

# Software Requirements Specification for Slope Stability Analysis

Henry Frankis

May 8, 2018

## **Contents**

# 1 Reference Material

This section records information for easy reference.

## 1.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, the table lists the symbol, a description and the SI name.

Symbol	Description
°	angle (degree)
m	length (metre)
N	force (newton)
Pa	pressure (pascal)

## 1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. Throughout the document, values with a subscript  $i$  implies that the value will be taken at and analyzed at a slice or slice interface composing the total slip mass.

Symbol	Description	Units
$\{x_{cs}, y_{cs}\}$	The Set of X and Y Coordinates: describe the vertices of the critical slip surface	m
$(x, y)$	Cartesian Position Coordinates: y is considered parallel to the direction of the force of gravity and x is considered perpendicular to y	m
$a$	Constant: FIXME: missing discription	m
$A$	Constant: FIXME: missing discription	m
$b$	Base Width of a Slice: in the x-ordinate direction only for slice index i	m
$c'$	Effective Cohesion: internal pressure that sticks particles of soil together	Pa
$C1$	Interslice Normal Force Function: FIXME: missing discription	Nm
$C2$	Interslice Shear Force Function: FIXME: missing discription	Nm
$E$	Elastic Modulus: The ratio of the stress exerted on a body to the resulting strain.	Pa

$E$	Interslice Normal Force: exerted between adjacent slices for interslice index $i$	N
$F$	Force: An interaction that tends to produce change in the motion of an object	N
$F_x$	X-Component of the Net Force: FIXME: missing discription	N
$F_y$	Y-Component of the Net Force: FIXME: missing discription	N
$f$	Scaling Function: magnitude of interslice forces as a function of the x coordinatefor interslice index $i$ ; can be constant or a half-sine	
$FS$	Factor of Safety: The global stability of a surface in a slope	
$FS_{Loc,i}$	Local Factor of Safety: for slice index $i$	
$FS_{min}$	Minimum Factor of Safety: The minimum factor of safety	
$H$	Interslice Water Force: exerted in the x-ordinate direction between adjacent slices for interslice index $i$	N
$\Delta H$	Difference Between Interslice Forces: exerted in the x-ordinate direction between adjacent slices for interslice index $i$	N
$h$	Midpoint Height: distance from the slip base to the slope surface in a vertical line from the midpoint of the slice for slice index $i$	m
$i$	Index: used to show a quantity applies to only one slice	
$K_{bA}$	Effective Base Stiffness a: for rotated coordinates of a slice base surface, for slice index $i$	$\frac{\text{Pa}}{\text{m}}$
$K_{bB}$	Effective Base Stiffness a: for rotated coordinates of a slice base surface, for slice index $i$	$\frac{\text{Pa}}{\text{m}}$
$K$	Stiffness: The extent a body resists strain.	$\frac{\text{N}}{\text{m}}$
$K_{bn}$	Normal Stiffness: for a slice base surface, without length adjustment for slice index $i$	$\frac{\text{Pa}}{\text{m}}$
$K_{bt}$	Shear Stiffness: for a slice base surface, without length adjustment for slice index $i$	$\frac{\text{Pa}}{\text{m}}$
$K_c$	Earthquake Load Factor: proportionality factor of force that weight pushes outwards; caused by seismic earth movements	
$K_{no}$	Normal Stiffness: residual strength	$\frac{\text{Pa}}{\text{m}}$

$K_{sn}$	Normal Stiffness: for an interslice surface, without length adjustment for interslice index i	$\frac{\text{Pa}}{\text{m}}$
$K_{st}$	Shear Stiffness: for interslice surface, without length adjustment for interslice index i	$\frac{\text{Pa}}{\text{m}}$
$K_{tr}$	Shear Stiffness: residual strength	$\frac{\text{Pa}}{\text{m}}$
$M$	Moment of a Body: assumed 2D allowing a scalar	Nm
$N$	Normal Force: total reactive force for a soil surface subject to a body resting on it	N
$N'$	Effective Normal Force: for a soil surface, subtracting pore water reactive force from total reactive force	N
$n$	Number of Slices: the slip mass has been divided into	
$N^*$	Effective Normal Force: for a soil surface, without the influence of interslice forces	N
$p$	Pressure: A force exerted over an area	Pa
$P$	Resistive Shear Force: Mohr Coulomb frictional force that describes the limit of mobilized shear force the slice i can withstand before failure	N
$Q$	Imposed Surface Load: a downward force acting into the surface from midpoint of slice i	N
$R$	Resistive Shear Force: without the influence of interslice forces for slice index i	N
$S$	Mobilized Shear Force: for slice index i	N
$s$	Mobilized Shear Stress: acting on the base of a slice	Pa
<i>SpencerFixme1Please</i>	Fixme: What is this value?	
<i>SpencerFixme2Please</i>	Fixme: What is this value?	
$T$	Mobilized Shear Force: without the influence of interslice forces for slice index i	N
$U_b$	Base Hydrostatic Force: from water pressure within the slice for slice index i	N
$U_t$	Surface Hydrostatic Force: from water pressure acting into the slice from standing water on the slope surface for slice index i	N
$u$	Local Index: used as a bound variable index in calculations	
$v$	Local Index: used as a bound variable index in calculations	
$W$	Weight: downward force caused by gravity on slice i	N
$x$	X Ordinate: refers to either slice i midpoint, or slice interface i	m

$x_{slip}$	X Ordinate: distance of the slip surface at i, refers to either slice i midpoint, or slice interface i	m
$x_{us}$	X Ordinate: distance of the edge of the slope at i, refers to either slice i midpoint, or slice interface i	m
$X$	Interslice Shear Force: exerted between adjacent slices for interslice index i	N
$y_{slip}$	Y Ordinate: height of the slip surface at i, refers to either slice i midpoint, or slice interface i	m
$y_{us}$	Y Ordinate: height of the top of the slope at i, refers to either slice i midpoint, or slice interface i	m
$y_{wt}$	Y Ordinate: height of the water table at i, refers to either slice i midpoint, or slice interface i	m
$y$	Y Ordinate: refers to either slice i midpoint, or slice interface i	m
$z$	Center of Slice Height: the distance from the lowest part of the slice to the height of the centers of slice	m
$\alpha$	Angle: base of the mass relative to the horizontal for slice index i	°
$\beta$	Angle: surface of the mass relative to the horizontal for slice index i	°
$\gamma$	Dry Unit Weight: The weight of a dry soil/ground layer divided by the volume of the layer.	$\frac{N}{m^3}$
$\gamma_{Sat}$	Saturated Unit Weight: The weight of saturated soil/ground layer divided by the volume of the layer.	$\frac{N}{m^3}$
$\gamma_w$	Unit Weight of Water: The weight of one cubic meter of water.	$\frac{N}{m^3}$
$\delta$	Displacement: generic displacement of a body	m
$\delta n$	Displacement: for the element parallel to the surface for slice index i	m
$\delta t$	Displacement: for the element normal to the surface for slice index i	m
$\delta u$	Displacement: shear displacement for slice index i	m
$\delta v$	Displacement: normal displacement for slice index i	m
$\delta x$	Displacement: in the x-ordinate direction for slice index i	m
$\delta y$	Displacement: in the y-ordinate direction for slice index i	m
$\varepsilon$	Displacement: in rotated coordinate system	m
$\kappa$	Constant: FIXME: missing discription	Pa

$\lambda$	Interslice Normal/shear Force Ratio: applied to all interslices	
$\mu$	Pore Pressure: from water within the soil	Pa
$\nu$	Poisson's Ratio: The ratio of perpendicular strain to parallel strain.	
$\sigma$	Normal Stress: The stress exerted perpendicular to the plain of the object	Pa
$\tau$	Resistive Shear Stress: acting on the base of a slice	Pa
$\Upsilon$	Function: generic minimization function or algorithm	
$\varphi'$	Effective Angle of Friction: The angle of inclination with respect to the horizontal axis of the Mohr-Coulomb shear resistance line	°
$\Phi$	Constant: converts resistive shear without the influence of interslice forces, to a calculation considering the interslice forces	N
$\Psi$	Constant: converts mobile shear without the influence of interslice forces, to a calculation considering the interslice forces	N
$\omega$	Angle: of imposed surface load acting into the surface relative to the vertical for slice index i	°
$\ell_b$	Total Base Length of a Slice: for slice index i	m
$\ell_s$	Length of an Interslice Surface: from slip base to slope surface in a vertical line from an interslice vertex for interslice index i	m

---

### 1.3 Abbreviations and Acronyms

Symbol	Description
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
SSA	Slope Stability Analysis
T	Theoretical Model

## 2 Introduction

A slope of geological mass, composed of soil and rock, is subject to the influence of gravity on the mass. For an unstable slope this can cause instability in the form of soil/rock movement. The effects of soil/rock movement can range from inconvenient to seriously hazardous, resulting in significant life and economic losses. Slope stability is of interest both when analyzing natural slopes, and when designing an excavated slope. Slope stability analysis is the assessment of the safety of a slope, identifying the surface most likely to experience slip and an index of its relative stability known as the factor of safety. The following section provides an overview of the Software Requirements Specification (SRS) for a slope stability analysis problem. The developed program will be referred to as the Slope Stability Analysis (SSA) program. This section explains the purpose of this document, the scope of the system, the organization of the document, and the characteristics of the intended reader.

### 2.1 Purpose of Document

The SSA program determines the critical slip surface, and its respective factor of safety as a method of assessing the stability of a slope design. The program is intended to be used as an educational tool for introducing slope stability issues, and will facilitate the analysis and design of a safe slope. This document will be used as a starting point for subsequent development phases, including writing the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized, including decisions on the numerical algorithms and programming environment. The verification and validation plan will show the steps that will be used to increase confidence in the software documentation and the implementation. Although the SRS fits in a series of documents that follow the so-called waterfall model, the actual development process is not constrained in any way. Even when the waterfall model is not followed, as Parnas and Clements point out, the most logical way to present the documentation is still to “fake” a rational design process.

### 2.2 Scope of Requirements

The scope of the requirements includes stability analysis of a 2 dimensional slope, composed of homogeneous soil layers. Given the appropriate inputs, the code for SSA is intended to identify the most likely failure surface within the possible input range, and find the factor of safety for the slope as well as displacement of soil that will occur on the slope.

## **2.3 Characteristics of Intended Reader**

Reviewers of this documentation should have a strong knowledge in solid mechanics. The reviewers should also have an understanding of undergraduate level 4 physics. The users of SSA can have a lower level of expertise, as explained in Section ??.

## **2.4 Organization of Document**

The organization of this document follows the template for an SRS for scientific computing software proposed by Koothoor as well as Smith and Lai. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom up approach, they can start reading the instance models in Section ?? and trace back to find any additional information they require. The goal statements are refined to the theoretical models, and the theoretical models to the instance models. The instance models provide the set of algebraic equations that must be solved iteratively to perform a Morgenstern Price Analysis.

# **3 General System Description**

This section provides general information about the system, identifies the interfaces between the system and its environment, and describes the user characteristics and the system constraints.

## **3.1 User Characteristics**

The end user of SSA should have an understanding of undergraduate Level 1 Calculus and Physics, and be familiar with soil and material properties.

## **3.2 System Constraints**

There are no system constraints.

# **4 Specific System Description**

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, definitions and finally the instance models that model the slope.



## 4.1 Problem Description

SSA is a computer program developed to evaluate the factor of safety of a slope's slip surface and to calculate the displacement that the slope will experience.

### 4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

Factor of Safety: The global stability of a surface in a slope

Critical Slip Surface: Slip surface of the slope that has the lowest global factor of safety, and therefore most likely to experience failure.

Stress: Forces that are exerted between planes internal to a larger body subject to external loading.

Strain: Stress forces that result in deformation of the body/plane.

Normal Force: A force applied perpendicular to the plane of the material.

Shear Force: A force applied parallel to the plane of the material.

Tension: A stress that causes displacement of the body away from its center.

Compression: A stress that causes displacement of the body towards its center.

Plane Strain: The resultant stresses in one of the directions of a 3 dimensional material can be approximated as 0. Results when the length of one dimension of the body dominates the others. Stresses in the dominate dimensions direction are the ones that can be approximated as 0.

### 4.1.2 Physical System Description

Analysis of the slope is performed by looking at properties of the slope as a series of slice elements. Some properties are interslice properties, and some are slice or slice base properties. The index convention for referencing which interslice or slice is being used is shown in Figure ??.

1. Interslice properties convention is noted by  $j$ . The end interslice properties are usually not of interest, therefore use the interslice properties from  $1 \leq i \leq n - 1$
2. Slice properties convention is noted by  $i$

A free body diagram of the forces acting on the slice is displayed in Figure ??.

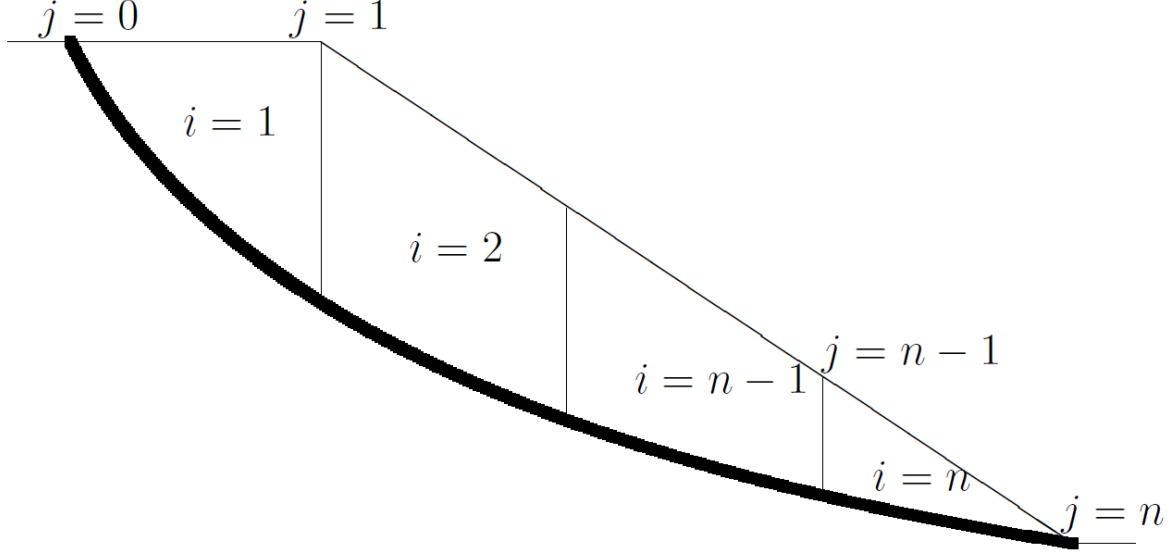


Figure 1: Index convention for numbering slice and interslice force variables

#### 4.1.3 Goal Statements

Given the geometry of the water table, the geometry of the layers composing the plane of a slope, and the material properties of the layers, the goal statements are:

GS1: Evaluate local and global factors of safety along a given slip surface.

GS2: Identify the critical slip surface for the slope, with the lowest factor of safety.

GS3: Determine the displacement of the slope.

## 4.2 Solution Characteristics Specification

The instance models that govern SSA are presented in Section ???. The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

### 4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the Theoretical Models [Section ??], General Definitions [Section ??], Data Definitions [Section ??], Instance Models [Section ??], or Likely Changes [Section ??], in which the respective assumption is used.

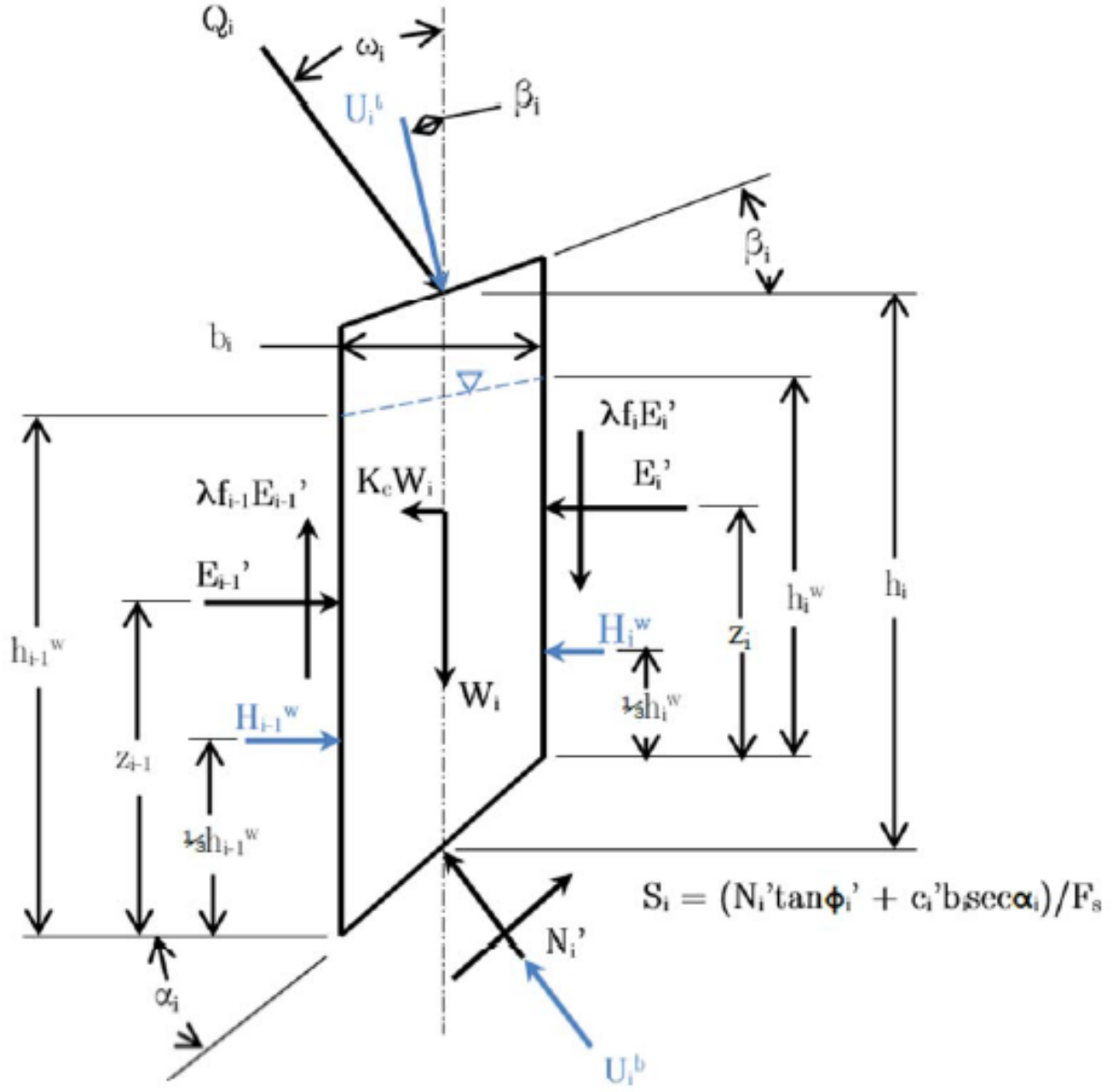


Figure 2: Forces acting on a slice

- A1: The slip surface is concave with respect to the slope surface. The  $(x, y)$  coordinates of the failure surface follow a monotonic function.
- A2: The geometry of the slope, and the material properties of the soil layers are given as inputs.
- A3: The different layers of the soil are homogeneous, with consistent soil properties throughout, and independent of dry or saturated conditions, with the exception of unit weight.
- A4: Soil layers are treated as if they have isotropic properties.
- A5: Interslice normal and shear forces have a linear relationship, proportional to a constant ( $\lambda$ ) and an interslice force function ( $f$ ) depending on  $x$  position.
- A6: Slice to base normal and shear forces have a linear relationship, dependent on the factor of safety  $FS$ , and the Coulomb sliding law.
- A7: The stress - strain curve for interslice relationships is linear with a constant slope.
- A8: The slope and slip surface extends far into and out of the geometry ( $z$  coordinate). This implies plane strain conditions, making 2D analysis appropriate.
- A9: The effective normal stress is large enough that the resistive shear to effective normal stress relationship can be approximated as a linear relationship.
- A10: The surface and base of a slice between interslice nodes are approximated as straight lines.

#### 4.2.2 Theoretical Models

This section focuses on the general equations and laws that SSA is based on.

Refname	T:fs.rc
Label	Factor of Safety
Equation	$FS = \frac{P}{S}$
Description	The stability metric of the slope, known as the factor of safety ( $FS$ ), is determined by the ratio of the shear force at the base of the slope ( $S$ ), and the resistive shear ( $P$ ).

<b>Refname</b>	<b>T:equilibrium</b>
Label	Equilibrium
Equation	$\sum F_x = \sum F_y = \sum M = 0$
Description	For a body in static equilibrium, the net forces and net moments acting on the body will cancel out. Assuming a 2D problem (A8) the x-component of the net force $F_x$ and y-component of the net force $F_y$ will be equal to 0. All forces and their distance from the chosen point of rotation will create a net moment equal to, 0 also able to be analyzed as a scalar in a 2D problem.
<b>Refname</b>	<b>T:mcShrStrgth</b>
Label	Mohr-Coulumb Shear Strength
Equation	$\tau = \sigma \tan (\varphi') + c'$
Description	For a soil under stress it will exert a shear resistive strength based on the Coulomb sliding law. The resistive shear is the maximum amount of shear a surface can experience while remaining rigid, analogous to a maximum normal force. In this model the resistive shear stress $\tau$ is proportional to the product of the normal stress on the plane $\sigma$ with it's static friction in the angular form $\tan (\varphi') = U_t$ . The $\tau$ versus $\sigma$ relationship is not truly linear, but assuming the effective normal force is strong enough it can be approximated with a linear fit (A9) where the cohesion $c'$ represents the $\tau$ intercept of the fitted line.

Refname	T:effStress
Label	Effective Stress
Equation	$\sigma = \sigma - \mu$
Description	$\sigma$ is the total stress a soil mass needs to maintain itself as a rigid collection of particles. The source of the stress can be provided by the soil skeleton $\sigma$ , or by the pore pressure from water within the soil $\mu$ . The stress from the soil skeleton is known as the effective stress $\sigma$ and is the difference between the total stress $\sigma$ and the pore stress $\mu$ .

Refname	T:hookesLaw
Label	Hooke's Law
Equation	$F = K\delta$
Description	Description Stiffness $K$ is the resistance a body offers to deformation by displacement $\delta$ when subject to a force $F$ , along the same direction. A body with high stiffness will experience little deformation when subject to a force.

### 4.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn are used to build the instance models.

<b>Refname</b>	<b>T:normForceEq</b>
Label	Normal Force Equilibrium
Equation	$N_i = (W_i - X_{i-1} + X_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \cos(\alpha_i) + (-K_c W_i - E_i + E_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \sin(\alpha_i)$
Description	<p>For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Rearranged to solve for the normal force of the surface <math>N</math>. Force equilibrium is derived from the free body diagram of Section ?? Index <math>i</math> refers to the values of the properties for slice/interslices following convention in Section ?. Force variable definitions can be found in DD1 to DD9.</p>
<b>Refname</b>	<b>T:bsShrFEq</b>
Label	Base Shear Force Equilibrium
Equation	$S_i = (W_i - X_{i-1} + X_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) + (-K_c W_i - E_i + E_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i)$
Description	<p>For a slice of mass in the slope the force equilibrium to satisfy T2 in the direction parallel to the base surface of the slice. Rearranged to solve for the shear force on the base <math>S</math>. Force equilibrium is derived from the free body diagram of Section ?? Index <math>i</math> refers to the values of the properties for slice/interslices following convention in Section ?. Force variable definitions can be found in DD1 to DD9.</p>

<b>Refname</b>	<b>T:resShr</b>
Label	Resistive Shear Force
Equation	$P_i = N'_i \tan(\varphi'_i) + c'_i b_i \sec(\alpha_i)$
Description	The Mohr-Coulomb resistive shear strength of a slice $\tau$ from T3 is multiplied by the area $b \sec(\alpha) 1$ to obtain the resistive shear force $P$ . Note the extra 1 is to represent a unit of width which is multiplied by the total base length of a slice $\ell_b$ of the plane where the normal occurs, where $\ell_b = b \sec(\alpha)$ and $b$ is the x width of the base. This accounts for the effective normal force $N' = N - U_b$ of a soil from T4 where the normal stress is multiplied by the same area to obtain the effective normal force $\sigma b \sec(\alpha) 1 = N'$ .
<b>Refname</b>	<b>T:mobShr</b>
Label	Mobile Shear Force
Equation	$S_i = \frac{P_i}{FS} = \frac{N'_i \tan(\varphi'_i) + c'_i b_i \sec(\alpha_i)}{FS}$
Description	From the definition of the factor of safety in T1, and the new definition of $P$ , a new relation for the net mobile shear force of the slice $T$ is found as the resistive shear $P$ (GD3) divided by the factor of safety $FS$ .
<b>Refname</b>	<b>T:normShrR</b>
Label	Interslice Normal/shear Relationship
Equation	$X = \lambda f E$
Description	The assumption for the Morgenstern Price method (A5) that the interslice shear force $x$ is proportional to the interslice normal force $E$ by a proportionality constant $\lambda$ and a predetermined scaling function $f$ , that changes the proportionality as a function of the x-coordinate position of the interslice. $f$ is typically either a half-sine along the slip surface, or a constant.



Refname	T:momentEq1
Label	Moment Equilibrium
Equation	$0 = -E_i \left( z_i - \frac{b_i}{2} \tan(\alpha_i) \right) + E_{i-1} \left( z_{i-1} - \frac{b_i}{2} \tan(\alpha_i) \right) - H_i \left( z_i - \frac{b_i}{2} \tan(\alpha_i) \right) + H_{i-1} \left( z_{i-1} - \frac{b_i}{2} \tan(\alpha_i) \right) - \frac{b_i}{2} (X_i + X_{i-1}) + \frac{K_c W_i h_i}{2} - U_{t,i} \sin(\beta_i) h_i - Q_i \sin(\omega_i) h_i$
Description	<p>For a slice of mass in the slope the moment equilibrium to satisfy T2 in the direction perpendicular to the base surface of the slice. Moment equilibrium is derived from the free body diagram of Section ???. Index i refers to the values of the properties for slice/interslices following convention in Section ???. Variable definitions can be found in DD1 to DD9.</p>
Refname	T:netForce
Label	Net X-Component Force
Equation	$F_{x,i} = -\Delta H_i - K_c W_i - U_{b,i} \sin(\alpha_i) + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)$
Description	

Refname	<b>T:netForce</b>
Label	Net Y-Component Force
Equation	$F_{y,i} = -W_i + U_{b,i} \cos(\alpha_i) - U_{t,i} \cos(\beta_i) - Q_i \cos(\omega_i)$
Description	<p>These equations show the net sum of the forces acting on a slice for the RFEM model and the forces that create an applied load on the slice. <math>F_x</math> refers to the load in the direction perpendicular to the direction of the force of gravity for slice <math>i</math>, while <math>F_y</math> refers to the load in the direction parallel to the force of gravity for slice <math>i</math>. Forces are found in the free body diagram of Section ?? . In this model the elements are not exerting forces on each other, so the interslice forces <math>E</math> and <math>X</math> are not a part of the model. Index <math>i</math> refers to the values of the properties for slice/interslices following convention in Section ?? . Force variable definitions can be found in DD1 to DD8.</p>
Refname	<b>T:hookesLaw2d</b>
Label	Hooke's Law 2D
Equation	$\begin{bmatrix} p_i \\ p_i \end{bmatrix} = \begin{bmatrix} K_{st,i} & 0 \\ 0 & K_{bn,i} \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$
Description	<p>A 2D component implementation of Hooke's law as seen in T5. <math>\delta n</math> is the displacement of the element normal to the surface and <math>\delta t</math> is the displacement of the element parallel to the surface. <math>P_{n,i}</math> is the net pressure acting normal to the surface, and <math>P_{t,i}</math> is the net pressure acting parallel to the surface. Pressure is used in place of force as the surface has not been normalized for it's length. The stiffness values <math>K_{n,i}</math> and <math>K_{t,i}</math> are then the resistance to displacement in the respective directions defined as in DD14. The pressure forces would be the result of applied loads on the mass, the product of the stiffness elements with the displacement would be the mass's reactive force that creates equilibrium with the applied forces after reaching the equilibrium displacement.</p>

Refname	T:displVect
Label	Displacement Vectors
Equation	$\varepsilon_i = \begin{bmatrix} \delta u_i \\ \delta v_i \end{bmatrix} = \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ -\sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} \delta_i = \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ -\sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$
Description	<p>Vectors describing the displacement of slice <math>i</math>. <math>\delta</math> is the displacement in the unrotated coordinate system, where <math>\delta x</math> is the displacement of the slice perpendicular to the direction of gravity, and <math>\delta y</math> is the displacement of the slice parallel to the force of gravity. <math>\varepsilon</math> is the displacement in the rotated coordinate system, where <math>\delta u</math> is the displacement of the slice parallel to the slice base, and <math>\delta v</math> is the displacement of the slice perpendicular to the slice base. <math>\varepsilon</math> can also be found by rotating <math>\delta</math> clockwise by the base angle, <math>\alpha</math> through a rotation matrix as shown.</p>

#### 4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. Definitions DD1 to DD8 are the force variables that can be solved by direct analysis of given inputs. The interslice forces DD9 are force variables that must be written in terms of DD1 to DD8 to solve.

Refname	DD:W.i
Label	Weight
Units	N
Equation	$W = b_i \begin{cases} (y_{us,i} - y_{slip,i}) \gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ (y_{us,i} - y_{wt,i}) \gamma + (y_{wt,i} - y_{slip,i}) \gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ (y_{us,i} - y_{slip,i}) \gamma, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	<p> <math>W</math> is the weight (N)  <math>b</math> is the base width of a slice (m)  <math>i</math> is the index  <math>y_{us}</math> is the y ordinate (m)  <math>y_{slip}</math> is the y ordinate (m)  <math>\gamma_{Sat}</math> is the saturated unit weight (<math>\frac{N}{m^3}</math>)  <math>y_{wt}</math> is the y ordinate (m)  <math>\gamma</math> is the dry unit weight (<math>\frac{N}{m^3}</math>) </p>
Refname	DD:U.b,i
Label	Base Hydrostatic Force
Units	N
Equation	$U_b = \ell_{b,i} \begin{cases} (y_{wt,i} - y_{slip,i}) \gamma_w, & y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	<p> <math>U_b</math> is the base hydrostatic force (N)  <math>\ell_b</math> is the total base length of a slice (m)  <math>i</math> is the index  <math>y_{wt}</math> is the y ordinate (m)  <math>y_{slip}</math> is the y ordinate (m)  <math>\gamma_w</math> is the unit weight of water (<math>\frac{N}{m^3}</math>) </p>

<b>Refname</b>	<b>DD:U.t,i</b>
Label	Surface Hydrostatic Force
Units	N
Equation	$U_t = \ell_{s,i} \begin{cases} (y_{wt,i} - y_{us,i}) \gamma_w, & y_{wt,i} > y_{us,i} \\ 0, & y_{wt,i} \leq y_{us,i} \end{cases}$
Description	<p><math>U_t</math> is the surface hydrostatic force (N)  <math>\ell_s</math> is the length of an interslice surface (m)  <math>i</math> is the index  <math>y_{wt}</math> is the y ordinate (m)  <math>y_{us}</math> is the y ordinate (m)  <math>\gamma_w</math> is the unit weight of water (<math>\frac{\text{N}}{\text{m}^3}</math>)</p>
<b>Refname</b>	<b>DD:H.i</b>
Label	Interslice Water Force
Units	N
Equation	$H = \begin{cases} \frac{(y_{us,i} - y_{slip,i})^2}{2} \gamma_{Sat} + (y_{wt,i} - y_{us,i})^2 \gamma_{Sat}, & y_{wt,i} \geq y_{us,i} \\ \frac{(y_{wt,i} - y_{slip,i})^2}{2} \gamma_{Sat}, & y_{us,i} > y_{wt,i} > y_{slip,i} \\ 0, & y_{wt,i} \leq y_{slip,i} \end{cases}$
Description	<p><math>H</math> is the interslice water force (N)  <math>y_{us}</math> is the y ordinate (m)  <math>i</math> is the index  <math>y_{slip}</math> is the y ordinate (m)  <math>\gamma_{Sat}</math> is the saturated unit weight (<math>\frac{\text{N}}{\text{m}^3}</math>)  <math>y_{wt}</math> is the y ordinate (m)</p>

<b>Refname</b>	<b>DD:alpha.i</b>
Label	Angle
Units	°
Equation	$\alpha = \frac{y_{slip,i} - y_{slip,i-1}}{x_{slip,i} - x_{slip,i-1}}$
Description	$\alpha$ is the angle (°) $y_{slip}$ is the y ordinate (m) $i$ is the index $x_{slip}$ is the x ordinate (m)
<b>Refname</b>	<b>DD:beta.i</b>
Label	Angle
Units	°
Equation	$\beta = \frac{y_{us,i} - y_{us,i-1}}{x_{us,i} - x_{us,i-1}}$
Description	$\beta$ is the angle (°) $y_{us}$ is the y ordinate (m) $i$ is the index $x_{us}$ is the x ordinate (m)
<b>Refname</b>	<b>DD:b.i</b>
Label	Base Width of a Slice
Units	m
Equation	$b = x_{slip,i} - x_{slip,i-1}$
Description	$b$ is the base width of a slice (m) $x_{slip}$ is the x ordinate (m) $i$ is the index

<b>Refname</b>	<b>DD:l.b,i</b>
Label	Total Base Length of a Slice
Units	m
Equation	$\ell_b = b_i \sec(\alpha_i)$
Description	$\ell_b$ is the total base length of a slice (m) $b$ is the base width of a slice (m) $i$ is the index $\alpha$ is the angle ( $^{\circ}$ )
<b>Refname</b>	<b>DD:l.s,i</b>
Label	Length of an Interslice Surface
Units	m
Equation	$\ell_s = b_i \sec(\beta_i)$
Description	$\ell_s$ is the length of an interslice surface (m) $b$ is the base width of a slice (m) $i$ is the index $\beta$ is the angle ( $^{\circ}$ )
<b>Refname</b>	<b>DD:K.c</b>
Label	Earthquake Load Factor
Units	Unitless
Equation	$K_c = K_c W_i$
Description	$K_c$ is the earthquake load factor $W$ is the weight (N) $i$ is the index

<b>Refname</b>	<b>DD:Q.i</b>
Label	Imposed Surface Load
Units	N
Equation	$Q = Q_i \omega_i$
Description	<p><math>Q</math> is the imposed surface load (N)  <math>i</math> is the index  <math>\omega</math> is the angle (<math>^{\circ}</math>)</p>
<b>Refname</b>	<b>DD:X.i</b>
Label	Interslice Shear Force
Units	N
Equation	$X = \lambda f_i E_i$
Description	<p><math>X</math> is the interslice shear force (N)  <math>\lambda</math> is the interslice normal/shear force ratio  <math>f</math> is the scaling function  <math>i</math> is the index  <math>E</math> is the interslice normal force (N)</p>



Refname	DD:R.i
Label	Resistive Shear Force
Units	N
Equation	$R = ((W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \cos(\alpha_i) + (-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \sin(\alpha_i) - U_{b,i}) / c'_i \sec(\alpha_i)$
Description	<p><math>R</math> is the resistive shear force (N)  <math>W</math> is the weight (N)  <math>i</math> is the index  <math>U_t</math> is the surface hydrostatic force (N)  <math>\beta</math> is the angle (<math>^\circ</math>)  <math>Q</math> is the imposed surface load (N)  <math>\omega</math> is the angle (<math>^\circ</math>)  <math>\alpha</math> is the angle (<math>^\circ</math>)  <math>K_c</math> is the earthquake load factor  <math>\Delta H</math> is the difference between interslice forces (N)  <math>U_b</math> is the base hydrostatic force (N)  <math>\varphi'</math> is the effective angle of friction (<math>^\circ</math>)  <math>c'</math> is the effective cohesion (Pa)  <math>b</math> is the base width of a slice (m)</p>

The resistive shear force of a slice is defined as  $P$  in GD3. The effective normal force in the equation for  $P$  of the soil is defined in the perpendicular force equilibrium of a slice from GD2, using the effective normal force  $N'$  of T4 shown in equation (1).

$$N'_i = (W_i - X_{i-1} + X_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \cos(\alpha_i) + (-K_c W_i - E_i + E_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \sin(\alpha_i) - U_{b,i} \quad (1)$$

The values of the interslice forces  $E$  and  $X$  in the equation are unknown, while the other values are found from the physical force definitions of DD1 to DD9. Consider a force equilibrium without the affect of interslice forces, to obtain a solvable value as done for  $N^*$  in equation (2).

$$N^*_i = (W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \cos(\alpha_i) + (-K_c W_i - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \sin(\alpha_i) - U_{b,i} \quad (2)$$

Using  $N_*$ , a resistive shear force without the influence of interslice forces for slice index  $i$  can be solved for in terms of all known values as done in equation (3).

$$\begin{aligned}
R_i &= N_* \tan(\varphi'_i) + c'_i b_i \sec(\alpha_i) \\
&= ((W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \cos(\alpha_i) \\
&\quad + (-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \sin(\alpha_i) - U_{b,i}) \tan(\varphi'_i) + c'_i b_i \sec(\alpha_i)
\end{aligned} \tag{3}$$

Refname	DD:T.i
Label	Mobilized Shear Force
Units	N
Equation	$T = \frac{(W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) - (-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i)}{\tan(\varphi'_i) + c'_i \sec(\alpha_i)}$
Description	<p><math>T</math> is the mobilized shear force (N)  <math>W</math> is the weight (N)  <math>i</math> is the index  <math>U_t</math> is the surface hydrostatic force (N)  <math>\beta</math> is the angle (<math>^\circ</math>)  <math>Q</math> is the imposed surface load (N)  <math>\omega</math> is the angle (<math>^\circ</math>)  <math>\alpha</math> is the angle (<math>^\circ</math>)  <math>K_c</math> is the earthquake load factor  <math>\Delta H</math> is the difference between interslice forces (N)</p>

The mobilized shear force acting on a slice is defined as  $S$  from the force equilibrium in GD2, also shown in equation (4).

$$\begin{aligned}
S_i &= (W_i - X_{i-1} + X_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) \\
&\quad + (-K_c W_i - E_i + E_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i)
\end{aligned} \tag{4}$$

The equation is unsolvable, containing the unknown interslice normal force  $E$  and interslice shear force  $X$ . Consider a force equilibrium without the influence of interslice forces, to obtain the mobilized shear force  $T$ , as done in equation (5).

$$\begin{aligned}
T_i &= (W_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) \\
&\quad - (-K_c W_i - \Delta H_i + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i)
\end{aligned} \tag{5}$$

The values of  $R$  and  $T$  are now defined completely in terms of the known force property values of DD1 to DD9.

<b>Refname</b>	<b>DD:pressure</b>
Label	Pressure
Units	Pa
Equation	$p = \begin{bmatrix} K_{st,i} & 0 \\ 0 & K_{bn,i} \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$

Description

$p$  is the pressure (Pa)  
 $K_{st}$  is the shear stiffness ( $\frac{\text{Pa}}{\text{m}}$ )  
 $i$  is the index  
 $K_{bn}$  is the normal stiffness ( $\frac{\text{Pa}}{\text{m}}$ )  
 $\delta x$  is the displacement (m)  
 $\delta y$  is the displacement (m)

<b>Refname</b>	<b>DD:pressure</b>
Label	Pressure
Units	Pa
Equation	$p = \begin{bmatrix} K_{bA,i} & K_{bB,i} \\ K_{bB,i} & K_{bA,i} \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix}$

Description

$p$  is the pressure (Pa)  
 $K_{bA}$  is the effective base stiffness A ( $\frac{\text{Pa}}{\text{m}}$ )  
 $i$  is the index  
 $K_{bB}$  is the effective base stiffness A ( $\frac{\text{Pa}}{\text{m}}$ )  
 $\delta x$  is the displacement (m)  
 $\delta y$  is the displacement (m)

Using the force-displacement relationship of GD8 to define stiffness matrix  $K_{st}$ , as seen in equation (6).

$$K_{st,i} = \begin{bmatrix} K_{st,i} & 0 \\ 0 & K_{bn,i} \end{bmatrix} \quad (6)$$

For interslice surfaces the stiffness constants and displacements refer to an unrotated coordinate system,  $\delta$  of GD9. The interslice elements are left in their standard coordinate system, and therefore are described by the same equation from GD8. Seen as  $K_{st}$  in DD12.  $K_{st}$  is the shear element in the matrix, and  $K_{sn}$  is the normal element in the matrix, calculated as in

DD14. For basal surfaces the stiffness constants and displacements refer to a system rotated for the base angle  $\alpha$  (DD5). To analyze the effect of force-displacement relationships occurring on both basal and interslice surfaces of an element  $i$  they must reference the same coordinate system. The basal stiffness matrix must be rotated counter clockwise to align with the angle of the basal surface. The base stiffness counter clockwise rotation is applied in equation (7) to the new matrix  $N^*$ .

$$\begin{aligned} K_{st,i} &= \begin{bmatrix} \cos(\alpha_i) & -\sin(\alpha_i) \\ \sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} K_{st,i} \\ &= \begin{bmatrix} K_{bt,i} \cos(\alpha_i) & -K_{bn,i} \sin(\alpha_i) \\ K_{bt,i} \sin(\alpha_i) & K_{bn,i} \cos(\alpha_i) \end{bmatrix} \end{aligned} \quad (7)$$

The Hooke's law force displacement relationship of GD8 applied to the base also references a displacement vector  $\varepsilon$  of GD9 rotated for the base angle of the slice  $\alpha$ . The basal displacement vector,  $\delta$  is rotated clockwise to align with the interslice displacement vector  $\delta$ , applying the definition of  $\varepsilon$  in terms of  $\delta$  as seen in GD9. Using this with base stiffness matrix  $K_{bt}$ , a basal force displacement relationship in the same coordinate system as the interslice relationship can be derived as done in equation (8).

$$\begin{aligned} \begin{bmatrix} p_i \\ p_i \end{bmatrix} &= K_{bt,i} \varepsilon \\ &= \begin{bmatrix} K_{bt,i} \cos(\alpha_i) & -K_{bn,i} \sin(\alpha_i) \\ K_{bt,i} \sin(\alpha_i) & K_{bn,i} \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} \cos(\alpha_i) & \sin(\alpha_i) \\ -\sin(\alpha_i) & \cos(\alpha_i) \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix} \\ &= \begin{bmatrix} K_{bt,i} \cos(\alpha_i)^2 + K_{sn,i} \sin(\alpha_i)^2 & (K_{bt,i} - K_{bn,i}) \sin(\alpha_i) \cos(\alpha_i) \\ (K_{bt,i} - K_{bn,i}) \sin(\alpha_i) \cos(\alpha_i) & K_{bt,i} \cos(\alpha_i)^2 + K_{sn,i} \sin(\alpha_i)^2 \end{bmatrix} \begin{bmatrix} \delta x_i \\ \delta y_i \end{bmatrix} \end{aligned} \quad (8)$$

The new effective base stiffness matrix  $K_{bt}$  as derived in equation (7) is defined in equation (9). This is seen as matrix  $K_{bt}$  in GD12.  $K_{bt}$  is the shear element in the matrix, and  $K_{bn}$  is the normal element in the matrix, calculated as in DD14. The notation is simplified by the introduction of the constants  $K_{bA}$  and  $K_{bB}$ , defined in equation (10) and equation (11) respectively.

$$\begin{aligned} K_{bt,i} &= \begin{bmatrix} K_{bt,i} \cos(\alpha_i)^2 + K_{sn,i} \sin(\alpha_i)^2 & (K_{bt,i} - K_{bn,i}) \sin(\alpha_i) \cos(\alpha_i) \\ (K_{bt,i} - K_{bn,i}) \sin(\alpha_i) \cos(\alpha_i) & K_{bt,i} \cos(\alpha_i)^2 + K_{sn,i} \sin(\alpha_i)^2 \end{bmatrix} \\ &= \begin{bmatrix} K_{bA,i} & K_{bB,i} \\ K_{bB,i} & K_{bA,i} \end{bmatrix} \end{aligned} \quad (9)$$

$$K_{bA,i} = K_{bt,i} \cos(\alpha_i)^2 + K_{bn,i} \sin(\alpha_i)^2 \quad (10)$$

$$K_{bB,i} = (K_{bt,i} - K_{bn,i}) \sin(\alpha_i) \cos(\alpha_i) \quad (11)$$

A force-displacement relationship for an element  $i$  can be written in terms of displacements occurring in the unrotated coordinate system  $\delta$  of GD9 using the matrix  $K_{bt}$ , and  $K_{bt}$  as seen in DD12.

Refname	DD:force
Label	Force
Units	N
Equation	$F = -\ell_{s,i-1}K_{sn,i-1}\delta_{i-1} + (\ell_{s,i-1}K_{sn,i-1} + \ell_{b,i}K_{bn,i} + \ell_{s,i}K_{sn,i})\delta_i - \ell_{s,i}K_{sn,i}\delta_{i+1}$
Description	$F$ is the force (N) $\ell_s$ is the length of an interslice surface (m) $i$ is the index $K_{sn}$ is the normal stiffness ( $\frac{\text{Pa}}{\text{m}}$ ) $\delta$ is the displacement (m) $\ell_b$ is the total base length of a slice (m) $K_{bn}$ is the normal stiffness ( $\frac{\text{Pa}}{\text{m}}$ )
Refname	DD:K.bt,i
Label	Shear Stiffness
Units	$\frac{\text{Pa}}{\text{m}}$
Equation	$K_{bt} = \frac{E}{2(1+\nu)} \frac{0.1}{b} + \frac{c'_i - \sigma \tan(\varphi'_i)}{ \delta u  + a}$
Description	$K_{bt}$ is the shear stiffness ( $\frac{\text{Pa}}{\text{m}}$ ) $E$ is the interslice normal force (N) $\nu$ is the Poisson's ratio $b$ is the base width of a slice (m) $c'$ is the effective cohesion (Pa) $i$ is the index $\sigma$ is the normal stress (Pa) $\varphi'$ is the effective angle of friction ( $^\circ$ ) $\delta u$ is the displacement (m) $a$ is the constant (m)

<b>Refname</b>	<b>DD:K.bn,i</b>
Label	Normal Stiffness
Units	$\frac{\text{Pa}}{\text{m}}$
Equation	$K_{bn} = \begin{cases} \frac{E(1-\nu)}{(1+\nu)(1-2\nu+b)}, & \nu < 0 \\ 0.01 \frac{E(1-\nu)}{(1+\nu)(1-2\nu+b)} + \frac{\kappa}{\delta v + A}, & \nu \geq 0 \end{cases}$
Description	<p> <math>K_{bn}</math> is the normal stiffness (<math>\frac{\text{Pa}}{\text{m}}</math>)  <math>E</math> is the interslice normal force (N)  <math>\nu</math> is the Poisson's ratio  <math>b</math> is the base width of a slice (m)  <math>\kappa</math> is the constant (Pa)  <math>\delta v</math> is the displacement (m)  <math>A</math> is the constant (m) </p>
<b>Refname</b>	<b>DD:fixme1</b>
Label	Fixme
Units	Unitless
Equation	$SpencerFixme1Please = E_i + E_{i-1}$
Description	<p> <math>SpencerFixme1Please</math> is the fixme  <math>E</math> is the interslice normal force (N)  <math>i</math> is the index </p>

Refname	DD:fixme2
Label	Fixme
Units	Unitless
Equation	$SpencerFixme2Please = H_i + H_{i-1}$
Description	<p><i>SpencerFixme2Please</i> is the fixme</p> <p><math>H</math> is the interslice water force (N)</p> <p><math>i</math> is the index</p>

#### 4.2.5 Instance Models

This section transforms the problem defined in Section ?? into one which is expressed in mathematical terms. It uses concrete symbols defined in Section ?? to replace the abstract symbols in the models identified in Section ?? and Section ??. The Morgenstern Price method is a vertical slice, limit equilibrium slope stability analysis method. Analysis is performed by breaking the assumed failure surface into a series of vertical slices of mass. Static equilibrium analysis using two force equilibrium, and one moment equation as in T2. The problem is statically indeterminate with only these 3 equations and one constitutive equation (the Mohr Coulomb shear strength of T3) so the assumption of GD5 is used. Solving for force equilibrium allows definitions of all forces in terms of the physical properties of DD1 to DD9, as done in DD10, DD11. The values of the interslice normal force  $E$  the interslice normal/shear force ratio  $\lambda$ , and the Factor of Safety ( $FS$ ), are unknown. Equations for the unknowns are written in terms of only the values in DD1 to DD9, the values of  $R$ , and  $T$  in DD10 and DD11, and each other. The relationships between the unknowns are non linear, and therefore explicit equations cannot be derived and an iterative solutions method is required.

Refname	T:fctSfty
Label	Factor of Safety
Equation	$FS = \frac{\sum_{v=1}^{n-1} R_v \prod_{u=i}^{n-1} \frac{\Psi_u}{\Phi_u} + R_n}{\sum_{v=1}^{n-1} T_v \prod_{u=i}^{n-1} \frac{\Psi_u}{\Phi_u} + T_n}$

Description      Equation for the Factor of Safety is the ratio between resistive and mobile shear of the slip surface. The sum of values from each slice is taken to find the total resistive and mobile shear for the slip surface. The constants  $\Phi$  and  $\Psi$  convert the resistive and mobile shear without the influence of interslice forces, to a calculation considering the interslice forces.

Using equation (21) from IM3, rearranging, and applying the boundary condition that  $E_0$  and  $E_n$  are equal to 0, an equation for the factor of safety is found as equation (12), also seen in IM1.

$$FS = \frac{\sum_{v=1}^{n-1} R_v \prod_{u=i}^{n-1} \frac{\Psi_u}{\Phi_u} + R_n}{\sum_{v=1}^{n-1} T_v \prod_{u=i}^{n-1} \frac{\Psi_u}{\Phi_u} + T_n} \quad (12)$$

The constants  $\Psi$  and  $\Phi$  described in equation (20) and equation (19) are functions of the unknowns: the interslice normal/shear force ratio  $\lambda$  (IM2) and the factor of safety  $FS$  (IM1).



Refname	T:nrmShrFor
Label	Normal/shear Force Ratio
Equation	$C1_i = \begin{cases} b_1 (E_1 + H_1) \tan(\alpha_1), \\ b_i (SpencerFixme1Please + SpencerFixme2Please) \tan(\alpha_i) + h (K_c W_i - \\ b_n (E_{n-1} + H_{n-1}) \tan(\alpha_{n-1}), \end{cases}$ $C2_i = \begin{cases} b_1 f_1 E_1, & i = 1 \\ b_i (f_i E_i + f_{i-1} E_{i-1}), & 2 \leq i \leq n-1 \\ b_n E_{n-1} H_{n-1}, & i = n \end{cases} \quad \lambda = \frac{\sum_{i=1}^n C1_i}{\sum_{i=1}^n C2_i}$

Description  $\lambda$  is the magnitude ratio between shear and normal forces at the interslice interfaces as the assumption of the Morgenstern Price method in GD5 The inclination function  $f$  determines the relative magnitude ratio between the different interslices, while  $\lambda$  determines the magnitude.  $\lambda$  uses the sum of interslice normal and shear forces taken from each interslice.

Taking the last static equation of T2 with the moment equilibrium of GD6 about the mid-point of the base and the assumption of GD5 results in equation (13).

$$\begin{aligned}
0 = & -E_i \left( z_i - \frac{b_i}{2} \tan(\alpha_i) \right) + E_{i-1} \left( z_{i-1} - \frac{b_i}{2} \tan(\alpha_i) \right) \\
& - H_i \left( z_i - \frac{b_i}{2} \tan(\alpha_i) \right) + H_{i-1} \left( z_{i-1} - \frac{b_i}{2} \tan(\alpha_i) \right) \\
& - \lambda \frac{b_i}{2} (E_i f_i + E_{i-1} f_{i-1}) + \frac{K_c W_i h_i}{2} - U_{t,i} \sin(\beta_i) h_i - Q_i \sin(\omega_i) h_i
\end{aligned} \tag{13}$$

The equation in terms of  $\lambda$  leads to equation (14).

$$\begin{aligned}
& \lambda \\
= & \frac{-E_i \left( z_i - \frac{b_i}{2} \tan(\alpha_i) \right) + E_{i-1} \left( z_{i-1} - \frac{b_i}{2} \tan(\alpha_i) \right) - H_i \left( z_i - \frac{b_i}{2} \tan(\alpha_i) \right) + H_{i-1} \left( z_{i-1} - \frac{b_i}{2} \tan(\alpha_i) \right) + \frac{K_c W_i}{2}}{\frac{b_i}{2} (E_i f_i + E_{i-1} f_{i-1})}
\end{aligned} \tag{14}$$

Taking a summation of each slice, and applying the boundary condition that  $E_0$  and  $E_n$  are equal to 0, a general equation for the constant  $\lambda$  is developed in equation (15), also found

in IM2.

$$\lambda_i \tag{15}$$

$$= \frac{\sum_{i=1}^n b_i (SpencerFixme1Please + SpencerFixme2Please) \tan(\alpha_i) + h_i (K_c W_i - 2U_{t,i} \sin(\beta_i) - 2Q_i \sin(\omega_i))}{\sum_{i=1}^n b_i (E_i f_i + E_{i-1} f_{i-1})}$$

equation (15) for  $\lambda$ , is a function of the unknown interslice normal force  $E$  IM3.

Refname	T:intsliceFs
Label	Interslice Forces
Equation	$E_i = \begin{cases} \frac{FST_1 - R_1}{\Phi_1}, & i = 1 \\ \frac{\Psi_{i-1} E_{i-1} + FST_i - R_i}{\Phi_i}, & 2 \leq i \leq n-1 \\ 0, & i = 0 \vee i = n \end{cases}$

Description The value of the interslice normal force  $E$  at interface  $i$  The net force. is the weight of the slices adjacent to interface  $i$  exert horizontally on each other.

Taking the normal force equilibrium of GD1 with the effective stress definition from T4 that  $N_i = N'_i - U_{b,i}$ , and the assumption of GD5 the equilibrium equation can be rewritten as equation (16).

$$N'_i = (W_i - \lambda f_{i-1} E_{i-1} + \lambda f_i E_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \cos(\alpha_i) + (-K_c W_i - E_i + E_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \sin(\alpha_i) - U_{b,i} \tag{16}$$

Taking the base shear force equilibrium of GD2 with the definition of mobilized shear force from GD4 and the assumption of GD5, the equilibrium equation can be rewritten as equation (17).

$$\frac{N_i \tan(\varphi'_i) + c'_i b_i \sec(\alpha_i)}{FS} = (W_i - \lambda f_{i-1} E_{i-1} + \lambda f_i E_i + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i)) \sin(\alpha_i) + (-K_c W_i - E_i + E_{i-1} - H_i + H_{i-1} + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i)) \cos(\alpha_i) \tag{17}$$

Substituting the equation for  $N'$  from equation (16) into equation (17) and rearranging results in equation (18).

$$E_i ((\lambda f_i \cos(\alpha_i) - \sin(\alpha_i)) \tan(\varphi'_i) - (\lambda f_i \sin(\alpha_i) - \cos(\alpha_i)) FS) = E_{i-1} ((\lambda f_{i-1} \cos(\alpha_i) - \sin(\alpha_i)) \tan(\varphi'_i) - (\lambda f_{i-1} \sin(\alpha_i) - \cos(\alpha_i)) FS) + FST_i - R_i \tag{18}$$

Where  $R$  and  $T$  are the resistive and mobile shear of the slice, without the influence of interslice forces  $E$  and  $X$ , as defined in DD10 and DD11 Making use of the constants, and with full equations found below in equation (19) and equation (20) respectively, then equation (18) can be simplified to equation (21), also seen in IM3.

$$\Phi_i = (\lambda f_i \cos(\alpha_i) - \sin(\alpha_i)) \tan(\varphi'_i) - (\lambda f_i \sin(\alpha_i) - \cos(\alpha_i)) FS \quad (19)$$

$$\Psi_i = (\lambda f_i \cos(\alpha_{i+1}) - \sin(\alpha_{i+1})) \tan(\varphi'_i) - (\lambda f_i \sin(\alpha_{i+1}) - \cos(\alpha_{i+1})) FS \quad (20)$$

$$E_i = \frac{\Psi_{i-1} E_{i-1} + FST_i - R_i}{\Phi_i} \quad (21)$$

The constants  $\Psi$  and  $\Phi$  described in equation (20) and equation (19) are functions of the unknowns: the interslice normal/shear force ratio  $\lambda$  (IM2) and the factor of safety  $FS$  (IM1).

Refname	T:forDisEqlb
Label	Force Displacement Equilibrium
Equation	$ \begin{aligned} -\Delta H_i - K_c W_i - U_{b,i} \sin(\alpha_i) + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i) &= \\ \delta x_{i-1} - \ell_{s,i-1} K_{sn,i-1} + \delta x_i (-\ell_{s,i-1} K_{sn,i-1} + \ell_{s,i} K_{sn,i} + \ell_{b,i} K_{bA,i}) + \\ \delta x_{i+1} - \ell_{s,i} K_{sn,i} + \delta y_i - \ell_{b,i} K_{bB,i} &= -W_i - U_{b,i} \cos(\alpha_i) + \\ U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i) &= \delta y_{i-1} - \ell_{s,i-1} K_{st,i-1} + \\ \delta y_i (-\ell_{s,i-1} K_{st,i-1} + \ell_{s,i} K_{sn,i} + \ell_{b,i} K_{bA,i}) + \delta y_{i+1} - \ell_{s,i} K_{st,i} + \\ \delta x_i - \ell_{b,i} K_{bB,i} \end{aligned} $
Description	<p>There is one set of force displacement equilibrium equations in the x and y directions for each element. System of equations solved for displacements ( <math>\delta x</math> and <math>\delta y</math> ) <math>\Delta H = H</math> is the net hydrostatic force across a slice. <math>K_c</math> is the earthquake load factor. <math>W</math> is the weight of the slice. <math>U_b</math> is the pore water pressure acting on the slice base. <math>U_t</math> is the pore water pressure acting on the slice surface. <math>\alpha</math> is the angle of the base with the horizontal. <math>\beta</math> is the angle of the surface with the horizontal. <math>\delta x</math> is the x displacement of slice i. <math>\delta y</math> is the y displacement of slice i. <math>\ell_s</math> is the length of the interslice surface i. <math>\ell_b</math> is the length of the base surface i. <math>K_{st}</math> is the interslice shear stiffness at surface i. <math>K_{st,i-1}</math> is the interslice normal stiffness at surface i. <math>K_{bA,i}</math>, and <math>K_{bB,i}</math> are the base stiffness values for slice i.</p>

Using the net force-displacement equilibrium equation of a slice from DD13 with the definitions of the stiffness matrices from DD12 and the force definitions from GD7 a broken down force displacement equilibrium equation can be derived. equation (22) gives the broken

down equation in the  $x$  direction, and equation (23) gives the broken down equation in the  $y$  direction.

$$\begin{aligned}
& -\Delta H_i - K_c W_i - U_{b,i} \sin(\alpha_i) + U_{t,i} \sin(\beta_i) + Q_i \sin(\omega_i) \\
& = \delta x_{i-1} - \ell_{s,i-1} K_{sn,i-1} + \delta x_i (-\ell_{s,i-1} K_{sn,i-1} + \ell_{s,i} K_{sn,i} + \ell_{b,i} K_{bA,i}) \\
& \quad + \delta x_{i+1} - \ell_{s,i} K_{sn,i} + \delta y_i - \ell_{b,i} K_{bB,i} \\
& = -W_i - U_{b,i} \cos(\alpha_i) + U_{t,i} \cos(\beta_i) + Q_i \cos(\omega_i) \\
& = \delta y_{i-1} - \ell_{s,i-1} K_{st,i-1} + \delta y_i (-\ell_{s,i-1} K_{st,i-1} + \ell_{s,i} K_{sn,i} + \ell_{b,i} K_{bA,i}) \\
& \quad + \delta y_{i+1} - \ell_{s,i} K_{st,i} + \delta x_i - \ell_{b,i} K_{bB,i}
\end{aligned} \tag{22}$$

Using the known input assumption of A2, the force variable definitions of DD1 to DD8 on the left side of the equations can be solved for. The only unknown in the variables to solve for the stiffness values from DD14 is the displacements. Therefore taking the equation from each slice a set of  $2n$  equations, with  $2n$  unknown displacements in the  $x$  and  $y$  directions of each slice can be derived. Solutions for the displacements of each slice can then be found. The use of displacement in the definition of the stiffness values makes the equation implicit, which means an iterative solution method, with an initial guess for the displacements in the stiffness values is required.

Refname	T:rfemFoS
Label	RFEM Factor of Safety
Equation	$FS_{Loc,i} = \frac{c'_i - K_{bn,i} \delta v_i \tan(\varphi'_i)}{K_{bt,i} \delta u_i} = \frac{\sum_{i=1}^n \ell_{b,i} (c'_i - K_{bn,i} \delta v_i \tan(\varphi'_i))}{\sum_{i=1}^n \ell_{b,i} K_{bt,i} \delta u_i}$
Description	<p><math>FS_{Loc,i}</math> is the factor of safety for slice <math>i</math>. <math>FS</math> is the factor of safety for the entire slip surface. <math>c'</math> is the cohesion of slice <math>i</math>'s base. <math>\varphi'</math> is the effective angle of friction of slice <math>i</math>'s base. <math>\delta v</math> is the normal displacement of slice <math>i</math>. <math>\delta u</math> is the shear displacement of slice <math>i</math>. <math>K_{bt}</math> is the length of the base of slice <math>i</math>. <math>K_{bn}</math> is the base normal stiffness at surface <math>i</math>. <math>n</math> is the number of slices in the slip surface.</p>

RFEM analysis can also be used to calculate the factor of safety for the slope. For a slice element  $i$  the displacements  $\delta x$  and  $\delta y$ , are solved from the system of equations in IM4. The definition of  $\varepsilon$  as the rotation of the displacement vector  $\delta$  is seen in GD9. This is used to find the displacements of the slice parallel to the base of the slice  $\delta u$  in equation (24) and normal to the base of the slice  $\delta v$  in equation (25).

$$\delta u_i = \cos(\alpha_i) \delta x_i + \sin(\alpha_i) \delta y_i \tag{23}$$

$$\delta v_i = -\sin(\alpha_i) \delta x_i + \sin(\alpha_i) \delta y_i \quad (24)$$

With the definition of normal stiffness from DD14 to find the normal stiffness of the base  $K_{bn}$  and the now known base displacement perpendicular to the surface  $\delta v$  from equation (25) the normal base stress can be calculated from the force-displacement relationship of T5. Stress  $\sigma$  is used in place of force  $F$  as the stiffness hasn't been normalized for the length of the base. Results in equation (26).

$$\sigma_i = K_{bn,i} \delta v_i \quad (25)$$

The resistive shear to calculate the factor of safety  $FS$  is found from the Mohr Coulomb resistive strength of soil in T3 Using the normal stress  $\sigma$  from equation (26) as the stress, the resistive shear of the slice can be calculated from equation (27).

$$s_i = c'_i - \sigma_i \tan(\varphi'_i) \quad (26)$$

Previously the value of the shear stiffness  $K_{bt}$  as seen in equation (28) was unsolvable because the normal stress  $\sigma$  was unknown. With the definition of  $\sigma$  from equation (26) and the definition of displacement shear to the base  $\delta u$  from equation (25), the value of  $K_{bt}$  becomes solvable.

$$K_{bt,i} = \frac{E_i}{2(1 + \nu_i)} \frac{0.1}{b_i} + \frac{c'_i - \sigma_i \tan(\varphi'_i)}{|\delta u_i| + a} \quad (27)$$

With shear stiffness  $K_{bt}$  calculated in equation (28) and shear displacement  $\delta u$  calculated in equation (24) values now known the resistive shear stress acting on the base of a slice  $\tau$  can be calculated using T5, as done in equation (29). Again, stress  $\tau$  is used in place of force  $F$  as the stiffness has not been normalized for the length of the base.

$$\tau_i = K_{bt,i} \delta u_i \quad (28)$$

The resistive shear stress acting on the base of a slice  $\tau$  acts as the mobile shear acting on the base. Using the definition Factor of Safety equation from T1, with the definitions of resistive shear strength of a slice  $s$  from equation (27) and resistive shear stress  $\tau$  from equation (29) the local factor of safety  $FS_{Loc,i}$  can be found from as seen in equation (30) and IM5.

$$\begin{aligned} FS_{Loc,i} &= \frac{s_i}{\tau_i} \\ &= \frac{c'_i - K_{bn,i} \delta v_i \tan(\varphi'_i)}{K_{bt,i} \delta u_i} \end{aligned} \quad (29)$$

The global Factor of Safety is then the ratio of the summation of the resistive and mobile shears for each slice, with a weighting for the length of the slice's base. Shown in equation

(31) and IM5.

$$\begin{aligned}
 FS &= \frac{\sum_{i=1}^n \ell_{b,i} s_i}{\sum_{i=1}^n \ell_{b,i} \tau_i} \\
 &= \frac{\sum_{i=1}^n \ell_{b,i} (c'_i - K_{bn,i} \delta v_i \tan(\varphi'_i))}{\sum_{i=1}^n \ell_{b,i} K_{bt,i} \delta u_i}
 \end{aligned} \tag{30}$$

Refname	T:crtSlpId
Label	Critical Slip Identification
Equation	$FS_{min} = \Upsilon(\{x_{cs}, y_{cs}\})$
Description	Given the necessary slope inputs, a minimization algorithm or function $\Upsilon$ will identify the critical slip surface of the slope, with the critical slip coordinates $\{x_{cs}, y_{cs}\}$ and the minimum factor of safety $FS_{min}$ that results.

#### 4.2.6 Data Constraints

Table ??, and Table ?? shows the data constraints on the input and output variables, respectively. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Typical Value	TU
$E$	$E > 0$	15000.0 Pa	0.1
$c'$	$c' > 0$	10.0 Pa	0.1
$\nu$	$0 < \nu < 1$	0.4	0.1
$\varphi'$	$0 < \varphi' < 90$	25.0 °	0.1
$\gamma$	$\gamma > 0$	20.0 $\frac{\text{N}}{\text{m}^3}$	0.1

$\gamma_{Sat}$	$\gamma_{Sat} > 0$	$20.0 \frac{\text{N}}{\text{m}^3}$	0.1
$\gamma_w$	$\gamma_w > 0$	$9.8 \frac{\text{N}}{\text{m}^3}$	0.1
$a$	None	0.0 m	0.1
$A$	None	0.0 m	0.1
$\kappa$	None	0.0 Pa	0.1

Table 2: Input Data Constraints

Var	Physical Constraints
$FS$	$FS > 0$
$(x, y)$	None
$\delta x$	None
$\delta y$	None

Table 3: Output Data Constraints

## 5 Requirements

This section provides the functional requirements, the business tasks that the software is expected to complete, and the non-functional requirements, the qualities that the software is expected to exhibit.

### 5.1 Functional Requirements

- R1: Read the input file, and store the data. Necessary input data summarized in Table ??.
- R2: Generate potential critical slip surface's for the input slope.
- R3: Test the slip surfaces to determine if they are physically realizable based on a set of pass or fail criteria.
- R4: Prepare the slip surfaces for a method of slices or limit equilibrium analysis.
- R5: Calculate the factors of safety of the slip surfaces.
- R6: Rank and weight the slopes based on their factor of safety, such that a slip surface with a smaller factor of safety has a larger weighting.
- R7: Generate new potential critical slip surfaces based on previously analysed slip surfaces with low factors of safety.

- R8: Repeat requirements R3 to R7 until the minimum factor of safety remains approximately the same over a predetermined number of repetitions. Identify the slip surface that generates the minimum factor of safety as the critical slip surface.
- R9: Prepare the critical slip surface for method of slices or limit equilibrium analysis.
- R10: Calculate the factor of safety of the critical slip surface using the Morgenstern Price method.
- R11: Display the critical slip surface and the slice element displacements graphically. Give the values of the factors of safety calculated by the Morgenstern Price method.

Symbol	Unit	Name
$(x, y)$	m	cartesian position coordinates
$E$	Pa	elastic modulus
$c'$	Pa	effective cohesion
$\nu$		Poisson's ratio
$\phi'$	°	effective angle of friction
$\gamma$	$\frac{\text{N}}{\text{m}^3}$	dry unit weight
$\gamma_{Sat}$	$\frac{\text{N}}{\text{m}^3}$	saturated unit weight
$\gamma_w$	$\frac{\text{N}}{\text{m}^3}$	unit weight of water
$a$	m	constant
$A$	m	constant
$\kappa$	Pa	constant

Table 4: Required Inputs

## 5.2 Non-Functional Requirements

SSA is intended to be an educational tool, so accuracy and performance are not priorities. Rather, the non-functional requirement priorities are correctness, understandability, reusability, and maintainability.

## 6 Likely Changes

## 7 Values of Auxiliary Constants

There are no auxiliary constants.



## 8 References