

Building Future Generations: The Macroeconomic Consequences of Family Policies*

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Job Market Paper

October 25, 2021

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Abstract

Most developed countries use family policies, but little is known about their macroeconomic consequences. This paper develops a heterogeneous agent overlapping generations framework that integrates child quantity (number of children) and quality (investment per child) trade-offs, a rich demographic structure, and childcare choices. Calibrating the model to U.S. data and validating it with quasi-experimental evidence, I find that raising aggregate fertility to the replacement level requires a \$30,000 cash reward to childbirth, similar to the net present value change of the maximum Child Tax Credit in the past decade. Contrary to conventional wisdom, such a policy reduces average human capital and intergenerational mobility. The key to this result is that the marginal cost of child quality rises with fertility, and fertility effects of the policy are larger among parents with lower income. Nevertheless, average well-being rises by 1.6% in the long run as the old-age dependency ratio drops, requiring lower taxes to sustain retirement benefits. This gain in long-run welfare comes with higher public spending in transition, which may hurt existing households depending on how the policy is funded. Compared with cash rewards, in-kind benefits are less cost-effective in raising fertility but have other advantages: subsidized childcare encourages parents to work, while expansions of public education improve children's human capital and intergenerational mobility.

JEL classification: E62, H31, H52, J13

Keywords: Family policies, quantity-quality trade-off, demographic structure

*I am deeply indebted to my advisors Dean Corbae, Ananth Seshadri, and Kenneth West for their invaluable guidance and support. I also thank Naoki Aizawa, Simeon Alder, Manuel Amador, David Baqaee, Corina Boar, Job Boerma, Jason Choi, Diego Daruich, Maya Eden, Sam Engle, Xinyu Fan, Chao Fu, Nicole Fu, Jeremy Greenwood, Nezih Guner, Joe Hazell, Joseph Kaboski, Cody Kallen, Tim Kehoe, Rishabh Kirpalani, Annie Lee, Zehao Li, Chang Liu, Rasmus Lentz, Paolo Martellini, Joseph Mullins, Cezar Santos, Jeff Smith, Lyman Stone, John Stromme, Anna Trubnikova, Joanna Venator, Michael Waugh, Matthew Wiswall, Alan Yang, and seminar and conference participants at the University of Wisconsin-Madison, Minnesota-Wisconsin Macro/International workshop, Virtual Family Macro Group, WEAI, and SEA for helpful comments and suggestions on this paper. I gratefully acknowledge the financial support from the Juli Plant Grainger Institute Summer Research Fellowships, Donald D. Hester Dissertation Fellowship, and Juli Plant Grainger Institute Outstanding Dissertation Fellowship. All errors are my own.

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1 Introduction

During the past decades, family policies have become increasingly important in developed economies. They are social programs and laws designed to promote and enhance family formation, reproduction, and child-rearing. These programs include both in-cash benefits (e.g. baby bonuses and child tax benefits) and in-kind ones (e.g. subsidized childcare, public pre-K). At the macro level, family policies are one of the key policy instruments to address population aging, which poses a threat to the sustainability of the public pension system (Bongaarts, 2004). At the micro level, policymakers and economists believe that family policies are effective instruments to improve children’s outcomes and boost social mobility.¹ Since the 1980s, family policy expenditures have been growing steadily among OECD countries, exceeding 2% of GDP on average. Governments have also been increasingly relying on in-kind family policies.²

Despite the importance and wide implementation of large-scale family policies, little is known about their macroeconomic implications. Are family policies cost-effective in achieving stated macro-level goals of raising fertility, reducing the fiscal burden of the government, and improving social welfare? On a more micro level, do generous child benefits improve an average child’s outcome and raise social mobility? Given that changes in demographic structure take decades to realize, what do we know about the long-term effects versus outcomes in transition? With a variety of available policy instruments, how should policymakers choose among them, especially between in-cash and in-kind benefits? Answering these questions requires a unified framework to understand trade-offs under different policy instruments across households and over time. Such a framework also needs to provide quantitative predictions that can conform with data.

In this paper, I answer these questions by building a general equilibrium heterogeneous-agent model with overlapping generations. The innovation of this framework is to integrate three key ingredients that distinguish the predicted policy effects from existing work.

First, unlike most empirical studies that focus on policy effects on either fertility or children’s human capital while taking the other outcome as given, my model allows households to choose both the quantity and quality of children. From a normative perspective, both the quantity and quality margins are important because they correspond to the two main goals of family policies – raising fertility and improving children’s outcomes, respectively. From a theoretical perspective, the two margins are also closely linked in the model by the *quantity-quality trade-off* à la Becker and Lewis (1973). The theory of quantity-quality trade-off predicts that, when parents increase fertility in response to child benefits, the cost of maintaining the same level of child quality rises. Therefore, as I show in this paper, policies that effectively boost fertility may worsen children’s

¹For instance, see The White House (2021), Schanzenbach et al. (2021), and Pulliam and Reeves (2021).

²See <https://data.oecd.org/socialexp/family-benefits-public-spending.htm> for more details.

human capital.

In addition, the joint determination of child quality and quantity has implications for aggregate variables via *composition effects*. As we see in the model and data, parents with heterogeneous human capital and wealth respond differently in their fertility and child quality decisions to a given policy regime. Some parents increase fertility more than others, meaning that their children comprise a larger share of the population in the future. Because parents and children tend to have similar traits (e.g. human capital) due to intergenerational linkages, aggregate variables depend more on the traits of parents with stronger fertility responses.

Second, while most structural models with endogenous fertility consider agents that only live for two periods (e.g. [Barro and Becker, 1989](#); [de La Croix and Doepke, 2003](#); [Kim et al., 2021](#)), the framework in this paper captures a rich life cycle with childhood, working-age, and retirement. This allows me to quantify the *demographic structure effects* of family policies: following an increase in the population growth rate, the burden of pension payments under the pay-as-you-go (PAYG) system is relieved, but public expenditures for children rise. The rich demographic structure also highlights the distributional consequences of family policies across generations, especially along the transition path.

Third, unlike most models with endogenous fertility where each child costs a fixed amount of time (see [Jones et al. \(2008\)](#) for discussion), I consider *endogenous childcare arrangements* where parents can choose to either take care of children at home by themselves or utilize market childcare services. Besides allowing the model to generate childcare utilization patterns in the data that are otherwise hard to explain, this feature informs the ongoing discussion of choosing between family policies that are in-cash (e.g., baby bonuses or child allowances) versus those that are in-kind (e.g., subsidized childcare vouchers). Beyond the effects on children, distinguishing between in-cash and in-kind transfers is important for predicting the effects of policy on parents' working hours, which in turn affects wage inequality over the life cycle due to human capital accumulation on the job.

The quantitative strength of each of the mechanisms discussed above depends *fertility elasticities*, which measures the magnitude of fertility responses to financial incentives. Specifically, fertility elasticities are most affected by two key parameters in the model: the elasticity of intergenerational substitution (EGS)³ and the productivity of investment goods in children's human capital formation. Because EGS governs how the marginal benefits of having children varies with parents' human capital, I identify it using the income-fertility profile. As the productivity parameter dictates the benefit-cost ratio of education intervention programs, I calibrate it to match the investment returns estimated by [García et al. \(2020\)](#) using data from a randomized trial. Other parameters are chosen outside of the model or calibrated to match key empirical regularities in the United States

³As defined in [Córdoba et al. \(2016\)](#) and [Córdoba and Ripoll \(2019\)](#), the elasticity of intergenerational substitution (EGS) parameterizes the inter-personal willingness to substitute consumption across generations.

in 2010. To provide external validation, I compare the model predictions with estimates from a difference-in-differences analysis using data from the Alaska Permanent Fund Dividend (APFD). In addition, I show that quasi-experimental evidence from several other family policies also corroborates the magnitude of the calibrated fertility elasticities in the model.⁴

The policy counterfactual analyses reveal four main findings. First, a baby bonus (cash reward to childbirth) of \$30,000 boosts the average fertility rate from 1.9 children per woman to 2.1 children per woman, the replacement level.⁵ The amount of this transfer is similar to the expansions of the maximum payment of the Child Tax Credit (CTC) from 2010 to 2021 in net present value, taking the American Rescue Plan Act of 2021 into account. I find that parents with lower human capital respond more strongly to the baby bonus, raising fertility by a greater proportion. Surprisingly, such a child benefit is not effective in raising child human capital or social mobility. Under the calibrated EGS, parents optimally reduce child education investments by an average of 4% due to the quantity-quality trade-off, and this reduction is larger among low-income parents. The heterogeneous reductions in education investments, coupled with composition effects due to differential fertility responses, lower average human capital by 2% and intergenerational income mobility by 1.8% in the long-run steady-state equilibrium.

Second, despite reductions in average human capital and social mobility, long-run average welfare rises by 1.6% in consumption equivalents under the \$30,000 baby bonus.⁶ The key to this result is that as the old-age dependency ratio⁷ falls following an increase in fertility, the government is able to reduce tax rates while still balancing the fiscal budget.

Third, the welfare consequences for existing agents can be very different from those in transition or in the long-run steady state. During the first few decades of transition, the government will need to finance both direct policy expenses and higher child-related public expenditures as children make up a larger proportion of the population. While the old-age dependency ratio falls gradually, the total dependency ratio⁸ overshoots in the first few decades before converging to its long-run level. If the government, for example, chooses to balance the budget from period to period using changes in the consumption tax, most households in the current economy will bear higher tax burdens. These additional expenditures in transition are quantitatively significant. Therefore, policy discussions should not only focus on the long-run gains in alleviating pension liabilities, but should also devote

⁴Additional validation exercises include comparisons with the Australian baby bonus, the Spanish child benefits, and the Maternal Capital in Russia. See Appendix D for details.

⁵I use the replacement fertility as a benchmark policy goal since it is “widely considered among policy makers and the public as the desirable level of long term fertility” (Striessnig and Lutz, 2013). Section 6 discusses whether this goal is optimal in itself from a welfare point of view.

⁶Long-run welfare is defined as the average utility of a new-born under the veil of ignorance. See Section 6 for further discussion.

⁷Old-age dependency ratio is defined as the number of retired people divided by the size of the working population.

⁸Total dependency ratio is defined as the number of retired people plus the number of children, divided by the total working population.

more attention to the financing challenges and distributional consequences along the transition path.

Last, I use the model to study the macroeconomic impacts of alternative policies that are in-kind, such as subsidized childcare and expanded public education funding. I find that these policies are less cost-effective in raising fertility than the cash rewards but offer other advantages. In particular, subsidized childcare encourages parents to work in the labor market because it substitutes home production of childcare with market childcare services. The resulting increase in working hours, especially among parents with low human capital, reduces wage inequality. Among all policy instruments studied in this paper, public education most effectively improves children's outcomes and boosts social mobility. However, it also has the smallest effect on fertility.

Related Literature

This paper relates to and builds on three strands of literature. The first strand is a macroeconomic literature that studies the relationship between fertility and fiscal stability, economic growth, intergenerational mobility, and inequality. Notable examples include [Barro and Becker \(1989\)](#), [Doepke \(2004\)](#), [Erosa et al. \(2010\)](#), [De Silva and Tenreyro \(2020\)](#), [Jones \(2020\)](#), [Kim et al. \(2021\)](#), and [Cavalcanti et al. \(2021\)](#) among many others. My paper contributes to the literature by building a large-scale quantitative model to understand the macroeconomic consequences of policies that aim to raise fertility. In this line of research, [Kim et al. \(2021\)](#) is the most related paper. The authors propose an overlapping-generations model with status externality to explain the positive relationship between income and fertility in South Korea. Then, they use the model to analyze the effects of baby bonuses and education taxes. This paper differs from their work in two ways. First, I incorporate a rich life cycle to quantify the demographic structure effects of family policies. This is a key channel of the model as family policies affect the fiscal budget and taxes by changing the weight of each source of tax revenue and public expenditure. Second, I consider endogenous childcare choices where parents choose between home care and market care. This feature allows me to compare in-cash versus in-kind policies.

Second, this paper relates to a large body of empirical studies on the fertility effects of family policies (e.g., [Milligan, 2005](#); [Haan and Wrohlich \(2011\)](#), [Drago et al., 2011](#); [González, 2013](#); [Laroque and Salanié, 2014](#); [Olivetti and Petrongolo, 2017](#)). My paper complements this literature in two ways. First, I study policy effects beyond fertility, such as the effects on output, human capital, social mobility, and welfare. Second, my model generates predictions of the policy effects in the long run and in transition: children affected by these policies age and become parents in their own right, changing the distribution of the underlying population of interest.

Finally, the paper contributes to a growing literature evaluating the impacts of transfers to parents on children's outcomes. Prominent examples include [Dahl and Lochner \(2012\)](#), [Daruih \(2018\)](#), [Abbott et al. \(2019\)](#), [Mullins \(2019\)](#), [Guner et al. \(2020\)](#), and [García et al. \(2020\)](#) among

many others. Using both quasi-experimental methods and structural models with exogenous fertility, the conventional wisdom in the literature is that transfers to families with children have positive impacts on children’s outcomes, including improving health and education attainment and reducing criminal behavior (Schanzenbach et al., 2021). This paper differs by considering endogenous fertility responses. I show that in the quantitative model, introducing the “extensive margin” of fertility choice reverses the policy effects on average outcome of children and intergenerational mobility through both quantity-quality trade-off and composition effects.

The rest of the paper is organized as follows. In Section 2, I present the quantitative model. Section 3 describes the calibration of the model in detail. I conduct validation exercises in Section 4. Section 5 presents the main policy counterfactual results of the paper. Section 6 discusses optimal policy design and Section 7 concludes.

2 Model

I study a general-equilibrium overlapping generations (GE-OLG) economy. Households make choices over consumption and saving, child quality and quantity, and childcare. Representative firms hire labor and borrow capital from households. The government finances public education, pension payments, and other (exogenous) spending using distortionary taxes.

2.1 Household

Consider overlapping generations of households that live up to 90 years.⁹ I split the life cycle into 9 periods indexed by $j \in \{0, 1, 2, \dots, 8\}$, each representing 10 years. Period j therefore reflects age $j \times 10$ through $(j + 1) \times 10$.

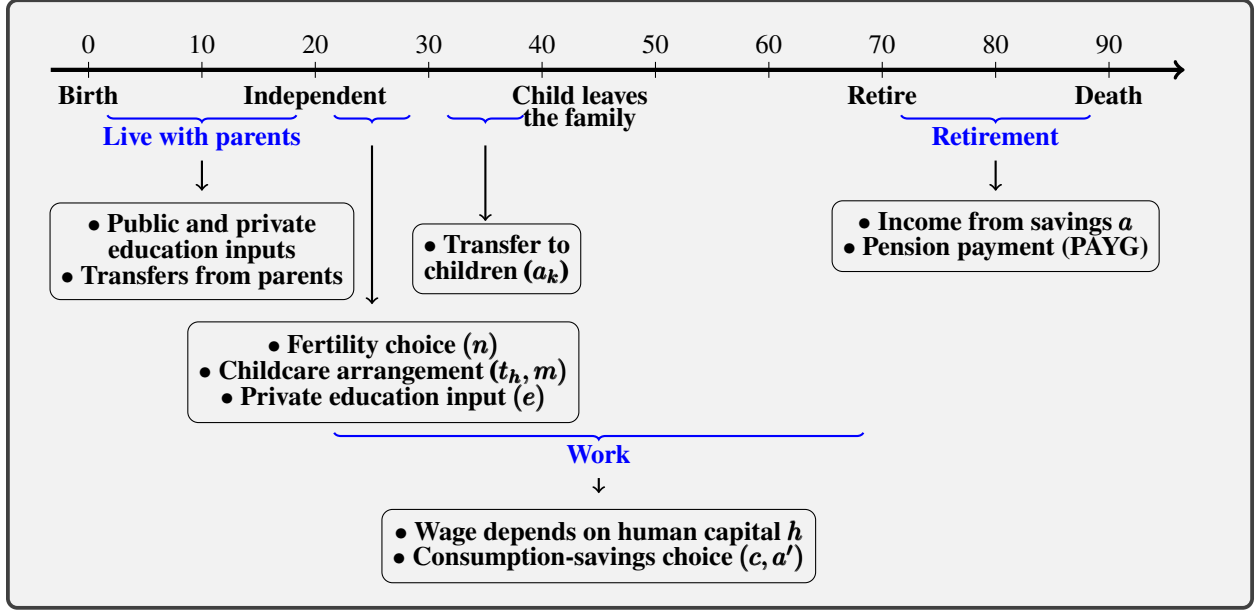
The life cycle of each agent in the model is presented in Figure 1. Children live with their parents from age 0 to 20, during which time they make no choices and receive human capital investments from both public and private sources. Children also receive inter vivos transfers from parents when they become independent and form families in their 20s. Households choose fertility, childcare arrangements, and private child human capital investments from age 20 to 30.¹⁰ Between ages 30 and 40, parents determine the amount of inter vivos transfers that their children would receive at the beginning of the next period.

People work from age 20 to 70, and their earnings depend on the amount of efficiency units, i.e., human capital, supplied to the market. Human capital for working adults evolves over time

⁹In this paper, I model unitary/collective decision-making within families and abstract away from intra-household bargaining. Hereafter, I use agents, households, and adults interchangeably.

¹⁰In Appendix A.3, I argue that the main results of this paper are robust to allowing for endogenous timing of childbirth where parents can also give birth in their 30s.

Figure 1: Model's Life Cycle



following an age-dependent learning-by-doing process:

$$h_{j+1} = L_j(h_j, t_w, z_{j+1}), \quad (1)$$

where z_{j+1} is an idiosyncratic, uninsurable shock to human capital that occurs at the beginning of period $j + 1$; h_j is the amount of human capital at period j ; and t_w is the time worked in period j . Agents have unit time endowment and supply labor inelastically, with the exception of period 2 where they trade off providing childcare at home against supplying labor in the market.¹¹

Agents retire at the end of period 6 (age 60 to 70). Retired adults receive income from savings and pension payments for the remainder of their lives. Households make consumption-savings choices in each period and face an age-specific survival rate δ_j .

At the beginning of period 2 (age 20 to 30), the state variables for young adults are the human capital level h and assets a . Both h and a are endogenously determined in equilibrium by the young adults' parents during previous periods. Agents choose consumption c , savings a' , fertility n , total time spent providing at-home childcare t_h , amount of market childcare purchased for each child m , and private education investment for each child e .¹² Parents' value $V_2(\cdot)$ is composed of flow

¹¹I have experimented with adding endogenous labor supply with leisure in the utility function. Quantitative results remain largely unchanged, but computation time is significantly increased, especially for the transition path. Therefore, I omit labor supply in the baseline model.

¹²Following Barro and Becker (1989), de La Croix and Doepke (2003), and most subsequent work, the fertility choice n is continuous in the model. An alternative approach is to use discrete fertility and add states or idiosyncratic

utility from consumption $u(c/\Lambda(n))$ and the discounted continuation value $\beta V_3(\cdot)$. Consumption expenditure c is divided by equivalence scale $\Lambda(n)$.¹³ Their maximization problem is given by

$$V_2(h, a) = \max_{c, a', n, t_h, m, e \geq 0} u(c/\Lambda(n)) + \beta \mathbb{E} V_3(h', a', n, \mathbb{E} h_k)$$

subject to

$$n \cdot \chi = \left(t_h^{v/\iota} + (n \cdot (m + \mathcal{S}))^v \right)^{1/v}, \quad [\text{time cost}]$$

$$y = wh \cdot (1 - t_h), \quad [\text{labor income}]$$

$$(1 + \tau_c)(c + p_m \cdot n \cdot m + e \cdot n) + a' = (1 + r)a + y - \mathcal{T}(y, a, n) + \mathcal{B} \cdot n, \quad [\text{BC}]$$

$$h' = L_2(h, 1 - t_h, z'), \text{ and} \quad [\text{learning OTJ}]$$

$$h_k = G(h, \mathcal{E}, e, \epsilon). \quad [\text{skill formation}]$$

To raise n children, parents need to produce $n \cdot \chi$ amount of childcare (in time units). Parents could satisfy this need either by spending their own time at home (t_h) or by purchasing market childcare services m . The government chooses to provide in-kind childcare $\mathcal{S} \in [0, \chi]$ free of charge to parents. These two forms of childcare are combined in a constant-elasticity-of-substitution (CES) production function where v governs the elasticity of substitution. Household production of childcare enjoys an economy of scale with parameter $\iota \in (0, 1)$. This captures the fact that taking care of two children at home simultaneously costs less than two times the hours needed to take care of a single child (Folbre, 2008).

The opportunity cost of home production of childcare t_h is hours spent in the labor market. The household's labor income y is the product of the market wage w , human capital h , and time worked $(1 - t_h)$. Total resources available to the parents consist of risk-free assets a multiplied by the gross interest rate $(1 + r)$, labor income y , net taxes paid $\mathcal{T}(y, a, n)$, and the total amount of baby bonuses received $\mathcal{B} \cdot n$ where \mathcal{B} is chosen by the government. The household divides resources into savings a' and different expenditures, including consumption c , total spending on market childcare $p_m \cdot n \cdot m$, and private education expenditures $e \cdot n$. I use p_m to denote the price of market childcare relative to consumption goods.¹⁴ All expenditures are subject to a proportional consumption tax τ_c .

shocks (e.g., Daruich and Kozlowski, 2020). Since the model is calibrated to match aggregate moments on income-fertility relationship, the second approach yields similar results once the additional state space are integrated over.

¹³As defined by Browning et al. (2013), "An equivalence scale is traditionally defined as the expenditures of the household divided by the expenditures of a single person that enjoys the same 'standard of living' as the household. Just as a true cost of living price index measures the ratio of costs of attaining the same utility level under different price regimes, equivalence scales are supposed to measure the ratio of costs of attaining the same utility level under different household compositions."

¹⁴In the model, the supply of childcare services is perfectly elastic at baseline price p_m . Since each period represents ten years, adjustments of capital and labor in and out of the childcare industry equate childcare prices to long-run

The child human capital production function $G(h, \mathcal{E}, e, \epsilon)$ combines parents' human capital h , public education \mathcal{E} , private investment e , and idiosyncratic ability shocks ϵ which are unknown to the parents. To keep the model tractable, I make a simplifying assumption that parental investments in child human capital take place when children are between age 0 and 10 while public education \mathcal{E} affects children throughout age 0 to 20. The function $G(\cdot)$ captures the overall human capital production function that spans age 0 to 20 without explicitly modeling several stages of production. Moreover, to the extent that the bulk of childcare needs $n \cdot \chi$ are non-educational (i.e., preparing food or changing diapers), childcare arrangements do not affect child human capital directly in the model.¹⁵ Parents can invest in their children's human capital through monetary investments e . As the government chooses public education expenditures \mathcal{E} , the model could replicate the public provision of high-quality childcare or pre-K by adopting high \mathcal{S} and \mathcal{E} at the same time.

In the economy, households face an inter-temporal borrowing constraint $a' \geq 0$ that is standard in the class of Aiyagari-Bewley-Huggett models. Moreover, parents cannot invest negative amounts of resources in children's education, i.e., $e \geq 0$. As a result, public investment \mathcal{E} serves as a lower bound of the total education investment received by each child.¹⁶

Parents' maximization problem from age 30 to 40 is given by

$$V_3(h, a, n, \mathbb{E}h_k) = \max_{c, a', a_k \geq 0} u(c/\Lambda(n)) + \beta \mathbb{E}V_4(h', a') + v(n, \mathbb{E}h_k, a_k)$$

subject to

$$y = wh,$$

$$(1 + \tau_c)c + a' + n \cdot a_k = (1 + r)a + y - \mathcal{T}(y, a, n), \text{ and} \quad (2)$$

$$h' = L_3(h, 1, z'),$$

where parents choose consumption, savings, and the amount of transfers to be received by each child

marginal costs so long as the industry is perfectly competitive. In the short run, of course, relative prices of childcare may change in response to family policies.

¹⁵As [Guryan et al. \(2008\)](#) have noted, educational childcare time accounts for at most one quarter of the time that parents spend with their children. Furthermore, [Bernal and Keane \(2011\)](#) find that among single mothers, the use of formal center-based childcare, resembling m in the model, do not affect children's cognitive achievements. In a recent paper, [Chaparro et al. \(2020\)](#) develop a model of childcare with both quality and quantity aspects of maternal and non-maternal care. In their model, effects on child human capital are proportional to both care quality and time "exposed" to each type of care, but they do not explicitly consider non-educational time spent with children. The model in this paper separates the essence of childcare into non-education chores χ and endogenous education investments e , which includes the premium paid to obtain high-quality childcare services.

¹⁶As public education funding depends on local taxation ([Kotera and Seshadri, 2017](#)), one might worry that the amount of education resources received by each child is not uniformly \mathcal{E} . The child human capital production function $G(\cdot)$ allows for this possibility through its dependency on parents' human capital h . In other words, $\partial G(\cdot)/\partial h$ captures not only the genetic transmission of ability but also ability transmitted via other channels such as residential segregation.

at the beginning of the next period (a_k) .¹⁷ Parents face intergenerational borrowing constraints so that they are not allowed to make negative transfers to children, i.e., $a_k \geq 0$. I use $v(n, \mathbb{E}h_k, a_k)$ to denote parents' preferences over child quantity, child human capital, and inter vivos transfers. I discuss the parametric form of $v(\cdot)$ in detail in Section 3.1.

For $j \in \{4, 5, 6\}$, households solve a simple consumption-savings problem with idiosyncratic shocks to human capital. The maximization problem is given by

$$V_j(h, a) = \max_{c, a' \geq 0} u(c/\Lambda(0)) + \beta \delta_j \mathbb{E} V_{j+1}(h', a') \quad j \in \{4, 5, 6\}$$

subject to

$$(1 + \tau_c)c + a' = (1 + r)a + y - \mathcal{T}(y, a, 0) \text{ and}$$

$$h' = L_j(h, 1, z).$$

Finally, retired agents solve

$$V_j(h, a) = \max_{c, a' \geq 0} u(c/\Lambda(0)) + \beta \delta_j V_{j+1}(h, a') \quad j \in \{7, 8\}$$

$$(1 + \tau_c)c + a' = (1 + r)a + \pi \cdot wh - \mathcal{T}(0, a, 0) \text{ and}$$

$$V_9(\cdot) \equiv 0,$$

where household income is composed of risk-free assets and pension payments $\pi \cdot wh$, and where π denotes the pension replacement rate.¹⁸

2.2 Firms

There is a representative firm in the economy that hires labor and borrows capital from households to produce final goods with Cobb-Douglas technology

$$Y = AK^\alpha H^{1-\alpha}. \quad (3)$$

In Equation (3), K is aggregate capital used in production and H is total efficiency units employed. Without loss of generality, we normalize total factor productivity A to one.¹⁹

¹⁷I discuss the robustness of the main results when I add end-of-life bequests to the model in Appendix A.4.

¹⁸For simplicity, I assume that pension payments are not subject to labor income taxes.

¹⁹I abstract away from population externalities that could affect aggregate production such as pollution (Bohn and Stuart, 2015) and idea creation (Jones, 2020). This choice is made since (1) the literature on the measurement of population externalities is still developing, and (2) the results will change in expected ways once positive/negative externalities are incorporated.

Physical capital depreciates at rate δ_K after use. With competitive factor markets, the equilibrium wage and risk-free interest rate are given by

$$r = \alpha \left(\frac{K}{H} \right)^{\alpha-1} - \delta_K \quad \text{and} \quad w = (1 - \alpha) \left(\frac{K}{H} \right)^{\alpha}.$$

2.3 Government

The government collects revenues from taxing labor income, capital income, and household expenditures (including spending on market childcare and children's education). Government expenditures include public education, family policies, pension payments, and other policy-invariant expenditures, denoted Ω , that depends on the mass of adults. The government balances the budget from period to period.²⁰

I use $\{\mu_j\}_{j=0}^8$ to denote the distribution of households across the state space and $\{\omega_j\}_{j=0}^8$ to denote the fraction of each age group in the population, with the total mass normalized to one at each date. The government budget is given by

$$\begin{aligned} & \underbrace{\left(\sum_{j=2}^6 \omega_j \int \mathcal{T}(y_j^*, a_j^*, n_j^*) d\mu_j \right)}_{\text{labor and capital income taxes}} + \underbrace{\tau_c \left(\sum_{j=2}^8 \omega_j \int c_j^* d\mu_j + \omega_2 \int n^* \cdot (p_m \cdot m^* + e^*) d\mu_2 \right)}_{\text{consumption taxes}} = \underbrace{(\omega_0 + \omega_1) \cdot \mathcal{E}}_{\text{mass of children} \cdot \text{public education expenditures}} \\ & + \underbrace{\pi \cdot \left(\sum_{j=7}^8 \omega_j \int wh d\mu_j \right)}_{\text{pension payments}} + \underbrace{\omega_2 \left(\underbrace{\int n^* \cdot \mathcal{B} d\mu_2}_{\text{baby bonus}} + \underbrace{\int (1 + \tau_c) \cdot n^* \cdot p_m \cdot \mathcal{S} d\mu_2}_{\text{subsidized childcare}} \right)}_{\text{family policy expenditures}} + \underbrace{\sum_{j=2}^8 \omega_j \cdot \Omega}_{\text{other spending}}. \end{aligned} \quad (4)$$

2.4 Equilibrium

I use t to denote time. The equilibrium of the economy is defined as a tuple composed of:

- decision rules $\{c_t^*, a_t^*, n_t^*, m_t^*, t_{h,t}^*, e_t^*, a_{k,t}^*\}_{t=0}^{\infty}$,
- prices²¹ $\{w_t^*, r_t^*\}_{t=0}^{\infty}$,
- government policies $\{\mathcal{T}_t(\cdot), \tau_{c,t}, \mathcal{B}_t, \mathcal{S}_t, \mathcal{E}_t\}_{t=0}^{\infty}$, and
- distribution of agents $\{\{\mu_{j,t}\}_{j=0}^8, \{\omega_{j,t}\}_{j=0}^8\}_{t=0}^{\infty}$,

²⁰I discuss the possibility of allowing for government borrowing in Appendix A.6.

²¹Recall that the relative price of childcare p_m is exogenous in the model.

such that households maximize utility subject to idiosyncratic shocks, prices clear labor and capital markets, the government balances the budget in each period, and the distribution of agents evolves following the dynamics shaped by household decision rules, exogenous human capital shocks z , and the ability shock for children ϵ .

The evolution of the human capital distribution from parents to children is given by

$$\mu'_2(h) = \frac{1}{N} \iint n^*(x) \mathbb{1}_{h_k^*(x, \epsilon) < h} d\mathcal{Q}(\epsilon) d\mu_2(x), \quad (5)$$

where \mathcal{Q} denotes the distribution of the child ability shock ϵ . Variable $h_k^*(x, \epsilon)$ is the human capital of a child whose ability shock is ϵ and parents' human capital is x . N is the aggregate fertility rate in the economy defined as:

$$N = \int n^*(x) d\mu_2(x). \quad (6)$$

In a stationary equilibrium of the economy, decision rules, prices, and distributions are unchanged over time. The size of the population, however, could vary over time as the aggregate fertility rate is not necessarily at the replacement level. Under mild conditions that are satisfied by the model, a stationary distribution exists and is unique (see [Mode, 1971](#); [Chu, 1990](#)).

2.5 Welfare

To facilitate comparisons between different government policies, I define social welfare in the long run (\mathcal{W}) as the *average value* of households at the beginning of their life cycle:

$$\mathcal{W} = \int V_2 d\mu_2, \quad (7)$$

where both V_2 and μ_2 are endogenous equilibrium objects. The welfare metric \mathcal{W} measures the expected utility of a newborn child under the veil of ignorance by summing up the discounted utility flow from later life periods. When making welfare comparisons, I convert the changes in \mathcal{W} into percentage changes in consumption equivalents. Since the welfare metric \mathcal{W} does not reflect the policy effects for households who are *already alive* when the policy is enacted, or for those who will be born in the *transition phase*, I also assess how these people are affected by family policies in [Section 5.3](#).

In the model, government policies have the potential to improve the well-being of agents for two reasons. First, childbearing and child-rearing carry fiscal externalities as parents do not internalize the effects of having an additional child or investing in child human capital on the future tax base and government revenues. Atomic parents take the age structure $\{\omega_j\}_{j=0}^8$, distribution $\{\mu_j\}_{j=0}^8$, and tax rates as given, but these objects will change when a mass of parents adjust their decisions on fertility

and education investments. Following this line of logic, [Schoonbroodt and Tertilt \(2014\)](#) argue that due to parents' lack of property rights over children's future output, the equilibrium level of fertility is too low relative to the planner's solution. Different from [Schoonbroodt and Tertilt \(2014\)](#), in a model with heterogeneous agents and public transfers (i.e., taxes, education, and pension), the equilibrium fertility could be either too high or too low depending on parental characteristics.²²

Second, parents face both inter-temporal and intergenerational borrowing constraints due to imperfections in capital markets. Parents cannot borrow against their children's future income or their own future income to finance current expenditures. Government policies can overcome these inefficiencies by providing in-cash or in-kind transfers ([Daruich, 2018](#); [Abbott et al., 2019](#)).

2.6 Discussion of Mechanisms

In this section, I discuss the mechanisms through which family policies affect the economy. In particular, I highlight how the features of this model (namely, fertility and child quality choice, endogenous demographic structure, and childcare arrangements) make the model's predictions on the overall impacts of family policies distinct from those in standard models.

2.6.1 Quantity-Quality Trade-Off

Consider the effect of an increase in the baby bonus \mathcal{B} on private education investment into child human capital formation e , which determines children's expected human capital $\mathbb{E}h_k$.²³ The first-order condition of e is given by

$$\underbrace{MU_c \cdot n}_{\text{marginal costs of } e} = \underbrace{\frac{\partial v(n, \mathbb{E}h_k, a_k)}{\partial \mathbb{E}h_k} \times \frac{\partial \mathbb{E}h_k}{\partial e}}_{\text{marginal benefits of } e}. \quad \text{FOC } [e]$$

When fertility is exogenous, i.e., n is fixed, the increase in \mathcal{B} is an *income transfer*, which implies that e rises unambiguously due to income effects. As the marginal utility of consumption MU_c decreases, e needs to increase to keep the first-order condition satisfied.

When fertility is endogenous, however, the increase in \mathcal{B} is a *price change*. The direction in which e changes is ambiguous. As fertility n rises due to more generous child benefits,²⁴ it affects the first-order condition in three ways. First, it interacts with e in parents' preferences $v(\cdot)$. If quality

²²The policy implications of this observation, however, are less straightforward than one might think. Due to intergenerational persistence of human capital, a tempting conclusion is that the policymaker should restrict fertility among the poor ([Chu and Koo, 1990](#)). Setting aside the rampant ethical concerns over such a policy, I show that this argument is not valid in Sections 5.1 and 6.3.

²³The same argument applies to inter vivos transfers a_k .

²⁴This holds if child quantity is not a Giffen good. See [McDonald \(2006\)](#) and [Stone \(2020\)](#) for supporting evidence.

and quantity are complements, parents demand higher *e ceteris paribus*, and vice versa.²⁵ Second, rising quantity could potentially offset the income effect since higher n raises the marginal utility of consumption via the change in the equivalence scale $\Lambda(n)$. Last, the marginal cost of e rises because it is proportional to fertility n due to their interaction in parents' budget constraint (see Equation (2)).²⁶ Considering endogenous fertility allows for the possibility that e - hence children's human capital h_k - could fall when family policies become more generous. In Section 5.1, I show that both indeed fall in the calibrated model.

2.6.2 Composition Effects

Because parents differ by human capital h and asset holdings a , they respond differently to the same baby bonus \mathcal{B} or subsidized childcare \mathcal{S} .²⁷ With an endogenous quantity-quality trade-off, children from families with stronger fertility responses account for a larger fraction of the future population. Combined with the intergenerational transmission of traits, differential fertility responses results in *composition effects* whereby aggregate variables gravitate towards those of the households with the largest fertility responses.

For example, consider the average human capital of children in the economy

$$\underbrace{\bar{h}_k}_{\text{average } h_k} = \iint \underbrace{\frac{n^*(h, a)}{N}}_{\text{fertility weight}} \cdot \underbrace{h_k^*(h, a, \cdot)}_{\text{individual child's } h_k} d \underbrace{\mu_2}_{\text{parents' dist.}} d\epsilon,$$

where N is the aggregate fertility rate defined in Equation (6). Family policies change fertility weights $n^*(h, a)/N$ across households. As a result, even when $h_k^*(h, a, \cdot)$ stays unchanged, \bar{h}_k converges towards families with rising fertility weights.

From a theoretical perspective, composition effects generate an interesting scenario where aggregate human capital decreases even if policy effects on individual children's human capital are positive. Empirically speaking, composition effects are shown to be important for the evolution of aggregate variables such as economic growth (de La Croix and Doepke, 2003) and public opinion on family values (Vogl and Freese, 2020).

²⁵Whether child quantity and quality are complements or substitutes remains an open question in the literature. Standard Barro-Becker models usually assume they are complements (e.g., Córdoba et al., 2016; Daruich and Kozłowski, 2020). On the other hand, there are also models using separable preferences (e.g. de La Croix and Doepke, 2003; Bar et al., 2018), or quality and quantity being substitutes (Jones and Schoonbroodt, 2010).

²⁶Becker and Lewis (1973) named this last effect the quantity-quality trade-off. In this paper, I abuse this term slightly to denote the overall effect of an increase in fertility on child quality. Using twin births as instruments, some recent papers (e.g., Black et al., 2005 and Angrist et al., 2010) find little evidence of such a trade-off. A recent study by Mogstad and Wiswall (2016), however, overturns that conclusion by relaxing the linear specification constraint. They find evidence of a trade-off between quality and quantity for larger families and complementarities in small families.

²⁷See Drago et al., 2011 and Table 3 for empirical evidence.

2.6.3 Demographic Structure Effects

With endogenous fertility, family policies change the population growth rate, and, as a result, the demographic structure $\{\omega_j\}_{j=0}^8$. This has profound implications for the government budget constraint (4) because demographic structure determines how each source of revenue or expenditure is weighted.

Most macroeconomic structural models with endogenous fertility assume two-period-lived agents. From the government budget perspective, an increase in the population growth rate leads to an unambiguous increase in the fiscal burden in this class of models since there are fewer tax-paying adults to finance public education expenditures for children. Therefore, tax rates need to rise to balance the budget. Most developed countries that actually adopt family policies, however, have another story in mind where tax rates should be *lower* instead of higher in the long run. The key missing piece in this analysis is the presence of retired households receiving pension payments.

By considering a rich life cycle with childhood, working-age, and retirement in the model, higher fertility rates reduce the old-age dependency ratio. Therefore, the model allows for the possibility that the government may *reduce tax rates* in the long run after family policies are adopted. I denote the effects of family policies through changes in the mass of each age group as *demographic structure effects*.

2.6.4 Childcare Choices

Consider a comparison between two family policies, a baby bonus \mathcal{B} and subsidized childcare \mathcal{S} . In standard macroeconomic models with endogenous fertility, each child costs a fixed amount of time for parents.²⁸ Since subsidized childcare reduces time costs, total income in this class of models is given by

$$y = wh \underbrace{(1 - (\chi - \mathcal{S}) \cdot n)}_{\text{hours worked}} + n \cdot \mathcal{B}.$$

For parents with human capital h , a baby bonus \mathcal{B} can be replicated by subsidized childcare with $\mathcal{S} = \frac{\mathcal{B}}{wh}$ in these models. Therefore, parents should be indifferent between in-cash and in-kind child benefits of the same face value. They are also expected to react to each policy in the same way in terms of labor supply and child-related decisions.

These predictions from traditional models are not supported by empirical evidence which indicate that parents react differently to in-cash versus in-kind benefits. Upon receiving a baby bonus, parents do not increase their labor supply but rather choose to stay at home and use less market

²⁸Modeling child costs as time costs rather than goods costs is crucial for explaining the negative income-fertility relationship (Jones et al., 2008). See the discussion in Doepke and Tertilt (2016). A notable exception that considers the use of market childcare is Bar et al. (2018). Those authors show that the adoption of market childcare services and rising income inequality explain the recent rise in fertility among high-income married women in the U.S.

childcare ([González, 2013](#)). In contrast, upon receiving subsidized childcare, parents increase their labor supply drastically and substitute home care for (subsidized) market care ([Milligan, 2005](#)). Last, use of market childcare is increasing in maternal education.²⁹ Models without childcare arrangements are unable to account for this pattern.

In this model, considering parents’ childcare arrangements provides an explanation for patterns observed in the data and offers a natural way to compare family policies of different kinds. With market childcare, a substitute for home care, available at a uniform price, richer parents use more market childcare services than poorer ones. A government-offered childcare subsidy \mathcal{S} will therefore be equivalent to a baby bonus \mathcal{B} of the same face value for parents with high human capital who are already spending more than \mathcal{S} on market childcare – subsidized childcare simply reduces their out-of-pocket expenditures. On the other hand, parents with low human capital who receive a childcare subsidy \mathcal{S} would rather have an in-cash transfer \mathcal{B} of the same face value since they prefer spending the money on consumption or education. As a result, subsidized childcare \mathcal{S} is “binding” for low-human capital parents in the sense that the in-kind policy pushes them into the labor market if they want to take up the benefits. Their welfare improvements will be smaller than if they were offered a baby bonus \mathcal{B} of the same face value. As they work more, however, these parents accumulate more human capital through learning-by-doing, and the economy sees a reduction in the life-cycle wage inequality across parents.

2.7 Summary of Mechanisms

As discussed above, the framework contains a unique mix of modeling ingredients. Therefore, the model explains more observed data patterns (e.g., childcare usage) relative to standard macroeconomic models with endogenous fertility. It also speaks to several important mechanisms through which family policies affect the economy.

The quantitative magnitude of these mechanisms depends crucially on the fertility responses to family policies (the fertility elasticities). For instance, if child quantity is fixed, then family policies will have (1) a simple income effect on children’s outcomes, (2) no composition effects, and (3) no demographic structure effects. In [Section 3](#), fertility elasticities across households are disciplined by the calibrated model parameters. In [Section 4](#), I show that the model predictions on fertility responses are consistent with existing empirical evidence.

²⁹See [Figure 5](#) for evidence. Also see [Bar et al. \(2018\)](#) and [Chaparro et al. \(2020\)](#).

3 Calibration

In this section, I discuss the parameterization, calibration procedures, and the model fit. Following the literature, I choose some parameters exogenously, most of which either have standard values or have observable counterparts. The other 14 parameters (listed in Table 2) are calibrated inside the model by matching steady-state moments to the United States in 2010.

3.1 Preferences over Quantity and Quality

Parents' preference over child quantity and quality is given by

$$v(n, \mathbb{E}h_k, a_k) = \underbrace{\Psi(n)}_{\text{child discounting}} \cdot \underbrace{(\theta \cdot u(\mathbb{E}h_k) + \nu \cdot u(a_k))}_{\text{utility from quality}}, \quad (8)$$

$$\Psi(n) = 1 - \exp(-\psi n), \text{ and} \quad (9)$$

$$u(x) = \frac{x^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0, 1), \quad x \in \{\mathbb{E}h_k, a_k, c\}. \quad (10)$$

where parents value child quality weighted by the child discounting function $\Psi(n)$ (c.f., [Barro and Becker, 1989](#); [de La Croix and Doepke, 2003](#); [Kim et al., 2021](#)). The utility function of child quality and consumption is governed by parameter γ , which determines the elasticity of intergenerational substitution (EGS), i.e., $1/\gamma$, defined in [Córdoba et al., 2016](#) and [Córdoba and Ripoll \(2019\)](#).³⁰ I use $\theta \cdot u(\mathbb{E}(h_k)) + \nu \cdot u(a_k)$ as a first-order approximation to general preferences over child quality and transfers. In Appendix A.2, I argue that results in this paper are robust to other common specifications used in the literature, including separable preferences ([de La Croix and Doepke, 2003](#)), quality and quantity being substitutes ([Jones and Schoonbroodt, 2010](#)), and dynastic altruism ([Daruich and Kozłowski, 2020](#)).

I calibrate parameters $\theta = 2.85$ and $\nu = 0.29$ to match the average human capital investment as a share of income and average inter vivos transfers of \$48,381 in 2000 dollars ([Daruich, 2018](#)).³¹ I calibrate $\psi = 2.3$ to match the average fertility calculated using the CPS Fertility Supplement data from 2008 to 2014.

³⁰Since the EGS is designed to match interactions across generations, its magnitude and interpretations are different from that of the elasticity of intertemporal substitution (EIS) used to capture risk-aversion in business cycle models. More specifically, when utilities from consumption and child quality are separable, it is common to assume that EIS and EGS are the same to ensure the existence of long-run steady states (see [Barro and Becker, 1989](#); [Soares, 2005](#)). For recent work modeling EGS and EIS jointly with non-separable utilities, see [Córdoba and Ripoll \(2019\)](#).

³¹[Daruich \(2018\)](#) uses the PSID and the CEX to estimate that average expenditures on each child below age 12 include 18 hours of "quality" hours per week and \$1,966 (in 2000 dollars) spent on childcare and education investments. These two expenditures combined amount to 13.4% of median family income. The calibration targets this moment by computing average education investments e plus expenditures on market childcare services m . These moments are also similar to those reported in [Lee and Seshadri \(2019\)](#).

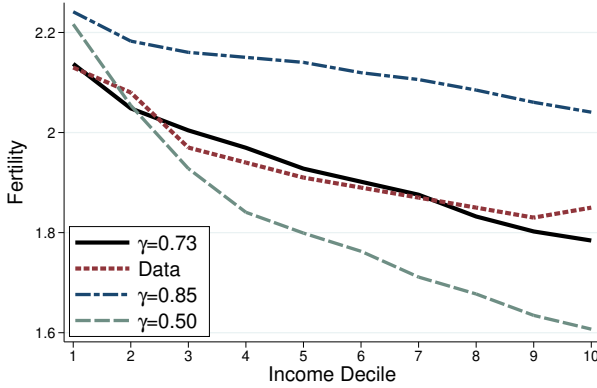
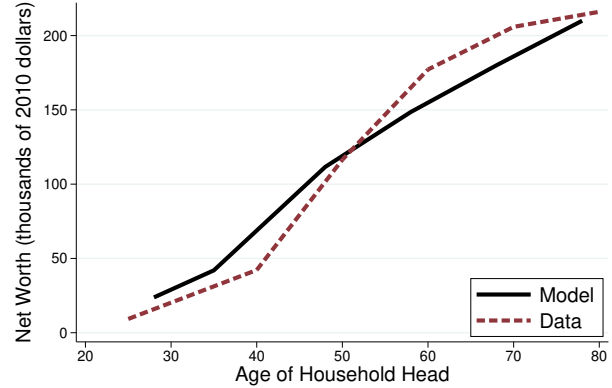
Figure 2: Identification of γ 

Figure 3: Non-Targeted - Net Worth by Age



Notes: Figure 2 plots the relationship between family income and fertility in the model under different γ . Fertility rates by income are calculated using the number of live births ever had for married women between 40 and 55 years old in the CPS Fertility Supplement data from 2008 to 2014.

— Figure 3 plots the median net worth by age in the model and in the data. Median net worth in the model is calculated using net asset position a integrated over the equilibrium distribution of households by age. In the data, median net worth by age of the household head is collected from the [Federal Reserve System](#) estimated using the Survey of Consumer Finances (SCF).

Conditional on $\{\theta, \nu, \psi\}$ and other parameters of the model, γ governs the fertility elasticity. Higher γ leads to a faster decay of the marginal utility from $\{\mathbb{E}h_k, a_k, c\}$. Hence when the “price” of child falls, parents have larger fertility responses to balance the first-order-condition.³² The same logic provides the identification of γ following [Córdoba et al. \(2016\)](#): controlling for other parameters, higher γ implies a faster decay in the marginal utility of consumption as parents get richer, which makes having children more valuable for them as an “extensive margin” of adjustment. As a result, higher γ leads to a flatter income-fertility profile. Figure 2 shows a numerical illustration of the identification. I choose $\gamma = 0.73$ such that the model best fits the observed relationship between fertility and income.³³

Even though γ is restricted to be between $(0, 1)$, the model generates a surprisingly good fit to the data in terms of life-cycle asset accumulation. Figure 3 shows the median net worth profile from the model and from the Survey of Consumer Finances (SCF). The model generates a non-targeted life-cycle profile of net worth similar to the data both in trend and in levels.

³²The illustrative model in Appendix E.2 highlights this logic. [Soares \(2005\)](#) uses a similar argument in discussing fertility responses to changes in adult longevity and child mortality.

³³See Appendix A.5 for discussion and tests of robustness to alternative measures of fertility.

3.2 Child's Human Capital Production

I parameterize the child human capital production function as

$$h_k = G(h, \mathcal{E}, e, \epsilon) = \underbrace{Z}_{\text{scalar}} \cdot \underbrace{\epsilon}_{\text{shock}} \cdot \underbrace{h^\rho}_{\text{spillover}} \cdot \left(\underbrace{\mathcal{E}^\xi}_{\text{public education}} + \underbrace{e^\xi}_{\text{private input}} \right)^{\kappa/\xi}, \quad (11)$$

where child ability shock ϵ follows $\log(\epsilon) \sim \mathcal{N}\left(-\frac{\sigma_\epsilon^2}{2}, \sigma_\epsilon^2\right)$

Parameter Z is a scaling parameter that governs the overall scale of the economy. I choose $Z = 2.5$ to normalize the median income of families in the model to one, corresponding to the Census-estimated household median income of \$49,445. Parameter σ_ϵ governs the dispersion of idiosyncratic shocks to children's ability. I pick $\sigma_\epsilon = 0.58$ to match the dispersion of earnings for young households.³⁴ I calibrate $\rho = 0.30$ to match the rank-rank intergenerational mobility estimated by [Chetty et al. \(2014\)](#). Figure 4 shows that the model generates a good fit both in absolute upward mobility and relative mobility.

I denote ξ as the elasticity of substitution between public and private education inputs. $\xi = 0.9$ is calibrated to match the relationship between education spending and household income using data from the Consumer Expenditure Survey (CEX) which includes tuition, test preparation, tutoring, books, and supplies. Public education \mathcal{E} in the baseline economy is chosen to match the \$12,000 annual expenditure per student reported by the National Center of Education Statistics (NCES). This value translates to 0.16 under the normalization of median income to be 1.

The last parameter to be calibrated is κ . It governs the productivity of education investments. I identify γ using RCT evidence from [García et al. \(2020\)](#). [García et al. \(2020\)](#) evaluate early childhood programs (ABC/CARE) from the 1970s. The yearly cost of the program was \$18,514 per participant (in 2014 dollars) for five years. Treated children were followed into adulthood with education and incomes observed by researchers. [García et al. \(2020\)](#) estimate that children's lifetime labor income increases by 1.3 dollars in net present value for every dollar invested. I take this moment on benefit-cost ratio and compute the model counterpart. More specifically, I apply the same policy in the model by expanding existing public education \mathcal{E} by \$17,000 (in 2010 dollars) for five years. The policy targets children with parents at the 10th percentile of earnings. Moreover, the counterfactual is evaluated at a small scale so that prices and taxes remain unchanged. I calibrate $\kappa = 0.13$ by matching the benefit-cost ratio in the model to that in [García et al. \(2020\)](#).

³⁴I calibrate σ_ϵ to match the Gini coefficient of income among married households age 23 to 29 in the CPS-ASEC data (2008-2014).

3.3 Costs of Children and Childcare

The household equivalence scale $\Lambda(n)$ is taken from the OECD standard:

$$\Lambda(n) = 1.7 + 0.5 \cdot n,$$

where n is the number of children residing with the family.³⁵

Recall that the childcare requirement, i.e., time cost of children, is given by

$$n \cdot \chi = \left(t_h^{v/\iota} + (n \cdot (m + \mathcal{S}))^v \right)^{1/v}.$$

I choose $\chi = 0.18$ following estimates by [Folbre \(2008\)](#) (Table 6.2), calculated using data from the American Time Use Survey (ATUS).³⁶ I calibrate the economies of scale in providing childcare at home $\iota = 0.7$ to match the estimates by [Folbre \(2008\)](#) (Table 6.4).³⁷ The parameter governing the elasticity of substitution between home care and market care v is calibrated to be 0.38 to match the average expenditure on childcare as a fraction of total family income using estimates from [Herbst \(2018\)](#).³⁸ Last, I choose the price of full-time market care for a child aged 0 to 10, p_m , to be \$6,860 per year in 2010 following the statistic reported by the National Association of Child Care Resource & Referral Agencies ([NACCRRA, 2011](#)).³⁹ Figure 5 shows that the model generates the pattern of childcare usage by parent income decile that fits the data from the National Survey of Early Care and Education (NSECE), a non-targeted moment.

3.4 Other Parameters

Human capital of adults evolves with age according to

$$h_{j+1} = L_j(h_j, t_w, z') = \exp(z') [h_j + \zeta_j(h_j \cdot t_w)^\eta] \text{ and} \quad (12)$$

³⁵The base level $\Lambda(0)$ is 1.7 since I model the decision of married households.

³⁶[Folbre \(2008\)](#) documents that an average child in a two-parent household spends about 22 hours per week in active non-parental care, and 23 hours per week in active parental care (adjusting for presence of both parents). Therefore, I calculate χ , the total active care required by one child as a fraction of parents' total time endowment, as $\chi = (22 + 23)/((24 - 6.5) \cdot 7 \cdot 2) \approx 0.18$ assuming 6.5 hours of sleep for each parent ([Richter et al. \(2019\)](#)).

³⁷[Folbre \(2008\)](#) reports that for two-parent families, the active parental time per child is 1.5 times higher in one-child families than that of two-child families.

³⁸[Herbst \(2018\)](#) uses Wave 8 of the 2008 SIPP panel, which covers winter and spring of 2011. The results show that average childcare expenditures in the whole sample are \$134.44 per week in 2013 dollars, which translates to 16% of the median household income. The calibration targets this moment by computing the average share of income spent on market childcare services, measured by $n \cdot m \cdot (1 + \tau_c)$ integrated over the equilibrium distribution.

³⁹The average annual costs of full-time childcare across all U.S. states are \$9,303 (infant), \$7,377 (4-year-old), and \$4,753 (school-age) for child care centers; family child care, on the other hand, costs \$6,926 (infant), \$6,131 (4-year-old), and \$4,405 (school-age) ([NACCRRA, 2011](#) Appendix 1). I take an age-weighted average of these costs to calculate full-time childcare costs for children aged 0 to 10.

Figure 4: Model Matches Mobility

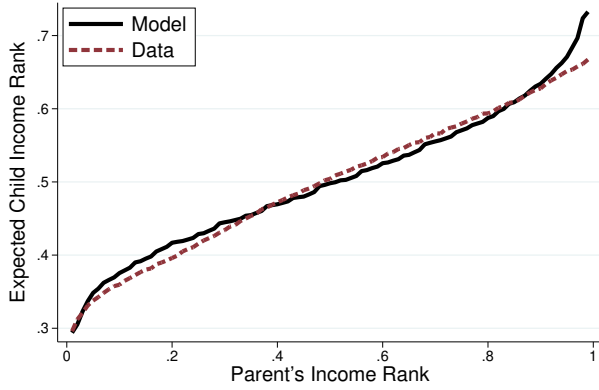
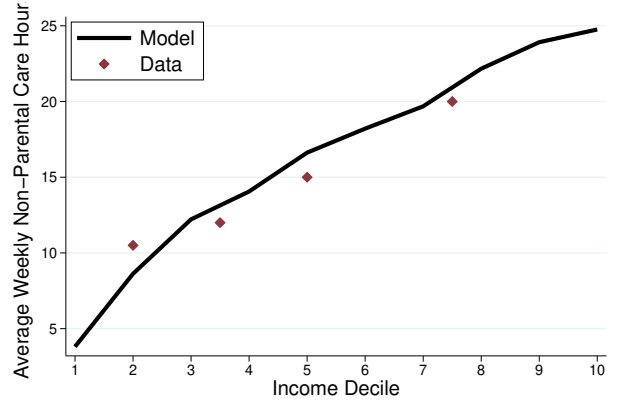


Figure 5: Non-Targeted: Childcare Usage



Notes: Figure 4 plots the relationship between parents' income rank and children's expected income rank in the calibrated model and that estimated by Chetty et al. (2014).

— Figure 5 plots the average weekly non-parental care received by children aged 0-2 conditional on parents' income decile. The data estimate is obtained using the public-use file of the National Survey of Early Care and Education (NSECE) wave 2012 by Flood et al. (2021). They calculate the average weekly non-parental care hours in the data by maternal education categorized into “less than high school”, “high school”, “some college”, and “bachelor's degree and above”. I map the education category into income deciles using median income by education and income distribution estimated from the Current Population Survey (CPS).

$$\log(z) \sim \mathcal{N}(\mu_z, \sigma_z).$$

I calibrate $\eta = 1.22$ to match the heterogeneous growth rate of income by initial income decile (see Figure 6).⁴⁰ Parameters $\{\zeta_j\}_{j=2}^5$ and $\sigma_z = 0.38$ are calibrated to match the life-cycle profile of average household income and its dispersion.⁴¹ I choose $\mu_z = -0.23$ exogenously so that human capital depreciates at an annual rate of 2%.

Following Heathcote et al. (2017), I parameterize income taxes as

$$\mathcal{T}(y, a, n) = y \cdot (1 - \tau_y^n y^{-\lambda_y^n}) + \tau_a r a, \quad (13)$$

where $\{\tau_y^n, \lambda_y^n\}$ denote the level and progressivity of taxes depending on the number of children residing in the household while τ_a denotes the linear capital income taxes. I obtain $\{\tau_y^n, \lambda_y^n\}$ using simulated data from TAXSIM provided by the NBER (see Figure 7) and use linear interpolation to calculate $\{\tau_y^n, \lambda_y^n\}$ when n is not an integer. Having an additional child significantly reduces the tax burden faced by households through various programs such as the Earned Income Tax Credit (EITC), Dependent Care Tax Credit (DCTC), and the Child Tax Credit (CTC). The tax system is

⁴⁰This model reproduces the well-documented hump-shaped life-cycle earnings profile as well as a Gini coefficient of human capital that is increasing in age (Huggett et al., 2006). These two facts are reconciled by considering a profile of learning ability $\{\zeta_j\}_{j=2}^5$ that is decreasing in age and the self-production of human capital with $\eta > 1$.

⁴¹I calculate average income by age and the Gini coefficient of income by age using data from married households in the CPS-ASEC data (2008-2014). I have experimented with panel methods à la Huggett et al. (2011) and the results are quantitatively similar.

Figure 6: Income Growth By Initial Decile



Figure 7: Labor Income Taxes



Notes: Figure 6 plots the growth rate of average income from age 25 to 35, depending on the initial income decile at age 25. For married households in the CPS-ASEC data (2008-2014), I calculate the growth rate of average household income by four education categories: “less than high school”, “high school”, “some college”, and “bachelor’s degree and above”. The initial income decile of each education category is calculated using the income distribution at age 25. — Figure 7 plots average tax rates depending on household income and number of dependent children residing with the family calculated TAXSIM.

progressive, with lower-income households receiving subsidies. As household income grows, labor income taxes converge to 40%. Following [McDaniel \(2007\)](#) and [Darulich and Fernández \(2020\)](#), I choose capital income tax $\tau_a = 0.27$ and consumption taxes $\tau_c = 0.07$. The pension replacement rate π is set to 40%.

In the production function of the representative firm, I choose the capital share α to be 0.33 following the standard in the literature and set the capital depreciation rate δ_k at 4% per year. Last, I choose the annual discount rate $\beta = 0.98$ and calculate the age-specific mortality rate $\{\delta_j\}_{j=0}^8$ using the [actuarial life table](#) from the Social Security Administration.

Table 1 summarizes the model parametrization, and Table 2 contains the list of identifying moments and the model fit. As all the moments affect all the parameters, it is difficult to pinpoint which moment is identifying a certain parameter. Nevertheless, Table 4 in Appendix A displays the sensitivity of model parameters to change in moments. Table 5 in Appendix A displays the elasticity of moments to model parameters to illustrate the identification argument.

4 Validation

In this section, I discuss the main external validation of the fertility responses to financial incentives. The purpose of the validation exercises is to lend extra credibility to the quantitative predictions of the model before evaluating policy counterfactual. In Appendix D, I provide further confirmation of the model’s predictions using evidence from the Australian baby bonus, the Spanish child benefits, and the Maternal Capital in Russia.

Table 1: Calibrated Parameters

Interpretation	Value	Source	Interpretation	Value	Source
Preferences			Child human capital production		
β	discount rate (annual)	0.98	standard		
γ	elasticity of substitution	0.73	CPS		
ψ	fertility preference	2.30	CPS		
θ	quality preference	2.85	PSID		
ν	transfer preference	0.29	PSID		
Childcare arrangement			Adults' human capital evolution		
χ	childcare cost	0.18	ATUS		
ι	economies of scale at home	0.7	ATUS		
ν	substitutability of care	0.38	SIPP		
p_m	price of full-time care	\$6,860	NACCRA (2011)		
Taxes and pension			Firm production function		
τ_y^n, λ_y^n	tax levels and progressivity	misc.			
τ_c	consumption tax	0.07	McDaniel (2007)	1	normalization
τ_a	capital income tax	0.27	McDaniel (2007)	0.33	standard
π	pension replacement rate	0.40	OECD Database	0.04	standard

Notes: This table displays the list of parameters used in the model. Parameters in red are calibrated within the model while those in black are chosen exogenously.

Table 2: Identifying Moments and Model Fit

Parameter	Interpretation	Moment	Data	Model
γ	elasticity of substitution	fertility differential	0.12	0.12
ψ	fertility preference	average fertility	1.92	1.92
θ	human capital preference	average investment as % of income	13.4	13.5
ν	transfer preference	average transfer	\$48,381	\$48,400
ι	economies of scale at home	childcare time by # children	1.5	1.5
v	substitutability of care	average care spending as % of income	16	16
Z	normalizing scalar	median income = 1	N/A	N/A
σ_ϵ	ability shock dispersion	Gini of earnings at $j = 2$	0.29	0.29
ρ	intergenerational spillover	intergenerational elasticity of earnings	0.34	0.33
ξ	substitution of education	investment by parents' education	misc.	misc.
κ	input productivity	return on per dollar investment (NPV)	\$1.3	\$1.29
η	learning curvature	income growth by initial decile	0.1	0.09
$\{\zeta\}_{j=2}^5$	learning level	income growth by age	misc.	misc.
σ_z	shock dispersion	Gini of earnings at $j = 6$	0.39	0.39

Notes: This table displays the list of parameters, identifying moments, and the model fit. See text for more discussion on the source of each moment.

The main validation exercise exploits empirical evidence from the Alaska Permanent Fund Dividends (APFD). The dividend was officially established in 1982 after the discovery of petroleum increased state revenues. Every year, it gives uniform transfers to all residents regardless of income, employment or age. In particular, the program allows a parent, guardian, or other authorized representative to claim a dividend on behalf of a child, while Alaska law imposes no requirements whatsoever on how parents use a child's dividend. As a result, the policy has pro-natal effects even though it is not explicitly advertised as a family policy that encourages fertility.

The APFD is an ideal policy environment to validate fertility elasticities in the model for three reasons. First, compared with other family policies which are usually less than a few thousand dollars in net present value per child (McDonald, 2006; Luci-Greulich and Thévenon (2013)), the net present value that parents could receive and use with an additional child under the APFD is almost \$20,000.⁴² With sizable benefits, it is more likely that the APFD would have meaningful and observable impacts on people's behavior, especially when it comes to the important and irreversible decision of having a child. Second, unlike family policies which are typically means-tested or depend on birth order, the APFD has a simple implementation, with more than 91% of the state population filing for the application historically. In contrast, the Census Bureau estimates that the

⁴²I calculated the average payment to be around \$1,500 per year. Since the amount of the dividend is tied to the performance of the overall stock market, future payment uncertainties might affect people's responses. Given that childbirth is an irreversible decision, I argue that a mean-preserving spread of the dividend payment would reduce households' fertility responses. Therefore, since the model generates fertility elasticities that are consistent with people's choices under uncertainty, the results on child benefits without uncertainty is likely conservative.

national participation rate of the Earned Income Tax Credit (EITC) is [lower than 80%](#). Moreover, the APFD mimics a universal basic income for parents plus a fully refundable Child Tax Credit (CTC) for children without income requirements. Given that the APFD is not advertised as a policy that aims to encourage childbirth, its effects on fertility provide a conservative benchmark for family policies that are explicitly pro-natal, and thus may change parents' behaviors through preferences or information. Last, [Cowan and Douds \(2021\)](#) find that the migration effect, also known as “population magnets effect,” of the APFD was not large empirically, with net migration rates around one-tenth of a percent in the sample period.

To implement the APFD in the model, I first re-calibrate the model to match the aggregate data in the 1980s.⁴³ Then, I implement the APFD by transferring \$1,500 (annually) to every household member, including both parents and children. Parents will receive this transfer (for themselves) every period until they die. They are also entitled to receive the children's share of dividends before their children become independent. I normalize the amount of the transfer by median household income in Alaska relative to the total U.S. I conduct the policy experiment in the partial equilibrium without changing prices and distribution. The model predicts that the completed fertility rate, i.e., the total number of children that parents end up having, increases by 0.16 compared with the scenario without the APFD.⁴⁴ The model also predicts heterogeneous fertility responses across households. Since parents are entitled to claim children's dividends and these dividends are uniform in size, the model predicts that parents with lower human capital will have a larger increase in fertility.

Using the CPS Fertility Supplement data from 1982 to 2018, I collect micro-level data on the completed fertility rate, i.e., the total number of live births ever had, among women aged 40 to 55.⁴⁵ I divide the Alaskan sample into three groups based on the survey years. The “not treated” group contains the data prior to year 1987, as these women had already passed their childbearing years when the APFD was enacted. The “partially treated” group contains the sample from 1987 to 2005 because the APFD affected some, but not all, of their childbearing years. Last, the “fully treated” group contains the observations from 2006 to 2018 since these women fully took the policy into account when making fertility choices. Women in other states are in the control group.

To estimate the policy effects on the completed fertility rate, I use a difference-in-differences strategy by regressing the completed fertility rates on state fixed effects, year fixed effects, and

⁴³I re-calibrate the tax function $\mathcal{T}(y, a, n)$ using the TAXSIM, and I re-estimate the fertility parameters $\{\psi, \gamma\}$ to match the aggregate fertility level and fertility differential across income groups. Further, I turn off the access to market childcare services by letting $p_m = \infty$. The identification argument is identical to that in Section 3.

⁴⁴I derive this result by calculating the difference in the aggregate fertility rate between the new stationary equilibrium and the baseline economy without the APFD. Results from the transition path (see Figure 11a) indicate that the policy effect on fertility is almost unchanged if I use short-run effects in the transition phase instead of long-run ones.

⁴⁵Figure B.2 displays the time series of the completed fertility rate in Alaska and the rest of the U.S.

treatment dummies:

$$\text{fertility} = \beta_0 + \beta_1 T_1 + \beta_2 T_2 + \text{State FE} + \text{Year FE} + \epsilon, \quad (14)$$

where T_1 is a dummy variable for being in the “partially treated” group, and T_2 is a dummy variable for being in the “fully treated” group. The standard errors are clustered at the state level. Since the model prediction concerns the long-run impacts of policies, coefficient β_2 is the one of interest. To explore heterogeneous treatment effects, I also estimate the specification in Equation (14) separately for women with or without high education, which is defined as having at least one year of college experience.⁴⁶ Table 3 reports the regression results.

Table 3: Effects of the APFD on the Completed Fertility Rates

	(1)	(2)	(3)	Model Predictions		
	Full Sample	Low Educ.	High Educ.	Average	Low Educ.	High Educ.
β_1	0.098 (0.027)	0.216 (0.036)	0.074 (0.021)			
β_2	0.172 (0.032)	0.296 (0.041)	0.105 (0.025)	0.16	0.31	0.09
# Obs.	146,804	69,511	77,293			

Notes: This table reports the treatment effects simulated using the model and the regression results of specification (14) using data from the CPS and state, year fixed effects. Standard errors, in parentheses, are clustered at the state level. Column (1) shows the results with the full sample. Column (2) shows the estimated coefficients among women without any college experience. Column (3) shows the estimated coefficients among women with at least one year of college experience. The next three columns show model predictions of changes in fertility for the average women, women with low education (30th percentile of human capital), and women with high education (70th percentile of human capital).

Column (1) of Table 3 shows that the estimated effects of the APFD on the completed fertility rates is $\hat{\beta}_2 = 0.172$ children per woman with a 95% confidence interval of (0.109, 0.235).⁴⁷ The model-predicted policy effects of 0.16 children per woman is within this range. Results on $\hat{\beta}_2$ from columns (2) and (3) also confirms the model’s prediction on heterogeneous treatment effects: women without high education responds more strongly to the APFD.⁴⁸ Quantitatively, these estimates are consistent with the model predictions for women with low and high education (mapped to the 30th and 70th percentiles of human capital).

⁴⁶Figure B.3 displays the time series of the completed fertility rate by state of residence and education.

⁴⁷This finding echos the results in Yonzan et al. (2020). Using synthetic control methods on Natality files, they conclude that the total fertility rate in Alaska increases by 13.1% due to the APFD.

⁴⁸This finding echos the results in Cowan and Douds (2021). They find larger increases in fertility among Alaska natives and women without a high school degree.

5 Counterfactuals

In this section, I use the model to evaluate family policies of different sizes in a general equilibrium where population distribution adjusts. I focus on a uniform cash reward to childbirth (i.e. baby bonuses B) for two reasons. First, the policy’s structure is simple and similar to expanding the fully refundable Child Tax Credit (CTC).⁴⁹ The simplicity in policy structure facilitates better exposition of the model mechanisms. Second, such cash rewards have already been widely adopted in the developed world (see [Drago et al., 2011](#); [González, 2013](#)). I discuss the long-run implications of baby bonuses in Section 5.1 where the government balances the budget by adjusting consumption taxes.⁵⁰ The outcome variables of interest consists of fertility, human capital, average income, intergenerational mobility (measured by the inverse of IGE),⁵¹ and social welfare. In Section 5.2, I compare the baby bonus with subsidized childcare and public education expenditures. Last, I study the transition path of the baby bonus and its distributional effects across generations in Section 5.3.

5.1 Baby Bonus: Long-run Implications

This section evaluates baby bonuses of different sizes ranging from \$0 to \$50,000 in 2010 dollars. I compute the long-run macroeconomic implications of these policies by comparing the long-run steady state of the economy with the benchmark economy where the baby bonus is zero.

Figures 8a and 8b show the fertility effects and the direct fiscal costs of baby bonuses. To reach the replacement fertility level (2.1 children per family on average), the model predicts that it would require a baby bonus of \$30,000, or approximately 1.6% of GDP in the new steady-state economy. This amount is similar to the increase in the maximum benefit of the Child Tax Credit (CTC) from 2010 to 2021 in net present value, taking the American Rescue Plan Act of 2021 into account. It offsets 19% of the average cost of raising one child as estimated by the USDA. These results confirm the common perception by demographers that using financial incentives to raise fertility is not “cheap.” Under realistic magnitudes of fertility elasticities and reasonable policy scales, the effects of financial incentives provided by the government are small relative to historical changes in preferences, social norms, contraceptive technologies, and perhaps most important of all, changes in the skill premium. These results, however, suggest that using cash transfers to prevent further

⁴⁹There are two main differences between baby bonuses and the CTC. First, the CTC has an income requirement and phase-out region, whereas a baby bonus is usually not means-tested. As is discussed later, the main results are stronger when family policies target low-income households. Second, a baby bonus is a lump-sum transfer when the child is born, while the CTC is an annual transfer to parents when the child is below age 18. With borrowing constraints, low-income parents prefer baby bonuses to a CTC of the same net present value because they can replicate the latter through savings.

⁵⁰I discuss the robustness of the results to alternative funding methods in Appendix A.6.

⁵¹Intergenerational Elasticity of Earnings (IGE) is calculated by simulating parent-child pairs and evaluating the correlation between their earnings at $j = 2$.

crashes in fertility, or even to raise the fertility rate, is feasible in practice.

Figure 8c shows that parents in the lower half of the human capital distribution (low h) have larger fertility responses to baby bonuses than the ones with higher human capital (high h). This is intuitive since the same level of \mathcal{B} translates into a larger proportional change in the “shadow price” of children, which is composed of the opportunity costs of time wh and private education investments $e^*(h)$. This prediction is also consistent with the estimated fertility effects of APFD presented in Table 3.

Figure 8d shows the quantity-quality trade-off channel discussed in Section 2.6.1. Baby bonuses reward parents for higher fertility, and parents respond by having more children, but they reduce private investment in children’s education. Average e falls by 4% when fertility reaches the replacement rate. Further, due to their larger fertility responses, parents with lower human capital reduce investment in their children’s education by more than parents with high human capital. Figure 8d also displays the model predictions when fertility is exogenous.⁵² In that case, parents spend more on e when they receive the baby bonus as an *income transfer*, with larger increases among parents with low human capital. As can be seen, implications of family policies are qualitatively different with endogenous fertility.

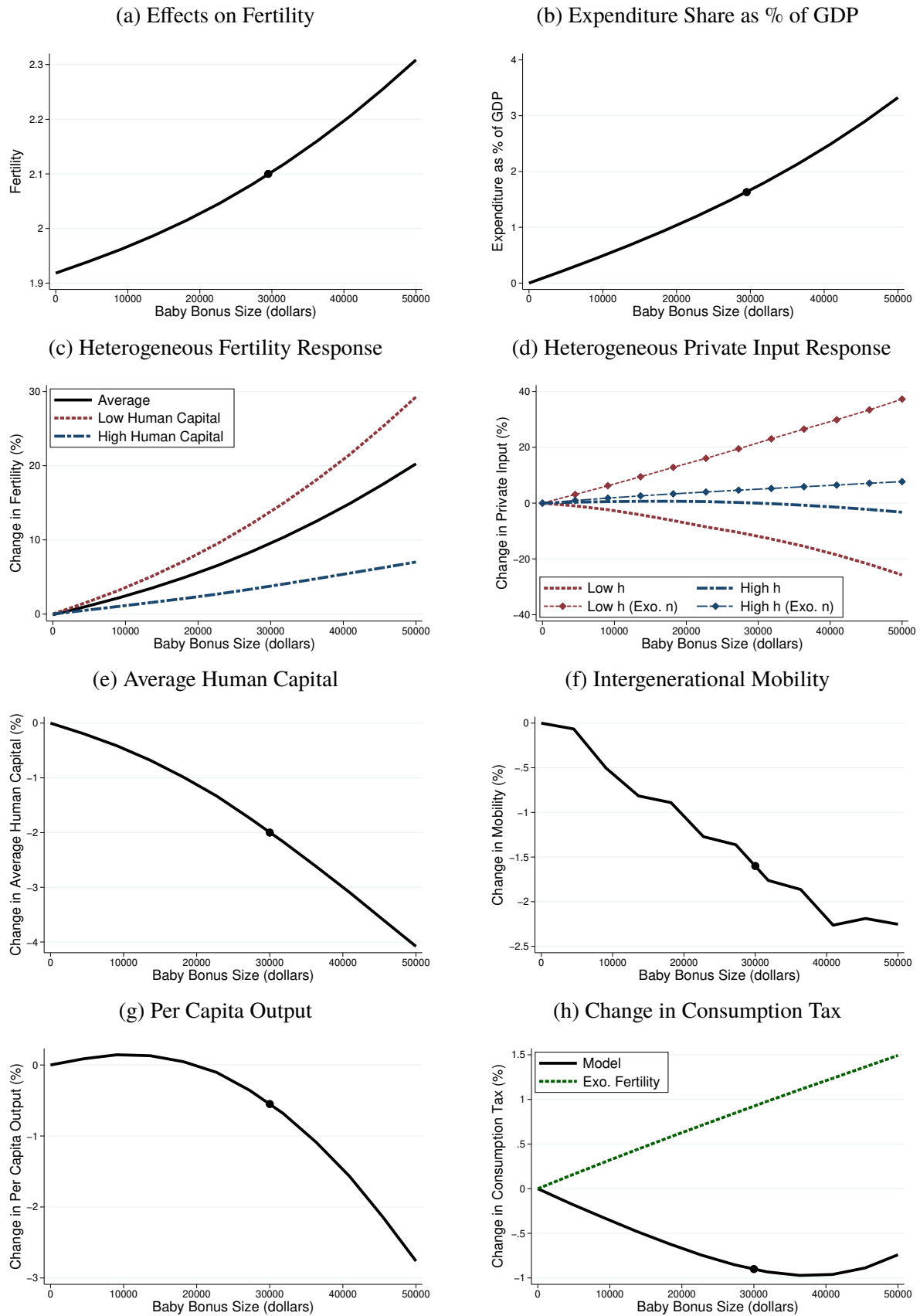
Due to intergenerational transmission of human capital, heterogeneous responses in fertility induce gravitation of aggregate variables towards low-income families. Composition effects, in addition to the reductions in private investment per child, lead to a reduction of the average human capital by 2% under a \$30,000 baby bonus (see Figure 8e).⁵³ As the decrease in education investments is larger among low-income households, Figure 8f shows that the \$30,000 baby bonus also reduces intergenerational mobility by 1.8% rather than boosting it as policymakers have hoped. Due to the quantity-quality trade-off mechanism, the negative policy effects on social mobility would be even stronger if the baby bonus is exclusively given to low-income households.

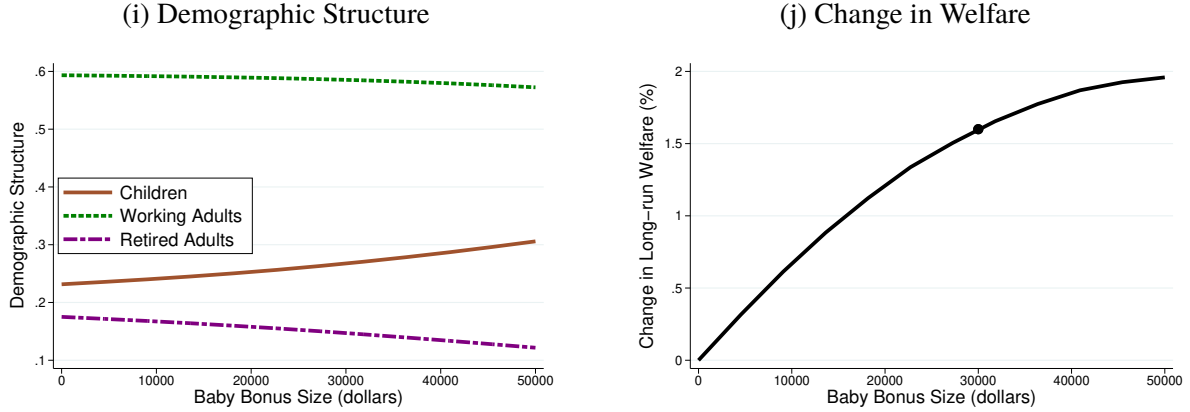
Figure 8g shows that per capita output declines by 0.6% at $\mathcal{B} = \$30,000$. Nevertheless, Figure 8h indicates that in the long-run steady-state, the government is able to reduce the consumption tax by 0.9%. This is a result of several forces shaping the government budget constraint in Equation (4). First, the change in human capital distribution affects tax revenues. Second, as the share of children increases, the government needs to allocate more resources to public education. Further, as the share of retired households decreases (see Figure 8i), the government is less burdened by pension payments. This result stands in contrast with predictions under exogenous fertility (dashed line in Figure 8h) or those from models with endogenous fertility, but agents only live for two periods.

⁵²To compute this, I do not allow parents to make endogenous fertility choices and assign the number of children that corresponds to the level observed in the data (see Figure 2). Parents still make private education investments and transfer choices as in the baseline model.

⁵³The 2% reduction in average human capital is smaller than that of the private investment (-4%) because public education investment \mathcal{E} is unchanged.

Figure 8: Long-Run Effects of Baby Bonuses





Notes: These figures plot macroeconomic impacts of baby bonuses of different sizes by comparing long-run steady-states under policies with the baseline steady-state economy. In Figure 8d and 8c, I plot the average responses as well as heterogeneous responses by parents' human capital levels.

Figure 8j plots the welfare effects in consumption equivalents.⁵⁴ I find that the average utility of new-borns under the veil of ignorance increases by 1.6% in the long-run economy when aggregate fertility reaches the replacement level. Quantitatively, the reduction of consumption taxes by 0.9% explains more than half of the welfare improvements. The rest of the welfare gain is explained by (1) higher fertility, (2) insurance benefits *ex ante*, and (3) changes in the distribution of agents.

5.2 Comparing Policies: Subsidized Childcare and Education Expenditures

In this section, I highlight results from policy comparisons. For more details on subsidized childcare and public education expenditures, see Appendices C.1 and C.2.

Compared with baby bonuses, subsidized childcare is less cost-effective in raising fertility. While it takes a baby bonus equal to 1.6% of GDP to boost aggregate fertility to the replacement level, the government needs to spend 2.5% of GDP if it uses subsidized childcare to achieve the same goal (see Figure 9a). This result is consistent with evidence from historical policies (Luci-Greulich and Thévenon, 2013; Stone, 2020). It is also intuitive; compared with cash benefits, subsidized childcare offers fewer benefits per child (in consumption equivalents) to parents with low human capital (see Figure 10a) given that they would prefer to spend the money elsewhere.

Different from baby bonuses, subsidized childcare encourages the combination of childbearing and working, especially among parents with low human capital.⁵⁵ This in turn fosters adults' hu-

⁵⁴Consumption equivalents measure the percentage changes of consumption in the original economy that would make households indifferent between the original allocation and that with the policy.

⁵⁵Hours worked among parents with low human capital fall after the implementation of a baby bonus for two reasons. First, these parents are most responsive in fertility, which drives up the total childcare time. Second, due to increasing economies of scale in home production of childcare, the increase in fertility decreases the relative price of home versus market childcare. See González (2013) for empirical evidence of these predictions. As discussed in Section 2.6.4,

man capital accumulation via on-the-job learning. As a result, subsidized childcare leads to more equitable income growth across agents with different initial human capital (see Figure 10b) and reduces life-cycle wage inequality.⁵⁶

The fertility effects of public education expenditures are an order of magnitude smaller than those from baby bonuses for two reasons. First, public education expenditures only indirectly affect quantity choices through changes in child quality. Second, in the long run, education expenditures increase the share of parents with higher human capital, who have a fewer children. Thus, the government should not expect to use education expenditures as an effective pro-natal policy instrument. While a \$30,000 baby bonus boosts aggregate fertility from 1.92 to 2.1 (the replacement level), an education expansion of the same size only raises aggregate fertility by 0.011 (see Figure 9a).

Similar to baby bonuses, private investments in children’s education also fall under public education expansion, but for a different reason. Parents spend less on children’s education under an expansion of public education expenditure due to crowding-out effects. This is because public and private monetary inputs into child human capital formation are substitutes.

Despite mild fertility effects, expanding public education is the most effective policy among those evaluated at raising social mobility and improving children’s outcomes. Figure 9b indicates that different from baby bonus or subsidized childcare, an expansion of public education that is 1.5% of GDP raises intergenerational mobility by 5.2%.⁵⁷ This effect will be larger if the public education expenditure is targeted at low-income households.

5.3 Baby Bonus: Transition Path

In this section, I discuss the transition path results following the implementation of a \$30,000 baby bonus that leads to the replacement level fertility rate in the long run. The policy is enacted unexpectedly at period $t = 1$ and stays in place for all subsequent periods. The government changes consumption taxes along the transition path to balance the budget in each ten-year period.⁵⁸

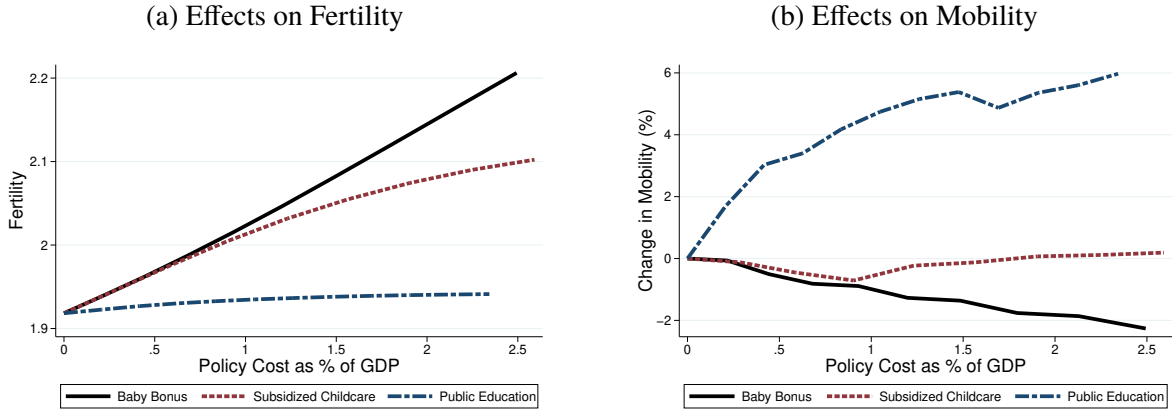
hours worked among parents with low human capital increase under subsidized childcare as the policy “pushes” them from home production of childcare to market work if they want to benefit from the in-kind transfers.

⁵⁶The quantitative effects on wage inequality are nevertheless mild, as most of the gap is driven by differences in initial human capital rather than hours worked.

⁵⁷These values are considerably smaller than the projected policy effects by Daruich (2018) for two reasons. First, Daruich (2018) does not consider the existence of public education expenditures in the benchmark economy. As a result, additional public education expenditures have significantly higher marginal effects on child human capital. Second, Daruich (2018) does not consider endogenous fertility which dampens the policy effects on child quality through the quantity-quality trade-off.

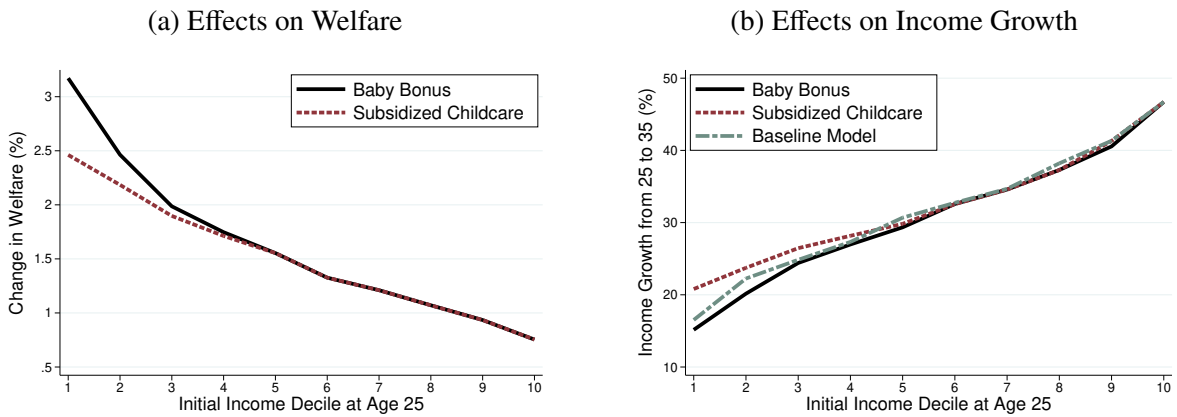
⁵⁸I discuss an alternative way of using government borrowings to fund fiscal expenditures in Appendix A.6. Furthermore, finding the optimal transition is interesting but rarely implemented in quantitative models since the dimensionality of policy instruments is immense. One notable exception is Bakış et al. (2015) where they compute the optimal path of taxes along the transition path in a stylized Aiyagari-Bewley-Huggett model. This would be interesting in my richer model but is beyond the scope of this paper.

Figure 9: Compare Policies: Effects on Fertility and Mobility



Notes: This figure compares baby bonus \mathcal{B} with subsidized childcare \mathcal{S} and public education \mathcal{E} on the basis of effects on fertility and intergenerational mobility.

Figure 10: Baby Bonuses and Subsidized Childcare: Effects on Welfare and Income Growth



Notes: This figure compares baby bonus \mathcal{B} with subsidized childcare \mathcal{S} . The direct expenditure on each policy is 1.6% of GDP in the long-run steady-state.

Figure 11a shows that the policy effects on fertility are immediate and persistent. Figure 11b indicates that the old-age dependency ratio starts to decline in period 6, given that past fertility rates predetermine the ratio of the retired to working-age population in the short run. The total dependency ratio, however, increases immediately after the policy is adopted due to a large increase in the number of children born due to the baby bonus.

As a result of this increase in the total dependency ratio, the government needs to finance additional expenditures in the first few periods. Such expenditures include education and transfers to parents with children through the tax system. These “induced” expenditures make the overall costs of family policies much larger than the direct expenditures.⁵⁹

In the baseline model, the government balances the budget period by period using consumption taxes. As a result, Figure 11c shows that consumption taxes rise at first before falling in the long run. Hence, welfare changes of new-born agents in transition are positive but significantly smaller than those in the long-run steady state (see Figure 11d). Most existing agents in the economy at $t = 1$ do not benefit from the baby bonus, but they are still required to pay higher consumption taxes, leaving them worse off.

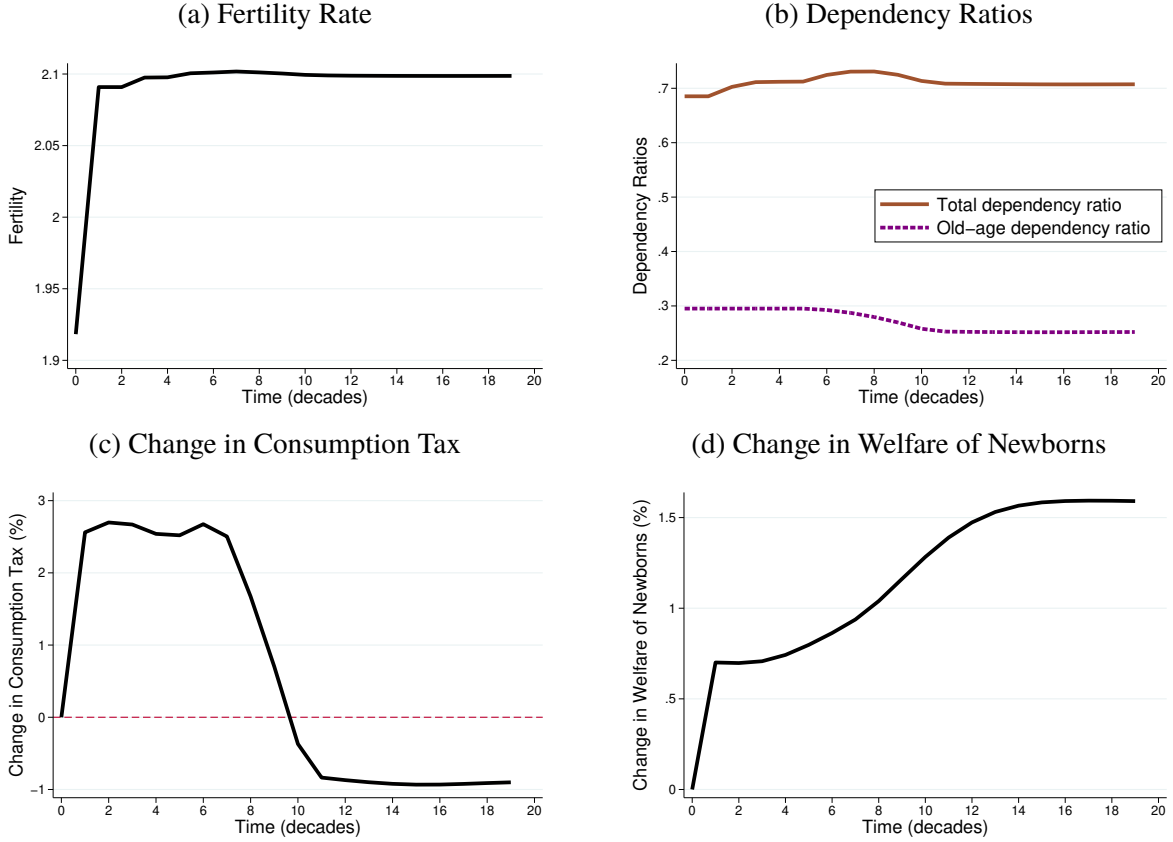
The transition path results shed light on two insights that hold more generally beyond the case of a baby bonus. First, the overall fiscal burden induced by family policies is usually much higher than the policy expenditure itself, especially in the first few decades in the transition path. When more children are born, the government needs to finance additional expenditures for existing child-related policies. The potential fiscal benefits of reductions in the old-age dependency ratio, on the other hand, will be realized much later than the upfront costs. Second, the amount of political support for family policies depends on how the fiscal costs are distributed across generations. The government will have a hard time gathering enough agents to support family policies that benefit the economy in the long run at the cost of existing households. In that sense, family policies are similar to climate change mitigation policies where countries wait “too long” to act due to frictions caused by intergenerational public finance (Sachs, 2014).

6 Optimal Policy

In this section, I study the optimal baby bonus to address the externalities of childbearing and inefficiencies caused by borrowing constraints. I begin with a discussion of welfare criteria under heterogeneous agents and endogenous fertility. Then, I show optimal policy results and propose principles for designing family policies in general.

⁵⁹For example, while the direct cost of a \$30,000 baby bonus is 1.6% of GDP, the government needs to raise consumption taxes by 2.6% for the first seven decades of the transition.

Figure 11: Transition Path of a \$31,000 Baby Bonus



Notes: These figures plot the evolution of aggregate variables under a baby bonus of \$31,000. Each period represents 10 years. The policy is enacted at period 1.

6.1 Welfare Criteria

Welfare criteria in models with endogenous fertility are complicated both conceptually and philosophically. Unlike standard comparisons between allocations where the set of agents is fixed, in this context there will be agents born in one economy but not in the other. As a result, the standard Pareto principle cannot be used to conduct welfare analysis in this context. The field of population ethics is devoted to understanding and resolving this question.⁶⁰

As the debate on welfare criteria is far from being settled, I adopt two definitions in studying the optimal policy. The first criterion is the *long-run average welfare* \mathcal{W} used in previous sections.

⁶⁰For instance, Parfit (1984) derives the famous “repugnant conclusion” showing that under a set of intuitively appealing assumptions, one can prove “for any perfectly equal population with very high positive welfare, there is a population with very low positive welfare which is better, other things being equal.” Golosov et al. (2007) propose two criteria called \mathcal{A} -efficiency and \mathcal{P} -efficiency which differ by whether the planner evaluates the welfare of those who are not born. De la Croix and Doepke (2021) consider the optimal welfare from a soul’s perspective where one needs to consider both the utility of being born and the average “waiting time” for incarnation.

It evaluates the expected utility of a newborn child in the long-run stationary equilibrium under the veil of ignorance. The second criterion assesses the *average utility of existing agents* when the policy is adopted.⁶¹ I decompose this into welfare changes for existing parents ($j = 2$) who receive the baby bonus and for other households ($j > 2$) who are not direct beneficiaries of the policy. The latter criterion has two features. First, it is forward-looking, as it incorporates tax changes in later periods that affect these households' utility. Second, with this criterion, the unique solution to the planning problem corresponds to the notion of \mathcal{A} -efficiency defined in [Goloso et al. \(2007\)](#) which focuses on the welfare of those already alive.

Rather than computing the unconstrained optimum, I follow the Ramsey tradition and allow the government/planner to use only a certain policy instrument (a baby bonus) that is uniform across households and birth order. As in previous sections, I consider the scenario where the government adopts the baby bonus at the beginning of time period $t = 1$. The policy change is permanent and financed by consumption tax changes. I use \mathcal{B}_{lr}^* to denote the optimal baby bonus that maximizes average utility in the long run, and I use \mathcal{B}_{sr}^* to denote the optimal baby bonus that maximizes the average utility of existing households when the policy is adopted.

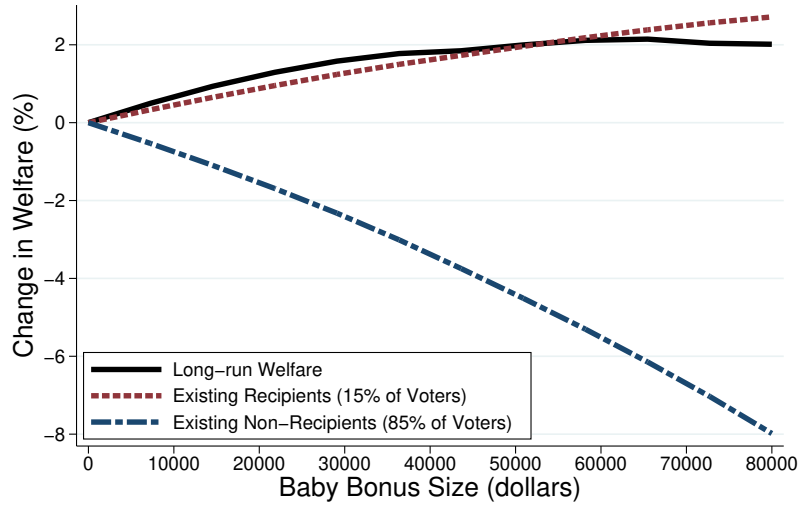
6.2 Optimal Policy Results

Figure 12 shows the changes in welfare under different baby bonuses for two welfare criteria. From a long-run perspective, the optimal baby bonus is $\mathcal{B}_{lr}^* = \$60,000$. \mathcal{B}_{lr}^* boosts aggregate fertility to 2.4 children per woman, raises long-run welfare by 2%, and costs around 4.1% of GDP.

The baby bonus that maximizes the welfare of the median voter among existing households, however, is $\mathcal{B}_{sr}^* = \$0$. Current parents who receive the baby bonus prefer larger \mathcal{B} for two reasons. First, despite the higher taxes in transition, these households are subsidized by older households ($j > 2$) on net. Second, the baby bonus redistributes towards poorer parents, which improves average welfare. Older households in the economy, however, oppose the baby bonus because they do not benefit from the bonus but still pay higher taxes for the remainder of their lives. If each household has the same voting power, then $\mathcal{B}_{sr}^* = \$0$ is the most likely outcome from a political perspective. As discussed in Section 5.3, this observation explains the observed puzzle where many countries with extremely low fertility rates fail to implement large-scale family policies despite knowing the dire consequences in the long run. Even when they do implement these policies, governments often renege on the promises due to fiscal pressures. For example, the Australian baby bonus was significantly downsized in 2014.

⁶¹Following standard practice in quantitative macroeconomic literature, I use equal weights as a benchmark. Another approach is to use the Negishi weights, which put a greater weight on households with higher human capital and initial assets. This would eliminate the redistributive benefits of baby bonuses. See [Kim et al. \(2021\)](#) for an example.

Figure 12: Optimal Baby Bonus by Welfare Measure



Notes: This figure plots the changes in welfare under baby bonuses of different sizes. I consider two welfare criteria: (1) average utility of a new-born child in the long run and (2) average utility of existing households when the policy is enacted, decomposed into agents who receive the baby bonus and agents who do not receive it.

6.3 Discussion

Even though this paper restricts the study of optimal policy to choosing the optimal level of a baby bonus, it highlights the major trade-offs in policy design that are broadly applicable to other instruments and goals.⁶² On the one hand, subsidizing childbirth among parents with low human capital leads to larger fertility changes per dollar spent on family policies. This in turn leads to larger welfare improvements due to changes in the demographic structure. On the other hand, subsidizing childbirth among parents with high human capital improves the equilibrium human capital distribution via intergenerational transmission of skills, thereby raising welfare due to composition effects. At the aggregate level, the government balances population growth, average human capital, and distortions through the taxation by weighing the quantity-quality trade-off, composition effects, demographic structure effects, and changes in childcare arrangements.

With countervailing forces present in the unified model, the first-order stochastic dominance (FSD) in equilibrium human capital distribution is neither necessary nor sufficient for choosing better policies. For instance, the equilibrium human capital distribution under a \$30,000 baby bonus is first-order stochastically dominated by that in the baseline economy, but average well-being improves in the long run. The key insight is that besides comparing human capital distributions

⁶²For instance, it is straightforward to incorporate subsidized childcare and public education expenditures, shown in Appendices C.1 and C.2. Instead of choosing a permanent policy change to the baby bonus, one could also conduct dynamic optimal policy design where the size of the bonus varies over time.

across economies, one also needs to consider differences in the *age distribution*. Thus, this paper provides a novel counterargument to the common conclusion in the existing literature on family policies arguing for childbirth restrictions among parents with low human capital (e.g., [Chu and Koo, 1990](#)).⁶³ These parents, with higher fertility responses to per dollar benefit, could be key to solving population aging problems.

7 Conclusion

Facing aging populations, family policies have been widely pursued to encourage childbirth. On the other hand, evidence of the effects of transfers to parents on children's outcomes lead policymakers and economists alike to believe that family policies are good instruments to "lift children out of poverty today and help them tomorrow" ([Schanzenbach et al., 2021](#)).

In this paper, I study the aggregate impacts of family policies in a heterogeneous agent overlapping generations model. Relative to previous studies on family policies, this paper is the first to combine the quantity-quality trade-off, an endogenous demographic structure, and childcare choices in a quantitative structural framework.

In the calibrated model, I find that when governments design family policies that reward having more children, parents respond by increasing child quantity but optimally reduce child quality. Compared with education subsidies, child benefits are not ideal instruments if the government aims to raise child human capital or boost social mobility. However, the pro-natal effect of family policies could lead to long-run welfare gains: lower old-age dependency ratio allows the government to cut taxes. I also show that the long-run gains in welfare require a transition path where the government needs to finance higher child-related expenditures during the first few decades. Depending on how the government decides to finance these higher costs, the welfare of existing households could fall.

The tractability of the model allows for several extensions and applications, which are left for future research. First, one can calibrate the model to match the institutional details of other countries and conduct policy counterfactuals on a case-by-case basis. Moreover, it would be interesting to consider the optimal policy design with additional or jointly-used policy instruments under different welfare criteria. For instance, when the government uses a baby bonus to raise fertility, it could expand public education at the same time to offset the reduction in private education investments due to the child quantity-quality trade-off.

⁶³[Córdoba and Liu \(2016\)](#) make a complementary argument invoking the Lucas' Critique by showing that family policies have direct effects on households' utility.

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A Robustness

A.1 More on Calibration

Table 4: Sensitivity of Model Parameters to Moments

	γ	ψ	θ	ν	v	σ_ϵ	ρ	ξ	κ	η	σ_z
n Gap	-0.50	-0.28	-0.52	-0.41	0.00	0.00	-0.03	0.03	0.03	-0.01	-0.03
TFR	0.56	-0.56	1.60	1.06	0.12	-0.07	-0.01	0.02	0.09	-0.04	0.04
e	0.46	0.49	1.14	0.34	-0.47	0.05	-0.08	0.28	-0.07	0.01	-0.02
a_k	-0.03	-0.06	-0.04	0.75	0.00	0.00	0.00	0.00	0.00	0.00	0.00
m	-0.41	-0.35	-0.31	-0.29	1.05	-0.11	-0.08	0.07	0.13	-0.05	0.03
Gini ₂	0.02	0.37	1.00	-0.39	-0.48	1.37	0.71	-0.61	-2.49	1.19	0.95
IGE	-0.27	-0.24	-0.71	-0.26	-0.23	-0.03	1.20	-0.29	-0.37	0.20	0.27
e_{low}	-0.07	-0.08	-0.19	-0.05	-0.01	0.00	0.02	-0.15	0.00	0.01	0.01
B/C Ratio	0.31	-0.12	-0.61	0.30	0.25	-0.11	-0.36	0.26	1.75	-0.22	-0.30
h Profile	-0.11	0.06	0.22	-0.31	-0.32	0.10	0.28	-0.19	-1.05	0.50	-0.05
Gini ₆	-0.03	0.06	0.20	-0.05	-0.09	0.03	0.02	-0.06	-0.40	0.19	0.36

Notes: This table displays the sensitivity of model parameters to moments (see [Andrews et al., 2017](#)). Each cell shows the percentage change of the parameter (column) when the corresponding estimation moment (row) changes by one percent. Bold entries report maximum (of absolute value) by each row, highlighting the parameter that is most sensitive to moment changes. See Table 2 for definitions of parameters and identifying moments.

Table 5: Elasticity of Target Moments to Parameters

	n Gap	TFR	e	a_k	m	Gini ₂	IGE	e_{low}	B/C Ratio	h Profile	Gini ₆
γ	-2.10	0.38	-3.50	-1.36	-1.73	0.00	-0.26	-7.58	0.04	0.07	0.20
ψ	-0.67	-0.96	2.36	0.73	1.21	-0.03	0.15	4.81	-0.01	-0.08	-0.10
θ	0.63	0.19	1.90	-0.14	0.89	0.02	0.12	4.03	-0.02	-0.01	-0.19
ν	-0.09	-0.05	0.16	1.33	0.08	0.00	0.01	0.34	0.00	-0.01	-0.04
v	-0.92	-0.12	0.02	0.07	0.99	0.08	0.00	0.10	-0.02	0.00	-0.38
σ_ϵ	0.76	0.05	-0.29	0.03	-0.32	0.85	-0.19	-1.36	0.03	-1.08	-2.13
ρ	-0.10	-0.02	0.90	0.02	0.52	0.06	0.93	0.45	-0.02	-0.17	-0.92
ξ	0.61	0.09	-1.96	-0.06	-0.98	0.01	0.05	-11.08	0.11	0.03	0.32
κ	0.65	-0.06	1.93	0.06	0.84	0.06	0.30	4.74	0.74	-0.04	0.20
η	0.19	-0.09	1.00	0.61	0.89	-0.03	0.06	1.87	1.59	2.14	1.60
η_z	-0.02	0.03	-0.49	-0.24	-0.31	0.01	0.16	-1.17	-0.01	-1.05	2.50

Notes: This table displays the elasticity of model moments with respect to changes in model parameters. Each cell shows the percentage change of the value of model moment (column) when the corresponding parameter (row) changes by one percent. Bold entries report maximum (of absolute value) by row, highlighting the moment that is most sensitive to parameter changes. See Table 2 for definitions of parameters and identifying moments.

A.2 Alternative Preferences

There are alternative ways to model parents’ preferences over child quantity and quality. I argue that the modeling choice in Section 3.1 results in a conservative benchmark relative to other parameterizations.

Separable preferences between quality and quantity (see [de La Croix and Doepke, 2003](#); [Bar et al., 2018](#); [Vogl, 2016](#)) is a commonly-used assumption where

$$v(n, \mathbb{E}h_k, a_k) = \log(n) + \theta \log(\mathbb{E}h_k) + \nu \log(a_k).$$

Compared with our benchmark case where quality and quantity are complements, parents will raise fertility and reduce child quality even more strongly when the government rewards childbirth. This is because the interaction in preferences $v(n, \mathbb{E}h_k, a_k)$ is not present when quantity and quality are separable. I show this result with closed-form solutions in Appendix E.1. For the same reason, if quantity and quality are substitutes, the results in this paper will be conservative ([Jones and Schoonbroodt, 2010](#)).

Another way of modeling parents’ preferences is with dynastic altruism (see [Córdoba et al., 2016](#); [Daruich and Kozłowski, 2020](#)) where

$$v(n, \mathbb{E}h_k, a_k) = \Psi(n)V_2(\mathbb{E}h_k, a_k).$$

This formulation is similar to the one used in this paper with the exception that utilities from child quality are pinned down recursively. Dynastic altruism is appealing aesthetically but faces unresolved challenges in fitting observed transfers between parents and children (see [Altonji et al., 1997](#); [Barczyk and Kredler, 2020](#)). Paternalistic motives are often added to improve data fit (e.g., [Abbott et al., 2019](#)). I argue that the results will also be stronger under dynastic altruism: parents endogenize the fact that returns to child quality become lower when the government creates a “social safety net” for children with low human capital by providing generous family benefits. As a result, parents would further increase child quantity and reduce quality investments. For instance, [Daruich and Fernández \(2020\)](#) argue that universal basic income reduces the human capital of future generations due to this mechanism.

A.3 Endogenous Timing of Childbirth

In the baseline model, I abstract away from birth timing by assuming that parents only make fertility choices from age 20 to 30. In reality, parents can decide when to give birth, and family

policies could affect that decision. This is known as the *tempo effects* of family policies.⁶⁴

I argue that adding endogenous timing will make my baseline results stronger. As the model matches effects on the completed fertility rate (known as the *quantum effects*) in the validation exercise, policy effects on the total fertility rate (quantum plus tempo) will be larger. Suppose that in addition to increasing the number of children, some parents shift birth from their 30s to their 20s in response to a baby bonus. This shift in birth timing will likely be detrimental to child human capital for two reasons. First, early birth reduces the spillover that the children could receive because parents' human capital grows rapidly from age 20 to 40. Second, family policies of realistic sizes fall short of offsetting the income differences of parents between early and late births. Hence, these children are born into households with fewer resources on average, which would also reduce child human capital as investment falls. As a result, family policies will have a larger observed fertility impact on the economy with endogenous timing of birth, but the outcomes of children will be even worse.

A.4 Bequests

The model restricts the set of timings that parents could make transfers to children to $j = 3$. I make this simplifying assumption to keep the model tractable – if parents can leave end-of-life bequests to their children, then children need to keep track of parents' state space, including human capital and assets.

I argue that this simplification does not significantly affect my model results. Because the key mechanisms of the model (i.e., quantity-quality trade-off and childcare choices) occur in period $j \in \{2, 3\}$, the possibility of receiving a bequest at $j \in \{5, 6\}$ does not matter for households' choices if they cannot borrow against it due to life-cycle credit constraints. In the calibrated model, most households, especially those with low human capital who are most responsive to family policies, hold few assets in the beginning of their life cycles despite receiving the inter vivos transfers from their parents (see Figure 3).

A.5 Alternative Measure of Fertility

As the baseline model focuses on the total number of children that parents decide to have, the most appropriate measure of fertility in this context is the completed fertility rate (CFR), measured using data on either “children ever born” in Census data (dropped after 1990) or “live births ever

⁶⁴An unresolved question is whether changes in timing are due to the relaxation of parents' constraints (e.g., down payments to buy a larger house) or the lack of commitment to policies by the government. Historically, governments often renege on family policies as fiscal conditions change. The significant downsizing of the Australian baby bonus in 2014 is one good example. This uncertainty creates an incentive for parents to shift birth timing while benefits are still in effect.

had” in the CPS June Fertility Supplement. As discussed in Section 4, the CFR has the advantage of being invariant to shifts in birth timing due to policy changes. For these reasons, in Figure 5.2 and the calibration, I use the CFR by household income decile, calculated using the sample of married women age 40-55 in the 2008-2014 CPS Fertility Supplement.⁶⁵

One disadvantage of using the CFR is that these women made their fertility decisions prior to 2010 where they may have faced different trade-offs. The total fertility rate (TFR) is an alternative measure that is more responsive to contemporaneous situations, calculated by summing the age-specific birth rates of all women alive in a given year. This measure is simple to compute and widely used in the literature (Kremer and Chen, 2002).

TFR by income decile, however, is less straightforward to compute than CFR by income decile. First, one does not want to assign income ranks that mix young women with older women due to life-cycle changes in the earnings profile. Second, given that the appropriate measure of income is total family income, one would not want to mix married women with single women. Daruich and Kozlowski (2020) provide a recent estimate of TFR by income decile using data from the 2000 Census. They restrict the sample to married women between ages 15-49 who are either themselves household heads or spouses of the household head. Income deciles are assigned by comparing the total family income of each woman with other observations in the same age group. To alleviate selection bias into marriage for younger age groups, they also drop age-income groups with a small number of observations. For each income decile, they calculate the TFR by adding up the age-specific fertility rates in that decile. Figure A.1 shows that their result is quantitatively similar to the CFR estimates used in the calibration.

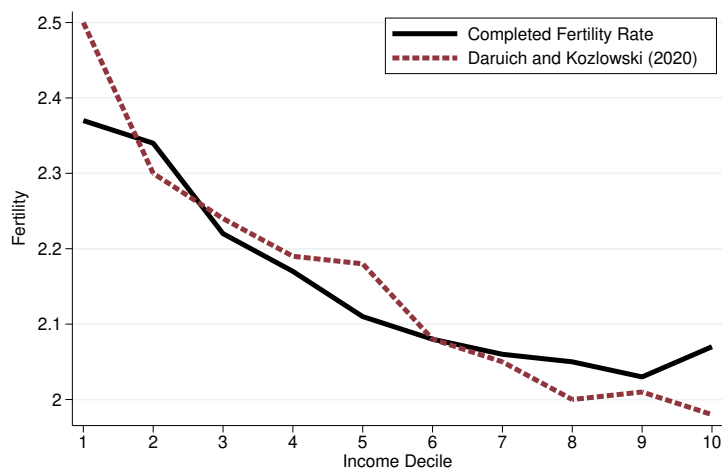
A.6 Government Borrowing

In the baseline policy counterfactual and the optimal policy analysis, the government changes consumption taxes τ_c to balance the budget in every period. This assumption is made primarily for computational reasons.

I argue that many results are unlikely to change when the government can borrow from domestic or international lenders. In particular, all of the main policy mechanisms (i.e., quantity-quality trade-off, composition effects, changes in demographic structure, and adjustments in childcare arrangements) are independent of whether or not the government is allowed to borrow. As a result, I

⁶⁵In a steady-state economy, fertility choices across cohorts are stationary, and hence the CFR coincides with the total fertility rate (TFR). For this reason, I normalize the CFR target in the calibration of ψ to match the total fertility rate in 2010, which was 1.92 children per woman. Model results are quantitatively similar if I omit this step. I have also experimented with a version of the model where a fraction of adults remain single and (exogenously) childless, meaning that the CFR in the economy is a weighted average between single adults without children and married households whose within-group CFR is higher than 1.92. The overall results are again similar.

Figure A.1: Fertility Rate by Income Decile



Notes: This figure plots the total fertility rate by income decile measured in [Darulich and Kozlowski \(2020\)](#) and the completed fertility rate used in model calibration.

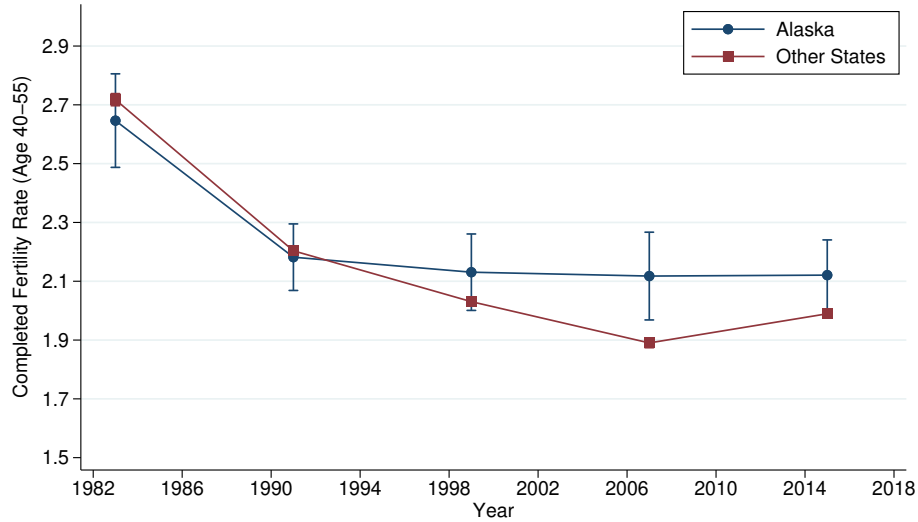
expect the model predictions on fertility, human capital, output, and mobility to be unchanged in a model with government borrowing.

Welfare implications, however, will likely change depending on how government expenditures are funded. For example, since the \$30,000 baby bonus generates welfare improvements for all agents in the long run, it is natural for the government to shift the fiscal burden to these beneficiaries via borrowing instead of asking existing elderly households to fund additional expenditures from which they derive little benefit (see Figure 12). If the government chooses to do so, then the welfare effect in the long-run economy will be smaller, while the effect on existing agents will be less negative. Such a policy change is also more likely to gain sufficient political support for implementation as it does not directly hurt existing agents who have voting power. On the other hand, if the government issues debt to finance large-scale family policies, the overall borrowing cost for the government could increase in general equilibrium if lenders require higher compensation for a larger debt-to-GDP ratio. This might crowd out other government borrowing that benefits existing households. As a result, whether current agents prefer government borrowing or changes in taxation also depends on the opportunity costs of family policies.

Despite the intuitive appeal of using government deficits to finance large-scale family policies that benefit future generations, family policies are often financed by adjusting fiscal revenues and expenditures in current policy proposals. For example, the American Families Plan vows to increase the tax rates among the very rich, while the Family Security Act proposed by Senator Romney would reform and consolidate outmoded federal programs. I leave the normative implications of such financing choices for future research.

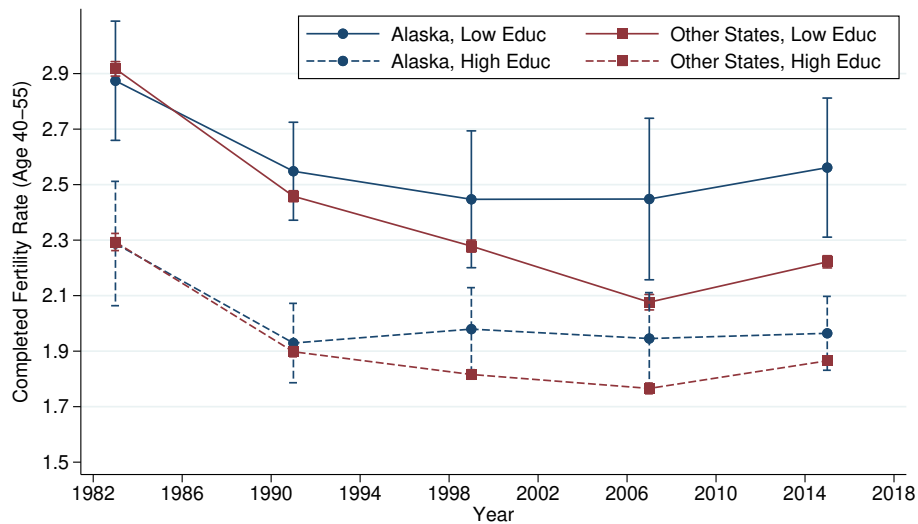
B Additional Figures on the APFD

Figure B.2: APFD and Completed Fertility Rate



Notes: This figure plots the average completed fertility rates for women aged 40-55 by state of residence from 1982 to 2018 using data from the CPS Fertility Supplement combined into 8-year bins. Bars around sample means show 90% confidence intervals.

Figure B.3: APFD and Completed Fertility Rate by Education



Notes: This figure plots the average completed fertility rates for women aged 40-55 by state of residence and education from 1982 to 2018 using data from the CPS Fertility Supplement combined into 6-year bins. I define women with high education as those who have at least one year of college experience. Bars around sample means show 90% confidence intervals.

C Additional Results

C.1 Public Childcare

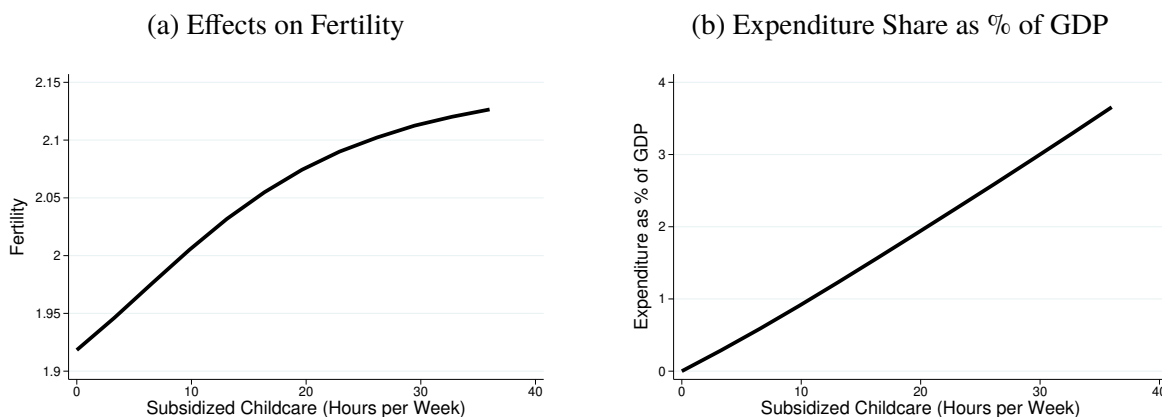
In this section, I evaluate childcare subsidies where the government offers $\mathcal{S} \in [0, 0.9 \cdot \chi]$ market childcare services for all families, equivalent to 0 to 36 hours of public childcare per week.

I find that subsidized childcare raises fertility, but at a higher cost than baby bonuses. The government needs to provide 25 hours of subsidized childcare to raise the aggregate fertility rate to 2.1. This would cost almost 2.5% of aggregate GDP in the long-run steady-state economy, which is 0.8 percentage point higher than a baby bonus that achieves the same fertility goal.

Figure C.4d shows that similar to the baby bonus counterfactual, parents reduce private education investments due to the quantity-quality trade-off. Figure C.4c shows that fertility responses are higher among low-income parents. Due to reductions in private investment and composition effects, average human capital in the economy decreases (see Figure C.4e). The overall effects on intergenerational mobility are small (see Figure C.4f).⁶⁶

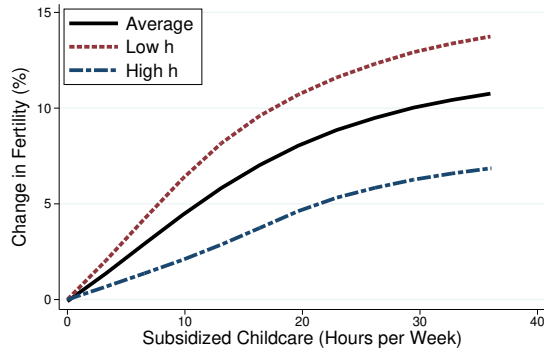
Figure C.4g shows that output per capita increases with subsidized childcare. Compared with the baby bonus, the output changes are larger for three reasons. First, composition effects are milder because the fertility differentials between parents with low and high human capital are smaller than those under the baby bonus (see Figure C.4c). Second, the childcare subsidy raises the labor supply by replacing home care with subsidized market childcare. Finally, the increased working time during age 20-30 translates to high human capital for adults in later periods due to learning-by-doing in the labor market. Relative to the original steady-state, Figure C.4i shows that the government can reduce consumption taxes by 0.6%. Long-run welfare could be raised by 2.5% when aggregate fertility is around the replacement level.

Figure C.4: Long-Run Effects of Childcare Subsidies

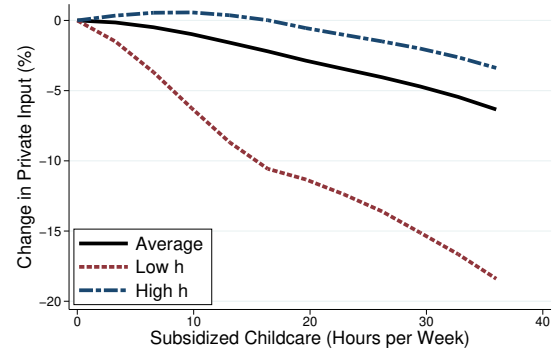


⁶⁶The effects on mobility are not monotonic because besides changing child human capital, subsidized childcare also changes hours worked, affecting household earnings.

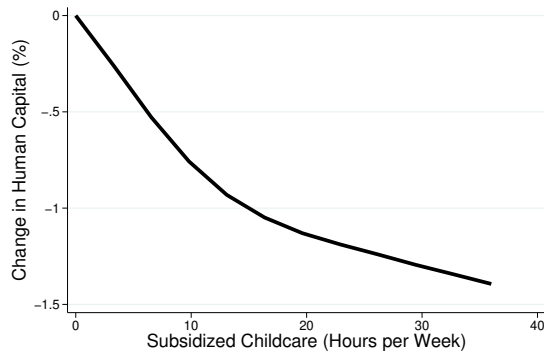
(c) Heterogeneous Fertility Response



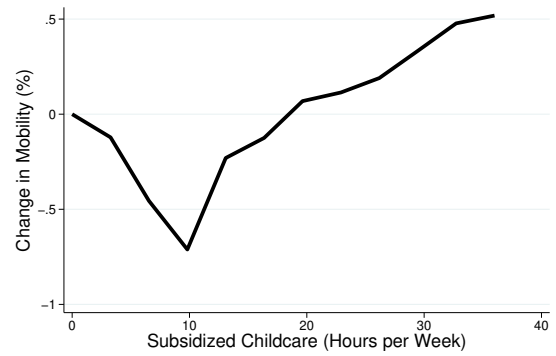
(d) Heterogeneous Private Input Response



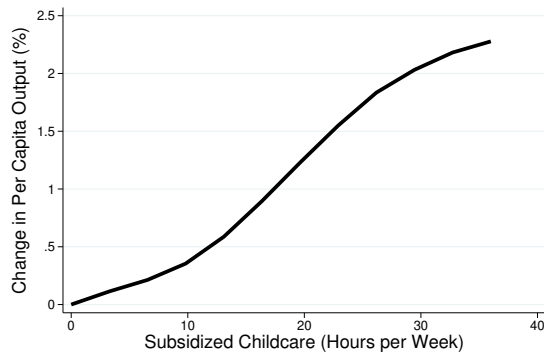
(e) Average Human Capital



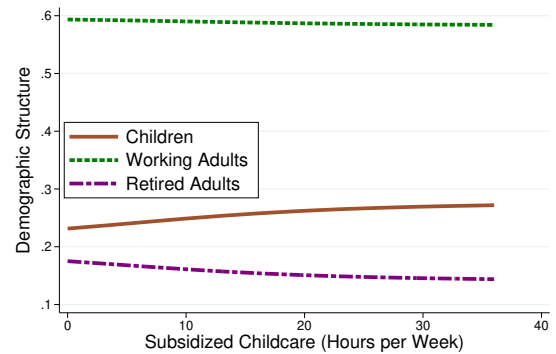
(f) Intergenerational Mobility



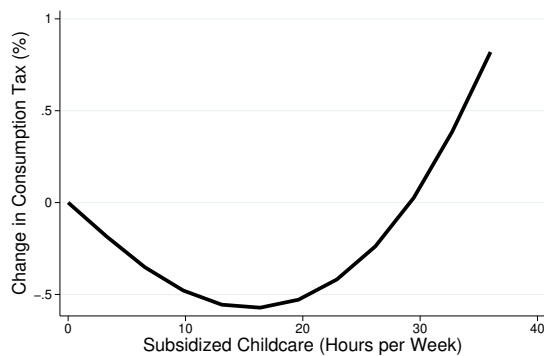
(g) Per Capita Output



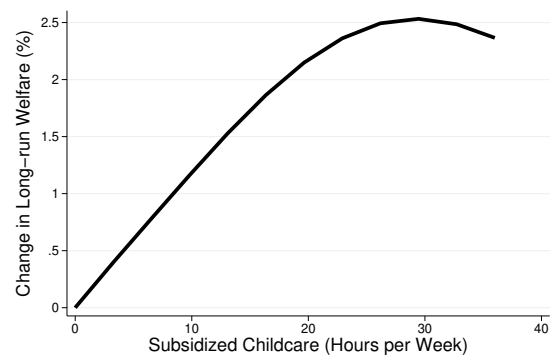
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of childcare subsidies of by comparing long-run steady-states under different policies to the baseline steady-state economy.

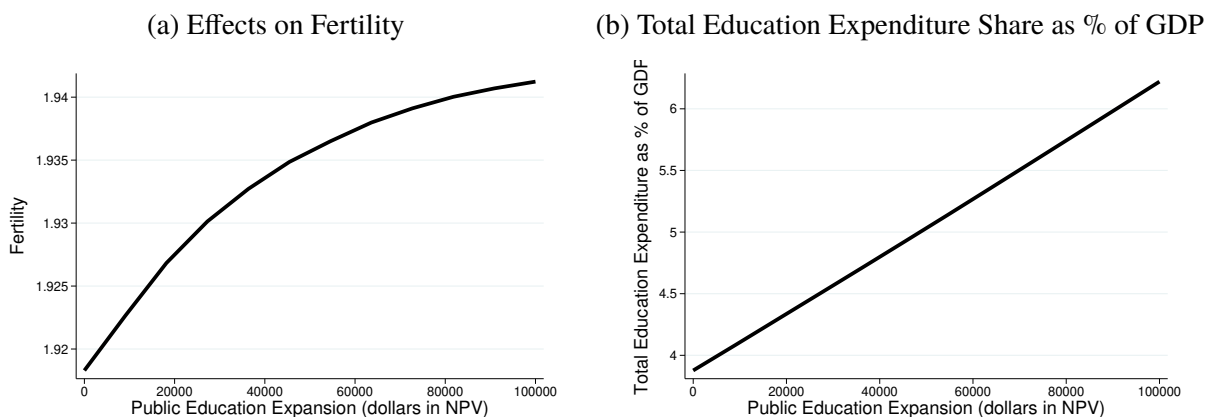
C.2 Public Education

In this section, I evaluate expansions of public education expenditures from the current level of \$12,000 per student per year. The size of the increment varies between \$0 and \$100,000 in net present value. Figure C.5a shows that the effect on fertility is positive: as public education raises children's human capital, they become more desirable to parents. The magnitude of the fertility effect here is much smaller than that of family policies. Compared with a \$30,000 baby bonus that raises fertility by 0.2 children per woman, an education expansion with costs being the same share of GDP only raises fertility by 0.011. The government should therefore not expect to use education policies to raise aggregate fertility to the replacement level.

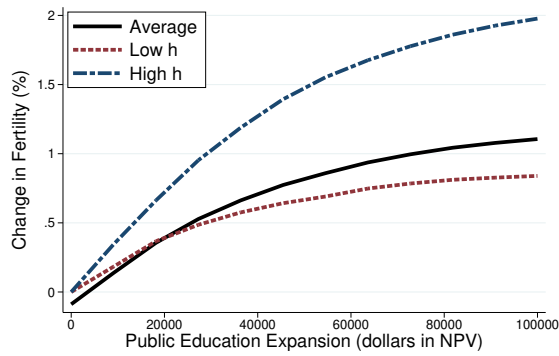
Figure C.5c shows that the fertility response is larger for households with higher human capital. An increase in public education expenditure also crowds out private input (see Figure C.5d). Due to strong crowding-out effects, average human capital only increases by 2.5% when the education expansion exceeds \$80,000 in net present value. It is costly to raise average human capital in the economy using uniform policies in general equilibrium because (1) crowding-out effects are strong and (2) investments face decreasing marginal returns as \mathcal{E} is already \$12,000 per year in the baseline economy.

Expanding public education, however, most effectively raises social mobility. As the expansion exceeds \$60,000 in net present value, intergenerational mobility rises by almost 5.2%. If the policy targets lower-income households, one can expect larger effects on mobility. Due to increased human capital, public education expansion of \$60,000 raises output by 2.3%. The government needs to increase the consumption tax by 0.75% to finance such expenditures. Figure C.5j shows that welfare could be increased by up to 2.5%.

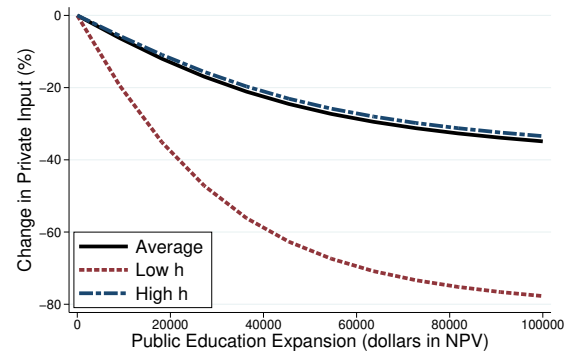
Figure C.5: Long-Run Effects of Public Education Expansions



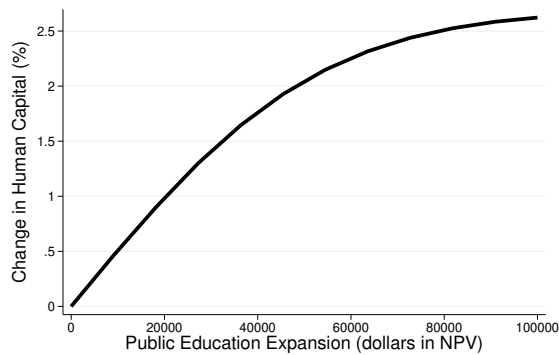
(c) Heterogeneous Fertility Response



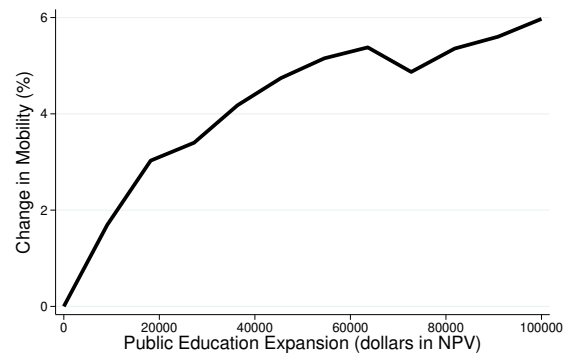
(d) Heterogeneous Private Input Response



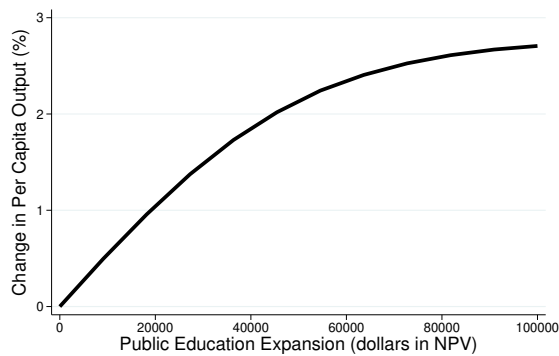
(e) Average Human Capital



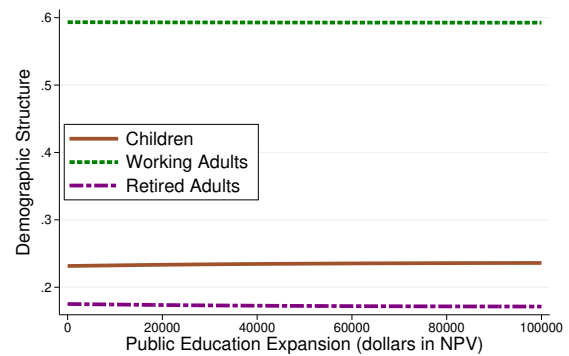
(f) Intergenerational Mobility



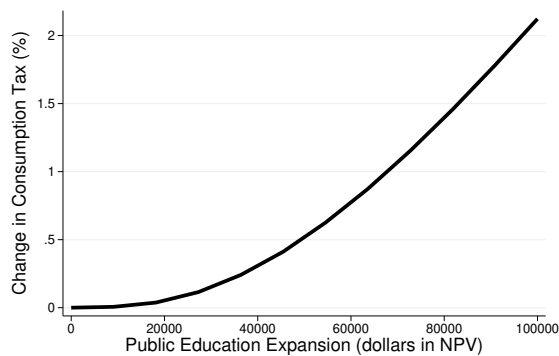
(g) Per Capita Output



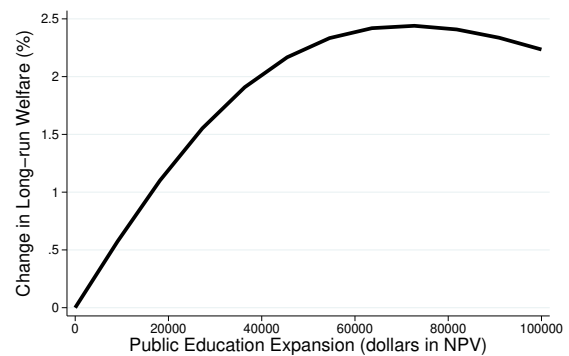
(h) Demographic Structure



(i) Change in Consumption Tax



(j) Change in Welfare



Notes: These figures plot macroeconomic impacts of expansions of public education expenditures by comparing long-run steady-states under policies to the baseline steady-state economy.

D Additional Validating Evidence

D.1 Australian Baby Bonus

This section compares the implications of the calibrated model with the empirical evidence from a recently adopted baby bonus in Australia.

The Australian government announced the introduction of a universal cash payment, i.e., a baby bonus, in its federal budget on May 12, 2004. It was designed to be a non-means-tested and non-taxable lump-sum payment of AU\$3,000 to encourage childbirth. The transfer would be paid to parents following the birth or the adoption of a child up to 2 years of age after July 1, 2004. Such payments were independent of family income, maternal employment status, or the number of existing children in the household. The payment amount was equivalent to 4 times the weekly average earnings in Australia, or \$2,800 in 2010 U.S. dollars.

[Drago et al. \(2011\)](#) conduct a comprehensive analysis of the fertility effects of the Australian baby bonus using household panel data ($N = 14,932$) from the Household, Income and Labour Dynamics in Australia Survey (HILDA) and a simultaneous equations approach. They first estimate the effect of the baby bonus on households' birth intentions in a linear regression, which, in turn, predicts actual births in a binomial probit regression. They exploit the panel structure of the data to test and correct for announcement effects, compression/tempo effects, and delayed effects. Regression results show that the baby bonus has significant and positive impacts on fertility. Moreover, the fertility responses are concentrated among low-income women.⁶⁷ [Risse \(2010\)](#) reports similar findings on women's birth intentions.

When I compare these empirical studies to the model, both the finding on the overall fertility effect and the heterogeneities across households are consistent with predictions in Section 5.1. [Drago et al. \(2011\)](#) estimate that the marginal cost to the government of an additional birth is at least A\$126,000, roughly 4 times the GDP per capita in 2004 Australia dollars. The calibrated model generates a quantitatively similar conclusion where the marginal costs for an additional birth using a US baby bonuses in 2010 are around 3.5 times the GDP per capita.⁶⁸

[Gaitz and Schurer \(2017\)](#) evaluate the effect of the Australian baby bonus on child human capital accumulation using high-quality panel data from the Kindergarten cohort (K-cohort) of the Longitudinal Study of Australian Children (LSAC). They find that the baby bonus, despite being

⁶⁷This finding is different from that of [Milligan \(2005\)](#) where the author evaluates a child benefit in Quebec and finds larger responses among parents with higher incomes. [Drago et al. \(2011\)](#) propose that the difference could be due to the fact that the Quebec baby bonus was significantly more generous for high-parity births. [Milligan \(2005\)](#) discusses some other possible explanations such as unobserved heterogeneities that are systematically related to income.

⁶⁸To provide additional corroborative evidence, leveraging a large-scale subsidy called Maternity Capital, [Sorvachev and Yakovlev \(2019\)](#) find that the costs of an additional birth in Russia is approximately \$50,000, around 3.5 times the GDP per capita of Russia in 2011.

significant in size, is not effective in boosting learning, socio-emotional or physical health outcomes of the average pre-school child. This finding is consistent with results in Section 5.1 where parents optimally reduce private education investments due to the quantity-quality trade-off.

D.2 Spanish Child Benefits

In this section, I provide additional evidence of the fertility effects of child benefits using a universal child benefits policy in Spain.

The Spanish government announced the new child benefit on July 3, 2007. The benefit was a one-time payment of €2,500 to the mother immediately following a child's birth on and after July 1, 2007. Since the cash benefit was universal and independent of the recipient's characteristics, it was essentially a universal baby bonus (c.f., the Australian baby bonus, and \mathcal{B} in the model). The size of the payment is 4.5 times the monthly gross minimum wage for a full-time worker or \$3,500 in 2010 U.S. dollars.

González (2013) studies the effect of the Spanish child benefit on fertility and mothers' labor supply using monthly vital statistics, monthly abortion statistics, and the 2008 Household Budget Survey ($N = 958$). In the paper, fertility effects are assessed by inspecting the time series of births and abortions, while effects on the labor supply are estimated using a regression-discontinuity design by comparing households who gave birth right before and right after the cutoff date. González (2013) finds that the fertility effects are positive and significant, with the policy increasing the total fertility rate by 6%. Around 80% of this increase is due to increased conception while the remaining 20% is due to reductions in abortion. The result suggests that (in Spain) the marginal cost of an additional birth using baby bonuses is 3.6 times the GDP per capita.⁶⁹ The model counterpart to this statistic is 3.5.

González (2013) also finds that the baby bonus reduced mothers' labor force participation. The calibrated model produces this effect via two channels. First, as the baby bonus raises fertility, parents need to generate more childcare services, increasing their time at home *ceteris paribus*. Second, higher fertility reduces the relative cost of home versus market care due to economies of scale in home production of childcare ($\iota < 1$). Therefore, the model predicts that parents optimally change their childcare arrangements so that their children spend less time in market care. González (2013) confirms this prediction by showing that Spanish parents reduce enrollment in formal childcare after the baby bonus.

⁶⁹Summarizing historical studies with both short-run and long-run effects on fertility, Stone (2020) show that one can multiply the policy effects on total fertility rates by 1/3 to derive a crude estimate of the effects on completed fertility rates.

D.3 Russian Maternal Capital

In this section, I provide additional evidence of the fertility effects of child benefits using a series of subsidies, called Maternity Capital, in Russia.

The Russian government introduced a sizable conditional child-subsidy program (called Maternity Capital) to address the decade-long decrease in fertility and depopulation. The program was implemented in two waves. In the first wave in 2007, the federal Maternity Capital program, a family that already had at least one child and gave birth to another became eligible for a one-time subsidy of approximately \$10,000 U.S. dollars. This figure was larger than the country's average 18-month wage or the sum of the minimum wage over a 10-year period. At the end of 2011, Russian regional governments introduced regional Maternity Capital programs that gave additional money on the top of the federal subsidy to all families with newborn children.

Using a regression-discontinuity design surrounding the program announcement date, [Sorvachev and Yakovlev \(2019\)](#) find that the program have significantly increased fertility both in the short-run and in the long-run. They estimate that the cost for an additional birth is around \$50,000, approximately 3.5 times the GDP per capita of Russia in 2011. Moreover, they find that the treatment effect is heterogeneous: while all educational groups are affected by the program, the increase in fertility is higher among low-educated mothers. In another work, [Slonimczyk and Yurko \(2014\)](#) build and estimate a structural dynamic programming model of fertility and labor force participation to evaluate the policy effects. They find that the program increased long-run fertility by about 0.15 children per women. This estimate suggest that the cost for an additional birth is around 4.6 times the GDP per capita, slightly higher than that in [Sorvachev and Yakovlev \(2019\)](#). They also find that the program “seems to have more strongly affected relatively underprivileged women”.

Subject to the caveat of not re-calibrating the model to match institutional details in Russia, these findings in both papers are consistent with the model's prediction. In particular, the model predicts that an additional birth requires a payment that is 3.5 times the national GDP per capita. Moreover, fertility responses are larger among parents with lower human capital (see Section 5.1).

E Illustrative Models

E.1 Simple Model of Quantity-Quality Trade-Off

In this section, I show the quantity-quality trade-off mechanism with closed-form solutions in a model that is adapted from [de La Croix and Doepke \(2003\)](#).

Agents in the economy live for two periods, child and adult. Adult parents solve the problem:

$$\max_{c,n,e} \log(c) + \theta \log(n \cdot h_k)$$

subject to

$$c + n \cdot e = 1 - n \cdot \chi, \text{ and}$$

$$h_k = e^\gamma, \quad \gamma \in (0, 1).$$

where c is consumption, n is fertility, h_k is human capital of children, e is private investment,⁷⁰ and χ is fixed cost per child.

When fertility is exogenous, i.e., n is given, parents maximize over c and e . The optimal investment is given by

$$e^* = \frac{\theta\gamma}{1 + \theta\gamma} \times \frac{1 - n\chi}{n}.$$

When the child cost χ decreases, e^* increases due to the income effect.

When fertility is endogenous, parents maximize over c , e , and n . The optimal fertility and investment decisions are given by

$$n^* = \frac{1}{\chi} \cdot \frac{\theta(1 - \gamma)}{1 + \theta}, \quad \text{and} \quad e^* = \frac{\gamma\chi}{1 - \gamma}.$$

When the cost of a child χ decreases, n^* increases while e^* decreases. The intuition for this result is simple. Parents increase fertility due to the substitution effect. The increase in n^* , in turn, raises the shadow price of investment e^* due to the interaction between n and e in the budget constraint. As a result, the optimal investment e^* falls.

Compared with the benchmark model where n and h_k are complements in parents' preferences, reductions in e^* in response to a change in χ are higher in this simple model. This is because the marginal utility in child quality h_k is independent of fertility n .

⁷⁰In [de La Croix and Doepke \(2003\)](#), children receive human capital endowments. This generates non-homotheticity over child quality for heterogeneous parents with different human capital and leads to a negative income-fertility relationship. I abstract away from steady-state heterogeneity across households in this simple model for clearer exposition of intuition in comparative statics, but all arguments carry through when human capital endowments are allowed.

E.2 Simple Model of Fertility Elasticity

In this section, I build a simple model to illustrate the relationship between the parameter γ , which determines the intergenerational elasticity of substitution (EGS), i.e., $1/\gamma$, and the magnitude of fertility responses to family policies.

Consider a simplified problem for parents with very low income so that child quality is generated by public investments alone:

$$\begin{aligned} \max_{c,n} \quad & u(c) + \Psi(n)u(\mathcal{E}) \\ \text{subject to} \quad & c + n \cdot \chi = 1. \end{aligned}$$

The first-order condition for n is therefore:

$$\underbrace{\Psi'(n) \cdot u(\mathcal{E})}_{\text{MB of } n} = \underbrace{u'(c) \cdot \chi}_{\text{MC of } n}.$$

By substituting in $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, I have

$$\Psi'(n) \cdot u(\mathcal{E}) = c^{-\gamma} \cdot \chi.$$

When the “price” of child χ falls, both n and c will rise. A higher γ results in a faster decay of the marginal utility change from rising consumption. To balance the first-order condition, the rise in n on the left-hand-side needs to be larger. In other words, higher γ leads to a greater elasticity of n with respect to changes in χ .

This argument holds equally for $\{\mathbb{E}h_k, a_k\}$ in the main model because they are also evaluated by parents through utility function $u(\cdot)$. [Soares \(2005\)](#) uses a similar argument in the context of mortality reductions of children.