

# Observations on Factoring Using the GNFS



# How Do I Factor - GNFS

1. Polynomial Selection
2. Sieving
3. Combine



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2. irreducible over rationals
3. interpreted mod  $n$  have common root mod  $m$



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1. Millions of pairs  $a, b$   
2. Such that  $b^d \cdot f(a/b)$  &  $b^e \cdot g(a/b)$  factor 'prettily' (are smooth)  
3. Via Lattice Sieving



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1. Filter Relations & Build Matrix
2. Linear Algebra using Lanczos
3. "Square Root Phase"



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Slow & Unparallelizable

512 Bit ~8 Core-Days

768 Bit ~155 Core-Years\*

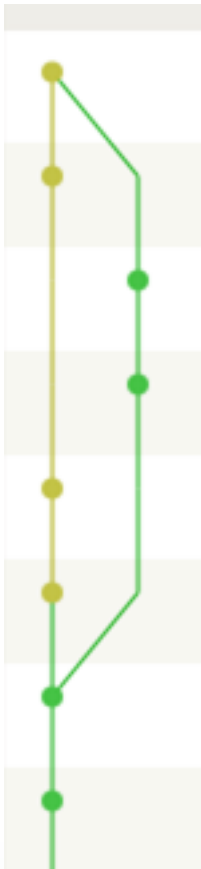
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# Some Details on Factoring



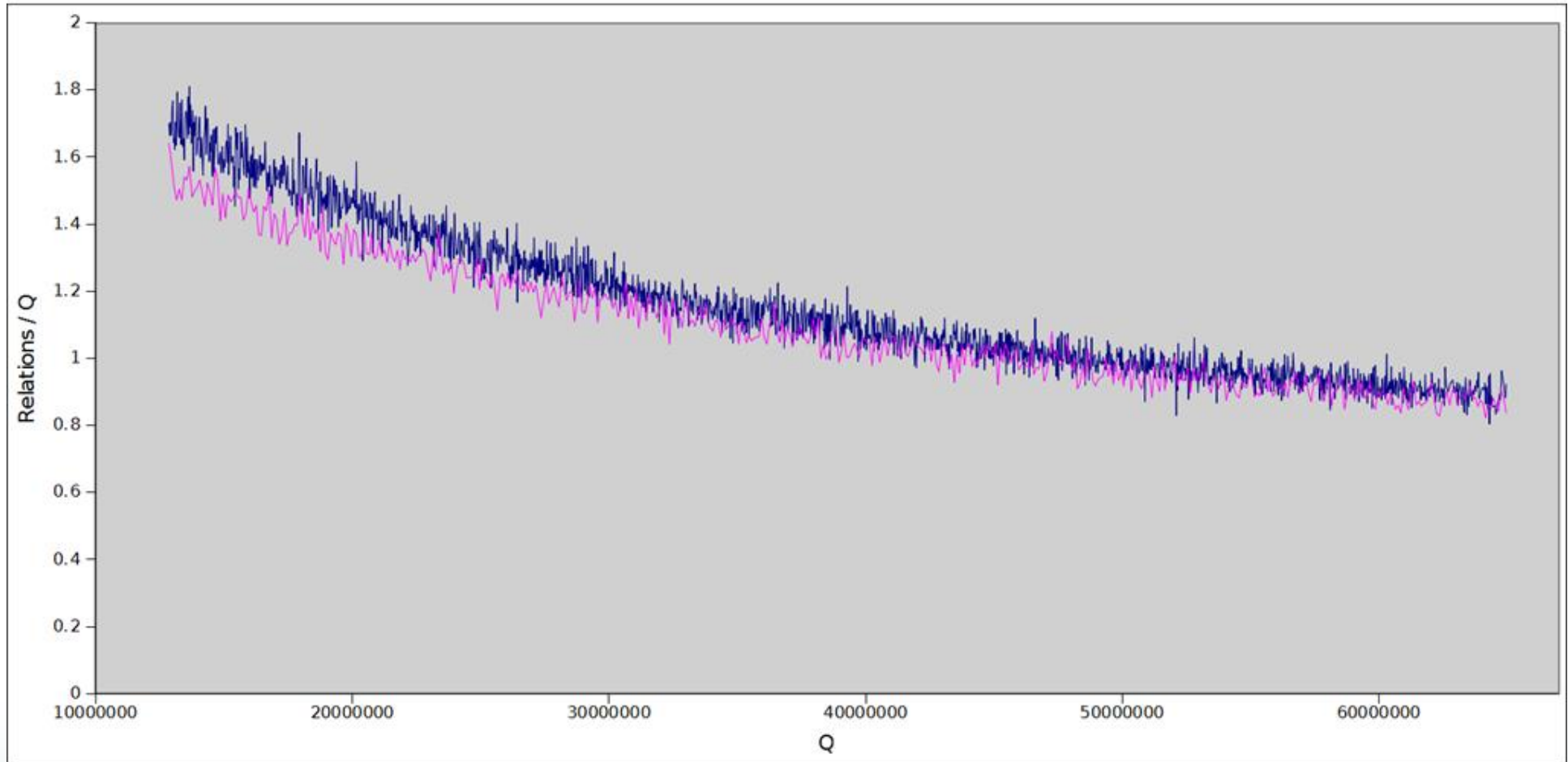
- Polynomial Selection
- Siever Comparisons
- Oversieving





# Misconceptions about Polynomials

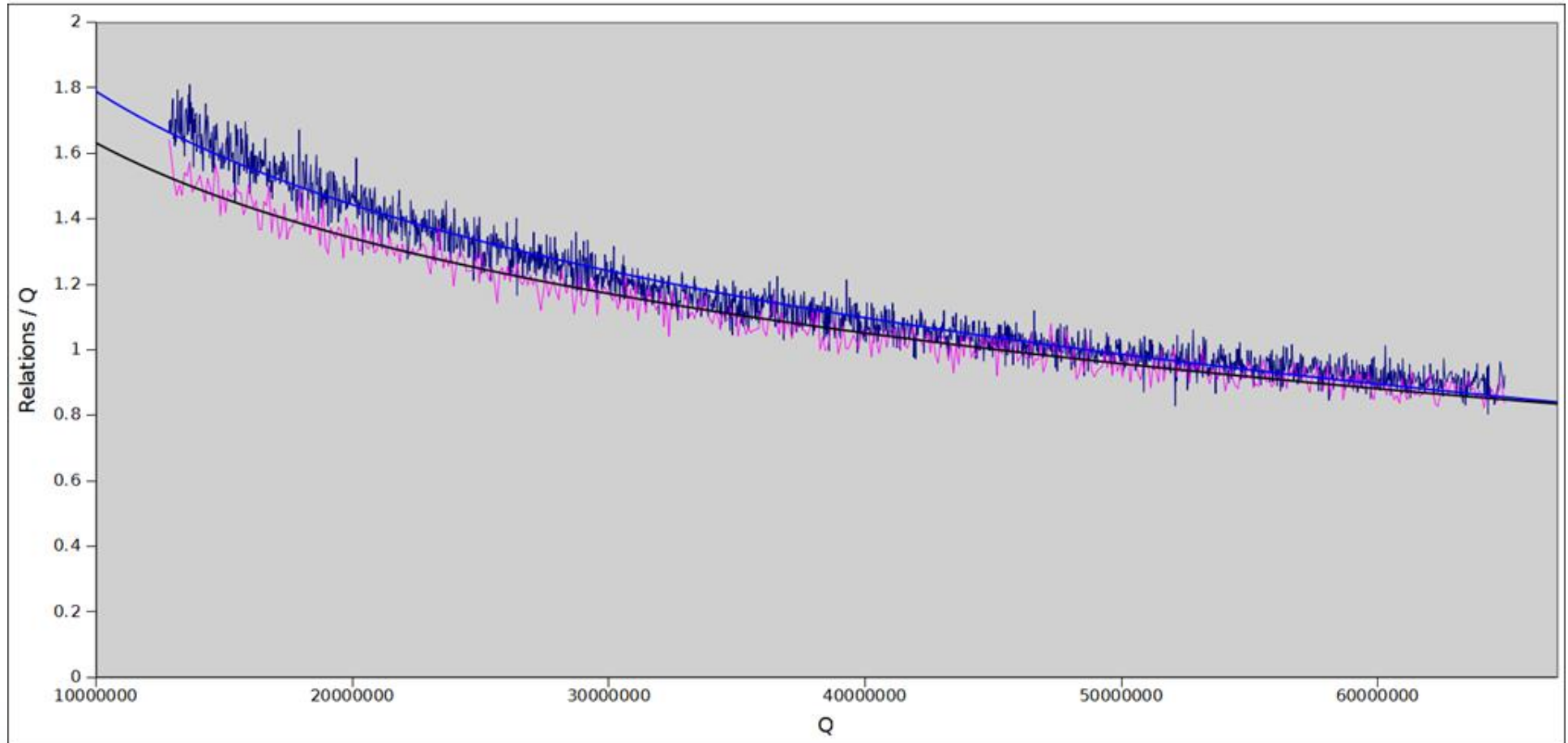
Relations / Q (higher is better)





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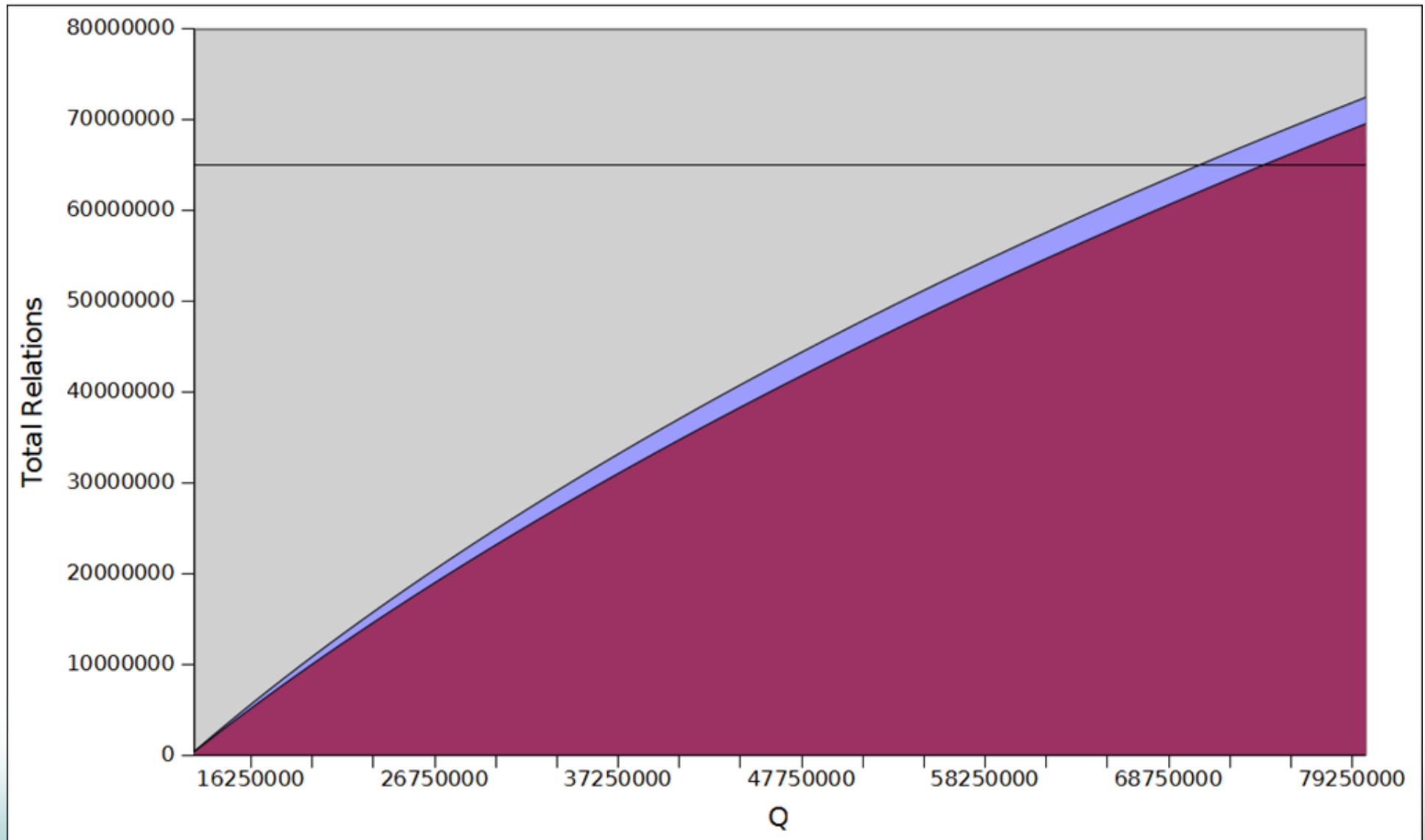
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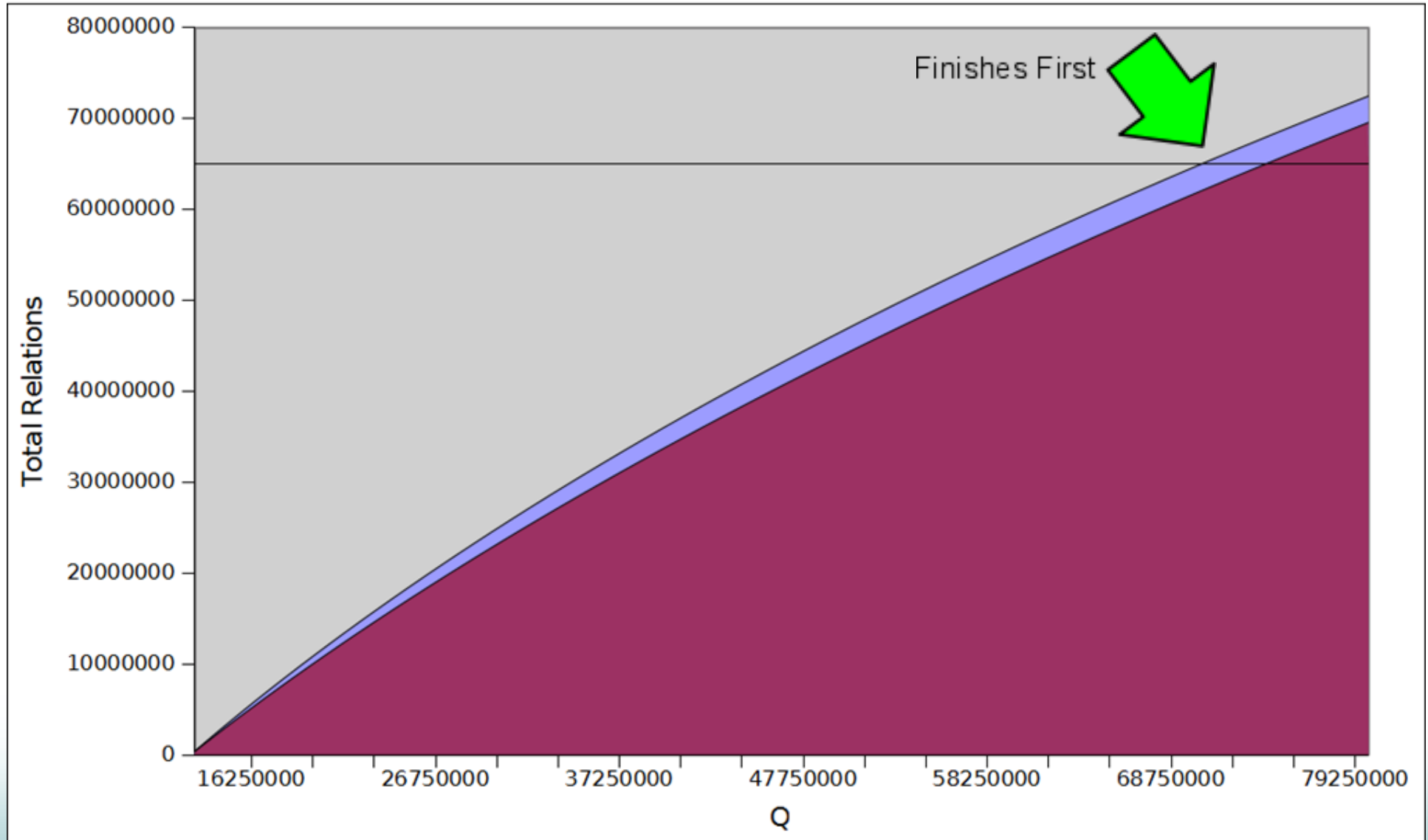
Total Relations By Q





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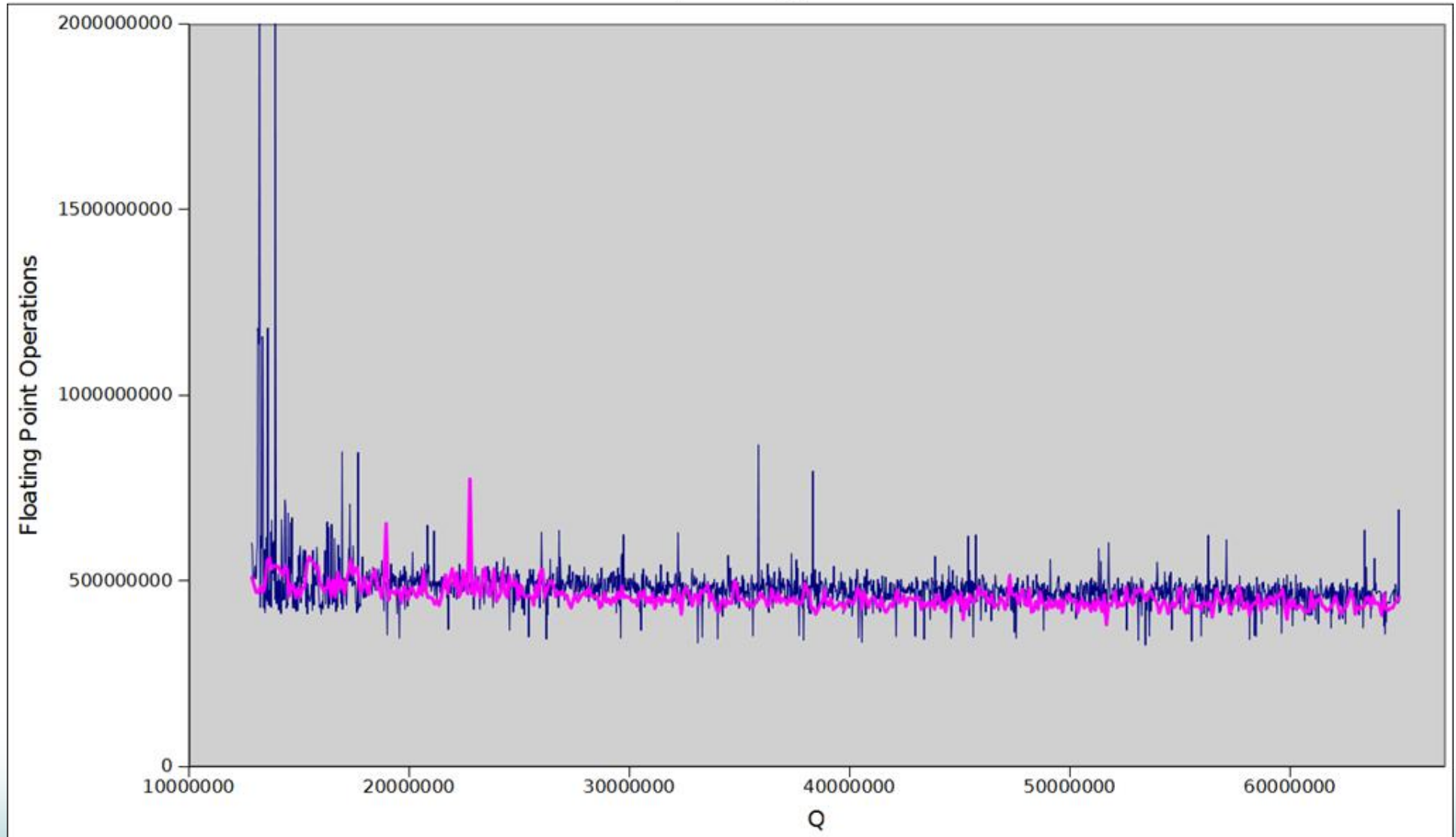
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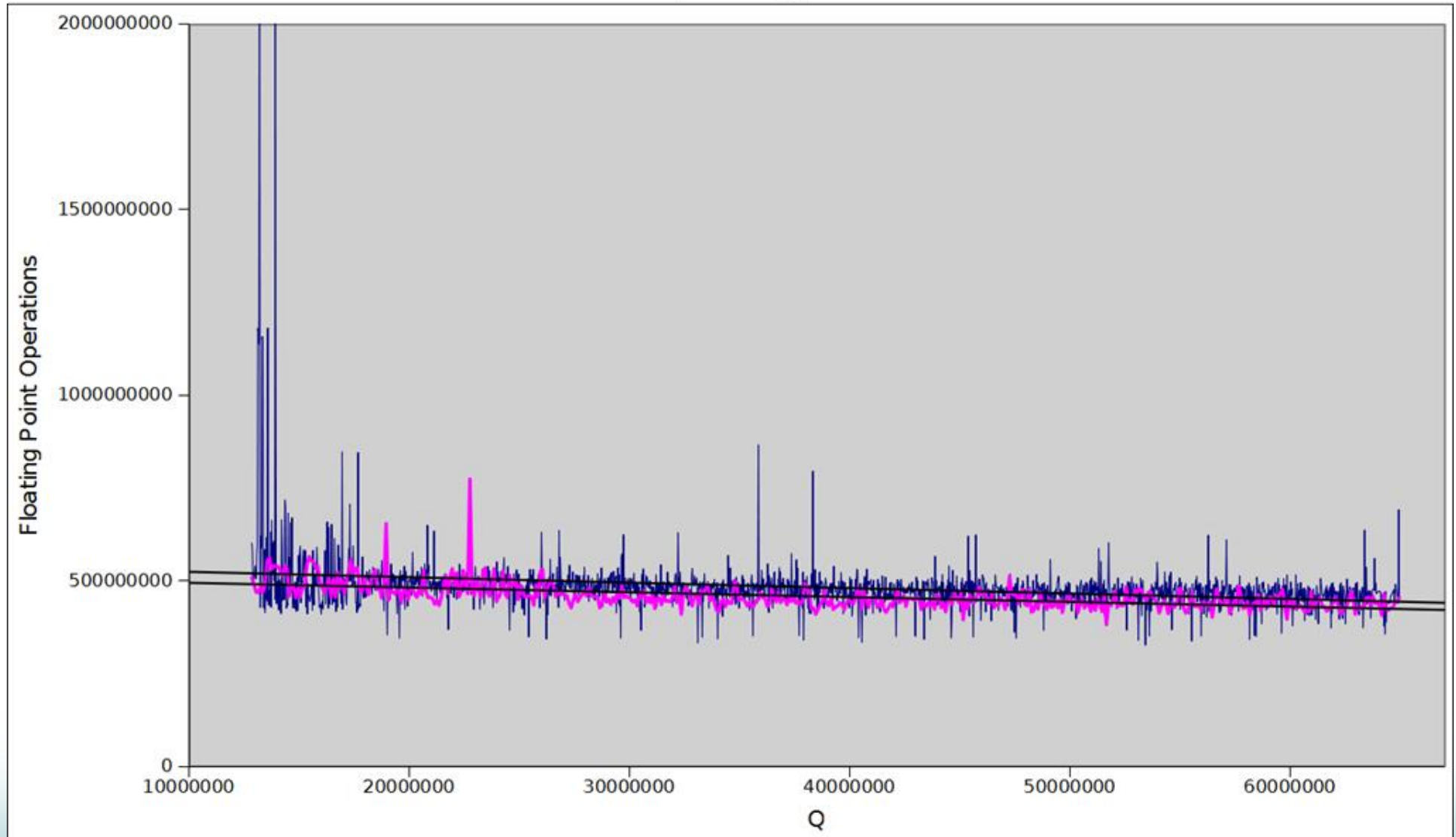
Operations/Q





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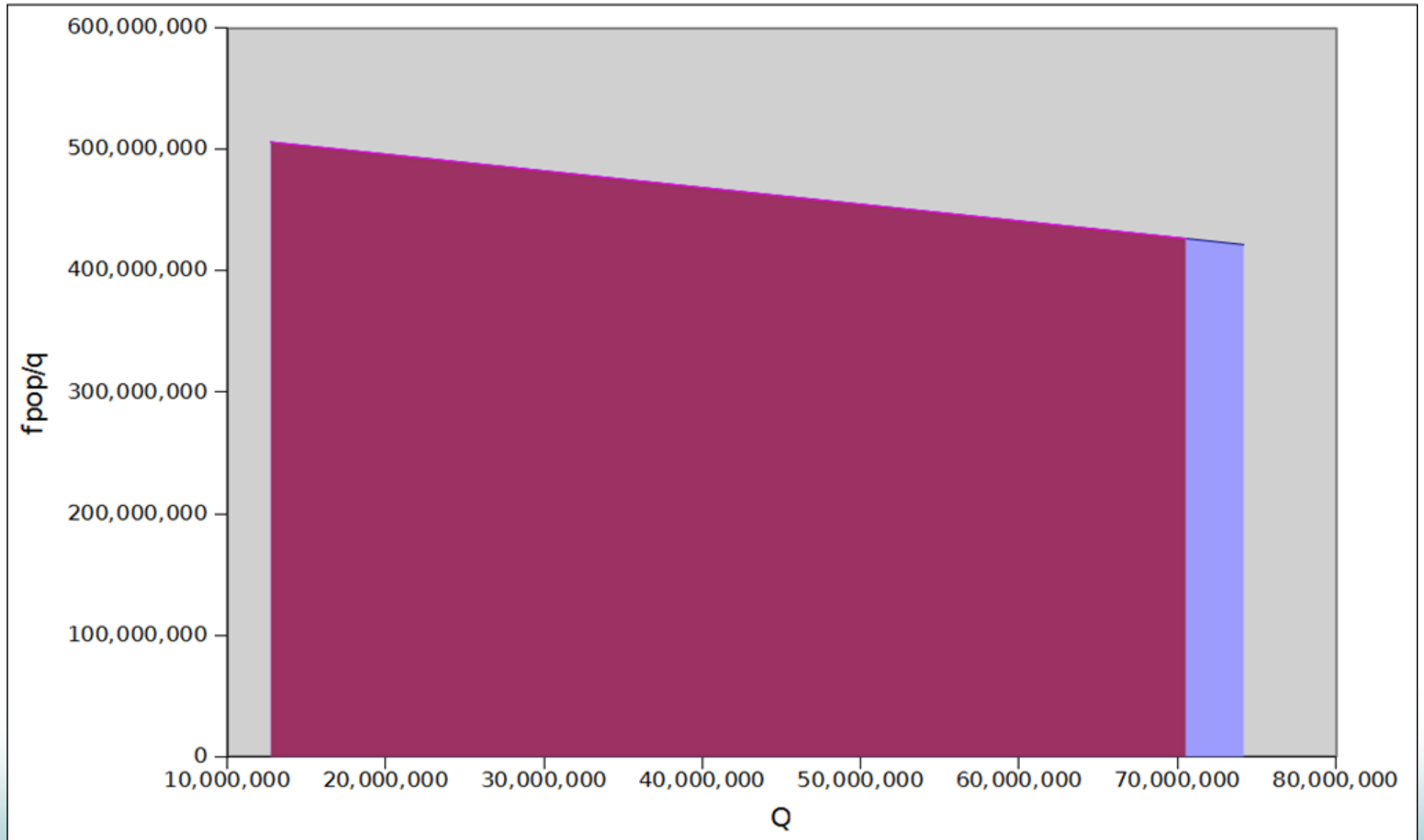
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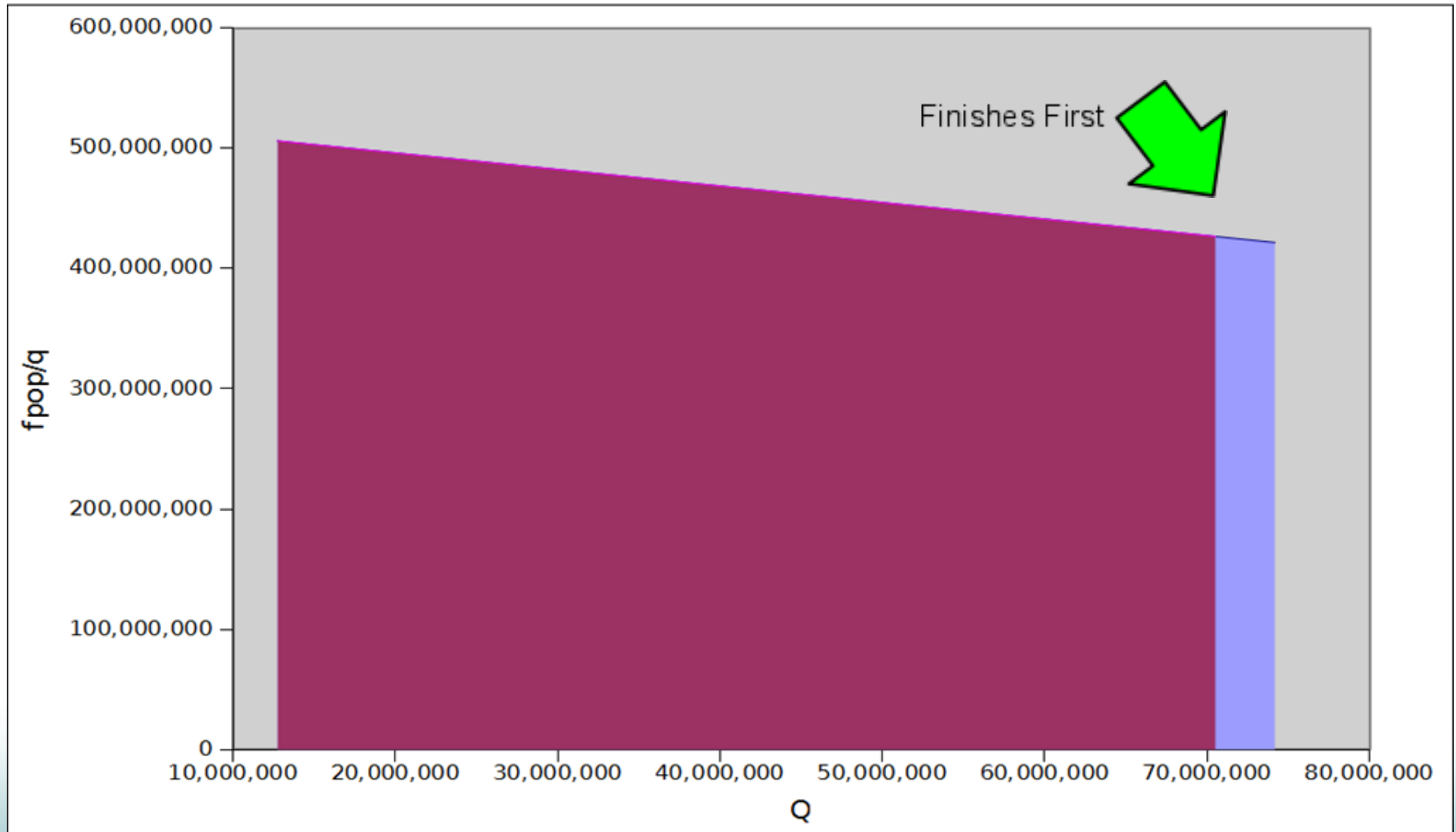
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Total Operations

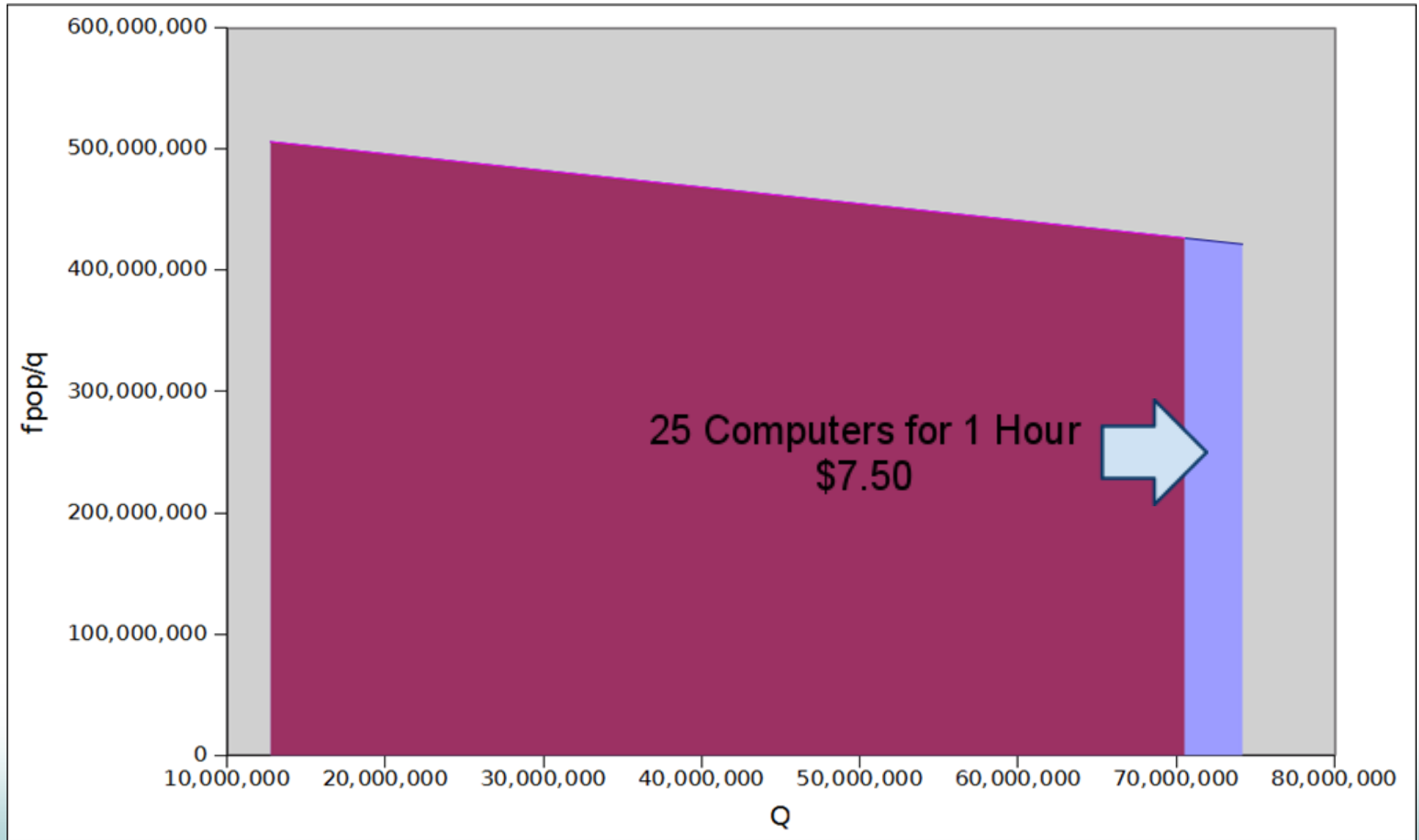






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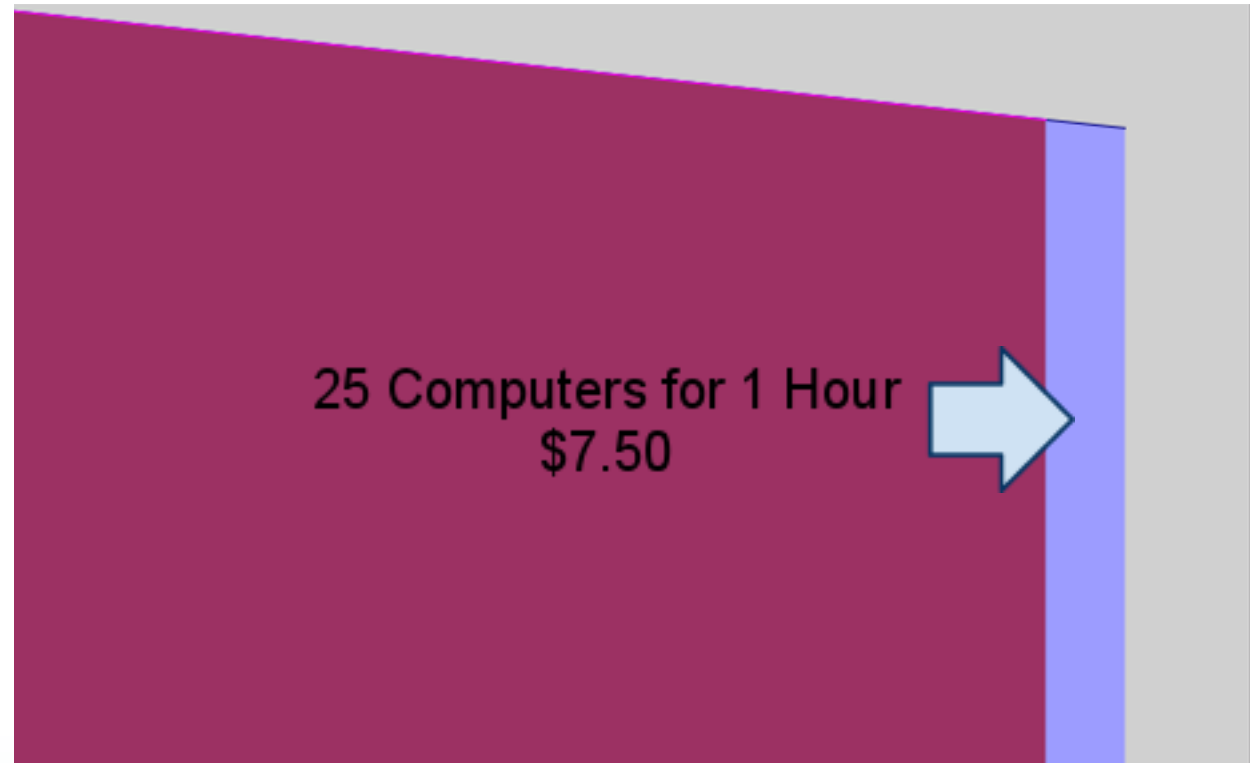
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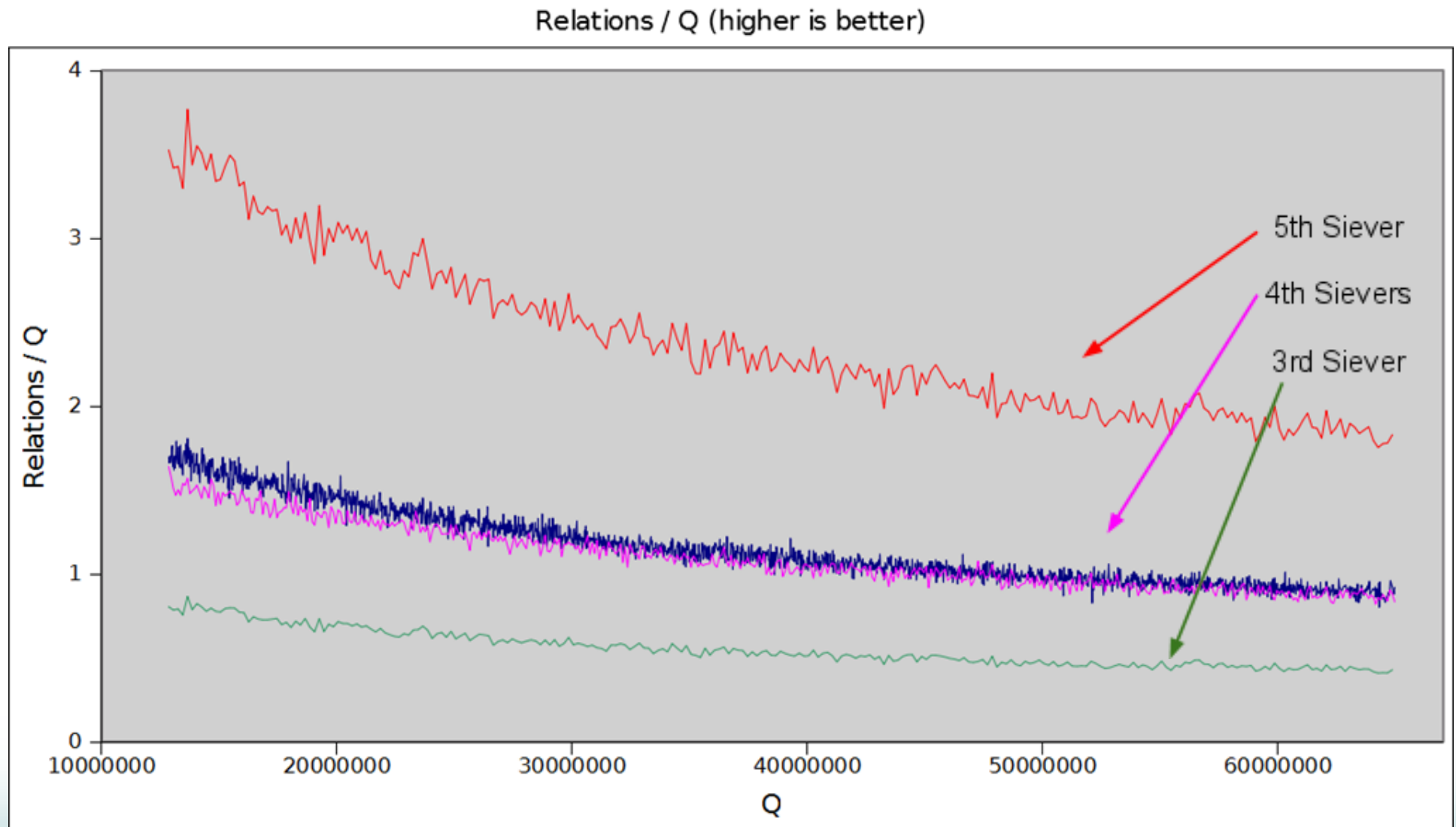
If time is more valuable to you than (a relatively little) money it is in your best interest to take the first polynomial you get and sieve with that, rather than doing another poly-selection run.



(this advice is only  
for 512-bit semiprimes.)



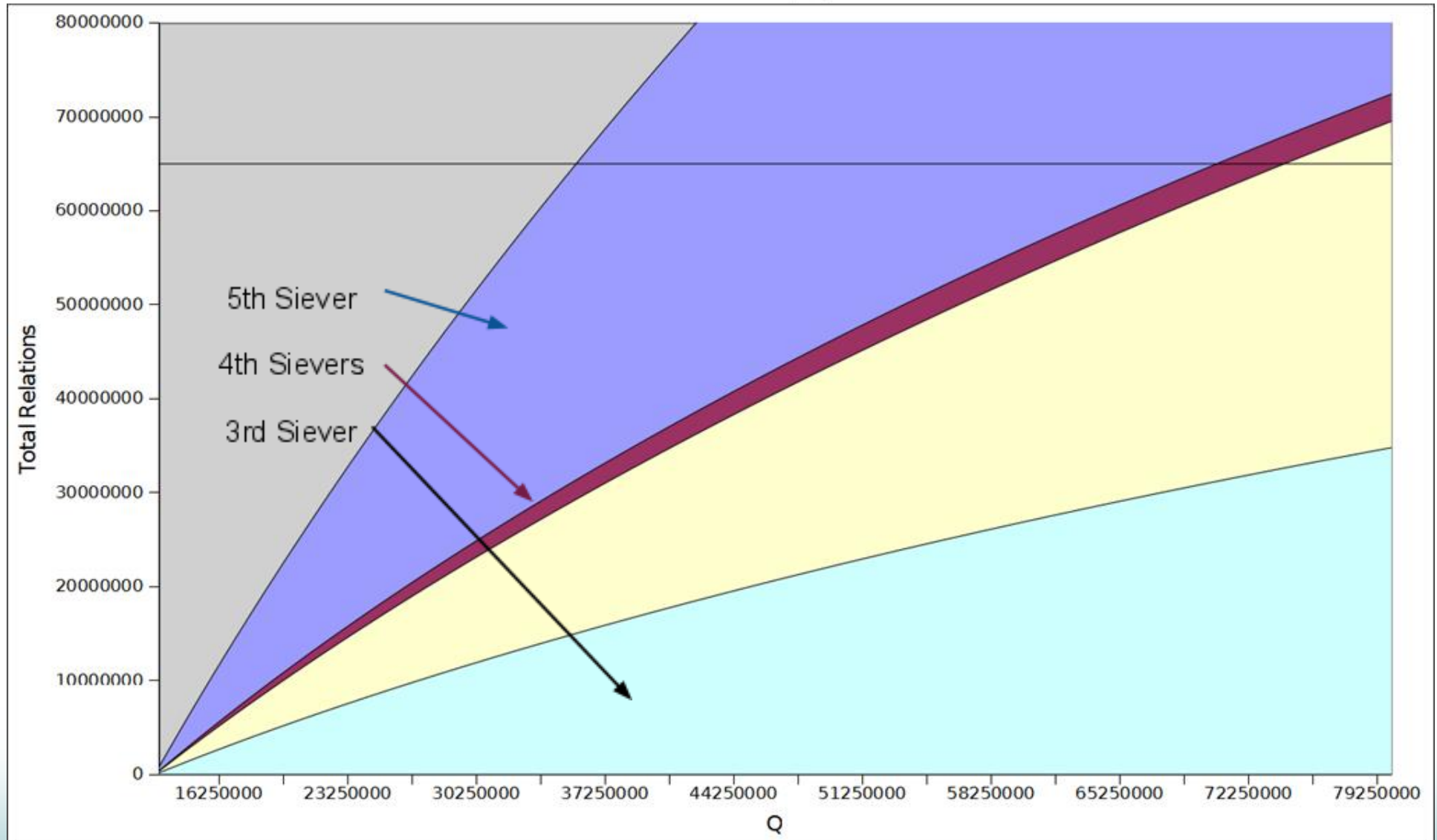
# Siever Comparisons





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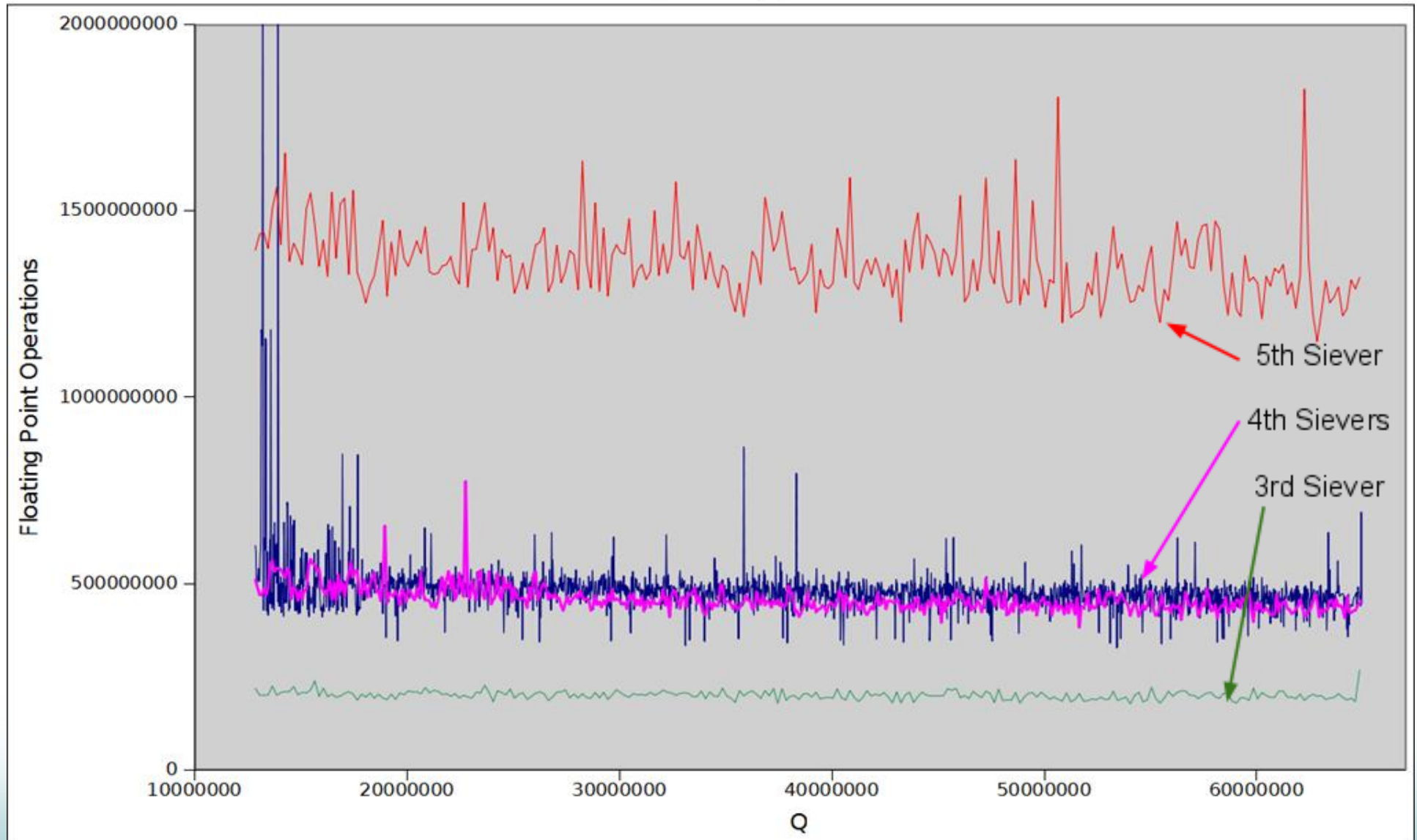
Total Relations By Q





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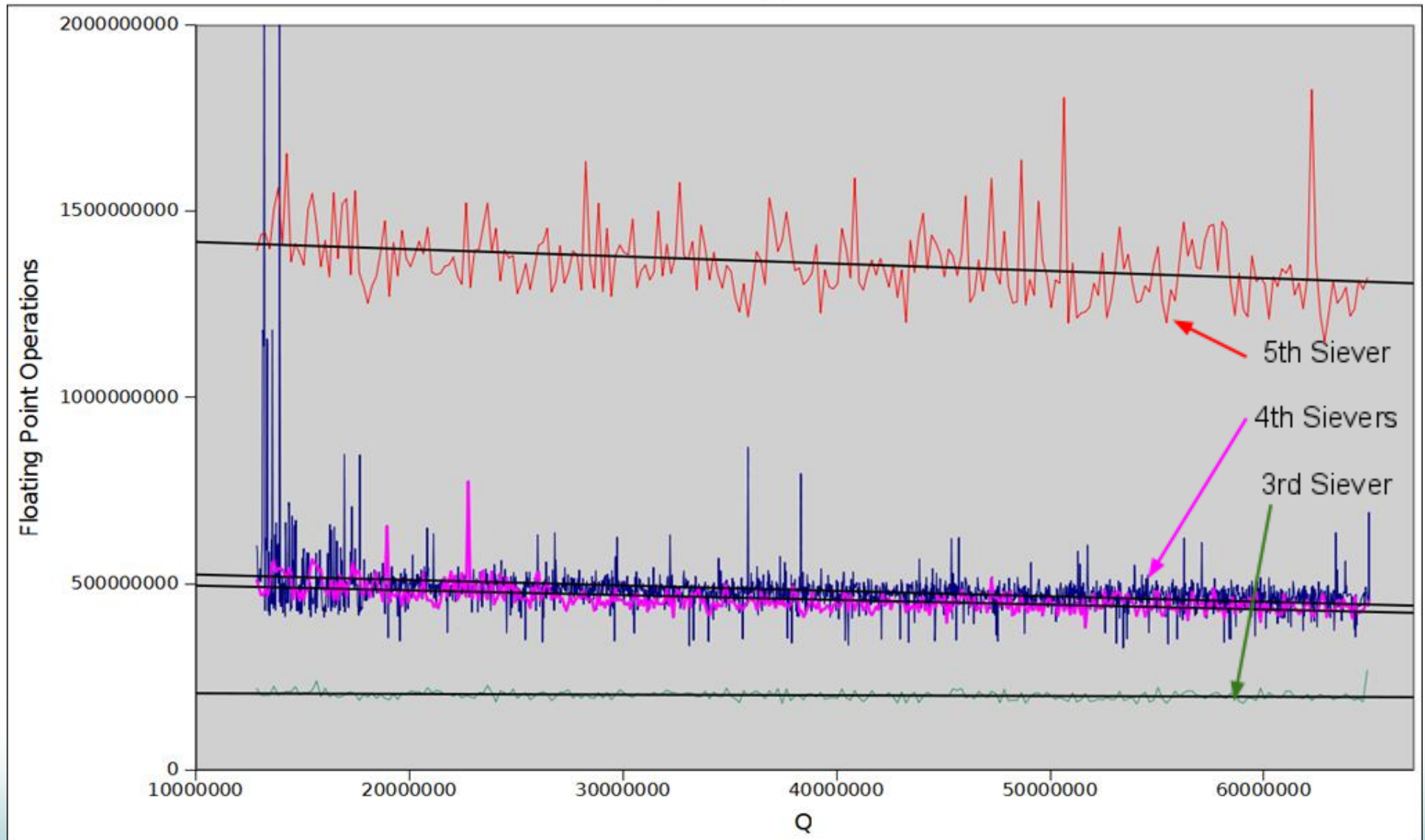
Operations/Q





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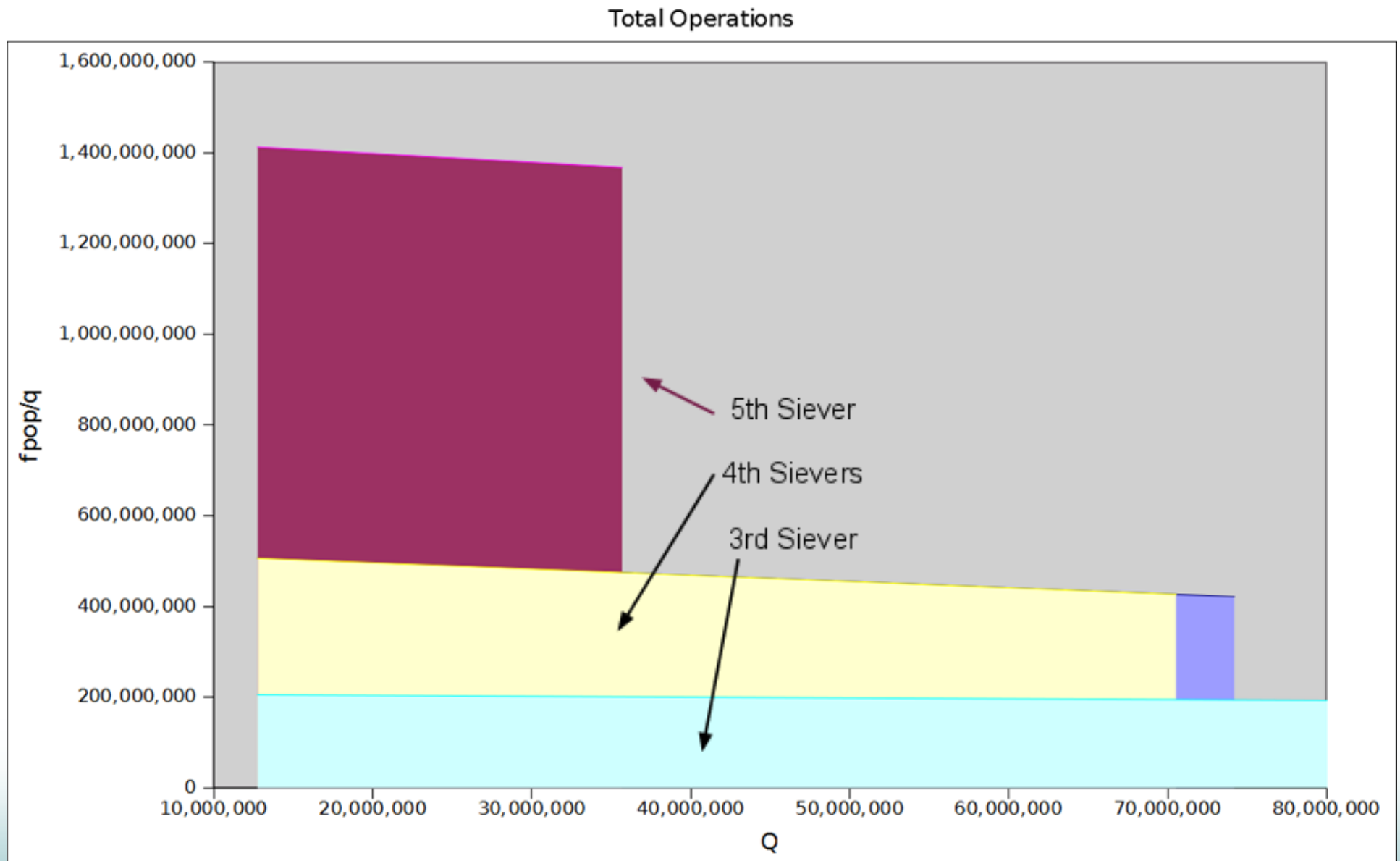
Operations/Q





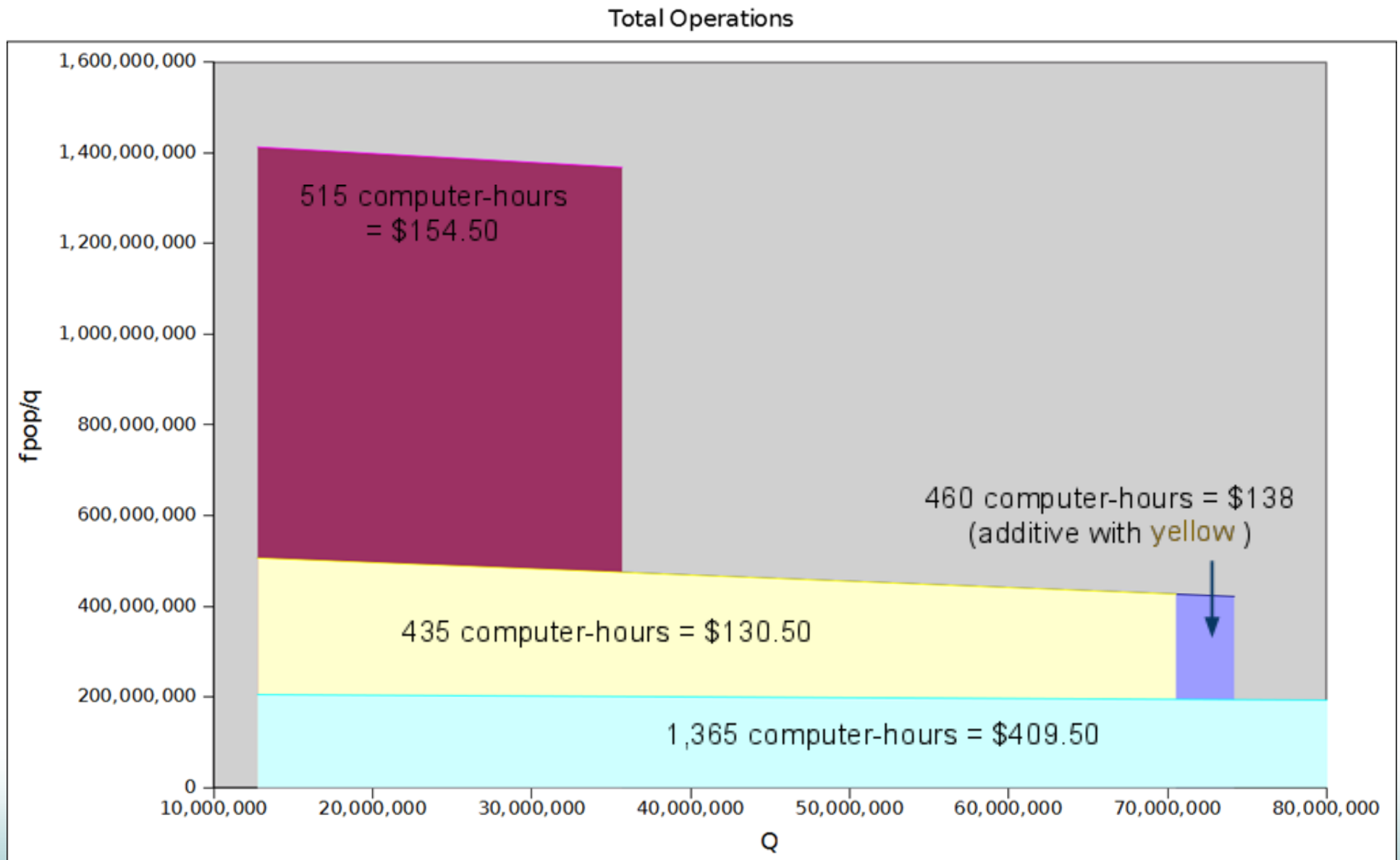


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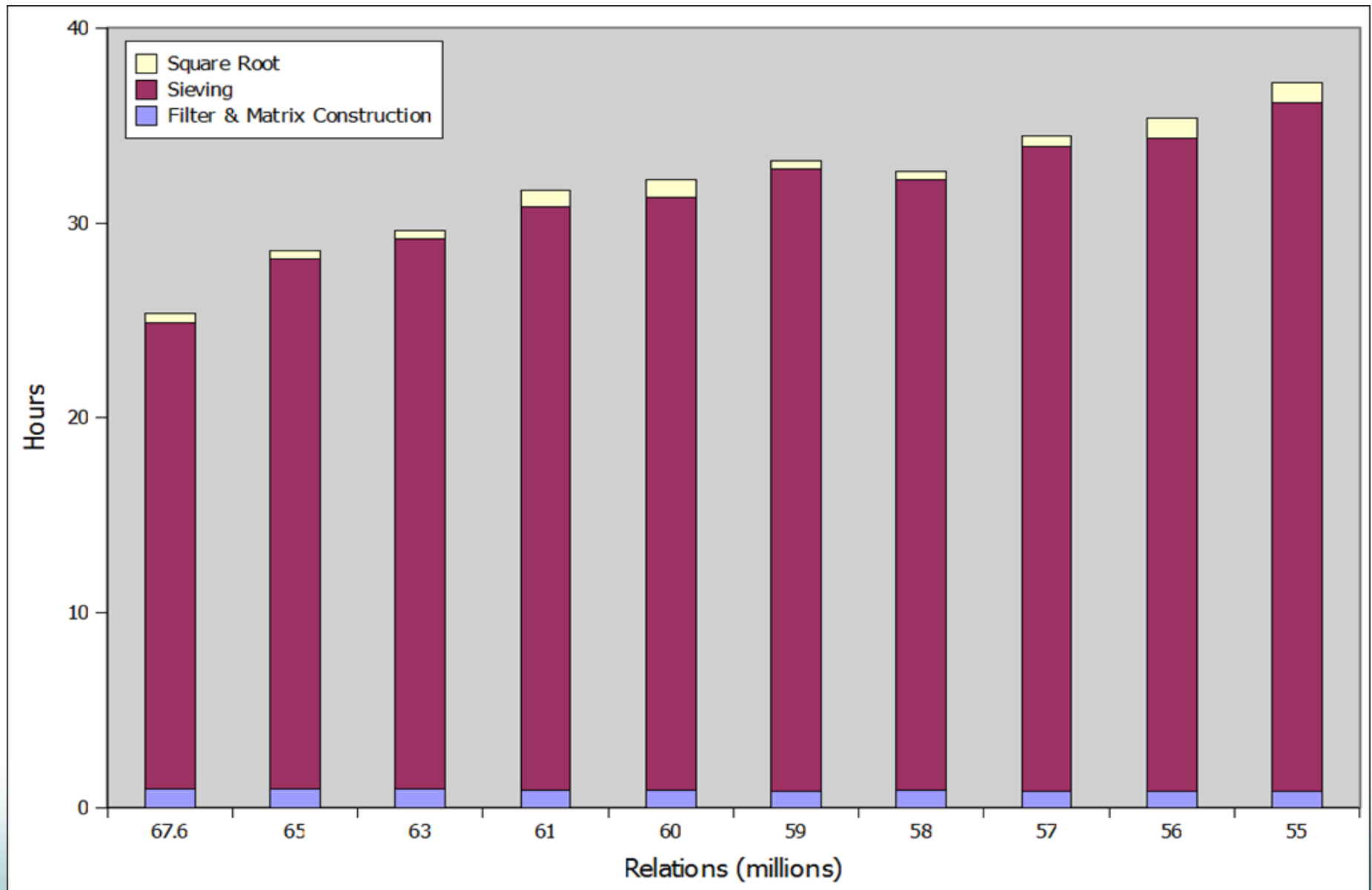
# Siever Comparisons







# Oversieving





# Obligatory Ending Slide

Fin

Thanks:

- GDS
- NYSec
- MersenneForum & jasonp

Tom Ritter

<http://ritter.vg>

(encrypted mail preferred)

Big Ups To:

- jasonp

<http://www.gdssecurity.com/>

<https://github.com/GDSecurity/cloud-and-control>