Quantitative Risk Management
Assignment 4

Due: October 22, 2019

Question 1: Suppose $X = \mu + \sqrt{W}Z$ where $Z \sim \mathcal{N}(0,1)$ is independent of W. W is a positive random variable such that $W \in \{k_1, \ldots, k_n\}$ with:

$$\mathbb{P}(W = k_i) = p_i$$

Construct a function of a variable v such that a root of the function v_0 satisfies $v_0 = VaR_\alpha(X)$. Argue that the function you construct has a unique root. (Start by writing down the CDF of X and breaking it up into different terms corresponding to different values of W.)

Question 2: Construct two random variables with zero correlation that are not independent. Prove that they satisfy these requirements.

Question 3: Download 5 years of historical prices of the following companies: International Business Machines Corporation (IBM), McDonald's Corp. (MCD), 3M Company (MMM), and Wal-Mart Stores Inc. (WMT). Also download 5 years of data of the SNP500 index (this can be found by entering the Stock Market Indexes menu on WRDS and selecting the variable "Level on S&P Composite Index"). All data should start from March 17, 2011.

1. Let $\boldsymbol{X}$ represent log returns of each of the stocks, and let F represent log returns of the S&P index. Perform a regression analysis to estimate a 1-factor model and find $\hat{\boldsymbol{a}}$ and $\hat{\mathcal{B}}$:

$$\boldsymbol{X} = \boldsymbol{a} + \boldsymbol{B}F + \boldsymbol{\epsilon}$$

2. Construct the matrix of residual errors $\hat{\mathcal{E}} = \mathcal{X} - \mathcal{F}\hat{\mathcal{B}}$ and compute the sample correlation matrix of these errors. Compare this to the sample correlation matrix of the original returns $\mathcal{X}$ and comment on the results (write down both matrices).

Question 4: Copy the following code into the beginning of a Matlab script:

```
rng(1);
N = 10000;
A = [ 1  0 0 0;
      1  1 0 0;
     -1  2 3 0;
      1 -1 1 1];
x = trnd(5,N,4);
X = (A*x')';
```

This code will generate an array, `X`, which consists of 10,000 rows and 4 columns. Each row represents a single data point observation of a 4-dimensional random vector. In this problem, assume the loss of a portfolio is equal to $L = \sum_{k=1}^{4} X_k$.

1. Based off of the 10,000 observations of $\boldsymbol{X}$, compute $VaR_\alpha(L)$ for $\alpha = 0.95$.

2. What is the eigenvector corresponding to the first principal component of $\boldsymbol{X}$? Can you find a link between the magnitude of some component of this vector and some component in the covariance matrix of $\boldsymbol{X}$? Which of the four components of $\boldsymbol{X}$ you would expect to contribute most to the 1st principle component?

3. Approximate $\boldsymbol{X}$ by using its first two principal components as factors (set the error terms to zero). Write down the steps you take and then recompute $VaR_\alpha(L)$ and compare with the previous result.

The following Matlab functions may be useful for this problem: `cov`, `eig`, `pca`. Be sure to read the documentation on these functions before using them. Matlab may use different conventions for eigenvalue ordering depending on the context.