HW13 - Dec 10 刘浚哲 1500011370

1. $f_1(x_1,x_2,x_3) = 3x_1 - \cos(x_2x_3) - \frac{1}{2}$

$f_2(x_1,x_2,x_3) = 4x_1^2 - 625x_2^2 + 2x_2 - 1 = 0$

$f_3(x_1,x_2,x_3) = e^{-x_1x_2} + 20x_3 + \frac{10\pi - 3}{3} = 0$

$F(x_1,x_2,x_3) = (f_1, f_2, f_3)^t$

Jacobian Matrix:

$$J(x_1,x_2,x_3) = \begin{bmatrix} 3 & x_3\sin(x_2x_3) & x_2\sin(x_2x_3) \\ 8x_1 & -1250x_2 + 2 & 0 \\ -x_2e^{-x_1x_2} & -x_1e^{-x_1x_2} & 20 \end{bmatrix}$$

$x^{(0)} = 0 \Rightarrow F(x^{(0)}) = (f_1(x^{(0)}), f_2(x^{(0)}), f_3(x^{(0)}))^t$

$= (-\frac{3}{2}, -1, \frac{10}{3}\pi)^t$

and $J(x^{(0)}) = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 20 \end{bmatrix}$, Solving the linear system: $J(x^{(0)})\, y^{(0)} = -F(x^{(0)})$:

$$y^{(0)} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{\pi}{6} \end{bmatrix} \Rightarrow x^{(1)} = x^{(0)} + y^{(0)} = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ \frac{\pi}{6} \end{bmatrix}$$

$$\Rightarrow F(x^{(1)}) = (f_1(x^{(1)}),\ f_2(x^{(1)}),\ f_3(x^{(1)}))^t$$

$$= \begin{bmatrix} -2.999989561 \\ -157.25 \\ 20.72275181 \end{bmatrix}$$

and
$$J(x^{(1)}) = \begin{bmatrix} 3 & -2.392451295\times10^{-3} & 2.284622699\times10^{-3} \\ -4 & 627 & 0 \\ 0.3894003915 & 0.3894003915 & 20 \end{bmatrix}$$

$$J(x^{(1)})\, y^{(1)} = -F(x^{(1)})$$

$$\Rightarrow y^{(1)} = \begin{bmatrix} 1.001009337 \\ 0.2571834726 \\ -1.060634629 \end{bmatrix} \Rightarrow x^{(2)} = x^{(1)} + y^{(1)} = \begin{bmatrix} 0.501009337 \\ -0.2428165274 \\ -0.5370358534 \end{bmatrix}$$

2. $G(\lambda, x) = F(x) - e^{-\lambda} F(x(0))$

$$\Rightarrow x'(\lambda) = -\left[\frac{\partial G(\lambda, x(\lambda))}{\partial x}\right]^{-1} \cdot \left[\frac{\partial G(\lambda, x(\lambda))}{\partial \lambda}\right]$$

$$= -\left[J(x(\lambda))\right]^{-1} \cdot e^{-\lambda} F(x(0))$$

Euler's method: $x(1) = x(0) + h\, x'(0)$

$h=1 \Rightarrow$ $\quad = x(0) + [J(x(0))]^{-1} \cdot F(x(0))$

$= x^{(0)} + y^{(0)}$

Therefore $x(1) = x^{(1)}$

where $x^{(1)}$ satisfies Newton's method.

3. $g(x_1, x_2) = 100\,(x_1^2 - x_2)^2 + (1-x_1)^2$

$$z(x) = \nabla g(x_1, x_2) = \left(\frac{\partial g}{\partial x_1}, \frac{\partial g}{\partial x_2}\right)^t$$

$$= \begin{bmatrix} 400x_1(x_1^2 - x_2) - 2(1-x_1) \\ -200(x_1^2 - x_2) \end{bmatrix}$$

$x^{(0)} = (0,0)^t \Rightarrow z(x^{(0)}) = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$

$\Rightarrow z_0 = \dfrac{z}{\|z\|_2} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$, $\quad \alpha_1 = 0, \quad \alpha_3 = 1$

$g_1 = g(x^{(0)}) = 1, \quad g_3 = g(x^{(0)} - \alpha_3 z_0) = 100$

$$\alpha_2 = 0.5, \quad g_2 = g(x^{(0)} - \alpha_2 z_0) = 6.5$$

$$h_1 = \frac{g_2 - g_1}{\alpha_2} = 11$$

$$h_2 = \frac{g_3 - g_2}{\alpha_3 - \alpha_2} = 198$$

$$h_3 = \frac{h_2 - h_1}{\alpha_3} = 187$$

$$\alpha_0 = 0.5\left(\alpha_2 - \frac{h_1}{h_3}\right) = \frac{15}{68} = 0.2205882353$$

$$g_0 = g(x - \alpha_0 z) = g\left(\frac{15}{68}, 0\right) = 0.8442542$$

$$\min(g_0, g_3) = 0.8442542$$

$$\alpha \Rightarrow g(x - \alpha z) = 0.8442542 = g$$

$$\Rightarrow \alpha = \alpha_0 = \frac{15}{68}$$

$$|g - g_1| = 0.1557458 > 0.005$$

continue:

$$x^{(1)} = \left(\frac{15}{68}, 0\right)^t, \quad g_1 = g(x^{(1)}) = 0.8442542$$

$$z = \begin{bmatrix} -0.4854594952 \\ -9.73183391 \end{bmatrix}$$

$$z = \frac{z}{\|z\|_2} = \begin{bmatrix} -0.04982171098 \\ -0.9987581274 \end{bmatrix}$$

$$\alpha_1 = 0, \quad \alpha_3 = 1, \quad g_3 = g(x^{(1)} - \alpha_3 z) =$$

$$= g(0.2704099463, 0.9987581274)$$

$$= 86.21261102$$

$$g_3 > g_1: \quad \alpha_3 = 0.5$$

$$g_3 = g(x - \alpha_3 z) = 19.85$$

$$\alpha_3 = 0.25 \Rightarrow g_3 = 4.4055636$$

$$\alpha_3 = 0.0125 \Rightarrow g_3 = 0.7393713278 < g_1$$

$$\alpha_2 = \frac{\alpha_3}{2} = 0.00625$$

$$\Rightarrow g_2 = g(x - \alpha_2 z) = 0.7880851327$$

$$\therefore h_1 = \frac{g_2 - g_1}{\alpha_2} = -8.987050768$$

$$h_2 = \frac{g_3 - g_2}{\alpha_3 - \alpha_2} = -7.794208784$$

$$h_3 = \frac{h_2 - h_1}{\alpha_3} = 95.42735872$$

$$\alpha_0 = 0.5 \left(\alpha_2 - \frac{h_1}{h_3}\right) = 0.050213439$$

$$g_0 = g(x - \alpha_0 z) = 0.6036036923$$

$$\min\{g_0, g_3\} = g_0 = 0.6036036293 = g$$