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IRCRM Pb Set 11

Exercise 1:

a) From the the lecture, we know that the value of the equity is:

$$S_0 = C^{BS}(t=0, V_0, \sigma_V, r, B) = V_0 \Phi(d_{0,1}) - Be^{-rT}\Phi(d_{0,2})$$

where
$$d_{t,1} = \frac{\log \frac{V_0}{B} + (r + \frac{1}{2}\sigma_V^2)(T-t)}{\sigma_V \sqrt{T-t}}$$

$$d_{t,2} = d_{t,1} - \sigma_V \sqrt{T-t}$$

And the value of the debt is:

$$B_0 = P_0(0,T) B \Phi(d_{0,2}) + V_0 \Phi(-d_{0,1})$$

where
$$P_0(t,T) = \exp(-r(T-t))$$

Therefore we figure out, with our python code:

$$\begin{cases} S_0 = 61.7 \\ B_0 = 38.3 \end{cases}$$

b) We plot the credit spreads with our python code as function of T and σ for different value of $L = B/V_0$:

$$c(t,T) = \frac{-1}{T-t} \log\left[\Phi(d_{t,2}) + \frac{V_t}{BP_0(t,T)}\Phi(-d_{t,1})\right]$$

We can observe that:

- when $T \to 0$, $c(0,T) \to 0$
- the higher the leverage is, the higher the credit spread is.
- concerning sigma, the higher the volatility is, the higher credit spread is.

c) Let's introduce:

$$f(T) = \Phi(d_2(T)) + \frac{V_0}{B} e^{rT} \Phi(-d_1(T))$$

Since $f(0) = 1$ and $c(0,T) = -\frac{\log(f(T))}{T}$ we have:

$$c(0,T) = -\frac{\log(f(T)) - 0}{T - 0} \xrightarrow[T \to 0]{} -\frac{f'(T)}{f(T)}$$

We also have:

$$f'(T) = d_2'(T)\,\phi(d_2(T)) + r\frac{V_0}{B} e^{rT}\Phi(-d_1(T)) - \frac{V_0}{B} e^{rT} d_1'(T)\,\phi(-d_1(T))$$

$$d_1'(T) = \frac{r + \frac{1}{2}\sigma_V^2}{\sigma_V} \cdot \frac{1}{\sqrt{T}} \xrightarrow[T \to 0]{} +\infty$$

$$d_1(T) \underset{T \to 0}{\sim} \log\frac{V_0}{B} \cdot \frac{1}{\sigma\sqrt{T}} \xrightarrow[T \to 0]{} +\infty$$

$$d_2'(T) = \frac{r - \frac{1}{2}\sigma_V^2}{\sigma_V} \frac{1}{\sqrt{T}} \xrightarrow[T \to 0]{} +\infty$$

$$d_2(T) \underset{T \to 0}{\sim} \log\frac{V_0}{B} \cdot \frac{1}{\sigma\sqrt{T}} \xrightarrow[T \to 0]{} +\infty$$

$$\Rightarrow \quad \Phi(d_2(T)) \xrightarrow[T \to 0]{} 1$$

$$\Phi(-d_1(T)) \xrightarrow[T \to 0]{} 0$$

And $\phi(d_2(T)) \underset{T \to 0}{\sim} 0$.

$\phi(d_1(T)) \underset{T \to 0}{\sim} 0$.

Therefore we have:

$$f'(T) \underset{T\to 0}{\longrightarrow} 0$$

$$f(T) \underset{T\to 0}{\longrightarrow} 1$$

So we have $\underline{c(0,T) \underset{T\to 0}{\longrightarrow} 1}$

Exercise 2

We want to compute the $\tilde{d}^j_k$ such that:

$$\mathbb{P}(\tilde{d}^j_k \leq V_T < \tilde{d}^j_{k+1}) = p_{jk} \quad \text{for } k \in \{0, \dots, n\}$$

where $0 = \tilde{d}^j_0 \leq \tilde{d}^j_1 \leq \tilde{d}^j_2 \leq \tilde{d}^j_3 \leq \tilde{d}^j_4 = +\infty$

We introduce:

$$X^j_T = \frac{\log V^j_T - \log V_0 - (\mu^j_V - \frac{1}{2}\sigma^{j^2}_V)T}{\sigma^j_V \sqrt{T}} \quad = \frac{W^j_T}{\sqrt{T}} \sim \mathcal{N}(0,1)$$

$$d^j_k = \frac{\log \tilde{d}^j_k - \log V_0 - (\mu^j_V - \frac{1}{2}\sigma^{j^2}_V)T}{\sigma^j_V \sqrt{T}}$$

Therefore: $d^j_k = \phi^{-1}\left(\sum_{l=0}^{k-1} p_{jl}\right)$ for $k = 1, \dots, n$

And $\tilde{d}^j_l = V_0 \exp\left(\sigma^j_V \sqrt{T} d^j_k + (\mu^j_V - \frac{1}{2}\sigma^{j^2}_V)T\right)$

We find out:

	d_1	d_2	d_3
A	0	0	71,29
B	74.64	86,9	169.89
C	97.54	128.19	$+\infty$