The Autumn of Patriarchy

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Abstract

This paper develops a unified model that explains the transition from patriarchal societies to egalitarian ones, featuring declines in fertility, marriage, and gender income gaps. I propose and empirically verify a novel Impossible Trinity hypothesis in family economics: high fertility, dual parenthood, and gender income equality cannot coexist. I also show that factor-neutral technological changes sow seeds of the inevitable demise of patriarchy by raising the opportunity cost of having children. The pace of the ultimate transition could vary across countries due to factors such as the social norm.

JEL classification: D13, J11, J12, J13, J16
Keywords: patriarchy, fertility, gender equality, family structure

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"...and the bells of glory that announced to the world the good news that the uncountable time of eternity had come to an end."

Gabriel García Márquez, *The Autumn of the Patriarch*

1. Introduction

After dominating human society over millennia, patriarchy has been tailing off in recent decades. Amidst the multifaceted transition towards a more egalitarian society (Doepke and Tertilt 2009, Folbre 2021), three key trends stand out: fertility rates have been falling (Guinnane 2011), marriage and dual parenthood have been declining (Stevenson and Wolfers 2007), and gender gaps in wage, income, and wealth have been converging (Goldin 2014).[1]

This paper develops a unified economic framework to account for all three trends jointly. I start with a static model where males and females make decisions on marriage, fertility, and labor supply. The three decisions are interconnected in the model by two simple yet intuitive assumptions. First, marriage is linked with fertility because a primary function of marriage is to share the costs of raising children. Second, fertility is linked with relative labor supply across genders because women shoulder a greater share of childcare responsibilities historically.

I characterize the model equilibrium by showing that individual optimization and marriage market clearing conditions uniquely pin down the fertility rate, marriage share, and within-marriage income transfers. Furthermore, the gender income gap is determined by the prevailing gender gaps in human capital and endogenous female labor supply.

Based on the static model, I propose a novel Impossible Trinity hypothesis in family economics: high fertility, high fraction of dual parenthood, and gender income equality cannot coexist in the same economy. In particular, I establish that achieving any pair

[1] See Figures A.4, A.5, and A.6 for the three trends in the data.

necessarily implies the opposite of the third. Therefore, while each outcome could be a desirable policy goal, the internal inconsistencies imply that policymakers cannot have them all and need to make trade-offs.

I empirically test the Impossible Trinity hypothesis using data from a panel of countries between 1970 and 2014 where all three outcomes can be measured. I divide countries into high fertility, dual parenthood, and gender income equality groups using sample averages of each aspect. Then, I plot the Venn diagram to inspect their intersections. I find that a negligible share of the observations achieved high fertility, dual parenthood, and gender income equality jointly. This finding is consistent with the hypothesis, underscoring the inherent conflicts among the three outcomes.

Then, I study the demise of patriarchy by extending the static model into a dynamic framework. To model the evolution of gender-specific human capital across generations, I incorporate a new fact established by the recent empirical literature: dual parenthood has differential impacts on the human capital of boys relative to girls.[2] This fact implies that changes in family structures have profound implications for future gender gaps in human capital, and hence marriage, fertility, and female labor supply decisions.

Based on the dynamic model, I show that the demise of patriarchy is driven by two channels. First, factor-neutral technological progress raises the opportunity cost of children and thus triggers declining fertility, falling marriage rates, and increasing female labor supply. Second, rising single parenthood and the narrowing of gender human capital gaps form a powerful dynamic feedback mechanism that propagates the impact of the first channel across generations. While the first channel applies uniformly across economies, the timing and magnitude of the second channel vary across countries due to differences in social norms. I illustrate this argument using the United Kingdom and Japan as examples.

Lastly, I discuss whether gender equality in childcare responsibilities could resolve the Impossible Trinity. I propose several arguments against that possibility.

[2] Quoting Wasserman (2020), "The evidence supports an emerging consensus that growing up in a family without biological married parents produces more adverse consequences for boys than for girls."

Related Literature

This paper is closely related to the literature on family economics and gender economics, especially the large body of papers that study historical changes in fertility, marriage, and gender gaps.[3] This paper makes four contributions.

First, while past papers propose distinct theories for each trend or study at most two trends at the same time (e.g., Galor and Weil 1996, Regalia and Rios-Rull 2001, Santos and Weiss 2016, Greenwood et al. 2016, Greenwood et al. 2023), I propose a simple but unified model that knits all three facts together and show that they can be driven by the same underlying force – the relentless rise in productivity.

Second, by taking a holistic approach, I propose and empirically test the Impossible Trinity hypothesis, a novel and central conjecture that links the scattered fields in the family economics literature. The hypothesis also points out an important boundary for policymakers: jointly achieving high fertility, dual parenthood, and gender income equality is unlikely to be feasible.

Third, I show that factor-neutral technological growth can simultaneously generate falling fertility, marriage, and gender income gaps. This mechanism complements existing theories that rely on factor-biased technological changes, such as the skill-biased technical change that favors child quality over child quantity (Galor and Weil 2000, Fernandez-Villaverde 2001), the household appliance revolution that favors single household over married ones (Greenwood et al. 2023), and structural transformation that favors the labor market prospects of women over men (Ngai and Petrongolo 2017, Cao et al. 2024).

Fourth, relative to the structural literature on demographic transition (e.g., Greenwood et al. 2023), I introduce a new mechanism that links marriage rates to gender gaps in human capital. While the differential effects of family structure on the outcomes of boys relative to girls are well documented in the empirical literature (e.g., see Bertrand and Pan 2013, Autor et al. 2019, Wasserman 2020, Reeves 2022, and Frimmel et al. 2024), this paper is the first to incorporate it into a dynamic macro model as a propagation

[3] Also see Greenwood et al. (2017) for an excellent review.

channel.

The rest of the paper is organized as follows: Section 2 presents the static model; Section 3 proposes and tests the Impossible Trinity hypothesis; Section 4 studies the demise of patriarchy in the dynamic model; Section 5 discusses whether equal sharing of childcare responsibilities could resolve the Impossible Trinity; and Section 6 concludes.

2. The Static Model

I first study a static economy. I keep the time subscript t so that the model can be readily extended to a dynamic setting in Section 4.

Individuals are indexed by gender $g \in \{♂, ♀\}$. For each gender, the utility from consumption c^g and fertility n is given by

$$u^g(c^g, n) = \left((1-\beta) \cdot (c^g)^{\frac{\rho-1}{\rho}} + \beta \cdot n^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} \tag{1}$$

where $\rho > 1$ following Jones and Schoonbroodt (2010) and Carlos Córdoba and Ripoll (2019) so that the utility for childless adults $u(c, 0)$ is well-defined.

Within each gender, I assume that individuals have the same amount of human capital within each generation denoted by $h_t^{♂}$ and $h_t^{♀}$ respectively. The gender gap in human capital at time t is defined as

$$\Gamma_t^h = \frac{h_t^{♂}}{h_t^{♀}} \tag{2}$$

Labor is the only productive factor in the economy. Therefore, $h_t^{♂}$ and $h_t^{♀}$ also determines wages and market income. I use A_t to denote total factor productivity (TFP) at time t. In the baseline analysis, I assume that A_t is exogenously given.[4]

[4]Allowing for endogenous A_t may lead to additional channels. For example, besides enhancing aggregate productivity A_t à la Hsieh et al. (2019), rising female labor supply could stimulate innovation and hence economic growth, i.e., $\dot{A}_t$.

2.1 Single Individuals

Single males consume their labor income but have no children. Their utility is given by

$$V_t^{\male,s} = u(A_t h_t^{\male}, 0) \tag{3}$$

where s in the superscript denotes "single."

Single females, on the other hand, can have children but do not receive any transfers or support from the absentee fathers. They choose consumption $c_t^{\female,s}$, fertility n_t^s, and labor supply n_t^s to solve

$$V_t^{\female,s} = \max_{c_t^{\female,s}, l_t^s, n_t^s} \quad u(c_t^{\female,s}, n_t^s) \tag{4}$$

subject to budget and time constraints

$$c_t^{\female,s} = A_t h_t^{\female} l_t^s, \quad \text{and} \quad l_t^s = 1 - \chi n_t^s$$

where χ is the time cost of raising each child. I follow the literature and assume that the fertility choice is continuous, i.e., $n_t^s \in \mathbb{R}_+$.

2.2 Married Individuals

I assume that once married, husbands supply one unit of labor inelastically and are required to transfer α_t share of their income to their wives. While individuals take α_t as given, it is an equilibrium object to be characterized in Section 2.4. Husbands derive utility from their remaining income and fertility – a public good shared with their wives. Therefore, the value of marriage for males is

$$V_t^{\male,m} = u(\underbrace{(1-\alpha_t) A_t h_t^{\male}}_{\text{remaining income}}, \underbrace{n_t^m}_{\text{fertility}}). \tag{5}$$

Because after transferring α_t, husbands do not directly bear the costs of children, they prefer as much fertility n_t^m as possible.[5]

Wives, on the other hand, need to balance fertility, consumption, and labor supply. Married women solves

$$V_t^{♀,m} = \max_{c_t^{♀,m}, l_t^m, n_t^m} \quad u(c_t^{♀,m}, n_t^m) \tag{6}$$

subject to budget and time constraints

$$c_t^{♀,m} = \underbrace{\alpha_t A_t h_t^{♂}}_{\text{transfer from husband}} + \underbrace{A_t h_t^{♀} l_t^m}_{\text{own labor income}}, \quad \text{and} \quad l_t^m = 1 - \chi n_t^m$$

where n_t^m and l_t^m are the fertility and labor supply of married women.[6] Motivated by Doepke and Kindermann (2019), childbirth is subject to veto. Therefore, wives are the key decision-makers regarding fertility within marriage.

2.3 Marriage Market

At the beginning of the period, I assume that each woman receives an idiosyncratic shock τ on the taste of marriage which follows a distribution $J(\tau)$. For a woman with taste shock τ, her utility from marriage becomes $\tau \cdot V_t^{♀,m}$. After receiving the shock, individuals decide whether or not to get married and the marriage market clears. The distribution $J(\tau)$ is a reduced-form way to capture other considerations of marriage that are not explicitly included in the model, such as mutual affection or risk-sharing.

For women, it is apparent that there exists a threshold τ_t^* above which they would prefer marriage over staying single. The value of τ_t^* can be defined using the condition

$$V_t^{♀,m} \cdot \tau^* = V_t^{♀,s}. \tag{7}$$

Therefore, imposing the marriage market clearing condition implies the equilibrium

[5] I discuss how the inclusion of childcare sharing across couples would change the results in Section 5.
[6] In Section 5, I discuss the possibilities where husbands share some of the childcare burden.

marriage rate

$$\mathcal{M}_t = 1 - J(\tau_t^*) \tag{8}$$

On the other hand, because males are homogeneous and are on the long side of the marriage market, the equilibrium imposes an indifference condition

$$V_t^{♂,m} = u((1-\alpha_t)A_t h_t^{♂}, n_t^m) = u(A_t h_t^{♂}, 0) = V_t^{♂,s} \tag{9}$$

where the share of transfers α_t acts as "prices" to clear the marriage market.

With marriage rate $\mathcal{M}_t$ defined, the model also gives expressions of other aggregate variables of interest. For example, aggregate fertility rate n_t is given by

$$n_t = \mathcal{M}_t \cdot n_t^m + (1 - \mathcal{M}_t) \cdot n_t^s \tag{10}$$

The share of children born with both parents, i.e., dual parenthood, is given by

$$\mathcal{D}_t = \frac{\mathcal{M}_t \cdot n_t^m}{n_t} \tag{11}$$

Average hours worked per female is

$$l_t^{♀} = \mathcal{M}_t \cdot l_t^m + (1 - \mathcal{M}_t) \cdot l_t^s = 1 - \chi n_t \tag{12}$$

The average labor income of females is

$$y_t^{♀} = A_t \cdot h_t^{♀} \cdot l_t^{♀}$$

which leads to a simple expression of the gender income gap

$$\Gamma_t^y = \frac{y_t^{♂}}{y_t^{♀}} = \frac{\Gamma_t^h}{l_t^{♀}} \tag{13}$$

2.4 Model Solution

In this section, I characterize the properties of the static model.

First, the indifference condition of males in the marriage market (9) implicitly defines α_t as a function of n_t^m:

$$(1-\beta)\cdot(A_t h_t^{\male})^{\frac{\rho-1}{\rho}}\left[1-(1-\alpha_t)^{\frac{\rho-1}{\rho}}\right]=\beta\cdot(n_t^m)^{\frac{\rho-1}{\rho}} \tag{14}$$

When $\rho>1$, using the implicit function theorem on Equation (14) reveals that the function $\alpha_t(n_t^m)$ is a strictly increasing and convex function. It takes the value of 0 when $n_t^m=0$, and shifts up when A_t rises.

On the other hand, the first-order condition of married women gives the optimality condition where n_t^m is a function of α_t:

$$n_t^m\cdot\left[\left(\frac{(1-\beta)A_t h_t^{\female}\chi}{\beta}\right)^{\rho}+A_t h_t^{\female}\chi\right]=(1+\alpha_t\Gamma_t^h)A_t h_t^{\female} \tag{15}$$

Equation (15) indicates that $n_t^m(\alpha_t)$ is an increasing and linear function. It takes a strictly positive value when $\alpha_t=0$ and shifts down when A_t rises.

Taking the properties of $\alpha_t(n_t^m)$ and $n_t^m(\alpha_t)$ together generates the first lemma.

Lemma 1: For given A_t, there is a unique fixed point of (α_t, n_t^m).

Proof: See Appendix.

Second, by comparing $V_t^{\female,s}$ and $V_t^{\female,m}$, Lemma 2 provides a condition for the marriage threshold τ_t^*.

Lemma 2: The marriage threshold $\tau_t^*=1/(1+\alpha_t\Gamma_t^h)$.

Proof: See Appendix.

Lemma 2 indicates that the marriage threshold, and hence the marriage rate $\mathcal{M}_t$, is determined by the economic gains from marriage from the women's perspective. The "transfer potential" of males is a product of the gender gap in human capital Γ_t^h and men's willingness to transfer α_t.

Together with Equation (26) in the Appendix, Lemma 2 also indicates that the fraction of dual parenthood $\mathcal{D}_t$, defined in (11), is monotonically increasing in the marriage rate $\mathcal{M}_t$. Therefore, I will use the $\mathcal{M}_t$ and $\mathcal{D}_t$ interchangeably when I analyze the Impossible Trinity in the next section.

3. The Impossible Trinity

In this section, I propose and empirically test the Impossible Trinity hypothesis.

3.1 Theory

Collecting the equilibrium conditions and results from Lemma 2, the relationship between fertility n_t, marriage $\mathcal{M}_t$, female labor supply $l_t^{♀}$, and gender income gap Γ_t^y can be summarized in the following three equations:

$$\mathcal{M}_t = 1 - J\left(\frac{1}{1+\alpha_t\Gamma_t^h}\right) \tag{16}$$

$$\Gamma_t^y = \frac{\Gamma_t^h}{l_t^{♀}} \tag{17}$$

$$l_t^{♀} = 1 - \chi n_t \tag{18}$$

Equations (16)-(18) illustrate the key tensions in the model. In particular, (16) shows that marriage rates are higher when there are larger gender gaps in human capital. But (17) implies that large gender gaps in human capital make it difficult to achieve gender income inequality unless the female labor supply is high. However, the opportunity cost of a high female labor supply is low fertility.

The Impossible Trinity hypothesis: high fertility, dual parenthood (or equivalently high marriage rate), and gender income inequality cannot coexist.

Proof: I prove the hypothesis in the context of the model by discussing three cases.

1. *High fertility and dual parenthood.* With high fertility n_t, female labor supply $l_t^{\female}$ is low from (18). To achieve a high marriage rate $\mathcal{M}_t$, gender human capital gap Γ_t^h cannot be too low from (16). Therefore, the gender income gap Γ_t^y is necessarily high from (17).

2. *High fertility and gender income equality.* With high fertility n_t, female labor supply $l_t^{\female}$ is low from (18). To achieve a low gender income gap Γ_t^y, it must be the case that Γ_t^h is very low from (17). But a very low gender gap in human capital Γ_t^h leads to a low marriage rate $\mathcal{M}_t$ from (16).

3. *Dual parenthood and gender income equality.* To achieve a high marriage rate $\mathcal{M}_t$, (16) implies that the gender gap in human capital Γ_t^h needs to be high. With high Γ_t^h, the only way to achieve a low gender income gap Γ_t^y is to have a high female labor supply $l_t^{\female}$. needs to be very high from (18). Therefore, fertility n_t is very low from (18).

The main takeaway from the proof is that although each of the three outcomes could be a desirable policy goal, it is difficult for policymakers to achieve them all due to the inherent incompatibility.

For example, consider family policies that change the cost of children χ (e.g., baby bonuses and child tax credits). If the policymaker raises χ, then it can achieve more gender equality because the female labor supply rises because $n \cdot \chi$ falls. But this comes at a cost of lower fertility. On the other hand, if the policymaker lowers χ, then fertility is higher, but the female labor supply falls and hence gender income gap widens.

Another example is policies that change the marriage decision such as tax benefits for married couples. In the model, these policies manifest themselves as shifts in $J(\cdot)$. While such policies do not directly involve the balancing between n_t and Γ_t^y, there is yet another tension between marriage rates $\mathcal{M}$ and gender human capital gaps Γ^h from the human capital formation side – I will introduce this tension in Section 4.

Lastly, it is worth pointing out that countries may have only one, or even none of the three outcomes in reality. To see this, I test the Impossible Trinity hypothesis using historical data in the next section.

3.2 Empirical Results

I collect data on (1) total fertility rates (TFR) from the United Nations, (2) the share of children born outside of marriage from the OECD database, and (3) gender gaps in median earnings from the OECD database. The resulting dataset is an unbalanced panel of 37 countries from 1970 to 2014 with 721 country-year observations in total.

I categorize observations based on sample averages of each variable.[7] Observations are labeled as

- "high fertility" if $\text{TFR}_{it} > 1.69$,
- "dual parenthood" if out of marriage$_{it} < 31.4\%$, and
- "gender income equality" if gap$_{it} < 17.2\%$.

After labeling each observation, I plot the Venn diagram to inspect the intersections. The results are shown in Figure 1. I find that less than 2% of the observations achieved high fertility, dual parenthood, and gender income equality jointly while there are large shares of observations in all other categories. This finding supports the Impossible Trinity hypothesis, highlighting the inherent conflicts among the three outcomes.

Table 1 gives some examples for each area of the Venn diagram. The only country that achieves high fertility, dual parenthood, and gender income equality according to our definition is Australia between 1991 and 2003. After 2003, the share of single parenthood rose sharply in Australia so it lost the "dual parenthood" status.

[7] I have also experimented with alternative cutoffs in defining "high fertility" (e.g., $\text{TFR}_{it} > 2$), "dual parenthood", and "gender income equality" categories. The main finding remains robust.

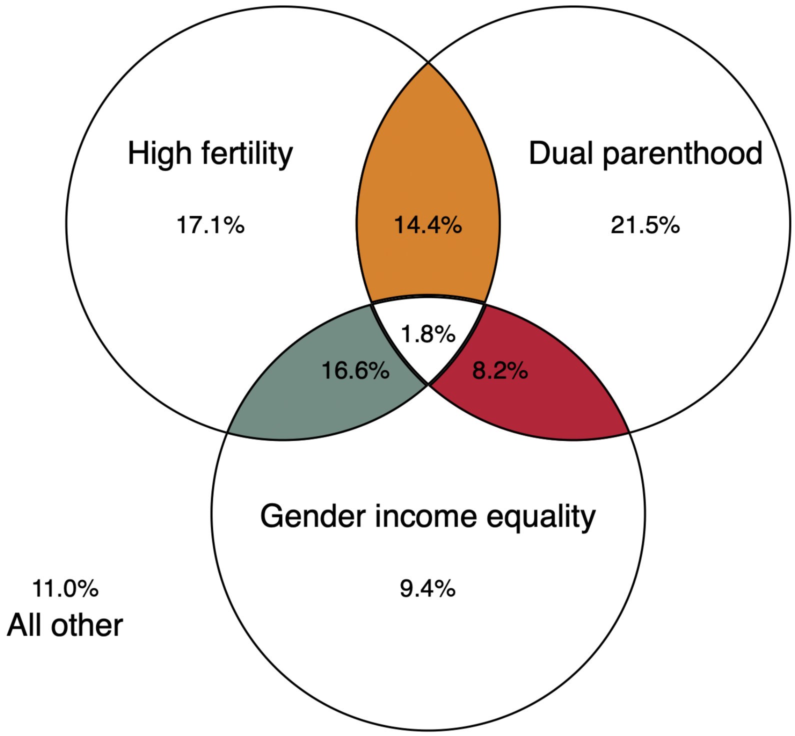

Figure 1: The Impossible Trinity in the Data

D – dual parenthood, G – gender income equality, F – high fertility	
Category	Countries
None	Austria, United Kingdom 1995-2003
Only D	Canada, Switzerland, Germany 1992-2006, Japan, South Korea
Only G	Germany 2009-2014, Hungary, Portugal
Only F	United States 1994-2013, Finland
$D+G$	Greece, Italy, Poland
$G+F$	Belgium, Norway, New Zealand, Sweden
$D+F$	United Kingdom 1970-1994, Israel, USA 1973-1993
$D+G+F$	Australia 1991-2003 ($G+F$ afterwards)

Table 1: Examples of Countries

4. The Autumn of Patriarchy

This section studies the transition from patriarchal societies to egalitarian societies in a dynamic model.

4.1 Human Capital Dynamics

I assume that the gender-specific human capital follows the law of motion[8] specified as

$$h_{t+1}^{\female} = (h_t^{\female})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{\female}} \tag{19}$$

$$h_{t+1}^{\male} = Z \cdot (h_t^{\male})^{\theta} \cdot (\mathcal{M}_t)^{\psi^{\male}} \tag{20}$$

where $Z > 1, \theta \in (0,1)$ and more importantly, $\psi^{\male} > \psi^{\female} > 0$.

The production functions (19) and (20) are motivated by a large empirical literature that has documented that growing up in a family without biological married parents leads to more adverse consequences for boys than for girls (e.g., see Bertrand and Pan 2013, Autor et al. 2019, Wasserman 2020, Reeves 2022, and Frimmel et al. 2024).

The difference between between $\psi^{\male}$ and $\psi^{\female}$ is economically sizable. For example, Autor et al. (2019) show that the racial differences in the ratio of single motherhood could explain the bulk of the black-white differences in gender gaps. Autor et al. (2023) find that a substantial fraction of the gender gap in high school outcomes can potentially be explained by the differential effect of family socioeconomic status, in particular family structure, on boys' medium-run outcomes.

Under the assumption that these empirical findings on differential sensitivity apply to economies generally, the prevailing marriage rates determine gender gaps in human capital in the next generation and hence the evolution of Γ^h. To see this, note that divid-

[8] I adopt Galton's approach to the intergenerational transmission of human capital for analytical and aggregation simplicity. As pointed out by Mulligan (1999), explicit modeling of parental human capital investment decisions, e.g., following Becker and Tomes (1979), often yields similar predictions.

ing (19) by (20) yields

$$\Gamma^h_{t+1} = Z \cdot (\Gamma^h_t)^{\theta} \cdot (\mathcal{M}_t)^{\psi^{♂} - \psi^{♀}}$$

which implies in steady-state

$$\Gamma^h = Z^{\frac{1}{1-\theta}} \cdot (\mathcal{M})^{\frac{\psi^{♂} - \psi^{♀}}{1-\theta}} \Longrightarrow \frac{d\Gamma^h}{d\mathcal{M}} > 0 \tag{21}$$

Therefore, higher marriage rates generates larger gender human capital gaps.

4.2 Mechanism

The following lemma presents the first channel that results in the demise of patriarchy.

Lemma 3: The levels of α_t and n^m_t are decreasing in A_t.

Proof: See Appendix.

The intuition behind Lemma 3 is simple: because consumption and fertility are substitutes in the utility function, a higher total factor productivity A raises the opportunity costs of having children and the substitution effect dominates the income effect. Therefore, n^m_t is decreasing in A. Because the amount of transfers males are willing to pay their wives depends positively on marital fertility n^m, transfer share α also falls as A rises.

The second channel that leads to the demise of patriarchy is a chain reaction between single parenthood and gender human capital gaps presented in the bottom-left part of Figure 2. When α falls, there is a decline in the economic gains from marriage for women ($\alpha\Gamma^h$). As a result, the marriage rate $\mathcal{M}$ drops. Because the decline in marriage hurts boys relatively more than girls, the gender gap in human capital Γ^h falls in the next generation, further dragging down the economic gains from marriage. The second channel propagates the effects of rising A_t over time, generating dynamic falls in marriage rates and human capital gaps.

More rigorously, the impact of the second channel is given by Lemma 4.

Lemma 4: Declining α_t reduces long-run $\mathcal{M}$ and Γ^h.

Proof: See Appendix.

Taking the two channels together, the rising female labor supply (due to falling fertility) and the shrinking gender gap in human capital (due to falling marriage) generate a converging gender income gap Γ^y.

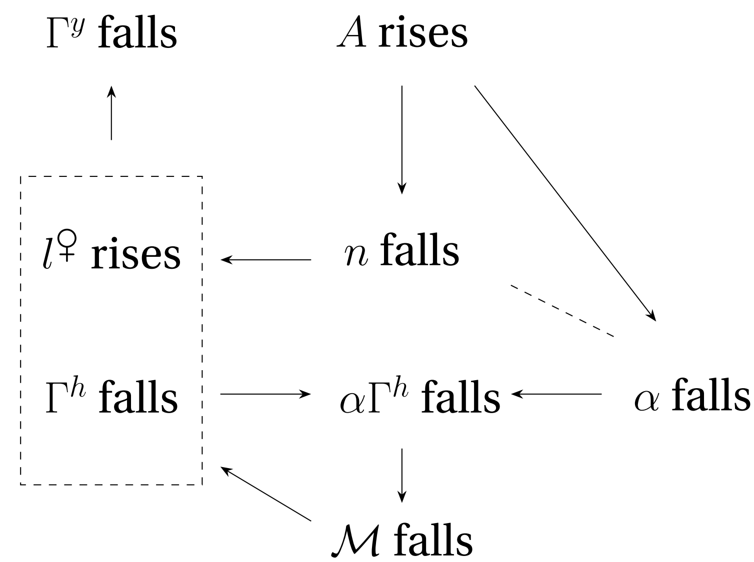

Figure 2: The Autopsy of Patriarchy

The upshot of Figure 2 is that an exogenous increase in A_t can generate an inevitable transition from patriarchal to egalitarian societies. Importantly, one does not need factor-biased technological changes to generate declines in fertility, dual parenthood, or gender income gaps.

4.3 The Role of Social Norms

Another message from Figure 2 is that while the effects of A_t on n_t and α_t are the same across countries, the final impacts on marriage and gender income gaps could be different across countries depending on the strength of the second channel.

To be more specific, the mapping from the "transfer potential" $\alpha\Gamma^h$ to marriage rates $\mathcal{M}$ depends on the distribution of idiosyncratic shocks $J(\tau)$. This distribution could vary across countries due to factors such as culture, religion, and social norms. Depending on the mass of individuals around the cutoff τ^*, responses in the marriage rate $\mathcal{M}$ could be either large or small. As a result, the timing and magnitude of the feedback mechanism between $\mathcal{M}$ and Γ^h could vary dramatically across countries.

To give some concrete examples, Figure 3 displays the case for the United Kingdom. As its fertility fell after the Baby Boom, single parenthood surged after the 1980s. Through the lens of the model, rising female labor supply and converging gender human capital gaps jointly contributed to the converging gender income gaps.

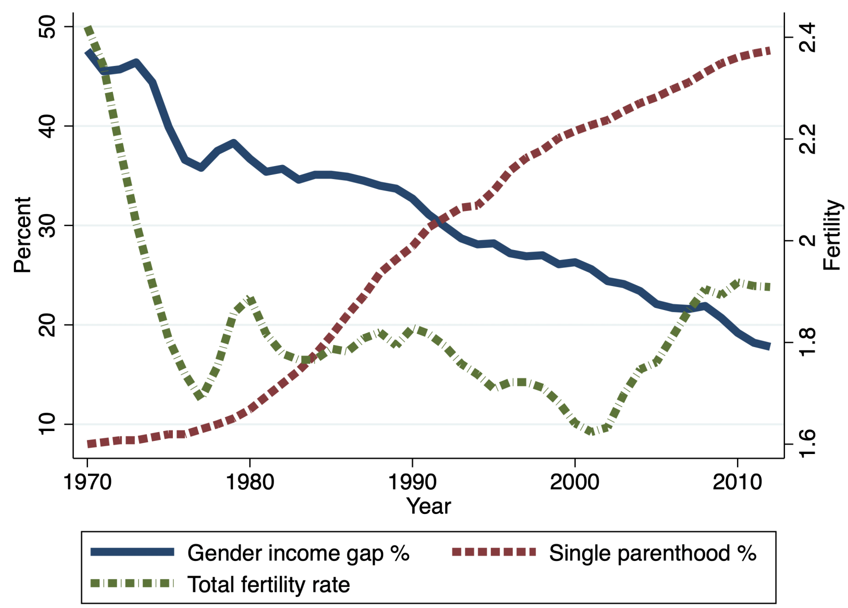

Figure 3: The Case of the U.K.

In contrast, Figure 4 displays the case of Japan. While fertility also fell during the rapid economic growth era in the 1980s, single parenthood barely rose, owing to the strong influence of the Confucian tradition that stigmatizes out-of-marriage births (Myong et al. 2021). Through the lens of the model, only the rising female labor supply contributed to the converging gender income gaps. As a result, the speed of gender gap convergence in Japan is much slower than that in the United Kingdom. Such differences can be attributed, at least partly, to the heterogeneous $J(\tau)$ distribution between Japan and the United Kingdom.

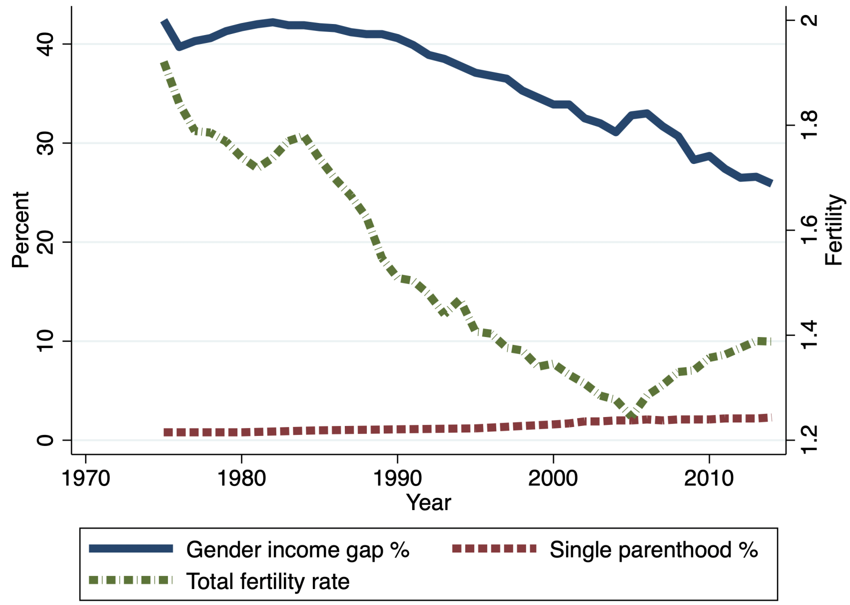

Figure 4: The Case of Japan

5. Discussions

An interesting and challenging question is whether gender equality in childcare responsibilities, which has been studied by many recent papers such as Doepke and Kindermann (2019), could resolve the Impossible Trinity. In particular, if both men and women participate in childcare, could countries achieve high fertility while preserving dual parenthood and gender income equality?

Through the lens of the model, if both genders share the same childcare burden, then the labor supply is the same across genders. As a result, the gender income gap Γ^y entirely depends on the gender human capital gap Γ^h. But with high marriage rates $\mathcal{M}$, the gender human capital gap Γ^h is also high due to the differential sensitivity assumption $\psi^{♂} > \psi^{♀}$. Therefore, to achieve both dual parenthood and gender income equality, men need to take *more* childcare responsibilities than women. This requirement, however, has three potential issues.

First, how large would the efficiency cost be for men to work less than women when their human capital is relatively higher? The efficiency cost could be even larger if women have an absolute advantage in childcare.

Second, because men have the outside option of staying single and having no children, the amount of transfer α needs to be very low for them to agree to take on more childcare responsibilities within marriage. But when α, and hence the economic gains from marriage, is small, more women would prefer to stay single, making high marriage rates an unlikely outcome.

Lastly, from an empirical point of view, even though there has been a lot of progress towards an equal sharing of childcare responsibilities, especially in many European countries, Figure 1 indicates that there hasn't been much evidence supporting it as a way out from the Impossible Trinity.

Due to the above-mentioned reasons, I argue that it is unlikely for gender equality in childcare responsibilities to resolve the Impossible Trinity.

6. Conclusion

Human society is undergoing an unprecedented transition in which patriarchy is withering away. Understanding the cause of the transition is a central question in economics, sociology, and anthropology.

In this paper, I present a unified framework about the joint decline of fertility, dual parenthood, and gender income gaps in this epoch.

The model offers three main insights. First, high fertility, dual parenthood, and gender income equality cannot coexist – a novel Impossible Trinity hypothesis in family economics. I also show that the hypothesis is supported by the data. Second, rising total factor productivity is sufficient to cause the demise of patriarchy – one does not need to assume factor-biased technological changes. Lastly, while the demise of patriarchy is inevitable, the pace of the transition could differ across countries due to social norms.

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A. Proofs

Proof of Lemma 1
Define function

$$f_1(\alpha_t) = A_t h_t^{\male} \cdot \left(\frac{1-\beta}{\beta} \cdot [1 - (1-\alpha_t)^{\frac{\rho-1}{\rho}}] \right)^{\frac{\rho}{\rho-1}}, \qquad \alpha \in [0,1]$$

For $\rho > 1$, $f_1(\alpha_t)$ is strictly increasing, convex, and $f_1(0) = 0$. Moreover, $n_t^m = f_1(\alpha_t)$ satisfies men's indifference condition (9).

Define function

$$f_2(\alpha_t) = \frac{(1 + \alpha_t \Gamma_t^h) A_t h_t^{\female}}{\left(\frac{(1-\beta) A_t h_t^{\female} \chi}{\beta} \right)^{\rho} + A_t h_t^{\female} \chi}, \qquad \alpha_t \in [0,1]$$

For $\rho > 1$, $f_2(\alpha_t)$ is strictly increasing, linear, and $f_2(0) > 0$. Moreover, $n_t^m = f_2(\alpha_t)$ satisfies women's optimality condition (15).

Thus, $f_3(\alpha_t) = f_1(\alpha_t) - f_2(\alpha_2)$ is strictly increasing, convex, and $f_3(0) < 0$. Therefore, there are two possibilities. If $f_3(\alpha_t)$ obtains the value of zero in the domain $\alpha \in [0,1]$, i.e., interior solution, then this solution is unique. Otherwise, there is a corner solution $\alpha_t = 1$, i.e., men strictly prefer marriage over being single and are willing to transfer the entirety of their income – a theoretically possible but empirically irrelevant case.

Figure A.1 provides a graphical illustration of the proof.

Proof of Lemma 2
For married women, the first-order condition is

$$(1-\beta) \cdot (c_t^{\female,m})^{-\frac{1}{\rho}} = \frac{\beta \cdot (n_t^m)^{-\frac{1}{\rho}}}{A_t h_t^{\female} \chi} \Longrightarrow c_t^{\female,m} = n_t^m \cdot \left(\frac{(1-\beta) A_t h_t^{\female} \chi}{\beta} \right)^{\rho} \tag{22}$$

Substituting (22) into the budget constraint, n_t^m satisfies

$$n_t^m \cdot \left(\frac{(1-\beta) A_t h_t^{\female} \chi}{\beta} \right)^{\rho} = \alpha_t \Gamma_t^h A_t h_t^{\female} + A_t h_t^{\female} (1 - \chi n_t^m)$$

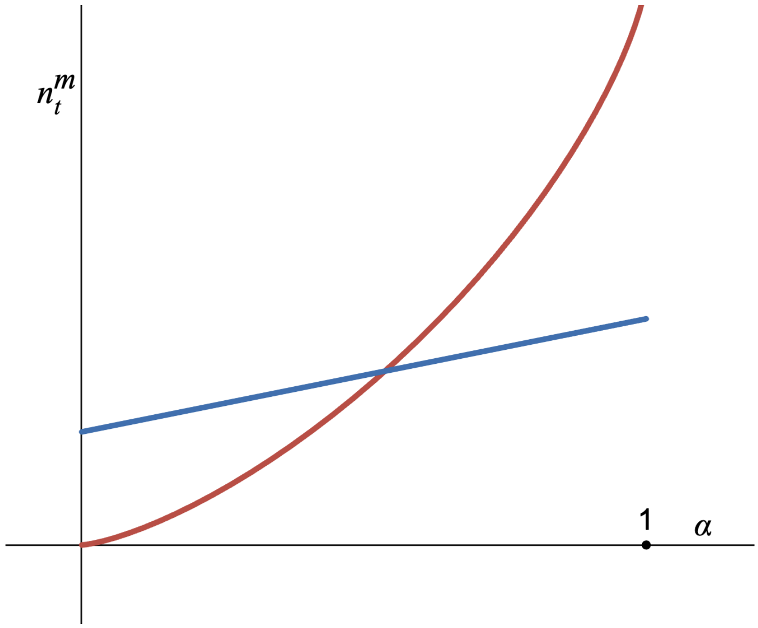

Figure A.1: $n_t^m(\alpha_t)$ (blue) and $\alpha_t(n_t^m)$ (red)

which is equivalent to

$$n_t^m \cdot \left[\left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} + A_t h_t^{♀} \chi \right] = (1 + \alpha_t \Gamma_t^h) A_t h_t^{♀} \tag{23}$$

For single women, the first-order condition is

$$(1-\beta) \cdot (c_t^{♀,s})^{-\frac{1}{\rho}} = \frac{\beta \cdot (n_t^s)^{-\frac{1}{\rho}}}{A_t h_t^{♀} \chi} \Longrightarrow c_t^{♀,s} = n_t^s \cdot \left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} \tag{24}$$

Substituting (24) into the budget constraint, $c_t^{♀,s}$ satisfies

$$n_t^s \cdot \left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} = A_t h_t^{♀} (1 - \chi n_t^s)$$

which is equivalent to

$$n_t^s \cdot \left[\left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho} + A_t h_t^{♀} \chi \right] = A_t h_t^{♀} \tag{25}$$

Take the ratio between (23) and (25) gives

$$\frac{n_t^m}{n_t^s} = 1 + \alpha_t \Gamma_t^h \tag{26}$$

which is independent of A_t.

On the other hand,

$$V_t^{♀,m}(\tau) = \tau \cdot n_t^m \cdot \left((1-\beta) \cdot \left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho-1} + \beta \right)^{\frac{\rho}{\rho-1}} \tag{27}$$

$$V_t^{♀,s} = n_t^s \cdot \left((1-\beta) \cdot \left(\frac{(1-\beta) A_t h_t^{♀} \chi}{\beta} \right)^{\rho-1} + \beta \right)^{\frac{\rho}{\rho-1}} \tag{28}$$

Combining (27), (28), and (26),

$$\tau^* = \frac{V_t^{♀,s}}{V_t^{♀,m}} = \frac{n_t^s}{n_t^m} = \frac{1}{1 + \alpha_t \Gamma_t^h} \tag{29}$$

Proof of Lemma 3

When A_t increases, $f_1(\alpha_t)$ shifts up while $f_2(\alpha_t)$ shifts down. Therefore, $f_3(\alpha_t) = f_1(\alpha_t) - f_2(\alpha_2)$ shifts up. As a result, the interior solution, i.e., the value of α_t such that $f_3(\alpha_t) = 0$, necessarily decreases.

Figure A.2 provides a graphical illustration of the proof.

Proof of Lemma 4

When α_t falls, $\mathcal{M}(\Gamma^h; \alpha)$ shifts down while $\Gamma^h(\mathcal{M})$ is unaffected. Figure A.3 provides a graphical illustration of the proof.

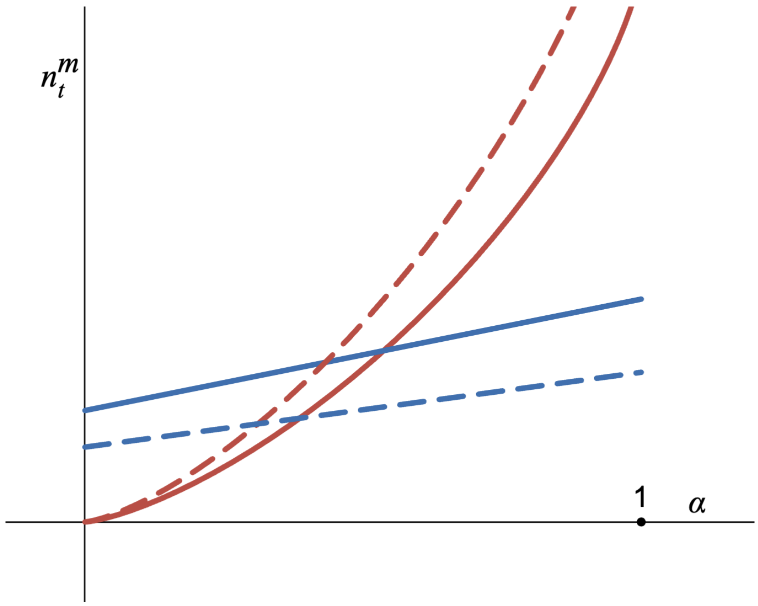

Figure A.2: $n_t^m(\alpha_t)$ (blue) and $\alpha_t(n_t^m)$ (red). Solid (before) and dashed (after)

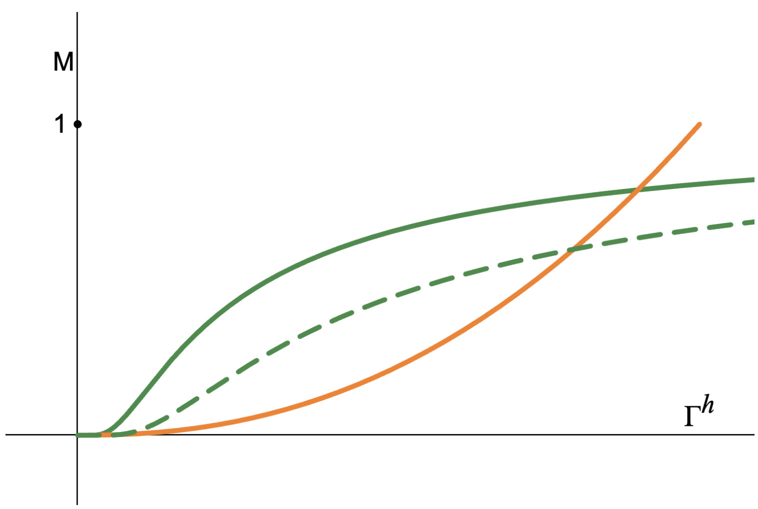

Figure A.3: $\mathcal{M}(\Gamma^h;\alpha)$ (green) and $\Gamma^h(\mathcal{M})$ (orange)

B. Figures

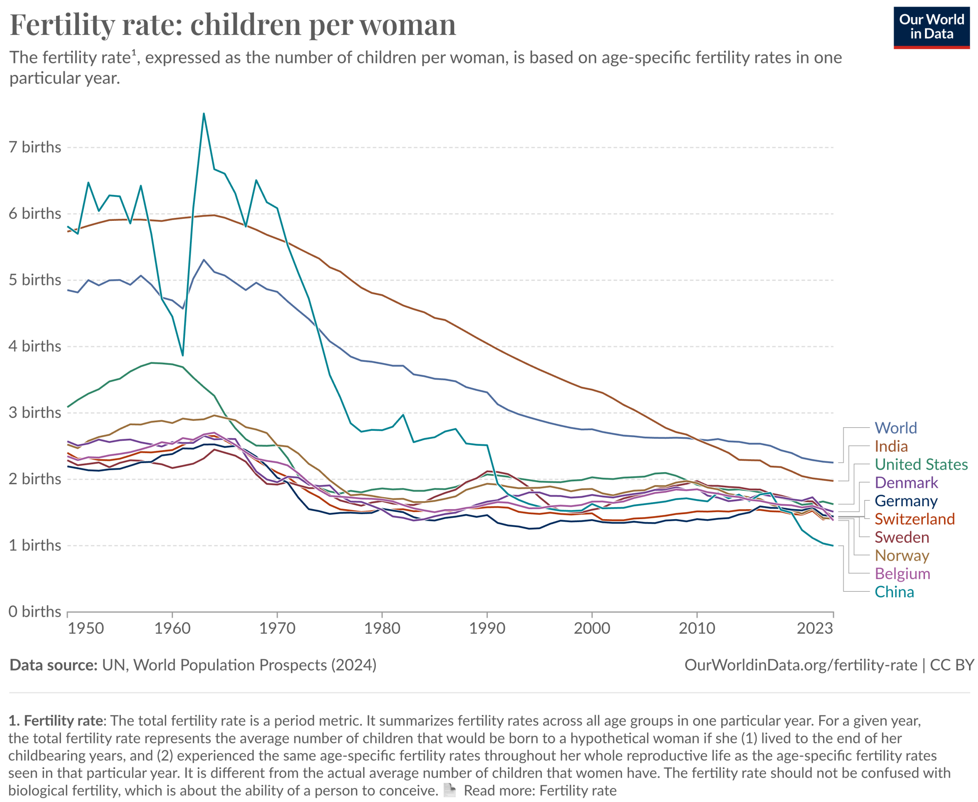

Figure A.4: Declining Fertility

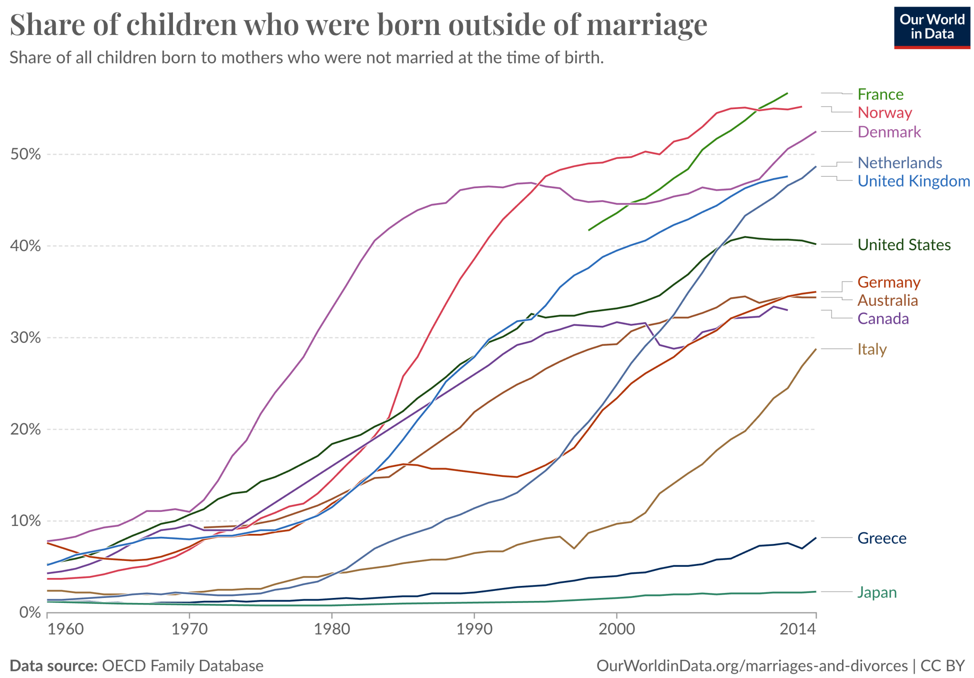

Figure A.5: Rising Single Parenthood

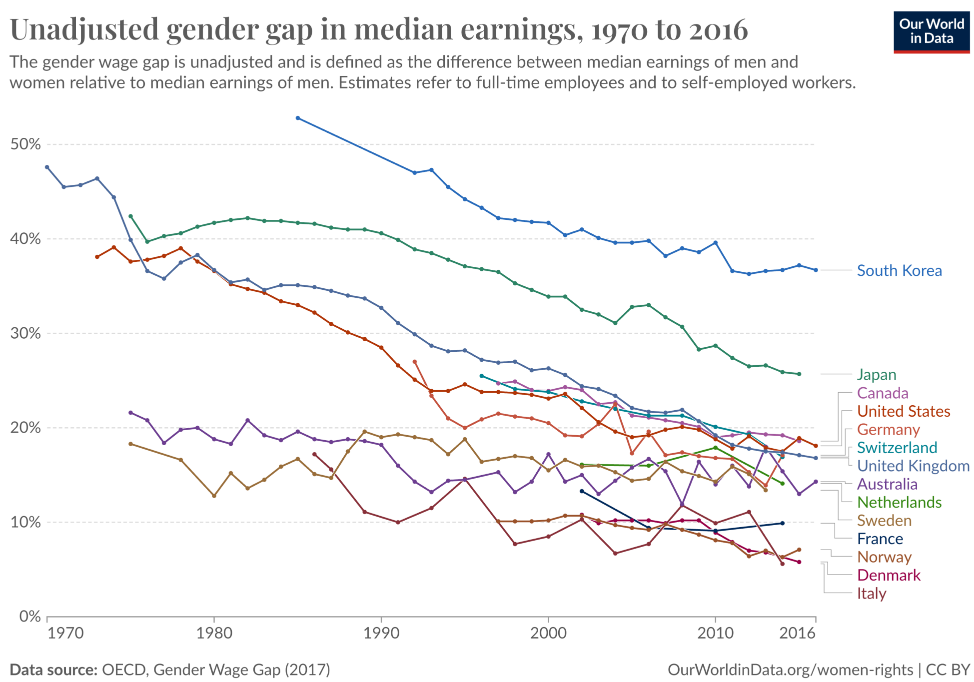

Figure A.6: Converging Gender Gaps