HW - oct 08

1. proof: Consider inequality:

$$x > \ln(1+x) \quad \text{for all } x > 0.$$

We consider serie: $\sum_{n=1}^{\infty} \ln(1+\frac{1}{n})$

$$\sum_{n=1}^{\infty} \ln(1+\frac{1}{n}) = \sum_{n=1}^{\infty} \ln \frac{n+1}{n} = \sum_{n=1}^{\infty} \ln(n+1) - \ln n$$

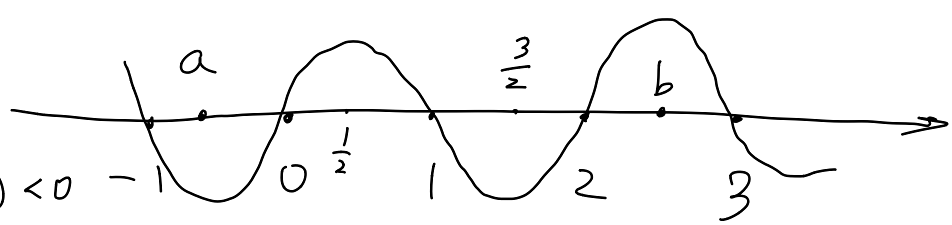
$$= (\ln 2 - \ln 1) + (\ln 3 - \ln 2) + \cdots + (\ln(i+1) - \ln i) + \cdots$$

$= \ln \infty - 0 \longrightarrow \infty$. therefore $\sum_{n=1}^{\infty} \ln(1+\frac{1}{n})$ diverge

since $\sum \frac{1}{n} > \sum \ln(1+\frac{1}{n})$,

so $\sum \frac{1}{n}$ diverges too.

2. Proof:



$p_0 = a \quad f(p_0) < 0$

$p_1 = \frac{a+b}{2}$ if $a+b < 2 \Rightarrow \frac{1}{2} < p_1 < 1$

then $f(p_1) > 0 \Rightarrow$ set $b = p_1 \Rightarrow$

p_n will converge to 0

if $a+b>2 \Rightarrow 1 < p_1 < \frac{3}{2}$

then $f(p_1)<0 \Rightarrow$ set $a = p_1$

$\Rightarrow p_n$ will converge to 1

if $a+b=2 \Rightarrow p_1 = 1 \Rightarrow f(p_1)=0$

then p_1 is the zero point.

3. (a) proof: $W(x) = \prod_{i=1}^{20} (x-i) = (x-1)(x-2)\cdots(x-20)$

$$\therefore W'(x) = \sum_{i=1}^{20} \frac{W(x)}{(x-i)} \Rightarrow x=16 \text{ is the root of } W(x)$$

therefore: $$W'(16) = \frac{(16-1)(16-2)\cdots(16-16)\cdots(16-20)}{(16-16)} + 0$$

$$= 15! \cdot (-1) \cdot (-2) \cdot (-3)(-4)$$

$$= 15! \cdot 4!$$

(b) note for every $i \in [1,20]$:

$$W'(i) = \frac{(x-1)\cdots(x-i)\cdots(x-20)}{(x-i)} + 0 = \frac{W(x)}{x-i}$$

$$= (x-1)\cdots(x-i-1)(x-i+1)\cdots(x-20)$$

4. proof: the fixed point iteration is the scheme: $p_n = g(p_{n-1})$

By assumption, $g(p) = p$ and $|g'(p)| < 1$,

so there exists a δ where:

$$|g'(x)| \le k \quad \forall x \in [p-\delta, p+\delta]$$

here k is in $(0,1)$.

If $x \in [p-\delta, p+\delta]$, the mean value theorem implies that for some ξ between x and p, so that:

$$|g(x) - g(p)| = |g'(\xi)| \cdot |x-p|.$$

therefore: $|g(x) - g(p)| = |g(x) - p| = |g'(\xi)| \cdot |x-p|$

$$\le k|x-p| < |x-p|$$

Since $x \in [p-\delta, p+\delta]$, $|x-p| < \delta \Rightarrow |g(x) - p| < \delta$

Hence $g(x) \in (p-\delta, p+\delta)$. when $x \in (p-\delta, p+\delta)$

In other word, in interval $[p-\delta, p+\delta]$, g maps into itself. and $|g'(x)| \le k$.

According to fixed-point theorem, p_0 coverges to p.