Quantitative Risk Management
Sample Midterm Exam

October 21, 2019

Question 1: Suppose $X = \mu + \sqrt{W}Z$ where $Z \sim \mathcal{N}(0,1)$ is independent of W. W is a positive random variable such that $W \in \{k_1, \dots, k_n\}$ with:

$$\mathbb{P}(W = k_i) = p_i$$

Construct a function of a variable v such that a root of the function v_0 satisfies $v_0 = VaR_\alpha(X)$. Argue that the function you construct has a unique root. (Start by writing down the CDF of X and breaking it up into different terms corresponding to different values of W.)

Question 2: Construct two random variables with zero correlation that are not independent. Prove that they satisfy these requirements.

Question 3: Let $X_1, \dots, X_n$ be independent and identically distributed normal random variables. Let $\hat{\mu}$ and $\hat{\sigma}^2$ be the sample mean and variance. Suppose that the true mean and variance of the X_i's are μ and σ^2. Compute the expected value of $\hat{\mu}$ and $\hat{\sigma}^2$.

Question 4:
Part a) A portfolio consists of three stocks with values $A_0 = 20$ CHF, $B_0 = 10$ CHF, and $C_0 = 50$ CHF. The vector of one day returns denoted $(X_A, X_B, X_C)^T$ is multivariate normal with mean zero and the following covariance matrix:

$$\begin{bmatrix} 0.0020 & 0.0010 & 0.0010 \\ 0.0010 & 0.0030 & 0.0010 \\ 0.0010 & 0.0010 & 0.0040 \end{bmatrix}$$

i) Compute the expected value of the loss after one day.

ii) Write an expression for the one day linearized loss in terms of A_0, B_0, C_0, X_A, X_B, and X_C. For the linearized loss, compute VaR_α and ES_α with $\alpha = 0.95$ and the expected value of the linearized loss. The following may be useful:

$$\Phi^{-1}(0.95) = 1.64\,, \qquad \frac{1}{1-0.95}\int_{\Phi^{-1}(0.95)}^{\infty} x\phi(x)dx = 2.06\,.$$

Part b) A portfolio consists of a single stock with value $S_0 = 100$ USD. Denote the USD-CHF exchange rate by F, where $F_0 = 1$, so that the value of the stock to a Swiss institution is FS. As usual, the risk factor change corresponding to the stock is the return, denoted X_S. Similarly, the risk factor change corresponding to the exchange rate is $X_F = \log(F_1/F_0)$. The risk factor changes are jointly normal with mean zero and the following covariance matrix:

$$\begin{bmatrix} \sigma_S^2 & \rho\sigma_S\sigma_F \\ \rho\sigma_S\sigma_F & \sigma_F^2 \end{bmatrix} = \begin{bmatrix} 0.0020 & 0.0005 \\ 0.0005 & 0.0005 \end{bmatrix}$$

Write an expression for the one day linearized loss in terms of S_0, F_0, X_S, and X_F. Compute VaR_α and ES_α of the linearlized loss for $\alpha = 0.95$.

Question 5:

Part a) Suppose you have access to computer software that can generate a standard uniform variable. How can you use this to generate a standard normal variable?

Part b) Suppose you have access to computer software that can generate independent standard normal variables. How can you use this to generate a multivariate normal random vector with covariance matrix $\mathbf{\Sigma}$ where $\mathbf{\Sigma}$ is a symmetric positive definite matrix?

Question 6:

Part a) Let $L \sim Exp(\lambda)$. Compute $VaR_\alpha(L)$ and $ES_\alpha(L)$.

Part b) Show that the expressions you find for $ES_\alpha(L)$ and $VaR_\alpha(L)$ tend to be the same for large values of α (hint: take the limit of their ratio when $\alpha \to 1$).