

Observations on Factoring Using the GNFS

How Do I Factor - GNFS

1. Polynomial Selection
2. Sieving
3. Combine

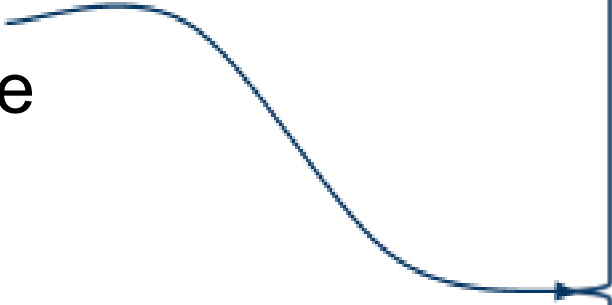
How Do I Factor - GNFS

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2. irreducible over rationals
3. interpreted mod n have common root mod m

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 2. Such that $b^d \cdot f(a/b)$ & $b^e \cdot g(a/b)$ factor 'prettily' (are smooth)
 3. Via Lattice Sieving

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1. Filter Relations & Build Matrix
2. Linear Algebra using Lanczos
3. "Square Root Phase"

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Slow & Unparallelizable

512 Bit ~8 Core-Days

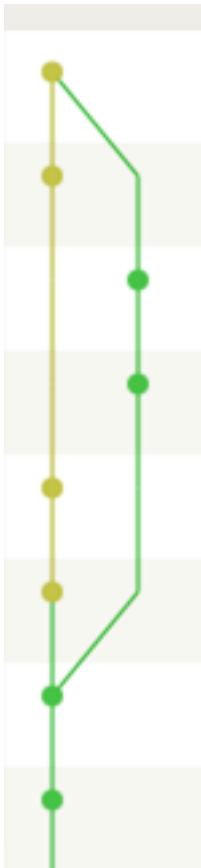
768 Bit ~155 Core-Years*

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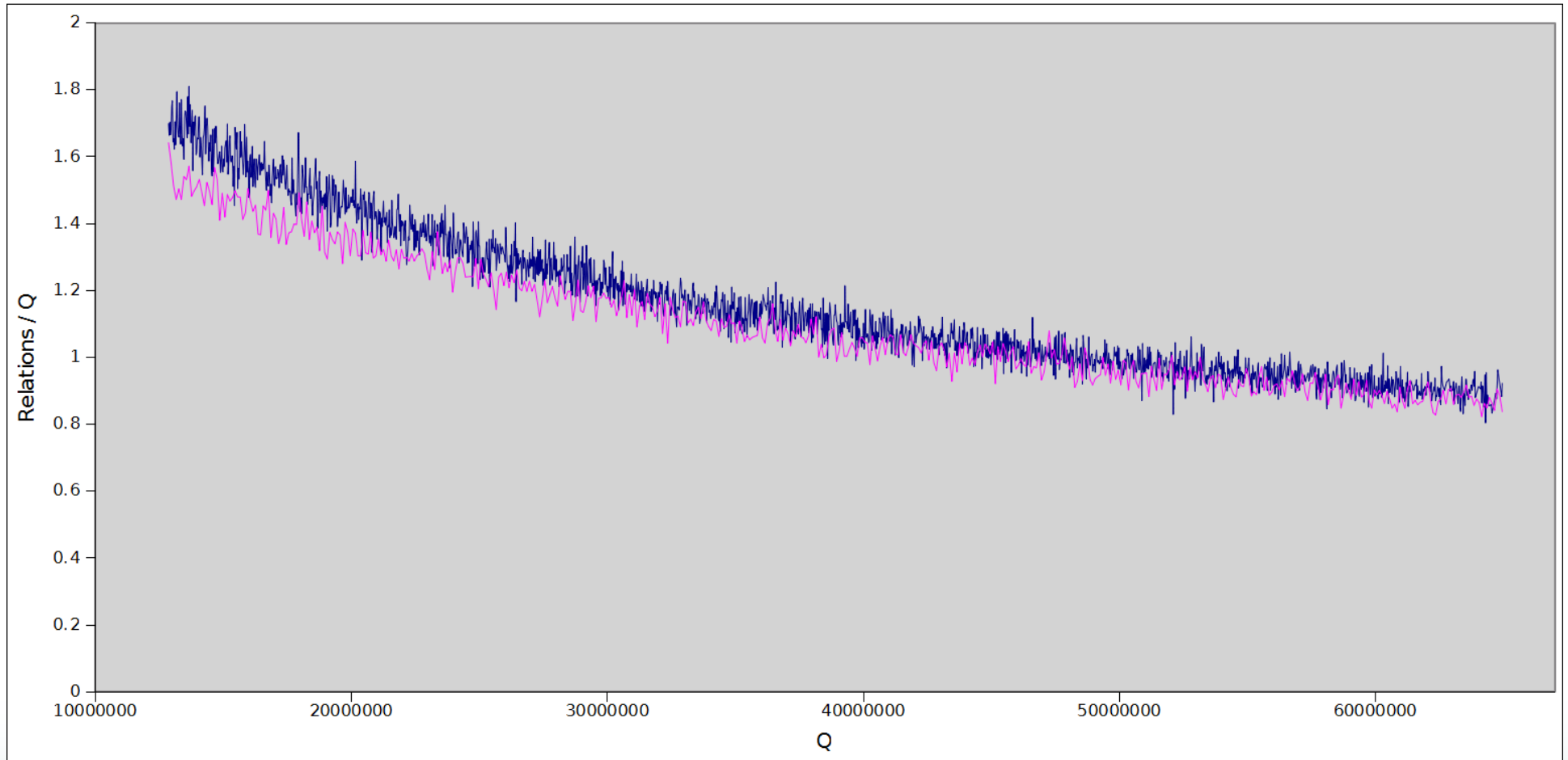
Some Details on Factoring



- Polynomial Selection
- Siever Comparisons
- Oversieving

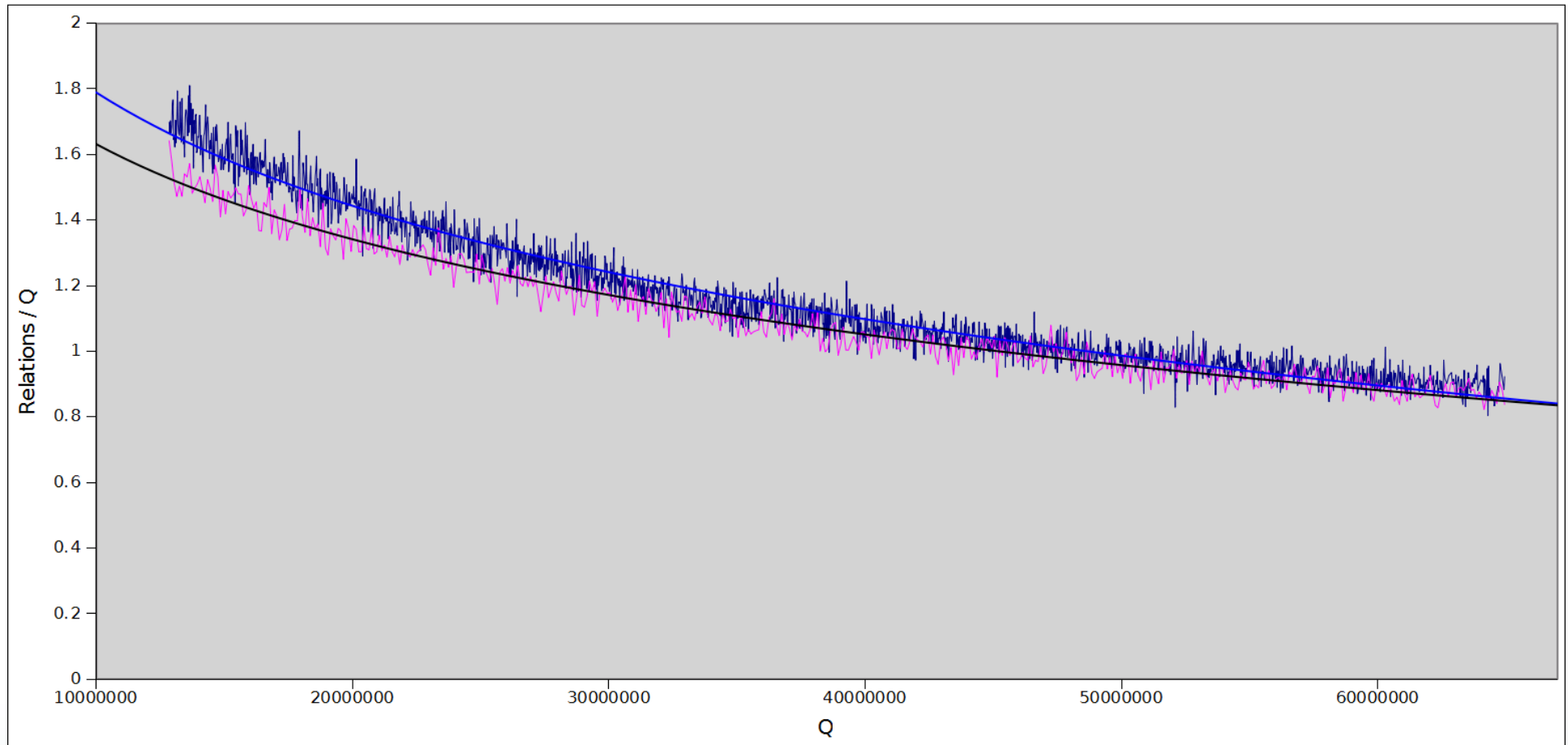
Misconceptions about Polynomials

Relations / Q (higher is better)



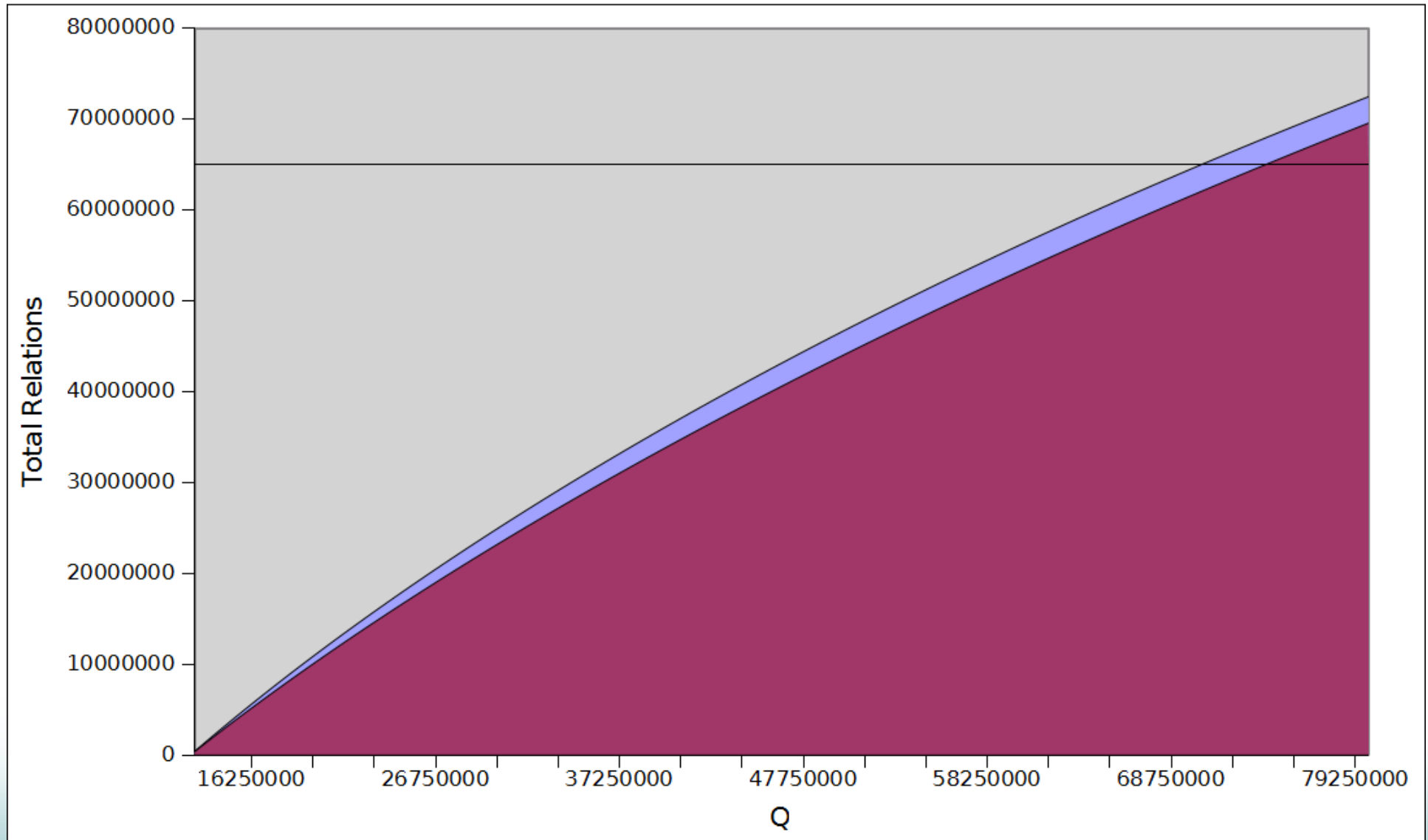
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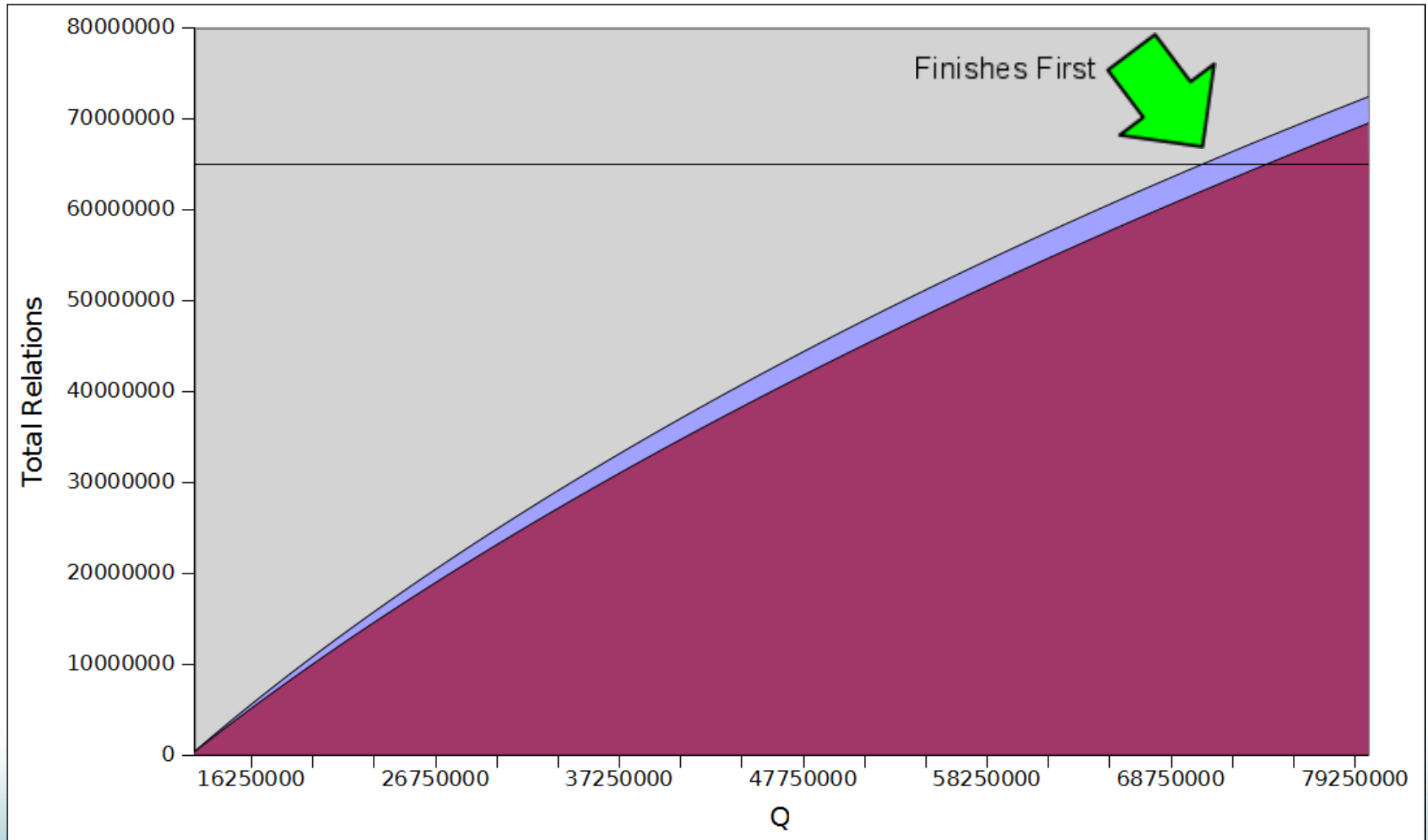
Misconceptions about Polynomials

Total Relations By Q



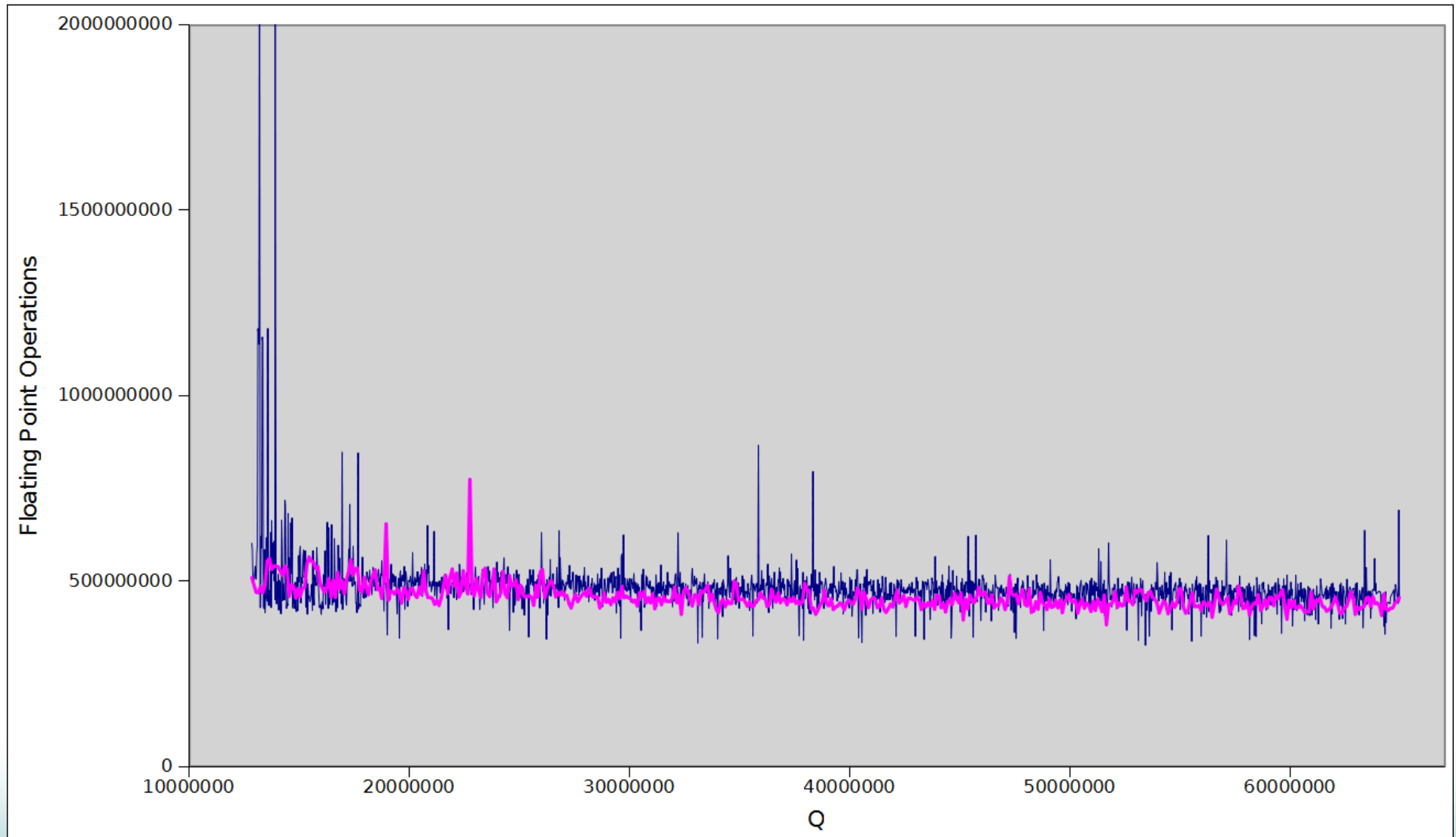
Misconceptions about Polynomials

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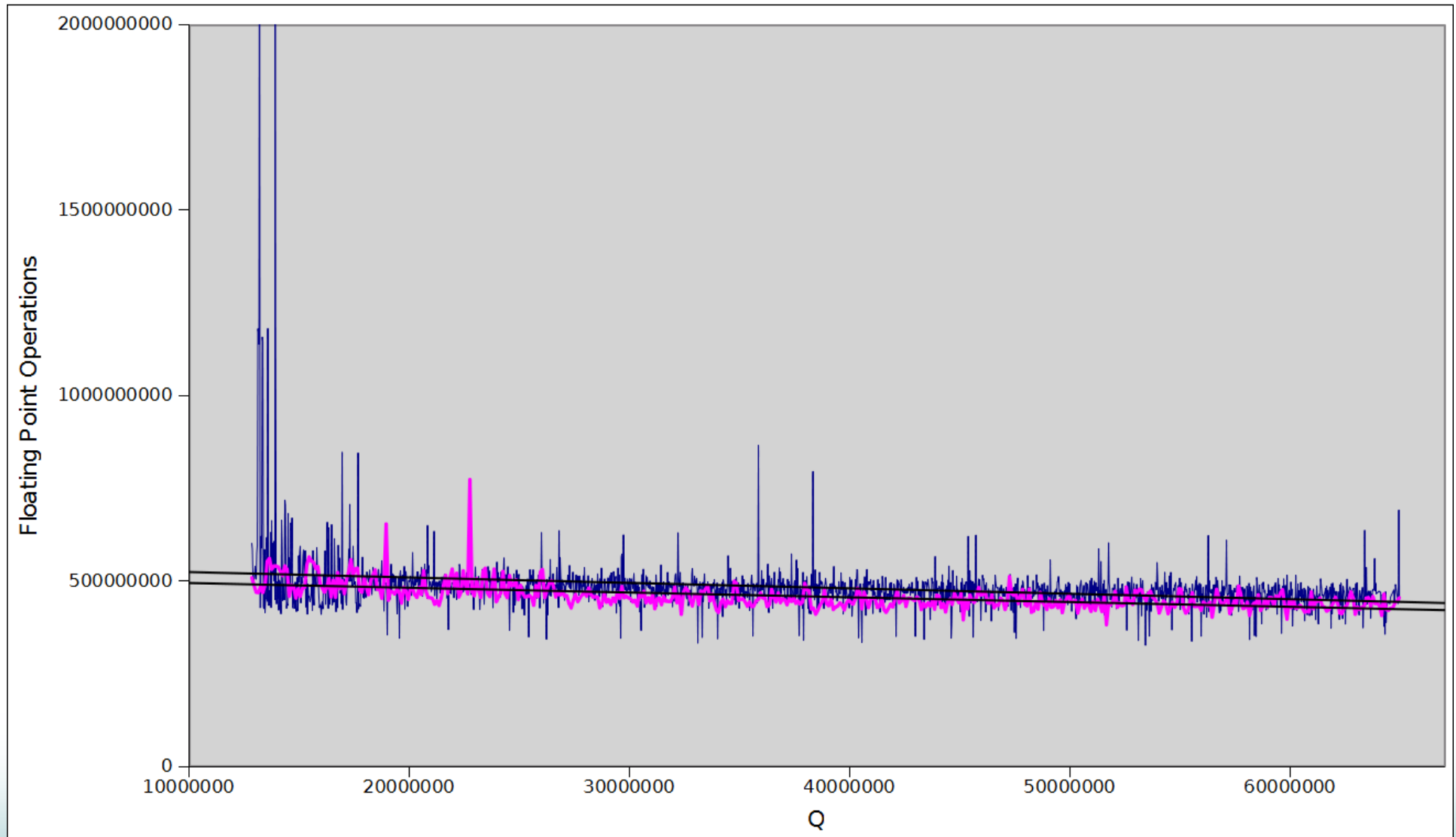
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Operations/Q



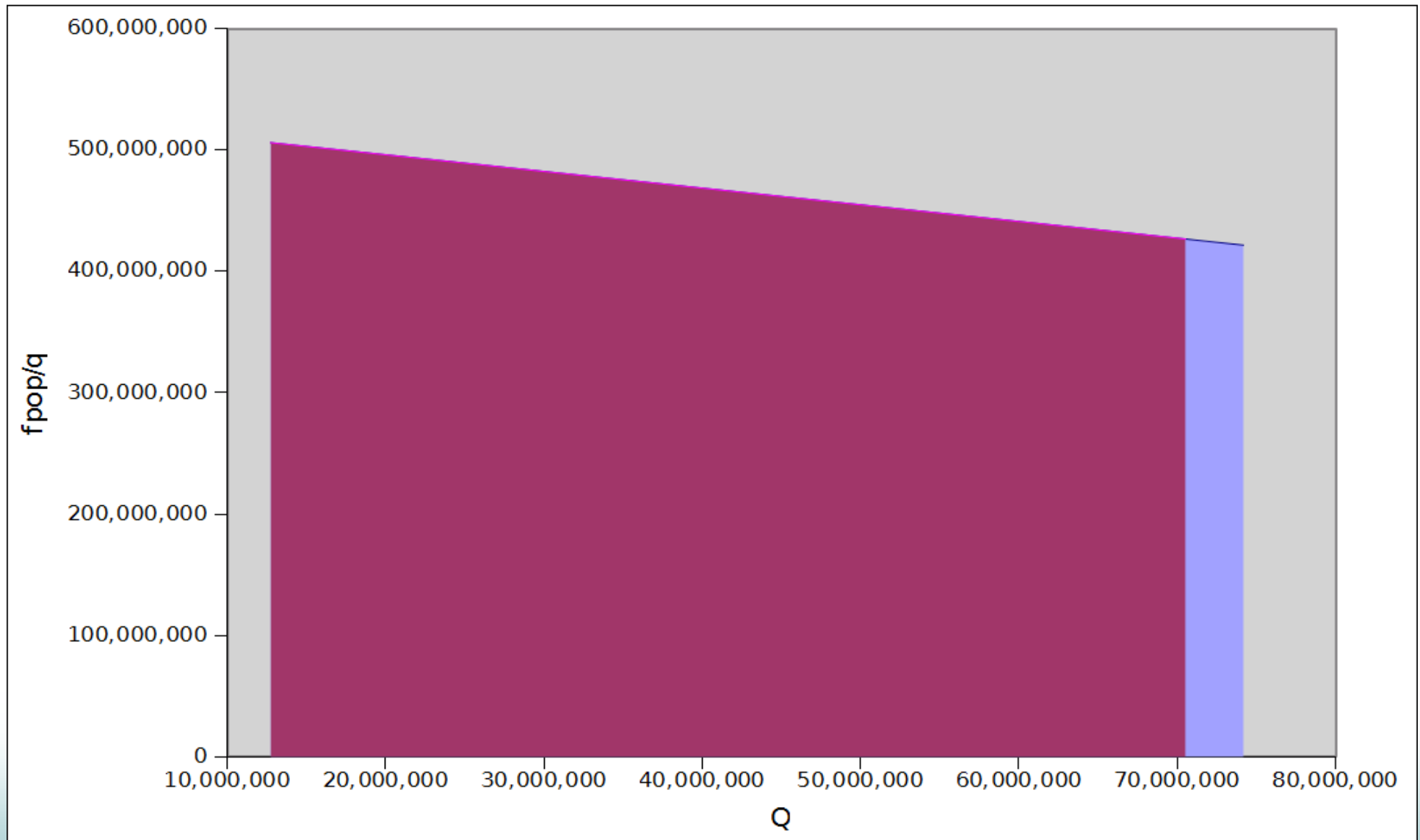
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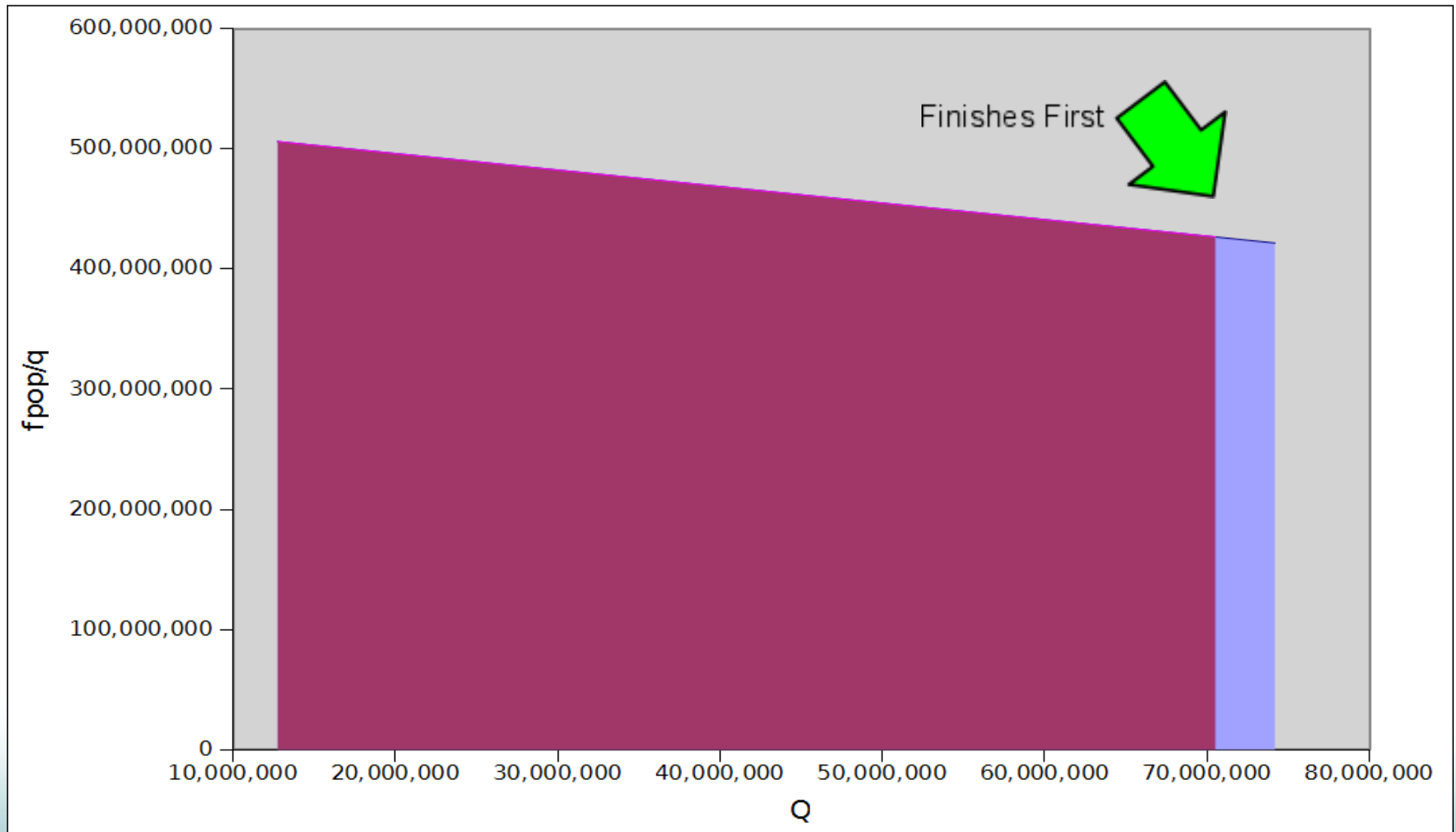
Misconceptions about Polynomials

Total Operations



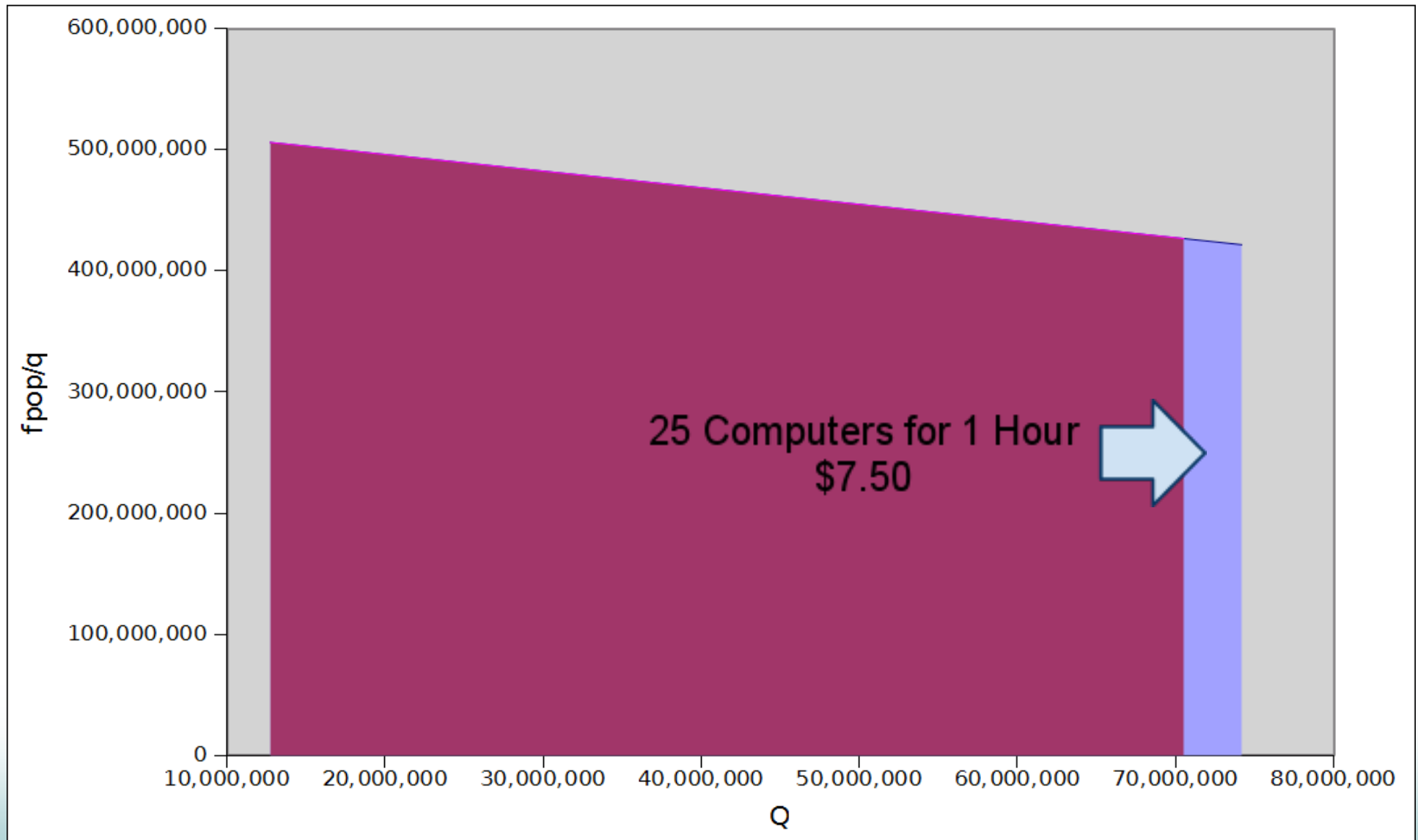
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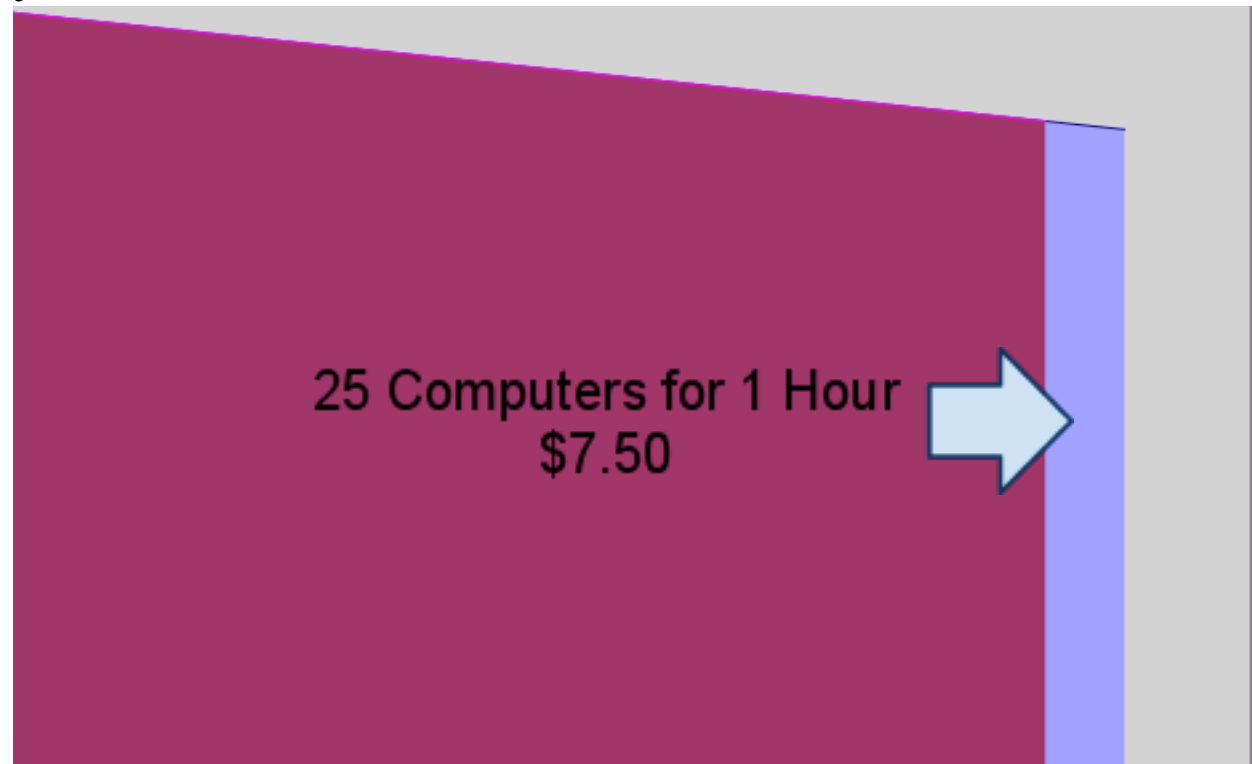
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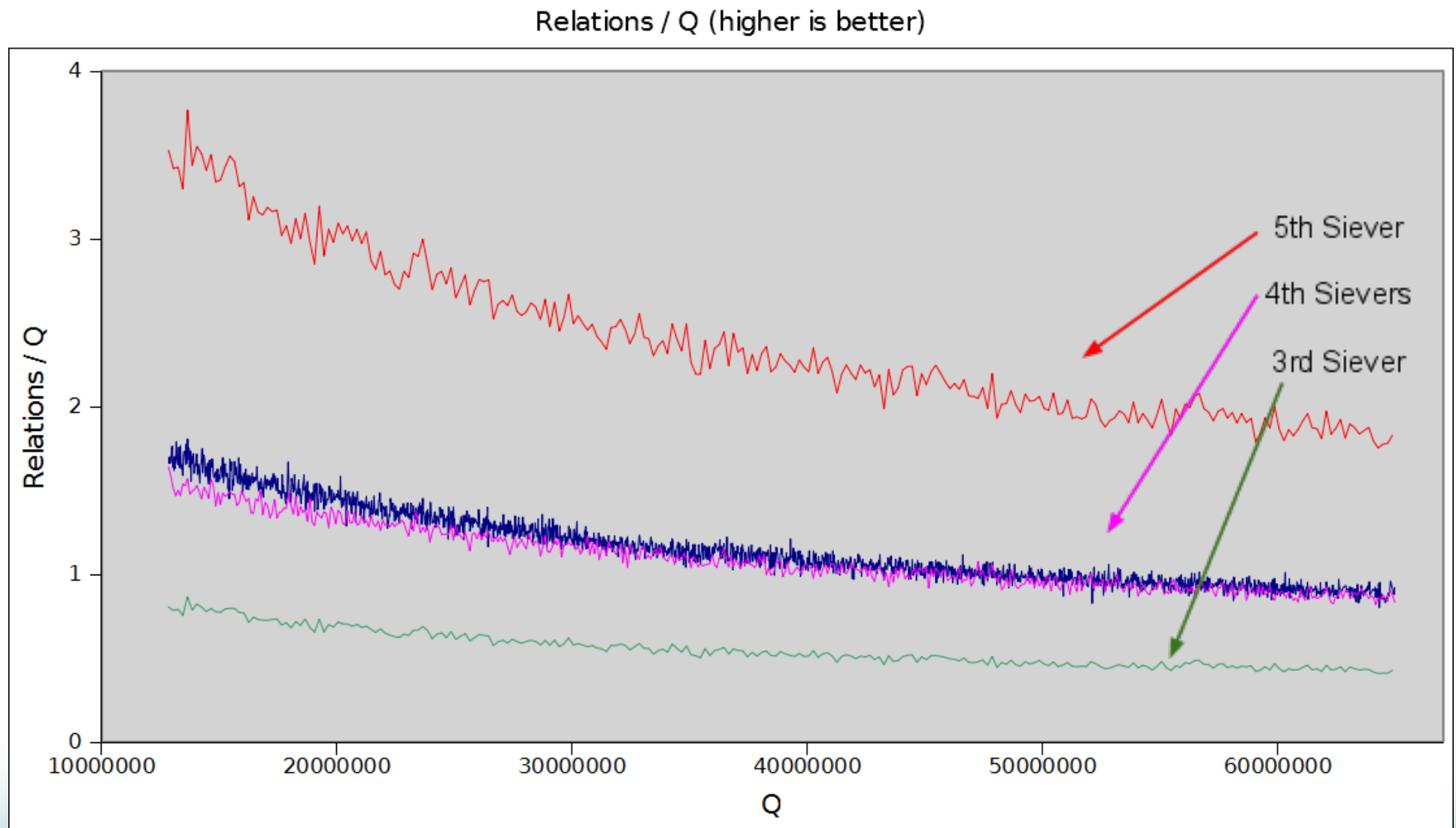
Misconceptions about Polynomials

If time is more valuable to you than (not much) money it is in your best interest to take the first polynomial you get and sieve with that, rather than doing another poly-selection run.



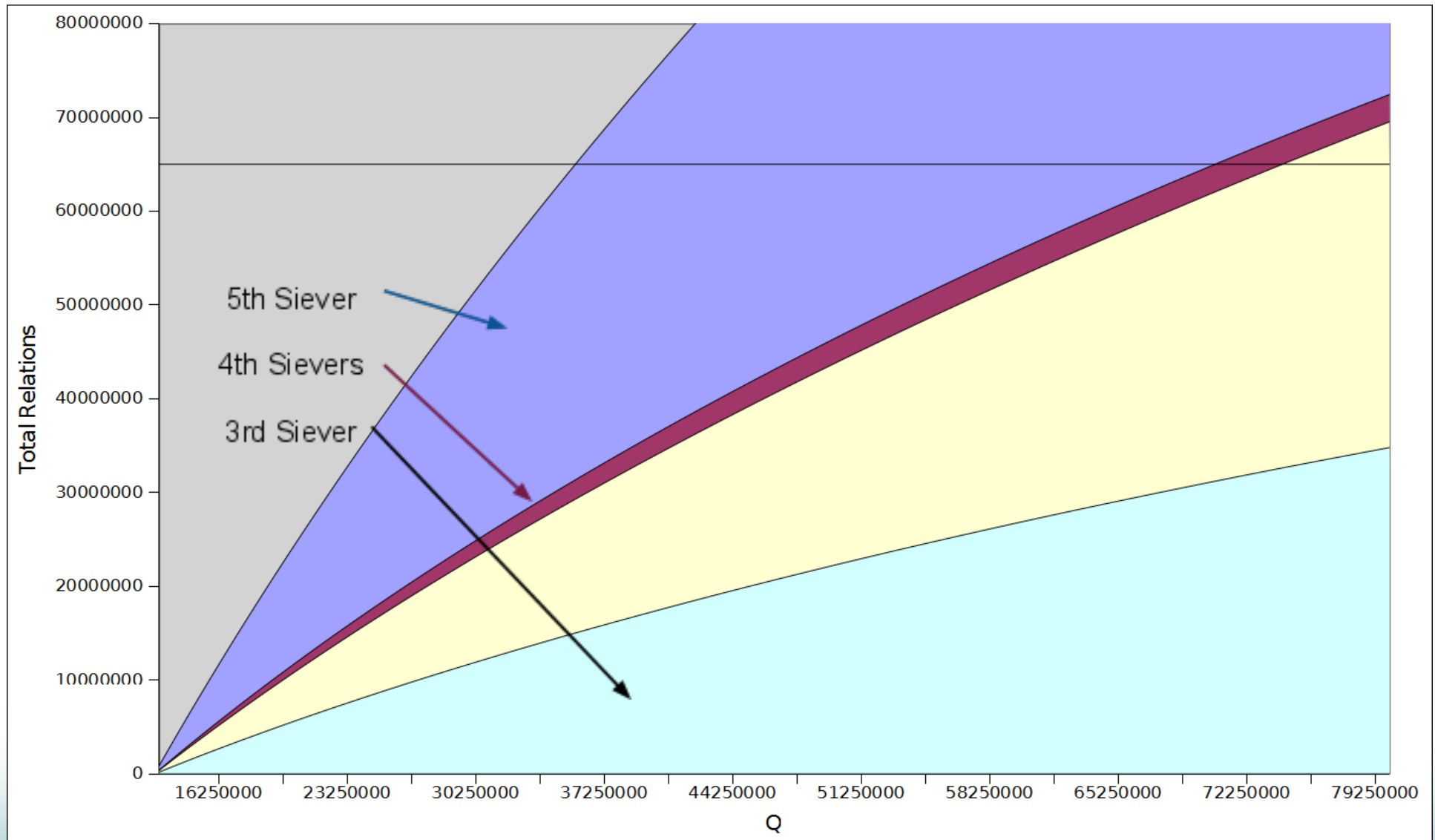
(this advice is only
for 512-bit semiprimes.)

Siever Comparisons



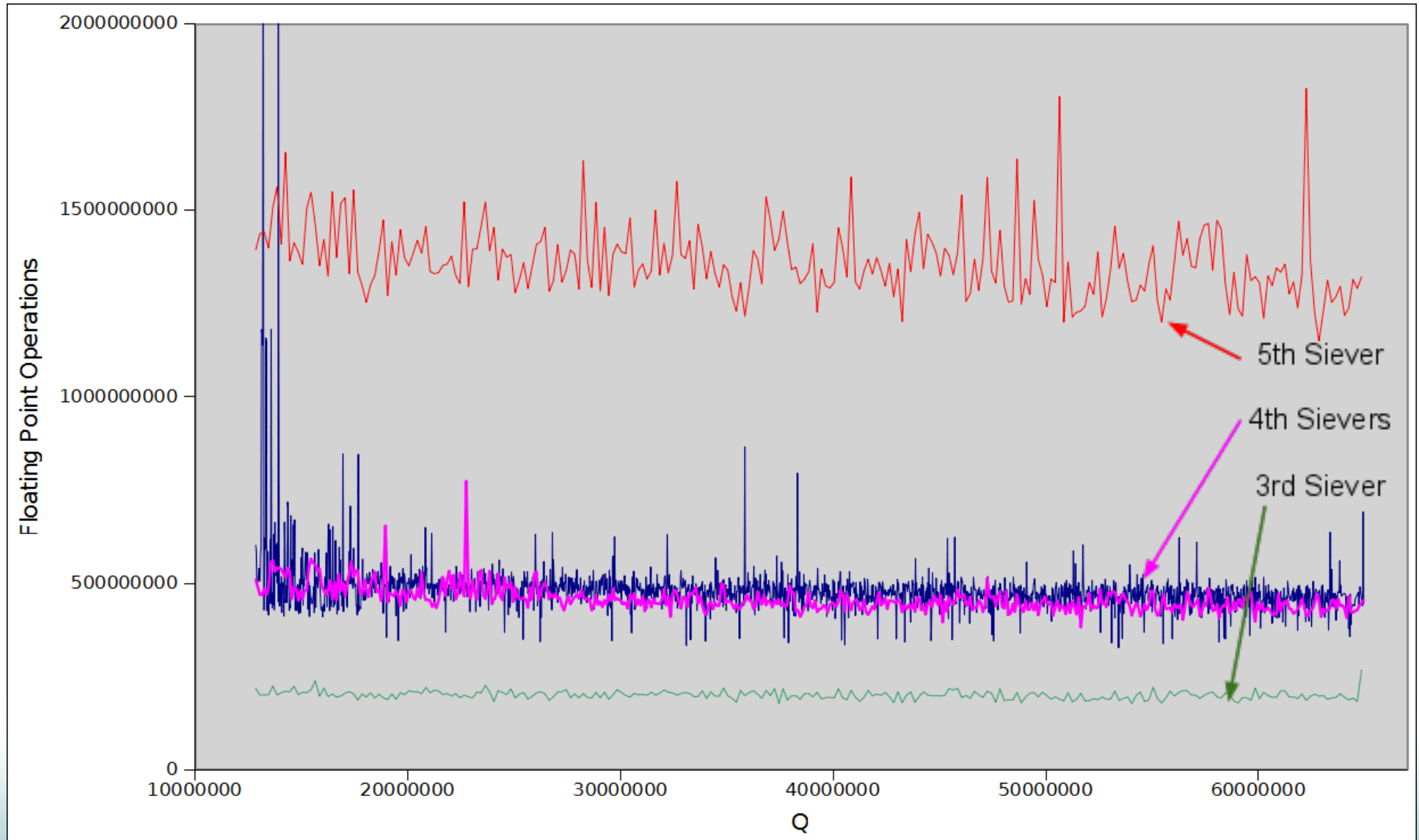
Siever Comparisons

Total Relations By Q



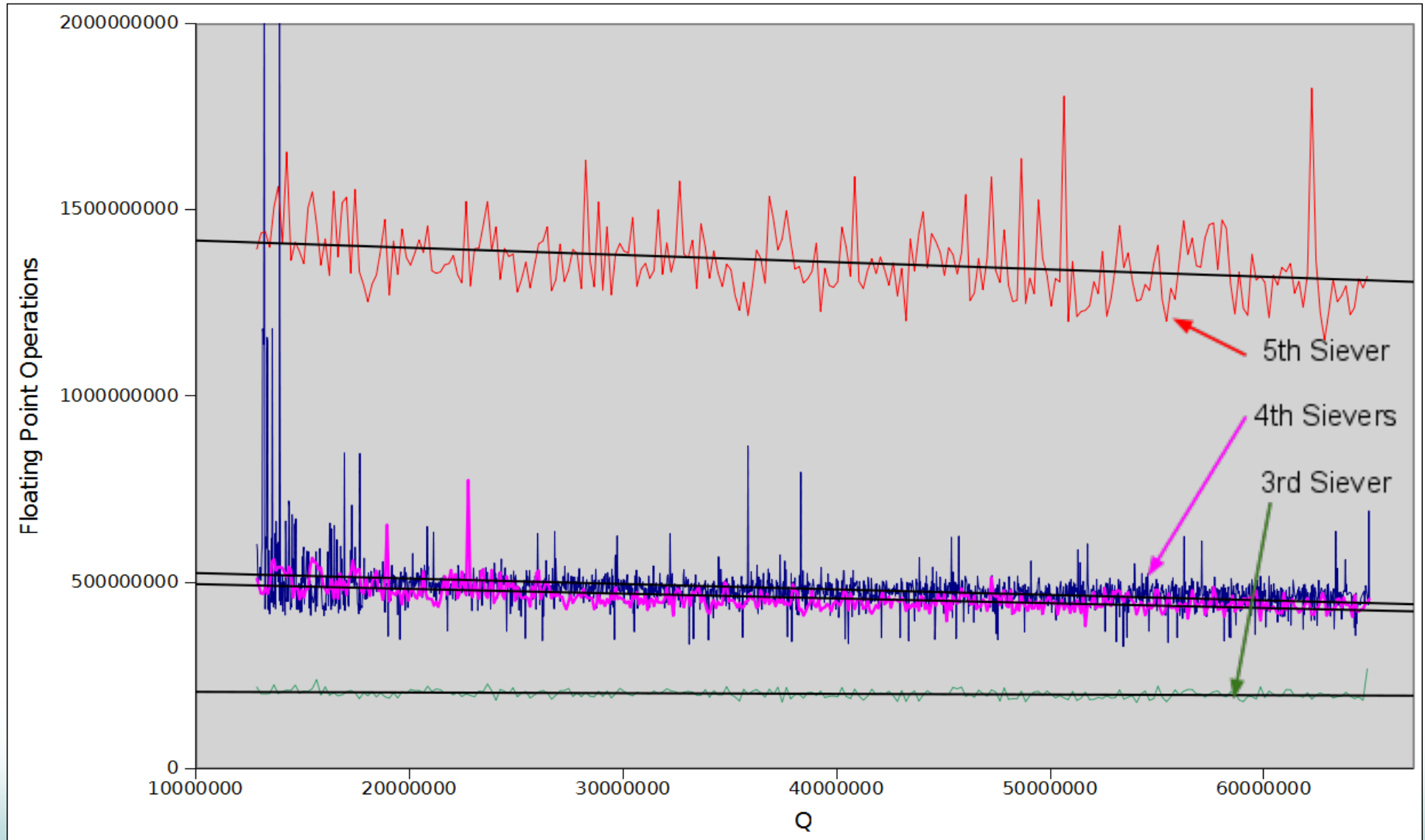
Siever Comparisons

Operations/Q

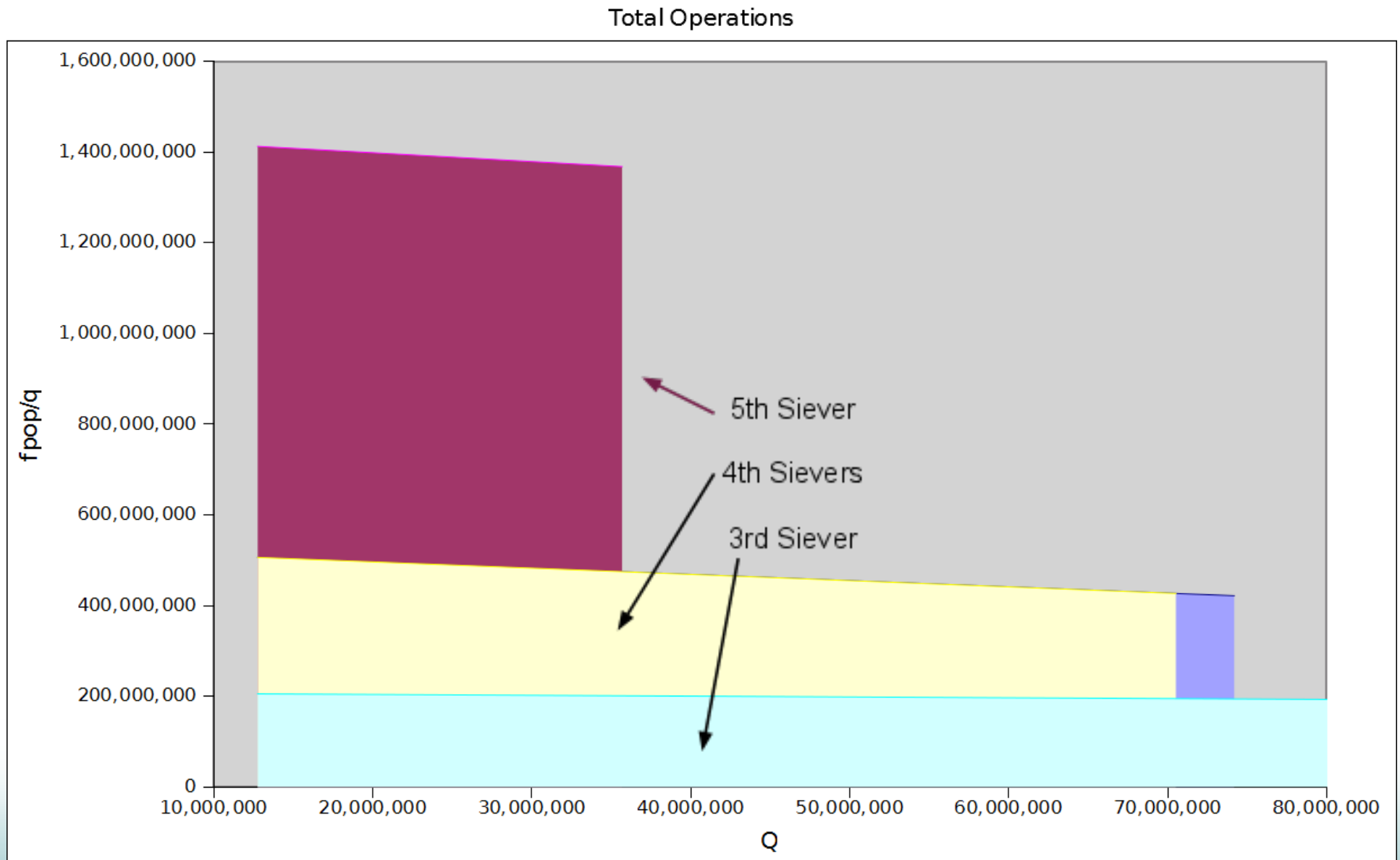


Siever Comparisons

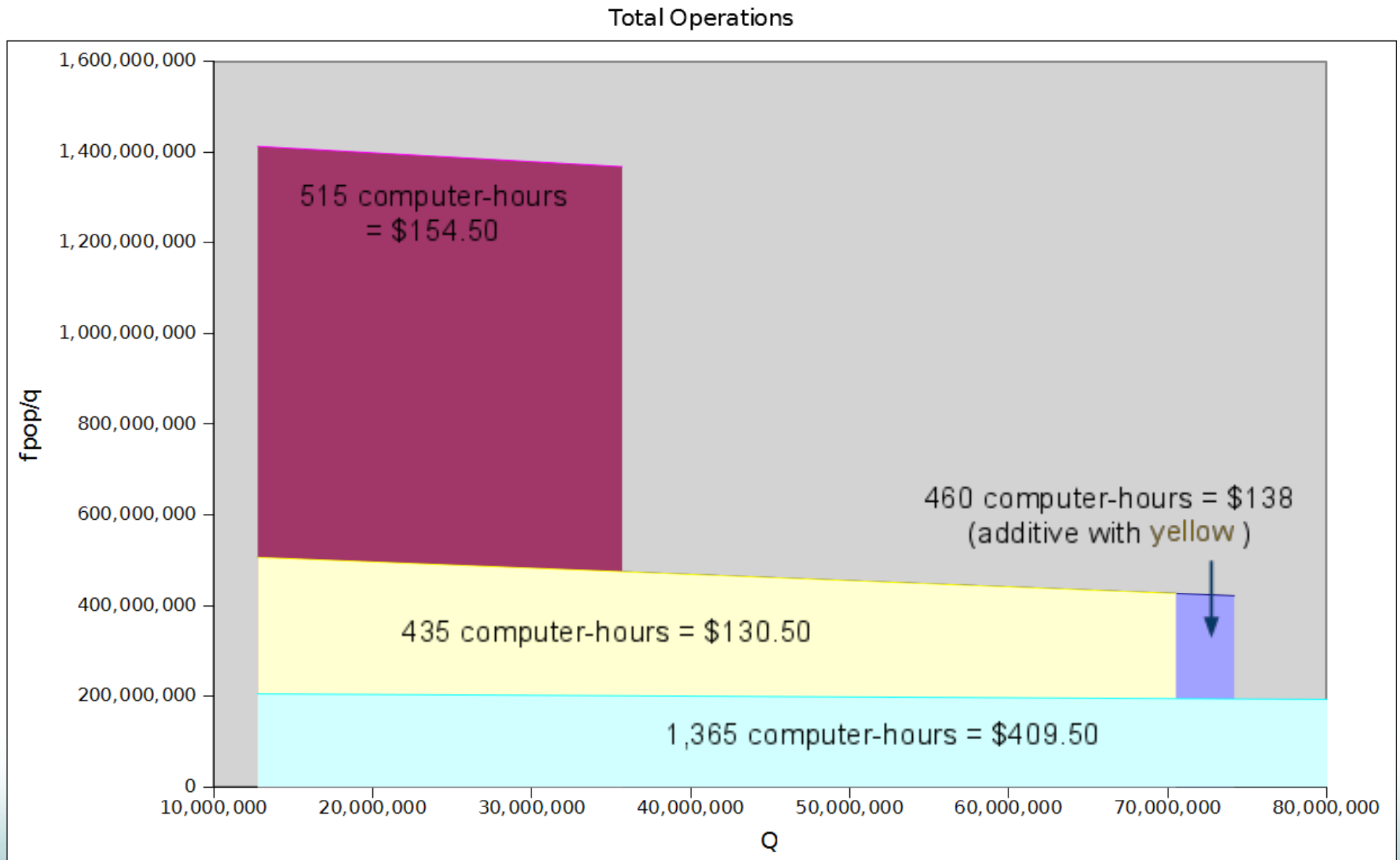
Operations/Q



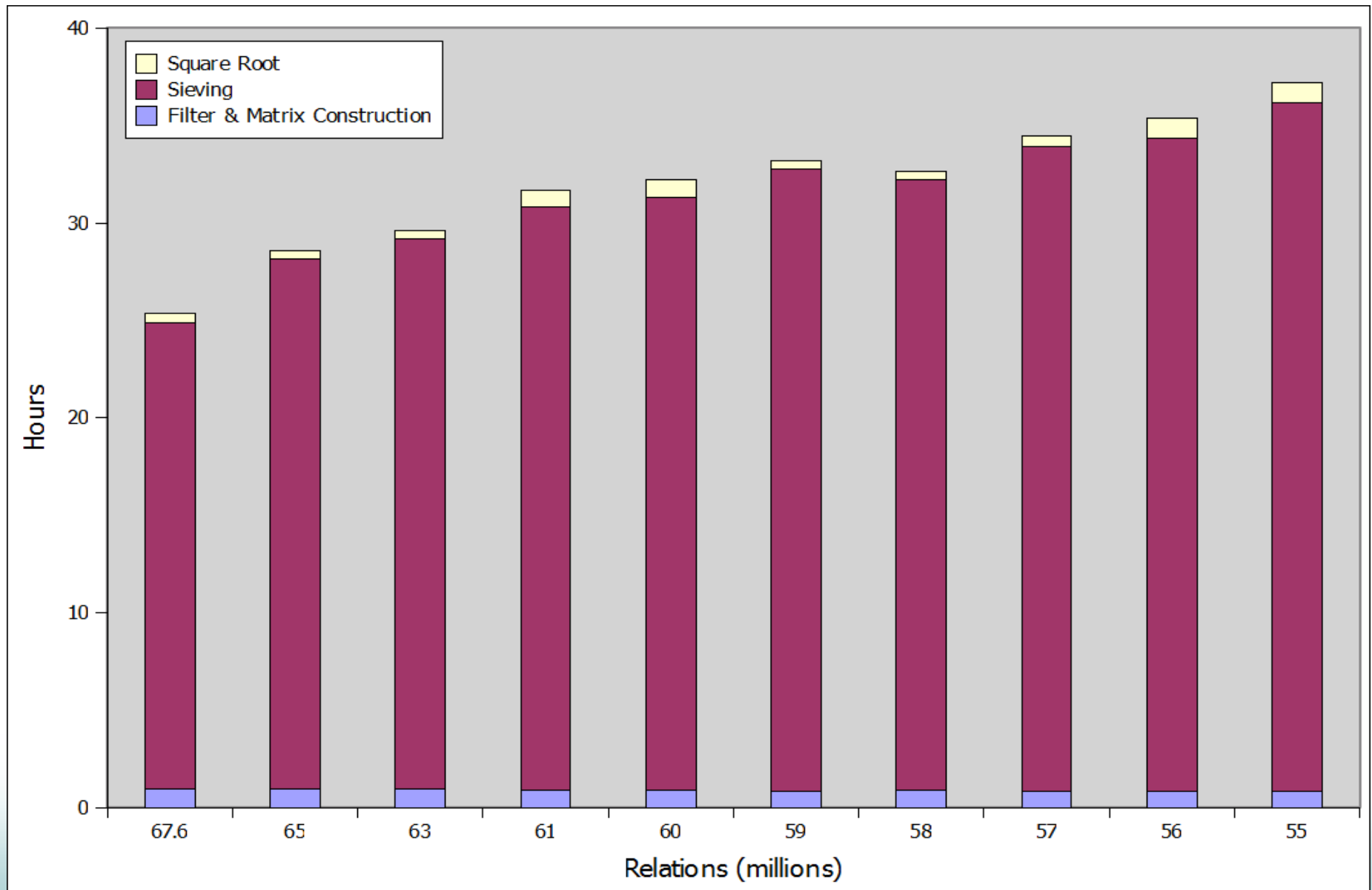
Siever Comparisons



Siever Comparisons



Oversieving



Obligatory Ending Slide

Fin

Thanks:

- GDS
- NYSec
- MersenneForum & jasonp

Tom Ritter

<http://ritter.vg>
(encrypted mail preferred)

Big Ups To:

- jasonp

<http://www.gdssecurity.com/>
<https://github.com/GDSSecurity/cloud-and-control>