

3.

$$f(u) = (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3$$

$$\begin{aligned}
 f_0 &= (1-u) r_0 + u r_1 \\
 &= (1-u)(1-u)r_0 + u((1-u)r_1 + u r_2) \\
 &= (1-u)[(1-u)[(1-u)p_0 + u p_1] + u[(1-u)p_1 + u p_2]] + u[(1-u)[(1-u)p_1 + u p_2] + u[(1-u)p_2 + u p_3]] \\
 &= (1-u)^3 p_0 + \underline{u(1-u)^2 p_1} + \underline{u(1-u)^2 p_1} + \underline{u^2(1-u)p_2} + \underline{u(1-u)^2 p_1} + \underline{u^2(1-u)p_2} + \underline{u^2(1-u)p_2} + u^3 p_3 \\
 &= (1-u)^3 p_0 + 3u(1-u)^2 p_1 + 3u^2(1-u)p_2 + u^3 p_3 = f(u)
 \end{aligned}$$

□