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SEARCHING TECHNIQUES IN SLOPE STABILITY ANALYSIS

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ABSTRACT

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Searching techniques for location of critical potential failure surfaces in two-dimensional slope stability analysis are presented. A computer program, STABL, developed at Purdue University, features unique random techniques for generation of circular as well as irregular trial failure surfaces in the search for the more critical ones.

The stability calculation in STABL is based on methods of slices. The simplified Bishop method is adopted for circular failure surfaces, and the simplified Janbu method for general shapes of failure surfaces. The Janbu procedure usually provides somewhat conservative results. Other procedures may be adapted to the program.

INTRODUCTION

Slope stability analysis by limiting equilibrium usually requires a search for the shape and location of the more critical failure surfaces. Different techniques are applied, such as the grid search for regular shapes of failure surfaces, and random search techniques for irregular shaped failure surfaces. The critical failure surfaces are the ones that give the lower values of the factor of safety.

GRID SEARCHES FOR REGULAR SHAPES

In the case of homogeneous soil, the shape of the critical failure surface is often assumed to be circular or that of a logarithmic spiral. For both of these shapes, a search for the most critical location of the failure surface may be performed by a grid search. The principle is shown in Fig.1 for a failure surface of circular shape. In order to locate the most critical surface, the centre of the circle or the pole of the logarithmic spiral is varied according to a chosen grid pattern and the radius (initial radius for a log. spiral) is varied between R_{max} and R_{min}. The use of the grid search is based on the observation that the centres or poles of different surfaces will form contour lines of equal factors of safety with the centre or pole of the most critical surface (the one having the least factor of safety) located at the lowest contour.

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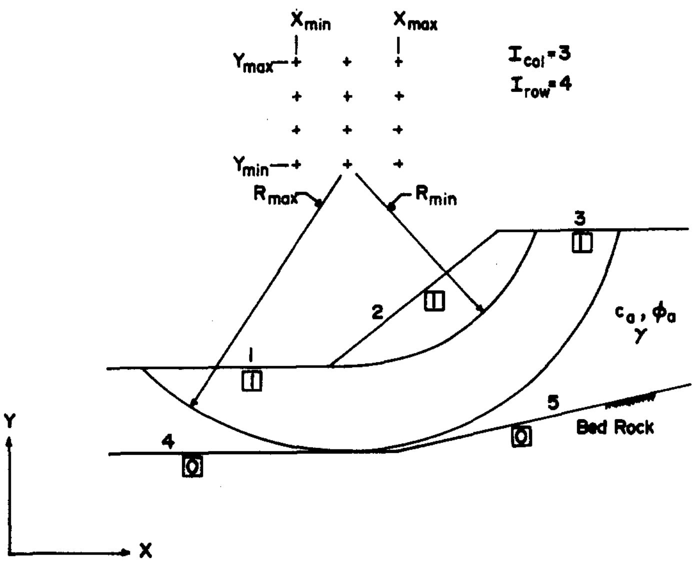

Fig.1. Grid search parameters for circular surface analysis. (From Carter, 1971.)

CARTER'S IRREGULAR FAILURE SURFACE SEARCH

Carter (1971) developed a first approach to a search for critical failure surfaces of irregular shapes. The surfaces are generated within certain limits and according to certain specifications. The limits are defined by X_0, Y_{min}, X_{min}, X_{max} and Y_{max}, see Fig.2. The irregular surface is composed of straight line segments connecting coordinate points (X_k, Y_k), where Y_k is defined by a cosine function, and X_k is determined in such a way that the slope of each succeeding line segment is in Fibonacci multiples of the slope of the initial line segment. The Fibonacci series is defined as follows:

$$T_1 = T_0 = 1$$
$$T_i = T_{i-1} + T_{i-2} \tag{1}$$

The surface terminates at the top of the slope between X_{min} and X_{max}.

If the (X_0, Y_{min}) point is defined below the ground surface, the surface is completed by an additional line segment directed back to the ground surface at angles of 30°, 45° and 60° to the horizontal. The surfaces so generated have a smooth shape as shown in Fig.2.

STABL'S RANDOM SURFACE GENERATION TECHNIQUES

The STABL program, developed at Purdue University (Siegel, 1975; Boutrup, 1978), has gone a step further in searching techniques for regular as well as irregular failure surfaces. Four surface generation routines are available: the circular shape; the sliding block shape, with randomly gener-

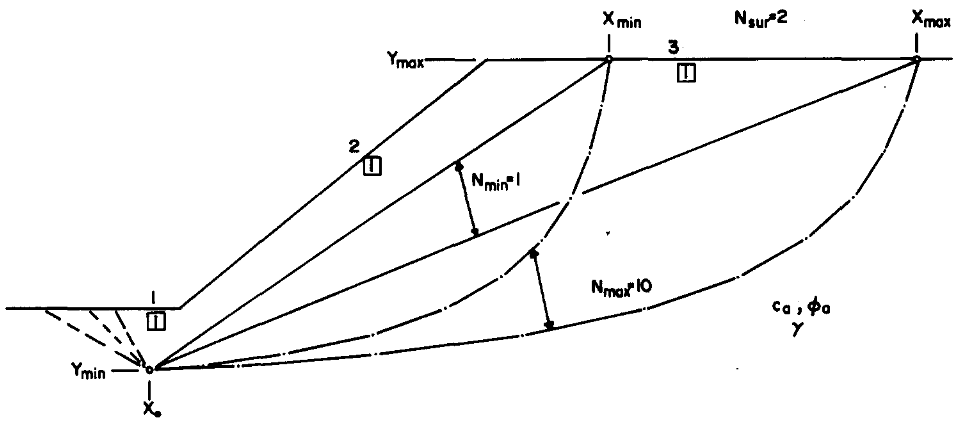

Fig.2. Irregular search parameters for irregular surface analysis. (From Carter, 1971.)

ated active and passive zones; the sliding block shape, with Rankine active and passive zones; and the general irregular shape. Each of the routines makes use of a pseudo-random number function, Ran$f(x)$, which generates real numbers in the range from 0 to 1. All surfaces are composed of straight line segments.

Circular and irregular surfaces

Trial failure surfaces of circular or irregular shape are generated from a number of initiation points with equal horizontal spacing along the ground surface at the base of the slope.

The direction, θ, of the first line segment defining a surface is chosen randomly between two direction limits. As shown on Fig.3:

$$\theta = \alpha_2 + (\alpha_1 - \alpha_2) R^2 \tag{2}$$

where α_1 and α_2 are counterclockwise and clockwise direction limits, and R represents the random function Ran$f(x)$. Using R squared introduces a bias in the random selection of θ so that angles closer to the clockwise direction limit are more likely.

The primary reason for introducing the bias is to obtain a good distribution of completed surfaces. When the initial line segment was given an equal likelihood of being oriented in any direction within the direction limits, poor distributions of completed surfaces were obtained.

For a circular shaped surface, each succeeding line segment is generated by changing the direction by some constant angle chosen randomly between limits defined by the termination limits for the surfaces at the top of the slope, as illustrated in Fig.4:

$$\begin{aligned} \Delta\theta &= (\Delta\theta_{\max} - \Delta\theta_{\min}) R^2 \\ \theta_{i+1} &= \theta_i + \Delta\theta \end{aligned} \tag{3}$$

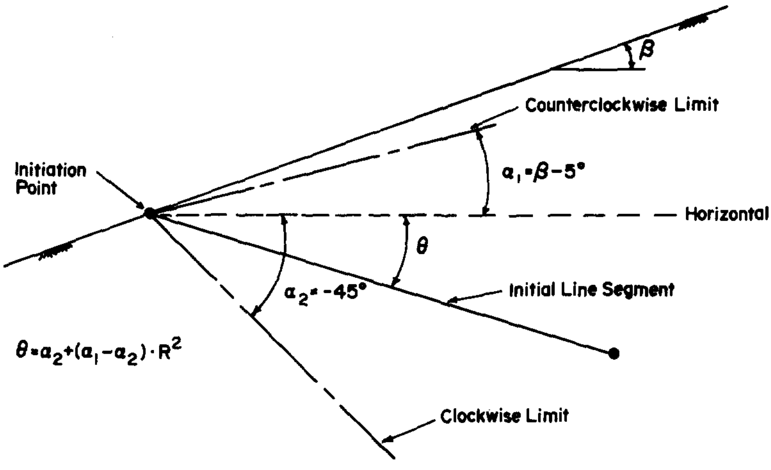

Fig.3. Selecting the initial line segment. (From Siegel, 1975.)

where θ_i is the inclination of the ith line segment of the circular failure surface.

For an irregular surface, the direction of each succeeding line segment is chosen randomly within limits determined by the direction of the preceding line segment, as shown in Fig.5.

The counterclockwise limit for the direction of a line segment, $\Delta\theta_1$, is normally deflected 45° counterclockwise from the projection of the preceding

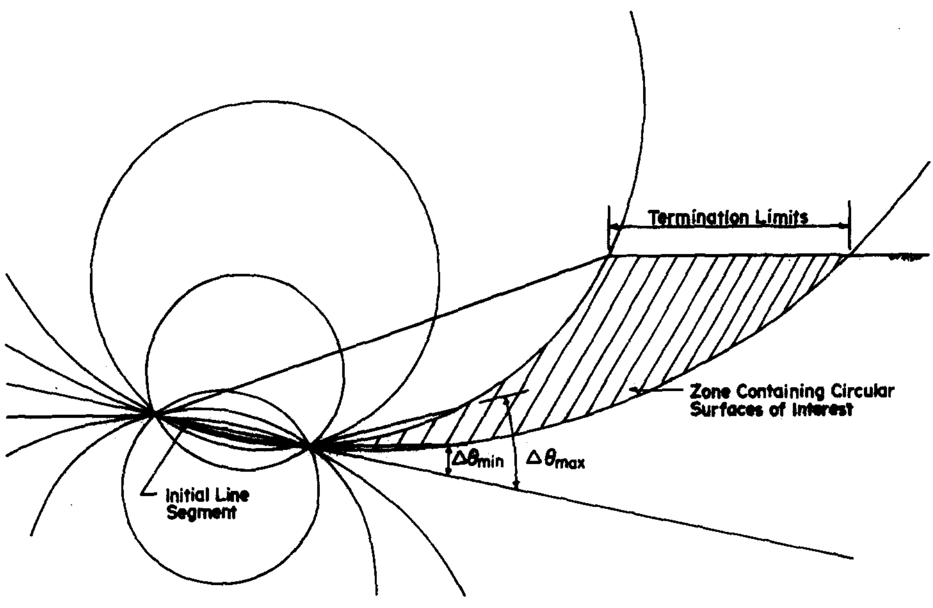

Fig.4. Family of circles having initial line segment as a chord. (From Siegel, 1975.)

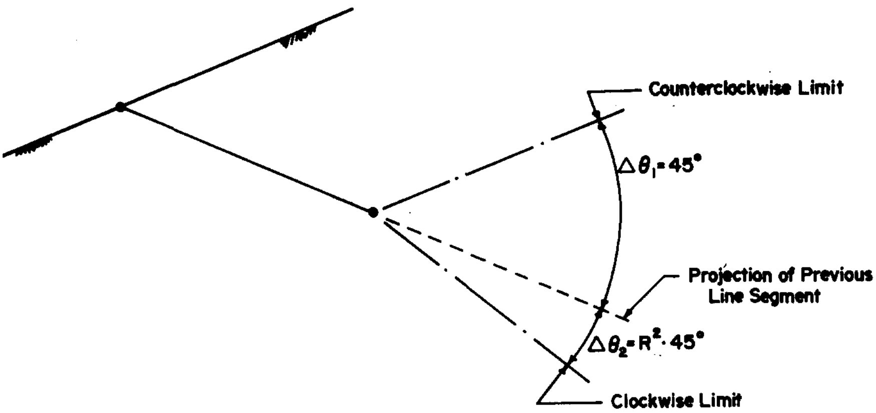

Fig.5. Direction limits for successive line segments of irregular surface. (From Siegel, 1975.)

line segment. If for a particular line segment the orientation of the counterclockwise direction limit, θ_1, is greater than 90° (measured with respect to the horizontal), the inclination of θ_1 is adjusted to 90°.

The clockwise deflection limit for the direction of a line segment, $\Delta\theta_2$, is randomly selected for each shear surface generated:

$$\Delta\theta_2 = R^2 \cdot 45^\circ \tag{4}$$

If for a particular line segment the inclination of the clockwise direction limit, θ_2, is less than —45°, it is set at this value.

The inclination of a line segment is then established by:

$$\theta = \theta_2 + (\theta_1 - \theta_2)R^{(1+R)} \tag{5}$$

where the angular extent of permissible directions, $\theta_1 - \theta_2$, is multiplied by a random number raised to a random power between 1 and 2 and added to the inclination of the clockwise direction limit, θ_2.

The above procedure, although somewhat arbitrary, does produce reasonable irregular surfaces of random shape and position. Reverse curvature of a shear surface is possible, but the frequency of occurrence is small. Unless very short line segments are used, 'kinkyness' of the resulting generated shear surfaces is not a problem.

Surfaces that violate the search limitations as defined either with respect to termination limits, depth limits, or other boundary limits (Fig.6) will be rejected and measures taken to avoid the further generation of such surfaces.

Sliding block type surfaces

This type of surface is used where a well defined weak zone exists within the soil profile. Two procedures are available. In both procedures the central block is obtained by randomly selecting points within defined boxes. Up to ten boxes may be specified, and not less than two. The points are connected

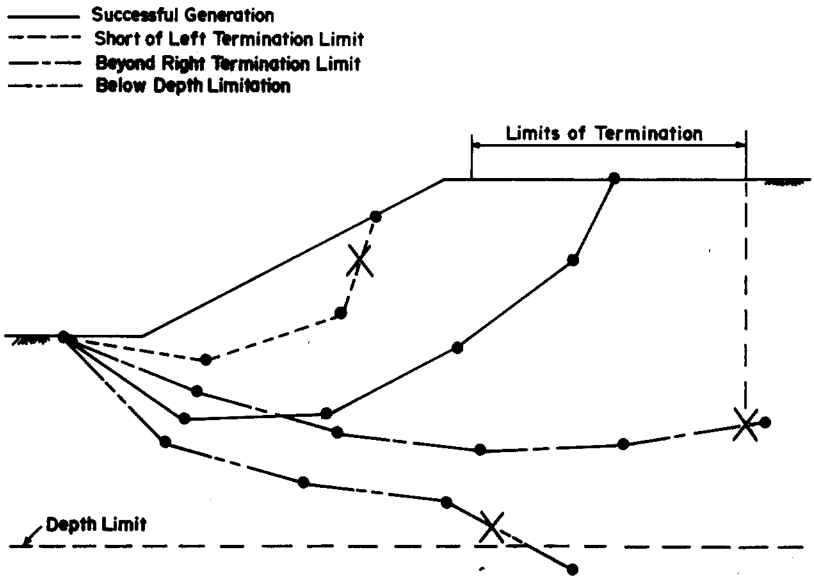

Fig.6. Trial failure surface acceptance criteria. (From Siegel, 1975.)

with straight line segments to form the central block, see Fig.7. The generation of active and passive portions of the surfaces differs for the two procedures. The BLOCK routine generates these zones in a biased random fashion between angle limits of 45° and 90° for the active zone and 0° and −45° for the passive zone as shown in Fig.8. The BLOCK2 routine, utilizing the Rankine earth pressure theory, generates active and passive portions of the failure surface at angles inclined at $(45° + \phi'/2)$ and $(45° - \phi'/2)$ with the horizontal, respectively, see Fig.9.

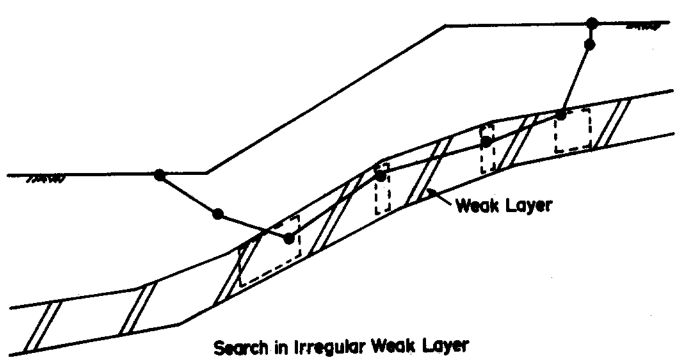

Fig.7. Sliding block generator using more than two boxes. (From Siegel, 1975.)

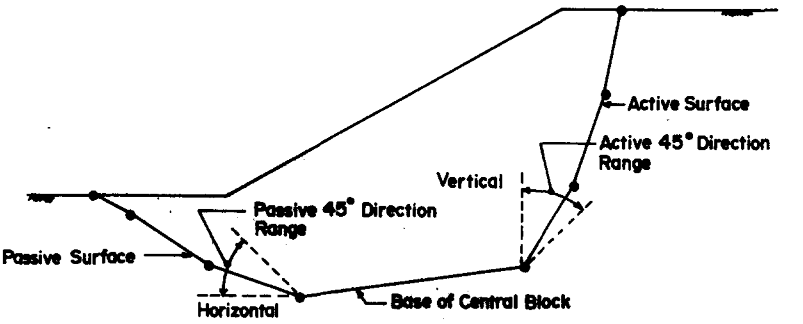

Fig.8. Generation of active and passive sliding surfaces. (From Siegel, 1975.)

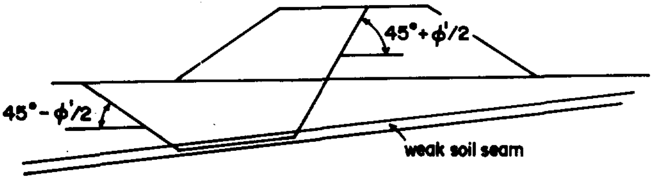

Fig.9. Sliding block failure surface with Rankine active and passive zones. (From Boutrup, 1978.)

Search for the critical failure surfaces

In the search for the most critical shape and location of a potential failure surface, one or several of the trial surface generation routines may be applied, generating a specified number of surfaces and calculating the corresponding factors of safety. For each surface type, the ten with the lowest safety factors are accumulated. High resolution plots are generated, one showing the problem with all surfaces generated (Fig.10) and another with the ten most critical surfaces (Fig.11), so the pattern may be studied.

The ten most critical surfaces will in most cases form a distinctive critical zone. Occasionally, more than one critical zone can be distinguished. In this case each zone should be further studied individually.

The compactness of the critical zone, its location within the zone containing all the surfaces generated, and the range of values for the factors of safety of the ten most critical surfaces, indicate the likelihood that a shear surface exists with a factor of safety significantly lower than any calculated. If the critical zone is narrow, if non-critical surfaces have been generated on both sides of the critical zone, and if the values of the ten most critical surfaces are nearly the same, the defined factor of safety of the most critical surface generated could be assumed, with reasonable confidence. On the other hand, if the critical zone is wide, and/or the more critical surfaces lie along one edge of the zone containing all the surfaces generated, and/or the range of factor of safety values for the critical surfaces is large, the possibility of a shear surface with a significantly smaller value for the factor of safety may require generation of additional surfaces.

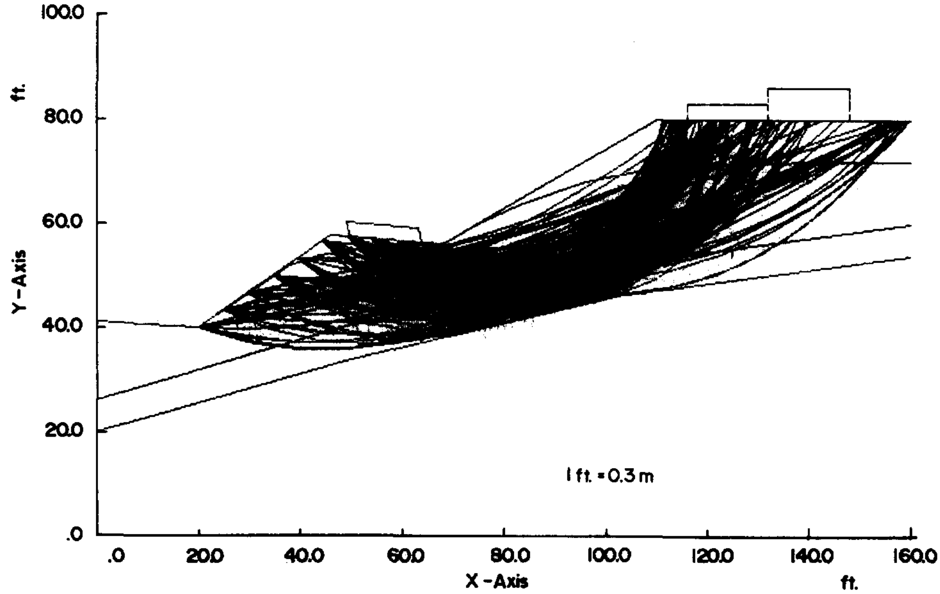

Fig.10. Example Gould plot with 100 circular surfaces. (From Siegel, 1975.)

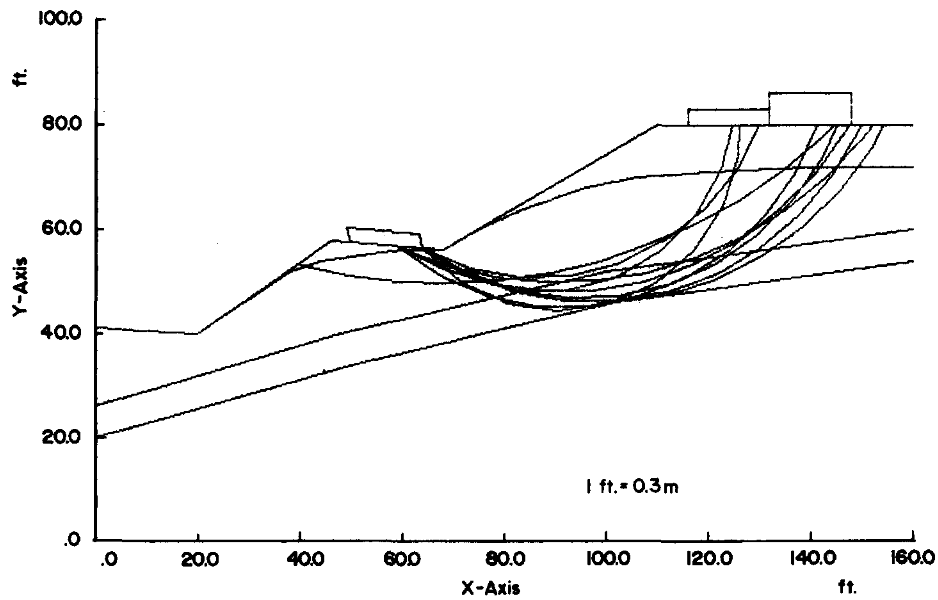

Fig.11. Ten most critical surfaces from Fig.10. (From Siegel, 1975.)

If more trial shear surfaces are required, the limitations can be revised to restrict the additional surfaces to a particular zone of interest. For example, additional shear surfaces generated for the problem shown in Fig.11 should all initiate from the trough of the bench on the slope. They should also be required to terminate at least 4.5 m behind the crest.

In this particular problem, the lower subsurface interface is defined by upward deflecting surface generation boundaries representing a competent slightly folded bedrock surface. Fig.10 demonstrates the control these boundaries have on surface generation.

SELECTION OF FACTOR OF SAFETY

The use of the circular (for $\phi = 0$) and logarithmic spiral (for $\phi \neq 0$) shape of failure surface provides a relatively simple way of calculating the factor of safety by taking moment around the center of the circle or the pole of the logarithmic spiral. These shapes are, however, only reasonable to assume for fairly homogeneous soil conditions. In the case of a soil profile with varying values of ϕ, the logarithmic spiral surface soon becomes impractical. And for circular surfaces in soils with $\phi \neq 0$, one has to start making assumptions with respect to the distribution of normal stresses along the shear surface in order to render the problem determinate.

For circular as well as irregular failure surfaces the soil mass above an assumed slip surface is divided into slices. Assumptions about interslice forces are needed to make the problem determinate. Various methods of slices have developed, based on different assumptions, some particularly applicable to surfaces of circular shape, others applicable to any shape of failure surface.

For circular surfaces, the simplified Bishop method (Bishop, 1955) has been found to provide results that are comparable to the more rigorous methods. Extending the simplified Bishop method to irregular surfaces, no specific point is obvious to use as a moment center. Carter (1971) found that the value of the factor of safety was independent of the x-coordinate of the moment center, but varied with the y-coordinate, and reached a minimum value for $y = \infty$. He concluded that $y = \infty$ was the only consistent moment center to use for irregular as well as circular slip surfaces. Taking moments around $y = \infty$ is the same as using a horizontal force equilibrium, and the equation obtained for the factor of safety is identical to that of the simplified Janbu method (Janbu, 1954). This method was adapted in the STABL program.

PRECAUTIONS

Force equilibrium methods are found in general to be less stable than moment equilibrium methods, and the simplified Janbu method is known to give somewhat conservative results. The difference is often within 10—15%, but larger differences may occur, e.g., where large differences in shear strength exist between soils.

It was further observed that STABL may give nonconservative and erroneous results for failure surfaces that intersect the top of the slope at steep angles, and where the strength of the soil is defined mainly in terms of a strength intercept $c(c')$. This contradicted Carter's earlier conclusion that taking moments around $y = \infty$ always gives a minimum value for the factor of safety.

Since this problem arose mainly for deep circular failure surfaces, it was solved by introducing the original simplified Bishop solution, with moments taken around the center of the circle. But precautions should be taken if the same situation occurs for irregular shaped failure surfaces. In any case it is always advisable to make a preliminary estimate of the factor of safety by means of simple slope stability charts for homogeneous soils.

Procedures other than the Janbu and Bishop methods may be applied in STABL. For example, the Spencer method, as adapted from Wright (1969), was utilized for some comparative example problems, but was not incorporated as a permanent option in the program because of difficulties with convergence.

SUMMARY

In limiting equilibrium slope stability analysis, searching techniques are used to locate the more critical potential failure surfaces. For regular shapes of the failure surface such as a circle or logarithmic spiral, a grid search technique is often applied. Carter (1971) introduced a searching technique for a smooth but irregular shape of failure surface. The STABL program, developed at Purdue University, features unique random techniques for generation of circular, sliding block type, and general irregular shaped surfaces in search for the more critical ones.

The simplified Bishop method of slices is adopted for circular failure surfaces, and the simplified Janbu method for general shapes of failure surfaces. The Janbu procedure usually provides somewhat conservative results. Other procedures may be adapted to the program.

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