Quantitative Risk Management
Assignment 2

Due: October 8, 2019

Question 1: This question deals with a portfolio of three stocks. At time t, the values of the stocks are $S_{1,t} = 100$, $S_{2,t} = 50$, and $S_{3,t} = 25$. The portfolio consists of 1 share of S_1, 3 shares of S_2, and 5 shares of S_3. The risk factors are logarithmic prices and the risk factor changes have mean zero and standard deviations 10^{-3}, $2 \cdot 10^{-3}$, and $3 \cdot 10^{-3}$ respectively. The risk factors are independent.

1. Compute VaR_α, VaR_α^{mean}, and ES_α using Monte Carlo with 10,000 simulations. Do this for $\alpha = \{0.90, 0.91, \ldots, 0.99\}$. Also use the following distributions for the risk factor changes:

   (a) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(3, \mu, \sigma)$ for appropriate values of μ and σ

   (b) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(10, \mu, \sigma)$ for appropriate values of μ and σ

   (c) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta} \sim t(50, \mu, \sigma)$ for appropriate values of μ and σ

   (d) For each $i \in \{1, 2, 3\}$, $X_{i,t+\Delta}$ has a normal distribution

   and plot the results.

2. Comment on the following:

   (a) The value of VaR_α compared to VaR_α^{mean}.

   (b) The value of VaR_α and ES_α as compared between the four distributions. Are the results what you expected?

Question 2: This question deals with a delta hedged call option. The following are the Black-Scholes parameters for a European call option at time $t = 0$:

$$T = 0.5$$
$$r_t = 0.05$$
$$\sigma_t = 0.2$$
$$S_t = 100$$
$$K = 100$$

The portfolio consists of a long position in the call option, and the corresponding position in the stock which makes the portfolio delta neutral. Let $\Delta = 1$day, $Z_1 = \log(S)$, and $Z_2 = \sigma$ (r will be considered unchanging in this problem). The risk factor changes are normally distributed with mean zero. Their standard deviations over one day are 10^{-3} and 10^{-4}, and their correlation is -0.5.

1. Compute VaR_α, VaR_α^{mean} and ES_α for $\alpha = 0.95$ and $\alpha = 0.99$ using the following methods:

   (a) Monte Carlo full revaluation with 10,000 simulations

   (b) Monte Carlo on the linearlized loss with 10,000 simulations

(c) Variance-covariance method

Do not neglect the time derivative in any linearization in this question.

Question 3: Let L have the Student t distribution with ν degrees of freedom. Derive the formula:

$$ES_\alpha(L) = \left(\frac{g_\nu(t_\nu^{-1}(\alpha))}{1 - \alpha}\right)\left(\frac{\nu + (t_\nu^{-1}(\alpha))^2}{\nu - 1}\right)$$

You will need to look up the probability density function of the distribution at hand.