CSCI 570 - Fall 2019 - HW 8

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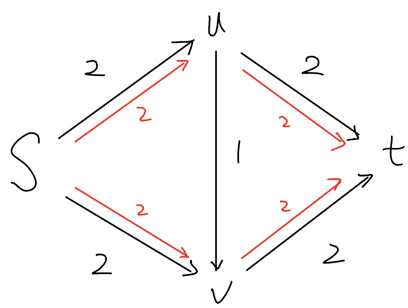
1 True or false: For any flow network G and any maximum flow on G, there is always an edge e such that increasing the capacity of e increases the maximum flow of the network. Justify your answer.

False. For instance, the following graph has a maximum flow with value of 4, 2 on each two arms and 0 on the central edge. But increasing any of them does not increase the maximum flow.



2 Suppose that you are given a flow network G, and G has edges entering the source s. Let f be a flow in G in which one of the edges (v, s) entering the source has $f(v, s) = 1$. Prove that there must exist another flow $f^{'}$ with $f^{'}(v, s) = 0$ such that $|f| = |f^{'}|$. Give an $O(|E|)$-time algorithm to compute$f^{'}$, given f, and assuming that all edge capacities are integers.

If there is a f that has some flow on the edge $< v, s >$, then we will have a cycle $s \rightarrow n_1 \rightarrow n_2 \ldots n_k \rightarrow v$ that carries at least 1 unit of flow, by reducing 1 unit of flow on each edge on this cycle, we can have $f^{'}$.

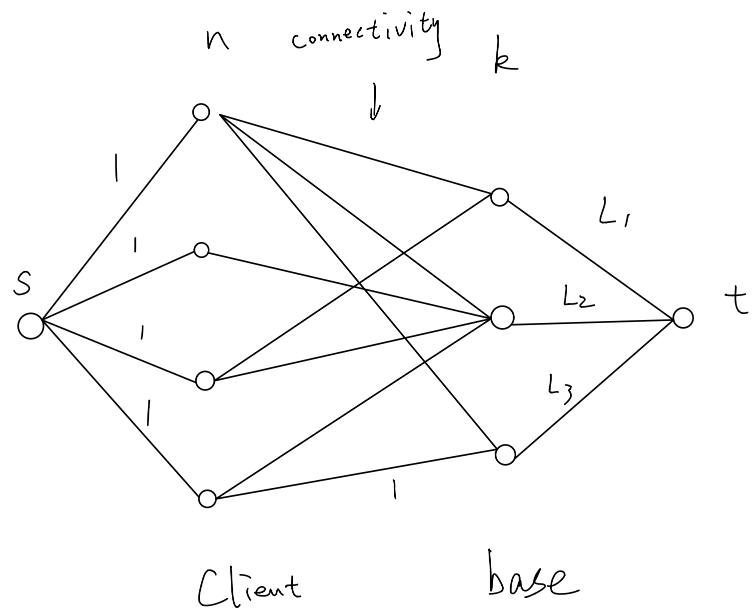
Proof: Note that all nodes is reachable from s, and since v has 1 unit of flow coming out of it, there must be a node pouring 1 unit of flow into v, by recursively conducting this process, we can eventually find a cycle from $s \rightarrow v$ that carries flow. To prove that there always exist another flow $f^{'}$, we only need to prove that the flow is still valid after we reduced the flow by 1.

We partition the nodes into A, B, where A is the set containing all the nodes in the cycle $s \rightarrow v$, and B contains the rest. Then the 1 unit flow on this cycle will be an inner flow inside the cut A, it has no contribution to the value of the flow $|f|$. Therefore by reducing the flow along the cycle by 1, the overall flow will still be valid and will not affect its value: $|f| = |f^{'}|$.

3 Suppose that you wish to find, among all minimum cuts in a flow network G with integral capacities, one that contains the smallest number of edges. Show how to modify the capacities of G to create a new flow network G' in which any minimum cut in G' is a minimum cut with the smallest number of edges in G.

4 Kleinberg and Tardos, Chapter 7, Exercise 7.

We will use network flow method to solve this problem. The network graph will be like following image:



The first step is to build up this graph. This graph is comprised of 1 source s, n clients, k base stations, and 1 sink t. There is an edge from source to every clients, and the weights are all 1; From every base station b_i, there is an edge link to the sink t with a weight equals to its load L_i. As for the edges from clients to the bases, it should be computed by the connectivity between every client-base pair, for example, if the distance between c_i and b_j is smaller than the range parameter r_j of the base station j, than c_i is connect to the base j, all the weights are equal to 1.

Then the second step is to find the maximum flow of this graph. If the maximum flow value is smaller than n, then not every client can be connected simultaneously, otherwise they can.