- Ang. massebevarelse
- Ang. energibevarelse
- Ang. Newton II (impulsbevarelse)
- Kinematikk og geometri
- Andre definisjoner
- Empiriske observasjoner etc

Formulae, TEP4100 Fluidmekanikk

All equation numbers refer to Cengel & Cimbala "Fluid Dynamics" 3rd edition. In a few cases the formula is slightly modified compared to the book.

Ideal gas law (2-4) :

$$P = \rho R T; \quad R_{\text{air}} = 287\,{}^{\text{Pa m}^3}/_{\text{kg K}}$$

Reynolds' number:

$$\text{Re} = \frac{\text{Velocity scale} \cdot \text{Length scale}}{\nu}.$$

Kinematic viscosity

$$\nu = \frac{\mu}{\rho}.$$

Shear stress (2-33)

$$\tau_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right).$$

Surface tension (soap bubble) (2-41)

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$$\Delta P = \frac{4\sigma_s}{R}$$

Hydrostatic pressure distribution (3-9)

$$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g.$$

Hydrostatic force on plane submerged surface (3-19)

$$F_{\text{R}} = (P_0 + \rho g h_{\text{C}})A = P_{\text{C}} A.$$

Centre of pressure (3-22 a,b)

$$y_{\text{P}} = y_{\text{C}} + \frac{I_{xx,\text{C}}}{[y_{\text{C}} + P_0/(\rho g \sin\theta)]\,A}; \quad y_{\text{P}} = y_{\text{C}} + \frac{I_{xx,\text{C}}}{y_{\text{C}} A}.$$

Pressure distribution in rigid body motion

$$\vec{\nabla} P = \rho \vec{g}_{\text{eff}}.$$

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where

$$\vec{g}_{\text{eff}} = \vec{g} - \vec{a}$$

or

$$\vec{g}_{\text{eff}} = \vec{g} + \omega^2 r \vec{e}_r$$

Acceleration (4-9, 4-11) (Cartesian)

$$\frac{\mathrm{D}\vec{V}}{\mathrm{D}t} = \frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V} = \frac{\partial \vec{V}}{\partial t} + u\frac{\partial \vec{V}}{\partial x} + v\frac{\partial \vec{V}}{\partial y} + w\frac{\partial \vec{V}}{\partial z}.$$

Along a streamline (4-15)

$$\frac{\mathrm{d}r}{V} = \frac{\mathrm{d}x}{u} = \frac{\mathrm{d}y}{v} = \frac{\mathrm{d}z}{w}.$$

Vorticity (4-28,29)

$$\vec{\zeta} = 2\vec{\omega} = \vec{\nabla} \times \vec{V}.$$

Reynolds transport theorem (4-41)

$$\frac{\mathrm{d}}{\mathrm{d}t} B_{\text{sys}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\text{CV}} \rho b \,\mathrm{d}\mathcal{V}\right) + \oint_{\text{CS}} \rho b \left(\vec{V}_{\text{r}} \cdot \vec{n}\right) \mathrm{d}A.$$

Volume flow rate through cross section A_c (5-8)

$$Q = \dot{\mathcal{V}} = \int_{A_c} \vec{V}_{\text{r}} \cdot \vec{n} \,\mathrm{d}A.$$

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Conservation of mass (5-17)

$$\frac{\mathrm{d}}{\mathrm{d}t}\int_{\text{CV}} \rho \,\mathrm{d}\mathcal{V} + \oint_{\text{CS}} \rho\left(\vec{V}_{\text{r}} \cdot \vec{n}\right) \mathrm{d}A = 0.$$

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Bernoulli equation along streamline, unsteady compressible flow (5-44)

$$\int \frac{\mathrm{d}P}{\rho} + \int \frac{\partial V}{\partial t}\mathrm{d}s + \frac{V^2}{2} + gz = \text{constant}.$$

Bernoulli equation along streamline, steady incompressible flow (5-48)

$$\frac{P}{\rho g} + \frac{V^2}{2g} + z = \text{constant}.$$

Energy equation (5-60)

$$\dot{Q}_{\text{net in}} + \dot{W}_{\text{shaft, net in}} = \frac{\mathrm{d}}{\mathrm{d}t}\int_{\text{CV}} e\rho \,\mathrm{d}\mathcal{V} + \oint_{\text{CS}} \left(\frac{P}{\rho} + e\right)\rho\left(\vec{V}_{\text{r}} \cdot \vec{n}\right)\mathrm{d}A,$$

where total energy per unit mass is (5-50)

$$e = u + \frac{V^2}{2} + gz.$$

ENERGIBEVARELSE

Energy equation for steady flow with one inlet and one outlet (5-77)

$$\frac{P_1}{\rho_1 g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump,u}} = \frac{P_2}{\rho_2 g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine,e}} + h_{\text{L}}.$$

Kinetic energy correction factor for pipe flow

$$\alpha \approx \begin{cases} 2, & \text{laminar} \\ 1.05, & \text{turbulent} \end{cases}$$

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Linear momentum equation (6-16)

$$\sum \vec{F} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{\text{CV}} \rho \vec{V} \,\mathrm{d}\mathcal{V}\right) + \oint_{\text{CS}} \rho \vec{V}\left(\vec{V}_{\text{r}} \cdot \vec{n}\right) \mathrm{d}A.$$

Net pressure force on closed CS

$$\vec{F}_{\text{press}} = -\oint_{\text{CS}} P_{\text{gage}}\, \vec{n} \,\mathrm{d}A$$

Critical Reynolds number, pipe flow (p.350)

$$\text{Re}_{\text{crit}} \approx 2300.$$

Entry length (8-6,7)

laminar $\quad \dfrac{L_{h,\text{ laminar}}}{D} \approx 0.05\text{Re},$

turbulent $\quad \dfrac{L_{h,\text{ turbulent}}}{D} \approx 1.359\text{Re}^{1/4}.$

Darcy friction factor for laminar pipe flow (8-23)

$$f = \frac{64}{\mathrm{Re}}.$$

Pipe head loss (8-24)

$$h_\mathrm{L} = f\frac{L}{D}\frac{V^2}{2g}.$$

Colebrook's formula (8-50)

$$\frac{1}{\sqrt{f}} = -2.0\log\left[\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right].$$

Haaland's formula (8-51)

$$\frac{1}{\sqrt{f}} \simeq -1.8\log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right].$$

Total head loss (8-58)

$$h_{\mathrm{L,\ total}} = h_{\mathrm{L,\ major}} + h_{\mathrm{L,\ minor}} = \sum f\frac{L}{D}\frac{V^2}{2g} + \sum K_\mathrm{L}\frac{V^2}{2g}$$

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Continuity equation (9-5)

$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot\left(\rho\vec{V}\right) = 0.$$

Continuity equation in cylindrical coordinates (9-12)

$$\frac{\partial\rho}{\partial t} + \frac{1}{r}\frac{\partial(r\rho u_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho u_\theta)}{\partial\theta} + \frac{\partial(\rho u_z)}{\partial z} = 0.$$

Incompressible continuity equation (9-16)

$$\vec{\nabla}\cdot\vec{V} = 0.$$

Incompressible stream function ψ (9-20,27,29)

$$\text{(Cartesian)}\ u = \frac{\partial\psi}{\partial y};\quad v = -\frac{\partial\psi}{\partial x};$$

$$\text{(Cylindrical)}\ u_r = \frac{1}{r}\frac{\partial\psi}{\partial\theta};\quad u_\theta = -\frac{\partial\psi}{\partial r};$$

$$\text{(Axisymmetric flow)}\ u_r = -\frac{1}{r}\frac{\partial\psi}{\partial z};\quad u_z = \frac{1}{r}\frac{\partial\psi}{\partial r}.$$

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Incompressible Navier-Stokes equation (9-60)

$$\rho\frac{\partial\vec{V}}{\partial t} + \rho\left(\vec{V}\cdot\vec{\nabla}\right)\vec{V} = -\vec{\nabla}P + \rho\vec{g} + \mu\nabla^2\vec{V}.$$

2D irrotational flow (10-28,30)

$$\nabla^2\psi = 0.$$

Plane flows: (10-36,37,43,46,50,51)

$$\begin{aligned}
&\text{(uniform stream)} && \psi = Vy\\
&\text{(source/sink)} && \psi = \frac{\dot{\mathcal{V}}/L}{2\pi}\theta\\
&\text{(line vortex)} && \psi = -\frac{\Gamma}{2\pi}\ln r\\
&\text{(doublet)} && \psi = -K\frac{\sin\theta}{r}
\end{aligned}$$

Displacement thickness (10-72)

$$\delta^* = \int_0^\infty\left(1 - \frac{u}{U}\right)dy.$$

Flat plate boundary layer thickness (Table 10-4, p.574)

$$\frac{\delta}{x} \approx \begin{cases}\frac{4.91}{\mathrm{Re}_x^{1/2}}, & \text{laminar}\\[4pt] \frac{0.16}{\mathrm{Re}_x^{1/7}}, & \text{turbulent}\end{cases}$$

Local skin friction coefficient (10-98)

$$C_{\mathrm{f,x}} = \frac{\tau_\mathrm{w}(x)}{\frac{1}{2}\rho U^2}.$$

Drag and lift coefficients (11-5,6)

$$C_\mathrm{D} = \frac{F_\mathrm{D}}{\frac{1}{2}\rho V^2 A};\quad C_\mathrm{L} = \frac{F_\mathrm{L}}{\frac{1}{2}\rho V^2 A}.$$