MATH 471 - Quantitative Risk Management
2016 Final Exam

NAME:____

Question 1: [10 marks]

Part a)
Let $X \sim \mathcal{N}(0, \sigma^2)$ and suppose you use a standard Monte Carlo simulation to estimate $\theta = \mathbb{E}[(X-K)_+]$. When the number of samples used is large, what fraction of the total samples will make the integrand evaluate to 0?

Part b)
Suppose an importance sampling method is used with the likelihood ratio chosen to be:

$$r(x) = e^{\frac{-2xK+K^2}{2\sigma^2}}.$$

When the importance sampling Monte Carlo simulations are performed, from what distribution is the variable X sampled? With this method, what is the modified integrand, and what percentage of the samples will make the integrand evaluate to 0?

Part c)
Compute the exact value of θ for $\sigma = 1$ and $K = 1$. You may find the following quantities useful:

$$\int_1^\infty x \exp\left\{-\frac{x^2}{2}\right\} dx = 0.6065 \qquad \Phi(1) = 0.8413$$

Question 2: [10 marks]

Let $\tau_1, \ldots, \tau_n$ denote the default times of n reference entities. The default intensities are constant with values $\lambda_1, \ldots, \lambda_n$ so that each default time has an exponential distribution.

Part a)
A 1^{st}-to-default swap is written on reference entities 1 and 2. Compute the fair spread of the swap assuming the default times are i) independent and ii) comonotonic.

Part b)
A 1^{st}-to-default swap is written on all n reference entities. Compute the fair spread of the swap assuming the default times are independent.

Part c)
A k^{th}-to-default swap is written on all n reference entities. Compute the fair spread of the swap assuming the default times are comonotonic.

Question 3: [10 marks]

Part a)
A portfolio consists of three stocks with values $A_0 = 20$ CHF, $B_0 = 10$ CHF, and $C_0 = 50$ CHF. The vector of one day returns denoted $(X_A, X_B, X_C)^T$ is multivariate normal with mean zero and the following covariance matrix:

$$\begin{bmatrix} 0.0020 & 0.0010 & 0.0010 \\ 0.0010 & 0.0030 & 0.0010 \\ 0.0010 & 0.0010 & 0.0040 \end{bmatrix}$$

i) Compute the expected value of the loss after one day.

ii) Write an expression for the one day linearized loss in terms of A_0, B_0, C_0, X_A, X_B, and X_C. For the linearized loss, compute VaR_α and ES_α with $\alpha = 0.95$ and the expected value of the linearized loss. The following may be useful:

$$\Phi^{-1}(0.95) = 1.64 \qquad \frac{1}{1-0.95}\int_{\Phi^{-1}(0.95)}^{\infty} x\phi(x)dx = 2.06$$

Part b)
A portfolio consists of a single stock with value $S_0 = 100$ USD. Denote the USD-CHF exchange rate by F, where $F_0 = 1$, so that the value of the stock to a Swiss institution is FS. As usual, the risk factor change corresponding to the stock is the return, denoted X_S. Similarly, the risk factor change corresponding to the exchange rate is $X_F = \log(F_1/F_0)$. The risk factor changes are jointly normal with mean zero and the following covariance matrix:

$$\begin{bmatrix} \sigma_S^2 & \rho\sigma_S\sigma_F \\ \rho\sigma_S\sigma_F & \sigma_F^2 \end{bmatrix} = \begin{bmatrix} 0.0020 & 0.0005 \\ 0.0005 & 0.0005 \end{bmatrix}$$

Write an expression for the one day linearized loss in terms of S_0, F_0, X_S, and X_F. Compute VaR_α and ES_α of the linearlized loss for $\alpha = 0.95$.

Question 4: [10 marks]

Part a)
Let $L \sim Exp(\lambda)$. Compute $VaR_\alpha(L)$ and $ES_\alpha(L)$.

Part b)
Let $X_i \sim F$ be a sequence of i.i.d. variables each having exponential distribution with parameter λ. Show that $F \in MDA(H_\xi)$ for some value of ξ.

Hint: recall that $F \in MDA(H_\xi)$ means there are sequences of constants a_n and b_n such that:

$$\lim_{n\to\infty} \mathbb{P}\left(\frac{M_n - a_n}{b_n} \le x\right) = H_\xi(x)$$

It may help to choose the sequence $a_n = \frac{\log(n)}{\lambda}$, and you should already know which value of ξ is valid.

Part c)
Let $X \sim F$ be exponentially distributed with parameter λ. Compute the excess distribution function over threshold u, $F_u(x)$.

Question 5: [10 marks]

Part a)
You hold a single non-defaultable bond with notional 100 CHF and maturity $T = 1$ year. The yield of the bond today is $y_0(T) = 0.05$. You model the yield tomorrow as $y_\Delta(T) = y_0(T) + X$ where X is the risk-factor change, and $\Delta = 1$ day $= \frac{T}{252}$. You do not make any distributional assumption on X, but you observe historical yields and compute the corresponding risk-factor changes over the last 50 days. The risk-factor changes which you observed, sorted from lowest to highest, are given below:

−0.019654	−0.018651	−0.015094	−0.014817	−0.013677
−0.012701	−0.011714	−0.011096	−0.010511	−0.010443
−0.008519	−0.008456	−0.007585	−0.006856	−0.006490
−0.006358	−0.005727	−0.005587	−0.005352	−0.004908
−0.004174	−0.003456	−0.003222	−0.002925	−0.002752
−0.002428	−0.001969	−0.001551	0.000660	0.001668
0.001784	0.004513	0.005864	0.005907	0.006033
0.006037	0.006121	0.007884	0.008003	0.008759
0.009262	0.009287	0.011752	0.011812	0.012708
0.014022	0.017737	0.017813	0.017972	0.020292

Using the technique of a historical simulation, compute VaR_α and ES_α of your portfolio for $\alpha = 0.95$. Do not neglect the change in time to maturity when computing the change in value of the bond. Thoroughly explain your methodology.

Note: To simplify this question, do not use the strict definition of ES_α. Instead you may compute $ES_\alpha = \mathbb{E}[L|L \geq VaR_\alpha]$.

Part b)
Repeat the exercise from Part a), but now the bond also has a probability of defaulting over the next day equal to 0.03. The default of the bond is independent of the change in yield and the recovery is zero.

Question 6: [each correct answer: 2 marks - each incorrect answer: -1 mark]

Part a) According to the readings, what is the "Formula that Destroyed Wall Street?"

- **A)** VaR_α for a Gaussian distribution
- **B)** CDS spread for constant default intensity
- **C)** The Gaussian copula function
- **D)** The Black-Scholes formula

Part b) The quantity VaR_α is not a coherent risk measure because it fails to satisfy:

- **A)** translation invariance
- **B)** subadditivity
- **C)** positive homogeneity
- **D)** monotonicity

Part c) In lecture and the first chapter of "Quantitative Risk Management", we introduce numerous challenges associated with the task. This topic is not identified as one of the challenges:

- **A)** Interdisciplinarity
- **B)** Modelling of extreme events
- **C)** Implementing legal and regulatory requirements
- **D)** Interdependence and concentration of risks

Part d) In "An Academic Response to Basel II", the authors believe the following criterion is not sufficiently addressed by the Basel Committee for Banking Supervision:

- **A)** Inconsistencies between the accord and financial laws in developing countries
- **B)** Endogeneity of risk and its destabilisation effects during crisis events
- **C)** Computational infeasibility of the proposed methods
- **D)** Model risk

Part e) In the article "Is GARCH(1,1) as good a model as the Nobel prize accolades would imply?" the conclusions of the author include:

- **A)** Rejection of the hypothesis that GARCH(1,1) is the data generating process for the S&P 500
- **B)** GARCH(1,1) shows no significant differences from a non-stationary non-parametric regression approach to next-day volatility forecasting
- **C)** Long term volatility forecasts using GARCH(1,1) have significantly larger errors than forecasts using a historical volatility approach
- **D)** All of the above