Assignment

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Question ST 31.2023
Two defective bulbs are present in a set of five bulbs. To remove the two defective bulbs, the bulbs are chosen randomly one by one and tested. If X denotes the minimum number of bulbs that must be tested to find out the two defective bulbs, then $\Pr(X = 3)$ (rounded off to two decimal places) equals
Solution:

RV	Values	Description
A	0	1^{st} Bulb defective
	1	1^{st} Bulb non-defective
B	0	2^{nd} Bulb defective
	1	2^{nd} Bulb non-defective
C	0	3^{rd} Bulb defective
	1	3^{rd} Bulb non-defective

TABLE I
RANDOM VARIABLE DECLARATION.

Here, the word ”minimum” does not signify anything. Therefore we get

$$\begin{aligned} p_X(2) &= p_{AB}(0,0) && (1)\\ &= \frac{2}{5} \times \frac{1}{4} && (2)\\ &= \frac{1}{10} && (3)\\ p_X(3) &= p_{ABC}(1,0,0) + p_{ABC}(0,1,0) + p_{ABC}(1,1,1) && (4)\\ &= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} && (5)\\ &= \frac{3}{10} && (6)\\ p_X(4) &= p_{ABC}(0,1,1) + p_{ABC}(1,0,1) + p_{ABC}(1,1,0) && (7)\\ &= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} && (8)\\ &= \frac{6}{10} && (9) \end{aligned}$$

Hence, The pmf of X is

$$p_X(k) = \begin{cases} 0 & k = 1 \\ \frac{1}{10} & k = 2 \\ \frac{3}{10} & k = 3 \\ \frac{6}{10} & k = 4 \\ 1 & k = 5 \end{cases} \tag{10}$$