Fixed Income Analysis
Solution 1

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This solution sheet only contains hints for solving the exercises and should not be taken as a reference for deserving full grades at an exam.

Exercise 1

a) We have $\Pi(t) = -P(t,T)K + P(t,S)Ke^{R^*(S-T)}$.

b) $\Pi(t) = 0$ is equivalent to $R^* = \frac{1}{(S-T)} \log\left(\frac{P(t,T)}{P(t,S)}\right)$, which is the definition of $R(t;T,S)$.

Exercise 2

$$e^{rT} = 2 \Leftrightarrow T = \frac{\log(2)}{r} = 34.6574 \approx 35$$

Exercise 3 The forward rates satisfy

$$f(t,T) = -\frac{\partial}{\partial T} \ln P(t,T),$$

from which it follows that

$$f(t,T) = \frac{\partial}{\partial T}\left(y(t,T)(T-t)\right) = \frac{\partial y(t,T)}{\partial T}(T-t) + y(t,T).$$

Thus for an increasing spot rate curve, $\frac{\partial y}{\partial T}(t,T) > 0$, we have $y(t,T) < f(t,T)$.

Exercise 4

a) First solve the following system for $P(t,t+3)$ and $\sum_{i=1}^{2} P(t,t+i)$:

$$\begin{cases} 95 & = 5\sum_{i=1}^{2} P(t,t+i) + 105P(t,t+3) \\ 108 & = 10\sum_{i=1}^{2} P(t,t+i) + 110P(t,t+3) \end{cases}$$

This results in:

$$P(t,t+3) = 0.82, \qquad \sum_{i=1}^{2} P(t,t+i) = 1.78.$$

From $P(t,t+3)$ you can now easily compute $y(t,t+3) = 6.62\%$.

b) First we calculate $P(t, t+2)$:

$$P(t, t+2) = \sum_{i=1}^{2} P(t, t+i) - P(t, t+1) = 1.78 - 0.92 = 0.86.$$

The present value of the cashflows can now easily be computed:

$$PV = 20 * P(t, t+1) + 35 * P(t, t+2) + 60 * P(t, t+3) = 97.7.$$

Exercise 5 Denote the maturity of the bond by T. Denote the yield prior to the increase by y_0 and after the increase by y_1. We can now compute the maturity as follows:

$$\frac{e^{-y_1 T}}{e^{-y_0 T}} = 0.8 \Leftrightarrow T = \frac{\log(0.8)}{-0.03} = 7.44$$