Assignment

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Two defective bulbs are present in a set of five bulbs. To remove the two defective bulbs, the bulbs are chosen randomly one by one and tested. If X denotes the minimum number of bulbs that must be tested to find out the two defective bulbs, then $\Pr(X = 3)$ (rounded off to two decimal places) equals

Solution:

RV	Values	Description
A	0	1^{st} Bulb defective
	1	1^{st} Bulb non-defective
B	0	2^{nd} Bulb defective
	1	2^{nd} Bulb non-defective
C	0	3^{rd} Bulb defective
	1	3^{rd} Bulb non-defective

TABLE I
RANDOM VARIABLE DECLARATION.

Therefore,

$$p_A(k) = \begin{cases} \frac{2}{5} & k = 0 \\ \frac{3}{5} & k = 1 \end{cases} \tag{1}$$

$$p_B(k) = \begin{cases} \frac{2}{4} & k = 0, 1 \text{ and } A = 1 \\ \frac{3}{4} & k = 1 \text{ and } A = 0 \\ \frac{1}{4} & k = 0 \text{ and } A = 0 \end{cases} \tag{2}$$

$$p_C k = \begin{cases} \frac{1}{3} & k = 0 \text{ and } A = 1, B = 0 \\ \frac{2}{3} & otherwise \end{cases} \tag{3}$$

Hence, pmf of X is

$$p_X(3) = p_{ABC}(1, 0, 0) + p_{ABC}(0, 1, 0) + p_{ABC}(1, 1, 1) \tag{4}$$

$$= \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} + \frac{2}{5} \times \frac{3}{4} \times \frac{1}{3} + \frac{3}{5} \times \frac{2}{4} \times \frac{1}{3} \tag{5}$$

$$= \frac{3}{10} \tag{6}$$