

## THE ACCURACY OF EQUILIBRIUM METHODS OF SLOPE STABILITY ANALYSIS

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### ABSTRACT

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Equilibrium methods of slope stability analysis all involve assumptions because the number of equilibrium equations available is smaller than the number of unknowns involved. Furthermore, a number of widely used methods do not satisfy all conditions of equilibrium, and thus do not employ all of the available equations of equilibrium. This paper discusses the inaccuracies which arise from these inevitable assumptions and these neglected conditions of equilibrium.

Comparative analyses have shown that all of the methods which satisfy all conditions of equilibrium result in the same value of safety factor with an accuracy no worse than  $\pm 5\%$ , which is perfectly acceptable for practical purposes. Furthermore, within this range of accuracy, this may be considered to be the 'correct' answer. Bishop's modified method, which does not satisfy all conditions of equilibrium, has been found to be as accurate as the methods which do so, and thus to be an effective and simple means of analyzing circular slip surfaces.

The ordinary method of slices, which satisfies only one condition of equilibrium, gives factors of safety which may be as much as 50% smaller than the correct value for flat slopes with high pore pressures. Force equilibrium procedures with ill-chosen side force assumptions may give factors of safety which are 30% larger than the correct value for slopes in cohesive soils.

The results of the study show how equilibrium methods may be selected which avoid significant errors arising from the mechanics of the analysis, and thus allow the engineer performing the analysis to devote his attention and effort to correct evaluation of shear strength.

### INTRODUCTION

Equilibrium analyses of slope stability are widely used in design of excavation and embankment slopes, and extensive experience has demonstrated their effectiveness and reliability. The accuracy of an equilibrium analysis of slope stability depends on the accuracy with which the strength properties and geometric conditions can be defined, and on the inherent accuracy of the method of analysis.

In most cases the uncertainties related to definition of geometry and soil properties are greater than those which arise from the approximations involved in the analytical technique, and the most accurate possible evaluation of shear strength is a critical aspect of all analytical studies of stability. Uncertainties in slope stability calculations also arise from the approximations made in developing the methods of analysis, and in some cases these are very significant.

The characteristics of the ordinary method of slices (Fellenius, 1927), Bishop's modified method (Bishop, 1955), Morgenstern and Price's method (Morgenstern and Price, 1965), Janbu's generalized procedure of slices (Janbu, 1957), Spencer's method (Spencer, 1967), the log spiral method (Wright, 1969), and the force equilibrium methods (Lowe and Karafiath, 1960) are reviewed in the following sections. It is shown that in some cases the errors resulting from inherent inaccuracies in the analyses may be very significant. However, if the method of analysis is suitable for the condition analyzed, the inaccuracy in the factor of safety engendered by the method of analysis need be no more than a few percent.

#### EQUILIBRIUM METHODS

All equilibrium methods of slope stability analysis have four characteristics in common:

- (1) They all use the same definition of the factor of safety ( $F$ ):

$$F = s/\tau \tag{1}$$

in which  $s$  = shear strength and  $\tau$  = shear stress required for equilibrium. The use of this definition of  $F$  is appropriate because the greatest uncertainties in most practical problems are related to evaluation of the shear strength. Thus defining  $F$  in terms of a factor on shear strength associates the factor of safety directly with what is almost always the most significant uncertainty in practical problems.

- (2) They all involve the implicit assumption that the stress-strain characteristics of the soils forming the slopes are non-brittle, and that the same value of shear strength ( $s$ ) may be mobilized over a wide range of strains along the slip surface. This assumption is necessary because there is no consideration of strains or deformations in these methods, and no assurance that the strains may not vary significantly from point to point along the slip surface. Thus, strictly speaking, these methods are not applicable to analysis of slopes in soils such as stiff-fissured clays and shales, which have residual strengths appreciably lower than their peak strengths. In practice this problem is overcome by using strengths lower than the peak in such cases, as indicated by experience with slopes which have failed (Skempton, 1977).

- (3) They all use some or all of the equations of equilibrium to calculate the average value of  $\tau$ , and to calculate the normal stress on the slip surface ( $p$ ) which is required to determine the shear strength using eq.2:

$$s = c + p \tan \phi \quad (2)$$

in which  $c$  and  $\phi$  are the Mohr-Coulomb strength parameters.

(4) They all involve explicit assumptions to supplement the equations of equilibrium. Since the number of equilibrium equations is smaller than the number of unknowns in the problem, all methods employ assumptions to make up the balance.

## EQUATIONS, UNKNOWNNS AND ASSUMPTIONS

Methods of analysis which are applicable to practical problems must be able to accommodate conditions where the slip surface is curved and the soil properties and pore pressures vary with location through the slope. For this reason, most of the equilibrium methods divide the freebody bounded by the slip surface into a number of vertical slices. The forces acting on a typical slice are shown in Fig.1. These forces are  $W$  = weight of slice,  $S$  = shear force on base of slice ( $S = \tau$  multiplied by  $l$ , where  $l$  is the length of the base),  $P'$  = effective normal force on base,  $U$  = water pressure force on base,  $X$  = vertical side force, and  $E$  = horizontal side force.  $c'$  and  $\phi'$  are the effective stress shear strength parameters and  $F$  is the factor of safety.

### *Methods satisfying all conditions of equilibrium*

Methods which use both force and moment equilibrium (see Table I) have three equations of equilibrium for each slice (horizontal force, vertical force, and moment). For  $N$  slices, this is a total of  $3N$  equations. The unknowns are:  $N$  values of  $P'$  (one for each slice),  $N-1$  values of  $X$  (one for each inter-slice boundary),  $N-1$  values of  $E$ ,  $N-1$  locations (moment arms) of  $E$ ,  $N$  locations of  $P'$ , and one value of  $F$ . In total there are  $5N-2$  unknowns. Thus for more than one slice, the equations exceed the unknowns and assumptions are required to make up the balance.

The location of  $P'$  on the base of the slice is not a critical unknown. Assuming that  $P'$  acts in the center of the base or below the center of gravity introduces very little uncertainty, especially when the slices are narrow, and each of the methods which satisfy all conditions of equilibrium make one of these two assumptions, reducing the number of unknowns to  $4N-2$ . This leaves  $N-2$  non-trivial assumptions required to make the equations balance the unknowns. The assumptions involved in various methods are summarized in Table II.

### *Force equilibrium methods*

Methods which use only force equilibrium have two equations for each slice (horizontal force and vertical force equilibrium). For  $N$  slices, this is  $2N$  equations. The unknowns are:  $N$  values of  $P'$ ,  $N-1$  values of  $X$ ,  $N-1$  values of  $E$ , and one value of  $F$ . In total there are  $3N-1$  unknowns. Thus, as for

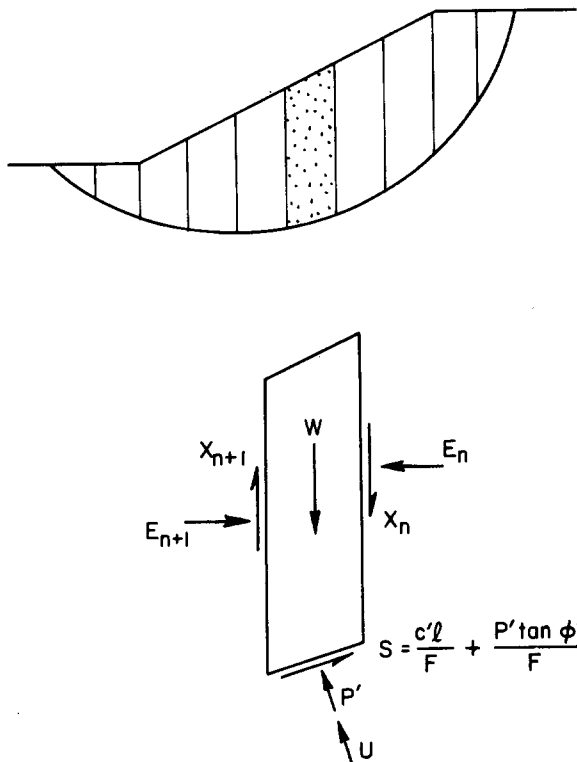


Fig.1. Forces on a typical slice.

methods using both force and moment equilibrium, the number of unknowns exceeds the number of equations if there is more than one slice, and assumptions are required to make up the difference between the number of equations and the number of unknowns.

### *The ordinary method of slices*

Some methods do not employ some of the equilibrium equations discussed above. For example, the ordinary method of slices satisfies only one condition of equilibrium, which is moment equilibrium around the center of the circular slip surface. (The method is only applicable to circular slip surfaces.) The method assumes that the resultant of all side forces acting on any slice acts parallel to the base of the slice. Then, by resolving forces normal to the base, the following equation is derived:

$$P = P' + U = W \cos \alpha \quad (3)$$

in which  $\alpha$  = inclination of the base of the slice. Because the direction in which forces are resolved varies from slice to slice (and because equilibrium is not satisfied in the direction parallel to the base of each slice), the ordinary method of slices does not satisfy either horizontal or vertical force equilibrium

TABLE I

## Characteristics of equilibrium methods

Procedure	Equilibrium conditions satisfied				Equations and unknowns*	Shape of slip surface	Practical for:	
	overall moment	ind. slice moment	vert. force	horiz. force			hand calc.	computer calc.
Ordinary method of slices	yes	no	no	no	1	circular	yes	yes
Bishop's modified method	yes	no	yes	no	$N + 1$	circular	yes	yes
Janbu's generalized procedure of slices	yes	yes	yes	yes	$3N$	any	yes	yes
Morgenstern and Price's method	yes	yes	yes	yes	$3N$	any	no	yes
Spencer's method	yes	yes	yes	yes	$3N$	any	no	yes
Force equilibrium	no	no	yes	yes	$2N$	any	yes	yes
Log spiral	yes	—	yes	yes	3	log spiral	yes	yes

\* $N$  = number of slices.

TABLE II

## Assumptions in equilibrium methods

Procedure	Assumptions employed
Ordinary method of slices	Resultant of side forces is parallel to base of each slice.
Bishop's modified method	Resultant of side forces is horizontal (no vertical side forces).
Janbu's generalized procedure of slices	Location of side force resultants on sides of slices (location can be varied).
Morgenstern and Price's method	Pattern of variation of side force inclination ( $\theta$ ) from slice to slice: $\theta = \lambda f(x)$ . The value of $f(x)$ is assumed at each interslice boundary, and the value of $\lambda$ is an unknown.
Spencer's method	Side forces are parallel ( $\theta = \text{constant}$ ). Corresponds to $f(x) = \text{constant}$ in Morgenstern and Price's method.
Force equilibrium methods	Inclinations of side force (value of $\theta$ ) at each interslice boundary.
Log spiral	Shape of slip surface is a logarithmic spiral.

for the mass above the slip surface. Thus the method involves only one equation (overall moment equilibrium around the center of the circle) and one unknown (the factor of safety).

The value of  $P$  given by eq.3 is a conservative (low) approximation, which leads to a conservative value of  $F$ . In cases of flat slopes with high pore pressures, the error in the value of  $F$  may be as much as 50%. In total stress analyses the error is not more than about 10%.

### *Bishop's modified method*

The simplified method developed by Bishop (1955) satisfies  $N + 1$  conditions of equilibrium. These are: overall moment equilibrium around the center of the circle (the method is applicable only to circular slip surfaces), and vertical equilibrium for each slice. Thus the method has  $N + 1$  equations and  $N + 1$  unknowns, the unknowns being the  $N$  values of  $P$  on the base of each slice and the factor of safety. The method does not satisfy horizontal force equilibrium or individual slice moment equilibrium. In spite of the fact that it does not satisfy all conditions of equilibrium, Bishop's modified method has been found to be an accurate method of analysis for circular slip surfaces, including flat slopes with high pore pressures.

### *Force equilibrium procedures*

These include: (1) the method described by Lowe and Karafiath (1960) which follows the work of Taylor; (2) the method developed by Seed and Sultan (1967); (3) various methods (commonly known as 'wedge' methods) which divide the slip surface into an active uphill wedge, a straight central section, and a passive downhill wedge; and (4) any other method which satisfies only force (not moment) equilibrium. In all such methods the analysis can be accomplished by trial-and-error graphical procedures wherein a value of  $F$  is assumed and a trial force polygon is drawn for each slice or wedge; if the last slice is in equilibrium, the assumed value of  $F$  is correct. The analyses may also be accomplished by a numerical equivalent of this graphical procedure.

In all force equilibrium procedures, the assumed quantities are the  $N - 1$  values of the side force inclinations. The factor of safety calculated using these procedures is very significantly affected by the assumed side force inclination. For example, an embankment on a soft clay foundation which the authors studied had a factor of safety equal to 2.27 as calculated by methods which satisfied all conditions of equilibrium. Detailed studies indicated that this value could be considered 'correct', at least within a range of  $\pm 15\%$ . However, a force equilibrium analysis, in which the side force was assumed to be parallel to the ground surface, resulted in a value of  $F = 2.98$ , about 30% higher than the correct value. Although this difference was not practically significant in the particular case studied because all methods indicated the embankment was stable, it could have been highly significant if the strengths of the embankment and the clay foundation (and thus the value of  $F$ ) had been lower. For example, for strength values low enough so that the correct value of  $F$  was 1.0, the force equilibrium method with side forces assumed parallel to the ground surface would have shown  $F = 1.3$ . On the basis of the force equilibrium analysis the embankment would have been judged to have a margin of safety, whereas it was actually on the verge of instability.

Of all the possible assumed inclinations for the side forces, the one

suggested by Lowe and Karafiath (1960) appears to be the most generally applicable. They proposed that the side force inclination at each interslice boundary should be assumed to be the average of: (a) the inclination of the ground surface at the top of the interslice boundary; and (b) the inclination of the slip surface at the bottom of the interslice boundary. This assumption leads to values of  $F$  which are about 10% too high for  $\phi = 0$  analyses. For larger values of  $\phi$  or  $\phi'$ , the assumption leads to more accurate values of  $F$ .

### *The log spiral method*

Unlike the methods discussed previously, the log spiral method is not a method of slices. By assuming that the slip surface has the shape of a logarithmic spiral, the equilibrium of the mass bounded by the slip surface can be satisfied completely without further assumption. The method involves three equations (overall moment, horizontal and vertical force equilibrium) and three unknowns (the magnitude and the direction of the resultant of the normal and the frictional shear forces, and the factor of safety). Because the log spiral method is applicable only to homogeneous conditions, it is not particularly useful for practical purposes, which nearly always involve greater complexity. The method is useful as a basis for comparison with the other equilibrium methods, all of which employ slices and thus involve fundamentally different types of assumptions.

## COMPARISONS OF FACTORS OF SAFETY

Although Tables I and II show that the various methods listed satisfy different conditions of equilibrium, and that they employ different assumptions to make up the balance between equations and unknowns, it remains to be determined whether these differences have large or small effects on the factors of safety calculated by the various methods. A number of comparative analyses have been made to evaluate these differences. Before these results are discussed, it is useful to consider a number of factors which influence factors of safety for slopes.

### *Minimum values of $F$*

In making comparisons of the various methods it is essential that the values of  $F$  which are compared are the minimum values for each method. Table III illustrates why this is so. In general, if a number of methods are used to analyze circular failure surfaces in the same slope, they will be found to have different critical circles and different values of  $F$  for the same circle. Thus, as shown in Table III, if a comparison was made for circles which are selected arbitrarily, the comparison might support any conclusion regarding which of the methods results in the lower factor of safety, i.e., it might be concluded that  $F_1 < F_2$ ,  $F_1 = F_2$  or  $F_1 > F_2$ . Obviously, only one of these can be correct in general. The only factor of safety for a given slope and

TABLE III

Illustration of the necessity of comparing minimum factors of safety

	Factor of safety calculated by:	
	method 1	method 2
Circle A (critical for method 1)	$F_1 = 1.5$ (min)	$F_2 = 1.7$
Circle B (not critical for either method)	$F_1 = 1.6$	$F_2 = 1.6$
Circle C (critical for method 2)	$F_1 = 1.8$	$F_2 = 1.4$ (min)
Comparisons:		
circle A:	$F_1 < F_2$	
circle B:	$F_1 = F_2$	
circle C:	$F_1 > F_2$	
min. values:	$F_{1,min} > F_{2,min}$	

method of analysis which has any general significance is the minimum factor of safety. Thus, in comparing factors of safety for various methods, the minimum values of  $F$  should be compared. This procedure has been followed in developing the comparisons discussed in the following paragraphs.

#### Definition of $\lambda_{c\phi}$

One problem in comparing methods of slope stability analysis is that quite a large number of parameters are involved in the problem. These include the slope height ( $H$ ), the slope angle ( $\beta$ ), the values of the strength parameters ( $c$  and  $\phi$ ), the unit weight of the soil ( $\gamma$ ), and the magnitudes of the pore-water pressures within the slope. The number of parameters is so large that making a systematic comparison of values of  $F_{min}$  for the methods discussed previously would be a very formidable task. Fortunately, Janbu (1954) defined a dimensionless parameter ( $\lambda_{c\phi}$ ) which combines four of the variables involved in the problem, and reduces the number of parameters which must be investigated from six to three. The definition of  $\lambda_{c\phi}$  is:

$$\lambda_{c\phi} = \gamma H(\tan\phi)/c \quad (4)$$

It may be seen that for a slope in soil with only cohesive shear strength ( $\tan\phi = 0$ ),  $\lambda_{c\phi}$  is equal to zero. As the value of  $c$  decreases, the value of  $\lambda_{c\phi}$  approaches infinity. The examples listed in Table IV show values of  $\lambda_{c\phi}$  for various conditions.

For values of  $\lambda_{c\phi} < 1$ , the critical slip circle passes beneath the toe of the slope, as shown in Fig.2, provided the slope angle is flatter than about  $50^\circ$ . Theoretically, the critical circle would extend infinitely deep in a layer of

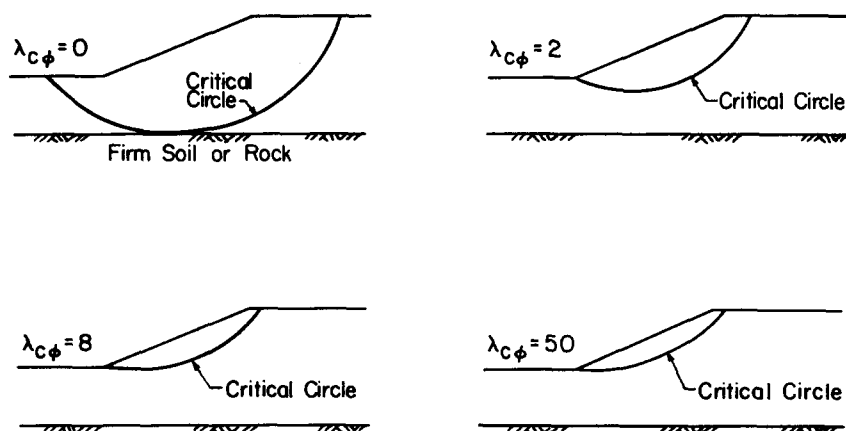


TABLE IV

Values of  $\lambda_{c\phi}$  for various slopes

$\gamma$ (t/m <sup>3</sup> )	$H$ (m)	$\tan\phi$ (dimensionless)	$c$ (t/m <sup>3</sup> )	$\lambda_{c\phi}$ (dimensionless)
any value	any value	0	any value	0
1.6	10	0.5	4	2
1.6	10	0.5	2	4
1.6	10	1.0	2	8
1.6	20	1.0	2	16
2.4	20	1.0	2	24
any value	any value	any value	0	$\infty$

$$\lambda_{c\phi} = \lambda H(\tan\phi)/c$$

Fig.2. Critical circle locations for four values of  $\lambda_{c\phi}$ .

constant shear strength with  $\lambda_{c\phi} < 1$ . Practically, the critical surface will extend to the top of firm soil or rock.

For values of  $\lambda_{c\phi} > 2$ , the critical slip circle passes through the toe of the slope, becoming more and more shallow as the value of  $\lambda_{c\phi}$  increases as shown in Fig.2. For  $\lambda_{c\phi}$  approaching infinity ( $c = 0$ ), the critical slip surface is a plane parallel to the surface of the slope. This plane parallel to the surface of the slope, and passing through the toe of the slope, may be thought of as the shallowest possible slip circle, which has infinite radius. For this special condition all of the methods described in Tables II and III give the same value of  $F$ , which may be shown to be:

$$F_{(c=0)} = \tan\phi / \tan\beta \quad (5)$$

for a slope with no pore pressures.

### *Definition of $r_u$*

The pore pressures within a slope affect its stability, and they also affect the relative values of factor of safety calculated by various methods. For purposes of examining the effects of pore pressure, it is convenient to examine conditions in which the ratio of the pore pressure to the overburden pressure is constant throughout the slope. This ratio, denoted by the parameter  $r_u$  (Bishop, 1954) is defined as:

$$r_u = u/\gamma h \quad (6)$$

in which  $r_u$  = pore pressure ratio,  $u$  = pore pressure at a point,  $\gamma$  = total unit weight, and  $h$  = depth of overburden at the point. Assuming that  $r_u$  = constant (or that the pore pressure is a constant fraction of the overburden pressure throughout the slope) is a simple and reasonable means of approximating the distribution of pore pressures in embankments at the end of construction, or within natural slopes or excavations for some conditions of seepage. Comparing factors of safety calculated by various methods for  $r_u$  = constant thus provides results which are relevant to practical conditions involving non-zero pore pressures.

### *Comparison of minimum values of $F$*

A number of homogeneous slopes varying from 1.5 on 1 (horizontal on vertical) to 3.5 on 1, with values of  $r_u$  varying from 0.0 to 0.6 were analyzed using each of the methods described in Tables I and II. The results for the 3.5 on 1 slopes are summarized in Table V (for  $r_u = 0$ ) and Table VI (for  $r_u = 1$ ). In each case the values of  $c$ ,  $\phi$ ,  $\gamma$  and  $H$  used in the analyses were adjusted so that  $F = 1.00$  for the log spiral method. Because the log spiral method is not a method of slices, and because it satisfies all conditions of equilibrium, it provides a convenient basis for comparison.

On the basis of these studies, a number of conclusions have been reached regarding the accuracy of these equilibrium methods of slope stability analysis:

(1) Methods which satisfy all conditions of equilibrium (log spiral, Janbu's, Spencer's, and Morgenstern and Price's methods) all give essentially the same value of  $F$ . Studies of non-homogeneous slopes and dams, and non-circular slip surfaces, show a slightly wider disparity in the values of  $F$  calculated by these methods. These studies indicate that for any practical slope stability problem, any method which satisfies all conditions of equilibrium will give a value of  $F_{\min}$  which differs by no more than  $\pm 5\%$  from what may be considered the 'correct' answer. Thus, although there is no mathematical proof that the values of  $F$  calculated by Janbu's, Spencer's, and Morgenstern and Price's methods are rigorously correct, from a practical point of view there is no doubt that they may be considered to be correct for all practical purposes.

(2) Bishop's modified method, which does not satisfy all conditions of equilibrium, gives virtually the same value of  $F$  as methods which satisfy all

TABLE V

Minimum values of  $F$  for a 3.5 on 1 (horizontal on vertical) slope with  $r_u = 0$ 

Analysis procedure	$\lambda_{c\phi}$					
	0	2	5	8	20	50
Log spiral	1.00	1.00	1.00	1.00	1.00	1.00
Ordinary method of slices	1.00	0.94	0.94	0.95	0.96	0.98
Bishop's modified method	1.00	1.00	1.00	1.00	1.00	1.00
Force equilibrium (Lowe and Karafiath's assumption)	1.09	1.02	1.01	1.00	1.00	1.00
Janbu's generalized procedure of slices	1.00	—	1.00	—	1.00	1.00
Spencer's procedure; also Morgenstern and Price's procedure with $f(x) = \text{constant}$	1.00	1.00	1.00	1.00	1.00	1.00

TABLE VI

Minimum values of  $F$  for a 3.5 on 1 (horizontal on vertical) slope with  $r_u = 0.6$ 

Analysis procedure	$\lambda_{c\phi}$					
	0	2	5	8	20	50
Log spiral	1.00	1.00	1.00	1.00	1.00	1.00
Ordinary method of slices	1.00	0.91	0.75	0.68	0.57	0.50
Bishop's modified method	1.00	1.00	1.00	1.00	0.99	0.99
Force equilibrium (Lowe and Karafiath's assumption)	1.09	1.03	1.02	1.01	1.00	1.00
Janbu's generalized procedure of slices	1.00	—	—	—	—	—
Spencer's procedure; also Morgenstern and Price's procedure with $f(x) = \text{constant}$	1.00	1.00	1.00	1.00	1.00	1.00

conditions of equilibrium. Thus, for analyses of circular slip surfaces, no more elaborate method need be used to insure that the errors arising from the mechanics of the method will be small.

(3) The ordinary method of slices, which is also applicable only to circular slip surfaces, gives values of  $F$  which are lower than those calculated by more accurate methods. For  $r_u = 0$  conditions (in practical terms these would be total stress analyses), the inaccuracy is no more than a few percent, which is certainly tolerable for practical purposes. For effective stress analyses with high pore pressures, as shown by the results for  $r_u = 0.6$  in Table VI, the inaccuracy may be as much as 50%. Thus, while the ordinary method of slices may be applied to total stress analyses, it should not be used for effective stress analyses with high pore pressure.

(4) For  $\phi = 0$  conditions ( $\lambda_{c\phi} = 0$  in Tables V and VI), any method which satisfies moment equilibrium around the center of a circular slip surface will give the correct value of  $F$  for this condition, regardless of what other equilibrium conditions it does or does not satisfy. This is true because: (a) for  $\phi = 0$ , shear strength is independent of the normal stress on the slip surface (see eq.2); and (b) the average value of shear stress required for equilibrium of a free body bounded by a circular arc is determined completely and uniquely by the equation of moment equilibrium around the center of the circle. Thus the ordinary method of slices, Bishop's modified method, Janbu's GPS, Spencer's method, and Morgenstern and Price's method all give exactly the same value of  $F$  for circular slip surfaces and  $\phi = 0$  conditions.

(5) Values of  $F$  calculated using procedures which satisfy force equilibrium only are sensitive to the assumed inclination of the side forces between slices. The assumption proposed by Lowe and Karafiath (side forces inclined at the average of: (a) the ground surface slope; and (b) the slope of the slip surface) appears to be quite accurate over a wide range of conditions. As shown in Table V and VI, this assumption is least accurate for  $\lambda_{c\phi} = 0$ , in which case the value of  $F$  is about 10% larger than the correct value. The more steeply inclined the side forces are assumed to be for  $\phi = 0$ , the more inaccurate the calculated value of  $F$ . If, for example, the side forces are assumed to be parallel to the ground surface, the value of  $F$  may be as much as 30% too high.

## SUMMARY

All equilibrium methods of slope stability analysis involve assumptions, because there are fewer equations of equilibrium than there are unknowns in the case of either a circular or non-circular slip surface, with the mass above divided into slices. Furthermore, some of the methods which are frequently used for practical purposes (the ordinary method of slices, force equilibrium procedures, and Bishop's modified method) do not satisfy all of the conditions of equilibrium.

Under some circumstances methods which do not satisfy all conditions of equilibrium may be highly inaccurate. The ordinary method of slices may give values of  $F$  which are 50% smaller than the correct value if used for effective stress analyses of slopes with high pore pressures. Force equilibrium procedures may give values of  $F$  which are 30% larger than the correct value if used for  $\phi = 0$  analyses with steeply inclined side forces between slices.

However, methods which satisfy all conditions of equilibrium give accurate results for all practical conditions. Regardless of the assumptions they employ, these methods (Janbu's, Spencer's, and Morgenstern and Price's methods) give values of  $F$  which differ by no more than  $\pm 5\%$  from the correct answer. Bishop's modified method is also equally accurate, even though it does not satisfy all conditions of equilibrium.

Based on these findings, equilibrium methods of stability analysis can be selected which can be relied on to produce results which involve no more

than  $\pm 5\%$  inaccuracy as a consequence of the approximations made in treating the mechanics of the problem. When this is the case, the engineer performing the analysis is justified in considering the factors of safety he calculates to be 'correct' in terms of the mechanics of the problem, and he can devote his attention and concern to accurate evaluation of the properties of the soils.

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#### REFERENCES

- Bishop, A.W., 1954. The use of pore pressure coefficients in practice. *Geotechnique*, (4): 148-152.
- Bishop, A.W., 1955. The use of the slip circle in the stability analysis of slopes. *Geotechnique*, 7(1):7-17.
- Fellenius, W., 1927. *Erdstatische Berechnungen*. Ernst, Berlin.
- Janbu, N., 1954. Stability analysis of slopes with dimensionless parameters. *Harvard Soil Mech. Ser.*, 46: 81 pp.
- Janbu, N., 1957. Earth pressures and bearing capacity calculations by generalized procedure of slices. *Proc. Intern. Conf. Soil Mech. Foundation Eng.*, 4th, London, 2: 207-212.
- Lowe III, J. and Karafiath, L., 1960. Stability of earth dams upon drawdown. *Proc. Pan Am. Conf. Soil Mech. Foundation Eng.*, 1st, Mexico City, 2:537-552.
- Morgenstern, N.R. and Price, V.E., 1965. The analysis of the stability of general slip surfaces. *Geotechnique*, 15(1):79-93.
- Seed, H.B. and Sultan, H.A., 1967. Stability analyses for a sloping core embankment. *J. Soil Mech. Foundations Div.* 93(4):69-83.
- Skempton, A.W., 1977. Slope stability of cuttings in brown London clay. *Intern. Conf. Soil Mech. Foundation Eng.*, Tokyo.
- Spencer, E., 1967. A method of analysis of the stability of embankments assuming parallel inter-slice forces. *Geotechnique* 17(1):11-26.
- Taylor, D.W., 1948. *Fundamentals of Soil Mechanics*. Wiley, New York, N.Y.
- Wright, S.G., 1969. A Study of Slope Stability and the Undrained Shear Strength of Clay Shales. Thesis, Univ. California, Berkeley, Calif.