1. Classes and Their Functions

Vertex

- Purpose: Represents a vertex (or node) in a graph.
- Attributes:
    - id: A unique identifier for the vertex.
    - point: Coordinates on the panel where the vertex is drawn.
    - radius: Size of the vertex.
    - color: Color of the vertex.
    - connectedEdges: A list of edges connected to this vertex.
    - neighbors: A list of neighboring vertices connected by edges.
- Key Methods:
    - Draw: Draws the vertex on the provided Graphics object.
    - MoveCoordinates: Changes the position of the vertex.
    - GetNeighbors: Returns a list of neighboring vertices.
    - IsObject: Checks if a point on the panel intersects with the vertex.
    - AddParallelEdge: Increments the count of parallel edges for this vertex.

Edge

- Purpose: Represents an edge (or link) in a graph, connecting two vertices.
- Attributes:
    - vertex1 and vertex2: The vertices connected by this edge.
    - edgeWidth: Width of the edge.
    - color: Color of the edge.
    - current: Indicates if the edge is selected.
    - parallelEdge: An index for identifying parallel edges.
- Key Methods:
    - Draw: Draws the edge on the provided Graphics object.
    - GetGraphicsPath: Returns the graphical path representing the edge.
    - IsObject: Checks if a point on the panel intersects with the edge.
    - CalculateParallelOffset: Calculates the offset for parallel edges.
    - BuildGraphicsPath: Builds the graphical path for drawing the edge.

Graph

- Purpose: Represents the graph structure, containing vertices and edges, and provides graph-related operations.
- Attributes:
    - edges: A list of all edges in the graph.
    - vertices: A list of all vertices in the graph.
    - currentVertex: The currently selected vertex.
    - currentEdges: A list of currently selected edges.

        - hasCurrentVertex: Indicates if a vertex is selected.
- Key Methods:
    - AddVertex: Adds a new vertex to the graph.
    - AddEdge: Connects two vertices with an edge.
    - UpdateSelectedVertex: Updates the selected state of a vertex.
    - SelectEdge: Selects or deselects an edge.
    - IsBipartite: Determines if the graph is bipartite.
    - GetAdjacencyMatrix: Returns the adjacency matrix representing connections between vertices.
    - GetConnectedComponentCount: Returns the count of connected components in the graph.
    - PerformDepthFirstSearch: Recursive depth-first search for graph traversal.
    - PaintGraph: Paints the graph for bipartite verification.

SketchPad (WinForms-based)

Purpose: Graphical interface for interacting with the graph, built using Windows Forms.

- Attributes:
    - graph: Instance of the Graph class.
    - vertexCount: Counter for tracking the number of vertices.
    - moveState: Indicates if a vertex is being moved.
    - deleteState: Indicates if objects are in deletion mode.
    - movingVertex: The vertex currently being moved.
    - lastVertexLocation: Last location of the moving vertex.
- Key Methods:
    - PanelPaint: Draws the vertices and edges on the panel.
    - PanelMouseClick: Handles mouse clicks to add/select objects.
    - PanelMouseMove: Handles mouse movements to drag vertices.
    - GraphPanelMouseDown: Begins dragging a vertex when the mouse is pressed.
    - GraphPanelMouseUp: Stops dragging a vertex when the mouse is released.
    - TryDeleteObject: Attempts to delete an object based on its position.
    - TryGetVertex and TryGetEdge: Retrieve a vertex or edge based on a given point.
    - UpdateGraphInfo: Updates displayed information about the graph.
    - DeleteButtonClick: Toggles the delete mode.
    - PaintButtonClick: Changes the color of selected objects.
    - ClearAllButtonClick: Clears the entire graph.
    - BipartiteTestButtonClick: Tests if the graph is bipartite.

2. Implemented Features

Graph Manipulation

- Vertex Operations: Add, move, select, and delete vertices.
- Edge Operations: Add, select, and delete edges; handle parallel edges and loops.
- Graph Operations: Retrieve adjacency matrix, check bipartite status, calculate connected components.

User Interaction

- Drawing: Visualize vertices and edges with custom colors.
- Selection: Select vertices and edges with right-click.
- Dragging: Move vertices by dragging.
- Deletion: Delete vertices and edges with delete mode.
- Color Customization: Change vertex and edge colors.

3. How to Use the GraphTheorySketchPad

1. Adding Vertices: Left-click on the panel to add a new vertex.
2. Adding Edges: Select two vertices by right-clicking, then an edge will be created between them.
3. Selecting Objects: Right-click on a vertex or edge to select or deselect it.
4. Moving Vertices: Left-click and hold on a vertex to move it. Drag to the desired position and release the mouse button.
5. Deleting Objects: Click the "Delete" button to enable delete mode. Left-click on a vertex or edge to delete it.
6. Changing Colors: Use the color selection button to choose a color. Click "Paint" to change the color of selected objects.
7. Bipartite Test: Click the "Test Bipartite" button to check if the graph is bipartite.
8. Clearing the Graph: Click the "Clear All" button to remove all vertices and edges.