CSCI 570 - Fall 2019 - HW 12

Due December 8th 11:59 p.m.

1 Graded Problems

1. A variation of the satisfiability problem is the MIN 2-SAT problem. The goal in the MIN 2-SAT problem is to find a truth assignment that minimizes the number of satisfied clauses. Give the best approximation algorithm that you can find for the problem.

2. Consider the following vertex cover algorithm for an undirected graph $G = (V, E)$.

   1. Initialize $C = Null$ set.
   2. Pick any edge $e = (u, v) \in E$, add u and v to C. Delete all edges incident on either u or v.
   3. If E is empty, output C and exit. Otherwise, go to step 1.

Show that C is a vertex cover and that the size of C is at most twice as big as the minimum vertex cover. (Thus we have a 2-approximation).

3. 720 students have pre-enrolled for the "Analysis of Algorithms" class in Fall. Each student must attend one of the 16 discussion sections, and each discussion section i has capacity for D_i students. The happiness level of a student assigned to a discussion section i is proportional to $\alpha_i(D_i - S_i)$, where α_i is a parameter reflecting how well the air-conditioning system works for the room used for section i (the higher the better), and S_i is the actual number of students assigned to that section. We want to find out how many students to assign to each section in order to maximize total student happiness. Express the problem as a linear programming problem.

4. Show that vertex cover remains $NP - Complete$ even if the instances are restricted to graphs with only even degree vertices.

2 Practice Problems

5. A set of n space stations need your help in building a radar system to track spaceships traveling between them. The ith space station is located in 3D space at coordinates (x_i, y_i, z_i). The space stations never move. Each space station i will have a radar with power r_i, where r_i is to be determined. You want to figure how powerful to make each space station's radar transmitter, so that whenever any spaceship travels in a straight line from one station to another, it will always be in radar range of either the first space station (its origin) or the second space station (its destination). A radar with power r is capable of tracking space ships anywhere the sphere with radius r centered at itself. Thus, a space ship is within radar range through its strip from space station i to space station j if every point along the line from (x_i, y_i, z_i) to (x_j, y_j, z_j) falls within either the sphere of radius r_i centered at (x_i, y_i, z_i) or the sphere of radius r_j centered at (x_j, y_j, z_j). The cost of each radar transmitter is proportional to its power, and you want to minimize the total cost of all of the radar transmitters. You are given all of the $(x_1, y_1, z_1), ..., (x_n, y_n, z_n)$ values, and your job is to choose values for $r_1, ..., r_n$. Express this problem as a linear program.

1. Describe your variables for the linear program.
2. Write out the objective function.
3. Describe the set of constraints for LP. You need to specify the number of constraints needed and describe what each constraint represents.