Quantitative Risk Management
Assignment 8

Due: November 26, 2019

Question 1: Download the spreadsheet posted on the course website. This is Fama-French data on international portfolios from http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/
Take the first two columns of the data. Transform this data to get a pseudo-sample of the copula by assuming the marginal distributions are the empirical distributions of this data. Show a scatter plot of the pseudo-sample and perform a maximum likelihood estimation for the Gumbel, Clayton, and Frank copulas on the pseudo-sample. Based on the results of your fits, which family of copulas do you think the original data was generated from?

You may find the Matlab function `copulapdf` useful, and you will likely have to use `fmincon` again (but not for the Frank copula). You may not use the function `copulafit` to solve this question.

Question 2: Let $X \sim \mathcal{N}(0,1)$, and let $Y = ZX$ where Z is independent of X with $Z = 1$ with probability p and $Z = -1$ with probability $1 - p$. Find the copula of (X, Y).

Question 3: Let G be a distribution function on $\mathbb{R}^+$ with $G(0) = 0$ and let $\hat{G}$ be the Laplace-Stieltjes transform of G, extended such that $\hat{G}(\infty) = 0$. Let V have distribution G, and let $U_1, \ldots, U_d$ be conditionally independent given V with conditional distribution:

$$F_{U_i|V}(u_i; v) = e^{-v\hat{G}^{-1}(u_i)}, \quad u_i \in [0,1]$$

Show that:

$$\mathbb{P}(U_1 \leq u_1, \ldots, U_d \leq u_d) = \hat{G}(\hat{G}^{-1}(u_1) + \cdots + \hat{G}^{-1}(u_d))\,.$$