Fixed Income Analysis
Exercise Sheet 9

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Please hand in your solution on Wednesday 27.11.2019 at the beginning of the lecture.

Exercise 1: Write a code for the Monte Carlo simulation with monthly timesteps of the LIBOR market model as outlined in the lecture, both for the risk-neutral measure $\mathbb{Q}^*$ as well as the forward measure $\mathbb{Q}^{T_M}$, and with tenor structure $T_i = i/2$ for $0 \leq i \leq M = 20$. If not mentioned otherwise, implement the two extremal correlation specifications I ($d = 1$ and $\ell_m = 1$) and II ($d = M - 1$ and $\ell_m = e_m^\top$).

(a) Calibrate the parametric specification $\sigma_m(t) = v_m e^{-\beta(T_m - t)}$ to the 3.5% strike caplet data in Table 2 (below) as a function of β. Report the values of v_m for $\beta = 0.07$.

(b) Compute via Monte Carlo simulation the 3.5% cap prices with maturities in $2, \ldots, 10$ years, using the initial forward LIBOR curve in Table 3 (below), and compare your result to the original quotes in Table 1 (below). Convince yourself that the computed cap prices are the same in both cases I and II.

(c) Compute the at-the-money 4×6-swaption price via Monte Carlo simulation as a function of β and the correlation specification. Report the prices for both specifications for the following values of β:

-0.4000	-0.3273	-0.2545	-0.1818	-0.1091	-0.0364
0.0364	0.1091	0.1818	0.2545	0.3273	0.4000

Note that the underlying swap has annual coupon payments.

(d) Compute this swaption price for a intermediary correlation matrix specified by $\rho_{mn} = e^{-\gamma |T_m - T_n|}$ for $\gamma \in \{0.1, 1, 2\}$.
(Hint: Use $d = M - 1$ and find the corresponding ℓ_m via Cholesky factorization).

For points b), c) and d) run the Monte Carlo algorithm under both the risk-neutral measure $\mathbb{Q}^*$ as well as the terminal forward measure $\mathbb{Q}^{T_M}$ and verify that the results are the same.

10 points

Table 1: Euro cap prices (in basis points) on 18 November 2008

T–K	3.5%	4%	4.5%	5%	5.5%	6%	6.5%	7%	7.5%
2	25.0	11.0	5.0	2.5	1.5	1.0	0.5	0.0	0.0
3	77.0	40.5	21.5	12.0	7.0	4.0	2.5	1.5	1.5
4	148.5	86.0	48.5	27.0	16.0	10.0	6.5	4.5	4.0
5	230.5	140.5	82.0	47.5	28.5	17.5	11.5	8.0	7.5
6	325.5	206.0	125.5	74.5	45.5	29.0	19.0	13.5	12.5
7	431.5	283.5	178.0	109.0	68.0	44.5	29.5	21.0	20.5
8	545.5	368.5	238.0	149.5	95.0	62.5	42.5	30.0	29.0
9	664.0	459.0	304.5	196.5	127.0	85.0	58.5	42.0	40.0
10	786.0	554.5	376.5	248.5	164.0	111.0	77.0	56.0	53.0

Table 2: Implied volatilities (in %) for caplets $Cpl(T_{i-1}, T_i)$ at strike rate 3.5%

i	1	2	3	4	5	6	7	8	9	10
$\sigma_{Cpl(T_{i-1},T_i)}$	n/a	29.3	29.3	29.3	20.8	20.8	18.3	18.3	17.8	17.8
i	11	12	13	14	15	16	17	18	19	20
$\sigma_{Cpl(T_{i-1},T_i)}$	16.3	16.3	16.7	16.7	16.1	16.1	15.7	15.7	15.7	15.7

Table 3: Forward LIBOR curve (in %) on 18 November 2008

T_i	0	0.5	1	1.5	2	2.5	3	3.5	4	4.5
$L_{T_i}(0)$	4.228	2.791	3.067	3.067	3.728	3.728	4.051	4.051	4.199	4.199
T_i	5	5.5	6	6.5	7	7.5	8	8.5	9	9.5
$L_{T_i}(0)$	4.450	4.450	4.626	4.626	4.816	4.816	4.960	4.960	5.088	5.088