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# QRM assignment 8

## Question 2

Let $X \sim \mathcal{N}(0,1)$ and $Y = ZX$ with $Z \perp X$

The marginal distribution of $(X, Y)$ is $\mathcal{N}(0,1)$ so we can write

$$C(v_1, v_2) = P(F(X) \leq v_1 \wedge F(Y) \leq v_2)$$

$$= P([F(X) \leq v_1 \cap F(X) \leq v_2] \cap Z = 1) + P([F(X) \leq v_1 \cap F(-X) \leq v_2] \cap Z = -1)$$

$Z \perp X$

$$= P(F(X) \leq v_1 \cap F(X) \leq v_2) P(Z=1) + P(F(X) \leq v_1 \cap F(-X) \leq v_2) P(Z=1)$$

$$= p\, P(F(X) \leq v_1 \wedge F(X) \leq v_2) + (1-p) P(F(X) \leq v_1 \cap F(-X) \leq v_2)$$

$$= \underline{p \min(v_1, v_2) + (1-p) \max(v_1 + v_2 - 1, 0)}$$

## Question 3

By the law of total probability we have.

$$P(U_1 \leq u_1, \dots, U_d \leq u_d) = \int_0^{+\infty} P(U_1 \leq u_1, \dots, U_d \leq u_d \mid V = v)\, dG(v)$$

$U_1 \dots U_d$ are conditional indep so we have

$$= \int_0^{+\infty} \prod_{i=1}^{d} F_{U_i|V}(u_i, v)\, dG(v)$$

$$= \int_0^{+\infty} \prod_{i=1}^{d} \exp(-v \hat{G}^{-1}(u_i))\, dG(v)$$

$$= \int_0^{+\infty} \exp(-v[\hat{G}^{-1}(u_1) + \dots \hat{G}^{-1}(u_d)])\, dG(v)$$

So we have $P(U_1 \leq u_1, \dots, U_d \leq u_d) = \hat{G}\left[\hat{G}^{-1}(u_1) + \dots + \hat{G}^{-1}(u_d)\right]$
By definition of the Laplace - Stieltjes transform of $G$.

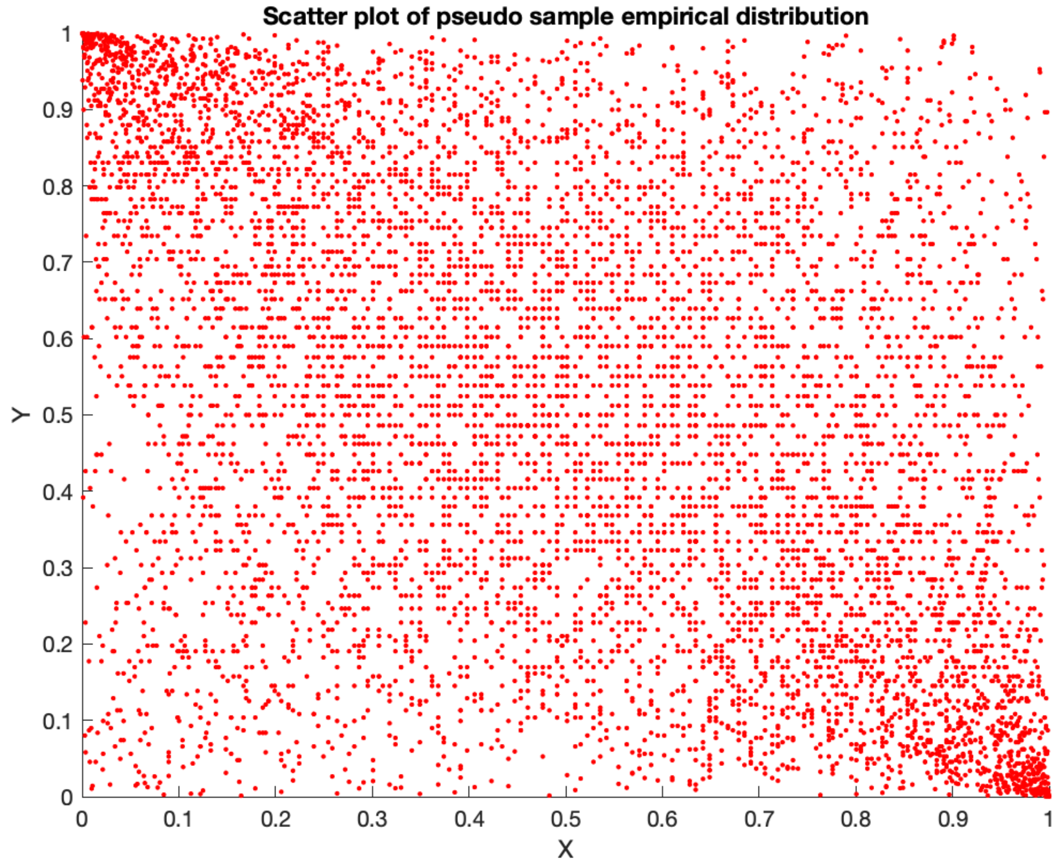


**Figure 1 : Pseudo-sample of empirical distribution between X (Mkt-Rf) and Y (SMB)**

| Distribution | Parameter value | ML value |
|---|---|---|
| Clayton | 2.45e-09 | 2.18e-05 |
| Gumbel | 1.00e+00 | -3.21e-04 |
| Frank | -3.13e+00 | 8.25e+02 |

**Figure 2 : Table of results of MLE for different bivariate distributions**

On the figure 2, we observe that the Frank copula is the one maximizing the maximum likelihood function by far with an optimal $\theta = -3.13$. Then it is more likely that data have been generated from Frank Copula.