SSY191 - Sensor Fusion and Nonlinear Filtering
Peer-Review of Home Assignment 01

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1 Transformation of Gaussian random variables

a) The figure plots, labels, captions and legends are very well done and easy to follow.

I would just add a remark that it is a bit confusing for me the way you mention "approximated transformation" throughout your text. I think the transformation is not approximated, it is a deterministic transformation. What we are doing is actually to approximate or estimate the mean and covariance of the *transformed distribution*, which we presume to be Gaussian and can therefore be represented by just a mean and a covariance matrix.

For example, when you wrote this:

"The more samples used and due to the fact that the transformation equation 2 in the Home assignment PM is linear the approximated transformation should converge to the real distribution by the law of large numbers."

I would have written in this way:

"The more samples are used, the better is the approximation of the mean and covariance in comparison to the analytical values. Due to the fact that the transformation equation 2 in the Home assignment PM is linear AND $\mathbf{x}$ is Gaussian, the transformed distribution will be also Gaussian. Therefore the transformed distribution can be well represented by only a mean and covariance. As we increase the number of samples, the estimation of mean and covariance are closer to the analytical values."

In my opinion, even if the distribution is not Gaussian, the mean and the covariance of the distribution should converge to the real one as we increase the number of samples. The point is that, if the distribution is not Gaussian, we may need more information to represent the distribution or it can not be simply represented by only a mean and a covariance matrix.

In addition, in the plot legend, I would replace "True and Approximated transformations" by "Analytical and approximated 3-σ ellipses". I would have added also the expressions "Gaussian distribution" and "covariance" in your text, which are not mentioned.

In sum, I got what you wanted to explain and I fully agree, but I would have written in a different way.

b) This explanation for me is very well written. I would just add a remark:

When you say "The mean can roughly be estimated", I think I do not fully agree. If you have enough samples, you could be able to estimate very well the mean of the distribution according to the law of large numbers, not matter the distribution of the random variable. Besides of that I fully agree with everything.

2 Snow depth in Norway

a) Your explanation in this item is very nice, but I am afraid you did not answer exactly what the question really was. I think it would have helped if you have plotted the two figures using the axis with same scale (`axis equal` command in Matlab). You would then have see that for Kvitfjell, the slope of the major axis is almost vertical, while for Hafjell it is close ot 45 degrees.

I agree with your analysis concerning the covariance of the posterior, which is higher for Kvitfjell. However, this analysis is not related to the slope of the major axis, but with the correlation factor. In my opinion, the analysis of the major axis is very important, because in this case the $\hat{x}_{MAP}$ and the $\hat{x}_{MMSE}$ will be always located on top of this line. If the major axis is close to be vertical, then the information of y will almost make no difference to the estimators $\hat{x}_{MAP}$ or $\hat{x}_{MMSE}$, because $E[x|y]$ will always result in the same value no matter of y. This is the case for Else at Kvitfjell.

I hope you got what I meant, and I am available to discuss this in more details if you want.

b) Nice explanation about how the posterior and the joint densities are related.

c) I fully agree with your explanation and reasoning. However, I would have written in a more formal/scientific way. For example, when you write:

"If the requirements are that guaranteed snow Hafjell is the better choice, but on the other hand if the depth of the snow is more important Kvitfjell is better, but with a chance of not having snow at all."

Maybe I would have written in this way:

"If the decision is made based on maximizing the expected value of snow depth, given the weather forecast and his friends report, then Kvitfjell is the better choice. However, as a drawback, one can see that the covariance of $p(x_K|y_E = 2)$ is too large, what causes the probability of finding snow below 0.5m at Kvitfjell to be too high, which on the other hand, is almost zero for Hafjell. Formally: $p(x_K < 0.5|y_E = 2) >> p(x_H < 0.5|y_A = 1) \approx 0$.

3 MMSE and MAP estimates for Gaussian mixture posteriors

Very nice plots and very nice explanation in my opinion.