Mathematical Model for Calculating IM3 in Glass-BR

May 30, 2017

Calculation of Demand q

From IM3, $q = q(w_{TNT}, SD)$, using interpolation from Fig 6 in the Spec

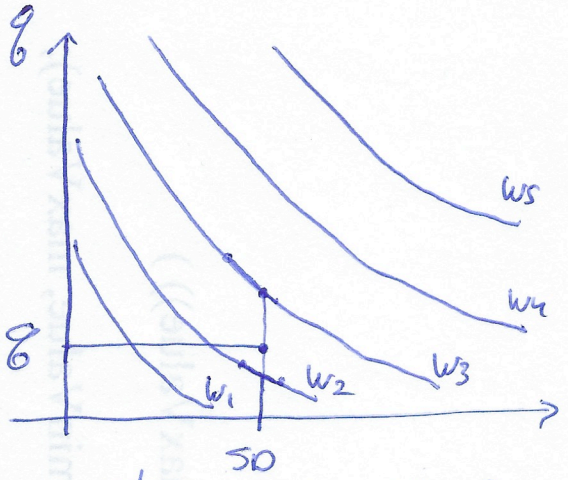


Fig 6

Steps Overview
① Find the appropriate w_j curves, say w_j and w_{j+1}
② Use interpolation on w_j and w_{j+1} to find q_1 and q_2 (the demand on w_j and w_{j+1}, respectively, at SD.)
③ Use interpolation between q_1 & q_2 for w_{TNT}

Goal - calculate q given the following:

① user inputs $w_{TNT} : \mathbb{R}$, $SD : \mathbb{R}$

② experimental curves from Figure 6, used for interpolation
- $\underline{w}$: seq. of $\mathbb{R}$ - the different charge weights, $\underline{w} = [4.5, 9.1, \ldots 410]$
- $n : \mathbb{N}$ - the number of $\underline{w}$ curves in Fig 6, $\underline{w} = [w_1, w_2, \ldots w_n]$
- S: seq. of seq. of $\mathbb{R}$ - stand off distances
- $S = [\underline{s}_1, \underline{s}_2, \ldots \underline{s}_n]$
- $\underline{s}_j$: seq of $\mathbb{R}$ - list of stand off distances for charge weight w_j
- the ith entry in $\underline{s}_j$ is referenced via the notation $\underline{s}_j(i)$ (Based on Gulob & VanLoan notation)

Q: seq. of (seq of $\mathbb{R}$) – demands

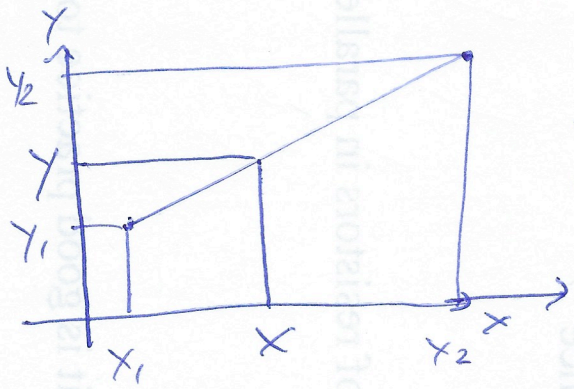
$Q = [q_1, q_2, \ldots q_n]$

q_j : seq of $\mathbb{R}$ – list of demands for charge weights w_j, matched with s_j to build curve

– ith entry is $q_j(i)$

Helper Functions

① Interpolation



$$y = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$$

$interp((x_1, y_1), (x_2, y_2), x) : \mathbb{R}^2 \times \mathbb{R}^2 \times \mathbb{R} \to \mathbb{R}$

$interp((x_1, y_1), (x_2, y_2), x) \equiv \left(\frac{y_2 - y_1}{x_2 - x_1}\right)(x - x_1) + y_1$

② indInSeq – find the index in the sequence that defines where an input value lies

$indInSeq(\underline{x}, v) : \text{seq of } \mathbb{R} \to \mathbb{N}$

$indInSeq(\underline{x}, v) \equiv i$ such that $c_i \leq v \leq c_{i+1}$

(at this time we will ignore the exception that would occur if $v \notin \underline{x}$)

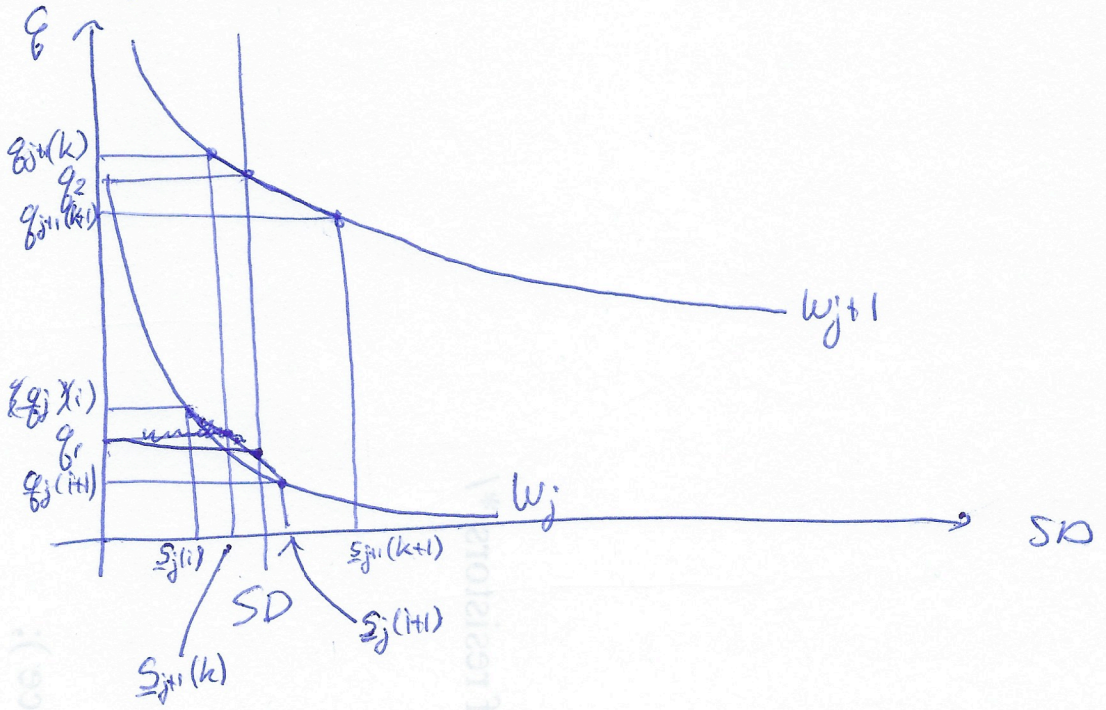
Detailed Steps

① Find the charge weights that bracket the given weight (w_{TWO}):

$$j = indInSeq(\underline{w}, w_{TWO})$$

→ the weights of interest are w_j and w_{j+1} these associated curves are defined by s_j, q_j and s_{j+1}, q_{j+1}

② For each of the charge weight curves j and j+1, find q (call it q_1 & q_2, respectively)



$i = \text{indInSeq}(s_j, SD)$

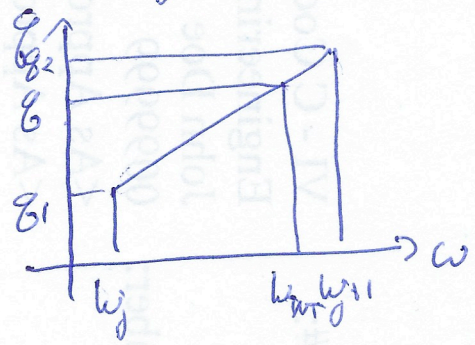
$k = \text{indInSeq}(s_{j+1}, SD)$

$q_1 = \text{interp}((s_j(i), q_j(i)), (s_j(i+1), q_j(i+1)), SD)$

$q_2 = \text{interp}((s_{j+1}(k), q_{j+1}(k)), (s_{j+1}(k+1), q_{j+1}(k+1)), SD)$

③ Use the interpolated q values on curve j & j+1 to find q

$$q = \text{interp}((w_j, q_1), (w_{j+1}, q_2), w_{TNT})$$



[this function defines the demand q for w_{TNT} & SD