GCN on Undirected Graph:

$G = (V, E, X)$

- V: node
- E: edge
- X: feature of node, $|V| \times d_0$

$$\begin{cases} H^{k+1} = f(\hat{A} H^k W^k) \\ H^0 = X \\ W^k \in R^{d_k \times d_{k+1}} \end{cases}$$

GCN on Multi-Relational Graph:

$G = (V, R, E, X)$

- R: relation

$$H^{k+1} = f(\hat{A} H^k W_r^k) \rightarrow h_v = f\left(\sum_{(u,r) \in N(v)} W_r h_u\right)$$

(u, v, r)

$u \xrightarrow{r \in R} v$

r^{-1}

CompGCN:

$G = (V, R, E, X, Z)$

- Z: initial relation feature, $|R| \times d_0$

$$\begin{cases} H_v^{k+1} = f\left(\sum_{(u,r) \in N(v)} W_{\lambda(r)}^k \phi(h_u^k, h_r^k)\right) \\ h_r^{k+1} = W_{rel}^k \cdot h_r^k \end{cases}$$

- $\lambda(r)$: out / in / self
- ϕ: composition operator

h_v^0, h_r^0: initial x_v, z_r

W_{rel}^k: $d_i \times d_o$