Comparison of slope stability methods of analysis[1]

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Received October 12, 1976
Accepted April 4, 1977

The paper compares six methods of slices commonly used for slope stability analysis. The factor of safety equations are written in the same form, recognizing whether moment and (or) force equilibrium is explicitly satisfied. The normal force equation is of the same form for all methods with the exception of the ordinary method. The method of handling the interslice forces differentiates the normal force equations.

A new derivation for the Morgenstern–Price method is presented and is called the 'best-fit regression' solution. It involves the independent solution of the force and moment equilibrium factors of safety for various values of λ. The best-fit regression solution gives the same factor of safety as the 'Newton–Raphson' solution. The best-fit regression solution is readily comprehended, giving a complete understanding of the variation of the factor of safety with λ.

L'article présente une comparaison des six méthodes de tranches utilisées couramment pour l'analyse de la stabilité des pentes. Les équations des facteurs de sécurité sont écrites selon la même forme, en montrant si les conditions d'équilibre de moment ou de force sont satisfaites explicitement. L'équation de la force normale est de la même forme pour toutes les méthodes à l'exception de la méthode des tranches ordinaires. La façon de traiter les forces intertranches est ce qui différencie les équations de force normale.

Une nouvelle dérivation appelée solution "best-fit regression" de la méthode de Morgenstern–Price est présentée. Elle consiste à déterminer indépendemment les facteurs de sécurité satisfaisant aux équilibres de force et de moment pour différentes valeurs de λ. La solution "best-fit regression" donne les mêmes facteurs de sécurité de la solution de Newton–Raphson. La solution "best-fit regression" est plus facile à comprendre et donne une image détaillée de la variation du facteur de sécurité avec λ.

[Traduit par la revue]

Can. Geotech. J., **14**, 429 (1977)

Introduction

The geotechnical engineer frequently uses limit equilibrium methods of analysis when studying slope stability problems. The methods of slices have become the most common methods due to their ability to accommodate complex geometrics and variable soil and water pressure conditions (Terzaghi and Peck 1967). During the past three decades approximately one dozen methods of slices have been developed (Wright 1969). They differ in (i) the statics employed in deriving the factor of safety equation and (ii) the assumption used to render the problem determinate (Fredlund 1975).

This paper is primarily concerned with six of the most commonly used methods:

(i) Ordinary or Fellenius method (sometimes referred to as the Swedish circle method or the conventional method)
(ii) Simplified Bishop method
(iii) Spencer's method
(iv) Janbu's simplified method
(v) Janbu's rigorous method
(vi) Morgenstern–Price method

The objectives of this paper are:

(1) to compare the various methods of slices in terms of consistent procedures for deriving the factor of safety equations. All equations are extended to the case of a composite failure surface and also consider partial submergence, line loadings, and earthquake loadings.

(2) to present a new derivation for the Morgenstern–Price method. The proposed derivation is more consistent with that used for the other methods of analysis but utilizes the elements of statics and the assumption proposed by Morgenstern and Price (1965). The Newton–Raphson numerical technique is not used to compute the factor of safety and λ.

(3) to compare the factors of safety obtained by each of the methods for several example problems. The University of Sas-

[1]Presented at the 29th Canadian Geotechnical Conference, Vancouver, B.C., October 13–15, 1976.

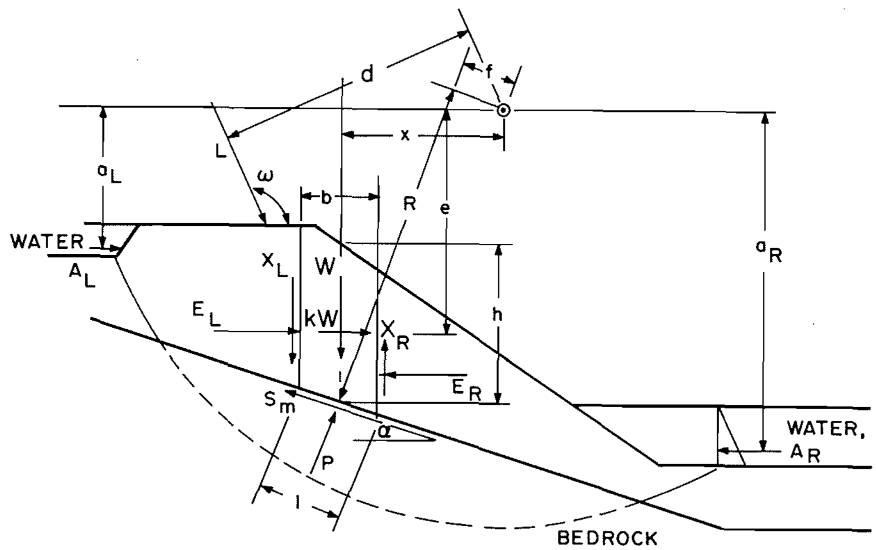

FIG. 1. Forces acting for the method of slices applied to a composite sliding surface.

katchewan SLOPE computer program was used for all computer analyses (Fredlund 1974).

(4) to compare the relative computational costs involved in using the various methods of analysis.

Definition of Problem

Figure 1 shows the forces that must be defined for a general slope stability problem. The variables associated with each slice are defined as follows:

W = total weight of the slice of width b and height h

P = total normal force on the base of the slice over a length l

S_m = shear force mobilized on the base of the slice. It is a percentage of the shear strength as defined by the Mohr–Coulomb equation. That is, $S_m = l\{c' + [P/l - u] \tan \phi'\}/F$ where c' = effective cohesion parameter, ϕ' = effective angle of internal friction, F = factor of safety, and u = porewater pressure

R = radius or the moment arm associated with the mobilized shear force S_m

f = perpendicular offset of the normal force from the center of rotation

x = horizontal distance from the slice to the center of rotation

α = angle between the tangent to the center of the base of each slice and the horizontal

E = horizontal interslice forces

L = subscript designating left side

R = subscript designating right side

X = vertical interslice forces

k = seismic coefficient to account for a dynamic horizontal force

e = vertical distance from the centroid of each slice to the center of rotation

A uniform load on the surface can be taken into account as a soil layer of suitable unit weight and density. The following variables are required to define a line load:

L = line load (force per unit width)

ω = angle of the line load from the horizontal

d = perpendicular distance from the line load to the center of rotation

The effect of partial submergence of the slope or tension cracks in water requires the definition of additional variables:

A = resultant water forces

a = perpendicular distance from the resultant water force to the center of rotation

Derivations for Factor of Safety

The elements of statics that can be used to derive the factor of safety are summations of forces in two directions and the summation of moments. These, along with the failure criteria, are insufficient to make the problem determinate. More information must be known about either the normal force distribution or the interslice force distribution. Either addi-

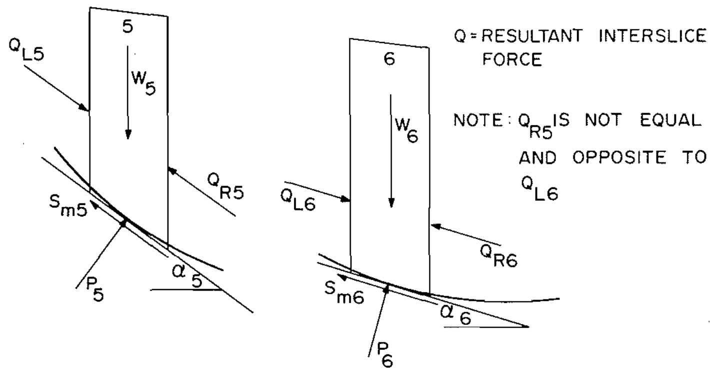

FIG. 2 Interslice forces for the ordinary method.

tional elements of physics or an assumption must be invoked to render the problem determinate. All methods considered in this paper use the latter procedure, each assumption giving rise to a different method of analysis. For comparison purposes, each equation is derived using a consistent utilization of the equations of statics.

Ordinary or Fellenius Method

The ordinary method is considered the simplest of the methods of slices since it is the only procedure that results in a linear factor of safety equation. It is generally stated that the interslice forces can be neglected because they are parallel to the base of each slice (Fellenius 1936). However, Newton's principle of 'action equals reaction' is not satisfied between slices (Fig. 2). The indiscriminate change in direction of the resultant interslice force from one slice to the next results in factor of safety errors that may be as much as 60% (Whitman and Bailey 1967).

The normal force on the base of each slice is derived either from summation of forces perpendicular to the base or from the summation of forces in the vertical and horizontal directions.

$$[1] \quad \sum F_V = 0$$

$$W - P\cos\alpha - S_m \sin\alpha = 0$$

$$[2] \quad \sum F_H = 0$$

$$S_m \cos\alpha - P\sin\alpha - kW = 0$$

Substituting [2] into [1] and solving for the normal force gives

$$[3] \qquad P = W\cos\alpha - kW\sin\alpha$$

The factor of safety is derived from the summation of moments about a common point (*i.e.* either a fictitious or real center of rotation for the entire mass).

$$[4] \quad \sum M_0 = 0$$

$$\sum Wx - \sum S_m R - \sum Pf + \sum kWe \pm Aa + Ld = 0$$

Introducing the failure criteria and the normal force from [3] and solving for the factor of safety gives

$$[5] \quad F = \frac{\sum\{c'lR + (P - ul)R\tan\phi'\}}{\sum Wx - \sum Pf + \sum kWe \pm Aa + Ld}$$

Simplified Bishop Method

The simplified Bishop method neglects the interslice shear forces and thus assumes that a normal or horizontal force adequately defines the interslice forces (Bishop 1955). The normal force on the base of each slice is derived by summing forces in a vertical direction (as in [1]). Substituting the failure criteria and solving for the normal force gives

$$[6] \quad P = \left[W - \frac{c'l\sin\alpha}{F} + \frac{ul\tan\phi'\sin\alpha}{F}\right] / m_\alpha$$

where $m_\alpha = \cos\alpha + (\sin\alpha\tan\phi')/F$

The factor of safety is derived from the summation of moments about a common point. This equation is the same as [4] since the interslice forces cancel out. Therefore, the factor of safety equation is the same as for the ordinary method ([5]). However, the definition of the normal force is different.

Spencer's Method

Spencer's method assumes there is a constant relationship between the magnitude of the interslice shear and normal forces (Spencer 1967).

$$[7] \qquad \tan\theta = \frac{X_L}{E_L} = \frac{X_R}{E_R}$$

where θ = angle of the resultant interslice force from the horizontal.

Spencer (1967) summed forces perpendicular to the interslice forces to derive the normal force. The same result can be obtained by summing forces in a vertical and horizontal direction.

$$[8] \quad \sum F_V = 0$$

$$W - (X_R - X_L) - P\cos\alpha - S_m \sin\alpha = 0$$

$$[9] \quad \sum F_H = 0$$

$$-(E_R - E_L) + P\sin\alpha - S_m\cos\alpha + kW = 0$$

The normal force can be derived from [8] and then the horizontal interslice force is obtained from [9].

$$[10] \quad P = \left[W - (E_R - E_L)\tan\theta - \frac{c'l\sin\alpha}{F} + \frac{ul\tan\phi'\sin\alpha}{F}\right]/m_\alpha$$

Spencer (1967) derived two factor of safety equations. One is based on the summation of moments about a common point and the other on the summation of forces in a direction parallel to the interslice forces. The moment equation is the same as for the ordinary and the simplified Bishop methods (*i.e.* [4]). The factor of safety equation is the same as [5].

The factor of safety equation based on force equilibrium can also be derived by summing forces in a horizontal direction.

$$[11] \quad \sum F_H = 0$$

$$\sum(E_L - E_R) + \sum P\sin\alpha - \sum S_m\cos\alpha + \sum kW \pm A - L\cos\omega = 0$$

The interslice forces $(E_L - E_R)$ must cancel out and the factor of safety equation with respect to force equilibrium reduces to

$$[12] \quad F_f = \frac{\sum\{c'l\cos\alpha + (P - ul)\tan\phi'\cos\alpha\}}{\sum P\sin\alpha + \sum kW \pm A - L\cos\omega}$$

Spencer's method yields two factors of safety for each angle of side forces. However, at some angle of the interslice forces, the two factors of safety are equal (Fig. 3) and both moment and force equilibrium are satisfied.

The Corps of Engineers method, sometimes referred to as Taylor's modified Swedish method, is equivalent to the force equilibrium portion of Spencer's method in which the direction of the interslice forces is assumed, generally at an angle equal to the average surface slope.

Janbu's Simplified Method

Janbu's simplified method uses a correction factor f_0 to account for the effect of the interslice shear forces. The correction factor is related to cohesion, angle of internal friction, and the shape of the failure surface (Janbu *et al.* 1956). The normal force is derived from the summation of vertical forces ([8]), with the interslice shear forces ignored.

$$[13] \quad P = \left[W - \frac{c'l\sin\alpha}{F} + \frac{ul\tan\phi'\sin\alpha}{F}\right]/m_\alpha$$

The horizontal force equilibrium equation is used to derive the factor of safety (*i.e.* [11]). The sum of the interslice forces must cancel and the factor of safety equation becomes

$$[14] \quad F_0 = \frac{\sum\{c'l\cos\alpha + (P - ul)\tan\phi'\cos\alpha\}}{\sum P\sin\alpha + \sum kW \pm A - L\cos\omega}$$

F_0 is used to designate the factor of safety uncorrected for the interslice shear forces.

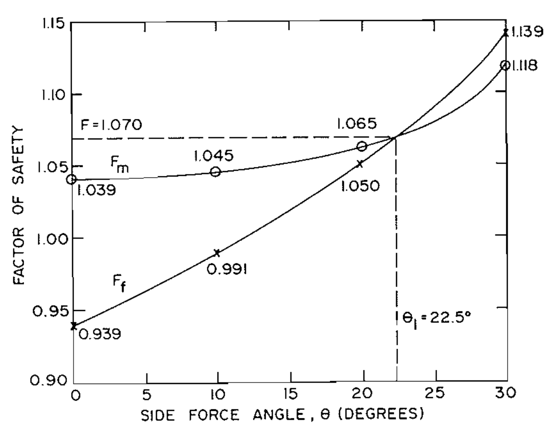

FIG. 3 Variation of the factor of safety with respect to moment and force equilibrium *vs.* the angle of the side forces. Soil properties: $c'/\gamma h = 0.02$; $\phi' = 40°$; $r_u = 0.5$. Geometry: slope = 26.5°; height = 100 ft (30 m).

The corrected factor of safety is

$$[15] \qquad F = f_0 F_0$$

Janbu's Rigorous Method

Janbu's rigorous method assumes that the point at which the interslice forces act can be defined by a 'line of thrust'. New terms used are defined as follows (see Fig. 4): t_L, t_R = vertical distance from the base of the slice to the line of thrust on the left and right sides of the slice, respectively; α_t = angle between the line of thrust on the right side of a slice and the horizontal.

The normal force on the base of the slice is derived from the summation of vertical forces.

$$[16] \quad P = \left[W - (X_R - X_L) - \frac{c'l \sin\alpha}{F} + \frac{ul \tan\phi' \sin\alpha}{F} \right] / m_\alpha$$

The factor of safety equation is derived from the summation of horizontal forces (*i.e.* [11]). Janbu's rigorous analysis differs from the simplified analysis in that the shear forces are kept in the derivation of the normal force. The factor of safety equation is the same as Spencer's equation based on force equilibrium (*i.e.* [12]).

In order to solve the factor of safety equation, the interslice shear forces must be evaluated. For the first iteration, the shears are set to zero. For subsequent iterations, the interslice forces are computed from the sum of the moments about the center of the base of each slice.

$$[17] \quad \sum M_c = 0$$

$$X_L b/2 + X_R b/2 - E_L\,[t_L + (b/2)\tan\alpha] + E_R\,[t_L + (b/2)\tan\alpha - b\tan\alpha_t] - kWh/2 = 0$$

After rearranging [17], several terms become negligible as the width b of the slice is reduced to a width dx. These terms are $(X_R - X_L)\ b/2$, $(E_R - E_L)\ (b/2)\ \tan\alpha$ and $(E_R - E_L)b$ $\tan\alpha_t$. Eliminating these terms and dividing by the slice width, the shear force on the right side of a slice is

$$[18] \quad X_R = E_R \tan\alpha_t - (E_R - E_L)\,t_R/b + (kW/b)(h/2)$$

The horizontal interslice forces, required for solving [18], are obtained by combining the summation of vertical and horizontal forces on each slice.

$$[19] \quad (E_R - E_L) = [W - (X_R - X_L)]\tan\alpha - S_m/\cos\alpha + kW$$

The horizontal interslice forces are obtained by integration from left to right across the slope. The magnitude of the interslice shear forces in [19] lag by one iteration. Each iteration gives a new set of shear forces. The vertical and horizontal components of line loads must also be taken into account when they are encountered.

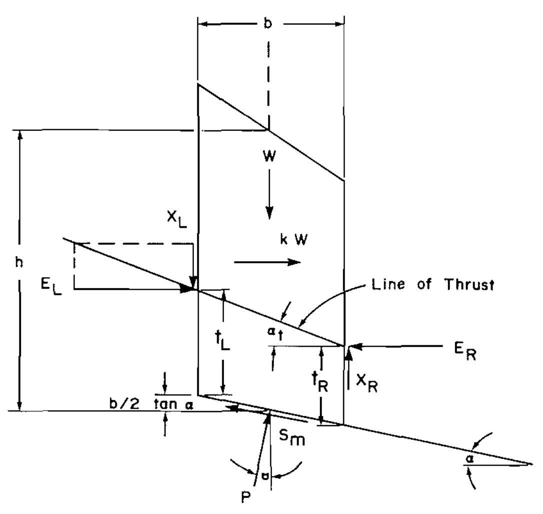

FIG. 4. Forces acting on each slice for Janbu's rigorous method.

Morgenstern–Price Method

The Morgenstern–Price method assumes an arbitrary mathematical function to describe the direction of the interslice forces.

$$[20] \qquad \lambda f(x) = X/E$$

where λ = a constant to be evaluated in solving for the factor of safety, and $f(x)$ = functional variation with respect to x. Figure 5 shows typical functions (*i.e.* $f(x)$). For a constant function, the Morgenstern–Price method is the same as the Spencer method. Figure 6 shows how the half sine function and λ are used to designate the direction of the interslice forces.

Morgenstern and Price (1965) based their solution on the summation of tangential and

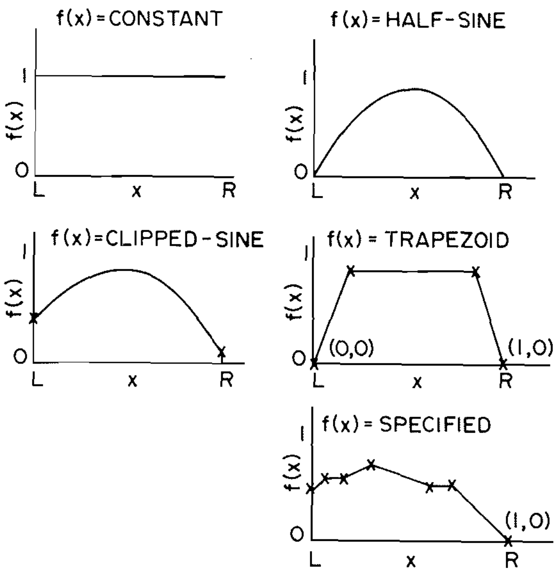

FIG. 5. Functional variation of the direction of the side force with respect to the *x* direction.

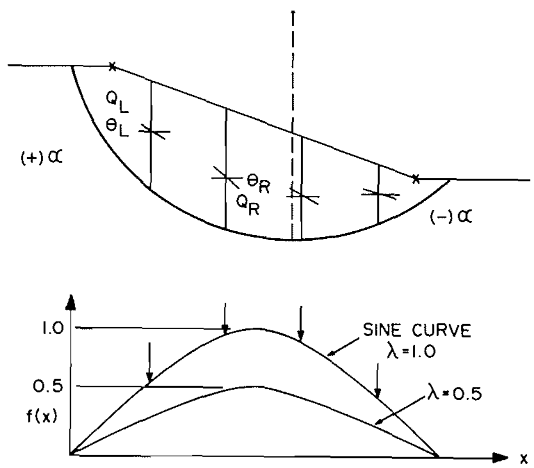

FIG. 6. Side force designation for the Morgenstern–Price method.

normal forces to each slice. The force equilibrium equations were combined and then the Newton–Raphson numerical technique was used to solve the moment and force equations for the factor of safety and λ.

In this paper, an alternate derivation for the Morgenstern–Price method is proposed. The solution satisfies the same elements of statics but the derivation is more consistent with that used in the other methods of slices. It also presents a complete description of the variation of the factor of safety with respect to λ.

The normal force is derived from the vertical force equilibrium equation ([16]). Two factor of safety equations are computed, one with respect to moment equilibrium and one with respect to force equilibrium. The moment equilibrium equation is taken with respect to a common point. Even if the sliding surface is composite, a fictitious common center can be used. The equation is the same as that obtained for the ordinary method, the simplified Bishop method, and Spencer's method ([4] and [5]). The factor of safety with respect to force equilibrium is the same as that derived for Spencer's method ([12]). The interslice shear forces are computed in a manner similar to that presented for Janbu's rigorous method. On the first iteration, the vertical shear forces are set to zero. On subsequent iterations, the horizontal interslice forces are first computed ([19]) and then the vertical shear forces are computed using an assumed λ value and side force function.

[21] $$X_R = E_R \lambda f(x)$$

The side forces are recomputed for each

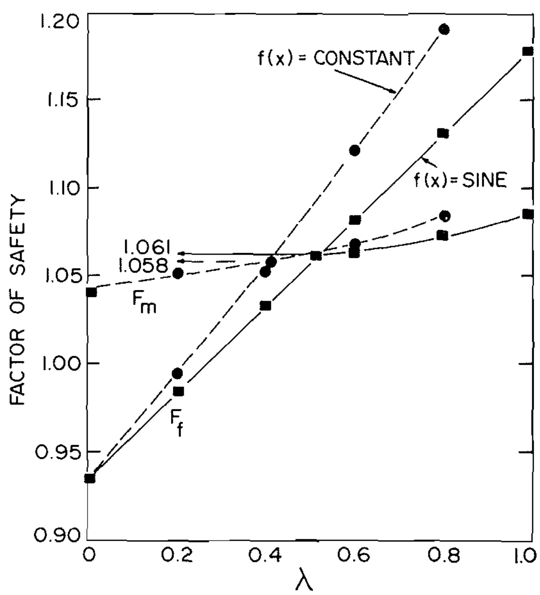

FIG. 7. Variation of the factor of safety with respect to moment and force equilibrium *vs.* λ for the Morgenstern–Price method. Soil properties: $c'/\gamma h = 0.02$; $\phi' = 40°$; $r_u = 0.5$. Geometry: slope = 26.5°; height = 100 ft (30 m).

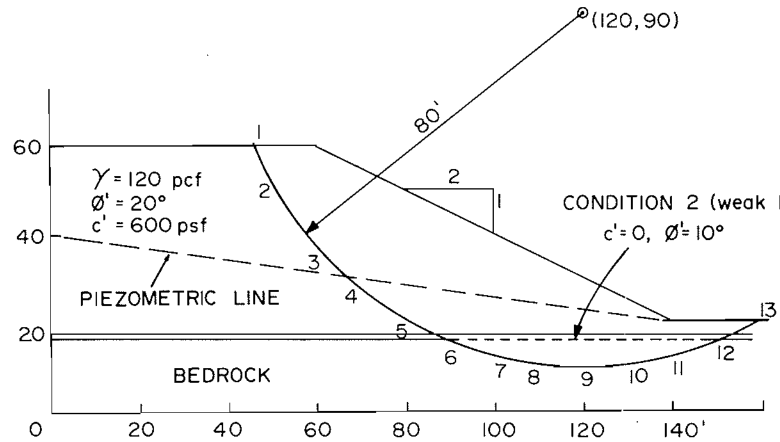

FIG. 8. Example problem.

iteration. The moment and force equilibrium factors of safety are solved for a range of λ values and a specified side force function. These factors of safety are plotted in a manner similar to that used for Spencer's method (Fig. 7). The factors of safety *vs.* λ are fit by a second order polynomial regression and the point of intersection satisfies both force and moment equilibrium.

Comparison of Methods of Analysis

All methods of slices satisfying overall moment equilibrium can be written in the same form.

$$[22]\quad F_m = \frac{\sum c'lR + \sum(P - ul)R \tan\phi'}{\sum Wx - \sum Pf + \sum kWe \pm Aa + Ld}$$

All methods satisfying overall force equilibrium have the following form for the factor of safety equation:

$$[23]\quad F_f = \frac{\sum c'l \cos\alpha + \sum(P - ul)\tan\phi' \cos\alpha}{\sum P \sin\alpha + \sum kW \pm A - L\cos\omega}$$

The factor of safety equations can be visualized as consisting of the following components:

	Moment equilibrium	Force equilibrium
Cohesion	$\Sigma c'lR$	$\Sigma c'l \cos\alpha$
Friction	$\Sigma(P\text{-}ul)R \tan\phi'$	$\Sigma(P\text{-}ul)\tan\phi' \cos\alpha$
Weight	ΣWx	—
Normal	ΣPf	$\Sigma P \sin\alpha$
Earthquake	ΣkWe	ΣkW
Partial submergence	Aa	A
Line loading	Ld	$L \cos\omega$

From a theoretical standpoint, the derived factor of safety equations differ in (i) the equations of statics satisfied explicitly for the overall slope and (ii) the assumption to make the problem determinate. The assumption used changes the evaluation of the interslice forces in the normal force equation (Table 1). All methods, with the exception of the ordinary method, have the same form of equation for the normal force.

$$[24]\quad P = \left[W - (X_R - X_L) - \frac{c'l \sin\alpha}{F} + \frac{ul \tan\phi' \sin\alpha}{F}\right] / m_\alpha$$

where $m_\alpha = \cos\alpha + (\sin\alpha \tan\phi')/F$.

It is possible to view the analytical aspects of slope stability in terms of one factor of safety equation satisfying overall moment equilibrium and another satisfying overall force equilibrium. Then each method becomes a special case of the 'best-fit regression' solution to the Morgenstern–Price method.

TABLE 1. Comparison of factor of safety equations

	Factor of safety based on		
Method	Moment equilibrium	Force equilibrium	Normal force equation
Ordinary or Fellenius	x		[3]
Simplified Bishop	x		[6]
Spencer's	x	x	[10]
Janbu's simplified		x	[13]
Janbu's rigorous		x	[16]
Morgenstern–Price	x	x	[24]

TABLE 2. Comparison of factors of safety for example problem

Case no.	Example problem*	Ordinary method	Simplified Bishop method	Spencer's method F	Spencer's method θ	Spencer's method λ	Janbu's simplified method	Janbu's rigorous method**	Morgenstern–Price method $f(x)$ = constant F	Morgenstern–Price method $f(x)$ = constant λ
1	Simple 2:1 slope, 40 ft (12 m) high, $\phi' = 20°$, $c' = 600$ psf (29 kPa)	1.928	2.080	2.073	14.81	0.237	2.041	2.008	2.076	0.254
2	Same as 1 with a thin, weak layer with $\phi' = 10°$, $c' = 0$	1.288	1.377	1.373	10.49	0.185	1.448	1.432	1.378	0.159
3	Same as 1 except with $r_u = 0.25$	1.607	1.766	1.761	14.33	0.255	1.735	1.708	1.765	0.244
4	Same as 2 except with $r_u = 0.25$ for both materials	1.029	1.124	1.118	7.93	0.139	1.191	1.162	1.124	0.116
5	Same as 1 except with a piezometric line	1.693	1.834	1.830	13.87	0.247	1.827	1.776	1.833	0.234
6	Same as 2 except with a piezometric line for both materials	1.171	1.248	1.245	6.88	0.121	1.333	1.298	1.250	0.097

*Width of slice is 0.5 ft (0.3 m) and the tolerance on the nonlinear solutions is 0.001.
**The line of thrust is assumed at 0.333.

Figure 8 shows an example problem involving both circular and composite failure surfaces. The results of six possible combinations of geometry, soil properties, and water conditions are presented in Table 2. This is not meant to be a complete study of the quantitative relationship between various methods but rather a typical example.

The various methods (with the exception of the ordinary method), can be compared by plotting factor of safety *vs.* λ. The simplified Bishop method satisfies overall moment equilibrium with $\lambda = 0$. Spencer's method has λ equal to the tangent of the angle between the horizontal and the resultant interslice force. Janbu's factors of safety can be placed along the force equilibrium line to give an indication of an equivalent λ value. Figures 9 and 10 show comparative plots for the first two cases shown in Table 2.

The results in Table 2 along with those from other comparative studies show that the factor of safety with respect to moment equilibrium is relatively insensitive to the interslice force assumption. Therefore, the factors of safety obtained by the Spencer and Morgenstern–Price methods are generally similar to those computed by the simplified Bishop method. On the other hand, the factors of safety based on overall force equilibrium are far more sensitive to the side force assumption.

The relationship between the factors of

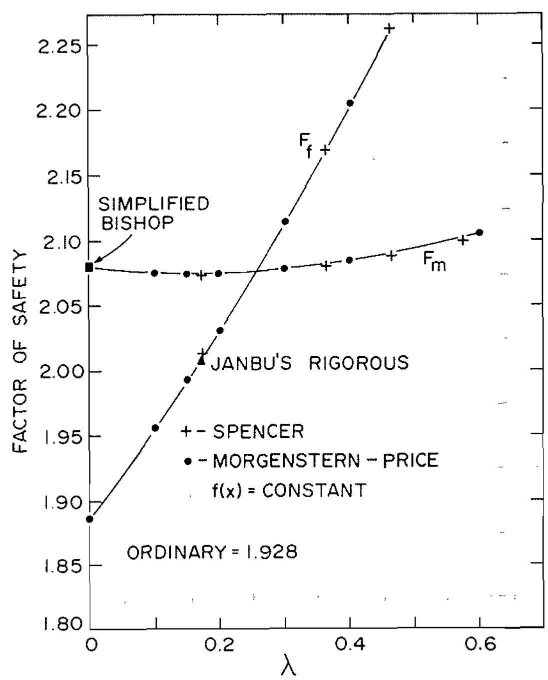

FIG. 9. Comparison of factors of safety for case 1.

TABLE 3. Comparison of two solutions to the Morgenstern–Price method**

Case no.	Example problem	University of Alberta program side force function: Constant F	Constant λ	Half sine F	Half sine λ	University of Saskatchewan SLOPE program side force function: Constant F	Constant λ	Half sine F	Half sine λ	Clipped sine* F	Clipped sine* λ
1	Simple 2:1 slope, 40 ft (12 m) high, $\phi' = 20°$, $c' = 600$ psf (29 kPa)	2.085	0.257	2.085	0.314	2.076	0.254	2.076	0.318	2.083	0.390
2	Same as 1 with a thin, weak layer with $\phi' = 10°$, $c' = 0$	1.394	0.182	1.386	0.218	1.378	0.159	1.370	0.187	1.364	0.203
3	Same as 1 except with $r_u = 0.25$	1.772	0.351	1.770	0.432	1.765	0.244	1.764	0.304	1.779	0.417
4	Same as 2 except with $r_u = 0.25$ for both materials	1.137	0.334	1.117	0.441	1.124	0.116	1.118	0.130	1.113	0.138
5	Same as 1 except with a piezometric line	1.838	0.270	1.837	0.331	1.833	0.234	1.832	0.290	1.832	0.300
6	Same as 2 except with a piezometric line for both materials	1.265	0.159	Not converging		1.250	0.097	1.245	0.101	1.242	0.104

*Coordinates $x = 0$, $y = 0.5$, and $x = 1.0$, $y = 0.25$.
**Tolerance on both Morgenstern–Price solutions is 0.001.

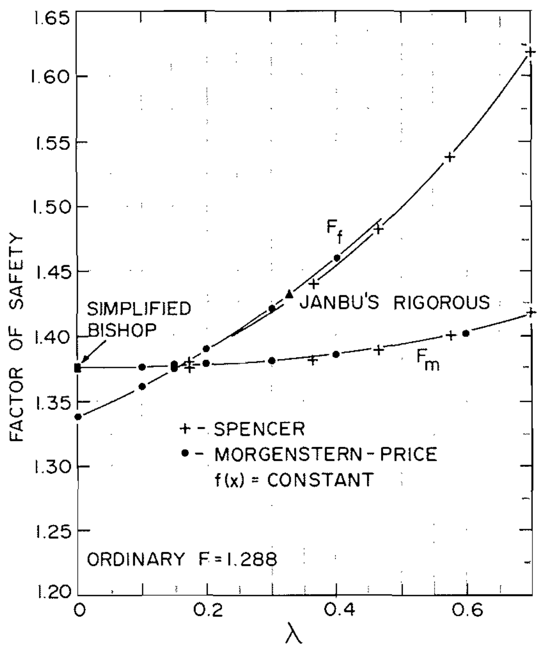

FIG. 10. Comparison of factors of safety for case 2.

safety by the various methods remains similar whether the failure surface is circular or composite. For example, the simplified Bishop method gives factors of safety that are always very similar in magnitude to the Spencer and Morgenstern–Price method. This is due to the small influence that the side force function has on the moment equilibrium factor of safety equation. In the six example cases, the average difference in the factor of safety was approximately 0.1%.

Comparison of Two Solutions to the Morgenstern-Price Method

Morgenstern and Price (1965) originally solved their method using the Newton-Raphson numerical technique. This paper has presented an alternate procedure that has been referred to as the 'best-fit regression' method. The two methods of solution were compared using the University of Alberta computer program (Krahn *et al.* 1971) for the original method and the University of Saskatchewan computer program (Fredlund 1974) for the alternate solution. In addition, it is possible to compare the above solutions with Spencer's method.

Table 3 shows a comparison of the two solutions for the example problems (Fig. 8). Figures 11 and 12 graphically display the comparisons. Although the computer programs use different methods for the input of the geometry and side force function and different techniques for solving the equations, the factors of safety are essentially the same.

The example cases show that when the side force function is either a constant or a half sine, the average factor of safety from the University of Alberta computer program differs from the University of Saskatchewan

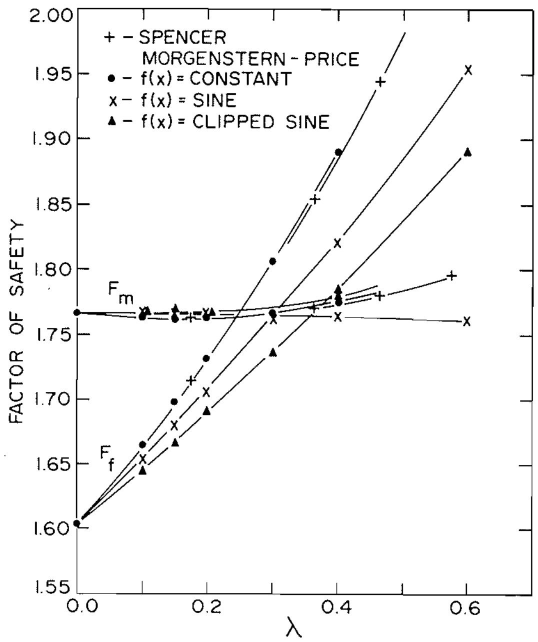

FIG. 11. Effect of side force function on factor of safety for case 3.

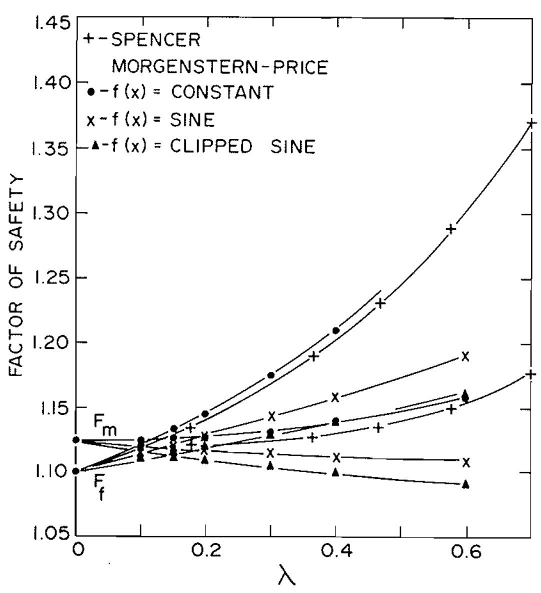

FIG. 12. Effect of side force function on factor of safety for case 4.

computer program by less than 0.7%. Using the University of Saskatchewan program, the Spencer method and the Morgenstern–Price method (for a constant side force function) differ by less than 0.2%. The average λ values computed by the two programs differ by approximately 9%. However, as shown above, this difference does not significantly affect the final factor of safety.

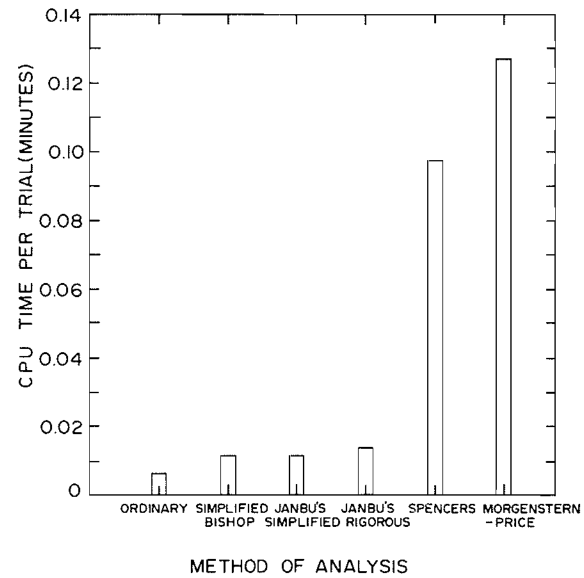

FIG. 13. Time per stability analysis trial for all methods of analysis.

Comparison of Computing Costs

A simple 2:1 slope was selected to compare the computer costs (i.e. CPU time) associated with the various methods of analysis. The slope was 440 ft long and was divided into 5-ft slices. The results shown in Fig. 13 were obtained using the Univeristy of Saskatchewan SLOPE program run on an IBM 370 model 158 computer.

The simplified Bishop method required 0.012 min for each stability analysis. The ordinary method required approximately 60% as much time. The factor of safety by Spencer's method was computed using four side force angles. The calculations associated with each side force angle required 0.024 min. The factor of safety by the Morgenstern–Price method was computed using six λ values. Each trial required 0.021 min. At least three estimates of the side force angle or λ value are required to obtain the factor of safety. Therefore, the Spencer or Morgenstern–Price methods are at least six times as costly to run as the simplified Bishop method. The above relative costs are slightly affected by the width of slice and the tolerance used in solving the nonlinear factor of safety equations.

Conclusions

(1) The factor of safety equations for all methods of slices considered can be written in the same form if it is recognized whether moment and (or) force equilibrium is explicitly satisfied. The normal force equation is of the same form for all methods with the exception of the ordinary method. The method of handling the interslice forces differentiates the normal force equations.

(2) The analytical aspects of slope stability can be viewed in terms of one factor of safety equation satisfying overall moment equilibrium and another satisfying overall force equilibrium for various λ values. Then each method becomes a special case of the best-fit factor of safety lines.

(3) The best-fit regression solution and the Newton–Raphson solution give the same factors of safety. They differ only in the manner in which the equations of statics are utilized.

(4) The best-fit regression solution is readily comprehended. It also gives a complete understanding of the variation of factor of safety with respect to λ.

Acknowledgements

The Department of Highways, Government of Saskatchewan, provided much of the financial assistance required for the development of the University of Saskatchewan SLOPE computer program. This program contains all the methods of analysis used for the comparative study.

The authors also wish to thank Mr. R. Johnson, Manager, Ground Engineering Ltd., Saskatoon, Sask., for the comparative computer time study.

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