MATH 471 - Quantitative Risk Management
Assignment 10

Due: December 10, 2019

Question 1: Consider a portfolio consisting of $N = 1000$ bonds. Let $L = \sum_{i=1}^{N} \mathbb{1}_{X_i \leq \pi}$ represent the number of bonds that has defaulted, where $X_i \leq \pi$ is the event that bond i defaults. Each X_i is a standard uniform so that π is the probability that any single bond defaults. Perform a Monte Carlo simulation with 10,000 samples and compute the 95^{th}, 99^{th}, and 99.9^{th} percentile of L for each of the following copulas of $\boldsymbol{X}$:

1. Gaussian copula with pairwise correlation of ρ.

2. Student-t copula with pairwise correlation of ρ and 3 degrees of freedom.

3. Student-t copula with pairwise correlation of ρ and 10 degrees of freedom.

In each case, use $\pi = 0.01$ and $\rho = 0.01$. In each case, on separate figures, plot the empirical distribution of the number of defaults. On each of these plots, also plot the distribution that would occur if all defaults were independent. Finally, compute the sample correlation of the variables $\mathbb{1}_{X_1 \leq \pi}$ and $\mathbb{1}_{X_2 \leq \pi}$ for each copula structure. How confident are you that these sample correlations are close to the true correlations of these variables?

Comment on the resulting quantiles for each distribution.

Question 2: Let L have distribution given by the function:

$$F_L(x) = \frac{1}{2}(\tanh(x) + 1),$$

so that the density is given by

$$f_L(x) = \frac{2}{(e^x + e^{-x})^2}.$$

The VaR_α for this loss distribution is the constant $F_L^{-1}(\alpha)$. You will perform a sequence of Monte Carlo simulations to compute an estimator of $ES_\alpha(L)$, as well as the variance of the estimator. In each Monte Carlo simulation, use 1000 trials. Obtain a distribution of $ES_\alpha(L)$ estimates by performing 1000 Monte Carlo simulations. The simulations are to be done using two different methods:

1. Standard Monte Carlo

2. Monte Carlo with importance sampling where the alternative distribution for L is given by:

$$G_L(x) = \begin{cases} 1 - e^{-(x - VaR_\alpha)} & x \geq VaR_\alpha \\ 0 & x < VaR_\alpha \end{cases}$$

   that is, L has a shifted exponential distribution with scale parameter 1.

Report the mean value of all $ES_\alpha(L)$ estimates using both methods, as well as the standard deviation of these estimates. Use $\alpha = 0.99$.

Question 3: Read the article "The Formula that Destroyed Wall Street" posted on the course website.