MODULE 2 FUNDAMENTALS OF COUNTING THEORY

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The Rule of Sum

If a first task can be performed in m ways, while a second task can be performed in n ways, and the two tasks cannot be performed simultaneously, then performing either task can be accomplished in any one of m+n ways.

Ex. A college library has 40 textbooks on Sociology and 50 textbooks dealing with Anthropology. By the rule of sum, a student at this college can select among 40+50 = 90 textbooks in order to learn more about one or the other of these two subjects.

Extension of Sum Rule

Extension of Sum Rule: If tasks $T1, T2, \cdots\cdots, Tm$ can be done in $n1, n2, n3, \cdots\cdots, nm$ ways respectively and no two of these tasks can be performed at the same time , then the number of ways to do one of these tasks is $n1 + n2 + \cdots + nm$.

Ex. If a student can choose a project either 20 from Mathematics or 35 from Computer Science or 15 from Engineering, then the student can choose a project in

20 + 35+15= 75 ways.

The Rule of Product

If a procedure can be broken down into first and second stages, and if there are m possible outcomes for the first stage and if, for each of these outcomes, there are n possible outcomes for the second stage, then the total procedure can be carried out, in the designated order, in mn ways.

Ex. The drama club of central University isis holding tryouts for a spring play. With 6 men and 8 women auditioning for the leading male and female roles. By the rule of product, the director can cast his leading couple in 6* 8= 48 ways

Extension of Product Rule

Extension of Product Rule: Suppose a procedure consists of performing tasks T1, T2,……, Tm in that order. Suppose task Ti can be performed in ni ways after the tasks T1, T2,……, Ti-1 are performed, then the number of ways the procedure can be executed in the designated order is n1, n2,……, nm.

Ex.Charmas brand shirt available in 12 colors, has a male and female version. It comes in four sizes for each sex, comes in three makes of economy, standard and luxury. Then the number of different types of shirts produced are 12*2*4*3=288 ways

PERMUTATIONS

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Def: Given a collection of n distinct objects, any (linear) arrangement of these objects is called a permutation of the collection. If there are n distinct objects and r is an integer, with $1 \le r \le n$, then by the rule of product, the number of permutations of size r for the n objects are

$$P(n, r) = n \times (n-1) \times (n-2) \times \cdots\cdots .. \times (n-r+1) = n!/\,(n-r)!$$

Ex. How many arrangements are there of all the letters in COMPUTER be arranged?

Sol: 8!

Ex. How many arrangements are there of all the letters in DATABASES be arranged?

Sol: 9!/ (2! 3!) = 30, 240.

Ex.(a) In how many ways can seven people be arranged about a circular table?

(7-1)!

(b) If two of the people insist on sitting next to each other, how many arrangements are possible?

H &W+5=6 members= (6- 1)!*2!=5! 2!

COMBINATIONS

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If we start with n distinct objects, each selection, or combination, of r of these objects, with no reference of order, corresponds to r! permutations of size r from the n objects. Thus the number of combinations of size r from a collection of size n is **$C(n, r) = P(n, r)/ r!= n!/ [r! (n-r)!]; 0 \le r \le n$.**

Ex. A student taking a history examination is directed to answer any seven of 10 essay questions. Student can answer the examination in $C(10, 7)= 120$ ways

COMB

Ex.A student taking a history examination is directed to answer three questions from the first five and four questions from the last five. Student can complete the examination in C(5, 3) * C(5, 4)= 50 ways. Ex. A student taking a history examination is directed to answer 7 of the ten questions where atleast three are selected from the first five. Then there are three cases (i) The student answers three of the first five questions and four of the last five questions, C(5,3) C(5,4)= 50 ways. (ii) C(5, 4) C(5,3)= 50 ways (iii) C(5,5) C(5, 2)= 10 ways Hence student can complete the examinations in 50+50+10 ways= 110 ways.