PS1_CRN_BEDANIAN_de_LESTABLE_RICHIARDI_SCHOENENBER

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1 Problem set 1

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[24]:
```
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats
import scipy.optimize
```

1.1 Exercie 3

[25]:
```
b= [np.exp(-0.06*1),np.exp(-0.058*2),np.exp(-0.0562*3),np.exp(-0.0546*4),np.
 ↪exp(-0.0533*5),np.exp(-0.0525*6),np.exp(-0.052*7),np.exp(-0.0516*8),np.
 ↪exp(-0.05125*9),np.exp(-0.05100*10)]
#Portfolio
portfolio=6*b[0]+8*b[1]+106*b[2]+7*b[3]+0*b[4]+102*b[5]+3*b[6]+3*b[7]+3*b[8]+110*b[9]
#Bond 1
bond1=4*b[0]+4*b[1]+4*b[2]+4*b[3]+4*b[4]+4*b[5]+104*b[6]
#Bond 2
bond2=10*b[0]+10*b[1]+10*b[2]+10*b[3]+10*b[4]+10*b[5]+10*b[6]+110*b[7]
```

[26]:
```
print(portfolio)
print(bond1)
print(bond2)
```

```
254.409295494
92.1756253141
129.513673737
```

[27]:
```
duration_portfolio=(1*6*b[0]+2*8*b[1]+3*106*b[2]+4*7*b[3]+5*0*b[4]+6*102*b[5]+7*3*b[6]+8*3*b[7
 ↪portfolio
duration_bond1=(1*4*b[0]+2*4*b[1]+3*4*b[2]+4*4*b[3]+5*4*b[4]+6*4*b[5]+7*104*b[6])/
 ↪bond1
```

```
duration_bond2=(1*10*b[0]+2*10*b[1]+3*10*b[2]+4*10*b[3]+5*10*b[4]+6*10*b[5]+7*10*b[6]+8*110*b[
 ↪bond2
```

[28]:
```
print(duration_portfolio)
print(duration_bond1)
print(duration_bond2)
```

```
5.76135022907
6.21212737952
6.16024913527
```

[29]:
```
convexity_portfolio=(1**2*6*b[0]+2**2*8*b[1]+3**2*106*b[2]+4**2*7*b[3]+5**2*0*b[4]+6**2*102*b[
 ↪portfolio
convexity_bond1=(1**2*4*b[0]+2**2*4*b[1]+3**2*4*b[2]+4**2*4*b[3]+5**2*4*b[4]+6**2*4*b[5]+7**2*
 ↪bond1
convexity_bond2=(1**2*10*b[0]+2**2*10*b[1]+3**2*10*b[2]+4**2*10*b[3]+5**2*10*b[4]+6**2*10*b[5]
 ↪bond2
```

[30]:
```
print(convexity_portfolio)
print(convexity_bond1)
print(convexity_bond2)
```

```
41.6565582217
41.4722797487
44.0374377641
```

[31]:
```
temp = np.eye(2)
temp[0,0]= -duration_bond1*bond1
temp[0,1]= -duration_bond2*bond2
temp[1,0]= convexity_bond1*bond1
temp[1,1]= convexity_bond2*bond2

sol = np.linalg.inv(temp) @␣
 ↪[duration_portfolio*portfolio,-convexity_portfolio*portfolio]
```

[32]:
```
sol
```

[32]:
```
array([ 0.44249593, -2.15472369])
```

[33]:
```
def value_curve(shift,q1,q2):
    b= [np.exp(-(0.06+shift)*1),np.exp(-(0.058+shift)*2),np.exp(-(0.
 ↪0562+shift)*3),np.exp(-(0.0546+shift)*4),np.exp(-(0.0533+shift)*5),np.
 ↪exp(-(0.0525+shift)*6),np.exp(-(0.052+shift)*7),np.exp(-(0.0516+shift)*8),np.
 ↪exp(-(0.05125+shift)*9),np.exp(-(0.05100+shift)*10)]
    port_coupon=[6.,8.,106.,7.,0.,102.,3.,3.,3.,110.]
    bond1_coupon=[4.,4.,4.,4.,4.,4.,104.,0.,0.,0.]
    bond2_coupon=[10.,10.,10.,10.,10.,10.,10.,110.,0.,0.]
    hedge_coupon=[]
    for i in range(0,10):
```

```
        temp= port_coupon[i]+q1*bond1_coupon[i]+q2*bond2_coupon[i]
        hedge_coupon=np.append(hedge_coupon,temp)

    hedge_value = np.sum(np.multiply(hedge_coupon,b))
    return hedge_value
```

[34]:
```
def value_hedged(shift):
    b= [np.exp(-(0.06+shift)*1),np.exp(-(0.058+shift)*2),np.exp(-(0.
↪0562+shift)*3),np.exp(-(0.0546+shift)*4),np.exp(-(0.0533+shift)*5),np.
↪exp(-(0.0525+shift)*6),np.exp(-(0.052+shift)*7),np.exp(-(0.0516+shift)*8),np.
↪exp(-(0.05125+shift)*9),np.exp(-(0.05100+shift)*10)]
    port_coupon=[6.,8.,106.,7.,0.,102.,3.,3.,3.,110.]
    bond1_coupon=[4.,4.,4.,4.,4.,4.,104.,0.,0.,0.]
    bond2_coupon=[10.,10.,10.,10.,10.,10.,10.,110.,0.,0.]
    hedge_coupon=[]


  ␣
↪portfolio=6*b[0]+8*b[1]+106*b[2]+7*b[3]+0*b[4]+102*b[5]+3*b[6]+3*b[7]+3*b[8]+110*b[9]
    #Bond 1
    bond1=4*b[0]+4*b[1]+4*b[2]+4*b[3]+4*b[4]+4*b[5]+104*b[6]
    #Bond 2
    bond2=10*b[0]+10*b[1]+10*b[2]+10*b[3]+10*b[4]+10*b[5]+10*b[6]+110*b[7]


  ␣
↪duration_portfolio=(1*6*b[0]+2*8*b[1]+3*106*b[2]+4*7*b[3]+5*0*b[4]+6*102*b[5]+7*3*b[6]+8*3*
↪portfolio

  ␣
↪duration_bond1=(1*4*b[0]+2*4*b[1]+3*4*b[2]+4*4*b[3]+5*4*b[4]+6*4*b[5]+7*104*b[6])/
↪bond1

  ␣
↪duration_bond2=(1*10*b[0]+2*10*b[1]+3*10*b[2]+4*10*b[3]+5*10*b[4]+6*10*b[5]+7*10*b[6]+8*110
↪bond2


  ␣
↪convexity_portfolio=(1**2*6*b[0]+2**2*8*b[1]+3**2*106*b[2]+4**2*7*b[3]+5**2*0*b[4]+6**2*102
↪portfolio

  ␣
↪convexity_bond1=(1**2*4*b[0]+2**2*4*b[1]+3**2*4*b[2]+4**2*4*b[3]+5**2*4*b[4]+6**2*4*b[5]+7*
↪bond1

  ␣
↪convexity_bond2=(1**2*10*b[0]+2**2*10*b[1]+3**2*10*b[2]+4**2*10*b[3]+5**2*10*b[4]+6**2*10*b
↪bond2


    temp = np.eye(2)
    temp[0,0]= -duration_bond1*bond1
    temp[0,1]= -duration_bond2*bond2
    temp[1,0]= convexity_bond1*bond1
```

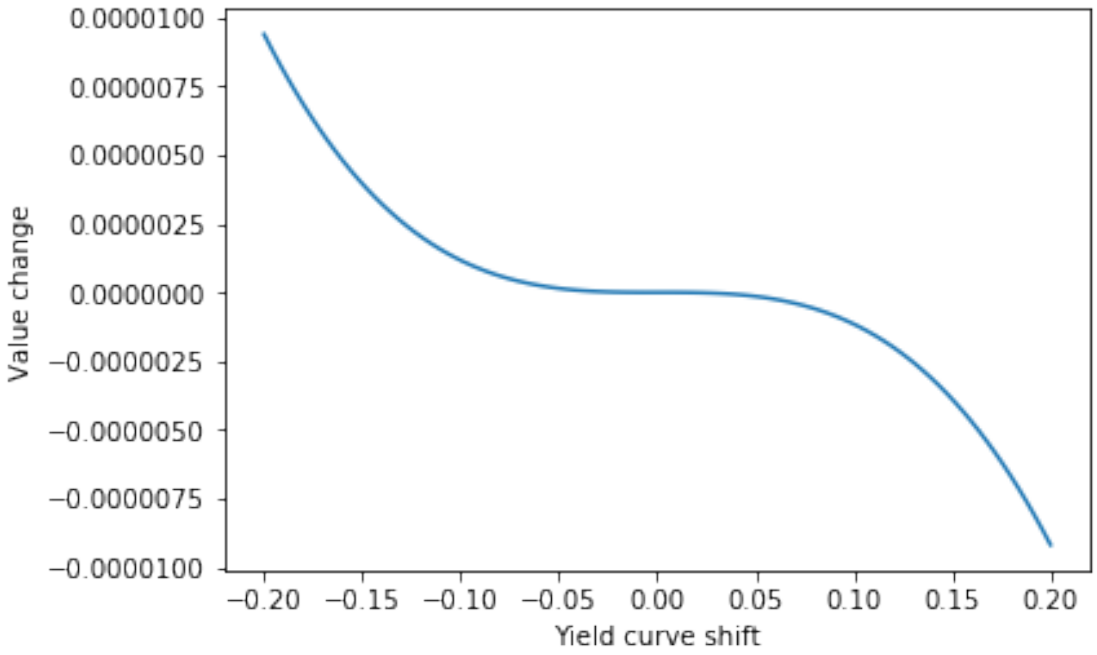
```
    temp[1,1]= convexity_bond2*bond2

    #q = np.linalg.inv(temp) @␣
↪[duration_portfolio*portfolio,-convexity_portfolio*portfolio]

    for i in range(0,10):
        temp= port_coupon[i]+sol[0]*bond1_coupon[i]+sol[1]*bond2_coupon[i]
        hedge_coupon=np.append(hedge_coupon,temp)

    hedge_value = np.sum(np.multiply(hedge_coupon,b))
    return hedge_value
```

[35]:
```
grid=np.linspace(-0.002,0.002,100)
result=[]
for i in range(0,len(grid)):
    temp=value_hedged(grid[i])-value_hedged(0)
    result=np.append(result,temp)
plt.plot(grid*100,result)
plt.xlabel("Yield curve shift")
plt.ylabel("Value change")
plt.show()
```



[36]:
```
##EXO 4
```

[37]:
```
def caplet_black(y,f,sigma,k,t,t0,t1):
    p = np.exp(-y*(t1-t))
```

```
    d = t1-t0
    d1 = (np.log(f/k)+ 0.5*sigma**2*(t0-t))/(sigma*np.sqrt(t0-t))
    d2 = (np.log(f/k)- 0.5*sigma**2*(t0-t))/(sigma*np.sqrt(t0-t))

    return d*p*(f*scipy.stats.norm.cdf(d1)-k*scipy.stats.norm.cdf(d2))
```

[38]:
```
caplet_black(0.03,0.04,0.4945,0.04,0,4,5)
```

[38]:
```
0.013049947392014823
```

[39]:
```
form=1*np.exp(-0.03*5)*0.04*(scipy.stats.norm.cdf(0.4945)-scipy.stats.norm.
 ↪cdf(-0.4945))
```

[40]:
```
sigma = form/(1*np.exp(-0.03*5)*2*(1/np.sqrt(2*np.pi)))
print(sigma)
```

```
0.0190025932205
```

1.1.1 Exercie 4

[41]:
```
delta = 1
t = 0
Ti_1 = 4
Ti = 5
k = np.array([1,1.5,2,3,3.5,4,5,6,7,8,9])/100
F045 = 0.04
y05 = 0.03
P05 = np.exp(-(Ti-t)*y05)
D = (F045*np.array([1]) - k)/(sigma*np.sqrt(Ti_1 - t))
caplet_bachelier = delta*P05*sigma*np.sqrt(Ti_1 - t)*(D*scipy.stats.norm.
 ↪cdf(D)+scipy.stats.norm.pdf(D))
```

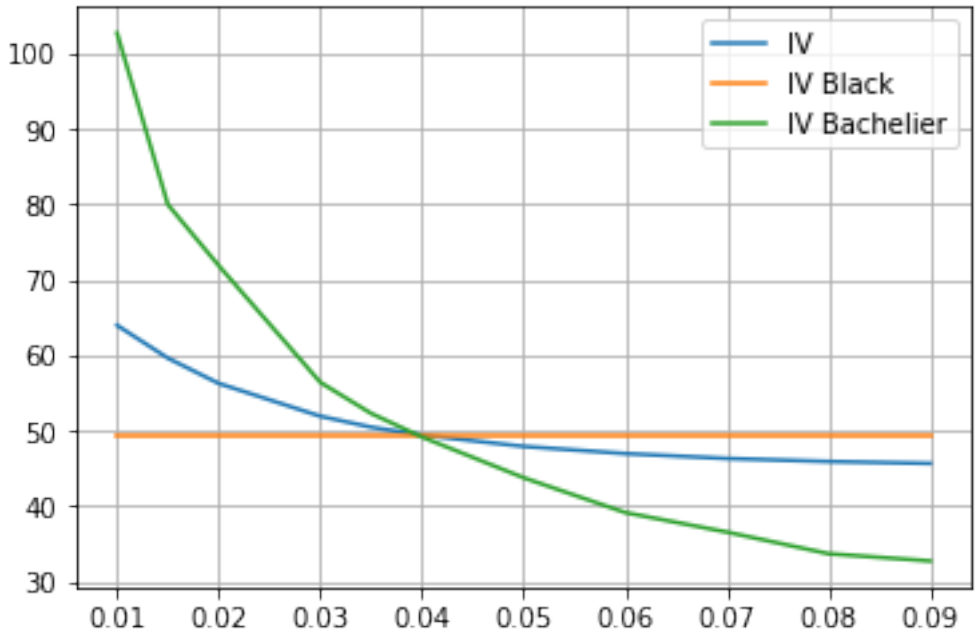
[42]:
```
caplet_bachelier
```

[42]:
```
array([ 0.02982761,  0.02653463,  0.02342343,  0.01780264,  0.01531449,
        0.01304995,  0.00919556,  0.00620927,  0.00400638,  0.00246367,
        0.00144053])
```

[43]:
```
def black_caplet(sigma):
    F045 = 0.04
    y05 = 0.03
    delta = 1
    t = 0
    Ti_1 = 4
    Ti = 5
    P05 = np.exp(-(Ti-t)*y05)
    d1 = (np.log(F045/k)+ 0.5*sigma**2*(Ti_1-t))/(sigma*np.sqrt(Ti_1-t))
    d2 = (np.log(F045/k) - 0.5*sigma**2*(Ti_1-t))/(sigma*np.sqrt(Ti_1-t))
    return (delta*P05*(F045*scipy.stats.norm.cdf(d1)-k*scipy.stats.norm.
 ↪cdf(d2)) - cp)**2
```

```
K = np.array([1,1.5,2,3,3.5,4,5,6,7,8,9])/100
res = []
for i in range(0,len(K)):
    k = K[i]
    cp = caplet_bachelier[i]
    res += [scipy.optimize.minimize(black_caplet,0.8).x]
```

[44]:
```
iv = [64,59.65,56.30,51.95,50.50,49.45,47.95,47,46.35,45.95,45.7]
black = [49.45]*len(iv)
res = 100*np.array(res)
plt.plot(K,iv,label = 'IV')
plt.plot(K,black, label = 'IV Black')
plt.plot(K,res, label = 'IV Bachelier')
plt.legend()
plt.grid()
plt.show()
```



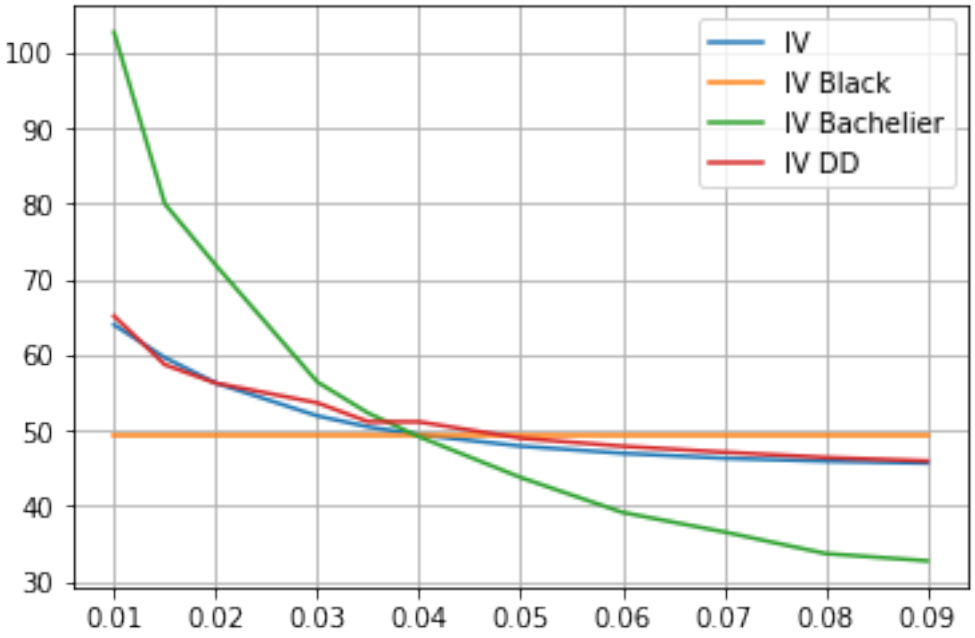
[45]:
```
sigma = 49.45/100
beta = 0.75
delta = 1
t = 0
Ti_1 = 4
Ti = 5
F045 = 0.04
y05 = 0.03
```

```
K = np.array([1,1.5,2,3,3.5,4,5,6,7,8,9])/100
dd = caplet_black(y05,F045/beta,beta*sigma,K+F045*(1/beta-1),t,Ti_1,Ti)
```

[46]:
```
def dd_caplet(sigma):
    F045 = 0.04
    y05 = 0.03
    delta = 1
    t = 0
    Ti_1 = 4
    Ti = 5
    P05 = np.exp(-(Ti-t)*y05)
    d1 = (np.log(F045/k)+ 0.5*sigma**2*(Ti_1-t))/(sigma*np.sqrt(Ti_1-t))
    d2 = (np.log(F045/k) - 0.5*sigma**2*(Ti_1-t))/(sigma*np.sqrt(Ti_1-t))
    return (delta*P05*(F045*scipy.stats.norm.cdf(d1)-k*scipy.stats.norm.
 ↪cdf(d2)) - Dd)**2

dd_iv = []
for i in range(0,len(K)):
    k = K[i]
    Dd = dd[i]
    dd_iv += [scipy.optimize.minimize(dd_caplet,1).x]
dd_iv = np.array(dd_iv)*100
```

[47]:
```
plt.plot(K,iv,label = 'IV')
plt.plot(K,black, label = 'IV Black')
plt.plot(K,res, label = 'IV Bachelier')
plt.plot(K,dd_iv, label = 'IV DD')
plt.legend()
plt.grid()
plt.show()
```


IV
IV Black
IV Bachelier
IV DD
100
90
80
70
60
50
40
30
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09