HW 11 - Dec 03 刘浚哲 1500011370

1. $x_1 + \frac{1}{2}x_2 = \frac{5}{21}$

$\frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{11}{84}$

a. $\begin{vmatrix} 1 & 0.5 & 0.24 \\ 0.5 & 0.33 & 0.13 \end{vmatrix} = \begin{vmatrix} 1 & 0.5 & 0.24 \\ 0 & 0.16 & 0.02 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0.18 \\ 0 & 1 & 0.13 \end{vmatrix}$

$\Rightarrow x_1 = 0.18, \ x_2 = 0.13$ therefore $(x_1 x_2) = \begin{bmatrix} 0.18 \\ 0.13 \end{bmatrix}$

b. $x^{(0)} = (0, 0, 0)^t \quad r^{(0)} = b - Ax^{(0)} = (0.24, 0.13)$

$w^{(0)} = C^{-1} r^{(0)} = (0.24, 0.13)^t$

$r^{(1)} = C^{-t} w = (0.24, 0.13)^{-t}$

$\alpha = \langle w \cdot w \rangle = 0.075$

$u = Av = (0.31, 0.16)^t$

$t^{(1)} = \frac{\alpha}{\langle v, u \rangle} = 0.79 \qquad x^{(1)} = x^{(0)} + t_1 v^{(1)} = (0.19, 0.10)^t$

2. $y' = 1 - y \quad 0 \le t \le 2$

$\left| \frac{\partial f}{\partial y}(t, y) \right| \le 1$ statisfies the Lipschitz criteria for constant $= 1$

therefore $y' = 1 - y$ has a unique solution

$\Rightarrow$ is well-posed

3. a. $y' = f(t, y(t))$

$$\Rightarrow \int_a^t y'(z)\,dz = \int_a^t f(z, y(z))\,dz$$

$$\Rightarrow y(t) - y(a) = \int_a^t f(z, y(z))\,dz$$

$$\Rightarrow y(t) = y(a) + \int_a^t f(z, y(z))\,dz$$

b. $y(0) = 1$

$$Y_1(t) = 1 + \int_0^t \tau\,d\tau = 1 + \frac{t^2}{2}$$

$$Y_2(t) = 1 + \int_0^t -(1 + \frac{1}{2}\tau^2) + \tau + 1 \;d\tau$$

$$= -\frac{t^3}{6} + \frac{t^2}{2} + 1$$

$$Y_3(t) = 1 + \int_0^t -\left(-\frac{\tau^3}{6} + \frac{\tau^2}{2} + 1\right) + \tau + 1 \, d\tau$$

$$= \frac{t^4}{24} - \frac{t^3}{6} + \frac{t^2}{2} + 1$$

C. $y(t) = 1 + \frac{t^2}{2} - \frac{t^3}{6} + \frac{t^4}{24} - \frac{t^5}{120} + \cdots$