Quantitative Risk Management
Assignment 5

Due: November 5, 2019

Question 1: Recall that a causal ARMA process can be represented as:

$$X_t = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i}$$

where

$$\sum_{i=0}^{\infty} |\psi_i| < \infty$$

and $(\epsilon_t)_{t\in\mathbb{Z}}$ is $WN(0, \sigma_\epsilon^2)$. Derive the autocorrelation function $\rho(h)$ for the process $(X_t)_{t\in\mathbb{Z}}$:

$$\rho(h) = \frac{\sum_{i=0}^{\infty} \psi_i \psi_{i+|h|}}{\sum_{i=0}^{\infty} \psi_i^2}$$

Question 2: Recall that an $ARMA(1,1)$ process satisfies:

$$X_t - \phi X_{t-1} = \epsilon_t + \theta \epsilon_{t-1}$$

As stated in class, this process is causal if $|\phi| < 1$ and $\phi \neq -\theta$, meaning that it can be written as:

$$X_t = \sum_{i=0}^{\infty} \psi_i \epsilon_{t-i}$$

where the coefficients satisfy:

$$\sum_{i=0}^{\infty} \psi_i z^i = \frac{1 + \theta z}{1 - \phi z}$$

Find ψ_i in terms of ϕ and θ. Also compute the autocorrelation function $\rho(h)$ for this process.

Question 3: Let $(X_t)_{t\in\mathbb{Z}}$ be an $ARCH(1)$ process and assume it has finite fourth moments. Compute $\mathbb{E}[X_t^4]$ in terms of α_0, α_1, and $\mathbb{E}[Z_t^4]$.