Assignment

Antalene (EE22BTECH11008)

Question 9.3.9

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

1) none
2) not more than one
3) more than one
4) at least one

will fuse after 150 days of use.

Solution:

Guassian :

Parameter	Value	Description
X	0,1,2,3,4,5	No. Of bulbs fused
n	5	Total no. Of bulbs
p	0.05	bulb fusing
q	0.95	not fusing
$\mu = np$	0.25	Mean of Binomial Distribution
$\sigma^2 = npq$	0.2375	Varience of binomial Distribution

TABLE I
RANDOM VARIABLE AND PARAMETER

Let Y be gaussain variable and normal distribution at $X = x$

$$\Pr(X \leq x) = Q(x) \tag{1}$$
$$= F_Y(x) \tag{2}$$
$$= \int_x^\infty \frac{1}{\sqrt{2\pi}} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \tag{3}$$
$$= \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \tag{4}$$

Binomial :

$$\Pr(X = k) = {}^nC_k p^k (1-p)^{n-k} \tag{5}$$
$$= {}^5C_k (0.05)^k (0.95)^{5-k} \tag{6}$$

CDF of X

$$F_X(k) = \Pr(X \leq k) \tag{7}$$
$$= \sum_{i=0}^k {}^{10}C_i (0.05)^i (0.95)^{5-i} \tag{8}$$

1) none defective

   a) By Gaussian,

$$\Pr(X = 0) = Q(-0.5298) \tag{9}$$
$$\approx 0.6960 \tag{10}$$

   b) By Binomial

$$\Pr(X = 0) = {}^5C_0 (0.05)^0 (0.95)^5 \tag{11}$$
$$= 0.773 \tag{12}$$

2) not more than one defective

   a) By Gaussian,

$$\Pr(X \leq 1) \approx Q(1.5896) \tag{13}$$
$$\approx 0.9948 \tag{14}$$

   b) By Binomial

$$\Pr(X \leq 1) = \sum_{k=0}^1 {}^5C_k (0.05)^k (0.95)^{5-k} \tag{15}$$
$$= 0.9774075 \tag{16}$$

3) more than one defective

   a) By Gaussian,

$$\Pr(X > 1) = 1 - \Pr(X \leq 1) \tag{17}$$
$$= 1 - Q(1.5896) \tag{18}$$
$$= 1 - 0.994 \tag{19}$$
$$= 0.006 \tag{20}$$

   b) By Binomial

$$\Pr(X > 1) = 1 - \Pr(X \leq 1) \tag{21}$$
$$= 1 - 0.9774075 \tag{22}$$
$$= 0.0226 \tag{23}$$

4) atleast one defective

   a) By Gaussian,

$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{24}$$
$$= 1 - Q(-0.5298) \tag{25}$$
$$= 1 - 0.6960 \tag{26}$$
$$= 0.304 \tag{27}$$

   b) By Binomial

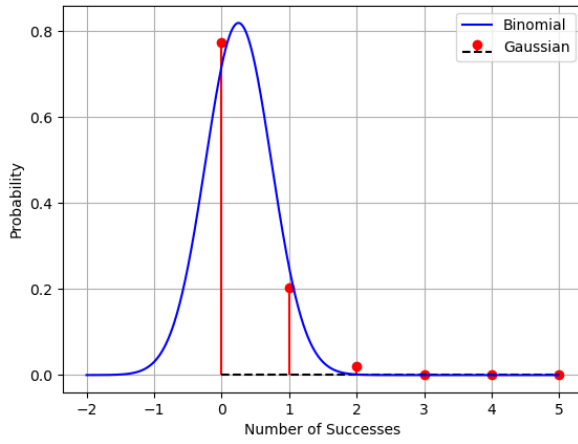$$\Pr(X \geq 1) = 1 - \Pr(X = 0) \tag{28}$$
$$= 1 - 0.773 \tag{29}$$
$$= 0.227 \tag{30}$$

The graph

Fig. 1. Binomial vs guassian