

# Fixed Income Analysis

## Exercise Sheet 1

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Please hand in your solutions on Wednesday 25.09.2019 at the beginning of the lecture.

**Exercise 1** Consider a forward rate agreement (FRA) with current, expiry and maturity time  $t < T < S$ , respectively, and cash flow to the lender:

- At time  $T$ :  $-K$ ,
- At time  $S$ :  $Ke^{R^*(S-T)}$ ,

for some predetermined principal  $K$  and interest rate  $R^*$ .

- a) Compute the value  $\Pi(t)$  at time  $t$  of the cash flow above in terms of zero-coupon bond prices.
- b) Show that in order for the value of the FRA to equal zero at  $t$ , the rate  $R^*$  has to equal the forward rate  $R(t; T, S)$ .

*1 point*

**Exercise 2** Suppose the short rate is constant over time and equals  $r = 2\%$ . If you invest in the money market account, how many years does it take to double your investment?

*2 points*

**Exercise 3** Find the relationship between the instantaneous forward rate  $f(t, T)$  and the continuously compounded spot rate  $y(t, T)$ . Show that, if the spot rate curve is increasing (i.e.,  $\frac{\partial y}{\partial T}(t, T) > 0$ ), forward rates will exceed spot rates.

*2 points*

**Exercise 4** Consider two coupon bonds with notional 100CHF, time to maturity 3 years and an annual coupon rate of 5% and 10% respectively. The first bond is trading at 95CHF and the second bond is trading at 108CHF.

- a) Compute the continuously compounded spot rate  $y(t, t + 3)$ .
- b) Suppose you also know that a zero-coupon bond with maturity 1 year and notional 100CHF is trading for 92CHF, what would the present value be of an instrument with a cash flow of 20CHF, 35CHF and 60CHF in 1 year, 2 years and 3 years respectively?

*3 points*

**Exercise 5** Suppose the price  $P(t, T)$  of a zero-coupon bond drops by 20% (relative change) when the continuously compounded spot rate  $y(t, T)$  rises by 3% (absolute change). Calculate the time to maturity  $T - t$  of this zero-coupon bond.

*2 points*