ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Quantitative Risk Management - Problem Set 10

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Question 1

We plot the empirical distribution of the number of defaults in each cases when the copula has a pairwise correlation of $\rho = 0.01$. On each of these plots, we plot the distribution that would occur if all defaults were independent.

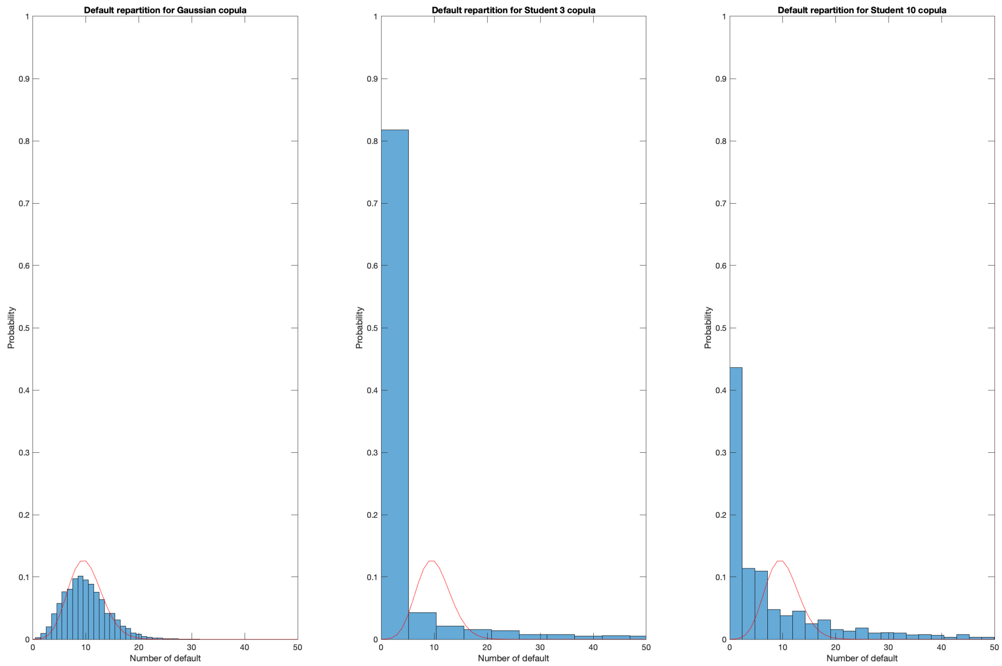

Figure 1: Empirical distribution of losses for different copulas. The red curve represents the distribution for independent losses.

The quantiles of interest are given below:

Copula	95^{th} percentile	99^{th} percentile	99.9^{th} percentile
Gaussian	18	22	26
Student-t, $\nu = 3$	66	194	384
Student-t, $\nu = 10$	41	82	146

Tableau 1: Quantiles of number of defaults for each copula.

We can notice that the quantiles behave in a very different way for each copula. For the gaussian, the 99.9^{th} percentile is 1.5 higher than the 95^{th} percentile. However for the student-t with $\nu = 3$ it's more a multiply by a coefficient 6. And for the last one it's by a coefficient of 3.5. Between the gaussian copula and the student-t with $\nu = 3$, for the 95^{th} percentile there is a factor 3.5 but for the 99.9^{th} percentile there is a factor of 15. Those differences show that the default dependence distribution is very sensitive to the nature of dependence.

Finally, compute the sample correlation of the variables $\mathbf{1}_{X_1 \leq \pi}$ and $\mathbf{1}_{X_2 \leq \pi}$ for each copula, we find out:

Copula	Sample Default Correlation
Gaussian	0.0003
Student-t, $\nu = 3$	0.1170
Student-t, $\nu = 10$	0.0107

Tableau 2: Sample correlations of default pairs for each copula.

The confidence should be very small since when we repeat the simulation we don't have the same results. Indeed we plot a distribution for each copula of the correlation, and we saw that it's widely spread.

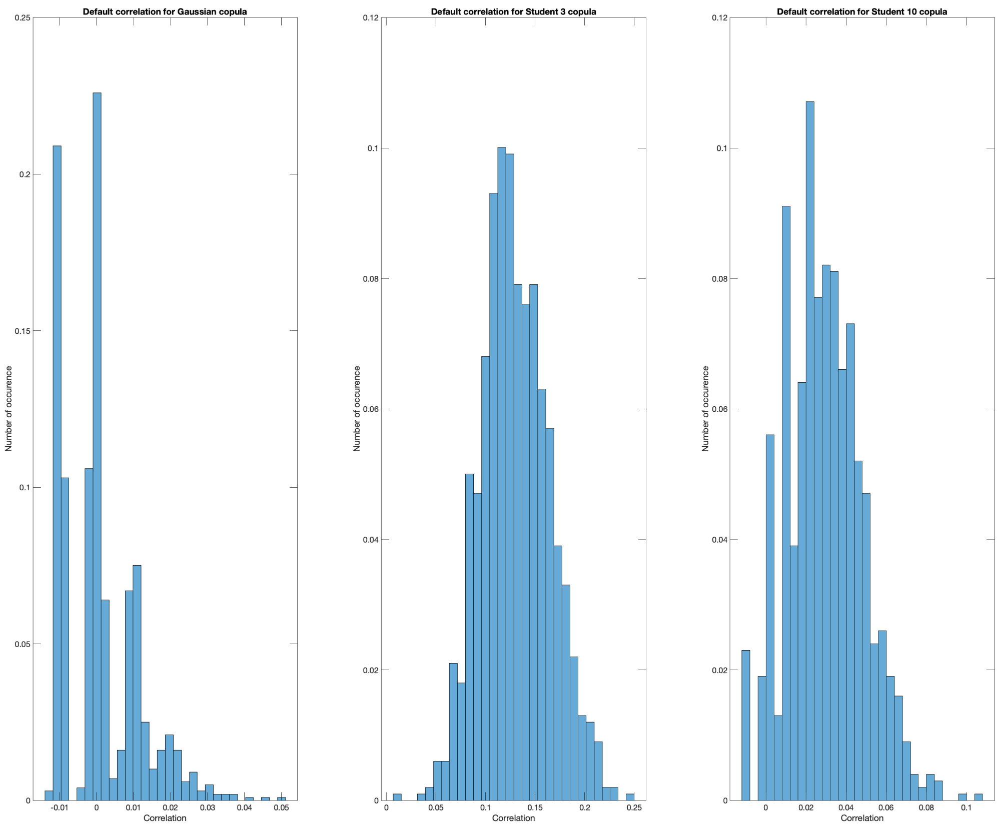

Figure 2: Distribution of sample correlations for each copula.

Question 2

1) To perform Monte Carlo we do it 1000 times and then we compute the expected shortfall.

$$ES_\alpha = \frac{1}{1-\alpha}\int_\alpha^1 VaR_u(L)du = \frac{1}{1-\alpha}\int_\alpha^1 F_L^{-1}(u)du$$

Where $F_L^{-1}(x) = arctanh(2x-1)$
Therefore, since we are interested only on the value over 0.99 we can approximate the expected

short fall by:

$$\hat{ES}_k = \frac{\sum_{i=1}^{1000} \mathbf{1}_{u_{i_k} \geq 0.99} F_L^{-1}\left(u_{i_k}\right)}{\left(\sum_{j=1}^{1000} \mathbf{1}_{u_{j_k} \geq 0.99}\right)}$$

Therefore the Monte-Carlo is:

$$MC\left(ES_\alpha\right) = \frac{1}{1000} \sum_{k=1}^{1000} \hat{ES}_k$$

The average of these estimates is 2.805 and the standard deviation is 0.167.

2) Now we have :

$$G_L^{-1}(x) = VaR_\alpha - \ln(1-x)$$

We use the same method as before but now we are taking all values of L which will be above VaR_α. We can now compute the likelihood ratio:

$$r(x) = \frac{f_L(x)}{g_L(x)}$$

Therefore, we can approximate the expected short fall with importance sampling by:

$$E\hat{S}^{IS}_{\alpha_k} = \frac{1}{1000(1-\alpha)} \sum_{i=1}^{1000} G_L^{-1}\left(u_{i_k}\right) r\left(G_L^{-1}\left(u_{i_k}\right)\right)$$

The average of these estimates is 2.801 and the standard deviation is 0.038.