

Observations on Factoring Using the GNFS



How Do I Factor - GNFS

1. Polynomial Selection
2. Sieving
3. Combine



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1. $f(x)$ & $g(x)$ of degree d, e
2. irreducible over rationals
3. interpreted mod n have common root mod m



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1. Millions of pairs a, b
2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
3. Via Lattice Sieving

Some more on this:

<http://mersenneforum.org/showthread.php?t=15796>



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1. Filter Relations & Build Matrix
2. Linear Algebra using Lanczos
3. "Square Root Phase"



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Slow & Unparallelizable

512 Bit ~8 Core-Days
768 Bit ~155 Core-Years*

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Why is it unparallelizable?

<http://www.mersenneforum.org/showthread.php?t=1536>

1

* is because the 768 bit semiprime used Block Weildmann as opposed to msieve's block lanczos algorithm.

<http://www.mersenneforum.org/showthread.php?t=1295>

8



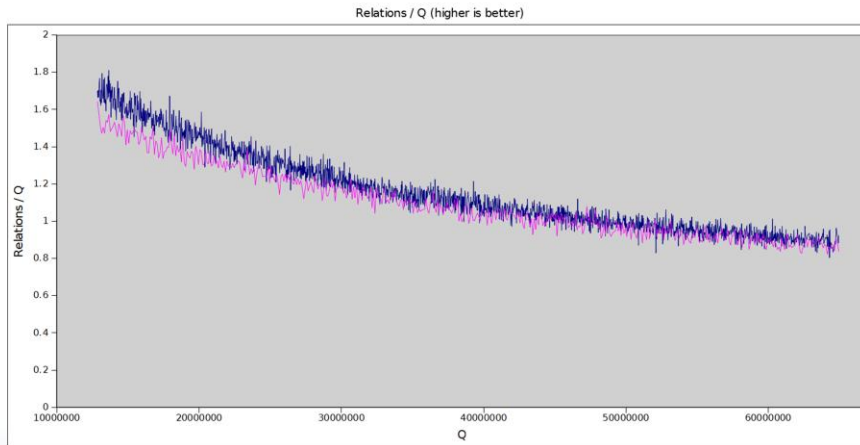
Some Details on Factoring



- Polynomial Selection
- Siever Comparisons
- Oversieving



Misconceptions about Polynomials

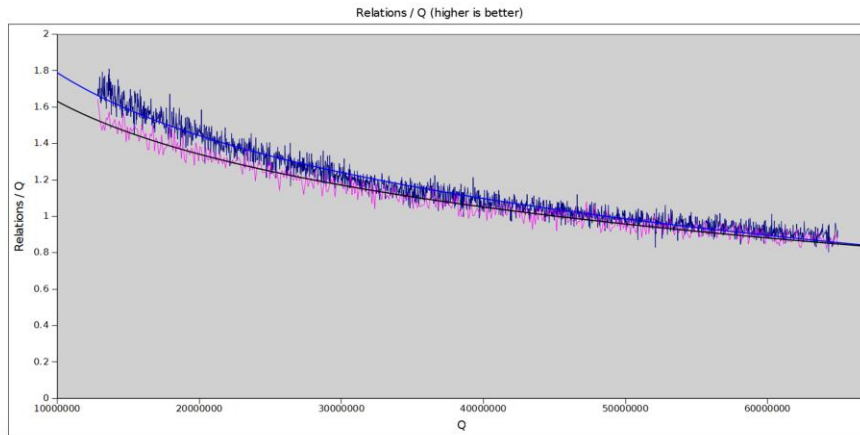


Comparison of sieve results for two polynomials

- Murphy 2.615×10^{-12}
- Murphy 3.023×10^{-12}



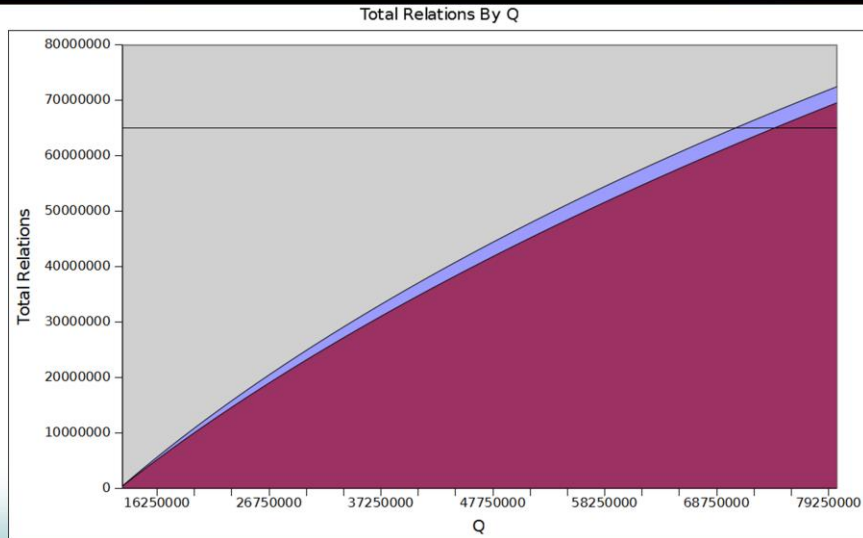
Misconceptions about Polynomials



Trend Lines. We can integrate under these curves to get...



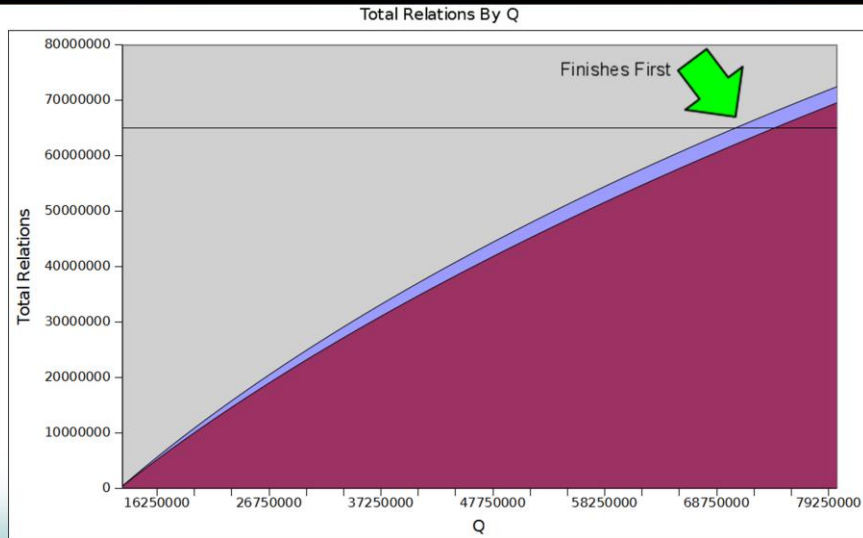
Misconceptions about Polynomials



The total sieve pairs as a function of Q

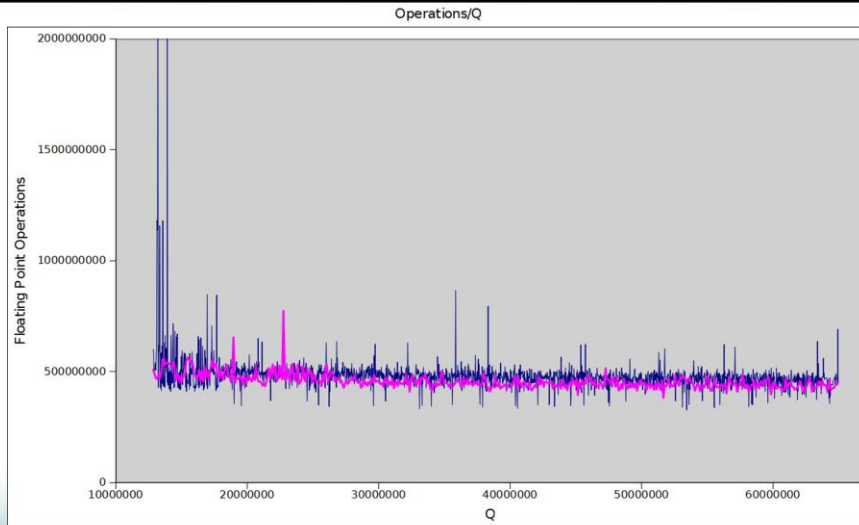


Misconceptions about Polynomials





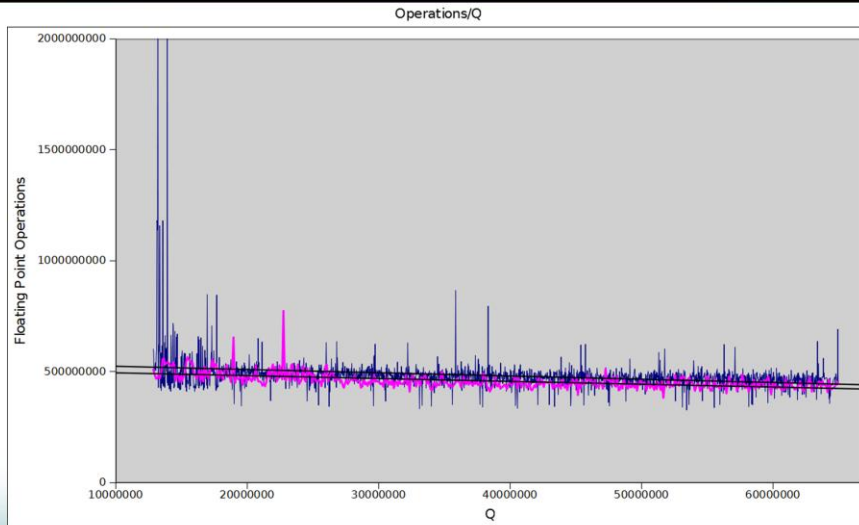
Misconceptions about Polynomials



Floating Point Operations per polynomial



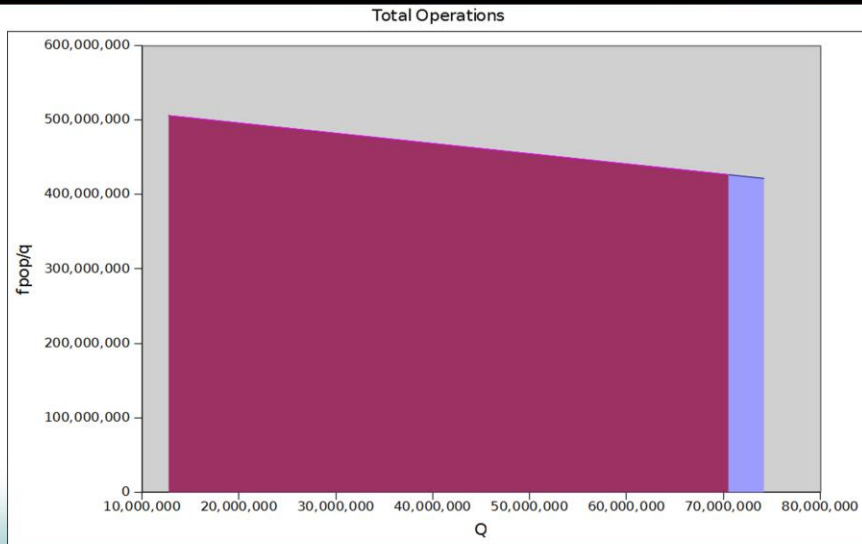
Misconceptions about Polynomials



They're pretty much the same.



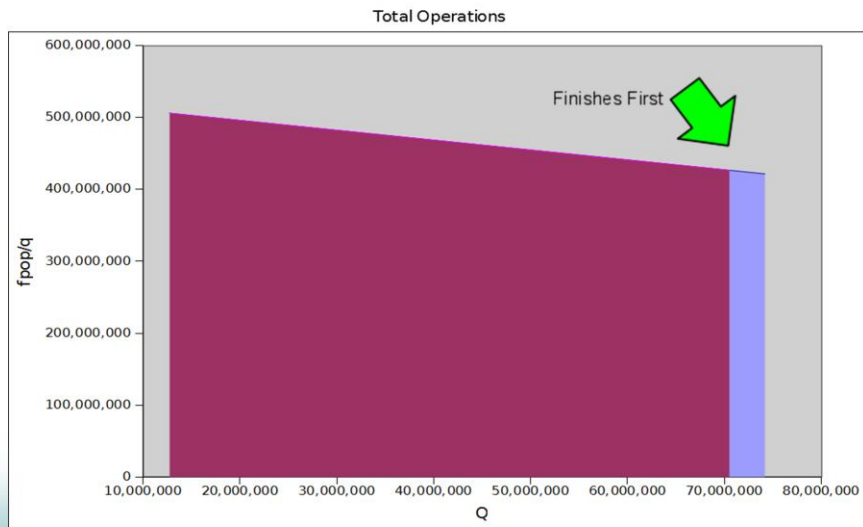
Misconceptions about Polynomials



Integrate under that line (the average actually) and cut it off when the polynomial finishes gathering enough relations, and we have the total amount of work done for each polynomial to achieve the requisite number of sieve pairs.

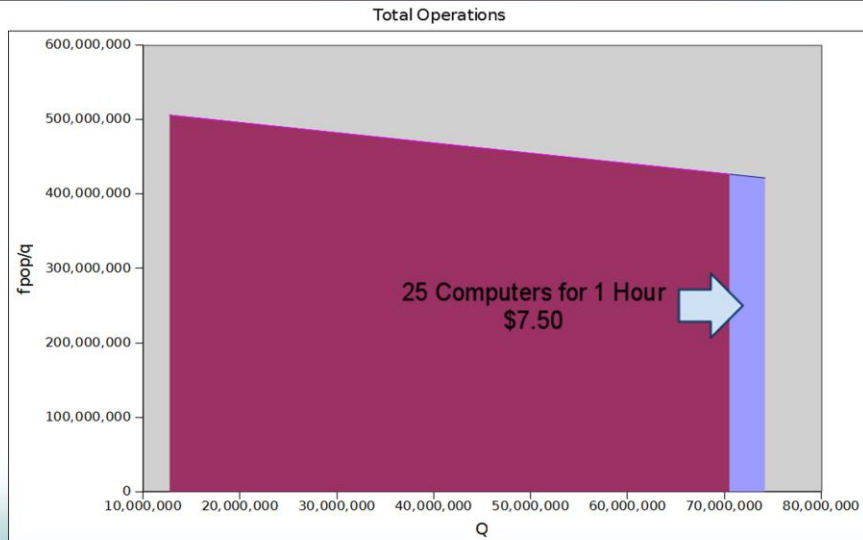


Misconceptions about Polynomials





Misconceptions about Polynomials



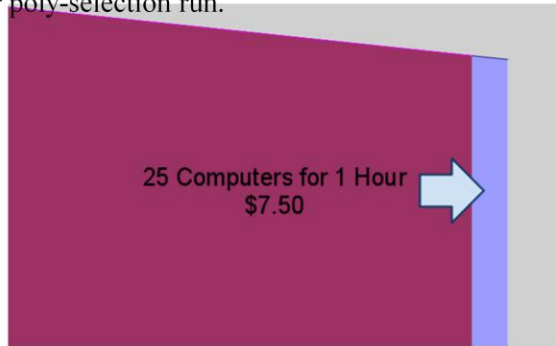
Now, because sieving scales horizontally perfectly, and I was working in EC2, that extra bit of work has a real dollar amount on it. And it's not very much.



Misconceptions about Polynomials

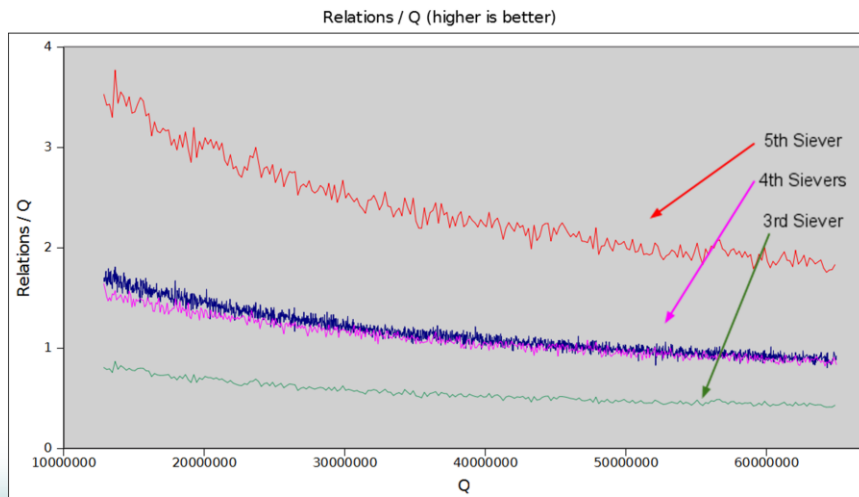
If time is more valuable to you than (not much) money it is in your best interest to take the first polynomial you get and sieve with that, rather than doing another poly-selection run.

(this advice is only
for 512-bit semiprimes.)





Siever Comparisons

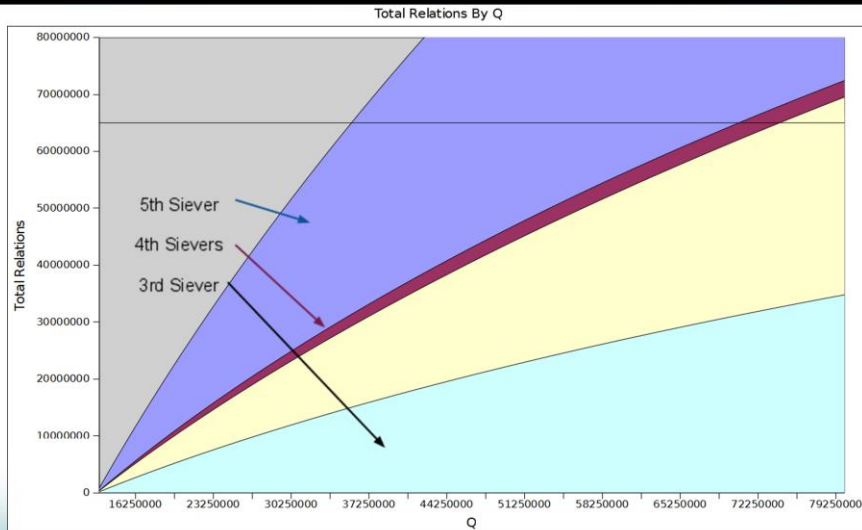


Comparison of

- gnfsลาสieve13e
- gnfsลาสieve14e for two polynomials
- gnfsลาสieve15e



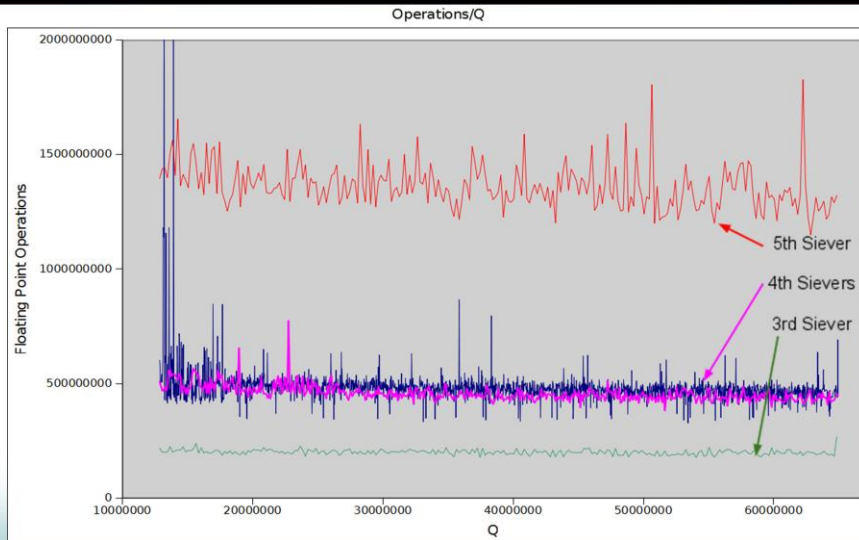
Siever Comparisons



We can again fit trend lines and see where they each finish.



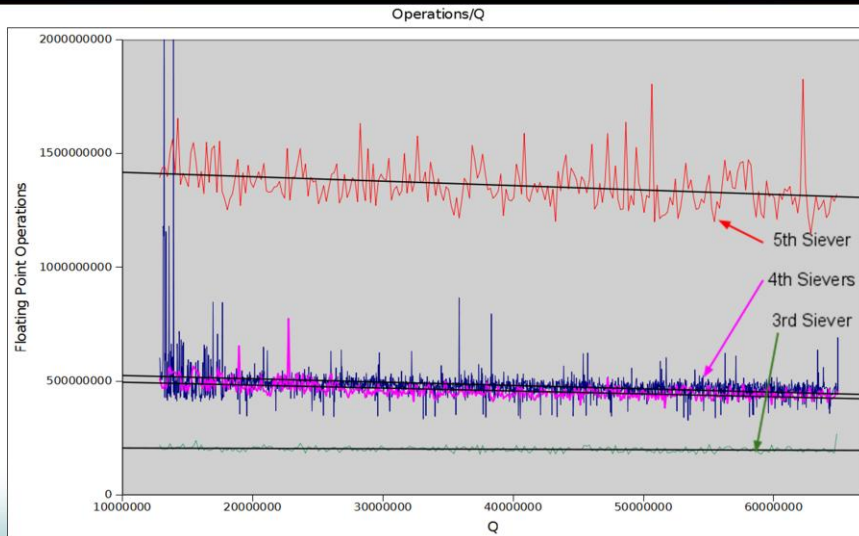
Siever Comparisons



But we need to do an apples-to-apples comparison. While the 5th siever gathers relations much earlier in Q, it also takes much more CPU time to gather those relations.



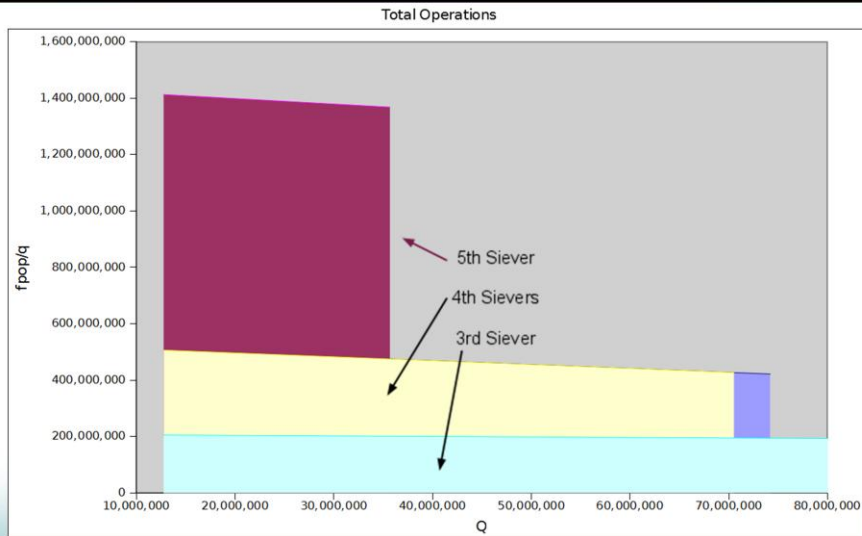
Siever Comparisons



Trend Lines



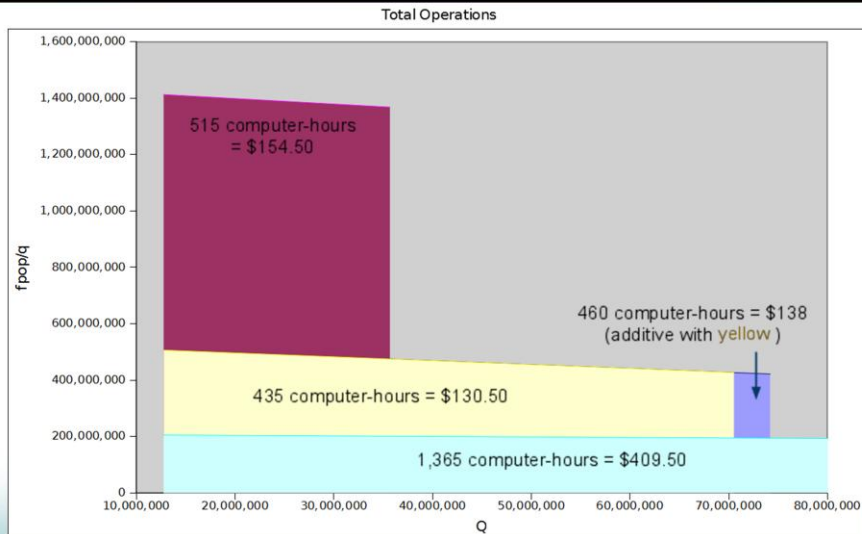
Siever Comparisons



By integrating under the floating point operations trend lines, and stopping when they achieve enough relations we can compare the sievers total work done.



Siever Comparisons



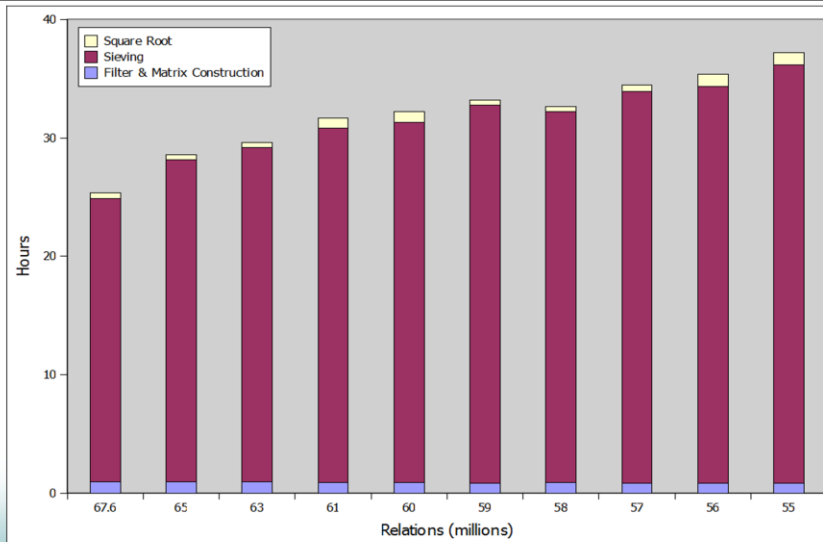
And again, because this is in EC2, we can put this into dollar figures.

This matches up reasonably well with:

<http://eprint.iacr.org/2011/254>



Oversieving



This is a comparison of the Combine done with various numbers of relations. It is clearly in your best interest to oversieve. Sieving scales perfectly out to more machines, and can save you 10 hours in the last step, which is not parallelizable.



Obligatory Ending Slide

Fin

Thanks:

- GDS
- NYSec
- MersenneForum & jasonp

Tom Ritter

<http://ritter.vg>
(encrypted mail preferred)

Big Ups To:

- jasonp

<http://www.gdssecurity.com/>
<https://github.com/GDSSecurity/cloud-and-control>