Problem set 1

Due date: September 24, 11:15am.

Exercise 1

Derive a formula for the price $O(t, S_1(t), S_2(t))$ of an **Outperformance option**, whose payout is

$$max\left\{0, \frac{S_1(T)}{S_1(0)} - \frac{S_2(T)}{S_2(0)}\right\} \tag{1}$$

the stocks S_1 and S_2 pay no dividends and follow the stochastic processes

$$dS_1 = \mu_1 S_1 dt + \sigma_1 S_1 dW_1 \tag{2}$$
$$dS_2 = \mu_2 S_2 dt + \sigma_2 S_2 dW_2 \tag{3}$$

with $E[dW_1 dW_2] = \rho dt$.
$S_1(0)$ and $S_2(0)$ are the stock prices at time 0 and should be treated as constants. Follow the same logic as for the pricing of the exchange option, done in class.

Exercise 2

A **quanto option** is an option on a stock S denominated in foreign currency, with payout in domestic currency. For example, consider have a stock denominated in USD with payout in Euro $max\{S - K, 0\}$. The stock price (in USD) follows the stochastic proicess

$$dS = \mu_S S dt + \sigma_S S dW_S \tag{4}$$

the foreign exchange rate f follows the stochastic process

$$df = \mu_f f dt + \sigma_f f dW_f \tag{5}$$

assume $[dW_S dW_f] = \rho dt$. The Euro-denominated risk-free asset is B_D, such that $dB_D = r_D B_D\, dt$. B_F is the dollar-denominated risk-free asset, such that $dB_F = r_F B_F dt$.
The assets available to the Euro investor are fS, B_D and fB_F. Find the PDE satisfied by the quanto option and the value of the option using a portfolio-replication argument.