

DATASCI 2G03 Project Proposal

Elastic Collisions between Contained Particles

Anthony Hunt

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Introduction

Simple 2D video games like pong, brick breaker, or even 8 ball pool have a great deal of both physical and computational concerns. In particular, this project aims to simulate the motion of moving particles and the collision interactions between them.

The main problems we will be solving are:

- Motion of several (round) particles with parametric starting configurations
- Elastic collisions between particles and a bounding box

The first part of this project will consist of modelling the interaction between many particles in a containing box the hopes of achieving a 2D Maxwell–Boltzmann distribution of velocities. Then, we will extend these interactions with the addition of charged particles to simulate non-constant acceleration.

Equations

The ODEs and other equations used to simulate this problem are listed below. The goal of this simulation is to track the position of objects (represented by s) over some length of time t . Each particle will have some mass m and starting velocity v .

Velocity (v) of one particle:

$$v = \frac{ds}{dt}$$

Acceleration (a) of one particle:

$$a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$$

Momentum of one particle:

$$p = mv$$

Kinetic energy of one particle:

$$E = \frac{1}{2}mv^2$$

Newton's second law:

$$F = ma$$

The below equations are concerned with calculating the vectors of velocity resulting from an elastic collision. Let v_1 represent the first particle's velocity and v_2 for the second particle's velocity. Additionally, let v' represent the velocity after the collision and v for the velocities prior to the collision.

$$\mathbf{v}'_1 = \mathbf{v}_1 - \frac{2m * 2}{m * 1 + m * 2} \frac{\langle \mathbf{v} * 1 - \mathbf{v}_2, \mathbf{x}_1 - \mathbf{x}_2 \rangle}{\|\mathbf{x}_1 - \mathbf{x}_2\|^2} (\mathbf{x}_1 - \mathbf{x}_2)$$
$$\mathbf{v}'_2 = \mathbf{v}_2 - \frac{2m_1}{m * 1 + m * 2} \frac{\langle \mathbf{v}_2 - \mathbf{v}_1, \mathbf{x}_2 - \mathbf{x}_1 \rangle}{\|\mathbf{x}_2 - \mathbf{x}_1\|^2} (\mathbf{x}_2 - \mathbf{x}_1)$$

Coulomb's law: Force of charged particles against one another:

$$F_{charge} = \frac{kq_1q_2}{r^2}$$

Configuration Parameters

- Starting position s of each particle
- Initial velocity v of each particle
- Mass m of each particle
- Radius of each particle r
- Charge of different particles q

Interesting Properties

- Number of times a particle is hit before stopping the simulation
- Speed of particles over time (momentum of all particles should remain constant) position of particles on a graph
- Distribution of particle speeds (Maxwell-Boltzmann distribution)
- Unique path of each particle

References

- Equations of motion: https://en.wikipedia.org/wiki/Equations_of_motion
- Elastic collisions: https://en.wikipedia.org/wiki/Elastic_collision
- Maxwell-Boltzmann distribution: https://en.wikipedia.org/wiki/Maxwell-Boltzmann_distribution