Bounding Fertility Elasticities

Anson Zhou

Preliminary and Incomplete

June, 2022

Abstract

I propose a new method to bound the magnitude of fertility responses to financial incentives. Under mild assumptions, I find that raising the fertility rate by 0.1 children per woman in the United States in 2010 requires the cost of children to fall by $7,514 to $35,966. This bound is tighter than the range provided by meta-studies of past estimates and is simple to compute for a different country and year. I also discuss the implications of this bound for models with and without endogenous fertility choices.

1 Introduction

How much does fertility respond to financial incentives? This question has significant implications for policymakers and economists. For many governments that plan to use financial transfers to raise fertility in order to combat population aging, it is vital to understand how cost-effective these policies are. For economists, price elasticity of fertility demand (hereafter fertility elasticity) provides a fundamental quantitative discipline to models with endogenous fertility choices. Furthermore, fertility elasticity enables a transparent evaluation of the exogenous fertility assumption which is widely adopted in structural models devoted to analyzing child-related policies.

Despite the great significance of the fertility elasticity and a large body of empirical studies,[1] little consensus has been reached (Stone (2020)). Estimation based on historical policies has proven

[1] See Milligan (2005), Laroque and Salanié (2008), Cohen et al. (2013), and González (2013) among many others. For review studies, see McDonald (2006), Bergsvik et al. (2020), and Stone (2020).

to be difficult for several reasons. First, there is a lack of large and persistent policies, making it unlikely for people to change their fertility behaviors, especially the total number of children, in a quantitatively significant way. Second, as pro-natal policies are typically nationwide or income-dependent, finding a control group is not straightforward (Gauthier (2005)). This difficulty is especially acute given that most pro-natal policies are adopted to address contemporaneous or future fertility decline, but it is unclear whether countries in the control group also face a similar situation (Castles (2003)). Third, many family policies come in bundles of incentives in excess of lowering the cost of children, such as measures encouraging women's labor force participation. Therefore, estimating the fertility elasticity of the policy bundle differs from estimating the price elasticity of fertility. Last but not least, it is unclear to what extent past estimates predict fertility elasticities in a different time or institutional context.

In this paper, I propose a new method to *bound* the fertility elasticities and discuss its implications. In Section 2, I show that under mild assumptions, a bound can be derived after knowing (1) the prevailing fertility rate and cost of children, (2) the maximum desired fertility, and (3) the cost of children such that potential parents would rather prefer not to have a child. In Section 3, I apply this method to the United States in 2010 and find that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman. I show that this bound is tighter than that in meta-studies of past estimates. Moreover, I demonstrate that the bound is simple to compute for a different country and year of interest. In Section 4, I show that this bound puts additional discipline on parameters in models with endogenous fertility choices. In addition, after comparing the bound with proposed policies in past research, I argue that fertility responses to these policies are non-negligible and could affect the mechanisms being studied.

2 Theory

Consider an economy populated by representative agents. I denote the Marshallian demand of fertility as $n(p; \boldsymbol{p}^{\text{other}}, y)$ where n is fertility, p is the cost of children, $\boldsymbol{p}^{\text{other}}$ is the price (vector) of other goods, and y is the household's lifetime income.

Assumption 1 The Marshallian demand of fertility $n(p; \boldsymbol{p}^{\text{other}}, y)$ is downward sloping, continuously differentiable, and convex.

This assumption is satisfied by most models of endogenous fertility, either with warm glow (e.g., De La Croix and Doepke (2003)) or with dynastic altruism (e.g., Barro and Becker (1989) and Córdoba and Ripoll (2019)). See Section 4 for examples.

Assumption 2 There exists $\overline{p}$ and $\overline{n}$ such that $n(\overline{p}; \boldsymbol{p}^{\text{other}}, y) = 0$ and $n(0; \boldsymbol{p}^{\text{other}}, y) = \overline{n}$.

This is a mild and realistic assumption. For example, let $\overline{p} = y$, the assumption requires that if having a child costs the parents' entire income and leaves no resources for the consumption of other goods, then the household would prefer not to have a child at all. The existence of $\overline{n}$ reflects satiation in preferences or biological constraints of childbearing.

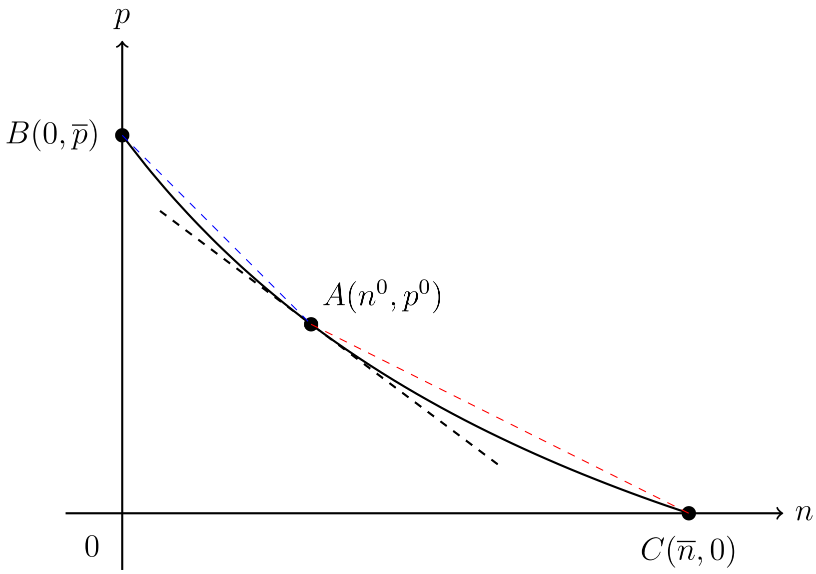
Proposition The fertility response to price around (n^0, p^0) is bounded by

$$\left.\frac{dn}{dp}\right|_{(n^0,p^0)} \in \left(\frac{n^0}{\overline{p}-p^0}, \frac{\overline{n}-n^0}{p^0}\right). \tag{1}$$

Proof Figure 1 shows the essence of the proof. The Marshallian demand of fertility is given by curve BAC where the coordinates of B and C are $(0, \overline{p})$ and $(\overline{n}, 0)$ correspondingly. Point A denotes the prevailing fertility and cost of children (n^0, p^0). The slope of the demand curve at A (black) is bounded by the slope of AC (red) and the slope of AB (blue) under the Mean Value Theorem and the assumption that curve BAC is decreasing, continuously differentiable, and convex.

In other words, while estimation using historical policies relies on local perturbations of p around (n^0, p^0), the method proposed here exploits the global properties of the demand curve under Assumptions 1 and 2 to bound the local slope.

Figure 1: Illustration of the Proof



Given the difficulties in identifying and quantifying a local shock of p in practice, the bounding approach provides a valuable alternative under mild assumptions. Adopting a large-scale pro-natal policy or a randomized control trial could lead to a more precise estimate for the specific country and year being studied. Nevertheless, the bound is still helpful as it provides a prediction of the program's cost-effectiveness *ex ante* and a check of the regression result *ex post*.

3 Quantification

In this section, I calculate bound for the United States, compare the results with prior studies, and show that the bound is simple to calculate for another country and year.

3.1 Calculating the Bound

I calculate the bound for the United States in 2010 where the prevailing fertility n^0 is 1.9 and the cost of one child p^0 is \$458,351 for a middle-income household (Córdoba and Ripoll (2019)). I make the following assumption on $\overline{n}$ and $\overline{p}$:

Assumption 3 Set $\overline{n} = 8$. Choose $\overline{p}$ such that an average household with one child lives in poverty for the rest of their lives, i.e.,

$$y - \overline{p} = y^{\text{poverty}}. \tag{2}$$

Proposition 1 makes it clear that the choice of $\overline{n}$ and $\overline{p}$ affects the tightness of the bound – as $\overline{n}$ or $\overline{p}$ decreases, the bound becomes tighter. The assumption that $\overline{n} = 8$ is probably conservative towards the high end as the prevailing ideal number of children is around 2.5 children per woman (Stone (2018)). The assumption that $\overline{p}$ is the cost of children that makes an average married household fall into poverty status after one childbirth is also a likely upper bound for $\overline{p}$. Thus, there are reasons to believe that the bound could be made tighter under alternative reasonable assumptions.

Following Córdoba and Ripoll (2019), I set $y =$ \$2,083,219. Using the federal poverty guideline in 2010, I set $y^{\text{poverty}} =$ \$18,310$\times$18+\$14,570$\times$42 $=$ \$941,520. This implies $\overline{p} =$ \$1,141,699.

Applying the proposition, I find that to raise the fertility by 0.1 children per woman, the change in p needed is between \$7,514 and \$35,966. This change in p can be brought about by policies such as a baby bonus, a universal basic income, or a fully-refundable Child Tax Credit (CTC) expansion.

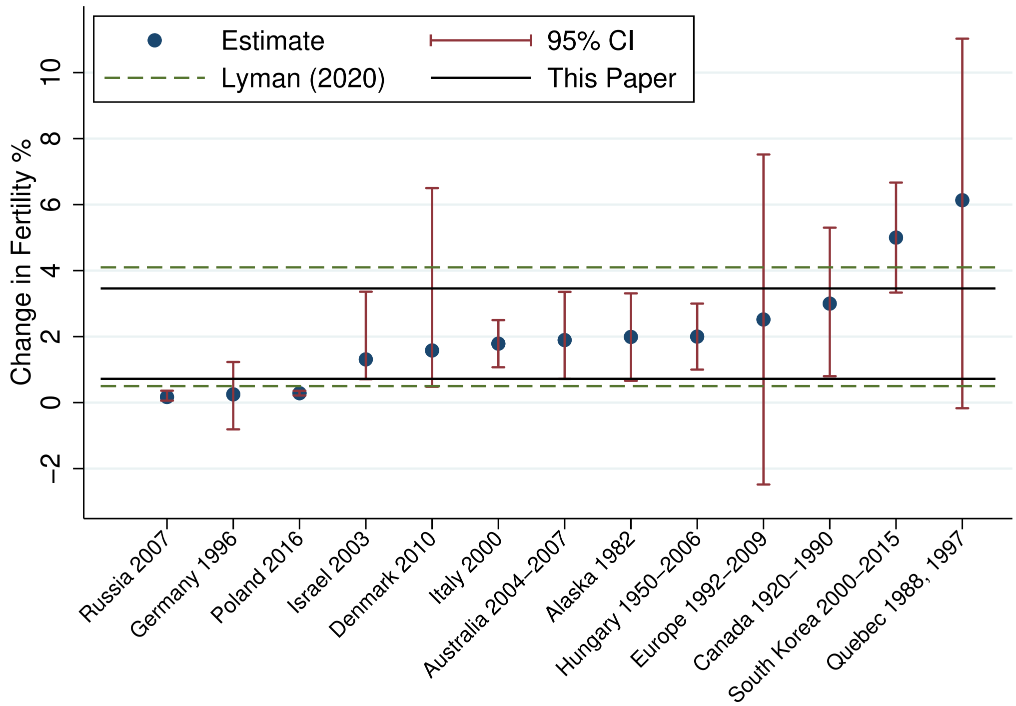
This bound can be used to validate models where the fertility demand is structurally modeled. For instance, Zhou (2022) develops a heterogeneous-agent OLG model with quantity-quality trade-off and calibrate the model to match the United States in 2010. In policy counterfactuals, it is shown that a baby bonus of \$15,000 raises the aggregate fertility rate by 0.1 children per woman. This number falls within the bound derived in this section.

3.2 Discussions

I compare my results to meta-studies of past estimates. Stone (2020) summarizes a large number of research estimating fertility elasticity from historical policy changes, mostly in low-fertility countries. He concludes that "an increase in the present value of child benefits equal to 10% of a household's (annual) income can be expected to produce between 0.5% and 4.1% higher birth rates" (see Figure 2).

To make the measures comparable, I convert the bound in this paper into percentages using

Figure 2: Estimates of Fertility Elasticities



Notes: This figure presents a summary of fertility elasticities estimated using historical policies and different bounds. As the goal of the paper is to pin down the price elasticity of fertility, I select policy changes including universal child benefits, baby bonuses, and universal basic income from the summary file compiled by Stone (2020). When there a multiple estimates exploiting the same policy change, I take the average across studies. The dots represent point estimates of fertility responses to a transfer with a net present value that is 10% of a household's annual income. The red intervals corresponds to 95% confidence intervals. The two horizontal dashed lines represent the bound of fertility elasticity suggested by Stone (2020). The two horizontal solid lines are the bound derived in this paper for the United States in 2010.

the median annual household income in 2010 ($49,445). The bound predicts that an increase in the present value of child benefits equal to 10% of a household's (annual) income can produce between 0.72% and 3.46% higher birth rates. Thus, the bound proposed here is tighter than existing estimates. Furthermore, another difference worth noting is that the bound proposed in this paper is a *theoretical* bound under assumptions while the ranges in Stone (2020) or other meta-studies are *statistical* bounds that summarizes past estimates.[2]

Another advantage of the bound is that it is simple to compute for a different country and year as long we know the prevailing (n^0, p^0), y, and y^{poverty}. In particular, p^0 acts as a *sufficient statistic*

[2]In Figure 2, there are some estimates that lie outside of the bound derived in this paper. This could arise because these policies were implemented in a different time and institutional setting.

that captures differences in policies and social norms that affect the costs of child-raising across time and space while n^0 is the revealed preference for children under prevailing prices.

For example, in the United Kingdom in 2016, the fertility rate is 1.79 children per woman and the cost of raising a child from birth to 21 years old is £231,843 (CEBR (2016)).[3] The lifetime income y is approximately £1,400,000. p^{poverty} is chosen to be 60% of y (£840,000) following the definition used by the British government's Department of Work and Pensions. Holding Assumption 3 unchanged and apply the proposition, it can be seen that to raise the fertility by 0.1 children per woman in the United Kingdom, the change in p required is between £3,733 and £18,333.

4 Further Implications

Beyond informing policymakers how costly it is to raise fertility, the bound provides a benchmark to discipline models with endogenous fertility and evaluate the exogenous fertility assumption in models that analyze child-related policies.

4.1 Endogenous Fertility w/ Dynastic Altruism

Consider a model of fertility choice with dynastic altruism following Barro and Becker (1989). Agents solve

$$U_0 = \max_{c_t, n_t, k_{t+1}} \sum_{t=0}^{\infty} \beta^t \left(\prod_{i=0}^{t-1} n_i \right)^{1-\varepsilon} \cdot \frac{c_t^{1-\sigma}}{1-\sigma}$$

subject to

$$w_t + (1+r_t)k_t = c_t + n_t(\chi_t + k_{t+1}) \quad \forall t$$

and the initial k_0. It is assumed that $\beta, \varepsilon, \sigma \in (0,1)$ to ensure that children are goods and $\varepsilon \leq \sigma$ for the second-order condition to hold.[4]

[3] Unlike Córdoba and Ripoll (2019), this cost does not include the opportunity costs of time in childraising. Thus, p^0 could be biased downward. As a result, the bound reported below will be wider than the case where the bias is eliminated.

[4] Jones and Schoonbroodt (2010) discuss an alternative set of assumptions which implies that the quantity and quality are substitutes rather than complements as in standard Barro-Becker models.

Using first-order conditions, the Marshallian demand of fertility in this economy is given by

$$n_t = (\beta(1+r_{t+1}))^{\frac{1}{\varepsilon}} \left[\frac{\chi_{t-1}(1+r_t) - w_t}{\chi_t(1+r_{t+1}) - w_{t+1}}\right]^{\frac{\sigma}{\varepsilon}} \tag{3}$$

where the net cost of creating a descendant at time t is $p_t \equiv \chi_t(1+r_{t+1}) - w_{t+1}$. Therefore, as the net cost of children p_t falls by 1 percent, fertility n_t increases by $\frac{\sigma}{\varepsilon}$ percent *ceteris paribus*.[5] Relating this elasticity to the numerical bound for the U.S. in 2010, the value of $\frac{\sigma}{\varepsilon}$ should lie between 0.67 and 3.23. Combining with the prior assumptions on σ and ε, the full set of restrictions are

$$0 < \varepsilon \leq \sigma < 1 \quad \text{and} \quad \sigma < 3.23 \cdot \varepsilon. \tag{4}$$

As can be seen, the bound puts further restrictions on the choice of σ and ε. For example, Manuelli and Seshadri (2009) satisfies these restrictions with $\sigma = 0.62$ and $\varepsilon = 0.35$; Córdoba (2015) considers $\sigma = 0.3$ and $\varepsilon = 0.288$; and Daruich and Kozlowski (2020) uses $\sigma = 0.5$ and $\varepsilon = 0.25$. Under different choices of $\overline{n}$ or $\overline{p}$, the restrictions induced by the bound could become more binding for this class of models.

4.2 Endogenous Fertility w/ Warm Glow Utility

Consider another model of fertility choice with warm glow utility. Agents solve

$$\max_{c,n} \quad c + \beta \cdot \frac{n^{1-\sigma}}{1-\sigma}$$

subject to

$$c + n \cdot p \leq y.$$

[5] A point worth noting here is that the model presented above generates a Marshallian demand that satisfies Assumption 1 in Section 2 but not Assumption 2. In light of Assumption 2, the isoelastic demand in Equation (3) could be interpreted as an *approximation* of the true underlying fertility demand around (n^0, p^0). The existence of $\overline{p}$ and $\overline{n}$, and hence the bound, puts restrictions on the *local properties* of $\frac{\sigma}{\varepsilon}$. The same interpretation applies to the endogenous fertility model with warm glow utility presented below.

As before, it is assumed that $\sigma \in (0, 1)$ to ensure that children delivers positive utility.

With interior solutions, the Marshallian demand of fertility is given by

$$n^* = \left(\frac{p}{\beta}\right)^{-1/\sigma}.$$

which implies that when p falls by 1 percent, n^* rises by $1/\sigma$ percent. Thus, if one calibrates this model to the U.S. economy in 2010, the bound suggests that the value of σ, which is typically chosen exogenously, should lie between 0.31 and 1.

In general, one can use the bound to validate comparative static results in other, potentially more complicated, models of endogenous fertility before conducting counterfactuals. For instance, Zhou (2022) shows that in a Huggett-Aiyagari model of quantity-quality trade-off calibrated to the U.S. economy, raising fertility rate by 0.1 children per woman requires the cost of children to fall by $15,000 - a number that is within the bound.

4.3 Models w/ Exogenous Fertility Assumption

The bound also permits an evaluation of the exogenous fertility assumption in structural models that analyze child-related policies. For example, Guner et al. (2020) consider a policy counterfactual where the per-child tax credit rises about $800 per child per year, amounting to about $12,000 in net present value terms for eligible households.[6] The optimal policy in Mullins (2019) is a Negative Income Tax on mothers that is equivalent to an additional $82 per week over 17 years, i.e., a transfer greater than $50,000 in net present value. Daruich (2022) shows that the welfare-maximizing early-childhood development subsidy is around $80,000.

A real-world policy example is the expansion of the Child Tax Credit in the American Rescue Plan that increases the annual transfer from $2,000 dollars to to $3,600 per child under age 6 and $3,000 per child ages 6 through 17. The net present value of this expansion is above $30,000 per child for eligible families.

[6]The net present values are calculated using a 4% annual discount rate.

With prior calculations, it can be seen a \$10,000 reduction in the cost of children increases fertility by 0.03 to 0.13 from the baseline level of 1.9 children per woman. Thus, the proposed policies are likely going to lead to non-negligible fertility responses and a dilution of family resources, triggering the quantity-quality trade-off mechanism à la Becker and Lewis (1973). As a result, these policies might not achieve the goal of raising children's human capital or improving social mobility (due to heterogeneities in fertility responses). Nevertheless, such policies could still benefit the society in the long-run through mechanisms such as changing the dependency ratio (Zhou (2022)).

5 Conclusion

Fertility elasticity is important to both policymakers and economists, yet pinning it down using historical policies has been proven difficult. In this paper, I propose a new method to bound the fertility elasticities. Under mild assumptions, I show that a transfer of size between \$7,514 and \$35,966 is required to raise the fertility rate by 0.1 children per woman in the United States in 2010. This bound is tighter than the range provided by meta-studies and is simple to compute for a different country and year. The bound puts further restrictions on parameters in endogenous fertility models and allows an evaluation of the exogenous fertility assumption in structural models that analyze child-related policies.

The bounding approach developed here is complementary to empirical inquiries into fertility elasticity using quasi-experimental variations and/or randomized control trials. It provides a prediction of the intervention's cost-effectiveness *ex ante* and a check of the regression result *ex post*.

References

Barro, R. J. and Becker, G. S. (1989). Fertility choice in a model of economic growth. *Econometrica*, pages 481–501.

Becker, G. S. and Lewis, H. G. (1973). On the interaction between the quantity and quality of children. *Journal of Political Economy*, 81(2, Part 2):S279–S288.

Bergsvik, J., Fauske, A., and Hart, R. K. (2020). Effects of policy on fertility: A systematic review of (quasi) experiments.

Castles, F. G. (2003). The world turned upside down: below replacement fertility, changing preferences and family-friendly public policy in 21 oecd countries. *Journal of European Social Policy*, 13(3):209–227.

CEBR (2016). Raising a child more expensive than buying a house. Link.

Cohen, A., Dehejia, R., and Romanov, D. (2013). Financial incentives and fertility. *Review of Economics and Statistics*, 95(1):1–20.

Córdoba, J. and Ripoll, M. (2019). The elasticity of intergenerational substitution, parental altruism, and fertility choice. *The Review of Economic Studies*, 86(5):1935–1972.

Córdoba, J. C. (2015). Children, dynastic altruism and the wealth of nations. *Review of Economic Dynamics*, 18(4):774–791.

Córdoba, J. C., Liu, X., and Ripoll, M. (2016). Fertility, social mobility and long run inequality. *Journal of Monetary Economics*, 77:103–124.

Daruich, D. (2022). The macroeconomic consequences of early childhood development policies. *Available at SSRN 3265081*.

Daruich, D. and Kozlowski, J. (2020). Explaining intergenerational mobility: The role of fertility and family transfers. *Review of Economic Dynamics*, 36:220–245.

De La Croix, D. and Doepke, M. (2003). Inequality and growth: why differential fertility matters. *American Economic Review*, 93(4):1091–1113.

Gauthier, A. H. (2005). Trends in policies for family-friendly societies. *The New Demographic Regime: Population Challenges and Policy Responses*, pages 95–110.

González, L. (2013). The effect of a universal child benefit on conceptions, abortions, and early maternal labor supply. *American Economic Journal: Economic Policy*, 5(3):160–88.

Guner, N., Kaygusuz, R., and Ventura, G. (2020). Child-related transfers, household labour supply, and welfare. *The Review of Economic Studies*, 87(5):2290–2321.

Jones, L. E. and Schoonbroodt, A. (2010). Complements versus substitutes and trends in fertility choice in dynastic models. *International Economic Review*, 51(3):671–699.

Laroque, G. and Salanié, B. (2008). Does fertility respond to financial incentives?

Manuelli, R. E. and Seshadri, A. (2009). Explaining international fertility differences. *The Quarterly Journal of Economics*, 124(2):771–807.

McDonald, P. (2006). Low fertility and the state: The efficacy of policy. *Population and Development Review*, pages 485–510.

Milligan, K. (2005). Subsidizing the stork: New evidence on tax incentives and fertility. *Review of Economics and Statistics*, 87(3):539–555.

Mullins, J. (2019). Designing cash transfers in the presence of children's human capital formation. *Job Market Paper.[235]*.

Stone, L. (2018). How many kids do women want? Link.

Stone, L. (2020). Pro-natal policies work, but they come with a hefty price tag. Link.

Zhou, A. (2022). The macroeconomic consequences of family policies. *Available at SSRN 3931927*.