# Assignment

Antalene (EE22BTECH11008)

Question 9.3.9
The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

1) none
2) not more than one
3) more than one
4) at least one

will fuse after 150 days of use.
**Solution:**
**Guassian :**
Let $X$ be a binomial Random variable with parameters $n$ and $p$ such that

$$n = 5 \tag{1}$$
$$p = 0.05 \tag{2}$$

The Mean and Varience of $X$ are

$$\mu = n \times p \tag{3}$$
$$= 0.25 \tag{4}$$
$$\sigma^2 = n \times p \times (1 - p) \tag{5}$$
$$= 0.2375 \tag{6}$$

Let $Z$ be a random variable with $\mu = 0$ and $\sigma^2 = 1$

$$Z = \frac{X - \mu + 0.5}{\sigma} \tag{7}$$

0.5 is correctional term
We can calculate the distribution of Z by assuming it be a set of discrete points on the Normal-Distribution $f(x)$

$$f(x) = \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{8}$$
$$\tag{9}$$

The Q-function from the Normal-Distribution

$$\Pr(Z > x) = Q(x) \tag{10}$$
$$= \int_x^\infty \frac{1}{\sqrt{2\pi}} \times e^{-\frac{x^2}{2}} \tag{11}$$

and for finding $\Pr\left(Z = \frac{X-\mu}{\sigma}\right)$ Using approximation,

$$\Pr\left(Z = \frac{X - \mu}{\sigma}\right) \approx \Pr\left(\frac{X + 0.5 - \mu}{\sigma} < Z < \frac{X - 0.5 - \mu}{\sigma}\right) \tag{12}$$
$$\approx \Pr\left(Z < \frac{X + 0.5 - \mu}{\sigma}\right) - \Pr\left(Z < \frac{X - 0.5 - \mu}{\sigma}\right) \tag{13}$$
$$\approx Q\left(\frac{X - 0.5 - \mu}{\sigma}\right) - Q\left(\frac{X + 0.5 - \mu}{\sigma}\right) \tag{14}$$

**Binomial :**

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \tag{15}$$
$$= {}^5C_k (0.05)^k (0.95)^{5-k} \tag{16}$$

CDF of X

$$F_X(k) = \Pr(X \le k) \tag{17}$$
$$= \sum_{i=0}^{k} {}^{10}C_i (0.05)^i (0.95)^{5-i} \tag{18}$$

1) none defective
   a) By Gaussian,
   $$\Pr(X = 0) = Q(-0.5298) \tag{19}$$
   $$\approx 0.6960 \tag{20}$$
   b) By Binomial
   $$\Pr(X = 0) = {}^5C_0 (0.05)^0 (0.95)^5 \tag{21}$$
   $$= 0.773 \tag{22}$$
2) not more than one defective
   a) By Gaussian,
   $$\Pr(X \le 1) \approx Q(1.5896) \tag{23}$$
   $$\approx 0.9948 \tag{24}$$
   b) By Binomial
   $$\Pr(X \le 1) = \sum_{k=0}^{1} {}^5C_k (0.05)^k (0.95)^{5-k} \tag{25}$$
   $$= 0.9774075 \tag{26}$$
3) more than one defective
   a) By Gaussian,
   $$\Pr(X > 1) = 1 - \Pr(X \le 1) \tag{27}$$
   $$= 1 - Q(1.5896) \tag{28}$$
   $$= 1 - 0.994 \tag{29}$$
   $$= 0.006 \tag{30}$$
   b) By Binomial
   $$\Pr(X > 1) = 1 - \Pr(X \le 1) \tag{31}$$
   $$= 1 - 0.9774075 \tag{32}$$
   $$= 0.0226 \tag{33}$$

4) atleast one defective

a) By Gaussian,

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{34}$$
$$= 1 - Q(-0.5298) \tag{35}$$
$$= 1 - 0.6960 \tag{36}$$
$$= 0.304 \tag{37}$$

b) By Binomial

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{38}$$
$$= 1 - 0.773 \tag{39}$$
$$= 0.227 \tag{40}$$

The graph

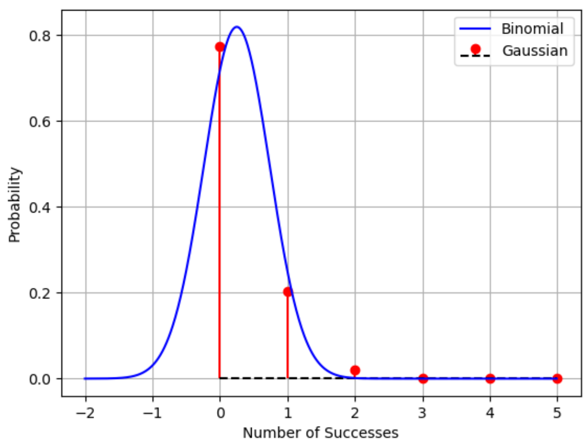


Fig. 1. Binomial vs guassian