EPFL

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Interest rate and credit risk models - Problem Set 9

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Exercise 1

a) We calibrate the parametric specification $\sigma_m(t) = v_m e^{-\beta(T_m - t)}$ to the 3.5% strike caplet data in Table 2 as a function of β with our Matlab code. Then we report the values of v_m for $\beta = 0.07$, we find out:

i	1	2	3	4	5	6	7	8	9
v_m	0.2981	0.3033	0.3085	0.2227	0.2264	0.2025	0.2058	0.2034	0.2067
10	11	12	13	14	15	16	17	18	19
0.1922	0.1952	0.2030	0.2061	0.2017	0.2046	0.2024	0.2053	0.2082	0.2111

Tableau 1: Values of v_m for $\beta = 0.07$

b) Via Monte Carlo simulation we compute the 3.5% cap prices with maturities in $2, \ldots, 10$ years, using the initial forward LIBOR curve in Table 3 under the risk neutral measure $\mathbb{Q}^*$ and the terminal forward measure $\mathbb{Q}^{T_M}$. We did it for both specifications I and II. We find out:

Maturities	2	3	4	5
Cap Price	24.4256	75.5958	146.1636	227.4935
6	7	8	9	10
320.0379	422.4798	531.5862	645.6939	762.9066

Tableau 2: Cap Price under $\mathbb{Q}^*$ for specification I

Maturities	2	3	4	5
Cap Price	25.1738	75.8284	146.9666	228.8342
6	7	8	9	10
324.1973	430.3406	544.2201	663.8520	784.7197

Tableau 3: Cap Price under $\mathbb{Q}^*$ for specification II

Maturities	2	3	4	5
Cap Price	24.2913	76.7131	149.9726	234.5198
6	7	8	9	10
332.3172	441.4088	558.0407	679.3189	802.7815

Tableau 4: Cap Price under $\mathbb{Q}^{T_M}$ for specification I

Maturities	2	3	4	5
Cap Price	24.3684	75.6646	146.8177	228.3647
6	7	8	9	10
323.3310	428.4075	541.1690	660.6793	781.0566

Tableau 5: Cap Price under $\mathbb{Q}^{T_M}$ for specification II

We see that the values under both measures and both specifications are very similar to the ones of the quoted Euro cap prices, which is expected. The difference can be explained by the fact that Monte-Carlo simulations tend to converge very slowly and that the difference would fade if we were to increase the number of simulated paths:

Maturities	2	3	4	5
Cap Price	25.0	77.0	148.5	230.5
6	7	8	9	10
325.5	431.5	545.5	664.0	786.0

Tableau 6: Euro cap prices with strike $K = 3.5\%$

c) We Compute the at-the-money 46-swaption price via Monte Carlo simulation as a function of β and the correlation specification. We report the prices for both specifications for the different values of β:

β	-0.4000	-0.3273	-0.2545	-0.1818	-0.1091	-0.0364
Swaption	389.2972	374.5190	357.2867	338.3110	319.3947	292.2253
β	0.0364	0.1091	0.1818	0.2545	0.3273	0.4000
Swaption	258.8728	222.4948	199.0751	173.2187	152.9986	130.9724

Tableau 7: Swaption price under $\mathbb{Q}^*$ for specification I

β	-0.4000	-0.3273	-0.2545	-0.1818	-0.1091	-0.0364
Swaption	108.7261	105.9507	104.4737	101.7526	93.4072	86.0592
β	0.0364	0.1091	0.1818	0.2545	0.3273	0.4000
Swaption	76.3402	67.8953	61.1729	55.0049	49.4063	46.3710

Tableau 8: Swaption price under $\mathbb{Q}^*$ for specification II

β	-0.4000	-0.3273	-0.2545	-0.1818	-0.1091	-0.0364
Swaption	333.4070	332.2819	317.9548	317.5667	306.8517	287.4133
β	0.0364	0.1091	0.1818	0.2545	0.3273	0.4000
Swaption	260.5240	234.5667	210.6885	181.6851	160.3287	136.4937

Tableau 9: Swaption price under $\mathbb{Q}^{T_M}$ for specification I

β	-0.4000	-0.3273	-0.2545	-0.1818	-0.1091	-0.0364
Swaption	108.4908	102.2348	104.7756	98.7736	92.9336	87.0501
β	0.0364	0.1091	0.1818	0.2545	0.3273	0.4000
Swaption	76.9744	71.3944	60.3368	55.4220	49.4449	45.8345

Tableau 10: Swaption price under $\mathbb{Q}^{T_M}$ for specification II

d) We compute this swaption price for a intermediary correlation matrix specified by $\rho_{mn} = e^{-\gamma|T_m - T_n|}$ for $\gamma \in \{0.1, 1, 2\}$, We find out:

γ	0.1	1	2
Swaption	225.7404	134.1434	105.0434

Tableau 11: Swaption price under $\mathbb{Q}^*$ for intermediary correlation matrix specified

γ	0.1	1	2
Swaption	231.6089	130.0646	104.1382

Tableau 12: Swaption price under $\mathbb{Q}^{T_M}$ for intermediary correlation matrix specified