Software Requirements Specification for Chipmunk2D

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Contents

1 Reference Material

This section records information for easy reference.

1.1 Table of Units

Throughout this document, SI (Système International d'Unités) is employed as the unit system. For each unit, the symbol is given followed by a description of the unit with the SI name.

symbol	unit	SI
m	length	meter
kg	mass	kilogram
s	time	second
N	force	Newton
rad	angle	radians

1.2 Table of Symbols

The table that follows summarizes the symbols used in this document along with their units. More specific instances of these symbols will be described in their respective sections. Throughout the document, symbols in **bold** will represent vectors, and scalars otherwise. The symbols are listed in alphabetical order.

symbol	unit	description
$\mathbf{a}$	$\mathrm{m\,s^{-2}}$	Acceleration
α	$\mathrm{rad\,s^{-2}}$	Angular acceleration
C_{R}	unitless	Coefficient of restitution
$\mathbf{F}$	N	Force
g	$\mathrm{m\,s^{-2}}$	Gravitational acceleration (9.81 $\mathrm{m\,s^{-2}}$)
G	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	Gravitational constant (6.673×10^{-11} $\mathrm{m^3\,kg^{-1}\,s^{-2}}$)
$\mathbf{I}$	$\mathrm{kg\,m^2}$	Moment of inertia
$\hat{\mathbf{i}}$	m	Horizontal unit vector
$\hat{\mathbf{j}}$	m	Vertical unit vector

j	$\mathrm{N\,s}$	Impulse (scalar)
$\mathbf{J}$	$\mathrm{N\,s}$	Impulse (vector)
L	m	Length
m	kg	Mass
n	unitless	Number of particles in a rigid body
$\mathbf{n}$	m	Collision normal vector
$\boldsymbol{\omega}$	$\mathrm{rad\,s^{-1}}$	Angular velocity
$\mathbf{p}$	m	Position
$\boldsymbol{\phi}$	rad	Orientation
r	m	Distance
$\mathbf{r}$	m	Displacement
t	s	Time
τ	$\mathrm{N\,m}$	Torque
$\boldsymbol{\theta}$	rad	Angular displacement
$\mathbf{v}$	$\mathrm{m\,s^{-1}}$	Velocity

1.3 Abbreviations and Acronyms

symbol	description
A	Assumption
CM	Center of Mass
DD	Data Definition
GD	General Definition
GS	Goal Statement
IM	Instance Model
LC	Likely Change
ODE	Ordinary Differential Equation
R	Requirement
SRS	Software Requirements Specification
T	Theoretical Model
2D	Two-dimensional

2 Introduction

Due to the rising cost of developing video games, developers are looking for ways to save time and money on their projects. Using an open source physics library that is reliable and free will cut down development costs and lead to better quality products.

The following section provides an overview of the Software Requirements Specification (SRS) for Chipmunk2D, an open source 2D rigid body physics library. It explains the purpose of this document, the scope of the system, and the organization of the document.

2.1 Purpose of Document

This document describes the modeling of an open source 2D rigid body physics library used for games. The goals and theoretical models used in Chipmunk2D are provided. This document is intended to be used as a reference to provide all necessary information to understand and verify the model.

This document will be used as a starting point for subsequent development phases, including the writing of the design specification and the software verification and validation plan. The design document will show how the requirements are to be realized. The verification and validation plan will show the steps that will be taken to increase confidence in the software documentation and implementation.

2.2 Scope of Requirements

The scope of the requirements includes the physical simulation of 2D rigid bodies acted on by forces. Given 2D rigid bodies, Chipmunk2D is intended to simulate how these rigid bodies interact with one another.

2.3 Organization of Document

The organization of this document follows the template for an SRS for scientific computing software proposed by [**?**] and [**?**]. The presentation follows the standard pattern of presenting goals, theories, definitions, and assumptions. For readers that would like a more bottom-up approach, they can start reading the instance models in Section 4.2.5 and trace back to find any additional information they require.

The goal statements are refined to the theoretical models, and theoretical models to the instance models.

3 General System Description

This section provides general information about the system, identifies the interfaces between the system and its environment, and describes the user characteristics and system constraints.

3.1 User Characteristics

The end user of Chipmunk2D should have an understanding of first year programming concepts and of high school physics.

3.2 System Constraints

There are no system constraints.

4 Specific System Description

This section first presents the problem description, which provides a high-level view of the problem to be solved. This is followed by the solution characteristics specification, which presents the assumptions, theories, and definitions that are used for the physics library.

4.1 Problem Description

Creating a gaming physics library is a difficult task. Games need physics libraries that can simulate objects acting under various physical conditions, while simultaneously being fast and efficient enough to work in soft real-time during the game. Developing a physics library from scratch takes a long period of time and is very costly, presenting barriers of entry which make it difficult for game developers to include physics in their products. There are a few free, open-source and high quality physics libraries available to be used for consumer products (Section 8). By creating a simple, lightweight, fast, and portable 2D rigid body physics library, game development will be more accessible to the masses and higher quality products will be produced.

4.1.1 Terminology and Definitions

This subsection provides a list of terms that are used in the subsequent sections and their meanings, with the purpose of reducing ambiguity and making it easier to correctly understand the requirements:

- Rigid Body: a solid body in which deformation is neglected.
- Elasticity: ratio of the velocities of two colliding objects after and before the collision.
- Center of Mass: the mean location of the distribution of mass of the object.

- Cartesian coordinates: a coordinate system that specifies each point uniquely in a plane by a pair of numerical coordinates.
- Right-handed coordinate system: a coordinate system where the positive z-axis comes out of the screen.

4.1.2 Goal Statements

GS1: Given the physical properties, initial positions and velocities, and forces applied on a set of rigid bodies, determine their new positions and velocities over a period of time (IM1).

GS2: Given the physical properties, initial orientations and angular velocities, and forces applied on a set of rigid bodies, determine their new orientations and angular velocities over a period of time. (IM2).

GS3: Given the initial positions and velocities of a set of rigid bodies, determine if any of them will collide with one another over a period of time.

GS4: Given the physical properties, initial linear and angular positions and velocities, determine the new positions and velocities over a period of time of rigid bodies that have undergone a collision (IM3).

4.2 Solution Characteristics Specification

4.2.1 Assumptions

This section simplifies the original problem and helps in developing the theoretical model by filling in the missing information for the physical system. The numbers given in the square brackets refer to the data definition, or the instance model, in which the respective assumption is used.

A1: All objects are rigid bodies.

A2: All objects are 2D (two-dimensional).

A3: The library uses a Cartesian coordinate system.

A4: The axes are defined using a right-handed coordinate system.

A5: All rigid body collisions are vertex-to-edge collisions.

A6: There is no damping involved throughout the simulation.

A7: There are no constraints and joints involved throughout the simulation.

4.2.2 Theoretical Models

This section focuses on the general equations and laws that the physics library is based on.

Number	T1
Label	**Newton's second law of motion**
Equation	$\mathbf{F} = m\mathbf{a}$
Description	The net force $\mathbf{F}$ (N) on a body is proportional to the acceleration $\mathbf{a}$ ($\mathrm{m\,s^{-2}}$) of the body, where m (kg) denotes the mass of the body as the constant of proportionality.
Source	
Ref. By	GD1, GD3 IM1

Number	T2
Label	**Newton's third law of motion**
Equation	$\mathbf{F}_1 = -\mathbf{F}_2$
Description	Every action has an equal and opposite reaction. In other words, the force $\mathbf{F}_1$ (N) exerted on the second body by the first is equal in magnitude and in the opposite direction to the force $\mathbf{F}_2$ (N) exerted on the first body by the second.
Source	
Ref. By	GD2

Number	T3
Label	**Newton's law of universal gravitation**
Equation	$\mathbf{F} = G\frac{m_1 m_2}{\|\|\mathbf{r}\|\|^2}\hat{\mathbf{r}} = G\frac{m_1 m_2}{\|\|\mathbf{r}\|\|^2}\frac{\mathbf{r}}{\|\|\mathbf{r}\|\|}$
Description	Two bodies in the universe attract each other with a force $\mathbf{F}$ (N) that is directly proportional to the product of their masses, m_1 and m_2 (kg), and inversely proportional to the square of the distance $\|\|\mathbf{r}\|\|^2$ (m^2) between them. The vector $\mathbf{r}$ (m) is the displacement between the centers of the bodies and $\|\|\mathbf{r}\|\|$ (m) represents the norm, or absolute distance between the two. $\hat{\mathbf{r}}$ denotes the unit displacement vector, equivalent to $\frac{\mathbf{r}}{\|\|\mathbf{r}\|\|}$. Finally, G is the gravitational constant $6.673 \times 10^{-11}\ \mathrm{m^3\,kg^{-1}\,s^{-2}}$.
Source	
Ref. By	GD3

Number	T4
Label	**Chasles' theorem**
Equation	$\mathbf{v}_\mathrm{B} = \mathbf{v}_\mathrm{O} + (\boldsymbol{\omega} \times \mathbf{r}_\mathrm{OB})$
Description	The linear velocity $\mathbf{v}_\mathrm{B}$ ($\mathrm{m\,s^{-1}}$) of a point B in a rigid body (A1) is the sum of the body's linear velocity $\mathbf{v}_\mathrm{O}$ ($\mathrm{m\,s^{-1}}$) at the origin (axis of rotation) and the resultant vector from the cross product of the body's angular velocity $\boldsymbol{\omega}$ ($\mathrm{rad\,s^{-1}}$) and the vector between the origin and point B, $\mathbf{r}_\mathrm{OB}$ (m).
Source	
Ref. By	DD8

Number	T5
Label	**Newton's second law for rotational motion**
Equation	$\tau = \mathbf{I}\alpha$
Description	The net torque τ ($\mathrm{N\,m}$) on a body (GD6) is proportional to its angular acceleration α ($\mathrm{rad\,s^{-2}}$). Here, $\mathbf{I}$ ($\mathrm{kg\,m^2}$) denotes the moment of inertia of the body (GD7). We also assume that all rigid bodies involved are two-dimensional (A2).
Source	
Ref. By	IM2

4.2.3 General Definitions

This section collects the laws and equations that will be used in deriving the data definitions, which in turn will be used to build the instance models.

Number	GD1
Label	**Impulse**
Units	N s
Equation	$\mathbf{J} = \int \mathbf{F}\, \mathrm{d}t = \Delta\mathbf{P} = m\Delta\mathbf{v}$
Description	An impulse $\mathbf{J}$ occurs when a force $\mathbf{F}$ acts over an interval of time. $\mathbf{J}$ is the resultant impulse applied on the body (N s). $\mathbf{F}$ is the force applied on the body (N). $\Delta\mathbf{P}$ is the change in momentum of the body (N s). m is the mass of the body (kg). $\Delta\mathbf{v}$ is the change in velocity of the body ($\mathrm{m\,s^{-1}}$).
Source	
Ref. By	GD2, DD8, IM3

Derivation of Impulse

Newton's second law of motion (T1) states:

$$\mathbf{F} = m\mathbf{a} = m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}$$

Rearranging:

$$\int_{t_1}^{t_2} \mathbf{F}\, \mathrm{d}t = m \int_{v_1}^{v_2} \mathrm{d}\mathbf{v}$$

Integrating the right hand side:

$$\int_{t_1}^{t_2} \mathbf{F}\, \mathrm{d}t = m\mathbf{v_2} - m\mathbf{v_1} = m\Delta\mathbf{v}$$

Number	GD2
Label	**Conservation of momentum**
Equation	$\sum_{k=0}^{n} m_k \mathbf{v}_{\mathrm{i}_k} = \sum_{k=0}^{n} m_k \mathbf{v}_{\mathrm{f}_k}$
Description	In an isolated system, where the sum of external impulses acting on the system is zero, the total momentum of the bodies is constant (conserved). m_k is the mass of the k-th body (kg). $\mathbf{v}_{\mathrm{i}_k}$ is the initial velocity of the k-th body ($\mathrm{m\,s^{-1}}$). $\mathbf{v}_{\mathrm{f}_k}$ is the final velocity of the k-th body ($\mathrm{m\,s^{-1}}$).
Source	
Ref. By	IM3

Derivation of the Conservation of Momentum

When bodies collide, they exert an equal force on each other in opposite directions. This is Newton's third law (T2):

$$\mathbf{F}_1 = -\mathbf{F}_2$$

The objects collide with each other for the exact same amount of time t:

$$\mathbf{F}_1 t = -\mathbf{F}_2 t \tag{1}$$

The above equation is equal to the impulse (GD1):

$$\mathbf{F}_1 t = \int \mathbf{F}_1 \, \mathrm{d}t = \mathbf{J}$$

The impulse is equal to the change in momentum:

$$\mathbf{J} = \Delta \mathbf{P} = m \Delta \mathbf{v} \tag{2}$$

Substituting 2 into 1 yields:

$$m_1 \Delta \mathbf{v}_1 = -m_2 \Delta \mathbf{v}_2$$

Expanding and rearranging the above formula gives:

$$m_1 \mathbf{v}_{\mathrm{i}_1} + m_2 \mathbf{v}_{\mathrm{i}_2} = m_1 \mathbf{v}_{\mathrm{f}_1} + m_2 \mathbf{v}_{\mathrm{f}_2}$$

Generalizing for multiple (k) colliding objects:

$$\sum_{k=0}^{n} m_k \mathbf{v}_{\mathrm{i}_k} = \sum_{k=0}^{n} m_k \mathbf{v}_{\mathrm{f}_k}$$

Number	GD3
Label	**Acceleration due to gravity**
Units	$\mathrm{m\,s^{-2}}$
Equation	$\mathbf{F}_\mathrm{g} = m\mathbf{g}$, where $\mathbf{g} = [-g_\mathrm{x}, -g_\mathrm{y}]$
Description	$\mathbf{F}_\mathrm{g}$ is the force due to gravity (N). m is the mass of a rigid body (kg). $\mathbf{g}$ is the acceleration due to gravity ($\mathrm{m\,s^{-2}}$).
Source	
Ref. By	IM1

Derivation of Gravitational Acceleration

From Newton's law of universal gravitation (T3), we have:

$$\mathbf{F} = G\frac{m_1 m_2}{||\mathbf{r}||^2}\hat{\mathbf{r}} \tag{3}$$

Equation 3 governs the gravitational attraction between two bodies. Suppose that one of the bodies is significantly more massive than the other, so that we concern ourselves with the force the massive body exerts on the lighter body. Further suppose that the coordinate system is chosen such that this force acts on a line which lies along one of the principal axes (A2). Then our unit vector $\hat{\mathbf{r}} = \frac{\mathbf{r}}{||\mathbf{r}||} = \hat{\mathbf{i}}$ or $\hat{\mathbf{j}}$ for the x or y axes (A3), respectively.

Given the above assumptions, let M and m be the mass of the massive and light body, respectively. Using 3 and equating this with Newton's second law (T1) for the force experienced by the light body, we get:

$$\mathbf{F}_\mathrm{g} = G\frac{Mm}{||\mathbf{r}||^2}\hat{\mathbf{r}} = m\mathbf{g} \tag{4}$$

where $\mathbf{g}$ is gravitational acceleration. Dividing 4 by m, and resolving this into separate x and y components:

$$G\frac{M}{||r_\mathrm{x}||^2}\hat{\mathbf{i}} = -g_\mathrm{x}\hat{\mathbf{i}}$$
$$G\frac{M}{||r_\mathrm{y}||^2}\hat{\mathbf{j}} = -g_\mathrm{y}\hat{\mathbf{j}}$$

Thus:

$$\mathbf{g} = [-g_\mathrm{x}, -g_\mathrm{y}]$$

Number	GD4
Label	**Relative velocity in collisions**
Units	$\mathrm{m\,s^{-1}}$
Equation	$\mathbf{v}^{\mathrm{AB}} = \mathbf{v}^{\mathrm{AP}} - \mathbf{v}^{\mathrm{BP}}$
Description	In a collision, the velocity of a rigid body A colliding with another body B relative to that body, $\mathbf{v}^{\mathrm{AB}}$, is the difference between the velocities of A and B at point P. $\mathbf{v}^{\mathrm{AB}}$ is the velocity of A relative to B ($\mathrm{m\,s^{-1}}$). P is the common collision point on both bodies (m). $\mathbf{v}^{\mathrm{AP}}$ is the velocity of point P in body A ($\mathrm{m\,s^{-1}}$). $\mathbf{v}^{\mathrm{BP}}$ is the velocity of point P in body B ($\mathrm{m\,s^{-1}}$).
Source	
Ref. By	GD5, DD8

Number	GD5
Label	**Coefficient of restitution**
Equation	$C_{\mathrm{R}} = -\frac{\mathbf{v}_{\mathrm{f}}^{\mathrm{AB}} \cdot \mathbf{n}}{\mathbf{v}_{\mathrm{i}}^{\mathrm{AB}} \cdot \mathbf{n}}$
Description	The coefficient of restitution C_{R} is a unitless, dimensionless quantity that determines the elasticity of a collision between two bodies. $C_{\mathrm{R}} = 1$ results in an elastic collision, while $C_{\mathrm{R}} < 1$ results in an inelastic collision, and $C_{\mathrm{R}} = 0$ results in a totally inelastic collision. C_{R} is the coefficient of restitution (unitless). $\mathbf{n}$ is the collision normal vector (m). Its signed direction is defined by (A4). $\mathbf{v}_{\mathrm{i}}^{\mathrm{AB}}$ is the initial relative velocity (GD4) of body A with respect to body B before collision ($\mathrm{m\,s^{-1}}$). $\mathbf{v}_{\mathrm{f}}^{\mathrm{AB}}$ is the final relative velocity (GD4) of body A with respect to body B after collision ($\mathrm{m\,s^{-1}}$).
Source	
Ref. By	DD8

Number	GD6
Label	**Torque**
Units	N m
Equation	$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}$
Description	The torque $\boldsymbol{\tau}$ on a body measures the tendency of a force to rotate the body around an axis or pivot. $\boldsymbol{\tau}$ is the torque on the body (N m). $\mathbf{F}$ is the force applied to the lever arm (N). $\mathbf{r}$ is a position vector of the point where the force is applied, measured from the axis of rotation (m).
Source	
Ref. By	T5, IM3

Number	GD7
Label	**Moment of inertia**
Units	kg m^2
Equation	$\mathbf{I} = \sum_{i=0}^{n} m_i r_{\mathrm{p}_i}^2$
Description	The moment of inertia $\mathbf{I}$ of a body measures how much torque is needed for the body to achieve an angular acceleration about axis of rotation. $\mathbf{I}$ is the moment of inertia (kg m^2). n is number of particles of the body. m_i is the mass of the i-th particle (kg). r_{p_i} is the distance between the i-th particle and the axis of rotation (m).
Source	
Ref. By	T5, DD8, IM3

4.2.4 Data Definitions

This section collects and defines all the data needed to build the instance models. The dimension of each quantity is also given.

Number	DD1
Label	**Center of mass**
Symbol	$\mathbf{p}_{\text{CM}}$
Units	m
Equation	$\mathbf{p}_{\text{CM}} = \frac{\sum_i m_i \mathbf{p}_i}{M}$
Description	The center of mass $\mathbf{p}_{\text{CM}}$ (m) of a rigid body (A1) is the mass-weighted average position of all its particles, or the unique point where all of its mass is concentrated. m_i is the mass of the i-th particle (kg). $\mathbf{p}_i$ is the position vector (A2) of the i-th particle (m). M is the total mass of the body (kg).
Sources	
Ref. By	IM1, IM3

Number	DD2
Label	**Linear displacement**
Symbol	$\mathbf{r}$
Units	m
Equation	$\mathbf{r}(t) = \frac{\mathrm{d}\mathbf{p}(t)}{\mathrm{d}t}$
Description	$\mathbf{r}(t)$ is the linear displacement of a body (A1, A2), without damping (A6), as a function of time t, also equal to the derivative of its linear position with respect to time t (m).
Sources	
Ref. By	IM1

Number	DD3
Label	**Linear velocity**
Symbol	$\mathbf{v}$
Units	$\mathrm{m\,s^{-1}}$
Equation	$\mathbf{v}(t) = \frac{\mathrm{d}\mathbf{r}(t)}{\mathrm{d}t}$
Description	$\mathbf{v}(t)$ is the linear velocity of a body (A1, A2), without damping (A6), as a function of time t, also equal to the derivative of its linear displacement with respect to time t ($\mathrm{m\,s^{-1}}$).
Sources	
Ref. By	IM1

Number	DD4
Label	**Linear acceleration**
Symbol	$\mathbf{a}$
Units	$\mathrm{m\,s^{-2}}$
Equation	$\mathbf{a}(t) = \frac{\mathrm{d}\mathbf{v}(t)}{\mathrm{d}t}$
Description	$\mathbf{a}(t)$ is the linear acceleration of a body (A1, A2), without damping (A6), as a function of time t, also equal to the derivative of its linear velocity with respect to time t ($\mathrm{m\,s^{-2}}$).
Sources	
Ref. By	IM1

Number	DD5
Label	**Angular displacement**
Symbol	$\boldsymbol{\theta}$
Units	rad
Equation	$\boldsymbol{\theta}(t) = \frac{\mathrm{d}\boldsymbol{\phi}(t)}{\mathrm{d}t}$
Description	$\boldsymbol{\theta}(t)$ is the angular displacement of a body (A1, A2), without damping (A6), as a function of time t, also equal to the derivative of its angular position with respect to time t (rad).
Sources	
Ref. By	IM2

Number	DD6
Label	**Angular velocity**
Symbol	$\boldsymbol{\omega}$
Units	$\mathrm{rad\,s^{-1}}$
Equation	$\boldsymbol{\omega}(t) = \frac{\mathrm{d}\boldsymbol{\theta}(t)}{\mathrm{d}t}$
Description	$\boldsymbol{\omega}(t)$ is the angular velocity of a body (A1, A2), without damping (A6), as a function of time t, also equal to the derivative of its angular displacement with respect to time t ($\mathrm{rad\,s^{-1}}$).
Sources	
Ref. By	IM2

Number	DD7
Label	**Angular acceleration**
Symbol	$\boldsymbol{\alpha}$
Units	$\mathrm{rad\,s^{-2}}$
Equation	$\boldsymbol{\alpha}(t) = \frac{\mathrm{d}\boldsymbol{\omega}(t)}{\mathrm{d}t}$
Description	$\boldsymbol{\alpha}(t)$ is the angular acceleration of a body (A1, A2), without damping (A6), as a function of time t, also equal to the derivative of its angular velocity with respect to time t ($\mathrm{rad\,s^{-2}}$).
Sources	
Ref. By	IM2

Number	DD8
Label	**Impulse for collision response**
Symbol	j
Units	$\mathrm{N\,s}$
Equation	$j = \frac{-(1 + C_{\mathrm{R}})\mathbf{v}_{\mathrm{i}}^{\mathrm{AB}} \cdot \mathbf{n}}{\left(\frac{1}{m_{\mathrm{A}}} + \frac{1}{m_{\mathrm{B}}}\right)\lvert\lvert\mathbf{n}\rvert\rvert^2 + \frac{\lvert\lvert\mathbf{r}_{\mathrm{AP}} \times \mathbf{n}\rvert\rvert^2}{\mathbf{I}_{\mathrm{A}}} + \frac{\lvert\lvert\mathbf{r}_{\mathrm{BP}} \times \mathbf{n}\rvert\rvert^2}{\mathbf{I}_{\mathrm{B}}}}$
Description	j is the impulse (scalar) used to determine collision response (A5) between two rigid bodies (A1, A2) . C_{R} is the coefficient of restitution (GD5). $\mathbf{n}$ is the collision normal vector (m). Its signed direction is defined by (A4). $\mathbf{v}_{\mathrm{i}}^{\mathrm{AB}}$ is the relative velocity (GD4) between body A and body B ($\mathrm{m\,s^{-1}}$). m_{A} and m_{B} are the masses of body A and B, respectively (kg). $\mathbf{r}_{\mathrm{AP}}$ and $\mathbf{r}_{\mathrm{BP}}$ are the displacement vectors between the centers of mass of body A and B, respectively, and the point of contact P (m). $\mathbf{I}_{\mathrm{A}}$ and $\mathbf{I}_{\mathrm{B}}$ are the moments of inertia (GD7) for body A and body B, respectively ($\mathrm{kg\,m^2}$).
Sources	
Ref. By	IM3

Derivation for Impulse for Collision Response

Rearranging the equation for the coefficient of restitution (GD5), we get:

$$\mathbf{v}_\mathrm{f}^\mathrm{AB} \cdot \mathbf{n} = -C_\mathrm{R}\mathbf{v}_\mathrm{i}^\mathrm{AB} \cdot \mathbf{n}$$

Expanding the relative velocity (GD4) on the left:

$$(\mathbf{v}_\mathrm{f}^\mathrm{AP} - \mathbf{v}_\mathrm{f}^\mathrm{BP}) \cdot \mathbf{n} = -C_\mathrm{R}\mathbf{v}_\mathrm{i}^\mathrm{AB} \cdot \mathbf{n}$$

Applying Chasles' Theorem (T4) and IM3 on the left-hand side:

$$\begin{aligned}
&(\mathbf{v}_\mathrm{f}^\mathrm{A} + \boldsymbol{\omega}_\mathrm{f}^\mathrm{A} \times \mathbf{r}_\mathrm{AP} - \mathbf{v}_\mathrm{f}^\mathrm{B} - \boldsymbol{\omega}_\mathrm{f}^\mathrm{B} \times \mathbf{r}_\mathrm{BP}) \cdot \mathbf{n} \\
&\implies \left(\mathbf{v}_\mathrm{i}^\mathrm{A} + \frac{j}{m_\mathrm{A}}\mathbf{n} + \left(\boldsymbol{\omega}_\mathrm{i}^\mathrm{A} + \frac{\mathbf{r}_\mathrm{AP} \times j\mathbf{n}}{\mathbf{I}_\mathrm{A}}\right) \times \mathbf{r}_\mathrm{AP} - \mathbf{v}_\mathrm{i}^\mathrm{B} + \frac{j}{m_\mathrm{B}}\mathbf{n} - \left(\boldsymbol{\omega}_\mathrm{i}^\mathrm{B} - \frac{\mathbf{r}_\mathrm{BP} \times j\mathbf{n}}{\mathbf{I}_\mathrm{B}}\right) \times \mathbf{r}_\mathrm{BP}\right) \cdot \mathbf{n}
\end{aligned}$$

Expanding and then collecting terms:

$$\begin{aligned}
&\Bigg[(\mathbf{v}_\mathrm{i}^\mathrm{A} + \boldsymbol{\omega}_\mathrm{i}^\mathrm{A} \times \mathbf{r}_\mathrm{AP}) - (\mathbf{v}_\mathrm{i}^\mathrm{B} + \boldsymbol{\omega}_\mathrm{i}^\mathrm{B} \times \mathbf{r}_\mathrm{BP}) \\
&+ j\left(\frac{1}{m_\mathrm{A}} + \frac{1}{m_\mathrm{B}}\right)\mathbf{n} + j\left(\frac{\mathbf{r}_\mathrm{AP} \times \mathbf{n} \times \mathbf{r}_\mathrm{AP}}{\mathbf{I}_\mathrm{A}} + \frac{\mathbf{r}_\mathrm{BP} \times \mathbf{n} \times \mathbf{r}_\mathrm{BP}}{\mathbf{I}_\mathrm{B}}\right)\Bigg] \cdot \mathbf{n} \\
&\implies (\mathbf{v}_\mathrm{i}^\mathrm{AP} - \mathbf{v}_\mathrm{i}^\mathrm{BP}) \cdot \mathbf{n} + j\left[\left(\frac{1}{m_\mathrm{A}} + \frac{1}{m_\mathrm{B}}\right)\mathbf{n} + \left(\frac{\mathbf{r}_\mathrm{AP} \times \mathbf{n} \times \mathbf{r}_\mathrm{AP}}{\mathbf{I}_\mathrm{A}} + \frac{\mathbf{r}_\mathrm{BP} \times \mathbf{n} \times \mathbf{r}_\mathrm{BP}}{\mathbf{I}_\mathrm{B}}\right)\right] \cdot \mathbf{n} \\
&\implies \mathbf{v}_\mathrm{i}^\mathrm{AB} \cdot \mathbf{n} + j\left[\left(\frac{1}{m_\mathrm{A}} + \frac{1}{m_\mathrm{B}}\right)\mathbf{n} \cdot \mathbf{n} + \left(\frac{\mathbf{r}_\mathrm{AP} \times \mathbf{n} \times \mathbf{r}_\mathrm{AP}}{\mathbf{I}_\mathrm{A}} + \frac{\mathbf{r}_\mathrm{BP} \times \mathbf{n} \times \mathbf{r}_\mathrm{BP}}{\mathbf{I}_\mathrm{B}}\right) \cdot \mathbf{n}\right] \\
&\implies \mathbf{v}_\mathrm{i}^\mathrm{AB} \cdot \mathbf{n} + j\left[\left(\frac{1}{m_\mathrm{A}} + \frac{1}{m_\mathrm{B}}\right)\mathbf{n} \cdot \mathbf{n} + \frac{(\mathbf{r}_\mathrm{AP} \times \mathbf{n}) \cdot (\mathbf{r}_\mathrm{AP} \times \mathbf{n})}{\mathbf{I}_\mathrm{A}} + \frac{(\mathbf{r}_\mathrm{BP} \times \mathbf{n}) \cdot (\mathbf{r}_\mathrm{BP} \times \mathbf{n})}{\mathbf{I}_\mathrm{B}}\right] \\
&\implies \mathbf{v}_\mathrm{i}^\mathrm{AB} \cdot \mathbf{n} + j\left[\left(\frac{1}{m_\mathrm{A}} + \frac{1}{m_\mathrm{B}}\right)||\mathbf{n}||^2 + \frac{||\mathbf{r}_\mathrm{AP} \times \mathbf{n}||^2}{\mathbf{I}_\mathrm{A}} + \frac{||\mathbf{r}_\mathrm{BP} \times \mathbf{n}||^2}{\mathbf{I}_\mathrm{B}}\right]
\end{aligned}$$

Finally, equating the left and right-hand sides back together and rearranging for j, we obtain:

$$j = \frac{-(1 + C_\mathrm{R})\mathbf{v}_\mathrm{i}^\mathrm{AB} \cdot \mathbf{n}}{\left(\frac{1}{m_\mathrm{A}} + \frac{1}{m_\mathrm{B}}\right)||\mathbf{n}||^2 + \frac{||\mathbf{r}_\mathrm{AP} \times \mathbf{n}||^2}{\mathbf{I}_\mathrm{A}} + \frac{||\mathbf{r}_\mathrm{BP} \times \mathbf{n}||^2}{\mathbf{I}_\mathrm{B}}}$$

4.2.5 Instance Models

This section transforms the problem defined in Section 4.1 into one expressed in mathematical terms. It uses concrete symbols defined in Section 4.2.4 to replace the abstract symbols in the models identified in Sections 4.2.2 and 4.2.3.

Number	IM1
Label	**Force on the translational motion of a set of 2D rigid bodies**
Input	$m_i, \mathbf{g}, \mathbf{p}_i(t_0), \mathbf{v}_i(t_0), \mathbf{F}_i(t_0)$
Output	$\mathbf{p}_i(t), \mathbf{v}_i(t)$, such that the following ODE is satisfied: $\mathbf{a}_i(t) = \frac{\mathrm{d}\mathbf{v}_i(t)}{\mathrm{d}t} = \mathbf{g} + \frac{\mathbf{F}_i(t)}{m_i}$
Description	The above equation expresses the total acceleration of the rigid body (A1, A2) i as the sum of gravitational acceleration (GD3) and acceleration due to applied force $\mathbf{F}_i(t)$ (T1). The resultant outputs are then obtained from this equation using DD2, DD3 and DD4. It is currently assumed that there is no damping (A6) or constraints (A7) involved. m_i is the mass of the i-th rigid body (kg). $\mathbf{g}$ is the acceleration due to gravity ($\mathrm{m\,s^{-2}}$). t is a point in time and t_0 denotes the initial time (s). $\mathbf{p}_i(t)$ is the i-th body's position (specifically, the position of its center of mass, $\mathbf{p}_{\mathrm{CM}}(t)$ (DD1)) at time t (m). $\mathbf{a}_i(t)$ is the i-th body's acceleration at time t ($\mathrm{m\,s^{-2}}$). $\mathbf{v}(t)$ is the i-th body's velocity at time t ($\mathrm{m\,s^{-1}}$). $\mathbf{F}(t)$ is the force applied to the i-th body at time t (N).
Sources	
Ref. By	GS1, R2, R5

Number	IM2
Label	**Force on the rotational motion of a set of 2D rigid body**
Input	$m_i, \mathbf{g}, \boldsymbol{\phi}_i(t_0), \boldsymbol{\omega}_i(t_0), \boldsymbol{\tau}_i(t_0), \mathbf{I}_i$
Output	$\boldsymbol{\phi}_i(t), \boldsymbol{\omega}_i(t)$, such that the following ODEs is satisfied: $\boldsymbol{\alpha}_i(t) = \frac{\mathrm{d}\boldsymbol{\omega}_i(t)}{\mathrm{d}t} = \frac{\boldsymbol{\tau}_i(t)}{\mathbf{I}_i}$
Description	The above equation for the total angular acceleration of the rigid body (A1, A2) i is derived from T5, and the resultant outputs are then obtained from this equation using DD5, DD6 and DD7. It is currently assumed that there is no damping (A6) or constraints (A7) involved. m_i is the mass of the i-th rigid body (kg). $\mathbf{g}$ is the acceleration due to gravity ($\mathrm{m\,s^{-2}}$). t is a point in time and t_0 denotes the initial time (s). $\boldsymbol{\phi}_i(t)$ is the i-th body's orientation at time t (rad). $\boldsymbol{\omega}_i(t)$ is the i-th body's angular velocity at time t ($\mathrm{rad\,s^{-1}}$). $\boldsymbol{\alpha}_i(t)$ is the i-th body's angular acceleration at time t ($\mathrm{rad\,s^{-2}}$). $\boldsymbol{\tau}_i(t)$ is the torque applied to the i-th body at time t (N m). Signed direction of torque is defined by (A4). $\mathbf{I_i}$ is the moment of inertia of the i-th body ($\mathrm{kg\,m^2}$).
Sources	
Ref. By	GS2, R6

Number	IM3
Label	**Collisions on 2D rigid bodies**
Input	$m_k, \mathbf{p}_k(t_0), \mathbf{v}_k(t_0), \boldsymbol{\phi}_k(t_0), \boldsymbol{\omega}_k(t_0), C_\mathrm{R}$
Output	$\mathbf{v}_k(t), \mathbf{p}_k(t), \mathbf{p}_2(t), \boldsymbol{\phi}_k(t), \boldsymbol{\omega}_k(t)$ such that momentum is conserved: $\sum_{k=0}^{n} m_k \mathbf{v}_{\mathrm{i}_k} = \sum_{k=0}^{n} m_k \mathbf{v}_{\mathrm{f}_k}$ (GD2) and that for any colliding pair of rigid bodies A and B, the following equations are satisfied: $\mathbf{v}_\mathrm{A}(t_c) = \mathbf{v}_\mathrm{A}(t) + \frac{j}{m_\mathrm{A}}\mathbf{n}$ $\mathbf{v}_\mathrm{B}(t_c) = \mathbf{v}_\mathrm{B}(t) - \frac{j}{m_\mathrm{B}}\mathbf{n}$ $\boldsymbol{\omega}_\mathrm{A}(t_c) = \boldsymbol{\omega}_\mathrm{A}(t) + \frac{\mathbf{r}_\mathrm{AP} \times j\mathbf{n}}{\mathbf{I}_\mathrm{A}}$ $\boldsymbol{\omega}_\mathrm{B}(t_c) = \boldsymbol{\omega}_\mathrm{B}(t) - \frac{\mathbf{r}_\mathrm{BP} \times j\mathbf{n}}{\mathbf{I}_\mathrm{B}}$
Description	This instance model is based on our assumptions regarding rigid body (A1, A2) collisions (A5). Again, this does not take damping (A6) or constraints (A7) into account. m_k is the mass of the k-th rigid body (kg). $\mathbf{I}_k$ is the moment of inertia of the k-th rigid body (kg m^2). t is a point in time, t_0 denotes the initial time, and t_c denotes the time at collision (s). $\mathbf{p}_i(t)$ is the i-th body's position (specifically, the position of its center of mass, $\mathbf{p}_\mathrm{CM}(t)$ (DD1)) at time t (m). $\mathbf{v}_k(t)$ is the k-th body's velocity at time t (m s^{-1}). $\boldsymbol{\phi}_k(t)$ is the k-th body's orientation at time t (rad). $\boldsymbol{\omega}_k(t)$ is the k-th body's angular velocity at time t (rad s^{-1}). $\mathbf{n}$ is the collision normal vector (m). Its signed direction is determined by (A4). j is the collision impulse (DD8) (N s). P is the point of collision (m). $\mathbf{r}_\mathrm{kP}$ is the displacement vector between the center of mass of the k-th body and point P (m).
Sources	
Ref. By	GS4, DD8, R3, R8

Collision Diagram

This section presents an image of a typical collision between two 2D rigid bodies labeled A and B, showing the position of the two objects, the collision normal vector $\mathbf{n}$ and the vectors from the approximate center of mass of each object to the point of collision P, $\mathbf{r}_{\mathrm{AP}}$ and $\mathbf{r}_{\mathrm{BP}}$. Note that this figure only presents vertex-to-edge collisions, as per our assumptions (A5).

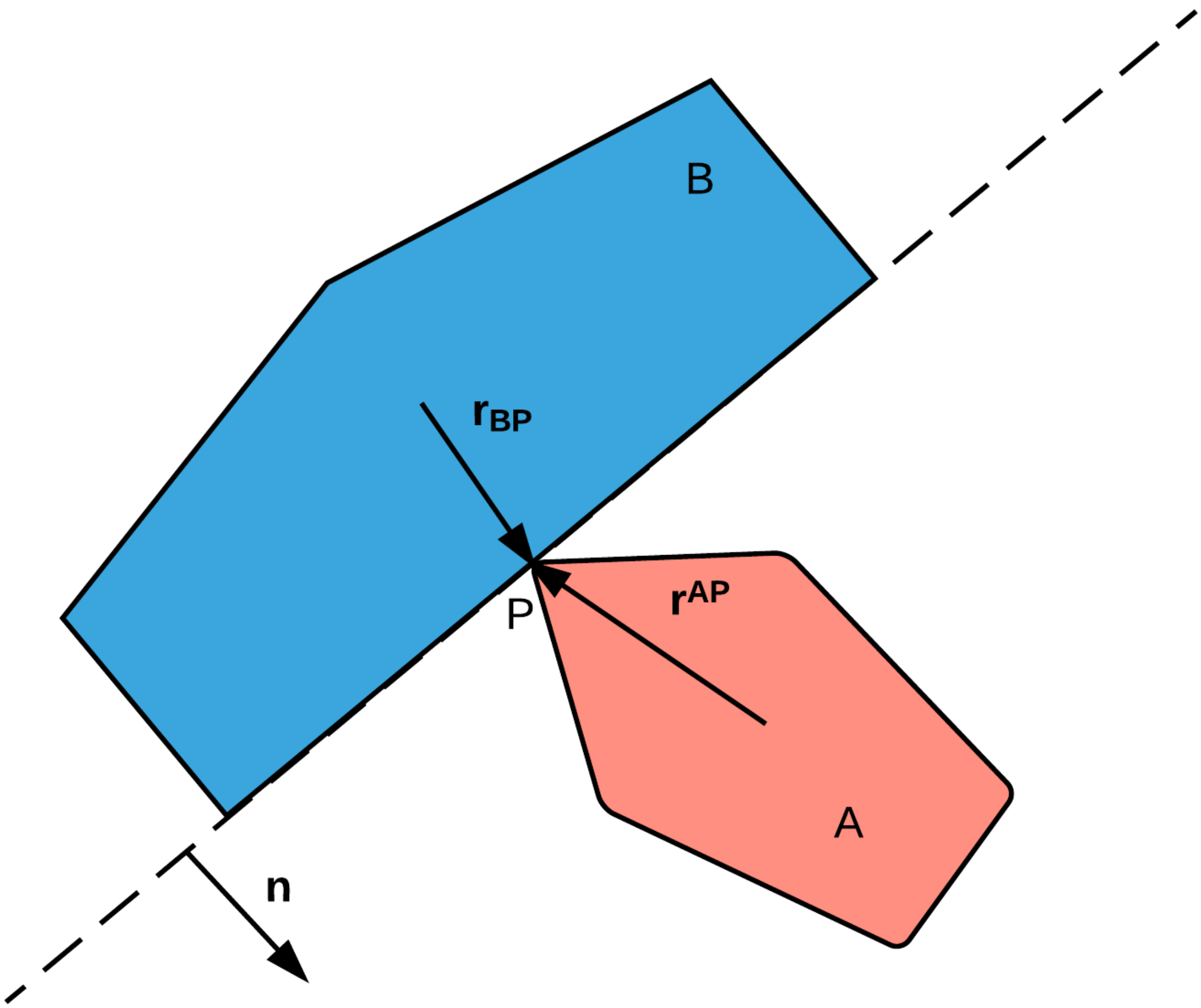

Figure 1: Collision between two rigid bodies

4.2.6 Data Constraints

Table 1 and 2 show the data constraints on the input and output variables, respectively. The "Physical Constraints" column gives the physical limitations on the range of values that can be taken by the variable. The constraints are conservative, to give the user of the model the flexibility to experiment with unusual situations. The column of typical values is intended to provide a feel for a common scenario.

Var	Physical Constraints	Typical Value
L	$L \geq 0$	44.2 m
m	$m \geq 0$	56.2 kg
$\mathbf{I}$	$\mathbf{I} \geq 0$	74.5 $\mathrm{kg\,m^2}$
g	None	9.8 $\mathrm{m\,s^{-2}}$
$\mathbf{p}$	None	(0.412, 0.502) m
$\mathbf{v}$	None	2.51 $\mathrm{m\,s^{-1}}$
C_{R}	$0 \leq C_{\mathrm{R}} \leq 1$	0.8
$\boldsymbol{\phi}$	$0 \leq \phi < 2\pi$	$\frac{\pi}{2}$ rad
$\boldsymbol{\omega}$	None	2.1 $\mathrm{rad\,s^{-1}}$
$\mathbf{F}$	None	98.1 N
$\boldsymbol{\tau}$	None	200 N m

Table 1: Input Variables

Var	Physical Constraints
$\mathbf{p}$	None
$\mathbf{v}$	None
$\boldsymbol{\phi}$	$0 \leq \phi < 2\pi$
$\boldsymbol{\omega}$	None

Table 2: Output Variables

5 Requirements

This section provides the functional requirements: the business tasks that the software is expected to complete, and the nonfunctional requirements: the qualities that the software is expected to exhibit.

5.1 Functional Requirements

R1: Create a space for all of the rigid bodies in the physical simulation to interact in.

R2: Input the initial mass, velocities, positions, orientations, angular velocities of, and forces applied on rigid bodies (IM1, IM2, R4).

R3: Input the surface properties of the bodies, such as friction or elasticity (IM3, R4),

R4: Verify that the inputs satisfy the required physical constraints (Section 4.2.6).

R5: Determine the position and velocities over a period of time of the 2D rigid bodies acted upon by a force (IM1).

R6: Determine the orientation and angular velocities over a period of time of the 2D rigid bodies (IM2).

R7: Determine if any of the rigid bodies in the space have collided (R1).

R8: Determine the position and velocities over a period of time of 2D rigid bodies that have undergone a collision (IM3, R7).

5.2 Nonfunctional Requirements

Games are resource-intensive, so performance is a high priority. Other non-functional requirements that are a priority are: correctness, understandability, portability, reliability, and maintainability.

6 Likely Changes

This section lists the likely changes to be made to the physics game library.

LC1: The internal ODE-solving algorithm used by the library may change in the future.

LC2: The library may be expanded to deal with edge-to-edge and vertex-to-vertex collisions (A5).

LC3: The library may be expanded to include motion with damping (A6).

LC4: The library may be expanded to include joints and constraints (A7).

7 Traceability Matrices and Graphs

The purpose of traceability matrices is to provide easy references on what has to be additionally modified if a certain component is changed. Every time a component is changed, the items in the column of that component that are marked with an "X" should be modified as well. Table 3 shows the dependencies of goal statements, requirements, instance models and data constraints with each other. Table 4 shows the dependencies of theoretical models, general definitions, data definitions and instance models on the assumptions. Finally, Table 5 shows the dependencies of the theoretical models, general definitions, data definitions and instance models on each other.

	IM1	IM2	IM3	R1	R4	R7	Data Constraints (4.2.6)
GS1	X						
GS2		X					
GS3							
GS4			X			X	
R1							
R2	X	X			X		
R3			X		X		
R4							X
R5	X						
R6		X					
R7				X			
R8			X			X	

Table 3: Traceability Matrix showing the connections between Goal Statements, Requirements, Data Constraints and Instance Models

	A1	A2	A3	A4	A5	A6	A7
T1							
T2							
T3							
T4	X						
T5							
GD1							
GD2							
GD3		X	X				
GD4							
GD5							
GD6							
GD7							
DD1	X	X					
DD2	X	X				X	
DD3	X	X				X	
DD4	X	X				X	
DD5	X	X				X	
DD6	X	X				X	
DD7	X	X				X	
DD8	X	X		X	X		
IM1	X	X				X	X
IM2	X	X		X		X	X
IM3	X	X			X	X	X
LC1							
LC2					X		
LC3						X	
LC4							X

Table 4: Traceability Matrix showing the connections between Assumptions and other items

	T1	T2	T3	T4	T5	GD1	GD2	GD3	GD4	GD5	GD6	GD7	DD1	DD2	DD3	DD4	DD5	DD6	DD7	DD8	IM1	IM2	IM3
T1																							
T2																							
T3																							
T4																							
T5											X	X											
GD1	X																						
GD2		X				X																	
GD3	X		X																				
GD4																							
GD5									X														
GD6																							
GD7																							
DD1																							
DD2																							
DD3																							
DD4																							
DD5																							
DD6																							
DD7																							
DD8				X		X			X	X		X											X
IM1	X							X					X	X	X	X							
IM2					X												X	X	X				
IM3						X	X				X	X	X							X			

Table 5: Traceability Matrix showing the connections between items of different sections

The purpose of the traceability graphs is also to provide easy references on what has to be additionally modified if a certain component is changed. The arrows in the graphs represent dependencies. The component at the tail of an arrow is depended on by the component at the head of that arrow. Therefore, if a component is changed, the components that it points to should also be changed. Figure 2 shows the dependencies of goal statements, requirements, instance models and data constraints with each other. Figure 3 shows the dependencies of theoretical models, general definitions, data definitions and instance models on the assumptions. Finally, Figure 4 shows the dependencies of the theoretical models, general definitions, data definitions and instance models on each other. Building a tool to automatically generate the graphical representation of the matrix by scanning the label and reference can be future work.

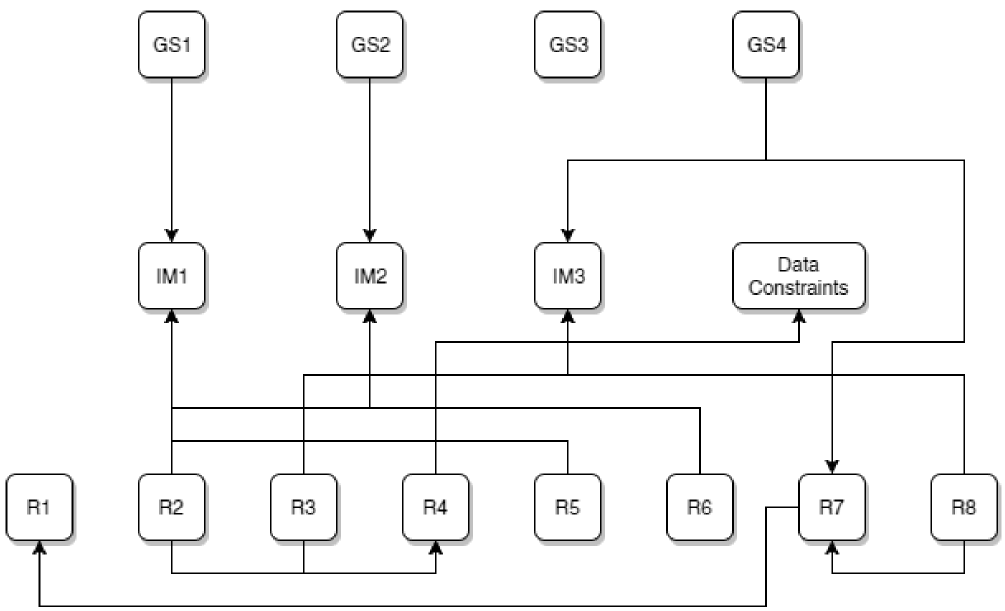

Figure 2: Traceability Graph showing the connections between Goal Statements, Requirements, Data Constraints and Instance Models

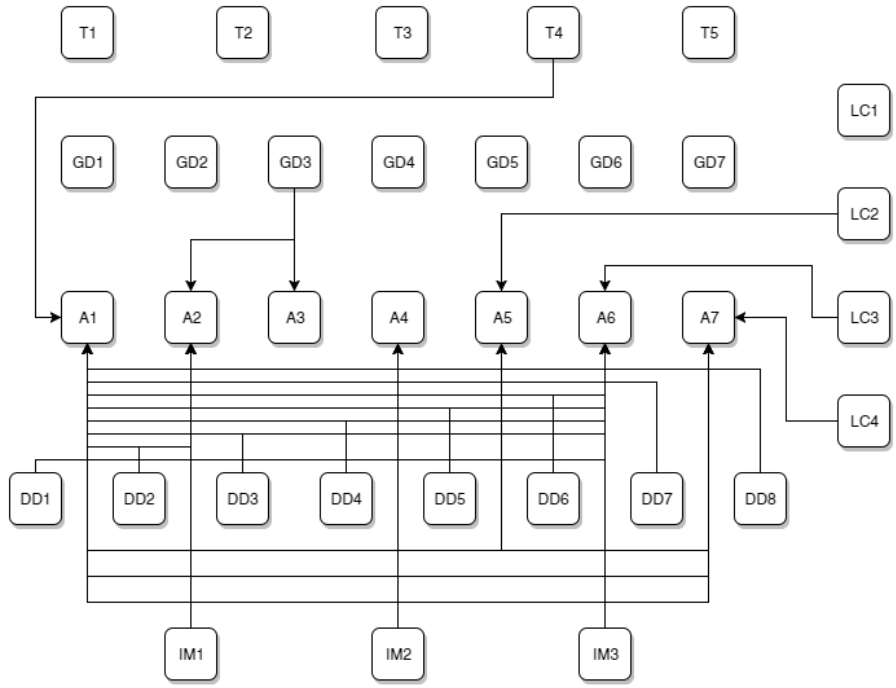

Figure 3: Traceability Graph showing the connections between Assumptions and other items

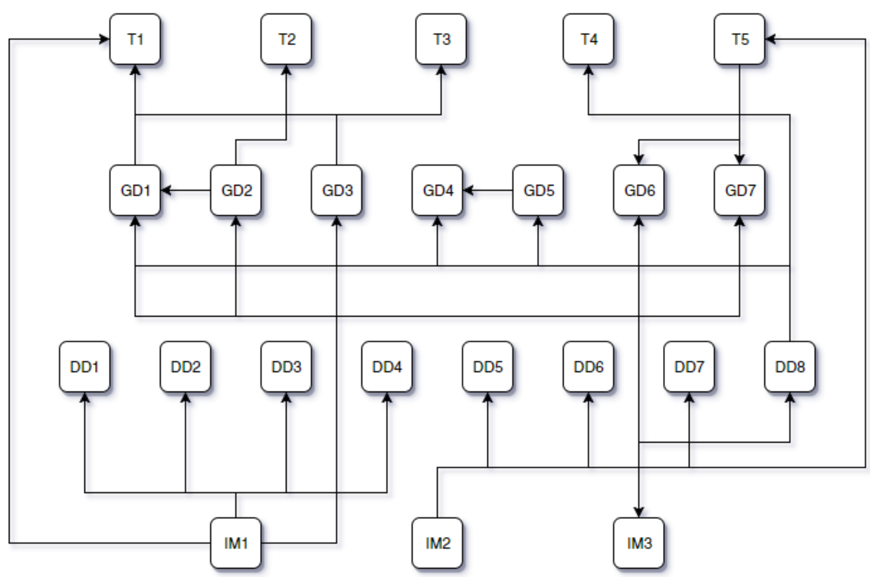

Figure 4: Traceability Graph showing the connections between items of different sections

8 Off the Shelf Solutions

As mentioned in section 4.1, there already exist free open source game physics libraries. Similar 2D physics libraries are:

- Box2D http://box2d.org/
- Nape Physics Engine http://napephys.com/

Free open source 3D game physics libraries include:

- Bullet http://bulletphysics.org/
- Open Dynamics Engine http://www.ode.org/
- Newton Game Dynamics http://newtondynamics.com/