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Assignment 1

① $\log_2 2048 = \log_{10} 2048 / \log_{10} 2$

$\approx 3.311 / 0.3$

≈ 11.036

$\boxed{= 11}$

② $\sum_{n=1}^{k} 2k+1 = \sum_{n=1}^{k} 2k + \sum_{n=1}^{k} 1 = 2(1+2+3+\ldots k) + k = \cancel{2} \cdot \frac{k(k+1)}{\cancel{2}} + k = \boxed{k^2+2k}$

$\sum_{n=1}^{k} i = \frac{n(n+1)}{2}$

③ $n^3 > 2^n$ for $n >= 1$

for $n = 1$, $(1)^3 > 2^1 \Rightarrow 1 \not> 2$

$\boxed{\text{false by proof by counterexample}}$

④ Assume n even, but n^2 odd

Since $2k$ = even, $n = 2k$, $n^2 = (2k)^2 = 4k^2$

$4k^2$ is not odd which contradicts our assumption that n^2 is odd, proven by contradiction

⑤ a) base case: $\sum^{1}(1)^3 = 1^2 \cdot (1+1)^2 / 4 = 1^2 \cdot \frac{4}{4}$

$1 = 1$ ✓

Induction: Assume $\sum^{k} k^2 = k^2 \cdot \frac{(k+1)^2}{4}$

$\sum^{k+1} k^2 = k^2 \cdot \frac{(k+1)^2}{4} + (k+1)^3 = (k+1)^2 \cdot \frac{(k+1+1)^2}{4}$

$\frac{k^2 \cdot (k+1)^2}{4} + \frac{4(k+1)^3}{4} = \frac{k^2+2k+1 \cdot k^2+4k+4}{4}$

$\frac{k^2 \cdot (k^2+2k+1)}{4} + \frac{4 \cdot (k^3+3k^2+3k+1)}{4} = \frac{k^2+2k+1 \cdot k^2+4k+4}{4}$

$\frac{k^4+2k^3+k^2+4k^3+12k^2+12k+4}{4} = \frac{k^4+4k^3+4k^2+2k^3+8k^2+8k+k^2+4k+4}{4}$

$\boxed{\text{Equal so proof by induction}}$

b) base case: $(1)^2 - 1$ is even

0 is even ✓

induction: Assume $(k)^2 - k$ is even

$\Rightarrow (k+1)^2 - (k+1) = k^2+2k+1-k-1$

$\Rightarrow k^2 - k$

$\boxed{k^2 - k \text{ is even so proof by induction}}$