

Assignment

Antalene (EE22BTECH11008)

Question 9.3.9

The probability that a bulb produced by a factory will fuse after 150 days of use is 0.05. Find the probability that out of 5 such bulbs

- 1) none
- 2) not more than one
- 3) more than one
- 4) at least one

will fuse after 150 days of use.

Solution:

Guassian :

let Y be a gaussian Random variable

Parameter	Value	Description
X	0,1,2,3,4,5	No. Of bulbs fused
n	5	Total no. Of bulbs
p	0.05	bulb fusing
q	0.95	not fusing
$\mu = np$	0.25	Mean of Binomial Distribution
$\sigma^2 = npq$	0.2375	Variance of binomial Distribution

TABLE I
RANDOM VARIABLE AND PARAMETER

$$Y \sim N(\mu, \sigma) \quad (1)$$

$$\sim N(1.25, 0.9375) \quad (2)$$

Due to continuity correction $\Pr(X = x)$ can be approximated using gaussian distribution as

$$p_Y(x) \approx \Pr(x - 0.5 < Y < x + 0.5) \quad (3)$$

$$\approx \Pr(Y < x + 0.5) - \Pr(Y < x - 0.5) \quad (4)$$

$$\approx F_Y(x + 0.5) - F_Y(x - 0.5) \quad (5)$$

CDF of Y is defined as:

$$F_Y(x) = \Pr(Y < x) \quad (6)$$

$$= \Pr\left(\frac{Y - \mu}{\sigma} < \frac{x - \mu}{\sigma}\right) \quad (7)$$

$$\Rightarrow \frac{Y - \mu}{\sigma} \sim N(0, 1) \quad (8)$$

$$= 1 - \Pr\left(\frac{Y - \mu}{\sigma} > \frac{x - \mu}{\sigma}\right) \quad (9)$$

$$= \begin{cases} 1 - Q\left(\frac{x - \mu}{\sigma}\right) & x \geq \mu \\ Q\left(\frac{\mu - x}{\sigma}\right) & x < \mu \end{cases} \quad (10)$$

Then probability in terms of Q function is

$$\Rightarrow p_Y(x) \approx Q\left(\frac{(x - 0.5) - \mu}{\sigma}\right) - Q\left(\frac{(x + 0.5) - \mu}{\sigma}\right) \quad (11)$$

Binomial :

$$\Pr(X = k) = {}^nC_k p^k (1 - p)^{n-k} \quad (12)$$

$$= {}^5C_k (0.05)^k (0.95)^{5-k} \quad (13)$$

CDF of X

$$F_X(k) = \Pr(X \leq k) \quad (14)$$

$$= \sum_{i=0}^k {}^{10}C_i (0.05)^i (0.95)^{5-i} \quad (15)$$

The solution

The graph

$\Pr(X = x)$	in term of Q	Numercal value	Binomial solution
$\Pr(X = 0)$	$Q(-1.53) - Q(0.512)$	0.6960	0.773
$\Pr(X \leq 1)$	$Q(1.5896)$	0.9948	0.9774075
$1 - \Pr(X = 0)$	$1 - (Q(-1.53) - Q(0.512))$	0.304	0.227
$1 - \Pr(X \leq 1)$	$1 - Q(1.5896)$	0.006	0.0226

TABLE II
RANDOM VARIABLE AND PARAMETER

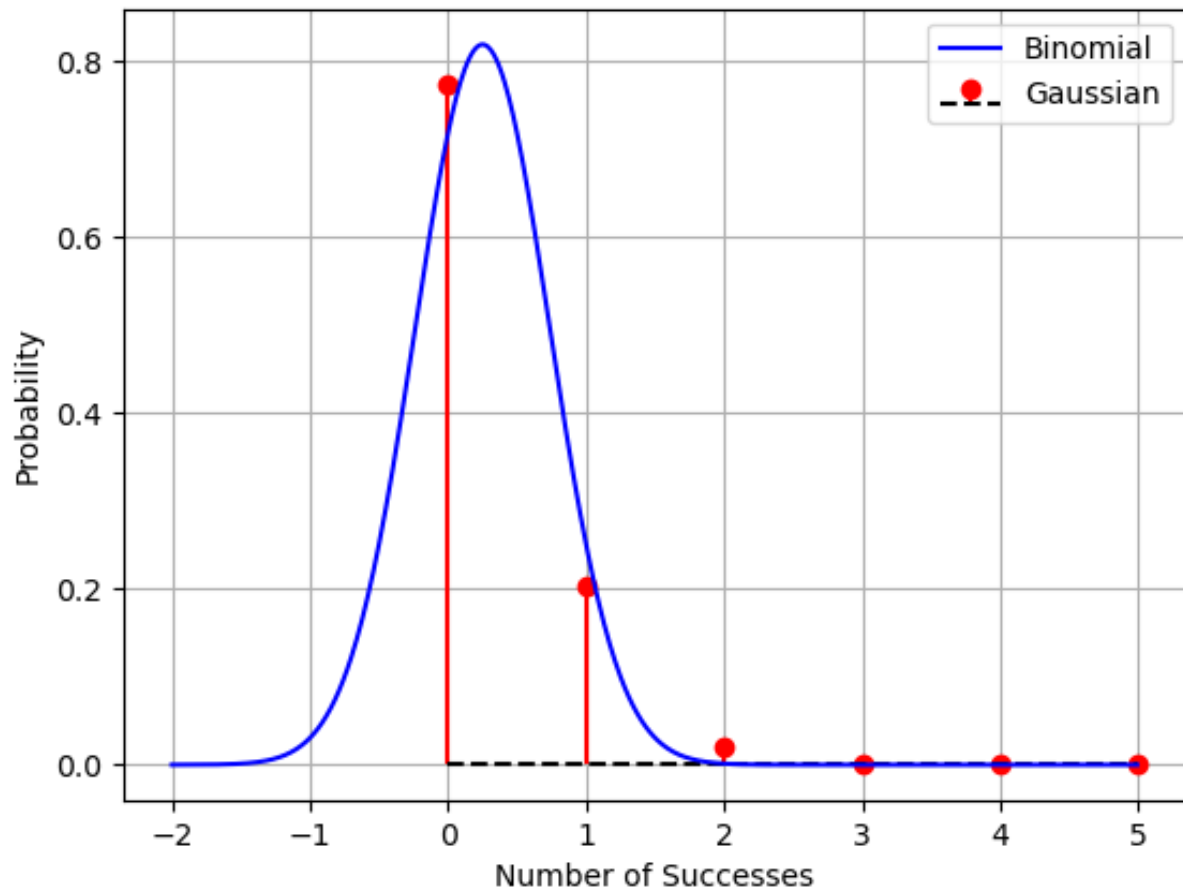


Fig. 1. Binomial vs gaussian