HW 10 - NOV 26

1、Proof: to prove that:

$$\frac{1}{K(A)} \leq \frac{||A-B||}{||A||}$$

We only need to prove that:

$$||A-B|| \geq \frac{1}{||A^{-1}||}$$

Since: $$||A-B|| = \max_{||x||=1} ||(A-B)x||$$

$$\geq \underset{||x||=1 \text{ and } Bx=0}{||Ax||} \geq \frac{||x||}{||A^{-1}||} = \frac{1}{||A^{-1}||}$$

Q.E.D.

2. solve:

$$\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} \frac{1}{63} \\ \frac{1}{168} \end{bmatrix}$$

so: $x - \tilde{x} = (0.00085714, -0.00066666)^t$

$$\|x - \tilde{x}\|_\infty = \frac{1}{7} - 0.142 \approx 0.00085714$$

$$K_\infty(A) = \frac{\|b - A\tilde{x}\|_\infty}{\|A\|_\infty} = \|A^{-1}\|_\infty \cdot \|b - A\tilde{x}\|_\infty$$

$$\Rightarrow b - A\tilde{x} = \begin{bmatrix} 0.00020634 \\ 0.00011904 \end{bmatrix}$$

therefore:

$$\|A^{-1}\|_\infty \cdot \|b - A\tilde{x}\|_\infty = 12 \cdot \left(\frac{1}{63} - \frac{47}{3000}\right)$$

$$= 0.00247619$$