Essentials of Slope Stability Analysis

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1 Introduction

The analysis of the stability of slopes of soil is a common problem in geotechnical engineering practice. Depending on the type of material, geometric characteristics of the slope, stratigraphy of the subsurface and presence of water, a slope may fail in various manners, as Figure 1 shows. A complete slope stability analysis has two key components:

1. Computing the relative stability of the slope, typically as a ratio between the forces and/or moments resisting failure and those driving failure. This ratio is often called the "factor of safety".
2. Determining the shape and location of the most "critical" failure surface, i.e. the surface that gives the lowest factor of safety.

Both parts of this problem are challenging. The first component involves analysis of a statically indeterminate problem, which means that the equations of force and moment equilibrium are not sufficient to compute a result. Therefore, one must either account for the stress-strain behaviour of the material or make simplifying assumptions to render the problem determinate. The second component is a highly nonlinear optimization problem, particularly in the most general case of a noncircular failure surface.

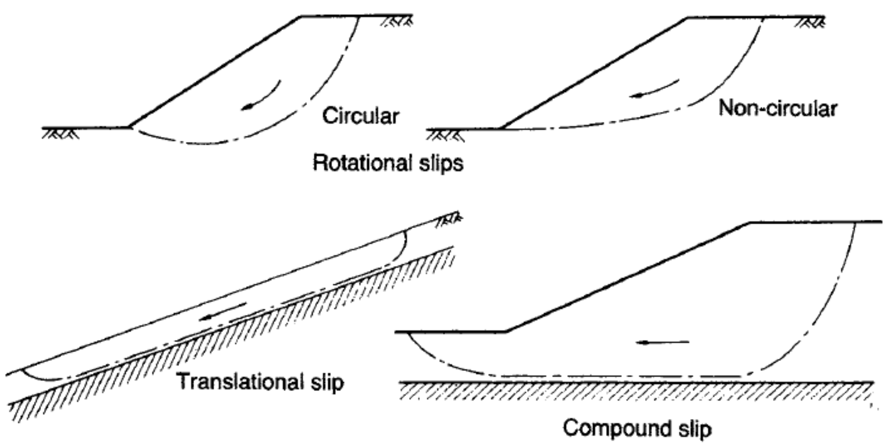

Figure 1. (Figure 9.1 from Craig 2004) Types of slope failure

2 Limit Equilibrium Analysis (or The Method of Slices)

One technique for computing the factor of safety for a slope involves making simplifying assumptions about how the forces are distributed within the failing soil mass, thereby rendering the problem statically determinate. Referring to Figure 2, we assume the shape and location of the failure surface and divide it into a finite number of slices. For a circular failure surface, its shape and location are determined by three parameters: the (x,y) coordinates of the circle's centre and the radius of the circle r. Other important information includes:

1. The geometry of the slope including the location of the water table, if present.
2. The strength properties of the soil. These are typically given as the internal angle of friction φ' and cohesion c'.
3. The unit weight of the soil including both the bulk unit weight above the water table γ and the saturated unit weight below the water table γ_{sat}, as necessary.

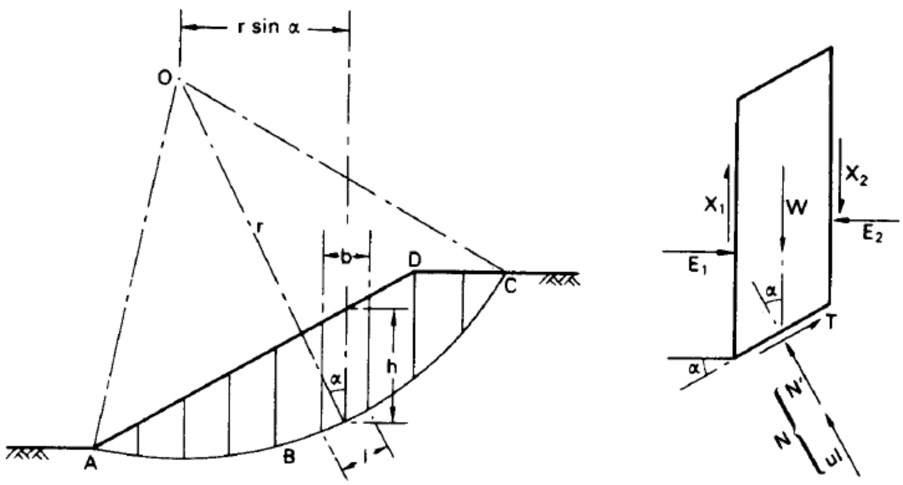

Figure 2 (Figure 9.5 from Craig 2004) The method of slices including a circular failure surface divided into slices (left) and a free body force diagram of a slice (right)

For a given failure surface divided into slices, the forces on it include the total weight W, the "effective" normal force at the base N', the water pressure force at the base $U = ul$, the shear force at the base T, the interslice normal forces E_i and the interslice shear forces X_i. The factor of safety is defined as a ratio $F = \tau_f / \tau_m$ where τ_f represents the available shear strength and τ_m is the actual applied or "mobilized" shear. A stable slope has $F > 1$. In general, if we assume an "effective stress analysis" and the Coulomb yield criterion we can define the factor of safety more specifically as (Craig 2004),

$$F = \frac{c' L_a + \tan\varphi' \Sigma N'}{\Sigma W \sin\alpha}$$

where L_a is the arc length of the failure surface (arc AC in Figure 2), α is the angle of the normal to the base of each slice relative to vertical and all other parameters are as defined previously. As was mentioned previously, the problem is statically determinate, which means that one cannot determine N' without making assumptions regarding the interslice forces. There exist several versions of the method of slices, and they differ primarily in these assumptions used to compute N' (Fredlund and Krahn 1977):

1. Ordinary / Fellenius / Swedish circle / conventional method
2. Simplified Bishop method
3. Spencer’s method
4. Janbu’s simplified method
5. Janbu’s rigorous method
6. Morgenstern-Price method

The Slope Stability Program (SSP) that will be the focus of the document driven design (DDD) project uses the Morgenstern-Price method, which is considered by many to be the most rigorous of the limit equilibrium approaches. The algorithm in the SSP for computing the Morgenstern-Price factor of safety is that proposed by Zhu et al. (2005).

For a simple introduction to the methods of slices, let us consider a worked example using the Fellenius method, which simply assumes that the balance of interslice forces is negligible, i.e. $E_1 + E_2 \approx 0$ and $X_1 + X_2 \approx 0$, so that we have $N' = W\cos\alpha - ul$ and the factor of safety becomes:

$$F = \frac{c' L_a + \tan\varphi' \Sigma\left(W\cos\alpha - ul\right)}{\Sigma W \sin\alpha}$$

The example comes from Craig (2004) and is numbered as Example 9.2 therein.

Example 9.2

Using the Fellenius method of slices, determine the factor of safety, in terms of effective stress, of the slope shown in Figure 9.6 for the given failure surface (a) using peak strength parameters $c' = 10\,\mathrm{kN/m^2}$ and $\phi' = 29°$ and (b) using critical-state parameter $\phi'_{cv} = 31°$. The unit weight of the soil both above and below the water table is $20\,\mathrm{kN/m^3}$.

(a) The factor of safety is given by Equation 9.4. The soil mass is divided into slices 1.5 m wide. The weight (W) of each slice is given by

$$W = \gamma b h = 20 \times 1.5 \times h = 30h\,\mathrm{kN/m}$$

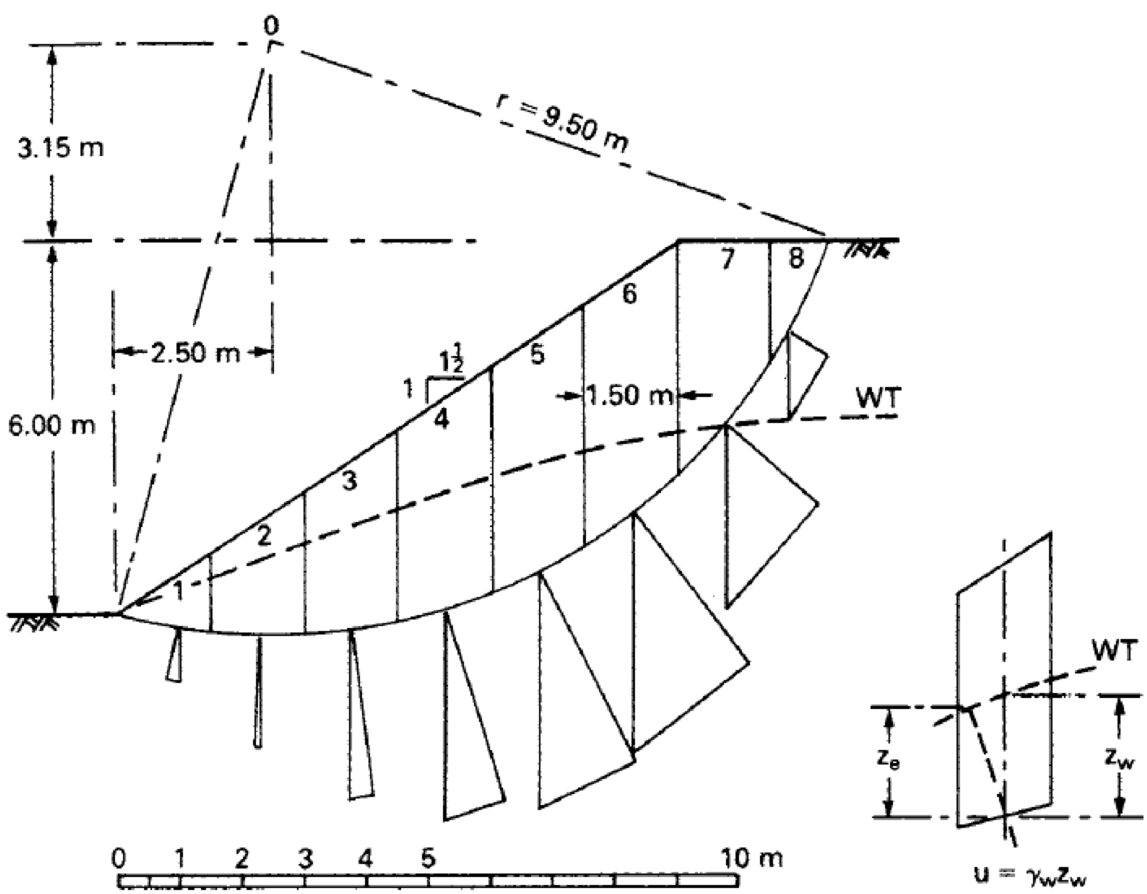

Figure 9.6 Example 9.2.

The height h for each slice is set off below the centre of the base, and the normal and tangential components $h \cos \alpha$ and $h \sin \alpha$, respectively, are determined graphically, as shown in Figure 9.6. Then

$$W \cos\alpha = 30h \cos\alpha$$

$$W \sin\alpha = 30h \sin\alpha$$

The pore water pressure at the centre of the base of each slice is taken to be $\gamma_w z_w$, where z_w is the vertical distance of the centre point below the water table (as shown in the figure). This procedure slightly overestimates the pore water pressure which strictly should be $\gamma_w z_e$, where z_e is the vertical distance below the point of intersection of the water table and the equipotential through the centre of the slice base. The error involved is on the safe side.

The arc length (L_a) is calculated as 14.35 m. The results are given in Table 9.1.

$$\Sigma W \cos\alpha = 30 \times 17.50 = 525\,\text{kN/m}$$

$$\Sigma W \sin\alpha = 30 \times 8.45 = 254\,\text{kN/m}$$

$$\Sigma(W \cos\alpha - ul) = 525 - 132 = 393\,\text{kN/m}$$

Table 9.1

Slice No.	$h \cos\alpha$ (m)	$h \sin\alpha$ (m)	u (kN/m^2)	l (m)	ul (kN/m)
1	0.75	−0.15	5.9	1.55	9.1
2	1.80	−0.10	11.8	1.50	17.7
3	2.70	0.40	16.2	1.55	25.1
4	3.25	1.00	18.1	1.60	29.0
5	3.45	1.75	17.1	1.70	29.1
6	3.10	2.35	11.3	1.95	22.0
7	1.90	2.25	0	2.35	0
8	0.55	0.95	0	2.15	0
	17.50	8.45		14.35	132.0

$$F = \frac{c' L_a + \tan\phi' \Sigma(W\cos\alpha - ul)}{\Sigma W \sin\alpha}$$

$$= \frac{(10 \times 14.35) + (0.554 \times 393)}{254}$$

$$= \frac{143.5 + 218}{254} = 1.42$$

3 Location of Critical Slip Surface

Assuming that one can compute a factor of safety for any given slip surface, the other part of the slope stability problem is the optimization of its location to minimize the factor of safety. This is because, all other things being equal, the slip surface with the lowest factor of safety is the most likely to fail. The SSP uses a genetic algorithm to perform this optimization, which is a direct search method good at solving optimization problems that are nonlinear and have many local minima. The specific algorithm is as presented by Li et al. (2010), with the specific implementation details described in the comments of the SSP code.

4 References

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