barycenter linear predictor $\hat{v}(t_i) = \langle v \rangle_n + \hat{a}_n (t_i - \langle t \rangle_n)$

for a Maximum Likelihood Estimation Distributional assumptions are required
for Least Squares Estimation Distributional assumptions are not required
Assumptions:

$$\begin{aligned} E[e_i] &= 0 \\ \mathrm{Var}[e_i] &= \sigma_i \\ e_i &= \text{indep} \end{aligned}$$

(Normality not required)

$$(v_1, v_2, \ldots v_N) \tag{1}$$

is a N-dimensional vector

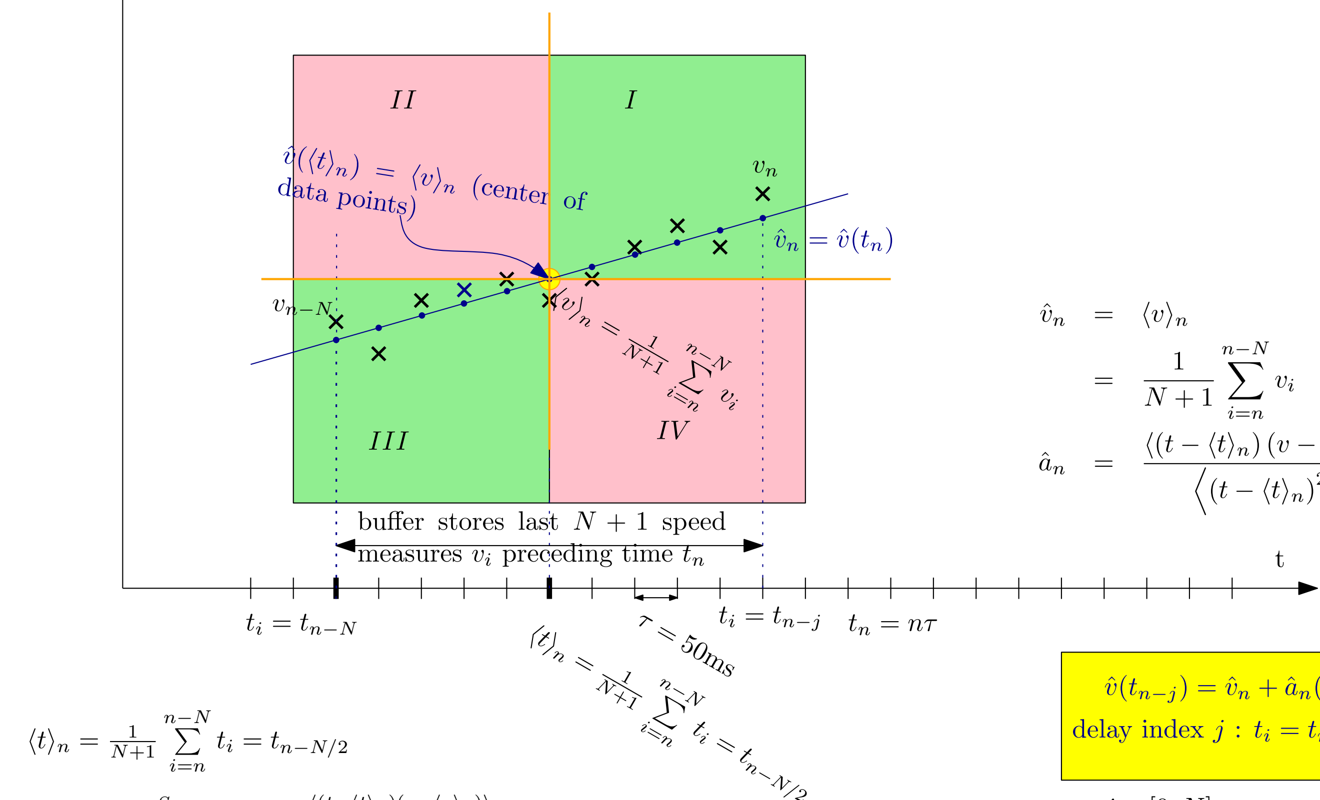
$$(\hat{v}(t_1), \hat{v}(t_2), \ldots \hat{v}(t_N)) \tag{2}$$

is N dimensional constrained vector with 2 degree of freedom.

$$\begin{aligned} \sum_{i=n}^{n-N} e_i &= 0 \\ \sum_{i=n}^{n-N} v_i &= \sum_{i=n}^{n-N} \hat{v}(t_i) \\ \sum_{i=n}^{n-N} t_i e_i &= 0 \\ \sum_{i=n}^{n-N} v_i e_i &= 0 \end{aligned} \tag{3}$$

$$\mathrm{Var}[t] = S_{t,t} = \left\langle (t - \langle t \rangle_n)^2 \right\rangle_n = \tfrac{1}{N+1} \sum_{i=n}^{n-N} (t_i - \langle t \rangle_n)^2 = \tfrac{N(N+2)}{12}$$

$$\mathrm{Cov}[t, v] = S_{t,v} = \langle (t - \langle t \rangle_n)(v - \langle v \rangle_n) \rangle_n = \tfrac{1}{N+1} \sum_{i=n}^{n-N} (t_i - \langle t \rangle_n)(v_i - \langle v \rangle_n)$$



$$\begin{aligned} \hat{v}_n &= \langle v \rangle_n \\ &= \frac{1}{N+1} \sum_{i=n}^{n-N} v_i \\ \hat{a}_n &= \frac{\langle (t - \langle t \rangle_n)(v - \langle v \rangle_n) \rangle_n}{\left\langle (t - \langle t \rangle_n)^2 \right\rangle_n} \end{aligned}$$

$\hat{v}(t_{n-j}) = \hat{v}_n + \hat{a}_n(-j\tau)$

delay index j : $t_i = t_{n-j} = t_n - j\tau$

$$\langle t \rangle_n = \tfrac{1}{N+1} \sum_{i=n}^{n-N} t_i = t_{n-N/2}$$

$$\mathrm{Cor}[t, v] = \frac{S_{t,v}}{\sqrt{S_{v,v} S_{t,t}}} = \frac{\langle (t - \langle t \rangle_n)(v - \langle v \rangle_n) \rangle_n}{\sqrt{\left\langle (v - \langle v \rangle_n)^2 \right\rangle_n \left\langle (t - \langle t \rangle_n)^2 \right\rangle_n}}$$

$$\mathrm{Var}[v] = S_{t,t} = \left\langle (v - \langle v \rangle_n)^2 \right\rangle_n = \tfrac{1}{N+1} \sum_{i=n}^{n-N} (v_i - \langle v \rangle_n)^2$$

$j \in [0, N]$ $i \in [n - N, n]$

$i = n - j$ absolute index of data point (t_i, v_i) $j = n - i$index over buffered sample