HW03 - Oct 15

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1. Solve:

We use 3 preceding iterates, x_{n-2}, x_{n-1} and x_n, with their function values $f(x_{n-2})$, $f(x_{n-1})$ and $f(x_n)$

Applying the Lagrange Interpolation on the inverse f yields:

$$x_{n+1} = f^{-1}(y) = x_{n-2} \cdot \frac{(y-f_n)(y-f_{n-1})}{(f_{n-2}-f_n)(f_{n-2}-f_{n-1})}$$

$$+ x_{n-1} \cdot \frac{(y-f_n)(y-f_{n-2})}{(f_{n-1}-f_n)(f_{n-1}-f_{n-2})}$$

$$+ x_n \cdot \frac{(y-f_{n-1})(y-f_{n-2})}{(f_n-f_{n-1})(f_n-f_{n-2})}$$

Since we're looking for a root of f, so we substitute $y=0$ in the above equation. where $f_k = f(x_k)$, and it gives out:

$$x_{n+1} = \frac{f_{n-1}\, f_n}{(f_{n-2}-f_{n-1})(f_{n-2}-f_n)} x_{n-2} + \frac{f_{n-2}\, f_n}{(f_{n-1}-f_{n-2})(f_{n-1}-f_n)} x_{n-1}$$

$$+ \frac{f_{n-2}\, f_{n-1}}{(f_n-f_{n-2})(f_n-f_{n-1})} x_n$$

Given 3 initial values x_0, x_1 and x_2, the root of f can be approximated by the above algorithm.

for $f = x - e^x$: $\varepsilon = 1\times10^{-4}$ and $N = 5$

x	0.3	0.4	0.5	0.6
$f(x)$	-0.441	-0.270	-0.106	0.051

We choose $x_0 = 0.4$ $x_1 = 0.5$, $x_2 = 0.6$ as initial values

$$\Rightarrow x_3 = \frac{f_1 f_2}{(f_0 - f_1)(f_0 - f_2)} x_0 + \frac{f_0 f_2}{(f_1 - f_0)(f_1 - f_2)} x_1 + \frac{f_0 f_1}{(f_2 - f_0)(f_2 - f_1)} x_2$$

$$= 0.5672 \Rightarrow f(x_3) = 8.887\times10^{-5} \quad |f(x_3) - 0| < \varepsilon.$$

Therefore zero point $p \approx x_3 = 0.5672$

2. Solve :

degree of 1 : choose $x_0 = 8.3$ and $x_1 = 8.6$

$$\therefore L_1(x) = f_0 \cdot \frac{x - x_1}{x_0 - x_1} + f_1 \frac{x - x_0}{x_1 - x_0}$$

$f(8.4) \approx L_1(8.4) = 17.87833$

degree of 2 : choose $x_0 = 8.1$, $x_1 = 8.3$, $x_2 = 8.6$.

$$\therefore L_2(x) = f_0 \cdot \frac{(x - x_1)(x - x_2)}{(x_0 - x_1)(x_0 - x_2)} + f_1 \frac{(x - x_0)(x - x_2)}{(x_1 - x_0)(x_1 - x_2)} + f_2 \frac{(x - x_0)(x - x_1)}{(x_2 - x_0)(x_2 - x_1)}$$

$f(8.4) \approx L_2(8.4) = 17.87713$

degree of 3 : choose $x_0 = 8.1$, $x_1 = 8.3$, $x_2 = 8.6$, $x_3 = 8.7$

$$\Rightarrow L_3(x) = f_0 \frac{(x - x_1)(x - x_2)(x - x_3)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)} + f_1 \frac{(x - x_0)(x - x_2)(x - x_3)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)} + f_2 \frac{(x - x_0)(x - x_1)(x - x_3)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)} + f_3 \frac{(x - x_0)(x - x_1)(x - x_2)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)}$$

$\Rightarrow f(8.4) \approx L_3(8.4) = 17.8771425$