# The Autumn of Patriarchy

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**Abstract**

This paper develops a unified theory to explain the concurrent declines in fertility, marriage, and gender income gaps observed during the grand gender convergence (Goldin 2014). The model integrates these phenomena through two key mechanisms: (1) the trade-off between fertility and women's labor supply, and (2) the role of children as a public good in family formation. The equilibrium conditions reveal a fundamental trilemma: high fertility, universal dual parenthood, and gender income equality cannot coexist in an economy—a theoretical prediction robustly supported by cross-country data. Furthermore, the model demonstrates that rising total factor productivity alone is sufficient to drive the decline of patriarchy, without requiring factor-biased technological change. However, the pace of this transition varies significantly across countries, shaped by differences in social norms. These findings provide a cohesive framework for understanding the economic and social forces reshaping family structure and gender dynamics in the modern era.

**JEL classification**: D13, J11, J12, J13, J16
**Keywords**: Gender equality, family structure, fertility

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"...and the bells of glory that announced to the world the good news that the uncountable time of eternity had come to an end."

Gabriel García Márquez, *The Autumn of the Patriarch*

# 1. Introduction

After dominating human society for millennia, patriarchy has been tailing off in recent decades, giving way to egalitarianism. Amidst the "grand convergence" (Goldin 2014), three key trends stand out: fertility rates have been falling (Greenwood et al. 2005a, Guinnane 2011), marriage and dual parenthood have been declining (Stevenson and Wolfers 2007, Folbre 2021), and gender gaps in wage, income, and wealth have been converging (Doepke and Tertilt 2009, Goldin 2014).

While extensive research has examined fertility, marriage, and gender income gaps as distinct phenomena, few studies have explored their interconnected dynamics or the mechanisms through which they mutually reinforce one another. This paper addresses this gap by proposing a novel unified framework that systematically integrates these three dimensions, offering new insights into their relationship.

I begin by outlining a static model in which opposite-gender individuals meet in a marriage market. Men face a choice between remaining single (and childless) or entering marriage, where they allocate a portion of their income to household consumption and child-rearing. Women, conversely, may opt for single motherhood or raise children within a marital union. Importantly, children are modeled as a public good in the process of family formation. In addition, fertility decisions within married households are subject to veto (Doepke and Kindermann 2019).

The model connects marriage, fertility, and labor supply decisions through two mechanisms. First, marriage and fertility are intertwined, as fertility plays a central role in shaping marital decisions. Second, fertility directly influences gender-based disparities in labor supply, driven by societal norms that disproportionately allocate childcare responsibilities to women, thereby constraining their workforce participation.

I characterize the model by demonstrating three key equilibrium outcomes: (1) individual optimization and marriage market clearing conditions jointly determine the equilibrium fertility rate and intra-household income transfers; (2) the prevalence of dual parenthood depends on the level of intra-household income transfers and the gender gap in human capital; and (3) the gender income gap is shaped by both the gender gap in human capital and the endogenous labor supply decisions of women.

Building on the static model, I identify a novel trilemma in family and gender economics: high fertility, a high prevalence of dual parenthood, and gender income equality cannot simultaneously co-exist within the same economy. Specifically, I demonstrate that achieving any two of these outcomes necessarily precludes the third. Even when policy interventions alter the underlying economic fundamentals, inherent tensions persist due to the equilibrium conditions of the model. While each outcome may be desirable as an independent policy objective, the trilemma underscores the inevitability of trade-offs, compelling policymakers to prioritize among competing goals. Although the paper refrains from addressing the normative dimensions of these trade-offs, it highlights a critical constraint that policymakers must navigate.

I empirically evaluate the trilemma using a panel dataset spanning a large set of countries from 1970 to 2014, where all three outcomes—fertility, dual parenthood, and gender income equality—are measured. To ensure the analysis does not artificially preclude the co-existence of these outcomes, I classify countries into high fertility, high dual parenthood, and high gender income equality groups based on the sample medians of each dimension. I then visualize their intersections using a Venn diagram. The results reveal that only a negligible fraction of observations simultaneously achieve high fertility, high dual parenthood, and high gender income equality, a proportion significantly lower than what would be expected under random chance. This empirical pattern aligns with the trilemma, highlighting the inherent trade-offs and conflicts among these three objectives

Next, I examine the grand gender convergence by extending the static model into a dynamic framework. To model the intergenerational evolution of gender-specific human capital, I integrate a key empirical insight from recent literature: single parenthood exerts differential effects on the human capital accumulation of boys compared to girls.[1] This finding underscores that shifts in family structures have far-reaching implications for future gender disparities in human capital, which in turn shape marriage patterns, fertility decisions, and female labor supply dynamics.

The dynamic model reveals that the decline of patriarchy is driven by two primary mechanisms. First, factor-neutral technological progress increases the opportunity cost of child-rearing, leading to reduced fertility, lower intra-household income transfers, declining marriage rates, and rising female labor force participation. Second, the rise in single parenthood and the narrowing of gender gaps in human capital create a self-reinforcing feedback loop, amplifying the effects of the first mechanism across generations. Together, these channels demonstrate that technological progress plays a central role in explaining the inevitable erosion of patriarchal structures.

While the first mechanism operates uniformly across economies, the timing and intensity of the second mechanism vary significantly due to cross-country differences in how intra-household income transfers influence the prevalence of single parenthood. For instance, in some societies experiencing declining fertility and narrowing gender income gaps, single parenthood remains rare due to persistent social norms. These institutional variations give rise to distinct transitional trajectories. I illustrate this heterogeneity using the contrasting cases of the United Kingdom and Japan as representative examples.

Finally, I explore whether equalizing childcare responsibilities between genders could resolve the trilemma. I present several arguments suggesting that such a policy intervention is unlikely to fully address the underlying trade-offs.

*Related Literature*

This paper is closely connected to the literature on family and gender economics, particularly the extensive body of work examining historical shifts in fertility, marriage, and gender inequality.[2] It makes four contributions to the literature.

[1]As highlighted by Wasserman (2020), "The evidence supports an emerging consensus that growing up in a family without biological married parents produces more adverse consequences for boys than for girls."

[2]For comprehensive reviews, see Greenwood et al. (2017) and Greenwood (2019).

First, while prior research has largely developed separate theories for each trend or examined at most two trends simultaneously (e.g., Galor and Weil 1996; Regalia and Rios-Rull 2001; Santos and Weiss 2016; Greenwood et al. 2016; Gayle et al. 2022; Greenwood et al. 2023), this paper introduces a unified framework that integrates all three phenomena, formalizing the inherent tensions among them.

Second, adopting a holistic perspective, I propose and empirically validate the trilemma hypothesis, a novel conjecture that connects previously disparate areas of the literature. This trilemma establishes an important boundary for policymakers: achieving high fertility, widespread dual parenthood, and gender income equality simultaneously is structurally unfeasible.

Third, I demonstrate that factor-neutral technological progress can simultaneously drive declines in fertility, marriage rates, and gender income gaps. This mechanism complements existing theories that emphasize factor-biased technological changes, such as skill-biased innovation favoring child quality over quantity (Galor and Weil 2000; Fernandez-Villaverde 2001), the household appliance revolution that reduced the demand for marriage and increased female labor force participation (Greenwood et al. 2005b; Greenwood et al. 2023), or structural transformation that shifted labor demand in favor of women (Galor and Weil 1996; Ngai and Petrongolo 2017; Cao et al. 2024).

Fourth, relative to structural models of demographic transition (e.g., Greenwood et al. 2023), I introduce a new propagation channel linking marriage rates to gender disparities in human capital formation. While the empirical literature has extensively documented the differential effects of family structure on boys versus girls (e.g., Bertrand and Pan 2013; Autor et al. 2019; Wasserman 2020; Reeves 2022; Frimmel et al. 2024), this paper is the first to incorporate these findings into a dynamic macroeconomic framework, highlighting how shifts in family structure amplify intergenerational gender gaps.

The remainder of the paper is structured as follows. Section 2 presents the static model of marriage and fertility decisions. Section 3 formulates the trilemma hypothesis and provides empirical evidence. Section 4 examines the decline of patriarchal structures through the dynamic model. Section 5 explores whether equalizing childcare responsibilities could resolve the trilemma. Section 6 concludes.

# 2. The Static Model

This section presents the static economy. Nevertheless, I keep the time subscript $t$ so that the model can be readily extended to a dynamic one in Section 4.

Individuals are indexed by gender $g \in \{♂, ♀\}$. Each gender comprises half of the population. Individuals derive utility from consumption $c$ and fertility $n$:

$$u(c,n) = \left((1-\beta)\cdot c^{\frac{\rho-1}{\rho}} + \beta \cdot n^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}}, \qquad \beta \in (0,1). \tag{1}$$

Following Jones and Schoonbroodt (2010) and Carlos Córdoba and Ripoll (2019), I assume that $\rho > 1$ so that the utility for childless individual $u(c,0)$ is well-defined.

To focus on across-gender disparities, I assume that individuals have the same amount of human capital within each gender denoted by $h_t^{♂}$ and $h_t^{♀}$ respectively. Labor is the only productive factor in the economy. Therefore, $h_t^{♂}$ and $h_t^{♀}$ determine wages. In the static model, $h_t^{♂}$ and $h_t^{♀}$ are exogenously given. The gender gap in human capital at time $t$ is defined as

$$\Gamma_t^h = \frac{h_t^{♂}}{h_t^{♀}}. \tag{2}$$

I use $A_t$ to denote total factor productivity (TFP) at time $t$. In the baseline analysis, I assume that $A_t$ is also exogenously given.[3]

## 2.1 Single Individuals

Single men consume their labor income and remain childless. They supply one unit of labor inelastically. Therefore, their utility is given by

$$V_t^{♂,s} = u(A_t h_t^{♂}, 0) \tag{3}$$

where $s$ in the superscript denotes "single."

[3]Allowing for endogenous $A_t$ leads to additional channels. For example, besides enhancing aggregate productivity $A_t$ à la Hsieh et al. (2019), rising female labor supply could stimulate innovation and hence economic growth (Chiplunkar and Goldberg 2021). Another example is Galor and Weil (1996) which discusses the feedback mechanism between fertility decline, which stimulates capital accumulation, and rising demand for female labor, which is more complementary to capital than male labor.

single women, on the other hand, can have children but do not receive any transfers or support from the absentee fathers.[4] They choose consumption $c_t^{♀,s}$, fertility $n_t^s$, and labor supply $n_t^s$ to solve the utility maximization problem

$$V_t^{♀,s} = \max_{c_t^{♀,s}, l_t^s, n_t^s} \quad u(c_t^{♀,s}, n_t^s) \tag{4}$$

subject to budget and time constraints

$$c_t^{♀,s} = A_t h_t^{♀} l_t^s, \quad \text{and} \quad l_t^s = 1 - \chi n_t^s$$

where $\chi$ is the time cost of raising each child. I follow the literature and assume that the fertility choice is continuous, i.e., $n_t^s \in \mathbb{R}_+$.

To summarize, the decisions of single women involve a simple consumption-fertility trade-off via endogenous labor supply – a margin that is commonplace in the female labor supply literature (e.g., Rosenzweig and Wolpin 1980).

## 2.2 Married Individuals

Once married, husbands in the economy supply one unit of labor inelastically and transfer $\alpha_t$ share of their income to their wives to fulfill the marital contract. This assumption captures the traditional role of marriage where husbands are the main breadwinners and provide income for the family. It is important that while individuals take $\alpha_t$ as given, it is an equilibrium object to be characterized in Section 2.5.

Husbands derive utility from fertility and consuming their remaining income. The former is a public good shared with their wives. Therefore, the value of married men is

$$V_t^{♂,m} = u(\underbrace{(1-\alpha_t) A_t h_t^{♂}}_{\text{remaining income}}, \underbrace{n_t^m}_{\text{fertility}}). \tag{5}$$

[4]This is a model simplification. If the biological father can be identified and located, it is possible to sue him for child support. Nevertheless, according to the calculation by the Annie E. Casey Foundation using data from the Current Population Survey wave 2020-2022, just 23% of U.S. female-headed families living with one or more children under age 18 reported receiving any amount of child support during the previous year.

Wives, on the other hand, trade-off fertility and consumption via endogenous labor supply. They solve the utility maximization problem

$$V_t^{♀,m} = \max_{c_t^{♀,m}, l_t^m, n_t^m} \quad u(c_t^{♀,m}, n_t^m) \tag{6}$$

subject to budget and time constraints

$$c_t^{♀,m} = \underbrace{\alpha_t A_t h_t^{♂}}_{\text{transfer from husband}} + \underbrace{A_t h_t^{♀} l_t^m}_{\text{own labor income}}, \quad \text{and} \quad l_t^m = 1 - \chi n_t^m$$

where $n_t^m$ and $l_t^m$ are the fertility and labor supply of married women. From the wives' perspective, the transfers from husbands generate a pure income effect which leads to higher consumption and fertility because both are normal goods in preferences. Rising wages for women, however, will generate a substitution effect, leading to a lower demand for children because consumption and fertility are substitutes in preferences, a mechanism emphasized by Greenwood et al. (2005a).

Given that husbands do not directly bear the costs of children after transferring $\alpha_t$ share of income, they prefer as many children $n_t^m$ as possible.[5] Wives, however, prefer to have fewer children because they directly shoulder the burden of childcare. This observation is supported by the empirical findings in Doepke and Tertilt (2018). Motivated by Doepke and Kindermann (2019), I assume that childbirth within marriage is subject to veto. Therefore, wives are the key decision-makers regarding fertility within marriage in this model.

## 2.3 Marriage Market

At the beginning of the period, each woman receives an idiosyncratic shock $\tau$ on the taste of marriage which follows a distribution $J(\tau)$.[6] For a woman with taste shock $\tau$, her

[5]The assumption that males do not share the cost of children is not crucial. In fact, all the results go through as long as wives shoulder *more* childcare responsibilities than their husbands–a pattern that holds widely across countries and over time (Kleven et al. 2019, Doepke et al. 2023). In Section 5, I discuss how gender equality in childcare would affect the results.

[6]Similar assumptions can be found in marriage models such as Greenwood et al. (2017).

utility from marriage is $\tau \cdot V_t^{♀,m}$. After receiving the shock, individuals decide whether or not to get married and the marriage market clears. The distribution $J(\tau)$ is a reduced-form way to capture other considerations of marriage that are not explicitly specified in the model, such as mutual affection, tax benefits, or risk-sharing.

For women, it is straightforward that there exists a threshold $\tau_t^*$ above which they would prefer marriage over staying single. The value of $\tau_t^*$ can be implicitly defined using the indifference condition

$$V_t^{♀,m} \cdot \tau^* = V_t^{♀,s}. \tag{7}$$

Therefore, the share of men or women that are married in the equilibrium of the monogamous economy is given by

$$\mathcal{M}_t = 1 - J(\tau_t^*) \tag{8}$$

On the other hand, because men are homogeneous and are on the long side of the marriage market, the equilibrium imposes an indifference condition for them between getting married and staying single:

$$V_t^{♂,m} = u((1-\alpha_t)A_t h_t^{♂}, n_t^m) = u(A_t h_t^{♂}, 0) = V_t^{♂,s}. \tag{9}$$

Therefore, the key assumption here is that the share of within-household transfers $\alpha_t$ and marital fertility $n_t^m$ acts as "prices" to clear the marriage market.

## 2.4 Aggregate Variables

The model provides simple expressions for other aggregate variables of interest. For example, aggregate fertility rate $n_t$ is a weighted average of marital and non-marital fertility:

$$n_t = \mathcal{M}_t \cdot n_t^m + (1 - \mathcal{M}_t) \cdot n_t^s \tag{10}$$

The share of children born with both parents, i.e., under dual parenthood, is given by

$$\mathcal{D}_t = \frac{\mathcal{M}_t \cdot n_t^m}{n_t}. \tag{11}$$

Average hours worked per female is

$$l_t^{♀} = \mathcal{M}_t \cdot l_t^m + (1 - \mathcal{M}_t) \cdot l_t^s = 1 - \chi n_t. \tag{12}$$

The average labor income of male and females are

$$y_t^{♂} = A_t \cdot h_t^{♂}, \qquad y_t^{♀} = A_t \cdot h_t^{♀} \cdot l_t^{♀}$$

which leads to a simple expression of the gender income gap

$$\Gamma_t^y = \frac{y_t^{♂}}{y_t^{♀}} = \frac{\Gamma_t^h}{l_t^{♀}}. \tag{13}$$

## 2.5 Model Solution

The equilibrium conditions of the static economy can be characterized in the following steps. First, the indifference condition of men in the marriage market (9) implicitly defines $\alpha_t$ as a function of $n_t^m$:

$$(1-\beta) \cdot (A_t h_t^{♂})^{\frac{\rho-1}{\rho}} \left[1 - (1-\alpha_t)^{\frac{\rho-1}{\rho}}\right] = \beta \cdot (n_t^m)^{\frac{\rho-1}{\rho}} \tag{14}$$

When $\rho > 1$, using the implicit function theorem on Equation (14) reveals that the function $\alpha_t(n_t^m)$ is strictly increasing and convex. It takes the value of $0$ when $n_t^m = 0$, and shifts up when $A_t$ rises.

On the other hand, the first-order condition of married women gives the optimality condition where $n_t^m$ is a function of $\alpha_t$:

$$n_t^m \cdot \left[\left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta}\right)^{\rho} + A_t h_t^{♀} \chi\right] = (1 + \alpha_t \Gamma_t^h) A_t h_t^{♀} \tag{15}$$

Equation (15) indicates that $n_t^m(\alpha_t)$ is an increasing and linear function. It takes a strictly positive value when $\alpha_t = 0$ and shifts down when $A_t$ rises.

Taking the properties of $\alpha_t(n_t^m)$ and $n_t^m(\alpha_t)$ together generates the first lemma.

**Lemma 1**: For given $A_t$, there is a unique fixed point of $(\alpha_t, n_t^m)$.

*Proof*: See Appendix.

Second, by comparing $V_t^{♀,s}$ and $V_t^{♀,m}$, Lemma 2 provides a condition for the cutoff $\tau_t^*$ above which women choose to get married.

**Lemma 2**: The marriage threshold $\tau_t^* = 1/(1+\alpha_t\Gamma_t^h)$.

*Proof*: See Appendix.

Lemma 2 indicates that the marriage threshold, and hence the marriage rate $\mathcal{M}_t$, is determined by the economic gains from marriage from the women's perspective. The "transfer potential" of men to women is a product of the gender gap in human capital (and hence wages) $\Gamma_t^h$ and men's willingness to transfer $\alpha_t$.

Together with Equation (26) in the Appendix, Lemma 2 also indicates that the fraction of dual parenthood $\mathcal{D}_t$, defined in (11), is monotonically increasing in the marriage rate $\mathcal{M}_t$. Therefore, I will use $\mathcal{M}_t$ and $\mathcal{D}_t$ interchangeably in the remaining analyses.

# 3. The Trilemma

In this section, I define and empirically test the trilemma.

## 3.1 Theory

Collecting the equilibrium conditions, the relationship between fertility $n_t$, marriage rate $\mathcal{M}_t$, female labor supply $l_t^♀$, and gender income gap $\Gamma_t^y$ can be summarized in the following three equations:

$$\mathcal{M}_t = 1 - J\left(\frac{1}{1+\alpha_t\Gamma_t^h}\right) \tag{16}$$

$$\Gamma_t^y = \frac{\Gamma_t^h}{l_t^♀} \tag{17}$$

$$l_t^♀ = 1 - \chi n_t \tag{18}$$

Equations (16)-(18) highlight the core tensions in the model. In particular, (16) shows that marriage rates are higher when there are larger gender gaps in human capital $\Gamma_t^h$. But (17) implies that large gender gaps in human capital make it difficult to achieve gender income inequality unless the female labor supply is high. However, the direct implication of a high female labor supply is low fertility from (18).

**The Trilemma**: high fertility, dual parenthood (or equivalently high marriage rate), and gender income inequality cannot co-exist in an economy.
*Proof*: I establish the trilemma by discussing three possible cases.

1. *High fertility and dual parenthood.*
   With high fertility $n_t$, female labor supply $l_t^{♀}$ is low from (18). To achieve a high marriage rate $\mathcal{M}_t$, gender human capital gap $\Gamma_t^h$ cannot be too low from (16). Therefore, the gender income gap $\Gamma_t^y$ is necessarily high from (17).
2. *High fertility and gender income equality.*
   With high fertility $n_t$, female labor supply $l_t^{♀}$ is low from (18). To achieve a low gender income gap $\Gamma_t^y$, it must be the case that $\Gamma_t^h$ is very low from (17). But a very low gender gap in human capital $\Gamma_t^h$ leads to a low marriage rate $\mathcal{M}_t$ from (16).[7]
3. *Dual parenthood and gender income equality.*
   To achieve a high marriage rate $\mathcal{M}_t$, (16) implies that the gender gap in human capital $\Gamma_t^h$ needs to be high. With high $\Gamma_t^h$, the only way to achieve a low gender income gap $\Gamma_t^y$ is to have a high female labor supply $l_t^{♀}$. needs to be very high from (18). Therefore, fertility $n_t$ is very low from (18).

## 3.2 Empirical Results

To empirically test the trilemma, I gather data from three key sources: (1) total fertility rates (TFR) from the United Nations, (2) the proportion of children born outside of marriage from the OECD database, and (3) gender gaps in median earnings, also from the

[7]One might wonder whether Nordic countries, with high levels of cohabitation, is an exception to the trilemma. Measures of living arrangements from the OECD Family Database, however, suggest that Norway, Finland, and Sweden also have a large share of children living with single parents that is well above the OCED average.

OECD database. This results in an unbalanced panel dataset comprising 37 countries from 1970 to 2014, with a total of 721 country-year observations.

To categorize the observations, I use the sample medians of each variable as thresholds to define ”high” or ”low” values. This approach ensures that each group contains a sufficient number of observations, thereby providing a balanced opportunity to observe the trinity. Arbitrarily strict cutoffs would make achieving the trinity impossible by design, which is why median-based categorization is more appropriate. Additionally, I have explored alternative cutoff definitions to test the robustness of the findings: (1) defining ”high fertility” as cases where $\text{TFR}_{it} > 2$, and (2) using the upper quartiles of each variable to define the categories. The results, illustrated in Figures A.4 and A.5, confirm that the main findings remain consistent across these alternative specifications.

Using this definition, observations are labeled as

- “high fertility” if $\text{TFR}_{it} > 1.69$,
- “dual parenthood” if out of $\text{marriage}_{it} < 31.4\%$, and
- “gender income equality” if $\text{gap}_{it} < 17.2\%$.

After categorizing the observations, I visualize their overlaps using a Venn diagram, as depicted in Figure 1. Since each variable is dichotomized at its sample median (i.e., ”high” or ”low”), the expected overlap of all three outcomes under statistical independence would be $0.5 \times 0.5 \times 0.5 = 12.5\%$. The empirical results, however, starkly diverge from this benchmark: fewer than 3% of observations simultaneously exhibit high fertility, dual parenthood, and gender income equality. This substantial discrepancy—less than a quarter of the random expectation—provides strong empirical support for the trilemma, underscoring the inherent incompatibility of these three outcomes. Moreover, Figure 1 reveals that a significant number of countries achieve only one or even none of the outcomes, further illustrating the systemic challenges in reconciling these societal goals.

able 1 provides illustrative examples for each segment of the Venn diagram. Notably, Australia stands out as the primary country that consistently achieved the trinity under

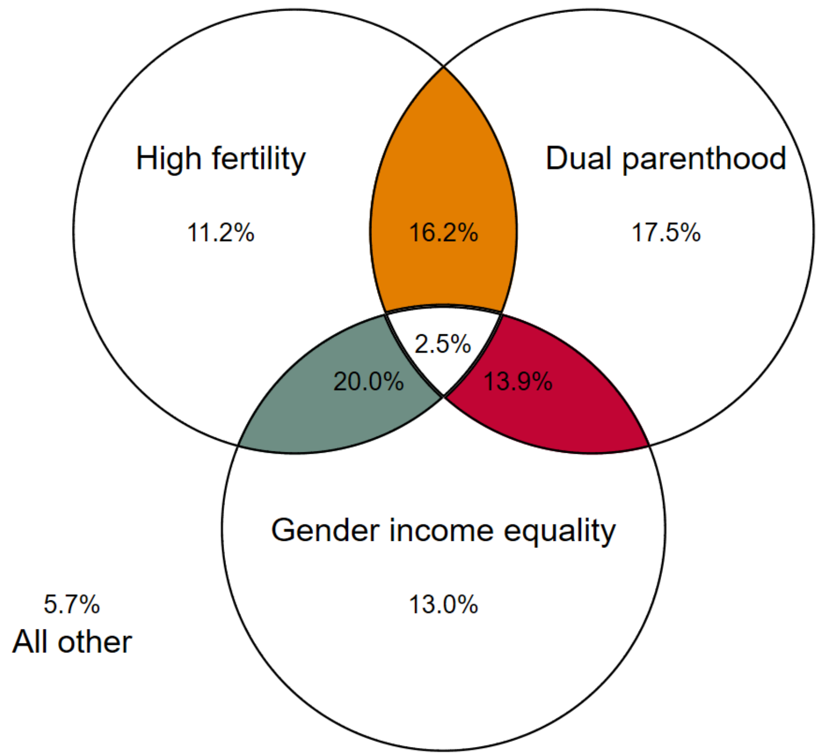



Figure 1: The Trilemma in the Data

| $D$ – dual parenthood, $G$ – gender income equality, $F$ – high fertility | |
|---|---|
| Category | Countries |
| None | Austria, United Kingdom 1995-2003 |
| Only $D$ | Canada, Switzerland, Germany 1992-2006, Japan, South Korea |
| Only $G$ | Germany 2009-2014, Hungary, Portugal |
| Only $F$ | United States 1994-2013, Finland |
| $D+G$ | Greece, Italy, Poland |
| $G+F$ | Belgium, Norway, New Zealand, Sweden |
| $D+F$ | United Kingdom 1970-1994, Israel, USA 1973-1993 |
| $D+G+F$ | Australia 1992-2002 ($G+F$ afterwards) |

Table 1: Examples of Countries

the given definition, maintaining this status from 1992 to 2002.[8] After 2003, Australia experienced a sharp rise in single parenthood, causing it to lose its ”dual parenthood” status and, consequently, its position within the trinity. A comprehensive overview of the number of outcomes achieved by each country over time is presented in Figure A.6, offering a broader perspective on the dynamics of these societal goals.

## 3.3 Implications

The main takeaway from the trilemma is that while each of the three outcomes—high fertility, dual parenthood, and gender income equality—may be desirable policy goals,[9] achieving all three simultaneously is inherently challenging due to the trade-offs between them.

To unpack the intuition behind this result, it is important to highlight two key tensions in the static model. The first tension arises between fertility and gender income equality, mediated by endogenous female labor supply. For example, consider family policies that alter the cost of children, $\chi$ (e.g., baby bonuses or child tax credits). If policymakers increase $\chi$, gender equality may improve as female labor supply rises, but this comes at the expense of lower fertility. Conversely, reducing $\chi$ may boost fertility, but it also decreases female labor supply, widening the gender income gap.

The second tension exists between dual parenthood and gender income equality. For instance, anti-discrimination policies that reduce $\Gamma_h$ and shrink gender wage gaps may lead to lower marriage rates, as the potential for income transfers from males diminishes. On the other hand, an increase in $\Gamma_h$ could raise marriage rates but worsen gender income equality. These trade-offs illustrate the inherent incompatibility of simultaneously achieving all three outcomes.

To take a step back, the two tensions in the static model may not be fully satisfactory for several reasons. First, due to the model's setup, it might not adequately capture the

[8] It is worth noting, however, that Australia's fertility rate during this period was below 1.9, meaning it would no longer qualify for the trinity if a stricter threshold of TFR $> 2$ were applied.

[9] For instance, numerous policies have been implemented to promote childbirth and gender equality. Additionally, although the model does not explicitly account for cross-sectional inequality, studies such as Kearney (2023) have demonstrated a strong connection between the prevalence of dual parenthood and inequalities in children's outcomes.

margins on leisure, thereby overlooking the possibility that certain government policies, such as subsidized childcare, could simultaneously increase both fertility and female labor supply (Baker et al. 2008). This could potentially resolve the first tension. Second, governments have the ability to directly influence the benefits of marriage, thereby altering $J(\cdot)$ and achieving higher marriage rates without compromising gender income equality, which addresses the second tension.

However, the result in Figure 1 suggests the presence of an additional mechanism. If such mechanisms did not exist, we would expect to see a greater number of countries achieving the trinity, especially given that subsidized childcare and marriage tax benefits are already available to governments and widely adopted in many developed economies.

To explore this further, in Section 4, I extend the model into a dynamic setting to uncover another intrinsic tension between dual parenthood and the gender income gap through the formation of gender-specific human capital. This dynamic model not only strengthens the argument for the trilemma but also provides a roadmap for the potential demise of patriarchy.

# 4. The Autumn of Patriarchy

This section studies the transition from patriarchal societies to egalitarian societies in a dynamic model.

## 4.1 Human Capital Dynamics

I assume that the gender-specific human capital follows the law of motion[10] specified as

$$h_{t+1}^{♀} = (h_t^{♀})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{♀}} \tag{19}$$

$$h_{t+1}^{♂} = Z \cdot (h_t^{♂})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{♂}} \tag{20}$$

[10] I adopt Galton's approach to the intergenerational transmission of human capital for analytical and aggregation simplicity. As pointed out by Mulligan (1999), explicit modeling of parental human capital investment decisions, e.g., following Becker and Tomes (1979), often yields similar predictions.

where $Z > 1, \theta \in (0,1)$ and more importantly, $\psi^{\male} > \psi^{\female} > 0$.

The production functions (19) and (20) are motivated by a large empirical literature that has documented that growing up in a family without biological married parents leads to more adverse consequences for boys than for girls (e.g., see Bertrand and Pan 2013, Chetty et al. 2016, Autor et al. 2019, Wasserman 2020, Reeves 2022, and Frimmel et al. 2024). This intriguing result could be due to (1) role model effects operating within genders, (2) differential sensitivity to parental inputs across genders, or (3) differential exposure or sensitivity to inputs from other social institutions such as neighborhoods or schools.

The difference between between $\psi^{\male}$ and $\psi^{\female}$ is economically sizable. For example, Autor et al. (2019) show that the racial differences in the ratio of single motherhood could explain the bulk of the black-white differences in gender gaps. Autor et al. (2023) find that a substantial fraction of the gender gap in high school outcomes can potentially be explained by the differential effect of family socioeconomic status, in particular family structure, on boys' medium-run outcomes.

Taking the results on differential sensitivity as given, the model implies that the prevailing marriage rates determine gender gaps in human capital in the next generation and hence the evolution of $\Gamma^h$. To see this, note that dividing (19) by (20) yields

$$\Gamma^h_{t+1} = Z \cdot (\Gamma^h_t)^{\theta} \cdot (\mathcal{M}_t)^{\psi^{\male} - \psi^{\female}}$$

which implies in steady-state

$$\Gamma^h = Z^{\frac{1}{1-\theta}} \cdot (\mathcal{M})^{\frac{\psi^{\male} - \psi^{\female}}{1-\theta}} \Longrightarrow \frac{d\Gamma^h}{d\mathcal{M}} > 0 \tag{21}$$

Therefore, higher marriage rates generates larger gender human capital gaps through the channel of human capital formation.

## 4.2 Mechanism

With all elements in the dynamic system defined, this section discusses the mechanisms that result in the demise of patriarchy.

**Lemma 3**: The levels of $\alpha_t$ and $n_t^m$ are decreasing in $A_t$.

*Proof*: See Appendix.

The logic of Lemma 3 hinges on the trade-off between consumption and fertility in household decision-making. When total factor productivity $A$ increases, it elevates wages, thereby raising the opportunity cost of child-rearing. Since consumption and fertility are substitutes in utility, households respond to higher productivity by prioritizing consumption over having children—the substitution effect (driven by increased costs of child-rearing) outweighs the income effect (which might otherwise encourage more children with higher income). This leads to a decline in marital fertility ($n_t^m$). Consequently, the share of transfers ($\alpha$) men offer their wives also decreases, as these transfers are tied to marital fertility. This mechanism is illustrated by the red arrows in Figure 2, linking rising $A$ to falling $\alpha$.

The demise of patriarchy is further accelerated by a self-reinforcing feedback loop between declining marriage rates and shrinking gender human capital gaps (illustrated by the blue arrows in Figure 2). Here's how this cycle unfolds:

1. Declining transfers reduce marriage incentives
   A fall in the transfer share ($\alpha$) directly reduces the economic surplus women gain from marriage ($\alpha\Gamma^h$), where $\Gamma^h$ measures the gender gap in human capital (e.g., men's historically higher earnings). As marriage becomes less financially advantageous, fewer women opt to marry, leading to a drop in the marriage rate ($\mathcal{M}$).

2. Disproportionate impact on boys' human capital
   The decline in marriage increases single-parenthood, which disproportionately harms boys' human capital accumulation. This could stem from societal biases (e.g., single mothers facing constraints in investing in sons' education) or labor market dynamics (e.g., boys in father-absent households lacking role models or networks). Consequently, the male advantage in human capital ($\Gamma^h$) narrows in

the next generation.

3. Feedback loop: smaller gender gap erodes marriage further
   A smaller $\Gamma^h$ reduces the economic rationale for marriage ($\alpha\Gamma^h$) even more, driving marriage rates ($\mathcal{M}$) down further. This creates a self-reinforcing cycle: lower marriage rates shrink the gender human capital gap, which in turn makes marriage even less appealing.

Rising productivity ($A_t$) thus sets off a chain reaction: the initial decline in $\alpha$ (from Lemma 3) kickstarts this feedback loop, which amplifies the effects of $A_t$ across generations. The result is a persistent, co-evolving decline in both marriage rates and gender gaps, eroding the structural pillars of patriarchal systems. More rigorously, the impact of the feedback loop is given by Lemma 4.

**Lemma 4**: Declining $\alpha_t$ reduces long-run $\mathcal{M}$ and $\Gamma^h$.

*Proof*: See Appendix.

Taking stock, Figure 2 indicates that the joint declines in fertility, marriage, and gender income can be explained by a unified framework that solely relies on rising total factor productivity $A_t$.

## 4.3 The Role of Social Norms

Figure 2 reveals an important nuance: although the first channel—where rising productivity $A_t$ uniformly reduces marital fertility ($n_t^m$) and lowers transfer shares ($\alpha_t$)—operates similarly across countries, the impacts on marriage rates ($\mathcal{M}$) and gender income gaps ($\Gamma^h$) diverge significantly in the long run and along the transition path. This divergence stems from cross-country differences in the quantitative strength of the second channel, driven by societal variations in how marriage decisions respond to shifts in economic incentives.

Central to this variation is the relationship between the "transfer potential" ($\alpha\Gamma^h$) and marriage rates ($\mathcal{M}$), which depends critically on the distribution of idiosyncratic shocks $J(\tau)$. These shocks reflect societal and institutional factors—such as cultural at-

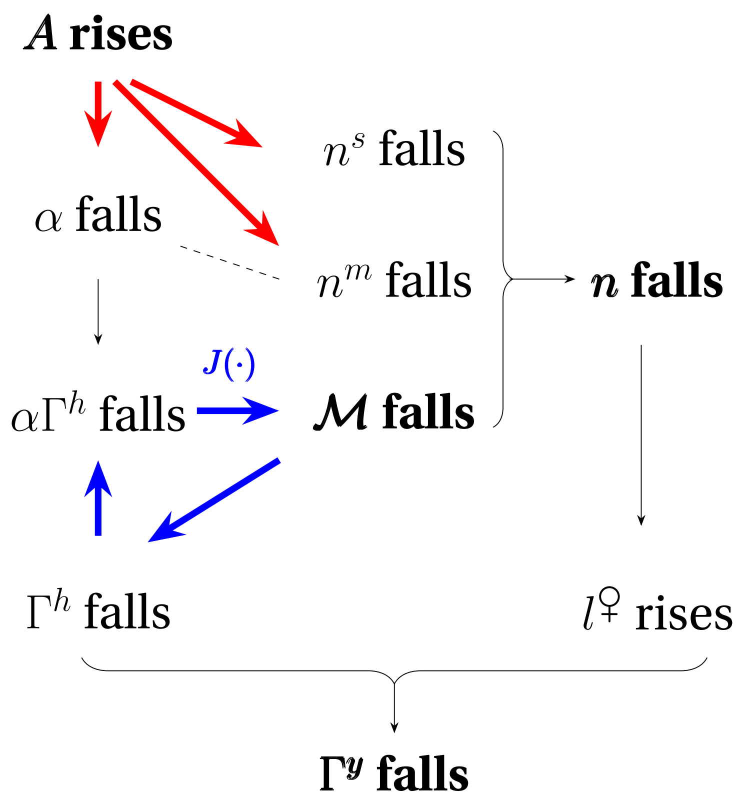



Figure 2: The Autopsy of Patriarchy

titudes toward marriage, religious norms, or the prevalence of practices like shotgun marriages—that shape individual thresholds for entering marriage. Crucially, the density of individuals near the marriage cutoff $\tau^*$ determines the sensitivity of $\mathcal{M}$ to changes in $\alpha\Gamma^h$. In societies where many individuals cluster near $\tau^*$ (e.g., cultures with weak marital norms or little stigma against single parenthood), even modest declines in $\alpha\Gamma^h$ can precipitate sharp, nonlinear drops in $\mathcal{M}$. In contrast, more traditional societies exhibit smaller, more gradual responses.

These differences in elasticity cascade into the feedback loop between marriage rates and human capital gaps. Where marriage declines are abrupt, the resulting rise in single parenthood disproportionately stunts boys' human capital accumulation, accelerating the narrowing of $\Gamma^h$ . This rapid narrowing further erodes the transfer potential ($\alpha\Gamma^h$), intensifying the cycle. Conversely, in societies with gradual marriage declines, the feedback between $\mathcal{M}$ and $\Gamma^h$ unfolds more slowly, dampening the pace of patriarchal decline.

To provide concrete examples, Figure 3a illustrates the case of the United Kingdom. Following the decline in fertility after the after-war Baby Boom, single parenthood experienced a significant rise starting in the 1980s. From the perspective of the model, this trend can be attributed to two key factors: the increasing female labor supply and the narrowing gender gaps in human capital. Together, these forces contributed to the convergence of gender income gaps in the UK.

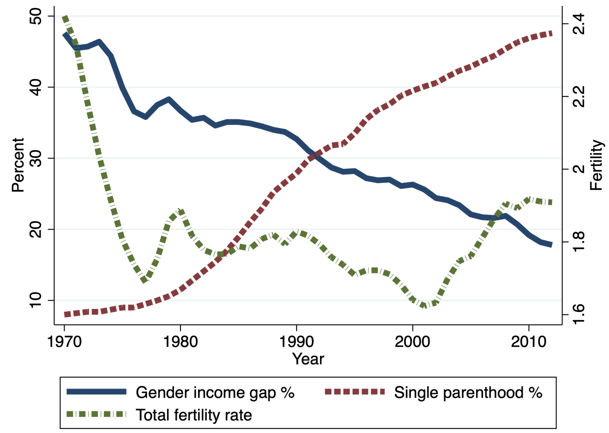
In contrast, Figure 3b depicts the case of Japan. While fertility also declined during the rapid economic growth of the 1980s, single parenthood remained relatively stagnant. This can be largely explained by the enduring influence of Confucian traditions, which stigmatize out-of-wedlock births (Myong et al. 2021). Through the lens of the model, only the rising female labor supply played a role in narrowing gender income gaps in Japan. Consequently, the pace of gender gap convergence in Japan has been significantly slower compared to that in the United Kingdom. These differences can be partly attributed to the distinct distributions of $J(\tau)$ in the two countries.

A similar comparative analysis can be extended to other cases, such as Hungary (Figure 3c), South Korea (Figure 3d), Australia (Figure 3e), and Poland (Figure 3f). Each of these examples reveals unique patterns and drivers of gender income gap convergence, shaped by their respective cultural, economic, and institutional contexts.

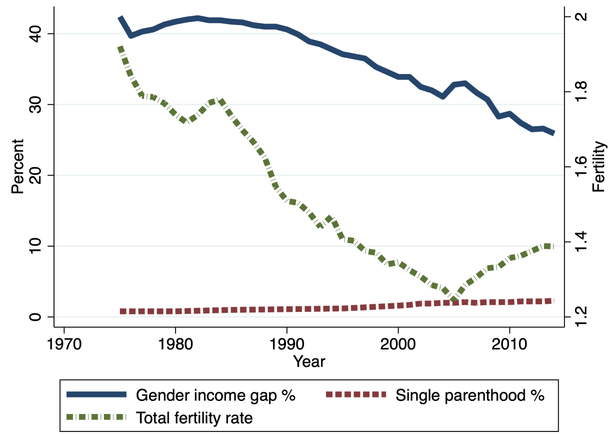
# 5. Discussions

A compelling and complex question arises: Could achieving gender equality in childcare responsibilities—a topic explored in recent studies such as Doepke and Kindermann (2019)—help resolve the trilemma? Specifically, if childcare responsibilities were more equally shared between men and women, could societies achieve higher fertility rates while simultaneously maintaining dual parenthood and gender income equality? This question lies at the heart of understanding how shifting gender norms and shared caregiving might address the intertwined challenges of fertility decline, family structure, and economic equity presented in this paper.

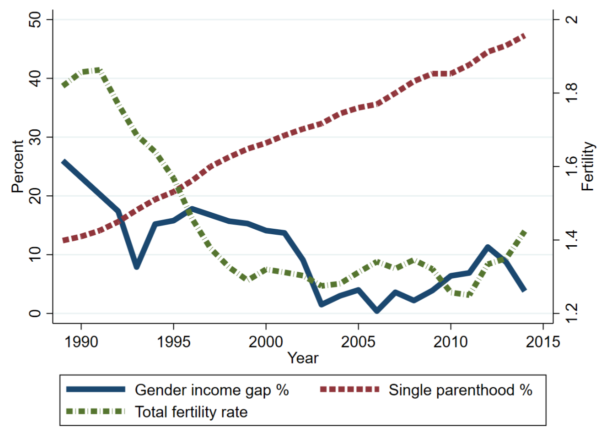
Through the lens of the model, if childcare responsibilities are equally shared be-



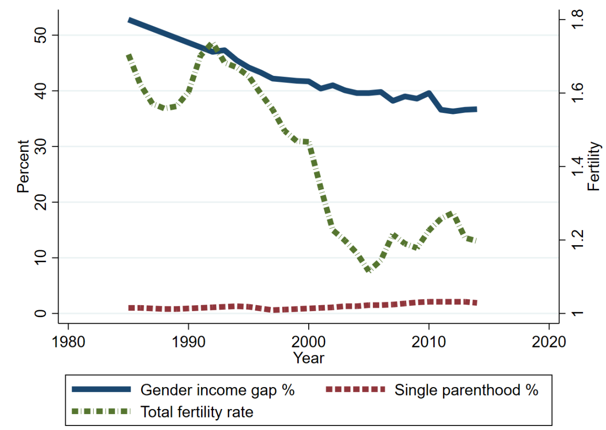

(a) The Case of the U.K.



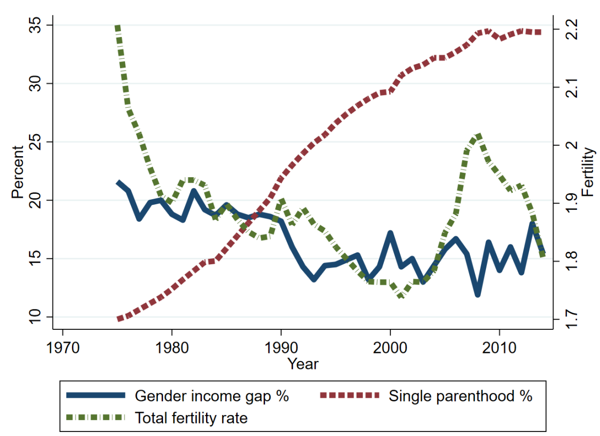

(b) The Case of Japan



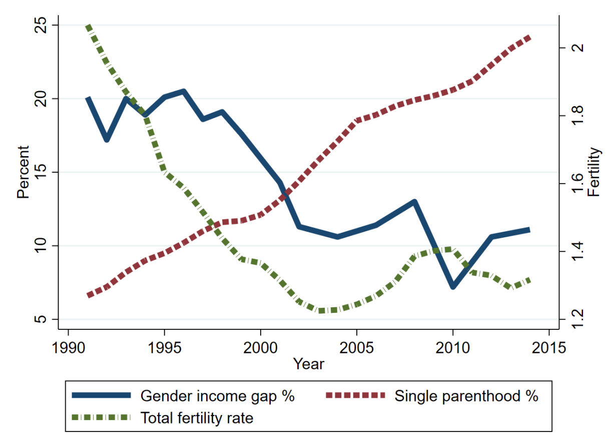

(c) The Case of Hungary




(d) The Case of South Korea




(e) The Case of Australia




(f) The Case of Poland

Figure 3: The Demise of Patriarchy: Some Examples

tween genders, the labor supply would be identical across genders regardless of fertility levels. Consequently, the gender income gap $\Gamma^y$ would depend entirely on the gender human capital gap $\Gamma^h$. However, under high marriage rates $\mathcal{M}$, the gender human capital gap $\Gamma^h$ tends to remain significant due to the differential sensitivity assumption $\psi^{♂} > \psi^{♀}$. This implies that, to achieve both dual parenthood and gender income equality, men would need to take on *more* childcare responsibilities than women. This requirement, however, raises three critical issues.

First, what would be the efficiency cost of men working less than women, particularly when their human capital is relatively higher? This cost could be further amplified if women possess an absolute advantage in childcare, as reallocating time from higher-productivity work to childcare could lead to greater economic inefficiencies.

Second, since men have the outside option of remaining single and childless, the transfer $\alpha$ (representing the economic gains from marriage) would need to be very low to incentivize them to take on additional childcare responsibilities within marriage. However, if $\alpha$ is small, the economic benefits of marriage diminish, making it less attractive for women as well. This could lead to lower marriage rates, undermining the goal of sustaining dual parenthood.

Third, from an empirical perspective, despite significant progress toward equal childcare sharing, particularly in many European countries, Figure 1 suggests that there is little evidence to support this approach as a viable solution to the trilemma. The data indicate that even in societies with more egalitarian childcare practices, the challenges of achieving high fertility, dual parenthood, and gender income equality persist.

For these reasons, I argue that achieving gender equality in childcare responsibilities is unlikely to resolve the trilemma. While it represents a step toward greater equity, it does not address the underlying structural and economic constraints that perpetuate the trade-offs between fertility, family structure, and gender income equality.

## 6. Conclusion

Human society is in the midst of a profound and irreversible transformation as the institution of patriarchy, which has shaped economic and social structures for millennia, gradually erodes. This paper develops a unified framework to analyze the dynamic interactions between fertility, dual parenthood, and gender income gaps during this historic transition. The model yields three central contributions. First, it identifies a fundamental trilemma in family economics: high fertility, dual parenthood, and gender income equality are mutually incompatible—a result that is both theoretically novel and empirically validated. Second, the analysis demonstrates that rising total factor productivity alone is sufficient to drive the decline of patriarchy, challenging the prevailing view that factor-biased technological change is a necessary condition for such a shift. Finally, while the demise of patriarchy appears inevitable in the long run, the pace of this transition is shown to vary significantly across societies, with social norms acting as a critical mediating factor.

These findings have far-reaching implications for economic theory and policy. The trilemma underscores the inherent trade-offs faced by modern societies as they strive to balance demographic sustainability, gender equality, and the growing demand for dual-earner households. The role of total factor productivity highlights the importance of broad-based economic growth in reshaping gender dynamics, while the heterogeneity in transition paths emphasizes the enduring influence of cultural and institutional contexts.

This paper not only advances our understanding of the forces driving the decline of patriarchy but also raises pressing questions for future research. How will societies navigate the trilemma in the face of declining fertility rates and rising gender equality? What institutional innovations might emerge to reconcile these competing objectives? And how will the uneven pace of transition shape global inequality in the decades to come? By shedding light on these questions, this study provides a foundation for rethinking the economic and social structures of a post-patriarchal world.

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# Appendix

## A. Proofs

**Proof of Lemma 1**

Define function

$$f_1(\alpha_t) = A_t h_t^{♂} \cdot \left( \frac{1-\beta}{\beta} \cdot [1 - (1-\alpha_t)^{\frac{\rho-1}{\rho}}] \right)^{\frac{\rho}{\rho-1}}, \qquad \alpha \in [0,1]$$

For $\rho > 1$, $f_1(\alpha_t)$ is strictly increasing, convex, and $f_1(0) = 0$. Moreover, $n_t^m = f_1(\alpha_t)$ satisfies men's indifference condition (9).

Define function

$$f_2(\alpha_t) = \frac{(1+\alpha_t \Gamma_t^h) A_t h_t^{♀}}{\left(\frac{(1-\beta)A_t h_t^{♀} \chi}{\beta}\right)^{\rho} + A_t h_t^{♀} \chi}, \qquad \alpha_t \in [0,1]$$

For $\rho > 1$, $f_2(\alpha_t)$ is strictly increasing, linear, and $f_2(0) > 0$. Moreover, $n_t^m = f_2(\alpha_t)$ satisfies women's optimality condition (15).

Thus, $f_3(\alpha_t) = f_1(\alpha_t) - f_2(\alpha_2)$ is strictly increasing, convex, and $f_3(0) < 0$. Therefore, there are two possibilities. If $f_3(\alpha_t)$ obtains the value of zero in the domain $\alpha \in [0,1]$, i.e., interior solution, then this solution is unique. Otherwise, there is a corner solution $\alpha_t = 1$, i.e., men strictly prefer marriage over being single and are willing to transfer the entirety of their income – a theoretically possible but empirically irrelevant case.

Figure A.1 provides a graphical illustration of the proof.

**Proof of Lemma 2**

For married women, the first-order condition is

$$(1-\beta) \cdot (c_t^{♀,m})^{-\frac{1}{\rho}} = \frac{\beta \cdot (n_t^m)^{-\frac{1}{\rho}}}{A_t h_t^{♀} \chi} \Longrightarrow c_t^{♀,m} = n_t^m \cdot \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} \tag{22}$$

Substituting (22) into the budget constraint, $n_t^m$ satisfies

$$n_t^m \cdot \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} = \alpha_t \Gamma_t^h A_t h_t^{♀} + A_t h_t^{♀} (1 - \chi n_t^m)$$

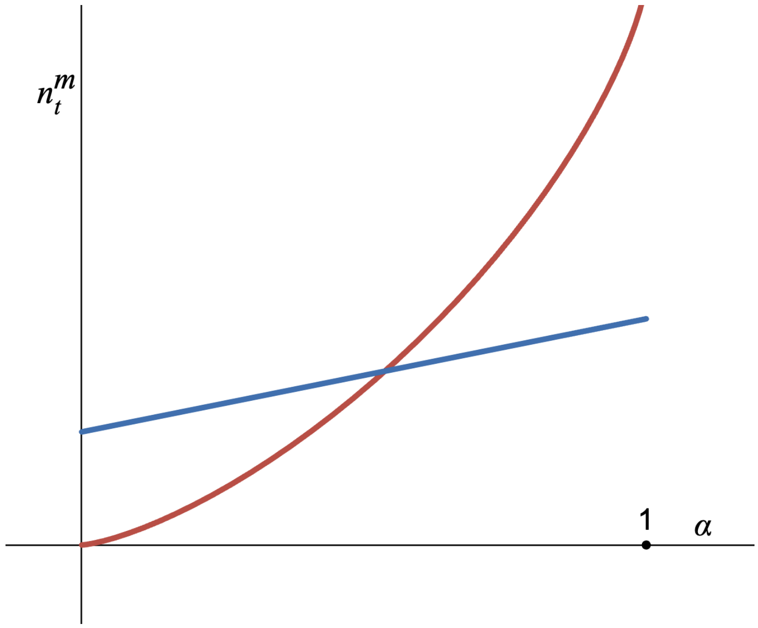


Figure A.1: $n_t^m(\alpha_t)$ (blue) and $\alpha_t(n_t^m)$ (red)

which is equivalent to

$$n_t^m \cdot \left[ \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} + A_t h_t^{♀} \chi \right] = (1 + \alpha_t \Gamma_t^h) A_t h_t^{♀} \tag{23}$$

For single women, the first-order condition is

$$(1-\beta) \cdot (c_t^{♀,s})^{-\frac{1}{\rho}} = \frac{\beta \cdot (n_t^s)^{-\frac{1}{\rho}}}{A_t h_t^{♀} \chi} \Longrightarrow c_t^{♀,s} = n_t^s \cdot \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} \tag{24}$$

Substituting (24) into the budget constraint, $c_t^{♀,s}$ satisfies

$$n_t^s \cdot \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} = A_t h_t^{♀} (1 - \chi n_t^s)$$

which is equivalent to

$$n_t^s \cdot \left[ \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} + A_t h_t^{♀} \chi \right] = A_t h_t^{♀} \tag{25}$$

Take the ratio between (23) and (25) gives

$$\frac{n_t^m}{n_t^s} = 1 + \alpha_t \Gamma_t^h \tag{26}$$

which is independent of $A_t$.

On the other hand,

$$V_t^{♀,m}(\tau) = \tau \cdot n_t^m \cdot \left( (1-\beta) \cdot \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho-1} + \beta \right)^{\frac{\rho}{\rho-1}} \tag{27}$$

$$V_t^{♀,s} = n_t^s \cdot \left( (1-\beta) \cdot \left( \frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho-1} + \beta \right)^{\frac{\rho}{\rho-1}} \tag{28}$$

Combining (27), (28), and (26),

$$\tau^* = \frac{V_t^{♀,s}}{V_t^{♀,m}} = \frac{n_t^s}{n_t^m} = \frac{1}{1 + \alpha_t \Gamma_t^h} \tag{29}$$

**Proof of Lemma 3**

When $A_t$ increases, $f_1(\alpha_t)$ shifts up while $f_2(\alpha_t)$ shifts down. Therefore, $f_3(\alpha_t) = f_1(\alpha_t) - f_2(\alpha_2)$ shifts up. As a result, the interior solution, i.e., the value of $\alpha_t$ such that $f_3(\alpha_t) = 0$, necessarily decreases.

Figure A.2 provides a graphical illustration of the proof.

**Proof of Lemma 4**

When $\alpha_t$ falls, $\mathcal{M}(\Gamma^h; \alpha)$ shifts down while $\Gamma^h(\mathcal{M})$ is unaffected. As a result, the intersection $(\alpha, \Gamma^h)$ falls.

Figure A.3 provides a graphical illustration of the proof.

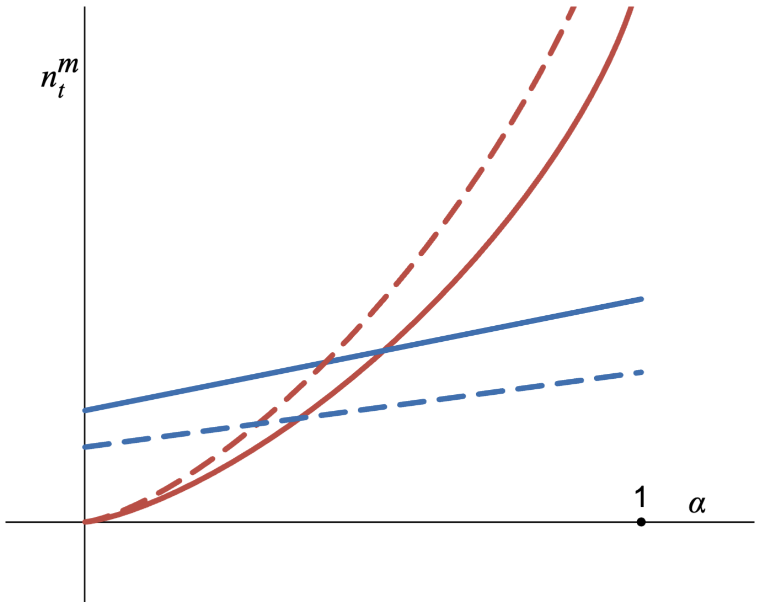


Figure A.2: $f_1(\alpha_t)$ (red) and $f_2(\alpha_t)$ (blue). Solid (before) and dashed (after)

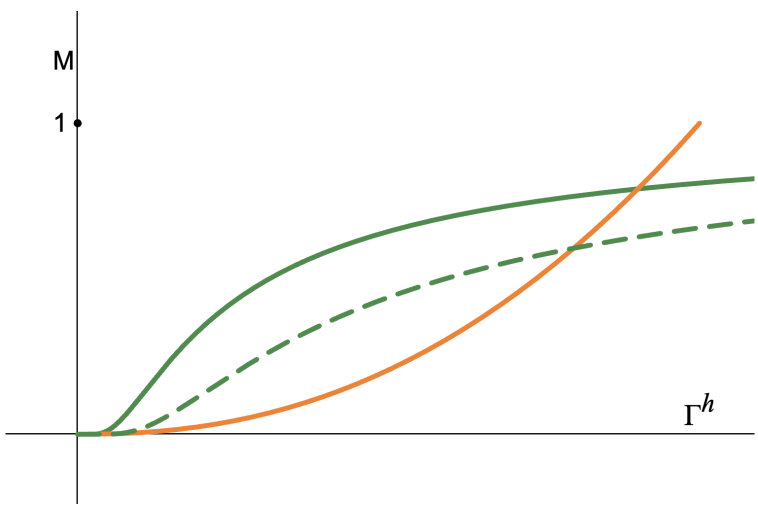


Figure A.3: $\mathcal{M}(\Gamma^h; \alpha)$ (green) and $\Gamma^h(\mathcal{M})$ (orange)

# B. Figures

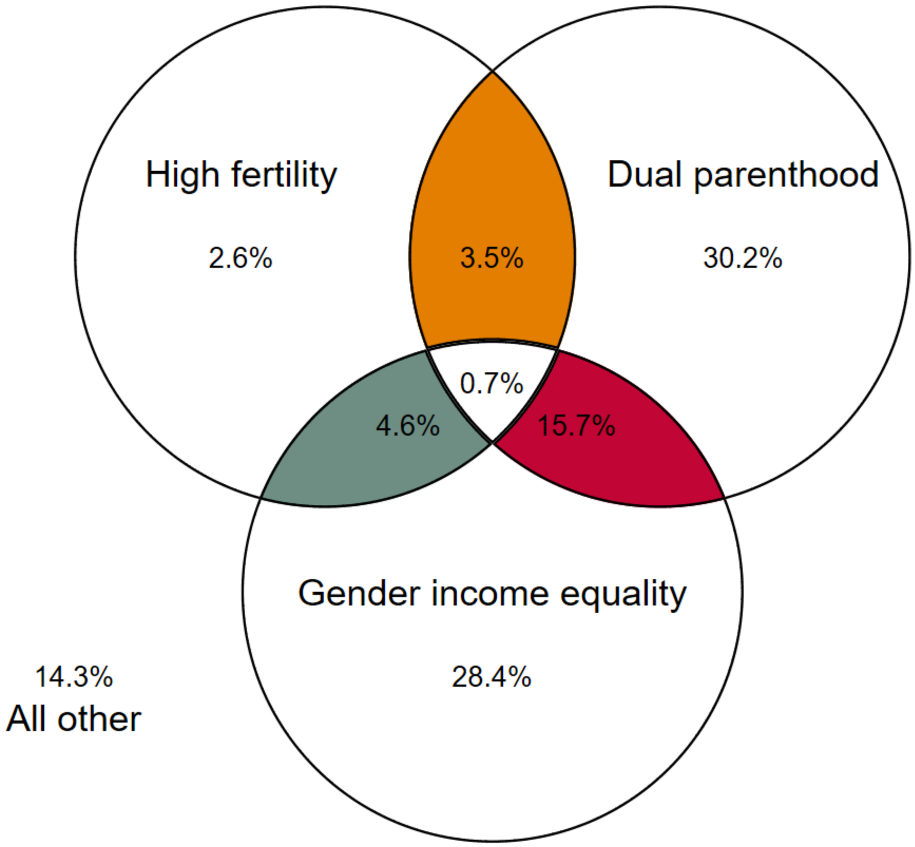


Figure A.4: Trilemma: “High fertility” if TFR¿2

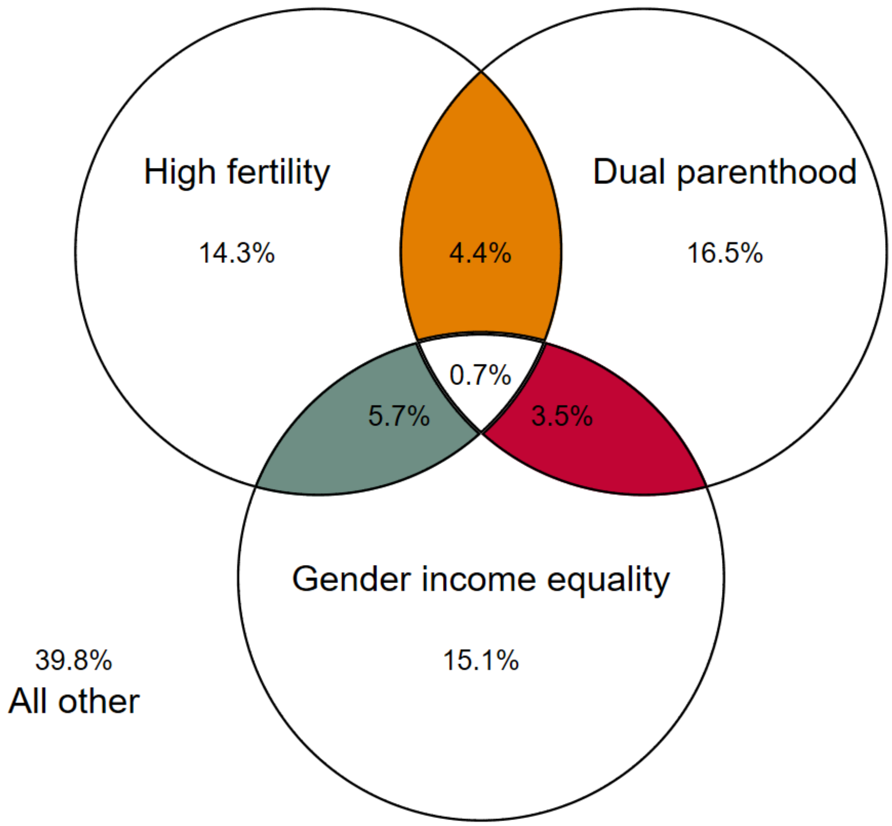


Figure A.5: Trilemma: Define Categories using Upper Quartiles

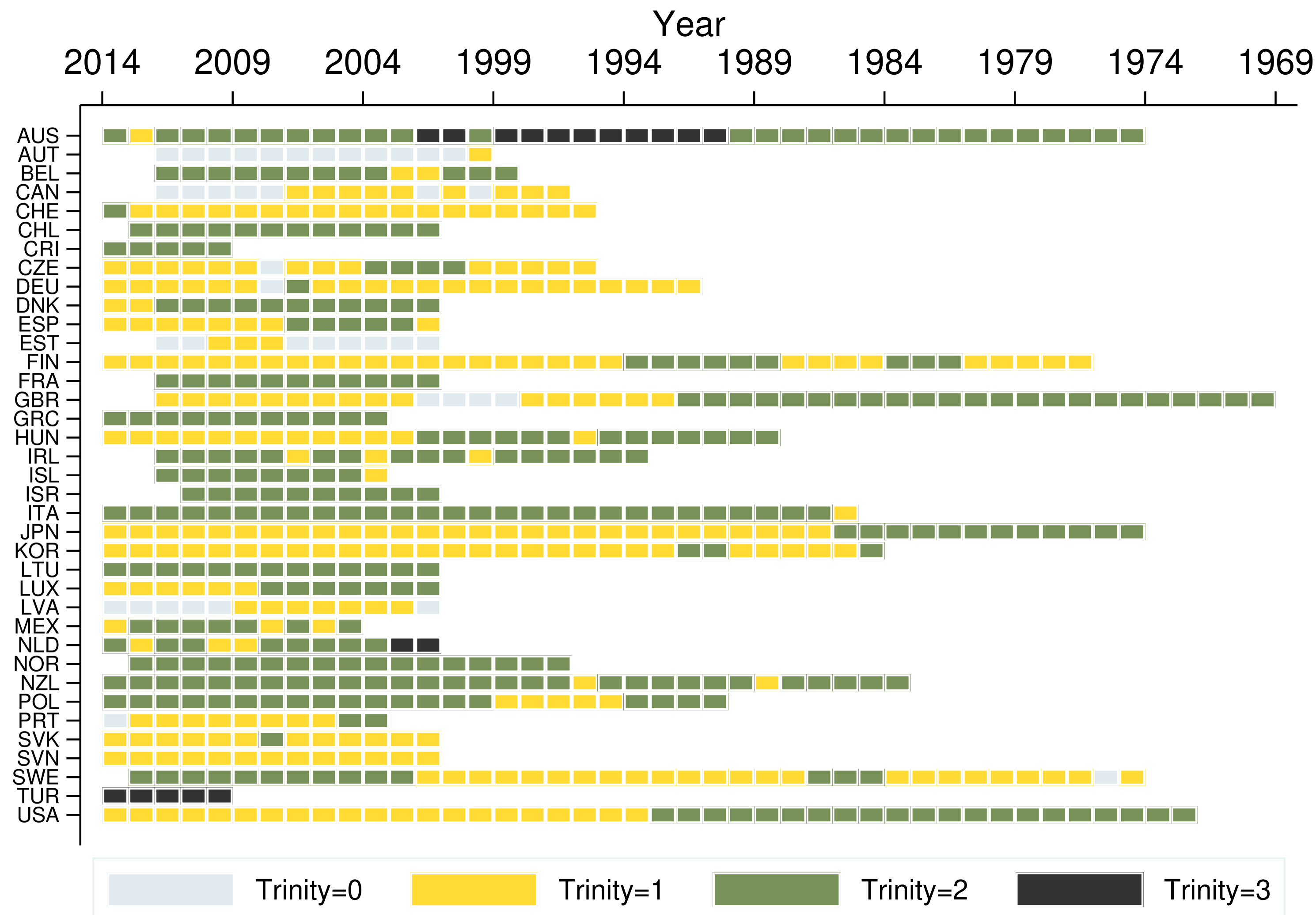


Figure A.6: Number of Outcomes Achieved by Country and Time