

3) The convexity of the 3 securities is given by the following formula:

$$C(o) = \frac{\sum_{i=1}^n c_i e^{-y_i T_i} (T_i)^2}{\text{Price}(o)}$$

A simple numerical application (Jupyter) yields:

$$\text{Convexity - portfolio}(o) \cong 18.29$$

$$\text{Convexity - bond 1}(o) \cong 41.47$$

$$\text{Convexity - bond 2}(o) \cong 17.06$$

b) Using the formula in the course, we need that:

$$\frac{d}{ds} \left( \Pi(s) + q_1 H_1(s) + q_2 H_2(s) \right) \Big|_{s=0} = 0$$

$$\frac{d^2}{ds^2} \left( \Pi(s) + q_1 H_1(s) + q_2 H_2(s) \right) \Big|_{s=0} = 0$$

This system has the following solution:

$$\begin{pmatrix} q_1 \\ q_2 \end{pmatrix} = \begin{pmatrix} -D_{H_1} H_1(o) & -D_{H_2} H_2(o) \\ C_{H_1} H_1(o) & C_{H_2} H_2(o) \end{pmatrix}^{-1} \begin{pmatrix} D_{\Pi} \Pi(o) \\ -C_{\Pi} \Pi(o) \end{pmatrix}$$

with Portfolio being  $\Pi$ , Bond 1  $H_1$   
and Bond 2  $H_2$