Stable roommate problem

Q. Do stable matchings always exist?

A. Not obvious a priori.

Stable roommate problem.

- $2n$ people; each person ranks others from 1 to $2n - 1$.
- Assign roommate pairs so that no unstable pairs.

	1st	2nd	3rd
A	B	C	D
B	C	A	D
C	A	B	D
D	A	B	C

no perfect matching is stable

A–B, C–D $\Rightarrow$ B–C unstable

A–C, B–D $\Rightarrow$ A–B unstable

A–D, B–C $\Rightarrow$ A–C unstable

Observation. Stable matchings need not exist.