# CBSE MATHEMATICS 2016

## 065 Set 1 N
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## 1  SECTION-A

**Qestion numbers 1 to 6 carry 1 mark each**

1.1. If $A = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix}$, find $\alpha$ satisfying $0 < \alpha < \frac{1}{2}$ when $A + A^{\mathrm{T}} = \sqrt{2}I_2$, where $A^{\mathrm{T}}$ is transpose of A

1.2. If $A$ is a $3 \times 3$ matrix and $\left|3A\right| = k\left|A\right|$ then write the value of k

1.3. For what value of $k$, the system of linear equations

$$x + y + z = 2$$
$$2x + y - z = 3$$
$$3x + 2y + kz = 4$$

has a unique solution?

1.4. Write the sum of intercepts cut off by the plane $\overrightarrow{r}.\left(2\hat{i} + \hat{j} - \hat{k}\right) - 5 = 0$ on the three axes.

1.5. Find $\lambda$ and $\mu$ if

$$\left(\hat{i} + 3\hat{j} + 9\hat{k}\right) \times \left(3\hat{i} - \lambda\hat{j} + \mu\hat{k}\right) = \overrightarrow{0}.$$

1.6. If $\overrightarrow{a} = 4\hat{i} - \hat{j} + \hat{k}$ and $\overrightarrow{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, then find a unit vector parallel to the vector $\overrightarrow{a} + \overrightarrow{b}$.

## 2  SECTION-B

**Qestion numbers 7 to 23 carry 4 mark each**

2.7. Solve for

$$x : \tan^{-1}(x - 1) + \tan^{-1}x + \tan^{-1}(x + 1) = \tan^{-1}3x$$

2.8. Prove that

$$\tan^{-1}\left(\frac{6x - 8x^3}{1 - 12x^2}\right) - \tan^{-1}\left(\frac{4x}{1 - 4x^2}\right) = \tan^{-1}2x;$$

$$\left|2x\right| < \frac{1}{\sqrt{3}}$$

2.9. A typist charges 145 rupees for typing 10 English and 3 Hindi pages, while charges for typing 3 English and 10 Hindi pages are 180 rupees. Using matrices, find the charges of typing one English and one Hindi page separately. However typist charged only 2 rupees per page from a poor student

Shayam for 5 Hindi pages. How mcuh less was charged from ths poor boy? which values are reflected in this problem?

2.10. If

$$f(x) = \begin{cases} \frac{\sin(a+1)x + 2\sin}{x} & , x < 0 \\ 2 & , x = 0 \\ \frac{\sqrt{1+bx}-1}{x} & , x > 0 \end{cases}$$

is continuous at $x = 0$, then find the values of a and b.

2.11. If $\cos(a + y) = \cos y$ then prove that
$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$. Hence show that
$\sin a \frac{d^2y}{dx^2} + \sin 2(a + y)\frac{dy}{dx} = 0$.

2.12. Find $\frac{dy}{dx}$ if $y = \sin^{-1}\left[\frac{6x - 4\sqrt{1-4x^2}}{5}\right]$

2.13. Find the equation of the tangents to the curve $y = x^3 + 2x - 4$ which are perpendicular to line $x + 14y + 3 = 0$

2.14. Find : $\int \frac{(2x-5)e^{2x}}{(2x-3)^3}dx$

2.15. Find : $\int \frac{x^2+x+1}{(x^2+1)(x+2)}dx$

2.16. Evaluate : $\int_{-2}^{2} \frac{x^2}{1+5^x}dx$

2.17. Find : $\int (x + 3)\sqrt{3 - 4x - x^2}dx$

2.18. Find the particular solution of difference equation :
$\frac{dy}{dx} = -\frac{x+y\cos x}{1+\sin x}$
given that $y = 1$ when $x = 0$.

2.19. Find the particualr solution of the differential equation

$$2ye^{\frac{x}{y}}dx + (y - 2xe^{\frac{x}{y}})dy = 0$$

given that $x = 0$ when $y = 1$.

2.20. Show that the four points $A(4, 5, 1)$, $B(0, -1, -1)$, $C(3, 9, 4)$ and $D(-4, 4, 4)$ are coplanar.

2.21. Find the coordinates of the foot of perpendicular drawn from the point $A(-1, 8, 4)$ to the line joining the points $B(0, -1, 3)$ and $C(2, -3, -1)$. Hence find the image of the point $A$ in the line $BC$.

2.22. A bag $X$ contains 4 white balls and 2 black balls, while another bag $Y$ contains 3 white balls and 3 black balls. Two balls are drawn (without replacement) at random from one of the bags and were found to be one white and one black. Find the probability that the balls were drawn from bag $Y$.

2.23. $A$ and $B$ throw a pair of dice alternately, till one of them gets a total of 10 and wins the game. Find their respective probabilities of winning, if $A$ starts first.

# 3 SECTION-C

**Qestion numbers 24 to 35 carry 4 mark each**

3.24. Three numbers are selected at random (without replacement) from first six positive integers. Let $X$ denote the largest of the three numbers obtained. Find the probability distribution of $X$. Also, find the mean and variance of the distribution.

3.25. Let $A = R \times R$ and $*$ be a binary operation on $A$ defined by

$$(a, b) * (c, d) = (a + c, b + d)$$

Show that $*$ is commutative and associative. Find the identity element for $*$ on $A$. Also find the inverse of every element $(a, b) \in A$.

3.26. Prove that $y = \frac{4 \sin \theta}{2 + \cos \theta} - \theta$ ia an ncreasing function of $\theta$ on $\left[0, \frac{\pi}{2}\right]$.

3.27. Show that semi-vertical angle of a cone of maximum volume and given slant height is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

3.28. Using the method of integration, find the area of the triangular region whose vertices are $(2, 2)$, $(4, 3)$ and $(1, 2)$.

3.29. Find the equation of the plane which contains the line of intersection of the planes

$$\vec{r} \cdot \left(\hat{i} - 2\hat{j} + 3\hat{k}\right) - 4 = 0 \text{ and}$$
$$\vec{r} \cdot \left(-2\hat{i} + \hat{j} + \hat{k}\right) + 5 = 0$$

and whose intercept on $x$-axis is equal to that of on $y$-axis.

3.30. A retired person wants to invest an amount of 50,000 rupees. His broker recommends investing in two type of bonds $A$ and $B$ yielding 10% and 9% return respectively on the invested amount. He decides to invest at least 20,000 rupees in bond $A$ and at least 10,000 rupees in bond $B$. He also wants to invest at least as much in bond $A$ as in bond $B$. Solve this linear programming problem graphically to maximise his returns.

3.31. Using properties of determinants, prove that

$$\begin{vmatrix} (x + y)^2 & zx & zy \\ zx & (z + y)^2 & xy \\ zy & xy & (z + x)^2 \end{vmatrix} = 2xyz(x + y + z)^3$$

3.32. If

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{pmatrix}$$

and $A^3 - 6A^2 + 7A + kI_3 = 0$ find $k$.