Chapter 5

Contact time: 0, when a collision is detected.

Contact set: Set of intersection points. Manifold when the set is rather a continuum of points (Cube face on a flat surface)

Find / Test - intersection - query.

Unconstrained Motion: No collisions

Newtonian: $m\ddot{x} = m\dot{v} = ma = F(t)$

State Vector: $S(t) = \begin{bmatrix} x \\ v \end{bmatrix}$

So the differential equation is

$$\frac{dS}{dt} = \frac{d}{dt}\begin{bmatrix} x \\ v \end{bmatrix} = \begin{bmatrix} v \\ a \end{bmatrix} = \begin{bmatrix} v' \\ \frac{F}{m} \end{bmatrix}$$

multiple particles $\frac{d}{dt}\begin{bmatrix} x_1 \\ v_1 \\ \vdots \\ x_n \\ v_n \end{bmatrix}$

Position and velocity: $\frac{dx(t)}{dt} = v(t)$

momentum: $p(t) = mv(t)$
is linear since object mass is constant.

☆ $Skew(u)r = u \times r$

$$\frac{d\,p(t)}{dt} = F(t)$$

- Process for determining positon of a center of mass given an applied force $F(t)$
  1. Compute p from F by integration
  2. Compute v from p by dividing by m
  3. Compute x from v by integrating

Rotation: Orientation: $R(t)$
Mass matrix (inertia): J
Angular Velocity: w
Angular momentum: L
Skew: Mirrored matrix with inverted signs

$$\begin{matrix} 0 & 4 & -3 \\ -4 & 0 & -1 \\ 3 & 1 & 0 \end{matrix}$$

$$\frac{d\,R(t)}{dt} = Skew(w(t))\,R(t)$$ (Change in pos = pos * vel)

Angular momentum = mass · Angular vel; $L(t) = J(t)\,w(t)$

Torque (force) = $\frac{d\,L(t)}{dt} = \tau(t)$

- Process

1. Compute L from τ by integrating equation
2. Compute w from L by multiplying by the inverse matrix J^{-1}
3. Compute R from w by integrating

☆ 226: 5.7 ??? 5.8

☆ Look at chapter 10 for quaternions

Integrating for R is prone to error buildup. Must use Gram-Schmidt occasionally to correct errors.

If $\hat{R} = [\hat{u}_0\ \hat{u}_1\ \hat{u}_2]$ we can get R like so:

$$u_0 = \frac{\hat{u}_0}{|\hat{u}_0|} \qquad u_1 = \frac{\hat{u}_1 - (\hat{u}_1 \cdot u_0)u_0}{|\hat{u}_1 - (\hat{u}_1 \cdot u_0)u_0|} \qquad u_2 = u_0 \times u_1$$

?

Quaternion? Like R but allows for imaginary?. Represented w/ q.

$$\frac{d\,q(t)}{dt} = \frac{1}{2}\,\omega(t)\,q(t)$$

why $\frac{1}{2}$? dunno

Where ω is the quaternion w/ corresponds to w

and normalize q by $\frac{\hat{q}}{|\hat{q}|}$

Now State vector:

$$s(t) = \begin{bmatrix} x(t) \\ q(t) \\ p(t) \\ L(t) \end{bmatrix} \quad \text{for entire system} \quad S(t) \begin{bmatrix} X_1(t) \\ q_1(t) \\ p_1(t) \\ L_1(t) \\ \vdots \\ X_n(t) \\ q_n(t) \\ p_n(t) \\ L_n(t) \end{bmatrix}$$

$$\frac{dS}{dt} = \begin{bmatrix} m^{-1}p \\ \omega\, q/2 \\ F \\ \tau \end{bmatrix}$$

Constrained Motion:

Bodies A and B collide at point P.

V_A (Velocity of body A) compared to the Normal of B at P determines the type of contact:

Dot product = speed of A in Normal direction

$N \cdot V_A < 0$ = colliding contact
$N \cdot V_A = 0$ = Resting contact
$N \cdot V_A > 0$ = Separation

Because contacts can have more than 1 (or infinite) contact points, we work with a reduced contact set: one point of contact.

Edge-Edge: Point where two edges meet, only for non parallel.
Edge-Face: Edge end point contained in face
Face-Face: Vertexes from each which are contained in the other's face

Distance between 2 points: $d(t) = N(t) \cdot (p_A(t) - p_B(t))$

Velocity in Normal direction:
$$\dot{d}(t) = N(t) \cdot (\dot{p}_A(t) - \dot{p}_B(t)) + \dot{N}(t) \cdot (p_A(t) - p_B(t))$$

At contact time $d(t_0) = 0$, and
$$\dot{d}(t) = N(t) \cdot (\dot{p}_A(t) - \dot{p}_B(t))$$
which is the $N \cdot V_A$ equation.

Velocity of point P =
$$v + w \times r$$
v: velocity of center of mass
w: angular velocity of body around center of mass
r: dist of point from center of mass

Impulse Force:
Because at the point of contact, the change in velocity is instantaneous
(i.e. $\dot{x}(t) = \begin{cases} v_0, & t < t_0 \\ -v_0, & t > t_0 \end{cases}$)
we must allow for discontinuity in linear momentum of the system

Relative velocity before impulse:
$$v^- = N^\perp + (N \cdot v^-) N$$
Relative velocity after
$$v^+ = N^\perp - \varepsilon (N \cdot v^-) N$$
where ε is the coefficient of restitution.
$\varepsilon \in [0,1]$ and represents loss of energy.

Pre/post impulse forces

$$\dot{P}^\pm = v^\pm + w^\pm \cdot r$$

$$v^+ = v^- + \frac{f N_0}{m_A}$$

$$w^+ = w^- + J_A^{-1}(r_A \times f N_0)$$

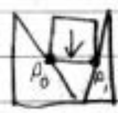
$$\dot{P}_A^+ = \dot{P}_A^- + f\left(\frac{N_0}{m_a} + J_A^{-1}(r_A \times N_0)\right) \times r_A$$

for B
-

Multiple Contact points:

Problem: Derive post impulse simultaneously? Sequentially?

Sequentially may not give the most accurate answer but is less time consuming. A system that solves simultaneously is more complex.



In this case, calculating one impulse or the other first gives a post impulse velocity that is blocked by the opposing triangle.

If P_0: $V_A^+ = V_A^- + \frac{f_0 N_0}{m_A}$

$= (0,-\lambda) + (\lambda,\lambda)$

$= (\lambda, 0)$

P_1: $= (0,-\lambda) + (-4\lambda/5, \lambda/5)$

$= (-4\lambda/5, -3\lambda/5)$

Simultaneous impulse calculation

$$V_A^- + \frac{f_0 N_0 + f_1 N_1}{2 m_A}$$

$$= (\lambda/10, -4\lambda/5)$$

In this case, either the object simply stops or derive an equation for the objects bounce

Simultaneous Processing for Contact points

To avoid interperetration of bodies at the contact point, Body B must exert a contact force C on A.

1. C acts only at instant of contact
2. C must be a repulsive force
3. C must prevent interpenetration
4. System cannot gain Kinetic energy from C.

1. is satisfied from C being an impulse force

2. is satisfied if f in $C = f N_0$ is positive so the force is in the direction of the Normal

3. f needs to be large enough so that given the equation.

$$\dot{d}^+ = \dot{d}^- + f\left(m_A^{-1} + (r_A \times N_0)^T J_A^{-1} (r_A \times N_0)\right)$$

$\dot{d}^+$ is at the very least 0. f can be larger than this value too.

4 However, to avoid adding energy to the system, we must have $|\dot{d}^+| \leqslant |\dot{d}^-|$. that means

$f \leqslant f_{max}$ where $f_{max} =$

$$\frac{-2\dot{d}^-}{m_A^{-1} + (r_A \times N_0)^T J_A^{-1} (r_A \times N_0)}$$

Generally, you need to analyze each situation individually to determine f.

If $d_i^- \geq 0$ our goal for d_i^+ is to make it as close to d_i as possible without negativity

If $d_i^- < 0$ goal is d_i^+ close to $-d_i$ as possible w/o negativity.

Collision Response For Resting Contact

We calculate the acceleration of body A relative to body B at point P.

$$\ddot{d}(t) = N(t) \cdot (\ddot{P}_A(t) - \ddot{P}_B(t)) + 2\dot{N}(t) \cdot (\dot{P}_A(t) - \dot{P}_B(t)) + \ddot{N} \cdot (P_A(t) - P_B(t))$$

if $\ddot{d}(t)$ is negative, A is attempting to accelerate into B.

Once again we need a contact Force C that must.

1. prevent the interpenetration of the bodies
2. must be repulsive
3. become zero when the bodies seperate.

$C = g N_0$ where g is non-negative and $g\,\ddot{d}(0)$ must $= 0$.