $\lambda$



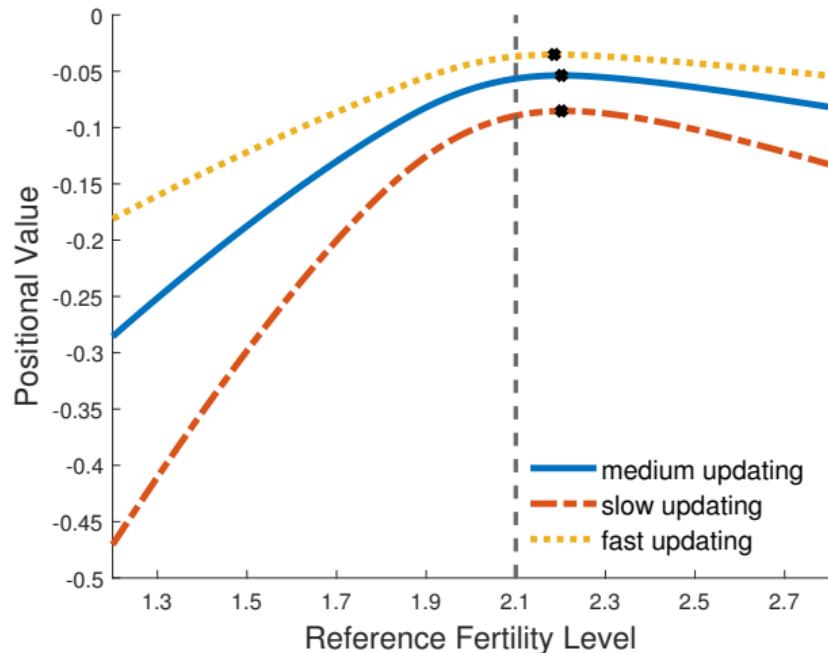
back

# The role of reference shock $\sigma_\epsilon$



back

# The role of reference updating speed $\phi$

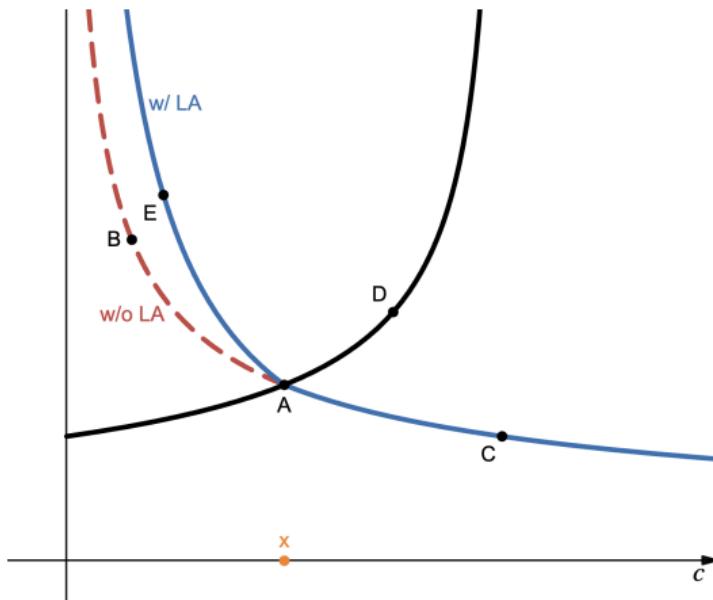


back

# Optimal choice

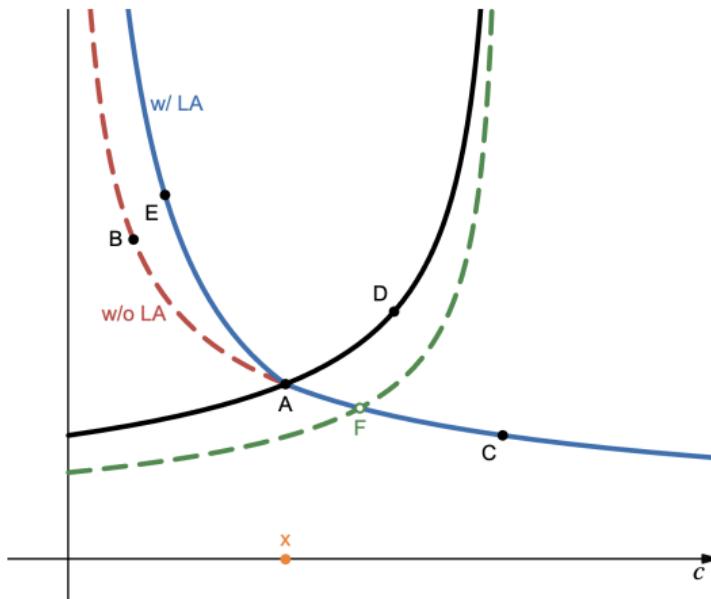
- The first-order condition of optimal consumption satisfies

$$(1 - \alpha)u'(c) + \alpha u'(c)G'(u(c) - u(x)) = \frac{1}{\chi}v' \left( \frac{y - c}{\chi} \right)$$



# Falling price of fertility $\chi$

- When  $\chi$  falls, optimal choices coincide with and without loss aversion



# Rising price of fertility $\chi$

- When  $\chi$  rises, optimal consumption falls *less* with loss aversion  $\Rightarrow$  fertility needs to reduce by *more* due to the budget constraint

