Assignment

Antalene (EE22BTECH11008)

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Two defective bulbs are present in a set of five bulbs. To remove the two defective bulbs, the bulbs are chosen randomly one by one and tested. If X denotes the minimum number of bulbs that must be tested to find out the two defective bulbs, then $\Pr(X = 3)$ (rounded off to two decimal places) equals

Solution: THerefore,

RV	Values	Description
A	0	1st Bulb defective
	1	1st Bulb non-defective
B	0	2nd Bulb defective
	1	2nd Bulb non-defective
C	0	3rd Bulb defective
	1	3rd Bulb non-defective

TABLE I

RANDOM VARIABLE DECLARATION.

$$p_X(3) = p_{ABC}(1, 0, 0) + p_{ABC}(0, 1, 0) + p_{ABC}(1, 1, 1) \tag{1}$$

$$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \tag{2}$$

$$= \frac{3}{10} \tag{3}$$

similarly,

$$p_X(2) = p_{AB}(0, 0) \tag{4}$$

$$= \frac{2}{5} \times \frac{1}{4} \tag{5}$$

$$= \frac{1}{10} \tag{6}$$

$$p_X(4) = p_{ABC}(0, 1, 1) \times \frac{1}{2} + p_{ABC}(1, 0, 1) \times \frac{1}{2} + p_{ABC}(1, 1, 0) \times \frac{1}{2} \tag{7}$$

$$= \frac{2}{5} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{2} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} + \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \tag{8}$$

$$= \frac{3}{10} \tag{9}$$

Hence, The pmf of X is

$$p_X(k) = \begin{cases} 0 & k = 0, 1 \\ \frac{1}{10} & k = 2 \\ \frac{3}{10} & k = 3, 4 \\ 1 & k = 5 \end{cases} \tag{10}$$