Commonality Analysis for a Family of Material Models

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Table of Units

Throughout this document consistent units are employed and a consistent notation is used. The unit system adopted is the "MLtT" dimension system, where M is the dimension of mass, L is length, t is time and T is temperature. This system corresponds nicely with the SI (Système International d'Unités), or modern metric, system, which uses units of kilogram (kg), meter (m), second (s) and Kelvin (K) for M, L, t and T, respectively. By leaving the units in this form any unit system can be adopted in the future, as long as the choice of specific units is consistent between different quantities. In addition to the basic units, several derived units are employed as described below. For each unit the symbol is given followed by a description of the unit with the SI equivalent in parentheses.

L	- length (metre, m)
M	- mass (kilogram, kg)
t	- time (second, s)
T	- temperature (Kelvin, K)
ForceU	- force, which has units of $\mathrm{M} \cdot \mathrm{L} \cdot \mathrm{t}^{-2}$ (Newton, $\mathrm{N} = \mathrm{kg} \cdot \mathrm{m} \cdot \mathrm{s}^{-2}$)
StressU	- stress, which has units of $\mathrm{L}^{-1}\mathrm{M}\mathrm{t}^{-2}$, or $\mathrm{ForceU}/\mathrm{L}^2$ (Pascal, $\mathrm{Pa} = \mathrm{N}/\mathrm{m}^2$)
angleU	- radian (rad)

Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. The choice of symbols was made with the goal of being consistent with the existing literature. Accomplishing this goal requires that some symbols are used for multiple purposes. Where this is the case, all possible meanings will be listed, with each on a separate line. When the symbol is later used in the body of the document, it will be defined on its first occurrence. In the other instances where the symbol occurs, its meaning should be clear from the context. The units are listed in two sets of brackets following the definition of the symbol. The first set of brackets shows the MLtT dimension system and the second set of brackets shows the equivalent SI units. In the cases where the symbol refers to entities with multiple components, such as vectors and matrices, the units given apply to each individual component of the entity. In the cases where the units are not listed, this is for one of several reasons. The symbol may not have any units associated with it, such as for strings and for a region of space (the location of the points within the space have units of length, but the set of points itself does not have an associated unit). Another reason for not including units is that the choice of units may be dependent on a particular instance of the symbol. For instance, the fluidity parameter (γ) is sometimes defined as $1/2\eta$, which will have units of $1/(\text{StressU} \cdot \text{t})$, but in other cases the fluidity parameter will have a different definition and different units. The potential ambiguity in units is a consequence of using a generic model of material behaviour.

Ω	- the region of space occupied by a body
x, y, z	- coordinates in Cartesian space (L) (m)
$\bar{\mathbf{T}}$	- surface traction (StressU) (Pa)

Superscripts

$\dot{}$	- time derivative

Subscripts

0	- original configuration or initial stress
$1, 2, 3$	- used to indicate different materials and used for indicating different coordinate axes

Prefixes

Δ	- finite change in following quantity
d	- infinitesimal change in the following quantity

Abbreviations and Acronyms

1D	- one dimensional
2D	- two dimensional
3D	- three dimensional
CA	- Commonality Analysis
DSL	- Domain Specific Language

Types

$\mathbb{R}$	- real numbers
$\mathbb{R}^+ : \{x : \mathbb{R} \mid x \geq 0 : x\}$	- positive real numbers

1 Introduction

1.1 Purpose of Document

1.2 Scope of the Family

1.3 Organization of the Document

2 General System Description

2.1 Potential System Context

2.2 Potential User Characteristics

2.3 Potential System Constraints

3 Commonalities

3.1 Background Overview

3.2 Terminology Definition

Each definition in this section uses the same table structure, with the following rows:

Number: All of the data definitions are assigned a unique number, which takes the form of a natural number with the prefix "D." This number will be used for purposes of cross-referencing and traceability within this document.

Label: The label is a short identifying phrase, each with the prefix "D_." This label provides a mnemonic that helps with quickly remembering which definition is being presented. Moreover, the label will useful when an external document needs to reference one of the definitions in this document.

Symbol: This field shows the symbol that is used to represent variables related to this concept. For instance, the natural strain rate tensor is represented by the symbol $\dot{\boldsymbol{\epsilon}}$. Later in the document when the symbol $\dot{\boldsymbol{\epsilon}}$ appears, possibly with superscripts such as e for elastic or vp for viscoplastic, this is an indication that the term in question is a measure of natural strain rate. If the symbol should appear without the dot over it, then it is referring to a total strain and not the rate of strain.

Type: Each variable designated by a symbol has a type associated with it, which is listed in this field of the data definition template. The type information helps to clarify the meaning of the symbol and the variables.

Units: Where applicable the units associated with the symbol are given. These units are given in terms of the mass (M), length (L), time (t) and temperature (T). For convenience the units are also given in SI.

Related Items: A related item is a data definition or assumption that is used by the current definition. That is, if the used data definition or assumption should change, then the current data definition will also need to be modified. As mentioned above, data definitions have a prefix "D." Assumptions will have the prefix "A."

Sources: This field lists references that can be consulted for additional information on the concept in question.

Description: The actual definition is given here. In some cases where the description is lengthy, some of the details are moved to a section following the table. When appropriate the description will reference the related definitions and assumptions.

History: Each data definition ends with a history of the definition, including the creation date and any subsequent modifications.

Number:	D1
Label:	D_Stress
Symbol:	$\boldsymbol{\sigma}$
Type:	array1DT
Units:	Each component of the stress tensor has units of StressU (Pa)
Related Items:	A1, A**??**, A**??**, A**??**
Sources:	Long (1961, pages 35–41); Malvern (1969, pages 64–119); Mase (1970, pages 44–76); Beer and Johnston (1985, pages 1–21)
Description:	The stress provides a measure of force per unit area associated with different directions at a point within a body. A detailed definition of the stress tensor is provided below. This definition is for the true stress, which is also sometimes called the Cauchy stress.
History:	Created – June 14, 2007

3.3 Goal Statement

Number:	G1
Label:	G_StressDetermination
Description:	Given the initial stress and the deformation history of a material particle, determine the stress within the material particle.
Related Items:	T**??**
History:	Created – June 8, 2007

3.4 Assumptions

Equation: Some of the assumptions include an equation, when this makes the description more precise. For each equation the types of each of the terms is listed.

Rationale: This field justifies the appropriateness of the assumption within the context of the current family. If changes in the assumptions are made in the future, it will be because the rationale is inadequate in some sense.

Number:	A1
Label:	A_ContinuumHypothesis
Related Items:	–
Equation:	–
Description:	The underlying molecular structure of matter is not considered and gaps and empty spaces within a material particle are ignored. The material is assumed to be continuous.
Rationale:	The continuum hypothesis allows for the definition of stress and strain at a point. Although not strictly true, the notion that matter is continuous fits with what one usually sees at the macroscopic scale. Even at small fractions of the scales that engineering problems typically deal with there is generally an enormous number of molecules, which makes the averages of the physical properties stable. As the final, and most important justification, the theories built using the continuum hypothesis often provide quantitative predictions that agree closely with experimental data.
Source:	Long (1961, 33–34); Malvern (1969, pages 1–2)
History:	Created – Aug 23, 2007

3.5 Theoretical Model

4 Variabilities

5 Dependence Graphs

6 Sample Family Members

References

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Robert R. Long. *Mechanics of Solids and Fluids.* Prentice Hall, Englewood Cliffs, New Jersey, 1961.

L. E. Malvern. *Introduction to the Mechanics of Continuous Medium.* Prentice Hall, 1969.

George E. Mase. *Schaum's Outline of Theory and Problems of Continuum Mechanics.* McGraw-Hill Publishing Company, 1970.