Fixed Income Analysis
Exercise Sheet 4

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Please hand in your solutions to exercises on Wednesday 16.10.2019 at the beginning of the lecture.

Exercise 1 On the Moodle page you can find the file

`statmon_E4_M1_M.xls`

containing monthly spot interest rates (that is, yields $y(t,T)$) for Swiss Confederation bonds for a time to maturity $(T-t)$ spectrum of 2, 3, 4, 5, 7, 10, 20 and 30 years.

a) Construct the instantaneous forward curve based on the July 2015 yields using the Representer Theorem for the following three values of the parameter: $\alpha = 1$, $\alpha = 0.1$, and $\alpha = 0.01$. Plot the three resulting yield curves $T \mapsto y(t,T)$. What can you conclude about the effect of the parameter α?

b) Perform a principal component analysis of the monthly yield curve changes (as shown in the slides for the forward curve changes) from the last ten years (August 2005 until and including July 2015). In particular, determine:

- the empirical covariance matrix;
- its eigenvectors and eigenvalues in decreasing order;
- the explained variances of the principal components.
- plot the first three eigenvectors (i.e. those that explain most of the variance).

4 points

Exercise 2 For this exercise we will use the same data file as Sheet 3 Exercise 3, that is `Bootstrap_data.xls` containing data from the Cash, Futures and Swap Euro markets, with spot date 03 October 2012 (available on Moodle). With the same assumptions and conventions, estimate the discount curve for the tenor dates as in the Excel sheet using the Nelson–Siegel exponential family, where you should use the following parametrization

$$f(0,T) = \beta_0 + \beta_1 e^{-aT} + \beta_2 \left(aTe^{-aT}\right),$$

and optimize over the parameters $\beta_0, \beta_1, \beta_2$ for a fixed. For the optimization you should minimize the squared error. Give the estimated parameters and plot the respective forward curves for $a \in \{0.06, 0.08, 0.1\}$.

2 points

Exercise 3 Let $x(1), \ldots, x(N)$ be a sample of a random vector $X = (X_1, \ldots, X_n)^\top$, and let $\hat{\mu}$ and $\hat{Q}$ denote the empirical mean and covariance matrix of x, respectively. Prove the following:

a) $\hat{Q}$ is symmetric and positive semi-definite.

b) The PCA decomposition given on the slide 180 of the lecture notes holds.

c) If $\hat{Q}$ is degenerate then one can express some x_i as a linear function of the other components x_j, $j \neq i$, plus a constant.

d) Assume that $\hat{Q}$ is non-degenerate. Find a sample of vectors $w(t) = (w_1(t), \ldots, w_n(t))^\top$, for $t = 1, \ldots, N$, such that

$$x(t) = \hat{\mu} + \sum_{i=1}^{n} \hat{a}_i \sqrt{\hat{\lambda}_i} w_i(t) \quad \text{and} \quad \mathrm{Cov}[w_i, w_j] = \begin{cases} 1, & \text{if } i = j, \\ 0, & \text{else,} \end{cases}$$

where $\hat{\lambda}_i$ is the ith eigenvalue of $\hat{Q}$, and $\hat{a}_i$ the ith vector of loadings of x.

2 points

Exercise 4 Consider the following Nelson–Siegel specification for the instantaneous forward curve (as on slide 207):

$$f(0,T) = \beta_0 + \beta_1 e^{-aT} + \beta_2 \left(aTe^{-aT} \right),$$

with $a = 1$, $\beta_0 = 0.07$, $\beta_1 = -0.02$, and $\beta_2 = 0.01$. Given is a coupon bond with maturity in 10 years, semi-annual coupon payments of 10, and nominal 100. Construct a static hedge for this coupon bond using three zero-coupon bonds (nominal 100 and maturity in 1 year, 3 years, and 5 years), such that the hedged portfolio is immunized against small changes in β_0, β_1, and β_2 (i.e., level, slope, and curvature).

2 points