

Formulae, TEP4100 Fluidmekanikk

All equation numbers refer to Cengel & Cimbala "Fluid Dynamics" 3rd edition. In a few cases the formula is slightly modified compared to the book.

Ideal gas law (2-4) :

$$P = \rho RT; \quad R_{\text{air}} = 287 \text{ Pa m}^3/\text{kg K}$$

Reynolds' number:

$$\text{Re} = \frac{\text{Velocity scale} \cdot \text{Length scale}}{\nu}$$

Kinematic viscosity

$$\nu = \frac{\mu}{\rho}.$$

Shear stress (2-33)

$$\tau_{xy} = \mu \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right).$$

Surface tension (soap bubble) (2-41)

$$\Delta P = \frac{4\sigma_s}{R}$$

Hydrostatic pressure distribution (3-9)

$$\frac{dP}{dz} = -\rho g.$$

Hydrostatic force on plane submerged surface (3-19)

$$F_R = (P_0 + \rho g h_C) A = P_C A.$$

Centre of pressure (3-22 a,b)

$$y_P = y_C + \frac{I_{xx,C}}{[y_C + P_0/(\rho g \sin \theta)] A}; \quad y_P = y_C + \frac{I_{xx,C}}{y_C A}.$$

Pressure distribution in rigid body motion

$$\vec{\nabla} P = \rho \vec{g}_{\text{eff}}.$$

where

$$\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$$

or

$$\vec{g}_{\text{eff}} = \vec{g} + \omega^2 r \vec{e}_r.$$

Acceleration (4-9, 4-11) (Cartesian)

$$\frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + (\vec{V} \cdot \nabla) \vec{V} = \frac{\partial \vec{V}}{\partial t} + u \frac{\partial \vec{V}}{\partial x} + v \frac{\partial \vec{V}}{\partial y} + w \frac{\partial \vec{V}}{\partial z}.$$

Along a streamline (4-15)

$$\frac{dr}{V} = \frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}.$$

Vorticity (4-28,29)

$$\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V}.$$

Reynolds transport theorem (4-41)

$$\frac{d}{dt} B_{\text{sys}} = \frac{d}{dt} \left(\int_{\text{CV}} \rho b \, dV \right) + \oint_{\text{CS}} \rho b (\vec{V}_r \cdot \vec{n}) \, dA.$$

Volume flow rate through cross section A_c (5-8)

$$Q = \dot{V} = \int_{A_c} \vec{V}_r \cdot \vec{n} \, dA.$$

Conservation of mass (5-17)

$$\frac{d}{dt} \int_{\text{CV}} \rho \, dV + \oint_{\text{CS}} \rho (\vec{V}_r \cdot \vec{n}) \, dA = 0.$$

Bernoulli equation along streamline, unsteady compressible flow (5-44)

$$\int \frac{dP}{\rho} + \int \frac{\partial V}{\partial t} ds + \frac{V^2}{2} + gz = \text{constant}.$$

Bernoulli equation along streamline, steady incompressible flow (5-48)

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}.$$

Energy equation (5-60)

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{d}{dt} \int_{\text{CV}} e \rho \, dV + \oint_{\text{CS}} \left(\frac{P}{\rho} + e \right) \rho (\vec{V}_r \cdot \vec{n}) \, dA,$$

where total energy per unit mass is (5-50)

$$e = u + \frac{V^2}{2} + gz.$$

Energy equation for steady flow with one inlet and one outlet (5-77)

$$\frac{P_1}{\rho_1 g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho_2 g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine,e}} + h_L.$$

Kinetic energy correction factor for pipe flow

$$\alpha \approx \begin{cases} 2, & \text{laminar} \\ 1.05, & \text{turbulent} \end{cases}$$

Linear momentum equation (6-16)

$$\sum \vec{F} = \frac{d}{dt} \left(\int_{\text{CV}} \rho \vec{V} \, dV \right) + \oint_{\text{CS}} \rho \vec{V} (\vec{V}_r \cdot \vec{n}) \, dA.$$

Net pressure force on closed CS

$$\vec{F}_{\text{press}} = - \oint_{\text{CS}} P_{\text{gage}} \vec{n} \, dA$$

Critical Reynolds number, pipe flow (p.350)

$$\text{Re}_{\text{crit}} \approx 2300.$$

Entry length (8-6,7)

$$\begin{array}{ll} \text{laminar} & \frac{L_{h, \text{laminar}}}{D} \approx 0.05 \text{Re}, \\ \text{turbulent} & \frac{L_{h, \text{turbulent}}}{D} \approx 1.359 \text{Re}^{1/4}. \end{array}$$

Darcy friction factor for laminar pipe flow (8-23)

$$f = \frac{64}{\text{Re}}.$$

Pipe head loss (8-24)

$$h_L = f \frac{L}{D} \frac{V^2}{2g}.$$

Colebrook's formula (8-50)

$$\frac{1}{\sqrt{f}} = -2.0 \log \left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}} \right].$$

Haaland's formula (8-51)

$$\frac{1}{\sqrt{f}} \simeq -1.8 \log \left[\frac{6.9}{\text{Re}} + \left(\frac{\varepsilon/D}{3.7} \right)^{1.11} \right].$$

Total head loss (8-58)

$$h_{L, \text{ total}} = h_{L, \text{ major}} + h_{L, \text{ minor}} = \sum f \frac{L}{D} \frac{V^2}{2g} + \sum K_L \frac{V^2}{2g}$$

Continuity equation (9-5)

$$\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot (\rho \vec{V}) = 0.$$

Continuity equation in cylindrical coordinates (9-12)

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u_r)}{\partial r} + \frac{1}{r} \frac{\partial (\rho u_\theta)}{\partial \theta} + \frac{\partial (\rho u_z)}{\partial z} = 0.$$

Incompressible continuity equation (9-16)

$$\vec{\nabla} \cdot \vec{V} = 0.$$

Incompressible stream function ψ (9-20,27,29)

$$(\text{Cartesian}) \quad u = \frac{\partial \psi}{\partial y}; \quad v = -\frac{\partial \psi}{\partial x};$$

$$(\text{Cylindrical}) \quad u_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}; \quad u_\theta = -\frac{\partial \psi}{\partial r};$$

$$(\text{Axisymmetric flow}) \quad u_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}; \quad u_z = \frac{1}{r} \frac{\partial \psi}{\partial r}.$$

Incompressible Navier-Stokes equation (9-60)

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho (\vec{V} \cdot \vec{\nabla}) \vec{V} = -\vec{\nabla} P + \rho \vec{g} + \mu \nabla^2 \vec{V}.$$

2D irrotational flow (10-28,30)

$$\nabla^2 \psi = 0.$$

Plane flows: (10-36,37,43,46,50,51)

$$(\text{uniform stream}) \quad \psi = Vy$$

$$(\text{source/sink}) \quad \psi = \frac{\dot{V}/L}{2\pi} \theta$$

$$(\text{line vortex}) \quad \psi = -\frac{\Gamma}{2\pi} \ln r$$

$$(\text{doublet}) \quad \psi = -K \frac{\sin \theta}{r}$$

Displacement thickness (10-72)

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U} \right) dy.$$

Flat plate boundary layer thickness (Table 10-4, p.574)

$$\frac{\delta}{x} \approx \begin{cases} \frac{4.91}{\text{Re}_x^{1/2}}, & \text{laminar} \\ \frac{0.16}{\text{Re}_x^{1/7}}, & \text{turbulent} \end{cases}$$

Local skin friction coefficient (10-98)

$$C_{f,x} = \frac{\tau_w(x)}{\frac{1}{2} \rho U^2}.$$

Drag and lift coefficients (11-5,6)

$$C_D = \frac{F_D}{\frac{1}{2} \rho V^2 A}; \quad C_L = \frac{F_L}{\frac{1}{2} \rho V^2 A}.$$

Equations in cylindrical coordinates

Vorticity vector:

$$\vec{\zeta} = \left(\frac{1}{r} \frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z} \right) \vec{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r} \right) \vec{e}_\theta + \frac{1}{r} \left(\frac{\partial(r u_\theta)}{\partial r} - \frac{\partial u_r}{\partial \theta} \right) \vec{e}_z$$

Incompressible continuity equation:

$$\frac{1}{r} \frac{\partial(r u_r)}{\partial r} + \frac{1}{r} \frac{\partial(u_\theta)}{\partial \theta} + \frac{\partial(u_z)}{\partial z} = 0$$

r-component of the incompressible Navier-Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_r}{\partial t} + u_r \frac{\partial u_r}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_r}{\partial \theta} - \frac{u_\theta^2}{r} + u_z \frac{\partial u_r}{\partial z} \right) \\ &= -\frac{\partial P}{\partial r} + \rho g_r + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_r}{\partial r} \right) - \frac{u_r}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_r}{\partial \theta^2} - \frac{2}{r^2} \frac{\partial u_\theta}{\partial \theta} + \frac{\partial^2 u_r}{\partial z^2} \right] \end{aligned}$$

θ -component of the incompressible Navier-Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_\theta}{\partial t} + u_r \frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r u_\theta}{r} + u_z \frac{\partial u_\theta}{\partial z} \right) \\ &= -\frac{1}{r} \frac{\partial P}{\partial \theta} + \rho g_\theta + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_\theta}{\partial r} \right) - \frac{u_\theta}{r^2} + \frac{1}{r^2} \frac{\partial^2 u_\theta}{\partial \theta^2} + \frac{2}{r^2} \frac{\partial u_r}{\partial \theta} + \frac{\partial^2 u_\theta}{\partial z^2} \right] \end{aligned}$$

z-component of the incompressible Navier-Stokes equation:

$$\begin{aligned} & \rho \left(\frac{\partial u_z}{\partial t} + u_r \frac{\partial u_z}{\partial r} + \frac{u_\theta}{r} \frac{\partial u_z}{\partial \theta} + u_z \frac{\partial u_z}{\partial z} \right) \\ &= -\frac{\partial P}{\partial z} + \rho g_z + \mu \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_z}{\partial r} \right) + \frac{1}{r^2} \frac{\partial^2 u_z}{\partial \theta^2} + \frac{\partial^2 u_z}{\partial z^2} \right] \end{aligned}$$

Viscous stress tensor:

$$\begin{pmatrix} \tau_{rr} & \tau_{r\theta} & \tau_{rz} \\ \tau_{\theta r} & \tau_{\theta\theta} & \tau_{\theta z} \\ \tau_{zr} & \tau_{z\theta} & \tau_{zz} \end{pmatrix} = \begin{pmatrix} 2\mu \frac{\partial u_r}{\partial r} & \mu \left[r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] & \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) \\ \mu \left[r \frac{\partial}{\partial r} \left(\frac{u_\theta}{r} \right) + \frac{1}{r} \frac{\partial u_r}{\partial \theta} \right] & 2\mu \left(\frac{1}{r} \frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r} \right) & \mu \left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right) \\ \mu \left(\frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r} \right) & \mu \left(\frac{\partial u_\theta}{\partial z} + \frac{1}{r} \frac{\partial u_z}{\partial \theta} \right) & 2\mu \frac{\partial u_z}{\partial z} \end{pmatrix}$$