Exercise 4

a) Recall first Black's formula for caplets:

$$Cpl_{Black}(t, T_0, T_1) = \delta P(t, T_1)\left(F(t, T_0, T_1)\Phi(d_1) - \kappa\Phi(d_2)\right)$$ for a given strike κ with

$$d_{1,2} = \frac{\log\left(\frac{F(t, T_0, T_1)}{\kappa}\right) \pm \frac{1}{2}\sigma^2_{Black}(T_0 - t)}{\sigma_{Black}\sqrt{T_0 - t}}$$

If the caplet is ATM, this formula simplifies too (since $F(t, T_0, T_1) = \kappa$):

$$Cpl_{Black}(t, T_0, T_1) = \delta P(t, T_1) F(t, T_0, T_1)\left(\Phi\left(\frac{1}{2}\sigma_{Black}\sqrt{T_0 - t}\right) - \Phi\left(-\frac{1}{2}\sigma_{Black}\sqrt{T_0 - t}\right)\right)$$

Plugging in $\delta = 1$, $F(t, T_0, T_1) = 0.04$,

$\sigma_{Black} = 0.4945$, $T_0 = 4$, $T_1 = 5$

$P(t, T_1) = e^{-0.03 \cdot 5}$, yields

$$Cpl_{Black} = e^{-0.03 \cdot 5} \cdot 0.04\left(\Phi(0.4945) - \Phi(-0.4945)\right)$$

$$= 0.13049947392014823$$