

# Software Requirements Specification for Pendulum

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# 1 Reference Material

This section records information for easy reference.

## 1.1 Table of Units

The unit system used throughout is SI (Système International d'Unités). In addition to the basic units, several derived units are also used. For each unit, [Tab:ToU](#) lists the symbol, a description and the SI name.

Symbol	Description	SI Name
°	angle	degree
Hz	frequency	hertz
kg	mass	kilogram
m	length	metre
N	force	newton
rad	angle	radian
s	time	second

Table 1: Table of Units

## 1.2 Table of Symbols

The symbols used in this document are summarized in [Tab:ToS](#) along with their units. Throughout the document, symbols in bold will represent vectors, and scalars otherwise. The symbols are listed in alphabetical order. For vector quantities, the units shown are for each component of the vector.

Symbol	Description	Units
$a_x$	$x$ -component of acceleration	$\frac{\text{m}}{\text{s}^2}$
$a_y$	$y$ -component of acceleration	$\frac{\text{m}}{\text{s}^2}$
<b>a</b>	Acceleration	$\frac{\text{m}}{\text{s}^2}$
<b>F</b>	Force	N
$f$	Frequency	Hz
<b>g</b>	Gravitational acceleration	$\frac{\text{m}}{\text{s}^2}$
<b>I</b>	Moment of inertia	$\text{kgm}^2$
$\hat{\mathbf{i}}$	Unit Vector	—
$L_{\text{rod}}$	Length of the rod	m
$m$	Mass	kg
$p_x$	$x$ -component of position	m
$p_x^i$	$x$ -component of initial position	m
$p_y$	$y$ -component of position	m

Symbol	Description	Units
$p_y^i$	$y$ -component of initial position	m
$\mathbf{p}$	Position	m
$T$	Period	s
$\mathbf{T}$	Tension	N
$t$	Time	s
$v_x$	$x$ -component of velocity	$\frac{\text{m}}{\text{s}}$
$v_y$	$y$ -component of velocity	$\frac{\text{m}}{\text{s}}$
$\mathbf{v}$	Velocity	$\frac{\text{m}}{\text{s}}$
$\alpha$	Angular Acceleration	$\frac{\text{rad}}{\text{s}^2}$
$\theta$	Angular Displacement	rad
$\theta_i$	Initial pendulum angle	rad
$\theta_p$	Displacement angle of the pendulum	°
$\pi$	Ratio of circumference to diameter for any circle	—
	Torque	Nm
$\Omega$	Angular frequency	s
$\omega$	Angular Velocity	$\frac{\text{rad}}{\text{s}}$

Table 2: Table of Symbols

### 1.3 Abbreviations and Acronyms

Abbreviation	Full Form
2D	Two-Dimensional
A	Assumption
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
PS	Physical System Description
R	Requirement
SRS	Software Requirements Specification
TM	Theoretical Model
Uncert.	Typical Uncertainty

Table 3: Abbreviations and Acronyms

## 2 Introduction

A pendulum consists of mass attached to the end of a rod and its moving curve is highly sensitive to initial conditions. Therefore, it is useful to have a program to simulate the motion of the pendulum to exhibit its chaotic characteristics. The program documented here is called pendulum.

The following section provides an overview of the Software Requirements Specification (SRS) for Pendulum. This section explains the purpose of this document, the scope of the requirements, the characteristics of the intended reader, and the organization of the document.

### 2.1 Scope of Requirements

The scope of the requirements includes the analysis of a two-dimensional (2D) pendulum motion problem with various initial conditions..

## 3 Specific System Description

This section first presents the problem description, which gives a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used.

### 3.1 Problem Description

A system is needed to efficiently and correctly to predict the motion of a pendulum.

#### 3.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meaning, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements.

- Gravity: The force that attracts one physical body with mass to another.
- Cartesian coordinate system: A coordinate system that specifies each point uniquely in a plane by a set of numerical coordinates, which are the signed distances to the point from two fixed perpendicular oriented lines, measured in the same unit of length (from [2]).

#### 3.1.2 Physical System Description

The physical system of Pendulum, as shown in [Fig:pendulum](#), includes the following elements:

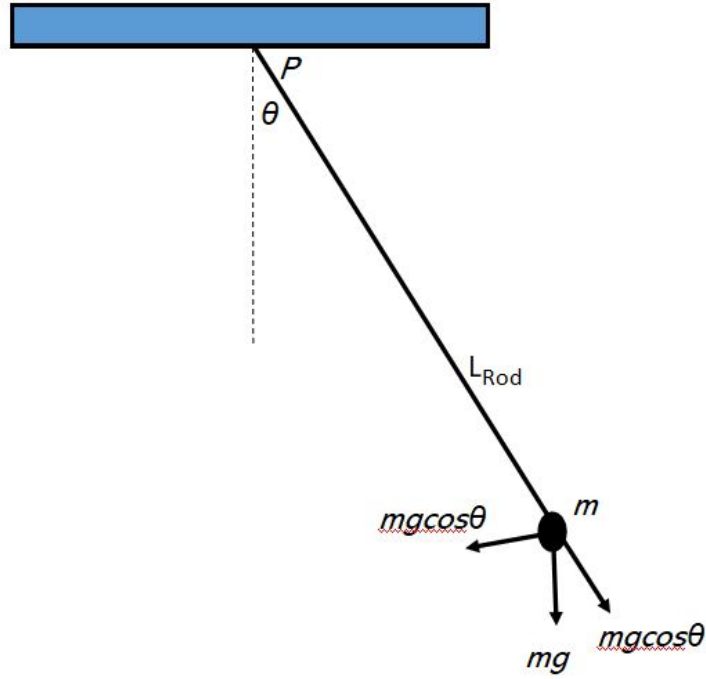


Figure 1: The physical system

PS1: The rod.

PS2: The mass.

### 3.1.3 Goal Statements

Given the mass and length of the rod, initial angle of the mass and the gravitational constant, the goal statements are:

Motion-of-the-mass: Calculate the motion of the mass.

## 3.2 Solution Characteristics Specification

The instance models that govern Pendulum are presented in [Sec:Instance Models](#). The information to understand the meaning of the instance models and their derivation is also presented, so that the instance models can be verified.

### 3.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical models by filling in the missing information for the physical system. The assumptions refine the scope by providing more detail.

pend2DMotion: The pendulum motion is two-dimensional (2D).

cartCoord: A Cartesian coordinate system is used.

cartCoordRight: The Cartesian coordinate system is right-handed where positive  $x$ -axis and  $y$ -axis point right up.

yAxisDir: The direction of the  $y$ -axis is directed opposite to gravity.

startOrigin: The pendulum is attached to the origin.

### 3.2.2 Theoretical Models

This section focuses on the general equations and laws that Pendulum is based on.

Refname	TM:acceleration
Label	Acceleration
Equation	$\mathbf{a} = \frac{d\mathbf{v}}{dt}$
Description	$\mathbf{a}$ is the acceleration ( $\frac{\text{m}}{\text{s}^2}$ ) $t$ is the time (s) $\mathbf{v}$ is the velocity ( $\frac{\text{m}}{\text{s}}$ )
Source	<a href="#">[1]</a>
RefBy	

Refname	TM:velocity
Label	Velocity
Equation	$\mathbf{v} = \frac{d\mathbf{p}}{dt}$
Description	<p><math>\mathbf{v}</math> is the velocity (<math>\frac{\text{m}}{\text{s}}</math>)</p> <p><math>t</math> is the time (s)</p> <p><math>\mathbf{p}</math> is the position (m)</p>
Source	<a href="#">[3]</a>
RefBy	

Refname	TM:NewtonSecLawMot
Label	Newton's second law of motion
Equation	$\mathbf{F} = m\mathbf{a}$
Description	<p><math>\mathbf{F}</math> is the force (N)</p> <p><math>m</math> is the mass (kg)</p> <p><math>\mathbf{a}</math> is the acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)</p>
Notes	The net force $\mathbf{F}$ on a body is proportional to the acceleration $\mathbf{a}$ of the body, where $m$ denotes the mass of the body as the constant of proportionality.
Source	—
RefBy	



Refname	TM:NewtonSecLawRotMot
Label	Newton's second law for rotational motion
Equation	$\tau = \mathbf{I}\alpha$
Description	<p><math>\tau</math> is the torque (Nm)</p> <p><math>\mathbf{I}</math> is the moment of inertia (<math>\text{kgm}^2</math>)</p> <p><math>\alpha</math> is the angular acceleration (<math>\frac{\text{rad}}{\text{s}^2}</math>)</p>
Notes	The net torque on a rigid body is proportional to its angular acceleration $\alpha$ , where $\mathbf{I}$ denotes the moment of inertia of the rigid body as the constant of proportionality.
Source	—
RefBy	IM:calOfAngularDisplacement and GD:angFrequencyGD

### 3.2.3 General Definitions

This section collects the laws and equations that will be used to build the instance models.

Refname	GD:velocityIX
Label	The $x$ -component of velocity of the pendulum
Units	$\frac{\text{m}}{\text{s}}$
Equation	$v_x = \omega L_{\text{rod}} \cos(\theta_p)$
Description	$v_x$ is the $x$ -component of velocity ( $\frac{\text{m}}{\text{s}}$ ) $\omega$ is the angular velocity ( $\frac{\text{rad}}{\text{s}}$ ) $L_{\text{rod}}$ is the length of the rod (m) $\theta_p$ is the displacement angle of the pendulum ( $^\circ$ )
Source	—
RefBy	

**Detailed derivation of the  $x$ -component of velocity:** At a given point in time, velocity may be defined as

$$\mathbf{v} = \frac{d\mathbf{p}}{dt}$$

We also know the horizontal position

$$p_x = L_{\text{rod}} \sin(\theta_p)$$

Applying this,

$$v_x = \frac{dL_{\text{rod}} \sin(\theta_p)}{dt}$$

$L_{\text{rod}}$  is constant with respect to time, so

$$v_x = L_{\text{rod}} \frac{d \sin(\theta_p)}{dt}$$

Therefore, using the chain rule,

$$v_x = \omega L_{\text{rod}} \cos(\theta_p)$$

Refname	GD:velocityIY
Label	The $y$ -component of velocity of the pendulum
Units	$\frac{\text{m}}{\text{s}}$
Equation	$v_y = \omega L_{\text{rod}} \sin(\theta_p)$
Description	$v_y$ is the $y$ -component of velocity ( $\frac{\text{m}}{\text{s}}$ ) $\omega$ is the angular velocity ( $\frac{\text{rad}}{\text{s}}$ ) $L_{\text{rod}}$ is the length of the rod (m) $\theta_p$ is the displacement angle of the pendulum ( $^\circ$ )
Source	—
RefBy	

**Detailed derivation of the  $y$ -component of velocity:** At a given point in time, velocity may be defined as

$$\mathbf{v} = \frac{d\mathbf{p}}{dt}$$

We also know the vertical position

$$p_y = -L_{\text{rod}} \cos(\theta_p)$$

Applying this again,

$$v_y = - \left( \frac{dL_{\text{rod}} \cos(\theta_p)}{dt} \right)$$

$L_{\text{rod}}$  is constant with respect to time, so

$$v_y = -L_{\text{rod}} \frac{d \cos(\theta_p)}{dt}$$

Therefore, using the chain rule,

$$v_y = \omega L_{\text{rod}} \sin(\theta_p)$$

Therefore, using the chain rule,

Refname	GD:accelerationIX
Label	The $x$ -component of acceleration of the pendulum
Units	$\frac{\text{m}}{\text{s}^2}$
Equation	$a_x = -\omega^2 L_{\text{rod}} \sin(\theta_p) + \alpha L_{\text{rod}} \cos(\theta_p)$
Description	<p><math>a_x</math> is the <math>x</math>-component of acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)</p> <p><math>\omega</math> is the angular velocity (<math>\frac{\text{rad}}{\text{s}}</math>)</p> <p><math>L_{\text{rod}}</math> is the length of the rod (m)</p> <p><math>\theta_p</math> is the displacement angle of the pendulum (<math>^\circ</math>)</p> <p><math>\alpha</math> is the angular acceleration (<math>\frac{\text{rad}}{\text{s}^2}</math>)</p>
Source	—
RefBy	

**Detailed derivation of the  $x$ -component of acceleration:** Our acceleration is:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

Earlier, we found the horizontal velocity to be

$$v_x = \omega L_{\text{rod}} \cos(\theta_p)$$

Applying this to our equation for acceleration

$$a_x = \frac{d\omega L_{\text{rod}} \cos(\theta_p)}{dt}$$

By the product and chain rules, we find

$$a_x = \frac{d\omega}{dt} L_{\text{rod}} \cos(\theta_p) - \omega L_{\text{rod}} \sin(\theta_p) \frac{d\theta_p}{dt}$$

Simplifying,

$$a_x = -\omega^2 L_{\text{rod}} \sin(\theta_p) + \alpha L_{\text{rod}} \cos(\theta_p)$$

Refname	GD:accelerationIY
Label	The $y$ -component of acceleration of the pendulum
Units	$\frac{\text{m}}{\text{s}^2}$
Equation	$a_y = \omega^2 L_{\text{rod}} \cos(\theta_p) + \alpha L_{\text{rod}} \sin(\theta_p)$
Description	<p> <math>a_y</math> is the <math>y</math>-component of acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)  <math>\omega</math> is the angular velocity (<math>\frac{\text{rad}}{\text{s}}</math>)  <math>L_{\text{rod}}</math> is the length of the rod (m)  <math>\theta_p</math> is the displacement angle of the pendulum (<math>^\circ</math>)  <math>\alpha</math> is the angular acceleration (<math>\frac{\text{rad}}{\text{s}^2}</math>) </p>
Source	–
RefBy	

**Detailed derivation of the  $y$ -component of acceleration:** Our acceleration is:

$$\mathbf{a} = \frac{d\mathbf{v}}{dt}$$

Earlier, we found the vertical velocity to be

$$v_y = \omega L_{\text{rod}} \sin(\theta_p)$$

Applying this to our equation for acceleration

$$a_y = \frac{d\omega L_{\text{rod}} \sin(\theta_p)}{dt}$$

By the product and chain rules, we find

$$a_y = \frac{d\omega}{dt} L_{\text{rod}} \sin(\theta_p) + \omega L_{\text{rod}} \cos(\theta_p) \frac{d\theta_p}{dt}$$

Simplifying,

$$a_y = \omega^2 L_{\text{rod}} \cos(\theta_p) + \alpha L_{\text{rod}} \sin(\theta_p)$$

Refname	GD:hForceOnPendulum
Label	Horizontal force on the pendulum
Units	N
Equation	$\mathbf{F} = ma_x = -\mathbf{T} \sin(\theta_p)$
Description	<p><math>\mathbf{F}</math> is the force (N)  <math>m</math> is the mass (kg)  <math>a_x</math> is the <math>x</math>-component of acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)  <math>\mathbf{T}</math> is the tension (N)  <math>\theta_p</math> is the displacement angle of the pendulum (<math>^\circ</math>)</p>
Source	—
RefBy	

**Detailed derivation of force on the pendulum:**

$$\mathbf{F} = ma_x = -\mathbf{T} \sin(\theta_p)$$

Refname	GD:vForceOnPendulum
Label	Vertical force on the pendulum
Units	N
Equation	$\mathbf{F} = ma_y = \mathbf{T} \cos(\theta_p) - m\mathbf{g}$
Description	<p><math>\mathbf{F}</math> is the force (N)  <math>m</math> is the mass (kg)  <math>a_y</math> is the <math>y</math>-component of acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)  <math>\mathbf{T}</math> is the tension (N)  <math>\theta_p</math> is the displacement angle of the pendulum (<math>^\circ</math>)  <math>\mathbf{g}</math> is the gravitational acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)</p>
Source	—
RefBy	

Detailed derivation of force on the pendulum:

$$\mathbf{F} = ma_y = \mathbf{T} \cos(\theta_p) - m\mathbf{g}$$



Refname	GD:angFrequencyGD
Label	The angular frequency of the pendulum
Units	s
Equation	$\Omega = \sqrt{\frac{\mathbf{g}}{L_{\text{rod}}}}$
Description	<p><math>\Omega</math> is the angular frequency (s)</p> <p><math>\mathbf{g}</math> is the gravitational acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)</p> <p><math>L_{\text{rod}}</math> is the length of the rod (m)</p>
Notes	The torque is defined in <a href="#">TM:NewtonSecLawRotMot</a> and frequency is $f$ is defined in <a href="#">DD:frequencyDD</a> .
Source	–
RefBy	<a href="#">GD:periodPend</a> and <a href="#">IM:calOfAngularDisplacement</a>

**Detailed derivation of the angular frequency of the pendulum:** Consider the torque on a pendulum defined in [TM:NewtonSecLawRotMot](#). The force providing the restoring torque is the component of weight of the pendulum bob that acts along the arc length. The torque is the length of the string  $L_{\text{rod}}$  multiplied by the component of the net force that is perpendicular to the radius of the arc. The minus sign indicates the torque acts in the opposite direction of the angular displacement:

$$= -L_{\text{rod}} m \mathbf{g} \sin(\theta_p)$$

So then

$$\mathbf{I}\alpha = -L_{\text{rod}}m\mathbf{g}\sin(\theta_p)$$

Therefore,

$$\mathbf{I}\frac{d\frac{d\theta_p}{dt}}{dt} = -L_{\text{rod}}m\mathbf{g}\sin(\theta_p)$$

Substituting for  $\mathbf{I}$

$$mL_{\text{rod}}^2\frac{d\frac{d\theta_p}{dt}}{dt} = -L_{\text{rod}}m\mathbf{g}\sin(\theta_p)$$

Crossing out  $m$  and  $L_{\text{rod}}$  we have

$$\frac{d\frac{d\theta_p}{dt}}{dt} = -\left(\frac{\mathbf{g}}{L_{\text{rod}}}\right)\sin(\theta_p)$$

For small angles, we approximate  $\sin \theta_p$  to  $\theta_p$

$$\frac{d\frac{d\theta_p}{dt}}{dt} = -\left(\frac{\mathbf{g}}{L_{\text{rod}}}\right)\theta_p$$

Because this equation, has the same form as the equation for simple harmonic motion the solution is easy to find. The angular frequency

$$\Omega = \sqrt{\frac{\mathbf{g}}{L_{\text{rod}}}}$$

Refname	GD:periodPend
Label	The period of the pendulum
Units	s
Equation	$T = 2\pi\sqrt{\frac{L_{\text{rod}}}{g}}$
Description	<p><math>T</math> is the period (s)</p> <p><math>\pi</math> is the ratio of circumference to diameter for any circle (Unitless)</p> <p><math>L_{\text{rod}}</math> is the length of the rod (m)</p> <p><math>g</math> is the gravitational acceleration (<math>\frac{\text{m}}{\text{s}^2}</math>)</p>
Notes	The frequency and period are defined in <a href="#">DD:frequencyDD</a> <a href="#">DD:periodSHMDD</a> respectively
Source	—
RefBy	

**Detailed derivation of the period of the pendulum:** The period of the pendulum can be defined from [GD:angFrequencyGD](#) equation

$$\Omega = \sqrt{\frac{g}{L_{\text{rod}}}}$$

Therefore from the equation [DD:angFrequencyDD](#), we have

$$T = 2\pi\sqrt{\frac{L_{\text{rod}}}{g}}$$

### 3.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models.

Refname	DD:positionIX
Label	$x$ -component of initial position
Symbol	$p_x^i$
Units	m
Equation	$p_x^i = L_{\text{rod}} \sin(\theta_i)$
Description	$p_x^i$ is the $x$ -component of initial position (m) $L_{\text{rod}}$ is the length of the rod (m) $\theta_i$ is the initial pendulum angle (rad)
Notes	$p_x^i$ is the horizontal position $p_x^i$ is shown in <b>Fig:pendulum</b> .
Source	—
RefBy	

Refname	DD:positionIY
Label	$y$ -component of initial position
Symbol	$p_y^i$
Units	m
Equation	$p_y^i = -L_{\text{rod}} \cos(\theta_i)$
Description	<p><math>p_y^i</math> is the <math>y</math>-component of initial position (m)</p> <p><math>L_{\text{rod}}</math> is the length of the rod (m)</p> <p><math>\theta_i</math> is the initial pendulum angle (rad)</p>
Notes	<p><math>p_y^i</math> is the vertical position</p> <p><math>p_y^i</math> is shown in <b>Fig:pendulum</b>.</p>
Source	—
RefBy	

Refname	DD:frequencyDD
Label	Frequency
Symbol	$f$
Units	Hz
Equation	$f = \frac{1}{T}$
Description	$f$ is the frequency (Hz) $T$ is the period (s)
Notes	$f$ is the number of back and forth swings in one second
Source	—
RefBy	GD:periodPend, DD:periodSHMDD, and GD:angFrequencyGD

Refname	DD:angFrequencyDD
Label	Angular frequency
Symbol	$\Omega$
Units	s
Equation	$\Omega = \frac{2\pi}{T}$
Description	<p><math>\Omega</math> is the angular frequency (s)</p> <p><math>\pi</math> is the ratio of circumference to diameter for any circle (Unitless)</p> <p><math>T</math> is the period (s)</p>
Notes	$T$ is from DD:periodSHMDD
Source	—
RefBy	GD:periodPend

Refname	DD:periodSHMDD
Label	Period
Symbol	$T$
Units	s
Equation	$T = \frac{1}{f}$
Description	$T$ is the period (s) $f$ is the frequency (Hz)
Notes	$T$ is from <a href="#">DD:frequencyDD</a>
Source	—
RefBy	<a href="#">GD:periodPend</a> and <a href="#">DD:angFrequencyDD</a>

### 3.2.5 Instance Models

This section transforms the problem defined in [Sec:Problem Description](#) into one which is expressed in mathematical terms. It uses concrete symbols defined in [Sec:Data Definitions](#) to replace the abstract symbols in the models identified in [Sec:Theoretical Models](#) and [Sec:General Definitions](#).



Refname	IM:calOfAngularDisplacement		
Label	Calculation of angular displacement		
Input	$L_{\text{rod}}, \theta_i, \mathbf{g}$		
Output	$\theta_p$		
Input Constraints	$L_{\text{rod}} > 0$ $\theta_i > 0$ $\mathbf{g} > 0$		
Output Constraints	$\theta_p > 0$		
Equation	$\theta_p(t) = \theta_i \cos(\Omega t)$		
Description	$\theta_p$ is the displacement angle of the pendulum (°) $t$ is the time (s) $\theta_i$ is the initial pendulum angle (rad) $\Omega$ is the angular frequency (s)		
Notes	The constraint $\theta_i > 0$ is required. The angular frequency is defined in <a href="#">GD:angFrequencyGD</a> .		
Source	—		
RefBy	<a href="#">FR:Output-Values</a> and <a href="#">FR:Calculate-Angular-Position-Of-Mass</a>		

**Detailed derivation of angular displacement:** When the pendulum is displaced to an initial angle and released, the pendulum swings back and forth with periodic motion. By applying Newton's second law for rotational motion in **TM:NewtonSecLawRotMot**, the equation of motion for the pendulum may be obtained:

$$= \mathbf{I}\alpha$$

Where  $\tau$  denotes the torque,  $\mathbf{I}$  denotes the moment of inertia and  $\alpha$  denotes the angular acceleration. This implies:

$$-m\mathbf{g} \sin(\theta_p) L_{\text{rod}} = mL_{\text{rod}}^2 \frac{d^2\theta_p}{dt^2}$$

And rearranged as:

$$\frac{d^2\theta_p}{dt^2} + \frac{\mathbf{g}}{L_{\text{rod}}} \sin(\theta_p) = 0$$

If the amplitude of angular displacement is small enough, we can approximate  $\sin(\theta_p) = \theta_p$  for the purpose of a simple pendulum at very small angles. Then the equation of motion reduces to the equation of simple harmonic motion:

$$\frac{d^2\theta_p}{dt^2} + \frac{\mathbf{g}}{L_{\text{rod}}} \theta_p = 0$$

Thus the simple harmonic motion is:

$$\theta_p(t) = \theta_i \cos(\Omega t)$$

### 3.2.6 Data Constraints

**Tab:InDataConstraints** shows the data constraints on the input variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable. The uncertainty column provides an estimate of the confidence with which the physical quantities can be measured. This information would be part of the input if one were performing an uncertainty quantification exercise. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Typical Value	Uncert.
$L_{\text{rod}}$	$L_{\text{rod}} > 0$	44.2 m	10%
$\theta_i$	$\theta_i > 0$	2.1 rad	10%

Table 4: Input Data Constraints

### 3.2.7 Properties of a Correct Solution

**Tab:OutDataConstraints** shows the data constraints on the output variables. The column for physical constraints gives the physical limitations on the range of values that can be taken by the variable.

Var	Physical Constraints
$\alpha$	$\alpha > 0$
$\theta_p$	$\theta_p > 0$

Table 5: Output Data Constraints

## 4 Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete, and the non-functional requirements, the qualities that the software is expected to exhibit.

### 4.1 Functional Requirements

This section provides the functional requirements, the tasks and behaviours that the software is expected to complete.

Input-Values: Input the values from **Tab:ReqInputs**.

Verify-Input-Values: Check the entered input values to ensure that they do not exceed the data constraints mentioned in **Sec:Data Constraints**. If any of the input values are out of bounds, an error message is displayed and the calculations stop.

Position-Of-Mass: Calculate the following values:  $\theta$  (from **IM:calOfAngularDisplacement**) and  $\theta_p$  (from **IM:calOfAngularDisplacement**).

Output-Values: Output  $L_{\text{rod}}$  (from **IM:calOfAngularDisplacement**) and  $L_{\text{rod}}$  (from **IM:calOfAngularDisplacement**).

Symbol	Description	Units
$L_{\text{rod}}$	Length of the rod	m
$m$	Mass	kg
$\alpha$	Angular Acceleration	$\frac{\text{rad}}{\text{s}^2}$
$\theta_i$	Initial pendulum angle	rad
$\theta_p$	Displacement angle of the pendulum	°

Table 6: Required Inputs following **FR:Input-Values**

## 4.2 Non-Functional Requirements

This section provides the non-functional requirements, the qualities that the software is expected to exhibit.

Correct: The outputs of the code have the properties described in [Sec:Properties of a Correct Solution](#).

Portable: The code is able to be run in different environments.

## 5 Traceability Matrices and Graphs

The purpose of the traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an “X” should be modified as well. [Tab:TraceMatAvsA](#) shows the dependencies of assumptions on the assumptions. [Tab:TraceMatAvsAll](#) shows the dependencies of data definitions, theoretical models, general definitions, instance models, requirements, likely changes, and unlikely changes on the assumptions. [Tab:TraceMatRefvsRef](#) shows the dependencies of data definitions, theoretical models, general definitions, and instance models with each other. [Tab:TraceMatAllvsR](#) shows the dependencies of requirements, goal statements on the data definitions, theoretical models, general definitions, and instance models.

	A:pend2DMotion	A:cartCoord	A:cartCoordRight	A:yAxisDir	A:startOrigin
A:pend2DMotion					
A:cartCoord					
A:cartCoordRight					
A:yAxisDir					
A:startOrigin					

Table 7: Traceability Matrix Showing the Connections Between Assumptions dependence of each other.

	A:pend2DMotion	A:cartCoord	A:cartCoordRight
DD:positionIX			
DD:positionIY			
DD:frequencyDD			
DD:angFrequencyDD			
DD:periodSHMDD			
TM:acceleration			
TM:velocity			
TM:NewtonSecLawMot			

	A:pend2DMotion	A:cartCoord	A:cartCoordRight
TM:NewtonSecLawRotMot			
GD:velocityIX			
GD:velocityIY			
GD:accelerationIX			
GD:accelerationIY			
GD:hForceOnPendulum			
GD:vForceOnPendulum			
GD:angFrequencyGD			
GD:periodPend			
IM:calOfAngularDisplacement			
FR:Input-Values			
FR:Verify-Input-Values			
FR:Calculate-Angular-Position-Of-Mass			
FR:Output-Values			
NFR:Correct			
NFR:Portable			

Table 8: Traceability Matrix Showing the Connections Between Assumptions and Items

	DD:positionIX	DD:positionIY	DD:frequencyDD	DD:angFr
DD:positionIX				
DD:positionIY				
DD:frequencyDD				
DD:angFrequencyDD				
DD:periodSHMDD			X	
TM:acceleration				
TM:velocity				
TM:NewtonSecLawMot				
TM:NewtonSecLawRotMot				
GD:velocityIX				
GD:velocityIY				
GD:accelerationIX				
GD:accelerationIY				
GD:hForceOnPendulum				
GD:vForceOnPendulum				
GD:angFrequencyGD			X	
GD:periodPend			X	
IM:calOfAngularDisplacement				X

DD:positionIX	DD:positionIY	DD:frequencyDD	DD:angFr
GS:Motion-of-the-mass			
FR:Input-Values			
FR:Verify-Input-Values			
FR:Calculate-Angular-Position-Of-Mass			
FR:Output-Values			
NFR:Correct			
NFR:Portable			

## 6 Values of Auxiliary Constants

There are no auxiliary constants.

## 7 References

- [1] Wikipedia Contributors. *Acceleration*. <https://en.wikipedia.org/wiki/Acceleration>. June 2019.
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- [4] R. C. Hibbeler. *Engineering Mechanics: Dynamics*. Pearson Prentice Hall, 2004.