Assignment_2 12.46.14 PM

October 8, 2019

[1]:
```
import numpy as np
import pandas as pd
import scipy.stats as stats
import matplotlib.pyplot as plt
```

1 Exercise 1

[2]:
```
S = np.array([100 , 50 , 25])
Q = np.array([1 , 3 , 5])
sig = np.log(S)
delta_sig_sd = np.array([1 , 2 , 3]) * 10**-3
N = 10**4
alpha = np.arange(.9,1,.01)
```

[3]:
```
col_names = ['t3_1' , 't10_1' , 't50_1' , 'norm_1' , 't3_2' , 't10_2' , 't50_2'␣
 ↪, 'norm_2' ,'t3_3' , 't10_3' , 't50_3' , 'norm_3']
df = pd.DataFrame(columns = col_names , index = range(N))
DoF = np.array([3 , 10 , 50])
for i in range(len(S)):
    scale_par = np.sqrt(DoF / (DoF - 2))/ delta_sig_sd[i]
    df[col_names[(i)*4]] = np.random.standard_t(DoF[0],N) / scale_par[0]
    df[col_names[(i)*4 +1]] = np.random.standard_t(DoF[1],N) / scale_par[1]
    df[col_names[(i)*4 +2]] = np.random.standard_t(DoF[2],N) / scale_par[2]
    df[col_names[(i)*4 +3]] = np.random.normal(0,delta_sig_sd[i],N)
```

[4]:
```
col_names2 = ['t3' , 't10' , 't50' , 'norm']
df2 = pd.DataFrame(columns = col_names2 , index = range(N))
df2['t3'] = -(S * Q * (np.exp(df[['t3_1','t3_2','t3_3']]) - 1)).sum(axis = 1)
df2['t10'] = -(S * Q*(np.exp(df[['t10_1','t10_2','t10_3']]) - 1)).sum(axis = 1)
df2['t50'] = -(S * Q*(np.exp(df[['t50_1','t50_2','t50_3']]) - 1)).sum(axis = 1)
df2['norm'] = -(S * Q*(np.exp(df[['norm_1','norm_2','norm_3']]) - 1)).sum(axis␣
 ↪= 1)
```

[5]:
```
df2
```

[5]:
```
            t3       t10       t50      norm
0     0.911191  0.314993  0.776305 -0.472226
```

```
1     0.061384 -0.148783 -0.445322  0.374284
2     0.040824  0.091908  0.196083 -0.669733
3    -0.352125 -0.287116 -0.103196 -0.376777
4     0.514142  0.424670 -0.182441  0.583193
...        ...       ...       ...       ...
9995 -0.706267  1.011115 -0.673325 -0.508916
9996  0.217687 -0.419483  0.633358  0.532454
9997 -0.173270  0.681698  0.591533 -0.091696
9998  0.760793  0.135042  0.594547 -0.082596
9999 -0.267970 -0.183098 -0.236401  0.670201

[10000 rows x 4 columns]
```

[6]:
```
df2.quantile(alpha)
```

[6]:
```
            t3       t10       t50      norm
0.90  0.502059  0.594705  0.620627  0.637345
0.91  0.531440  0.625773  0.649099  0.667323
0.92  0.566461  0.660457  0.680752  0.695948
0.93  0.602299  0.692509  0.718879  0.732331
0.94  0.645653  0.735445  0.753640  0.772068
0.95  0.698187  0.784473  0.793129  0.809305
0.96  0.760686  0.853577  0.853052  0.867344
0.97  0.858089  0.922974  0.918072  0.932555
0.98  0.993967  1.018249  1.010764  1.006069
0.99  1.281378  1.149354  1.111286  1.134843
```

[7]:
```
df2.quantile(alpha) - df2.mean()
```

[7]:
```
            t3       t10       t50      norm
0.90  0.500285  0.603726  0.621662  0.636216
0.91  0.529666  0.634793  0.650134  0.666194
0.92  0.564687  0.669477  0.681787  0.694819
0.93  0.600525  0.701529  0.719914  0.731202
0.94  0.643879  0.744465  0.754676  0.770939
0.95  0.696412  0.793494  0.794164  0.808176
0.96  0.758912  0.862597  0.854088  0.866215
0.97  0.856315  0.931994  0.919107  0.931426
0.98  0.992192  1.027269  1.011800  1.004940
0.99  1.279603  1.158374  1.112321  1.133714
```

[8]:
```
df_VaR = pd.DataFrame(columns = col_names2,index = alpha)
for i in range(len(alpha)):
    for j in range(len(col_names2)):
        #df_inter = df2[col_names2[j]].sort_values(ascending = False)[:
↪int(round(N * (alpha[i])))]
        #df_inter = df2.sort_values()[:int(round((N*(1-alpha[i]))))]
        df_VaR.at[alpha[i],col_names2[j]] = df2[col_names2[j]].
↪sort_values(ascending = False)[:int(round(N * (1 - alpha[i])))].mean()
```

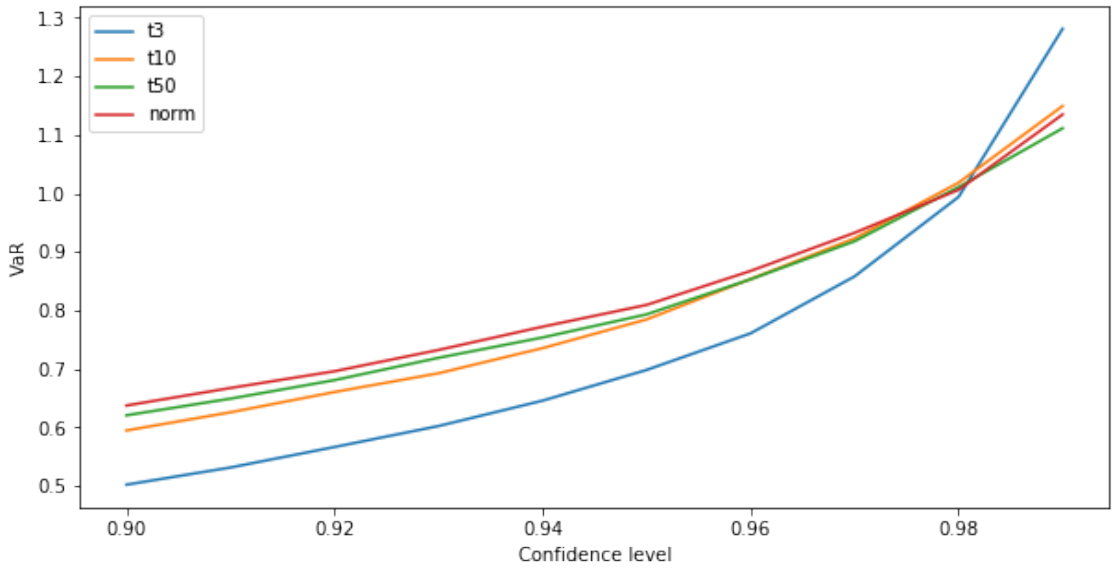
```
df_VaR
```

[8]:

	t3	t10	t50	norm
0.90	0.836826	0.852866	0.85408	0.866007
0.91	0.872615	0.879935	0.878316	0.889884
0.92	0.913045	0.909569	0.905129	0.915961
0.93	0.960175	0.942737	0.933849	0.944911
0.94	1.01599	0.980975	0.966941	0.977029
0.95	1.085	1.02558	1.00624	1.01407
0.96	1.17353	1.07744	1.05282	1.05904
0.97	1.29398	1.14034	1.10844	1.112
0.98	1.47937	1.22695	1.1799	1.18201
0.99	1.84503	1.37629	1.30409	1.30144

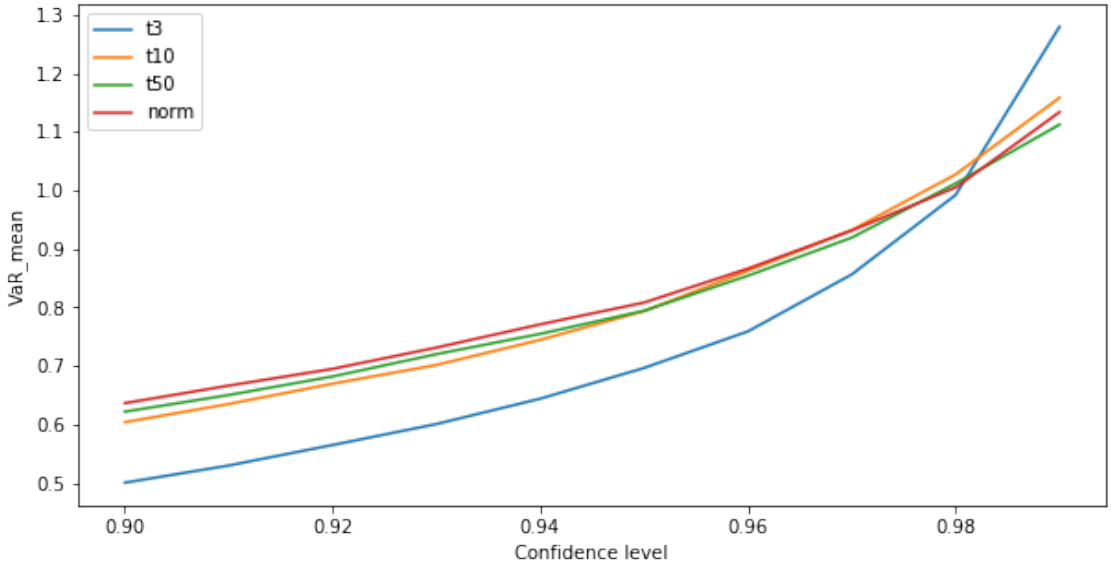
[9]:
```
#Plot VaR
plt.figure(figsize = (10,5))
plt.plot(df2.quantile(alpha))
plt.xlabel('Confidence level')
plt.ylabel('VaR')
plt.legend(df2.columns)
```

[9]:
```
<matplotlib.legend.Legend at 0x10b1bb650>
```



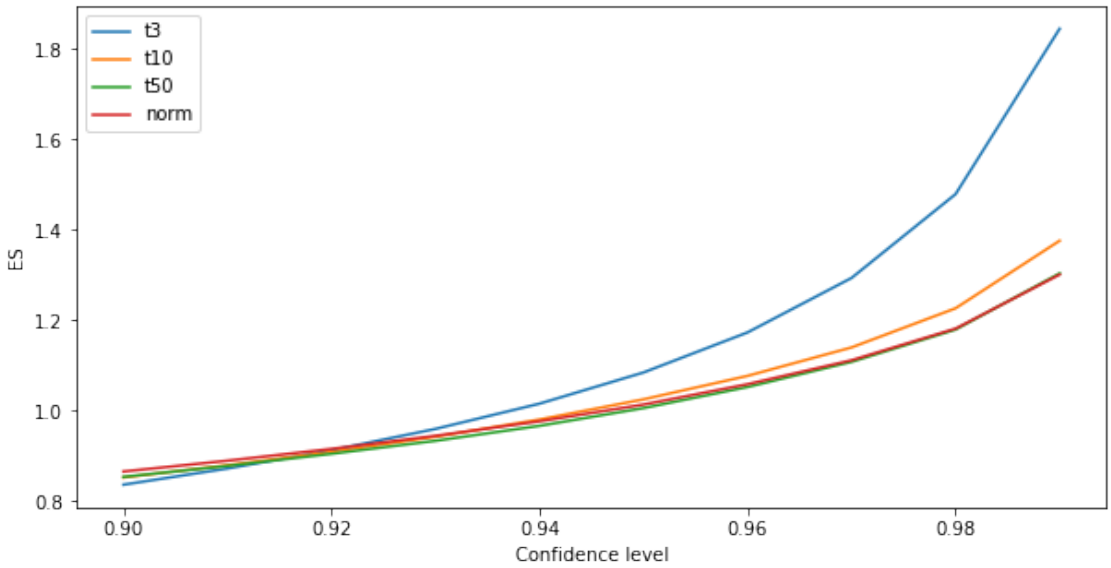
[10]:
```
#Plot VaR mean
plt.figure(figsize = (10,5))
plt.plot(df2.quantile(alpha) - df2.mean())
plt.xlabel('Confidence level')
plt.ylabel('VaR_mean')
plt.legend(df2.columns)
```

[10]: <matplotlib.legend.Legend at 0x10b25edd0>



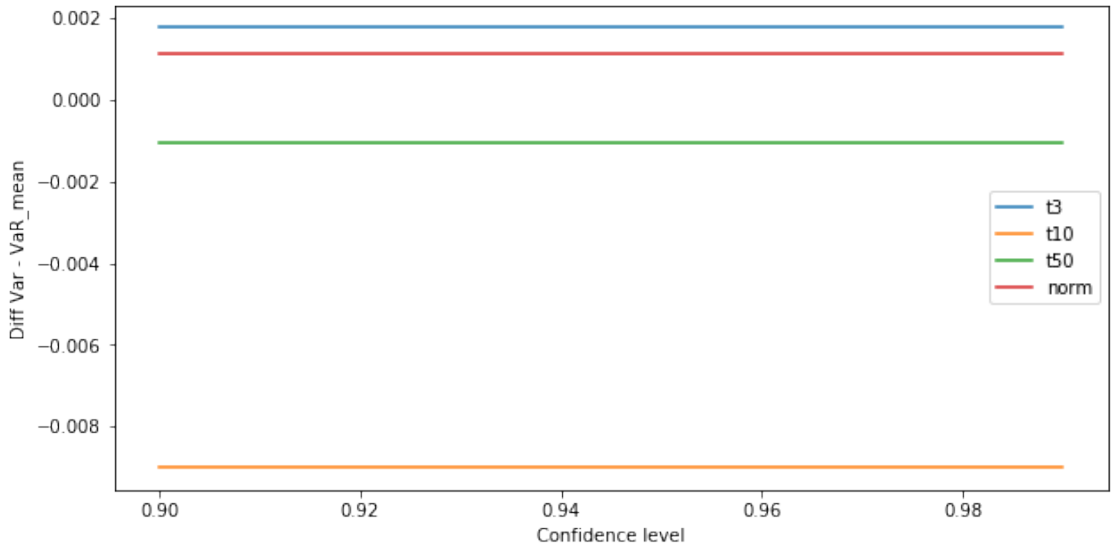
[11]:
```
#Plot ES
plt.figure(figsize = (10,5))
plt.plot(df_VaR)
plt.xlabel('Confidence level')
plt.ylabel('ES')
plt.legend(df_VaR)
```

[11]: <matplotlib.legend.Legend at 0x1277f9550>



[12]:
```
#Plot diff VaR and Var_mean
plt.figure(figsize = (10,5))
plt.plot(df2.quantile(alpha) - (df2.quantile(alpha) - df2.mean()))
plt.xlabel('Confidence level')
plt.ylabel('Diff Var - VaR_mean')
plt.legend(df2.columns)
```

[12]:
```
<matplotlib.legend.Legend at 0x125512090>
```



2 Exercise 2

[13]:
```
#Set parameters
T = 0.5
r = 0.05
sig = 0.2
S = 100
K = 100
dt = 1/252
q = 0
```

[14]:
```
alpha = np.array([0.95,0.99])
cov = np.array([[10**-6, -0.5 * 10**-7],[-0.5 * 10**-7, 10**-8]])
df = pd.DataFrame(np.random.multivariate_normal([0,0],cov,10**4),columns =␣
↪['X_1' , 'X_3'])
```

[15]:
```
def European_Call (S,K,T,q,r,sig):
    d1 = (np.log(S/K) + (r - q + sig**2/2) * T)/(sig * np.sqrt(T))
    d2 = d1 - sig * np.sqrt(T)
```

```
    return S * np.exp(-q * T) * stats.norm.cdf(d1) - K * np.exp(-r * T) * stats.
 ↪norm.cdf(d2)

def Theta(S,K,q,r,sig,delta):
    d1 = (np.log(S/K) + (r - q + sig**2 /2) * delta) / (sig * np.sqrt(delta))
    d2 = d1 - sig * np.sqrt(delta)
    return - np.exp(- q * delta) * S * stats.norm.pdf(d1) * sig / ( 2 * np.
 ↪sqrt(delta)) - r * K * np.exp(- r * delta) * stats.norm.cdf(d2) + q * S * np.
 ↪exp(-q * delta) * stats.norm.cdf(d1)

def Delta(S,K,q,r,sig,delta):
    d1 = (np.log(S/K) + (r - q + sig**2 /2) * delta) / (sig * np.sqrt(delta))
    return np.exp(-q*delta) * stats.norm.cdf(d1)

def Vega(S,K,q,r,sig,delta):
    d1 = (np.log(S/K) + (r - q + sig**2 /2) * delta) / (sig * np.sqrt(delta))
    return S * np.exp(-q * delta) * stats.norm.pdf(d1) * np.sqrt(delta)
```

[16]:
```
C0 = European_Call(S,K,T,q,r,sig)
df_real = pd.DataFrame(columns = ['Price' , 'Sig'])
df_real.Price = S * np.exp(df.X_1)
df_real.Sig = sig + df.X_3
```

[17]:
```
Val = pd.DataFrame(columns = ['Monte_Carlo' , 'Lin_Loss'])
```

[18]:
```
Val.Monte_Carlo = C0 - European_Call(df_real.Price, K , T , q , r , df_real.
 ↪Sig) - S + df_real.Price
```

[19]:
```
Val.Lin_Loss = - (Theta(S,K,q,r,sig,T) * dt + df.X_1 * S *(1 +␣
 ↪Delta(S,K,q,r,sig,T)) + df.X_3 * Vega(S,K,q,r,sig,T))
```

[20]:
```
VaR = pd.DataFrame(index = ['Monte_Carlo' , 'Lin_Loss' , 'VarCovar'],columns =␣
 ↪['VaR_0.05','VaR_mean_0.05','ES_0.05','VaR_0.01','VaR_mean_0.01','ES_0.01'])
```

[21]:
```
VaR.loc['Monte_Carlo'][0] = Val.Monte_Carlo.quantile(1 - alpha[0])
VaR.loc['Monte_Carlo'][1] = VaR.loc['Monte_Carlo'][0] - Val.Monte_Carlo.mean()
VaR.loc['Monte_Carlo'][2] = Val.Monte_Carlo.sort_values()[:int(round(N * (1 -␣
 ↪alpha[0])))].mean()
VaR.loc['Monte_Carlo'][3] = Val.Monte_Carlo.quantile(1 - alpha[1])
VaR.loc['Monte_Carlo'][4] = VaR.loc['Monte_Carlo'][3] - Val.Monte_Carlo.mean()
VaR.loc['Monte_Carlo'][5] = Val.Monte_Carlo.sort_values()[:int(round(N * (1 -␣
 ↪alpha[1])))].mean()
```

[22]:
```
VaR.loc['Lin_Loss'][0] = Val.Lin_Loss.quantile(1 - alpha[0])
VaR.loc['Lin_Loss'][1] = VaR.loc['Lin_Loss'][0] - Val.Lin_Loss.mean()
VaR.loc['Lin_Loss'][2] = Val.Lin_Loss.sort_values()[:int(round(N * (1 -␣
 ↪alpha[0])))].mean()
VaR.loc['Lin_Loss'][3] = Val.Lin_Loss.quantile(1 - alpha[1])
VaR.loc['Lin_Loss'][4] = VaR.loc['Lin_Loss'][3] - Val.Lin_Loss.mean()
```

```
VaR.loc['Lin_Loss'][5] = Val.Lin_Loss.sort_values()[:int(round(N * (1 -␣
 ↪alpha[1])))].mean()
```

[23]:
```
B = np.array([ (Delta(S,K,q,r,sig,T) + 1) * S , Vega(S,K,q,r,sig,T)])
VaR.loc['VarCovar'][0] = - Theta(S,K,q,r,sig,T) * dt + np.sqrt(np.matmul(B.
 ↪transpose(),np.matmul(B,cov))) * stats.norm.ppf(1 - alpha[0])
VaR.loc['VarCovar'][1] = VaR.loc['VarCovar'][0]
VaR.loc['VarCovar'][2] = - Theta(S,K,q,r,sig,T) * dt + np.sqrt(np.matmul(B.
 ↪transpose(),np.matmul(B,cov))) * stats.norm.pdf(stats.norm.ppf(1 -␣
 ↪alpha[0]))/alpha[0]
VaR.loc['VarCovar'][3] = - Theta(S,K,q,r,sig,T) * dt + np.sqrt(np.matmul(B.
 ↪transpose(),np.matmul(B,cov))) * stats.norm.ppf(1 - alpha[1])
VaR.loc['VarCovar'][4] = VaR.loc['VarCovar'][3]
VaR.loc['VarCovar'][5] = - Theta(S,K,q,r,sig,T) * dt + np.sqrt(np.matmul(B.
 ↪transpose(),np.matmul(B,cov))) * stats.norm.pdf(stats.norm.ppf(1 -␣
 ↪alpha[1]))/alpha[1]
```

[24]:
```
VaR
```

[24]:

	VaR_0.05	VaR_mean_0.05	ES_0.05	VaR_0.01	VaR_mean_0.01	\
Monte_Carlo	-0.0686048	-0.0689248	-0.0862567	-0.0977952	-0.0981152	
Lin_Loss	-0.227941	-0.258499	-0.289419	-0.331039	-0.361596	
VarCovar	-0.228377	-0.228377	0.0494053	-0.336341	-0.336341	

	ES_0.01
Monte_Carlo	-0.112361
Lin_Loss	-0.375782
VarCovar	0.0364712