FIN-417 - Quantitative Risk Management
Assignment 1

Question 0: Register for a WRDS account as soon as possible if you have not done so already: https://wrds-web.wharton.upenn.edu/wrds/

Question 1: This question refers to Example 1 from the lecture slides.

1. Take $d = 1$, and $\lambda_1 = 1$. Suppose $X_{1,t+\Delta}$ has mean zero and standard deviation 0.01, and $S_t = 100$. For each of the following distributions of $X_{1,t+\Delta}$, simulate 10,000 realizations of $L(t, t+\Delta)$ and plot the empirical distribution. Then compute the mean and standard deviation of $L(t, t+\Delta)$. Find the normal probability density function corresponding to this mean and standard deviation and plot it over the empirical distribution.

   (a) $X_{1,t+\Delta}$ is a scaled Student's t-distribution with 3 degrees of freedom

   (b) $X_{1,t+\Delta}$ is a scaled Student's t-distribution with 10 degrees of freedom

   (c) $X_{1,t+\Delta}$ is a scaled Student's t-distribution with 50 degrees of freedom

   (d) $X_{1,t+\Delta}$ has a normal distribution

   By "$X_{1,t+\Delta}$ is a scaled Student's t-distribution with ν degrees of freedom" we mean that $\alpha X_{1,t+\Delta}$ has the Student's t-distribution with ν degrees of freedom for some appropriate $\alpha \in \mathbb{R}$. You must find the appropriate value of α so that $X_{1,t+\Delta}$ has the correct standard deviation. The Matlab functions `randn` and `trnd` will be useful for this question.

   Which of the resulting distributions of $L(t, t+\Delta)$ above are normal distributions? How do you know?

2. For each of the distributions from the previous part, state the exact probability distribution of $L^\delta(t, t+\Delta)$.

Question 2: This question refers to Example 2 from the lecture slides.

1. Take $S_t = 100$, $r_t = 0.05$, and $\sigma_t = 0.2$. Suppose that $X_{1,t+\Delta}$ has a normal distribution with mean zero and standard deviation 0.01, $X_{2,t+\Delta}$ has a normal distribution with mean zero and standard deviation 10^{-4}, and $X_{3,t+\Delta}$ has a normal distribution with mean zero and standard deviation 10^{-3}. Further, $X_{2,t+\Delta}$ is independent from the other two risk factor changes, but $X_{1,t+\Delta}$ and $X_{3,t+\Delta}$ have correlation -0.5. The Matlab function `mvnrnd` may be useful for this question.

   Let $T = 1$, $K = 100$, and $\Delta = 1/252$.

   Simulate 10,000 realizations of $L(t, t+\Delta)$ and plot the empirical distribution. Note that there is positive probability for $r_{t+\Delta}$ and $\sigma_{t+\Delta}$ to be negative with the distributions they are assigned. State whether you think this is a problem and why, and how you circumvent it if necessary.

2. For the same distributions as the previous part, simulate 10,000 realizations of $L^\delta(t, t+\Delta)$ and plot the empirical distribution. Which of the three risk factors seems to contribute most to $L^\delta(t, t+\Delta)$, and how did you decide this?

Question 3: Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Derive the formula:

$$\mathbb{E}[e^X] = e^{\mu + \frac{1}{2}\sigma^2}$$

Question 4: Read "An Academic Response to Basel II" and Chapter 1 of Quantitative Risk Management.