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Exercise 2 IRMCRM

a) We have $V_0^{Prem}(x;\gamma^Q) = x\sum_{i=1}^{n}(t_i - t_{i-1})e^{-\int_0^{t_i} R(u)\,du}$

and $V_0^{def}(\gamma^Q) = \delta\int_0^{t_n}\gamma(s)e^{-\int_0^{s}R(u)du}\,ds$

So in this case

$$V_0^{Prem}(x;\gamma^Q) = \frac{x}{2}e^{-\int_0^{\frac{1}{2}}(2a+r)du} = \frac{x}{2}e^{-\frac{(2a+r)}{2}}$$

$$V_0^{def}(\gamma^Q) = \delta\int_0^{\frac{1}{2}} 2a\,e^{-(2a+r)s}\,ds$$

$$= \frac{\delta 2a}{2a+r}\left(1 - e^{-\frac{(2a+r)}{2}}\right)$$

Since we have that the contract is fairly priced, we have that the following relationship must hold

$$V_0^{Prem}(x;\gamma^Q) = V_0^{def}(\gamma^Q)$$

$$\Leftrightarrow \quad 0 = \frac{\delta 2a}{2a+r}\left(1 - e^{-\frac{(2a+r)}{2}}\right) - \frac{x}{2}e^{-\frac{(2a+r)}{2}}$$

Plugging into a Python solver this yields

$$a = 0.0246314$$

$$(\delta = 0.4,\ r = 0.01,\ x = 0.02)$$

b) again, starting from the same equations as in a), we have now

$$V_0^{\text{Prem}}(x; r^{Q}) = \frac{x}{2}\left(e^{-\int_0^{\frac{1}{2}} R(u)du} + e^{-\int_0^{1} R(u)du}\right)$$

$$= \frac{x}{2}\left(e^{-\frac{(2a+r)}{2}} + e^{-\left(\frac{2a+b}{2} + \frac{(2a)}{2} + r\right.}\right.$$

$$V_0^{\text{def}} = \delta\left(\int_0^{\frac{1}{2}} \gamma(s) e^{-\int_0^{s} R(u)du} ds + \int_{\frac{1}{2}}^{1} \gamma(s) e^{-\int_0^{\frac{1}{2}} R(u)du - \int_{\frac{1}{2}}^{s} R(u)du}\right)$$

$$= \delta\left(\frac{2a}{2a+r}\left(1 - e^{-\left(\frac{2a+r}{2}\right)}\right) + (2a+b)e^{-\frac{(2a+r)}{2}} \int_{\frac{1}{2}}^{1} e^{-\int_{\frac{1}{2}}^{s}(2a+b+r)du} ds\right.$$

$$= \delta\left(\frac{2a}{2a+r}\left(1 - e^{-\left(\frac{2a+r}{2}\right)}\right) + (2a+b)e^{-\frac{(2a+r)}{2}} \int_{\frac{1}{2}}^{1} e^{\frac{2a+b+r}{2} - (2a+b+r)} ds\right.$$

$$= \delta\left(\frac{2a}{2a+r}\left(1 - e^{-\left(\frac{2a+r}{2}\right)}\right) + (2a+b) e^{\frac{b}{2}} \int_{\frac{1}{2}}^{1} e^{-(2a+b+r)s} ds\right.$$

$$= \delta\left(\frac{2a}{2a+r}\left(1 - e^{-\left(\frac{2a+r}{2}\right)}\right) - \frac{(2a+b)e^{\frac{b}{2}}}{(2a+b+r)}\left(e^{-(2a+b+r)} - e^{-\frac{(2a+b+r)}{2}}\right)\right)$$

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b) (suite) by using the same scheme as in a),

with $\delta = 0.4$, $r = 0.01$, $x = 0.04$),

and letting $V_0^{prem}(x;\gamma^{0}) = V_0^{def}(\gamma^{0})$ and

solving using Python, we get

$$b = 0.10564099$$