Fixed Income Analysis
Exercise Sheet 8

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Please hand in your solutions on Wednesday 13.11.2019 at the beginning of the midterm exam.

Exercise 1 Give the stochastic differential equations that correspond to the three examples of admissible models given slide 364 - 366 using the Cholesky decomposition:

a) $(m, n) = (1, 2)$, a conditionally Gaussian model;

b) $(m, n) = (2, 1)$, a conditionally Gaussian model;

c) $(m, n) = (3, 0)$, a 3-factor square-root model.

4 points

Exercise 2 Let B be a Brownian motion and define the $\mathbb{R}^2_+$-valued process X by $X_i(t) = (\sqrt{x_i} + B(t))^2$, for $i = 1, 2$, for some $x \in \mathbb{R}^2_+$.

(a) Show that X satisfies

$$\begin{aligned} dX_1(t) &= dt + 2\sqrt{X_1(t)}\, dW(t) \\ dX_2(t) &= dt + 2\sqrt{X_2(t)}\, dW(t) \\ X(0) &= x, \end{aligned}$$

for some Brownian motion W. Is X an affine process? Your answer should be justified.

(b) Compute the characteristic function of $X(t)$ and verify your finding concerning the (supposed) affine property of X.
(*Hint:* For Z a noncentral χ^2-distributed with δ degrees of freedom and noncentrality parameter ζ, we have

$$\mathbb{E}[e^{uZ}] = \frac{e^{\frac{\zeta u}{1-2u}}}{(1-2u)^{\frac{\delta}{2}}}, \quad u \in \mathbb{C}_{-}.)$$

6 points