**Practical Introduction to Time-Frequency Analysis Using the Continuous Wavelet Transform**

**Summary**

This document provides a comprehensive and practical introduction to analyzing non-stationary signals using the **Continuous Wavelet Transform (CWT)**. Time-frequency analysis is essential for understanding signals whose frequency characteristics change over time, such as biomedical signals, communications, or geophysical data.

**1. Continuous vs. Discrete Wavelet Transform**

* **Continuous Wavelet Transform (CWT)** offers a finer frequency resolution than the Discrete Wavelet Transform (DWT).
* CWT uses multiple wavelets per octave and has a shift resolution of one sample, offering better time localization.
* **Complex-valued wavelets** in CWT also provide phase information.

**2. Filter Bank and Voices Per Octave**

* A **CWT filter bank** consists of several filters per octave (e.g., 8 or 10 voices).
* These filters maintain the **constant-Q property**, meaning bandwidth is proportional to center frequency.
* As a result, higher frequency wavelets are shorter in duration, while lower frequency wavelets are longer.

**3. Logarithmic Frequency Scaling**

* CWT filters are **logarithmically spaced** rather than linearly.
* This makes wavelets more appropriate than fixed-bandwidth filters (like in the STFT) for signals with both fast and slow variations.

**4. Application Examples**

* **Hyperbolic chirp**: CWT accurately tracks time-varying frequencies better than spectrogram.
* **ECG signal**: Transients and steady-state oscillations are localized effectively.
* **Solar magnetic field**: CWT identifies both sharp impulses and low-frequency waves.

**5. Advantages of CWT**

* **Multiresolution analysis**: High temporal resolution at high frequencies, high frequency resolution at low frequencies.
* **Transient localization**: Able to detect sharp changes, defects, and impulse events.
* **Synchrosqueezing**: Sharpens CWT by reassigning energy more accurately in frequency.

**6. Wavelet Coherence**

* Analyzes time-varying correlations between two signals.
* Useful for detecting phase shifts and relationships (e.g., El Nino and Indian rainfall data).

**7. MATLAB Tools**

* Functions like cwt, cwtfilterbank, wsst, wcoherence are used to compute and visualize wavelet transforms.
* Useful plots include **scalograms**, **wavelet coherence maps**, and **synchrosqueezed ridges**.

**Conclusion**

CWT is a powerful method for time-frequency analysis, particularly well-suited to non-stationary signals. Its variable resolution and adaptability make it superior to traditional methods in many applications. MATLAB’s Wavelet Toolbox provides a robust environment for applying CWT in real-world signal analysis tasks.