HW12 - Dec 5

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1. $y = y(t_i)$, $f = f(t_i, y(t_i))$, $f_t = f_t(t_i, y(t_i))$

$$\Rightarrow y + hf + \frac{h^2}{2}(f_t + f\cdot f_y) + \frac{h^3}{6}(f_{tt} + f_t f_y + 2 f f_{yt} + f f_y^2 + f^2 f_{yy})$$

$$= y + ahf + bh\left[f - h(f_t + f f_y) + \frac{h^2}{2}(f_{tt} + f_t f_y + 2 f f_{yt} + f f_y^2 + f^2 f_{yy})\right]$$

$$+ ch\left[f - 2h(f_t + f f_y) + 2h^2(f_{tt} + f_t f_y + 2 f f_{yt} + f f_y^2 + f^2 f_{yy})\right]$$

$$\Rightarrow \begin{cases} a + b + c = 1 \\ -b - 2c = \frac{1}{2} \\ \frac{b}{2} + 2c = \frac{1}{6} \end{cases} \Rightarrow \begin{cases} a = \frac{23}{12} \\ b = -\frac{4}{3} \\ c = \frac{5}{12} \end{cases}$$

2. a. $h=0.01$, $w_0=1$, $w_1=1-\ln(1-0.01e)=1.027559$

$w_2=1-\ln(1-0.02e)=1.05589929$

$w_{i+1}=w_i+\frac{h}{24}\left(9e^{w_i}+19e^{w_{i-1}}-5e^{w_{i-2}}+e^{w_{i-3}}\right)$

$\Rightarrow w_3=1.084747$ $w_4=1.114441$

$w_5=1.1450832$ $w_6=1.1765793$

$w_7=1.2091403$ $w_8=1.242784$

$w_9=1.2775582$ $w_{10}=1.3136084$

$w_{11}=1.3509827$ $w_{12}=1.3897608$

$w_{13}=1.4301111$ $w_{14}=1.4721449$

$w_{15}=1.5159289$ $w_{16}=1.561716$

$w_{17}=1.6096602$ $w_{18}=1.659973$

$w_{19}=1.7128968$ $w_{20}=1.7687157$

b. Newton's method will reduce the number of iteration per step from 3 to 2.

according to the stop criterion: $|w_i^{(k)}-w_i^{(k-1)}|\leq 10^{-6}$