HW06 — OCT 29 刘俊哲 1500011370

1. Proof:

Simpson's rule: for an integration in $[a, b]$.

it can be approximated by:

$$\int_a^b f(x)\,dx = \int_a^b P(x)\,dx = \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

We've proved the error term can be written as:

$$E(r) = -\frac{1}{90}\left(\frac{b-a}{2}\right)^5 f^{(4)}(\xi) \qquad \text{where } \xi \in (a, b)$$

Now we're considering a large interval $[a, b]$, we can divide the interval into many "small" interals:

$$x_i = a + ih, \quad h = \frac{b-a}{n}, \quad i = 0, 1, \cdots, n$$

and midpoint of each interval:

$$x_{i+1/2} = a + \left(i + \frac{1}{2}\right)h$$

in every small interval $[x_i, x_{i+1/2}]$ we can use simpson's rule, and it gives us Composite Simpson's rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{6}\sum_{i=0}^{n-1}\left[f(x_i) + 4f(x_{i+1/2}) + f(x_{i+1})\right] \triangleq S(h)$$

the error term is given by:

$$\left|\int_a^b f(x)\,dx - S(h)\right| \leqslant \frac{h^4}{2880}\cdot(b-a)\cdot M_4$$

$$= \frac{h^4}{2880} \cdot (b-a) \cdot \max_{\xi \in (a,b)} |f^{(4)}(\xi)|$$

$$\leq \frac{h^4}{2880} \cdot \max_{\xi \in (a,b)} \left| (b-a) \cdot f^{(4)}(\xi) \right|$$

$$= \frac{h^4}{2880} \cdot \left| f'''(b) - f'''(a) \right|$$

Here, h is defined as $h = \frac{b-a}{n}$, the length of every small interval.

2. Solve: $\int_0^1 f(x)\,dx = \frac{1}{2} f(x_0) + c_1 f(x_1)$

$2x_0^2 - 2x_0 + \frac{1}{3} = 0$

$x_0^2 + (1-x_0)^2 = \frac{2}{3}$

requires the smallest error.

$x_0 + x_1 = 1$

$x_0^2 + x_1^2 = \frac{2}{3}$

we let $f(x) = 1 \Rightarrow 1 = \frac{1}{2} + c_1$

$f(x) = x \Rightarrow \frac{1}{2} = \frac{1}{2} x_0 + c_1 \cdot x_1$

$f(x) = x^2 \Rightarrow \frac{1}{3} = \frac{1}{2} x_0^2 + c_1 x_1^2$

Solution $\Rightarrow$ $\begin{cases} c_1 = \frac{1}{2} \\ x_0 = \frac{3-\sqrt{3}}{6} \\ x_1 = \frac{3+\sqrt{3}}{6} \end{cases}$