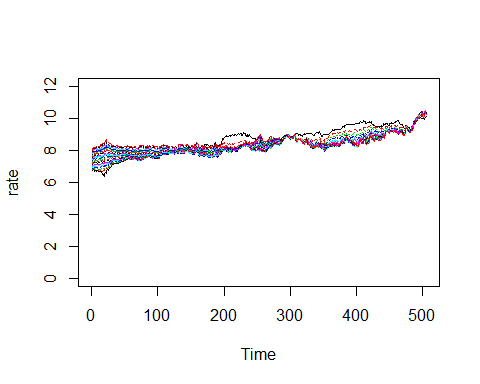
#excel with Maturity from 31.1.2018 till 01.02.2016  
  
library(readxl)  
yields <- read\_excel("C:/Users/User/Desktop/data.xlsx")

matplot(yields, type='l', ylim = c(0,12),ylab = 'rate', xlab = 'Time') #yield curve



returns = data.frame(diff(as.matrix(yields)))\*(-1) #daily zero-coupons increments  
head(returns)

## X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13  
## 1 0.01 -0.01 -0.02 -0.03 -0.05 -0.05 -0.06 -0.06 -0.08 -0.08 -0.08 -0.08 -0.08  
## 2 -0.06 -0.08 -0.07 -0.07 -0.06 -0.07 -0.07 -0.07 -0.06 -0.06 -0.06 -0.06 -0.06  
## 3 0.06 0.03 0.00 0.00 -0.01 0.01 0.02 0.02 0.01 0.01 0.00 0.00 0.00  
## 4 0.03 0.06 0.06 0.05 0.04 0.02 0.00 -0.01 0.00 -0.01 -0.01 -0.01 -0.01  
## 5 -0.01 -0.01 0.00 0.01 0.02 0.03 0.04 0.04 0.03 0.03 0.03 0.03 0.03  
## 6 0.01 0.00 -0.01 -0.02 -0.02 -0.02 -0.02 -0.02 -0.02 -0.02 -0.02 -0.03 -0.03  
## X14 X15 X16 X17 X18 X19 X20  
## 1 -0.08 -0.08 -0.09 -0.08 -0.09 -0.08 -0.09  
## 2 -0.06 -0.06 -0.06 -0.06 -0.06 -0.06 -0.06  
## 3 0.00 -0.01 -0.01 -0.01 -0.01 -0.01 -0.01  
## 4 -0.01 -0.01 -0.01 -0.02 -0.02 -0.03 -0.03  
## 5 0.03 0.03 0.03 0.03 0.04 0.04 0.04  
## 6 -0.04 -0.04 -0.04 -0.04 -0.05 -0.05 -0.05

Сorrelation

options(digits = 2)  
cor(returns)

## X1 X2 X3 X4 X5 X6 X7 X8 X9 X10 X11 X12 X13 X14 X15  
## X1 1.00 0.66 0.46 0.42 0.35 0.34 0.32 0.32 0.32 0.31 0.31 0.30 0.29 0.28 0.26  
## X2 0.66 1.00 0.91 0.82 0.71 0.66 0.61 0.59 0.58 0.56 0.54 0.53 0.51 0.50 0.48  
## X3 0.46 0.91 1.00 0.97 0.90 0.85 0.78 0.76 0.74 0.70 0.68 0.66 0.64 0.61 0.58  
## X4 0.42 0.82 0.97 1.00 0.97 0.94 0.90 0.87 0.85 0.81 0.79 0.76 0.74 0.70 0.67  
## X5 0.35 0.71 0.90 0.97 1.00 0.99 0.96 0.94 0.91 0.88 0.85 0.82 0.79 0.76 0.72  
## X6 0.34 0.66 0.85 0.94 0.99 1.00 0.99 0.97 0.96 0.92 0.90 0.88 0.85 0.81 0.77  
## X7 0.32 0.61 0.78 0.90 0.96 0.99 1.00 0.99 0.98 0.95 0.93 0.91 0.88 0.85 0.81  
## X8 0.32 0.59 0.76 0.87 0.94 0.97 0.99 1.00 0.99 0.98 0.96 0.94 0.92 0.89 0.86  
## X9 0.32 0.58 0.74 0.85 0.91 0.96 0.98 0.99 1.00 0.99 0.98 0.97 0.95 0.92 0.90  
## X10 0.31 0.56 0.70 0.81 0.88 0.92 0.95 0.98 0.99 1.00 0.99 0.99 0.97 0.95 0.93  
## X11 0.31 0.54 0.68 0.79 0.85 0.90 0.93 0.96 0.98 0.99 1.00 0.99 0.98 0.97 0.95  
## X12 0.30 0.53 0.66 0.76 0.82 0.88 0.91 0.94 0.97 0.99 0.99 1.00 0.99 0.98 0.97  
## X13 0.29 0.51 0.64 0.74 0.79 0.85 0.88 0.92 0.95 0.97 0.98 0.99 1.00 0.99 0.99  
## X14 0.28 0.50 0.61 0.70 0.76 0.81 0.85 0.89 0.92 0.95 0.97 0.98 0.99 1.00 1.00  
## X15 0.26 0.48 0.58 0.67 0.72 0.77 0.81 0.86 0.90 0.93 0.95 0.97 0.99 1.00 1.00  
## X16 0.25 0.47 0.57 0.65 0.70 0.75 0.79 0.84 0.88 0.91 0.94 0.96 0.98 0.99 1.00  
## X17 0.24 0.44 0.54 0.62 0.67 0.73 0.76 0.81 0.85 0.89 0.92 0.95 0.97 0.98 0.99  
## X18 0.23 0.43 0.52 0.60 0.65 0.70 0.73 0.78 0.82 0.86 0.90 0.93 0.95 0.97 0.98  
## X19 0.21 0.41 0.50 0.57 0.62 0.67 0.70 0.75 0.80 0.84 0.88 0.91 0.93 0.96 0.97  
## X20 0.20 0.39 0.48 0.55 0.60 0.64 0.68 0.73 0.78 0.82 0.86 0.89 0.92 0.94 0.96  
## X16 X17 X18 X19 X20  
## X1 0.25 0.24 0.23 0.21 0.20  
## X2 0.47 0.44 0.43 0.41 0.39  
## X3 0.57 0.54 0.52 0.50 0.48  
## X4 0.65 0.62 0.60 0.57 0.55  
## X5 0.70 0.67 0.65 0.62 0.60  
## X6 0.75 0.73 0.70 0.67 0.64  
## X7 0.79 0.76 0.73 0.70 0.68  
## X8 0.84 0.81 0.78 0.75 0.73  
## X9 0.88 0.85 0.82 0.80 0.78  
## X10 0.91 0.89 0.86 0.84 0.82  
## X11 0.94 0.92 0.90 0.88 0.86  
## X12 0.96 0.95 0.93 0.91 0.89  
## X13 0.98 0.97 0.95 0.93 0.92  
## X14 0.99 0.98 0.97 0.96 0.94  
## X15 1.00 0.99 0.98 0.97 0.96  
## X16 1.00 0.99 0.99 0.98 0.97  
## X17 0.99 1.00 0.99 0.99 0.98  
## X18 0.99 0.99 1.00 1.00 0.99  
## X19 0.98 0.99 1.00 1.00 1.00  
## X20 0.97 0.98 0.99 1.00 1.00

Сovariance

options(digits = 2)  
cov(returns)

## X1 X2 X3 X4 X5 X6 X7 X8 X9 X10  
## X1 0.00305 0.0018 0.0013 0.0012 0.0011 0.0011 0.0011 0.0011 0.0011 0.0010  
## X2 0.00176 0.0024 0.0023 0.0021 0.0020 0.0019 0.0018 0.0017 0.0017 0.0016  
## X3 0.00130 0.0023 0.0026 0.0026 0.0027 0.0025 0.0024 0.0023 0.0023 0.0022  
## X4 0.00122 0.0021 0.0026 0.0028 0.0030 0.0029 0.0029 0.0028 0.0027 0.0026  
## X5 0.00110 0.0020 0.0027 0.0030 0.0033 0.0033 0.0033 0.0032 0.0032 0.0030  
## X6 0.00109 0.0019 0.0025 0.0029 0.0033 0.0034 0.0035 0.0034 0.0033 0.0033  
## X7 0.00107 0.0018 0.0024 0.0029 0.0033 0.0035 0.0036 0.0036 0.0035 0.0035  
## X8 0.00106 0.0017 0.0023 0.0028 0.0032 0.0034 0.0036 0.0036 0.0035 0.0035  
## X9 0.00106 0.0017 0.0023 0.0027 0.0032 0.0033 0.0035 0.0035 0.0036 0.0036  
## X10 0.00105 0.0016 0.0022 0.0026 0.0030 0.0033 0.0035 0.0035 0.0036 0.0036  
## X11 0.00102 0.0016 0.0021 0.0025 0.0030 0.0032 0.0034 0.0035 0.0035 0.0036  
## X12 0.00100 0.0016 0.0020 0.0024 0.0029 0.0031 0.0033 0.0034 0.0035 0.0036  
## X13 0.00096 0.0015 0.0020 0.0024 0.0028 0.0030 0.0032 0.0033 0.0034 0.0036  
## X14 0.00093 0.0015 0.0019 0.0023 0.0027 0.0029 0.0031 0.0033 0.0034 0.0035  
## X15 0.00091 0.0014 0.0019 0.0022 0.0026 0.0028 0.0030 0.0032 0.0033 0.0035  
## X16 0.00089 0.0014 0.0018 0.0022 0.0026 0.0028 0.0030 0.0032 0.0033 0.0035  
## X17 0.00084 0.0014 0.0018 0.0021 0.0025 0.0027 0.0029 0.0031 0.0033 0.0034  
## X18 0.00082 0.0014 0.0017 0.0021 0.0024 0.0026 0.0029 0.0030 0.0032 0.0034  
## X19 0.00077 0.0013 0.0017 0.0020 0.0023 0.0026 0.0028 0.0030 0.0031 0.0033  
## X20 0.00075 0.0013 0.0017 0.0020 0.0023 0.0025 0.0027 0.0029 0.0031 0.0033  
## X11 X12 X13 X14 X15 X16 X17 X18 X19  
## X1 0.0010 0.0010 0.00096 0.00093 0.00091 0.00089 0.00084 0.00082 0.00077  
## X2 0.0016 0.0016 0.00151 0.00148 0.00144 0.00143 0.00137 0.00135 0.00129  
## X3 0.0021 0.0020 0.00198 0.00192 0.00186 0.00184 0.00177 0.00174 0.00167  
## X4 0.0025 0.0024 0.00237 0.00229 0.00221 0.00219 0.00211 0.00207 0.00199  
## X5 0.0030 0.0029 0.00277 0.00267 0.00259 0.00256 0.00247 0.00241 0.00233  
## X6 0.0032 0.0031 0.00300 0.00290 0.00282 0.00279 0.00271 0.00264 0.00256  
## X7 0.0034 0.0033 0.00321 0.00312 0.00304 0.00301 0.00292 0.00285 0.00277  
## X8 0.0035 0.0034 0.00332 0.00325 0.00319 0.00316 0.00309 0.00302 0.00295  
## X9 0.0035 0.0035 0.00344 0.00339 0.00334 0.00332 0.00326 0.00320 0.00314  
## X10 0.0036 0.0036 0.00355 0.00352 0.00349 0.00348 0.00343 0.00337 0.00332  
## X11 0.0036 0.0036 0.00360 0.00357 0.00358 0.00358 0.00354 0.00350 0.00346  
## X12 0.0036 0.0036 0.00361 0.00363 0.00365 0.00366 0.00364 0.00361 0.00359  
## X13 0.0036 0.0036 0.00367 0.00368 0.00372 0.00375 0.00373 0.00372 0.00371  
## X14 0.0036 0.0036 0.00368 0.00374 0.00380 0.00383 0.00383 0.00383 0.00383  
## X15 0.0036 0.0037 0.00372 0.00380 0.00388 0.00393 0.00394 0.00396 0.00398  
## X16 0.0036 0.0037 0.00375 0.00383 0.00393 0.00400 0.00401 0.00405 0.00406  
## X17 0.0035 0.0036 0.00373 0.00383 0.00394 0.00401 0.00406 0.00409 0.00414  
## X18 0.0035 0.0036 0.00372 0.00383 0.00396 0.00405 0.00409 0.00418 0.00422  
## X19 0.0035 0.0036 0.00371 0.00383 0.00398 0.00406 0.00414 0.00422 0.00430  
## X20 0.0035 0.0036 0.00373 0.00387 0.00402 0.00413 0.00420 0.00430 0.00438  
## X20  
## X1 0.00075  
## X2 0.00128  
## X3 0.00166  
## X4 0.00197  
## X5 0.00230  
## X6 0.00252  
## X7 0.00273  
## X8 0.00292  
## X9 0.00311  
## X10 0.00330  
## X11 0.00346  
## X12 0.00359  
## X13 0.00373  
## X14 0.00387  
## X15 0.00402  
## X16 0.00413  
## X17 0.00420  
## X18 0.00430  
## X19 0.00438  
## X20 0.00449

#necessary to standardize the variables because PCA is sensitive to the scale of the data  
model = prcomp(returns, scale = TRUE, center = TRUE)

Data frame which provides the data for the principal components analysis

x = data.frame(model$x)  
head(x)

## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10  
## 1 4.40 1.37 -0.386 0.076 -0.1418 0.029 0.0799 -0.134 -0.098 0.1168  
## 2 4.26 -1.01 0.608 -0.215 -0.0968 -0.217 -0.0422 0.095 0.062 -0.0197  
## 3 -0.65 1.01 -0.742 -0.710 0.2372 0.372 0.0087 -0.133 -0.081 -0.1018  
## 4 -0.66 2.29 -0.395 0.730 0.0052 -0.276 0.1718 0.199 0.090 0.0047  
## 5 -2.40 -0.50 0.380 -0.217 -0.1907 0.256 -0.0326 -0.134 -0.041 -0.0648  
## 6 1.59 0.91 -0.059 -0.334 0.2644 -0.020 -0.1080 0.026 -0.073 0.0599  
## PC11 PC12 PC13 PC14 PC15 PC16 PC17 PC18 PC19  
## 1 0.1415 0.0304 0.0846 -0.1027 0.0640 -1.1e-14 -5.4e-15 2.5e-15 -9.8e-15  
## 2 -0.0167 0.0247 -0.0033 0.0545 -0.0872 1.3e-14 6.9e-15 -3.2e-15 2.2e-14  
## 3 0.1044 -0.0250 -0.0627 -0.0043 0.1168 -4.3e-15 -2.2e-14 9.6e-16 -1.7e-14  
## 4 -0.1008 -0.0055 0.0436 -0.0181 -0.0314 3.3e-15 2.2e-14 8.3e-15 7.4e-15  
## 5 -0.0060 0.0114 0.0118 -0.0247 0.0012 -1.7e-15 4.7e-15 -8.5e-15 -3.4e-15  
## 6 -0.0062 -0.0709 0.0066 0.0790 -0.0253 6.4e-15 -4.9e-15 -1.2e-15 2.9e-15  
## PC20  
## 1 5.3e-15  
## 2 -4.2e-15  
## 3 -1.6e-15  
## 4 6.5e-15  
## 5 -6.1e-15  
## 6 -1.0e-15

tail(x)

## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10 PC11  
## 499 -6.0 -1.01 0.17 -0.575 -0.268 0.085 -0.073 -0.133 -0.0340 -0.025 -0.031  
## 500 -6.8 0.49 0.29 0.665 0.202 0.120 0.010 0.018 0.0021 0.019 0.008  
## 501 -1.9 1.41 0.87 -0.055 0.220 0.232 -0.137 0.059 0.0118 0.090 -0.025  
## 502 11.9 -0.38 -1.17 -2.110 -0.863 0.218 0.049 0.138 0.0908 -0.052 -0.068  
## 503 3.5 -1.13 0.60 0.774 0.755 -0.257 0.059 -0.133 -0.1689 -0.021 0.073  
## 504 -6.5 1.45 1.35 1.467 0.072 0.150 -0.038 0.056 0.1046 0.064 -0.015  
## PC12 PC13 PC14 PC15 PC16 PC17 PC18 PC19 PC20  
## 499 0.1178 0.020 0.0267 0.011 -4.4e-16 -8.3e-16 -2.5e-15 1.1e-15 1.7e-15  
## 500 0.0066 0.037 0.0477 -0.041 -1.0e-14 9.8e-15 1.7e-15 -5.9e-15 1.1e-16  
## 501 -0.1067 -0.036 -0.0460 0.092 -9.5e-15 3.0e-15 1.8e-14 4.0e-15 1.7e-15  
## 502 0.0216 0.033 -0.0085 -0.033 -3.3e-16 7.2e-15 -6.9e-16 -1.4e-14 -5.1e-15  
## 503 -0.0648 -0.073 -0.0226 -0.022 8.3e-16 -1.1e-14 -2.9e-14 8.9e-15 1.9e-15  
## 504 -0.0151 0.037 0.0285 -0.049 2.3e-15 6.1e-16 7.1e-16 -1.8e-15 1.1e-15

The matrix of variable loadings (i.e., a matrix whose columns contain the eigenvectors)

loadings = data.frame(model$rotation)   
head(loadings)

## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10 PC11  
## X1 -0.089 0.31 -0.716 -0.554 -0.257 -0.092 -0.011 -0.028 -0.0031 -0.016 0.0093  
## X2 -0.158 0.40 -0.357 0.367 0.550 0.432 0.072 0.180 0.0410 0.088 -0.0546  
## X3 -0.190 0.38 -0.032 0.425 -0.062 -0.406 -0.160 -0.395 -0.1052 -0.189 0.1626  
## X4 -0.211 0.33 0.098 0.225 -0.245 -0.246 -0.025 0.075 -0.0235 0.037 -0.0731  
## X5 -0.221 0.26 0.220 0.038 -0.370 -0.036 0.115 0.347 0.1527 0.198 -0.1372  
## X6 -0.229 0.20 0.230 -0.085 -0.229 0.231 0.160 0.134 0.0851 0.061 0.0103  
## PC12 PC13 PC14 PC15 PC16 PC17 PC18 PC19 PC20  
## X1 -0.018 0.0023 0.0019 -0.0025 -6.4e-16 -3.8e-17 -1.0e-15 6.9e-16 -9.6e-17  
## X2 0.100 0.0039 0.0183 -0.0115 2.5e-15 -1.3e-15 1.4e-15 -2.3e-15 1.1e-15  
## X3 -0.346 0.1550 0.2100 -0.1654 -1.5e-16 -1.5e-15 -5.5e-15 3.1e-15 -1.6e-15  
## X4 0.268 -0.3733 -0.5271 0.4078 4.2e-16 4.2e-15 8.5e-15 -3.9e-15 2.0e-15  
## X5 0.315 0.0241 0.2813 -0.5504 9.5e-15 -1.2e-15 -9.5e-16 2.5e-15 -6.7e-16  
## X6 -0.212 0.5086 0.2133 0.5812 -2.4e-14 9.3e-15 -5.4e-15 7.0e-15 -2.7e-15

tail(loadings)

## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8 PC9 PC10  
## X15 -0.24 -0.18 -0.081 0.024 0.084 -0.166 0.259 0.0066 0.0135 0.1068  
## X16 -0.24 -0.19 -0.100 0.063 0.019 -0.121 0.328 0.0064 -0.0038 0.0773  
## X17 -0.23 -0.22 -0.118 0.091 -0.055 -0.037 0.058 0.1300 -0.1045 0.5204  
## X18 -0.23 -0.24 -0.144 0.146 -0.149 0.135 -0.048 -0.1297 0.1013 -0.4342  
## X19 -0.22 -0.26 -0.161 0.172 -0.219 0.214 -0.307 -0.0077 0.0020 0.0032  
## X20 -0.22 -0.27 -0.177 0.205 -0.275 0.250 -0.232 -0.0077 -0.0142 -0.0233  
## PC11 PC12 PC13 PC14 PC15 PC16 PC17 PC18 PC19 PC20  
## X15 -0.050 -0.060 0.33 -0.367 -0.1594 0.2351 -0.28 -0.043 -0.24 -0.565  
## X16 -0.472 -0.338 -0.27 0.142 0.0475 0.0054 -0.22 0.033 -0.31 0.434  
## X17 0.356 -0.269 -0.18 0.014 -0.0018 0.3198 0.41 -0.079 0.24 0.029  
## X18 -0.436 0.148 0.16 -0.064 -0.0264 0.3244 0.42 -0.080 0.25 0.029  
## X19 0.371 0.211 0.25 -0.187 -0.0735 0.0056 -0.22 0.034 -0.32 0.451  
## X20 -0.036 -0.057 -0.32 0.293 0.1211 -0.3418 -0.20 0.048 0.07 -0.491

Standard deviation - this is simply the eigenvalues in our case since the data has been centered and scaled (standardized)

Proportion of Variance - the amount of variance the component accounts for in the data

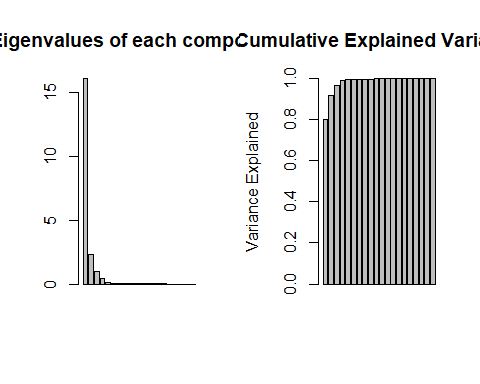
Cumulative Proportion - the accumulated amount of explained variance

Three principal components can explain almost 97% percent of the variance. Eigenvalues of first two components are larger than 1. Eigenvalue of the third component is almost 1.

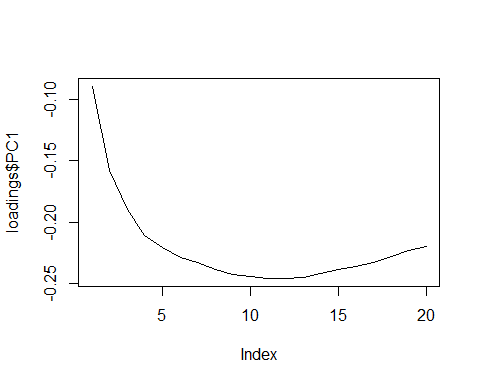
summary(model)

## Importance of components:  
## PC1 PC2 PC3 PC4 PC5 PC6 PC7 PC8  
## Standard deviation 4.009 1.514 0.9908 0.678 0.3224 0.17425 0.11337 0.09623  
## Proportion of Variance 0.804 0.115 0.0491 0.023 0.0052 0.00152 0.00064 0.00046  
## Cumulative Proportion 0.804 0.918 0.9674 0.990 0.9956 0.99709 0.99774 0.99820  
## PC9 PC10 PC11 PC12 PC13 PC14 PC15  
## Standard deviation 0.09302 0.08802 0.07930 0.07118 0.05678 0.05414 0.04605  
## Proportion of Variance 0.00043 0.00039 0.00031 0.00025 0.00016 0.00015 0.00011  
## Cumulative Proportion 0.99863 0.99902 0.99933 0.99959 0.99975 0.99989 1.00000  
## PC16 PC17 PC18 PC19 PC20  
## Standard deviation 1.06e-14 9.93e-15 8.9e-15 8.61e-15 7.74e-15  
## Proportion of Variance 0.00e+00 0.00e+00 0.0e+00 0.00e+00 0.00e+00  
## Cumulative Proportion 1.00e+00 1.00e+00 1.0e+00 1.00e+00 1.00e+00

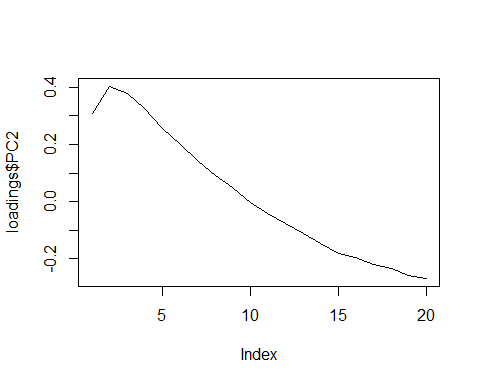
par(mfrow = c(1,2))  
barplot(model$sdev^2, main = 'Eigenvalues of each component')  
barplot(cumsum(model$sdev^2)/sum(model$sdev^2), main = 'Cumulative Explained Variance', ylab = 'Variance Explained')



plot(loadings$PC1, type = "line") #convexity



plot(loadings$PC2, type = "line") #slope change



plot(loadings$PC3, type = "line") #level change

