

# Fixed Income Analysis

## Exercise Sheet 11

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**Please hand in your solution on Wednesday 11.12.2019 at the beginning of the lecture.**

**Exercise 1:** Consider the Merton model with a constant risk-free rate  $r = 5\%$ .

- a) Compute the value of the equity and the debt at time  $t = 0$  when  $\mu = 0.15$ ,  $\sigma = 0.25$ , the assets are worth  $V_0 = 100$ , and the firm's debt consists of a single zero-coupon bond with face value  $B = 50$  to be reimbursed at date  $T = 5$ .
- b) Let  $L = B/V_0$  be the leverage of the firm. Plot the credit spread  $c(0, T)$  as a function of (i) the time to maturity  $T$  and (ii) the asset volatility  $\sigma$  for  $L = 0.3, 0.6$ , and  $0.9$  (i.e. you should have three 3D-plots). What do you observe?
- c) Prove that  $c(0, T) \rightarrow 0$  as  $T \rightarrow 0$  when  $V_0 > B$

*5 points*

**Exercise 2:** Consider the 1-year rating-transition probabilities:

	A	B	C	D
A	0.95	0.05	0	0
B	0.05	0.8	0.1	0.05
C	0	0.2	0.5	0.3
D	0	0	0	1

There are three firms with the following ratings at time  $t = 0$  and characteristics under the Merton's model:

- Firm 1:  $\mu_V = 0.20$ ,  $\sigma_V = 0.3$  and rated  $A$  at  $t = 0$ .
- Firm 2:  $\mu_V = 0.15$ ,  $\sigma_V = 0.25$  and rated  $B$  at  $t = 0$ .
- Firm 3:  $\mu_V = 0.1$ ,  $\sigma_V = 0.2$  and rated  $C$  at  $t = 0$ .

Compute the corresponding thresholds of transition  $\tilde{d}_{j,k}$  which are defined in slide 611.

*5 points*