Fixed Income Analysis
Solution 3

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This solution sheet only contains hints for solving the exercises and should not be taken as a reference for deserving full grades at an exam.

Exercise 1 Denote the cap rate by κ and the reset/settlement dates of the cap by $T_0 < T_1 < \cdots < T_M$. Recall that the swap rate is a convex combination of the simple forward rates:

$$R_{swap}(0) = \sum_{i=1}^{M} w_i(t) F(0, T_{i-1}, T_i), \quad w_i(t) = \frac{P(0,T_i)}{\sum_{j=1}^{M} P(0,T_j)}.$$

It is therefore possible that $R_{swap}(0) < \kappa$ and

$$F(0, T_{i-1}, T_i) > \kappa$$

for some $i \in \{1, \ldots, M\}$.

Exercise 2

a)

$$\delta(F(T_{i-1},T_i)-\kappa)^+ = \delta\left(\frac{1}{\delta}\left(\frac{1}{P(T_{i-1},T_i)}-1\right)-\kappa\right)^+ = \frac{(1+\delta\kappa)}{P(T_{i-1},T_i)}\left(\frac{1}{1+\delta\kappa}-P(T_{i-1},T_i)\right)^+.$$

b) Since the cash-flows of the corresponding caplets and floorlets at all times T_i sum up to the cash-flow of the swap in the following sense:

$$\delta(L(T_{i-1},T_i)-\kappa)^+ - \delta(L(T_{i-1},T_i)-\kappa)^- = \delta(L(T_{i-1},T_i)-\kappa),$$

by buying the cap and selling the floor we obtain the cash-flow of the swap, which means that the value of this strategy at time t, i.e. $Cp(t) - Fl(t)$, must equal the value of the swap $\Pi_p(t)$.

c) This follows from the fact that $\Phi(-a) = 1 - \Phi(a)$.

Exercise 3 With the bootstrapping method the forward rate between 04/10/2041 and 03/10/2042 is 2.56% and with the pseudo-inverse method it is 2.53%.

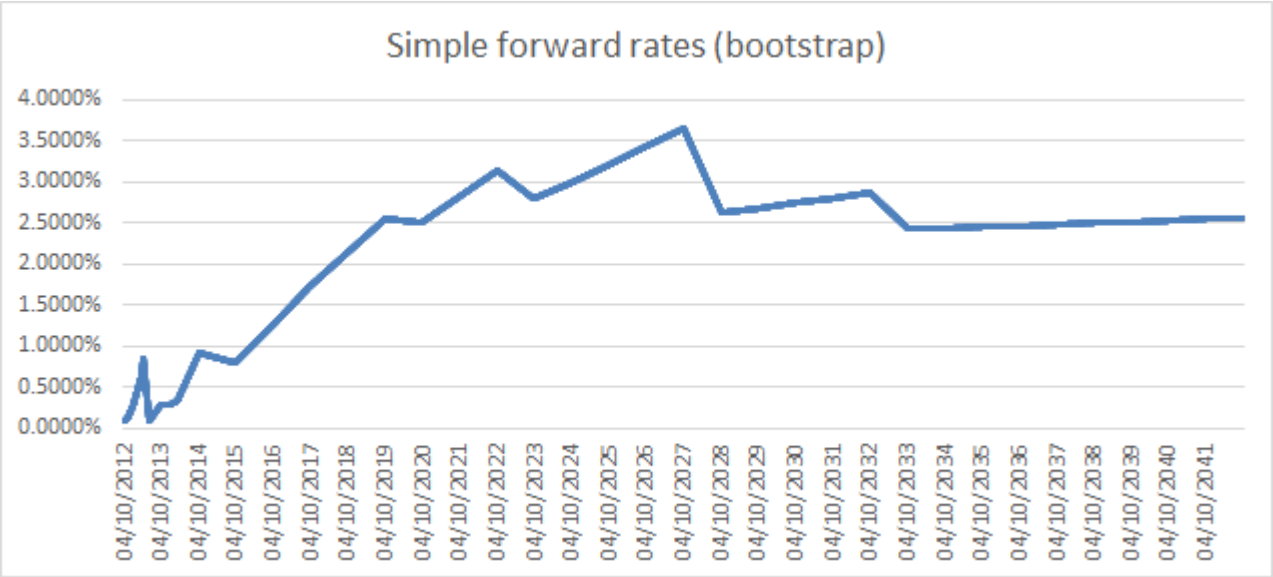

Figure 1: Simple forward rates obtained with the bootstrap method.

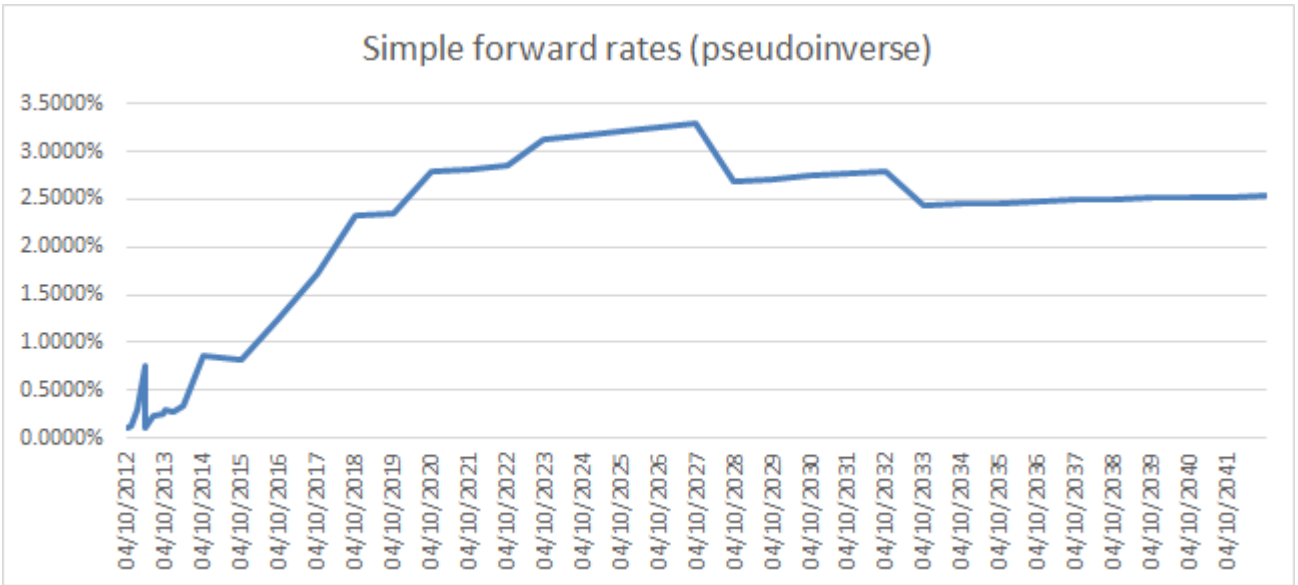

Figure 2: Simple forward rates obtained with the pseudo-inverse method.