Infrared Spectral Energy Distributions

November 2, 2020

1 Project 1: Infrared Spectral Energy Distributions

[1]:
```
# Date created: Oct 9, 2020
```

[23]:
```
# Import libraries
import csv
from os import listdir
import json
import numpy as np
import matplotlib.pyplot as plt
from scipy import optimize
import emcee
import corner
plt.ion()
plt.rcParams.update({'font.size': 16, 'text.usetex': True})
```

[24]:
```
# constants

h = 6.626e-34 #[J*s]
k = 1.381e-23 #[J/K]

DATA_PATH="data\\formatted_data.json"
```

[25]:
```
# properly formatting and defining a function to search through the data

path = "data\\"

files = listdir(path + 'raw\\') # lists all the file names in the 'path'␣
 ↪directory
num_files = len(files) # stores the number of files in the 'path' directory

floats = ["Photometry Measurement", "Frequency", "Flux Density", "Upper limit␣
 ↪of uncertainty", 'Lower limit of uncertainty']
ints = ["No."]
def return_csv(file):
    csv_dict = {}
    with open(file) as csv_file:
        csv_read = csv.reader(csv_file)

        i = 0
        for line in csv_read:
            if i == 0:
                # Initialize csv dictionary with header values as keys
                for key in line:
                    csv_dict[key] = []
                i += 1
                continue

            # Add the value in the csv to the corresponding key
            index = 0
            for key in csv_dict:
                if line[index] == "":
                    line[index] = None
                if key in floats and line[index] != None:
                    line[index] = float(line[index])
                if key in ints and line[index] != None:
                    line[index] = int(line[index])

                csv_dict[key].append(line[index])
                index+=1
            i +=1

    return csv_dict


data = {}

for file in files:
    name = file.replace('_Photometry_and_SED.csv', '')
    print(name)
    data[name] = return_csv(path + 'raw\\' + file)


with open(path + 'formatted_data.json', 'w') as f:
    json.dump(data, f, indent=2)




with open(DATA_PATH) as f:
    data = json.load(f)

keys = data.keys()
fields = None
```

```
print("Galaxies:")
for i in keys:
    fields = data[i].keys()
    print("\t- {} ({} datapoints across {} fields)".format(i, len(data[i]['No.
 ↪']), len(data[i])))

print("\nField List:")
print(fields)

# Returns all the data associated with a given set of number keys

def get_in_key(key, numbers):
    _ret = {"flux_density":[],"frequency":[],"uncertainty":[]}
    _not_added = []
    search = data[key]['No.']
    indices = []
    for i in numbers:
        found  = 0
        for j in range(len(search)-1,-1,-1):
            if search[j] == i and not found:
                _ret["flux_density"].append(data[key]['Flux Density'][j])
                _ret["frequency"].append(data[key]['Frequency'][j])
                _ret["uncertainty"].append(data[key]['Upper limit of␣
 ↪uncertainty'][j])
                found = 1
        if not found:
            _not_added.append(i)

    return _ret, _not_added
```

```
APM08279+5255
ARP220
NGC0958
Galaxies:
        - APM08279+5255 (50 datapoints across 21 fields)
        - ARP220 (270 datapoints across 21 fields)
        - NGC0958 (80 datapoints across 21 fields)

Field List:
dict_keys(['No.', 'Observed Passband', 'Photometry Measurement', 'Uncertainty',
'Units', 'Frequency', 'Flux Density', 'Upper limit of uncertainty', 'Lower limit
of uncertainty', 'Upper limit of Flux Density', 'Lower limit of Flux Density',
'NED Uncertainty', 'NED Units', 'Refcode', 'Significance', 'Published
frequency', 'Frequency Mode', 'Coordinates Targeted', 'Spatial Mode',
'Qualifiers', 'Comments'])
```

[26]:

```
# functions

def limit(nu, T, alpha, beta):
    return 3 + beta - ((10**nu)*h/(k*T))*np.exp(h*(10**nu)/(k*T))/(np.
 ↪exp(h*(10**nu)/(k*T))-1) + alpha

def model1(freq, fit):
    L, T, alpha, beta = fit
    nu_prime = optimize.newton(limit, 13, args=(T, alpha, beta))
    L1 = ((10**L)*(10**nu_prime)**(3+beta)/(np.exp(h*(10**nu_prime)/(k*T))-1)/
 ↪((10**nu_prime)**(-1*alpha)))
    predictions = []

    for nu in freq:
        if (nu < 10**nu_prime):
            predictions.append( 1e-26*(10**L)*(nu**(3+beta))/(np.exp(h*nu/
 ↪(k*T))-1) )
        else:
            predictions.append( 1e-26*L1*(nu**(-1*alpha)) )
    return predictions

def penalty(param, freq, flux, error):
    return np.sum((model1(freq, param)-flux)**2/error**2)

def lnprob(param, freq, flux, error):
     return -0.5*penalty(param, freq, flux, error)
```

[32]:

```
# initializing the data

# All you need to do is add which numbers you want for the data points needed␣
 ↪to be analyzed.
# Make sure the numbers are sorted in descending order to ensure the results␣
 ↪are sorted properly
# If there is no value in the table for a field, it will be set as 'None' and␣
 ↪will have to be manually fixed
# If the number searched for is not in the list, will be added to the␣
 ↪'omitted_numbers_N' list

f_NGC0958_list, omitted_numbers_1 = get_in_key('NGC0958', [99, 98, 97, 95, 92,␣
 ↪87,80])
f_ARP220_list, omitted_numbers_2 = get_in_key('ARP220', [284, 245, 221, 196,␣
 ↪189, 174, 134])
f_APM08279_5255_list, omitted_numbers_3 = get_in_key('APM08279+5255', [82, 72,␣
 ↪69, 58, 44, 42, 40])
```

```
print("NGC 0958 Data: \n\tData -> {}\n\tNumbers Not Found -> {}\n".
 ↪format(f_NGC0958_list, omitted_numbers_1))
print("ARP 220 Data: \n\tData -> {}\n\tNumbers Not Found -> {}\n".
 ↪format(f_ARP220_list, omitted_numbers_2))
print("APM 08279+5255 Data: \n\tData -> {}\n\tNumbers Not Found -> {}\n".
 ↪format(f_APM08279_5255_list, omitted_numbers_3))


frequencies_NGC = np.array(f_NGC0958_list["frequency"])
flux_densities_NGC = np.array(f_NGC0958_list["flux_density"])
errors_NGC = np.array(f_NGC0958_list["uncertainty"])

frequencies_Arp = np.array(f_ARP220_list["frequency"])
flux_densities_Arp = np.array(f_ARP220_list["flux_density"])

frequencies_APM = np.array(f_APM08279_5255_list["frequency"])
flux_densities_APM = np.array(f_APM08279_5255_list["flux_density"])
```

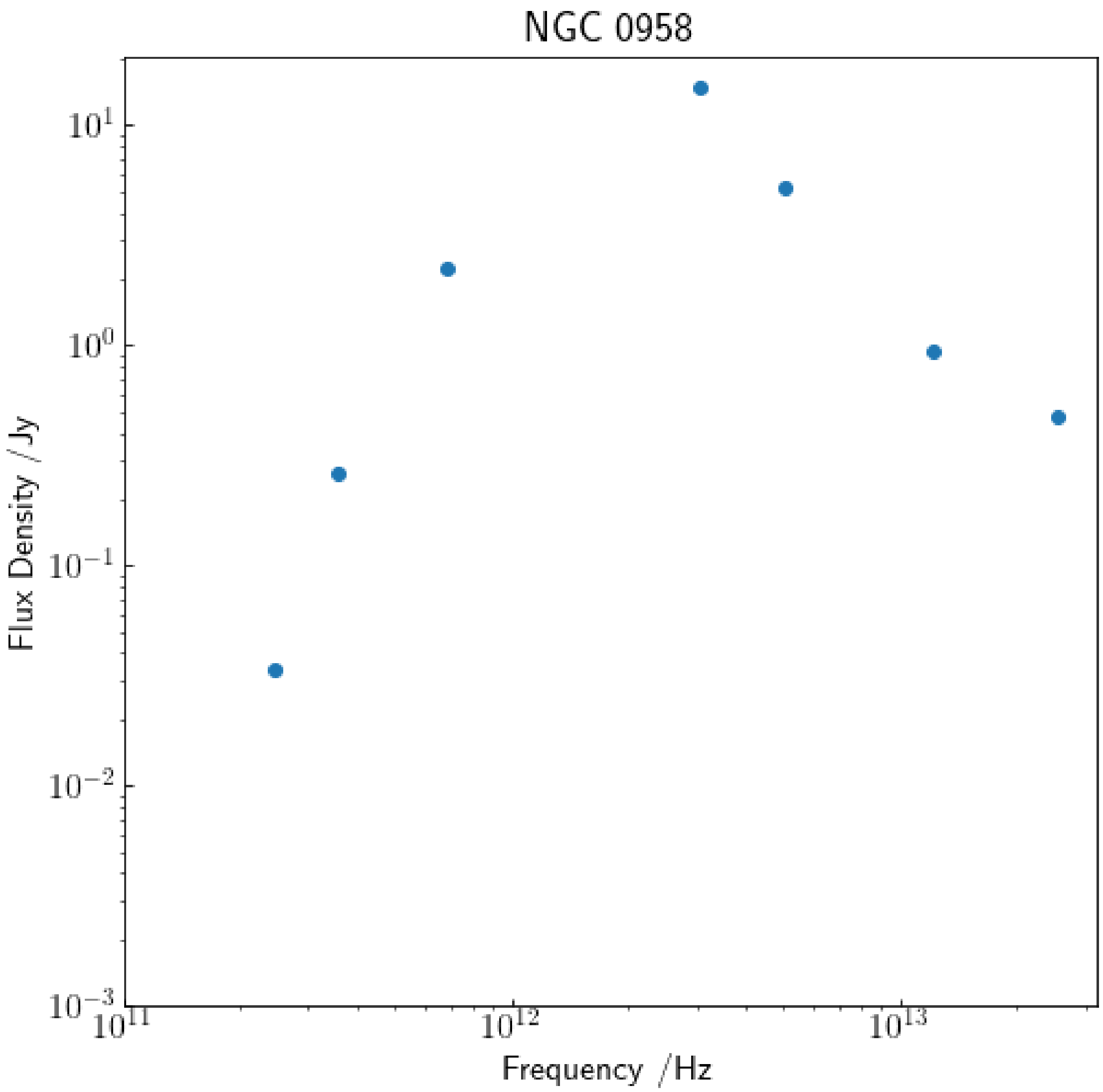
```
NGC 0958 Data:
        Data -> {'flux_density': [0.034, 0.262, 2.25, 15.0, 5.25, 0.94, 0.473],
'frequency': [240000000000.0, 353000000000.0, 666000000000.0, 3000000000000.0,
5000000000000.0, 12000000000000.0, 25000000000000.0], 'uncertainty': [0.007,
0.034, 0.428, 0.212, 0.263, 0.035, 0.0616]}
        Numbers Not Found -> []

ARP 220 Data:
        Data -> {'flux_density': [0.178, 0.832, 10.5, 112.0, 103.0, 7.92, 0.64],
'frequency': [217000000000.0, 353000000000.0, 857000000000.0, 3000000000000.0,
5000000000000.0, 12000000000000.0, 25000000000000.0], 'uncertainty': [0.032,
0.086, 3.3, 3.37, 0.144, 0.038, 0.029]}
        Numbers Not Found -> []

APM 08279+5255 Data:
        Data -> {'flux_density': [0.0012, 0.0266, 0.06, 0.342, 0.951, 0.511,
0.226], 'frequency': [90800000000.0, 237000000000.0, 302000000000.0,
666000000000.0, 3000000000000.0, 5000000000000.0, 12000000000000.0],
'uncertainty': [0.00013, 0.0013, 0.012, 0.026, 0.228, 0.0511, 0.0162]}
        Numbers Not Found -> []
```

2 NGC 0958

```
[7]: # NGC 0958

     # frequencies_NGC = (1 + 0.019)*np.array([2.4e+11, 3.53e+11, 6.66e+11, 3e+12,␣
      ↪5e+12, 1.2e+13, 2.5e+13])
```

```
# flux_densities_NGC = np.array([0.034, 0.262, 2.25, 14.99, 5.25, 0.94, 0.4735])
# errors_NGC = np.array( [0.007, 0.034, 0.428, 0.212, 0.2625, 0.035, 0.0616] )

# Accounting for redshift
frequencies_NGC = (1 + 0.019)*frequencies_NGC

f, ax = plt.subplots(1, figsize=(8,8))
ax.plot(frequencies_NGC, flux_densities_NGC, 'o')
ax.set_xscale('log')
ax.set_xticks([1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.001, 0.01, 0.1, 1, 10])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('NGC 0958')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
f.show()
```

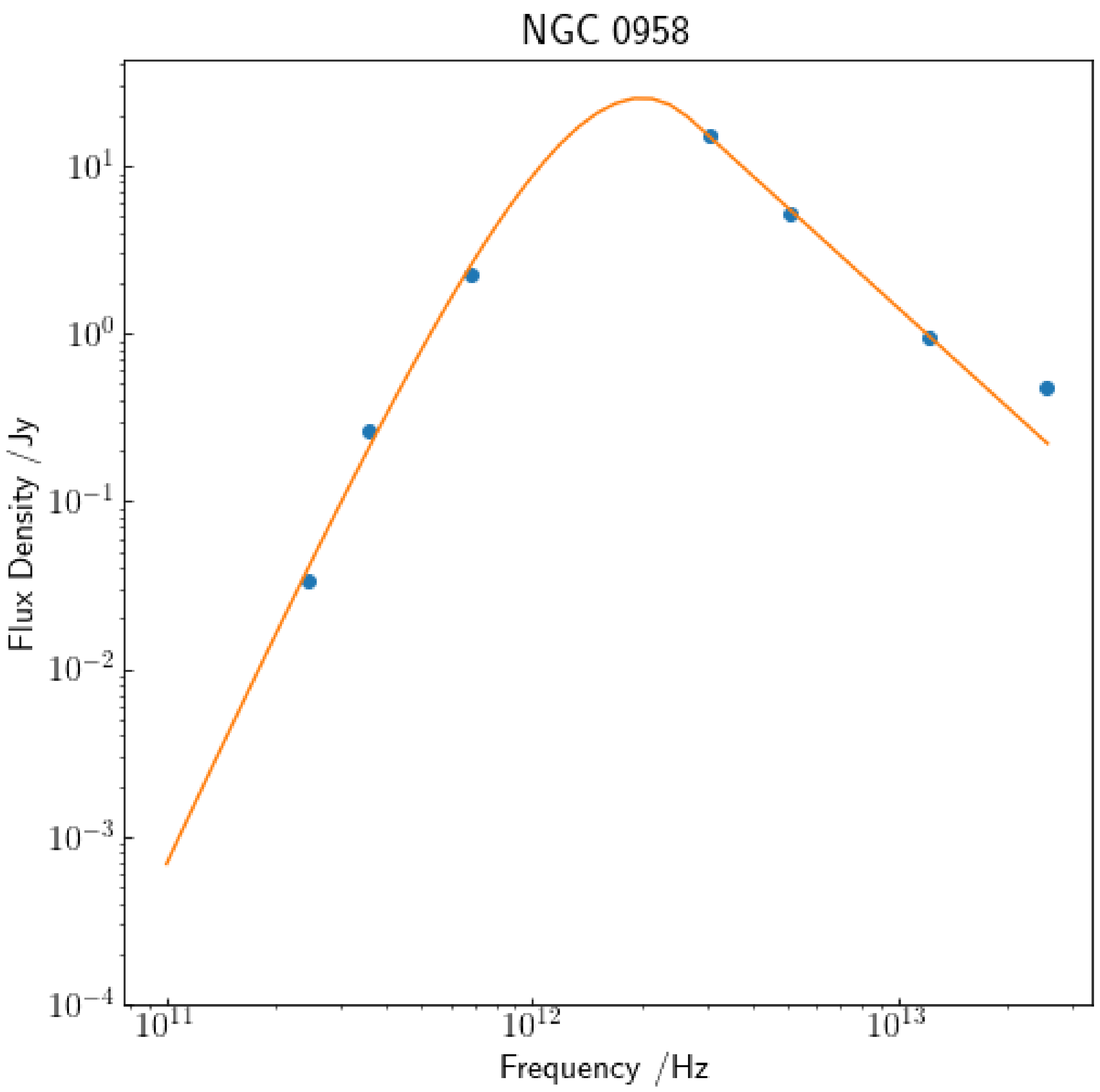
```
<ipython-input-7-6edb8d472222>:19: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```



```
[8]: fit = optimize.fmin(penalty, [-32, 28.8, 2.02, 1.5], args=(frequencies_NGC,␣
      ↪flux_densities_NGC, errors_NGC))
     ngc_fit = model1(frequencies_NGC, fit)
     print (fit)

     #plot fit for NGC 09958
     x = np.logspace(11,13.41000)

     f, ax = plt.subplots(1, figsize=(8,8))
     ax.plot(frequencies_NGC, flux_densities_NGC, 'o')
     ax.plot(x, model1(x,fit))
     ax.set_xscale('log')
     ax.set_xticks([1e+11, 1e+12, 1e+13])
     ax.set_yscale('log')
     ax.set_yticks([0.0001, 0.001, 0.01, 0.1, 1, 10])
     ax.tick_params(axis="y",direction="in")
     ax.tick_params(axis="x",direction="in")
     ax.set_title('NGC 0958')
     ax.set_ylabel('Flux Density /Jy')
     ax.set_xlabel('Frequency /Hz')
     f.show()
```

```
Optimization terminated successfully.
         Current function value: 20.526898
         Iterations: 458
         Function evaluations: 761
[-41.48681346  16.58910876   1.97908351   2.80462048]
```

```
<ipython-input-8-fc79847eab0c>:20: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

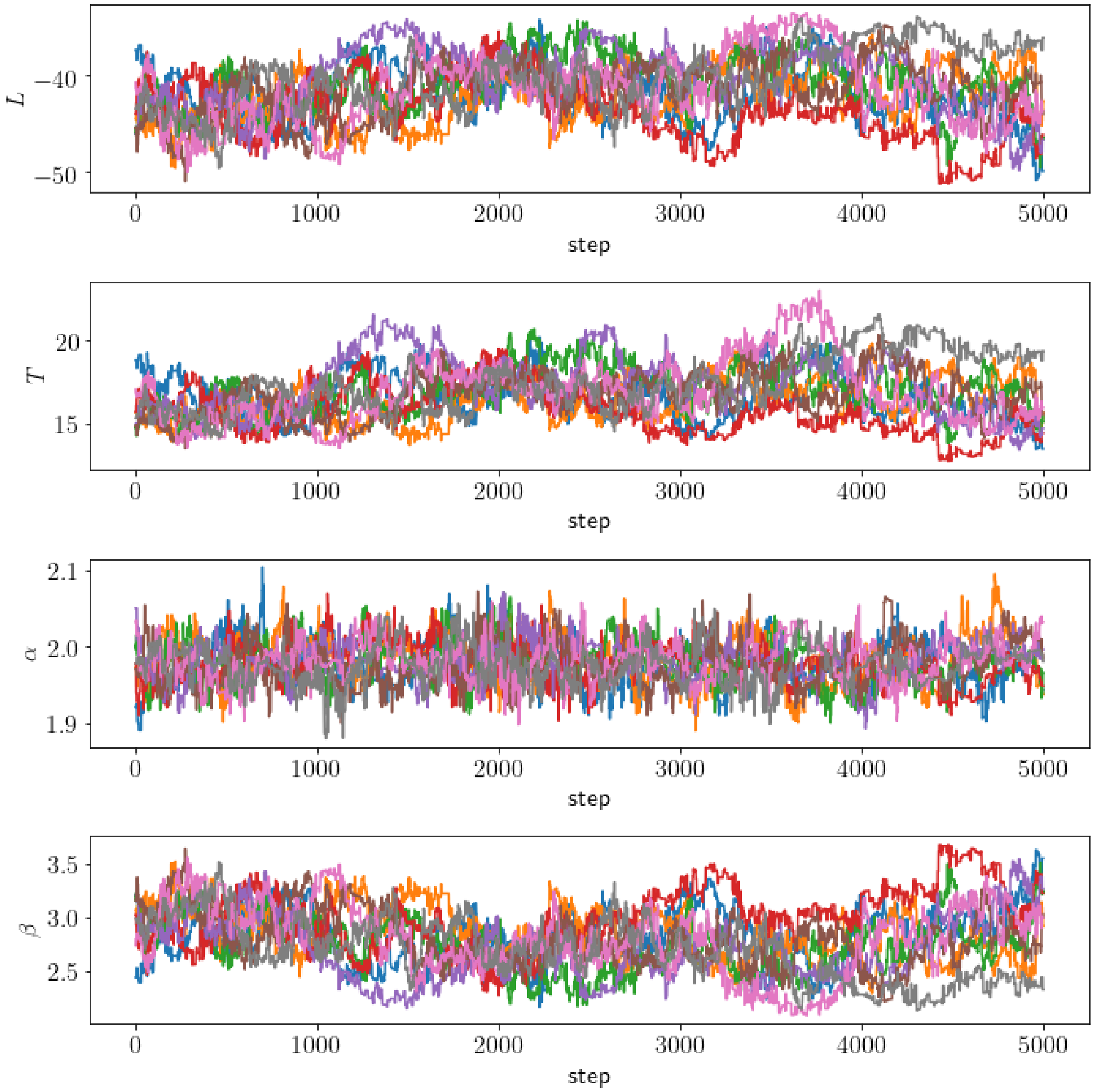


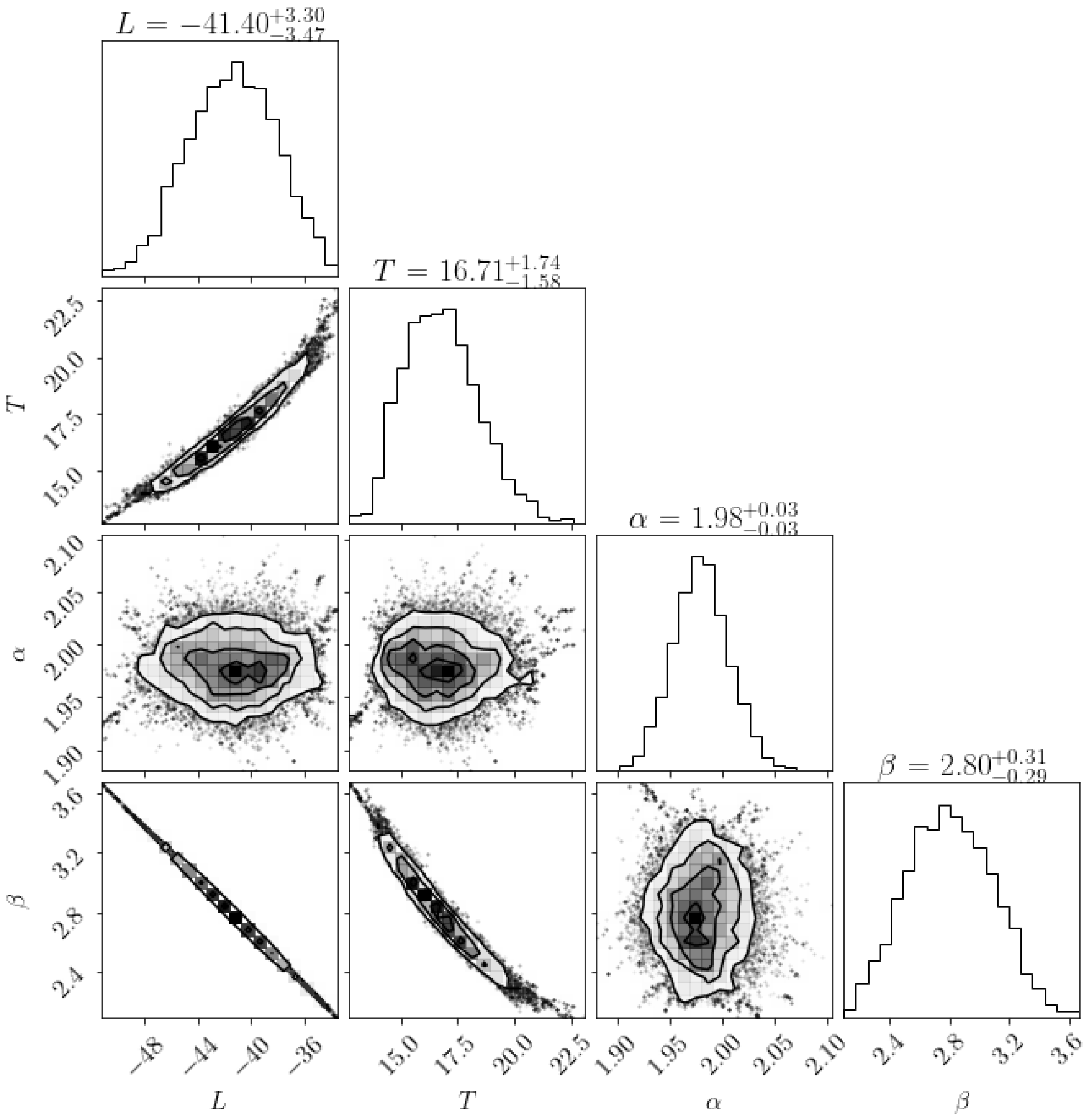
[9]:

```
# Emcee run for NGC 0958 model

ndim = 4
nwalk = ndim*2
nburn = 2000
nmain = 5000

# Random starting points
p0 = np.zeros((nwalk, ndim))
for i in range(nwalk):
    p0[i] = fit + np.random.uniform(low=-0.05, high=0.05, size=4)
```

```
sampler = emcee.EnsembleSampler(nwalk, ndim, lnprob, args=(frequencies_NGC,␣
 ↪flux_densities_NGC, errors_NGC))

# Burn-in run
pos,prob,state = sampler.run_mcmc(p0, nburn)

sampler.reset()

# Main run
res = sampler.run_mcmc(pos, nmain)
samples = sampler.chain.reshape((-1,ndim))

# plot the individual parameters for model
f, ax = plt.subplots(ndim, 1, figsize=(10, 10))
for idim in range(ndim):
    for iwalk in range(nwalk):
        ax[idim].plot(sampler.chain[iwalk,:,idim])
    ax[idim].set_xlabel('step')
ax[0].set_ylabel(r'$L$')
ax[1].set_ylabel(r'$T$')
ax[2].set_ylabel(r'$\alpha$')
ax[3].set_ylabel(r'$\beta$')
f.tight_layout()
f.show()

# Plot corner plot
f = corner.corner(samples, show_titles=True, labels=(r'$L$', r'$T$',␣
 ↪r'$\alpha$', r'$\beta$'))
f.show()
```

```
<ipython-input-9-d52d01e634d7>:36: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
<ipython-input-9-d52d01e634d7>:40: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

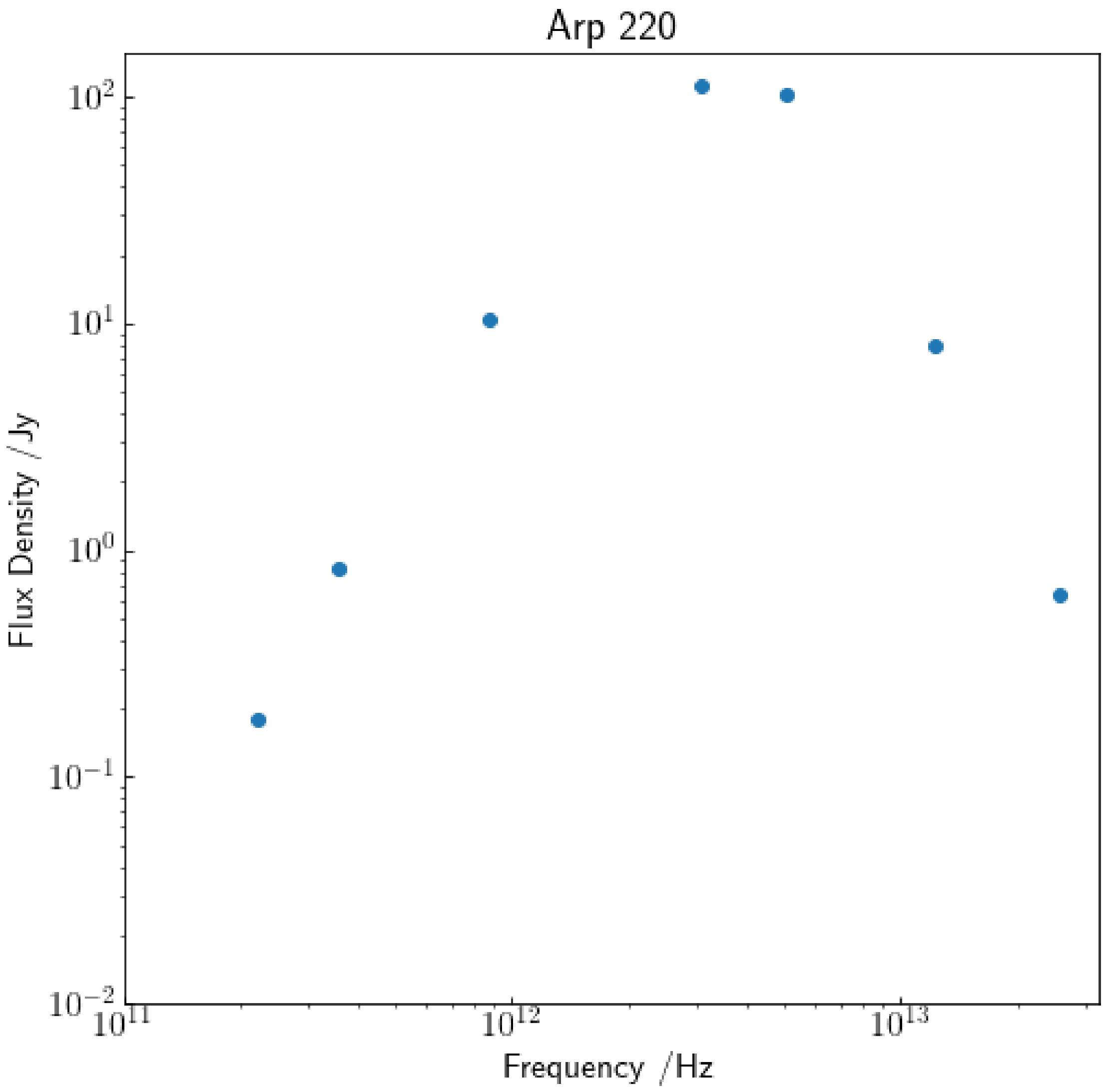

L
-40
-50
0
1000
2000
3000
4000
5000
step
T
20
15
0
1000
2000
3000
4000
5000
step
α
2.1
2.0
1.9
0
1000
2000
3000
4000
5000
step
β
3.5
3.0
2.5
0
1000
2000
3000
4000
5000
step



3 Arp 220

[28]:
```
# Arp 220

# Accounting for red shift
frequencies_Arp = (1 + 0.018)*np.array(frequencies_Arp)



f, ax = plt.subplots(1, figsize=(8,8))
ax.plot(frequencies_Arp, flux_densities_Arp, 'o')
ax.set_xscale('log')
ax.set_xticks([1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.01, 0.1, 1, 10, 100])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('Arp 220')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
f.show()
```

```
<ipython-input-28-844fb44b8bb0>:18: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

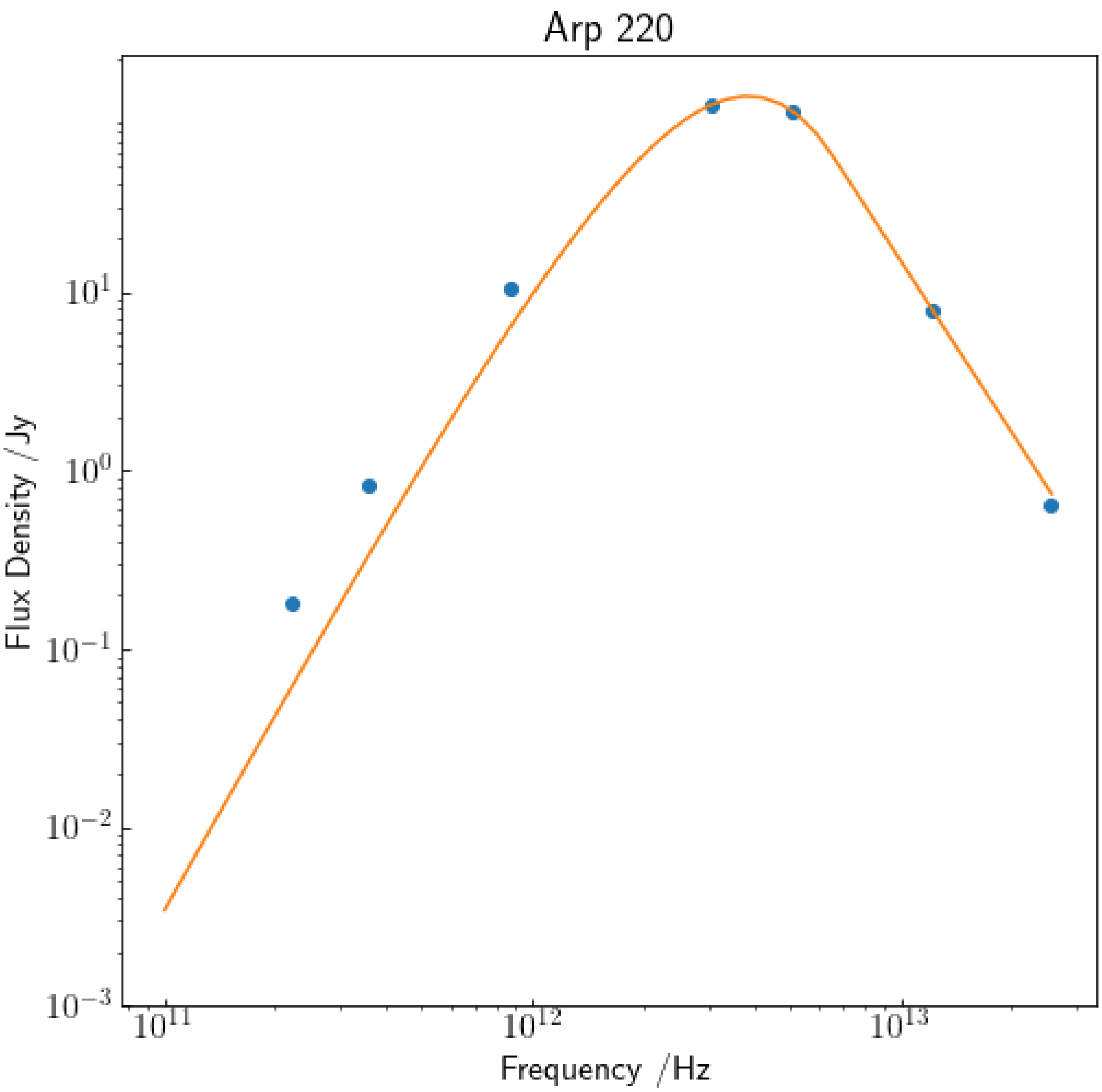


[29]:

```python
#fit for Arp 220
fit2 = optimize.fmin(penalty, [-31, 37.4, 2.9, 1.5], args=(frequencies_Arp,␣
 ↪flux_densities_Arp, errors_Arp))
arp_fit = model1(frequencies_Arp, fit2)
print (fit2)

#plot fit for Arp 220
x = np.logspace(11,13.41000)

f, ax = plt.subplots(1, figsize=(8,8))
ax.plot(frequencies_Arp, flux_densities_Arp, 'o')
ax.plot(x, model1(x,fit2))
ax.set_xscale('log')
ax.set_xticks([1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.001, 0.01, 0.1, 1, 10])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('Arp 220')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
f.show()
```

```
Optimization terminated successfully.
         Current function value: 66.249131
         Iterations: 159
         Function evaluations: 285
[-29.21771852  39.54718565   3.18605137   1.71495374]
```

```
<ipython-input-29-df0730857fd6>:21: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

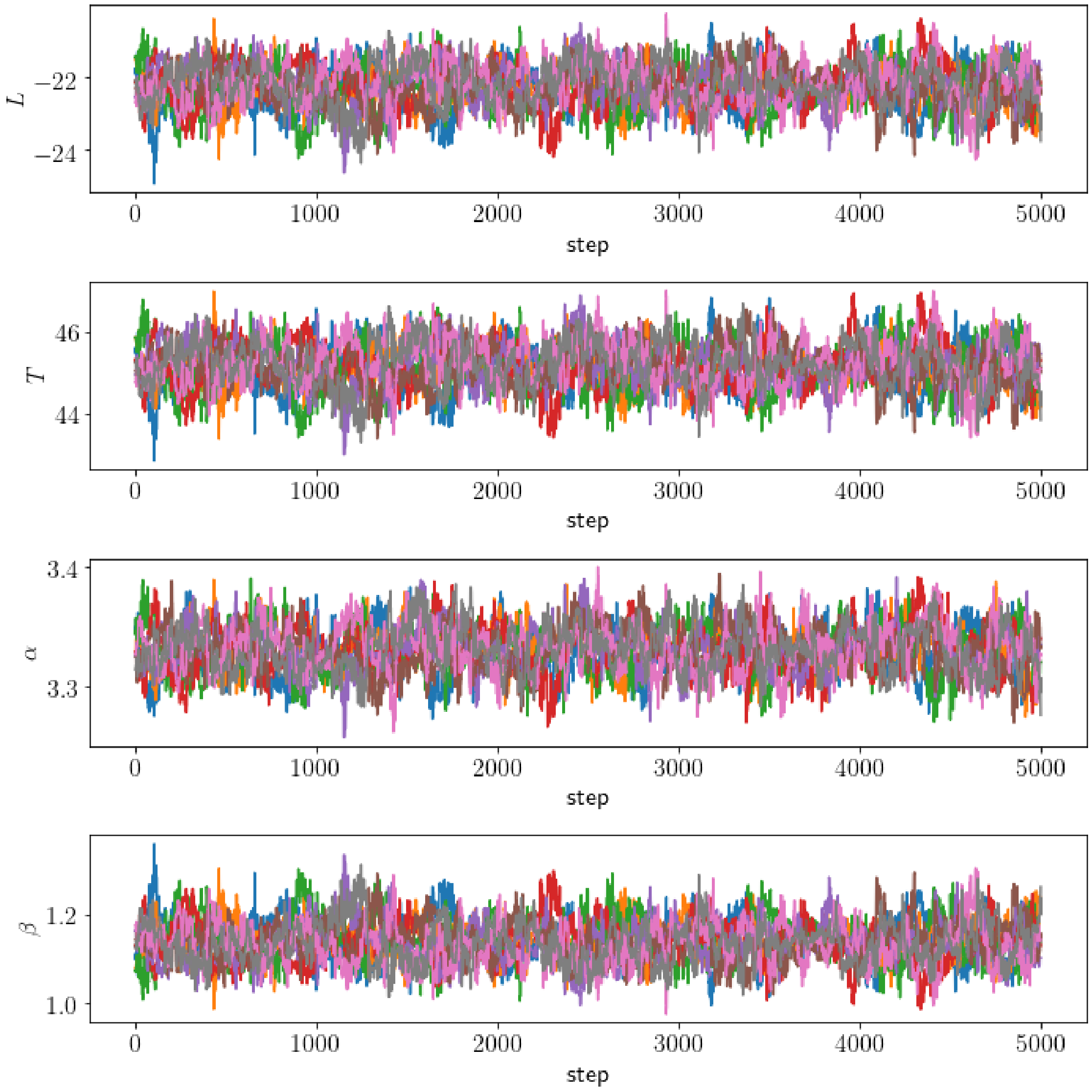


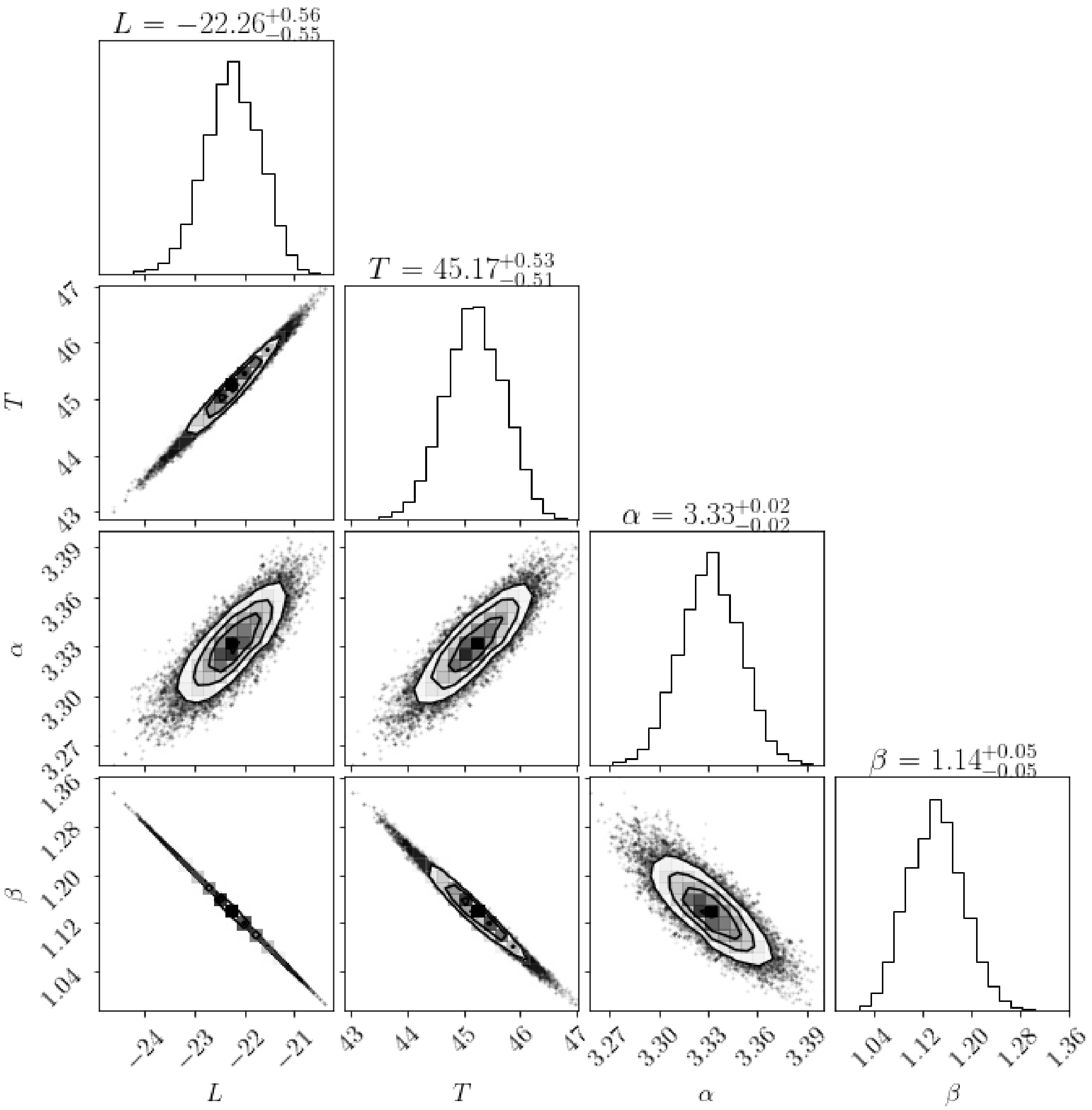
[30]:

```
# Emcee run for Arp 220 model

ndim = 4
nwalk = ndim*2
nburn = 1000
nmain = 5000

# Random starting points
p0 = np.zeros((nwalk, ndim))
for i in range(nwalk):
    p0[i] = fit2 + np.random.uniform(low=-0.05, high=0.05, size=4)
```

```
sampler = emcee.EnsembleSampler(nwalk, ndim, lnprob, args=(frequencies_Arp,␣
 ↪flux_densities_Arp, errors_Arp))

# Burn-in run
pos,prob,state = sampler.run_mcmc(p0, nburn)

sampler.reset()

# Main run
res = sampler.run_mcmc(pos, nmain)
samples = sampler.chain.reshape((-1,ndim))

# plot the individual parameters for model
f, ax = plt.subplots(ndim, 1, figsize=(10, 10))
for idim in range(ndim):
    for iwalk in range(nwalk):
        ax[idim].plot(sampler.chain[iwalk,:,idim])
    ax[idim].set_xlabel('step')
ax[0].set_ylabel(r'$L$')
ax[1].set_ylabel(r'$T$')
ax[2].set_ylabel(r'$\alpha$')
ax[3].set_ylabel(r'$\beta$')
f.tight_layout()
f.show()

# Plot corner plot
f = corner.corner(samples, show_titles=True, labels=(r'$L$', r'$T$',␣
 ↪r'$\alpha$', r'$\beta$'))
f.show()
```

```
<ipython-input-30-d1bb96ee9053>:36: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
<ipython-input-30-d1bb96ee9053>:40: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```


L
-22
-24
0
1000
2000
3000
4000
5000
step
T
46
44
0
1000
2000
3000
4000
5000
step
α
3.4
3.3
0
1000
2000
3000
4000
5000
step
β
1.2
1.0
0
1000
2000
3000
4000
5000
step

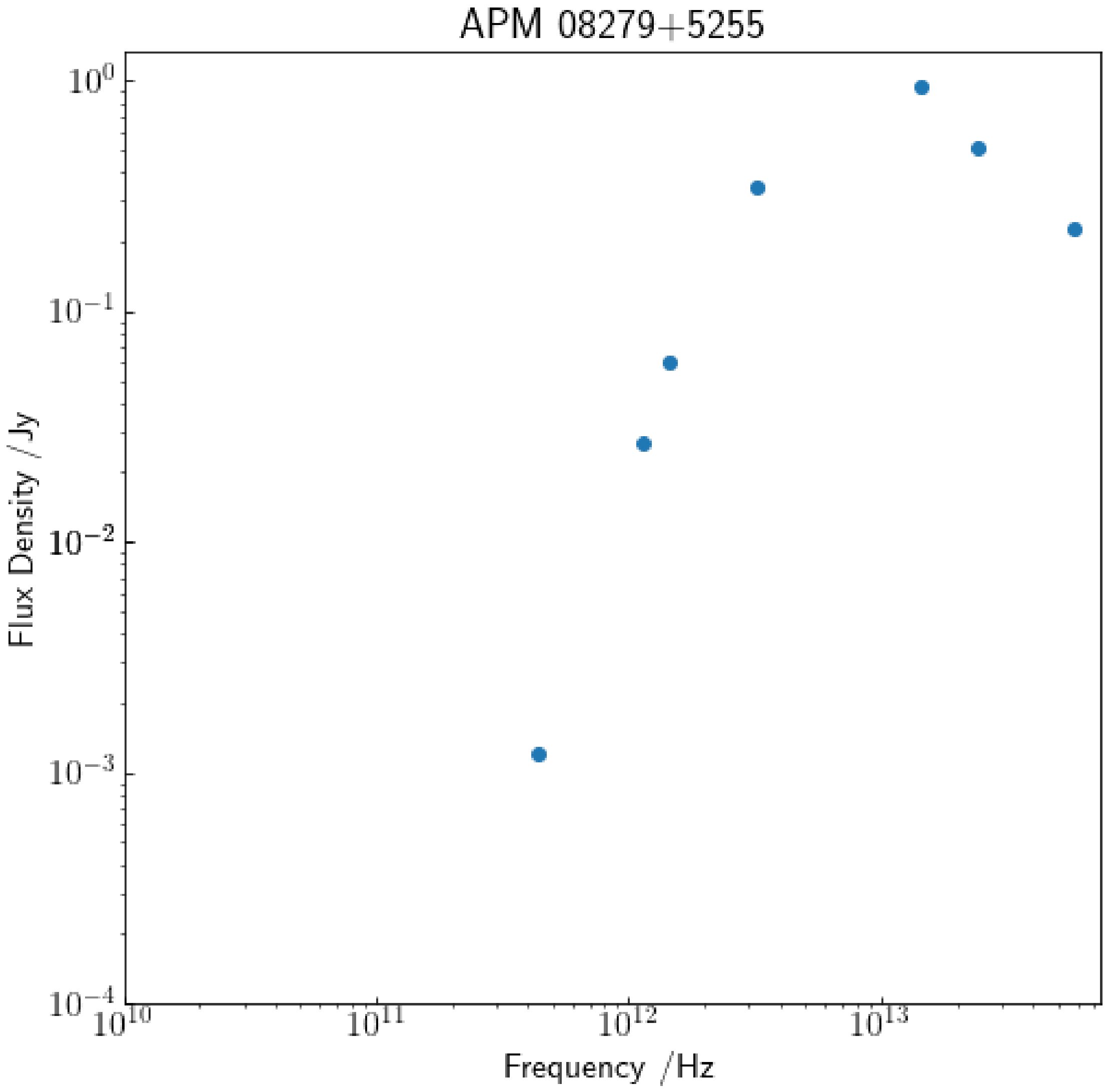


4 APM 08279+5255

[33]:

```
# APM 08279+5255

frequencies_APM = (1 + 3.8)*np.array(frequencies_APM)



f, ax = plt.subplots(1, figsize=(8,8))
ax.plot(frequencies_APM, flux_densities_APM, 'o')
ax.set_xscale('log')
ax.set_xticks([1e+10, 1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.0001, 0.001, 0.01, 0.01, 0.1, 1])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('APM 08279+5255')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
f.show()
```

```
<ipython-input-33-02d92e90822e>:17: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

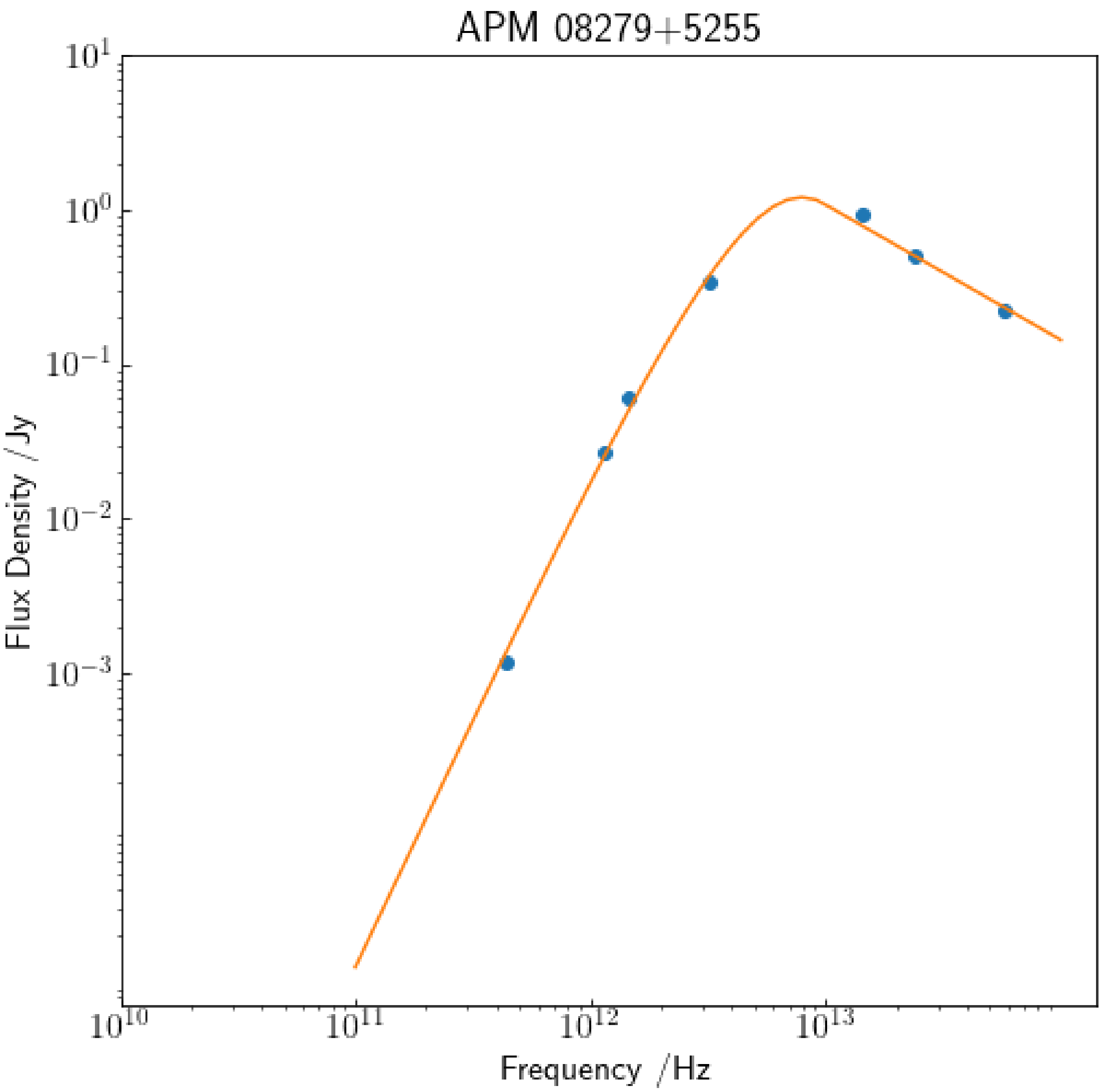


[14]:

```
#fit for APM 08279+5255
fit3 = optimize.fmin(penalty, [-31, 37.4, 2.9, 1.5], args=(frequencies_APM,␣
 ↪flux_densities_APM, errors_APM))
APM_fit = model1(frequencies_APM, fit3)
print (fit3)

#plot fit for APM 08279+5255
x = np.logspace(11,14)

f, ax = plt.subplots(1, figsize=(8,8))
ax.plot(frequencies_APM, flux_densities_APM, 'o')
ax.plot(x, model1(x,fit3))
ax.set_xscale('log')
ax.set_xticks([1e+10, 1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.001, 0.01, 0.1, 1, 10])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('APM 08279+5255')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
f.show()
```

```
Optimization terminated successfully.
         Current function value: 5.856039
         Iterations: 235
         Function evaluations: 404
[-26.83809238  91.16741976   0.87494301   1.24262506]
```

```
<ipython-input-14-04d6842d231d>:21: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

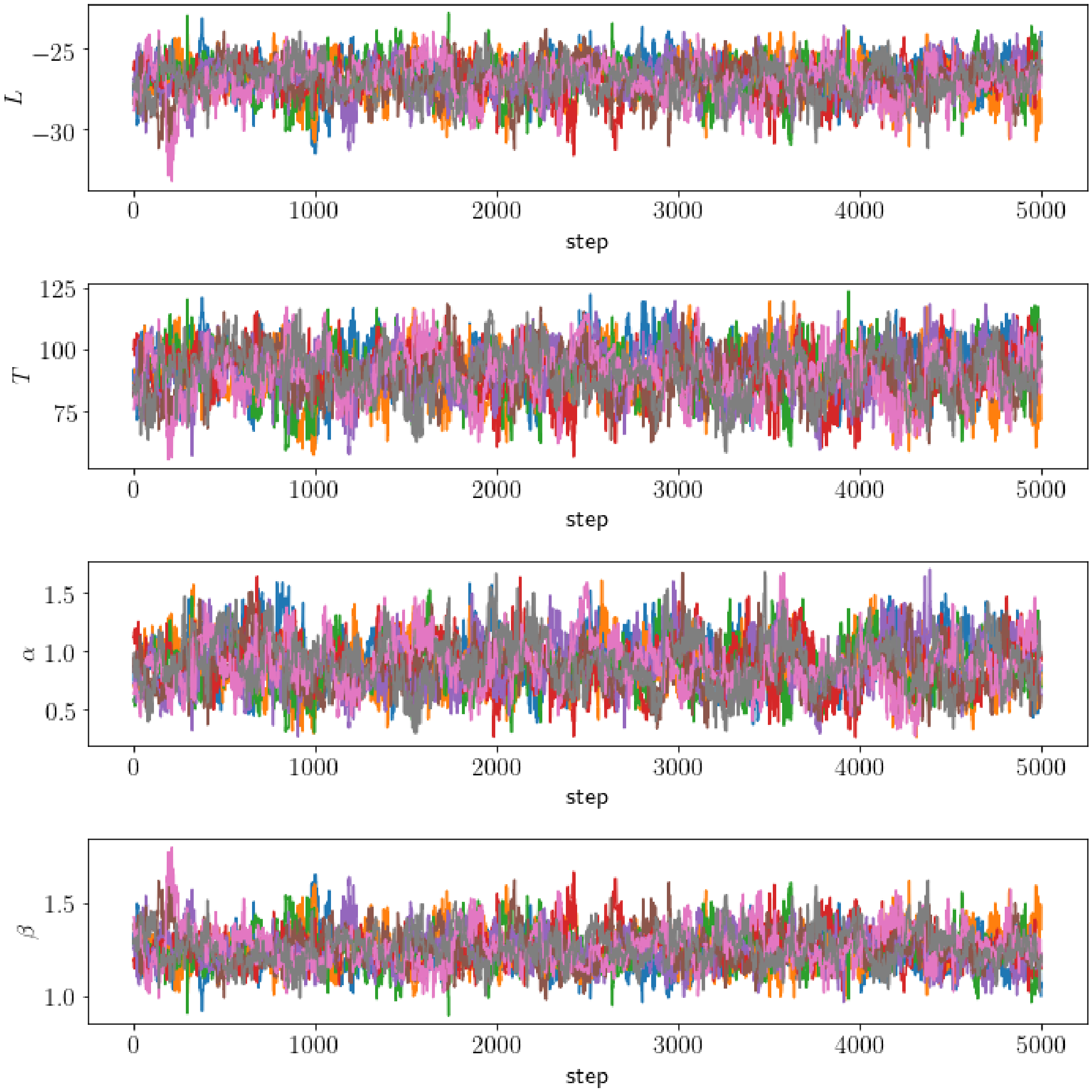


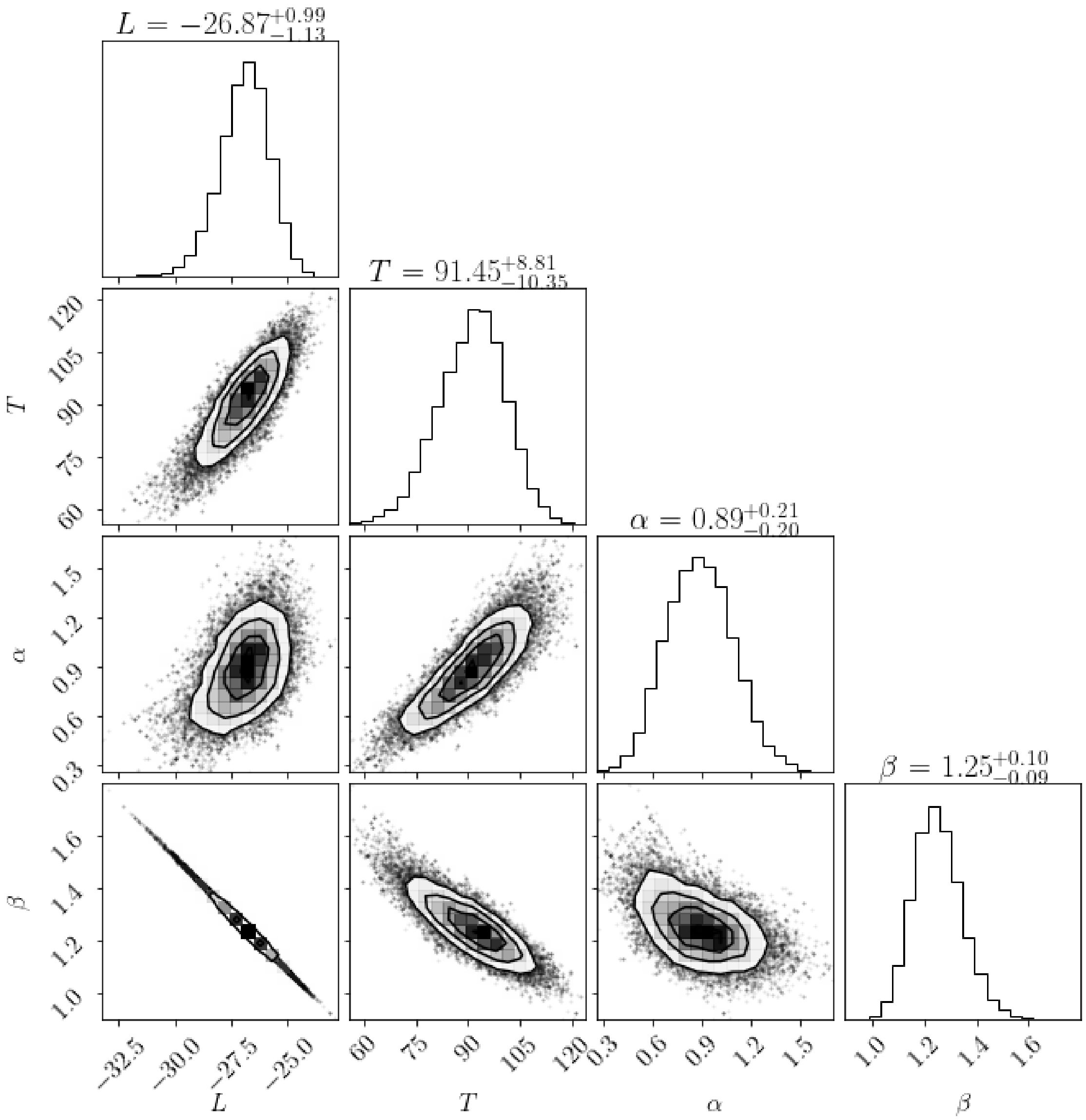
[15]:

```
# Emcee run for APM 08279+5255 model

ndim = 4
nwalk = ndim*2
nburn = 1000
nmain = 5000

# Random starting points
p0 = np.zeros((nwalk, ndim))
for i in range(nwalk):
    p0[i] = fit3 + np.random.uniform(low=-0.05, high=0.05, size=4)
```

```
sampler = emcee.EnsembleSampler(nwalk, ndim, lnprob, args=(frequencies_APM,␣
 ↪flux_densities_APM, errors_APM))

# Burn-in run
pos,prob,state = sampler.run_mcmc(p0, nburn)

sampler.reset()

# Main run
res = sampler.run_mcmc(pos, nmain)
samples = sampler.chain.reshape((-1,ndim))

# plot the individual parameters for model
f, ax = plt.subplots(ndim, 1, figsize=(10, 10))
for idim in range(ndim):
    for iwalk in range(nwalk):
        ax[idim].plot(sampler.chain[iwalk,:,idim])
    ax[idim].set_xlabel('step')
ax[0].set_ylabel(r'$L$')
ax[1].set_ylabel(r'$T$')
ax[2].set_ylabel(r'$\alpha$')
ax[3].set_ylabel(r'$\beta$')
f.tight_layout()
f.show()

# Plot corner plot
f = corner.corner(samples, show_titles=True, labels=(r'$L$', r'$T$',␣
 ↪r'$\alpha$', r'$\beta$'))
f.show()
```

```
<ipython-input-15-344d0cd00ef3>:36: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
<ipython-input-15-344d0cd00ef3>:40: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```


L
−25
−30
0
1000
2000
3000
4000
5000
step
T
125
100
75
0
1000
2000
3000
4000
5000
step
α
1.5
1.0
0.5
0
1000
2000
3000
4000
5000
step
β
1.5
1.0
0
1000
2000
3000
4000
5000
step



5 Subanalysis for fixed $\beta = 1.5$

5.0.1 NGC 0958

```
[16]: def model2(freq, fit):
          L, T, alpha = fit
          beta = 1.5
          nu_prime = optimize.newton(limit, 13, args=(T, alpha, beta))
          L1 = ((10**L)*(10**nu_prime)**(3+beta)/(np.exp(h*(10**nu_prime)/(k*T))-1)/
      ↪((10**nu_prime)**(-1*alpha)))
          predictions = []
```

```
    for nu in freq:
        if (nu < 10**nu_prime):
            predictions.append( 1e-26*(10**L)*(nu**(3+beta))/(np.exp(h*nu/
 ↪(k*T))-1) )
        else:
            predictions.append( 1e-26*L1*(nu**(-1*alpha)) )
    return predictions

def penalty2(param, freq, flux, error):
    return np.sum((model2(freq, param)-flux)**2/error**2)

def lnprob2(param, freq, flux, error):
     return -0.5*penalty2(param, freq, flux, error)
```

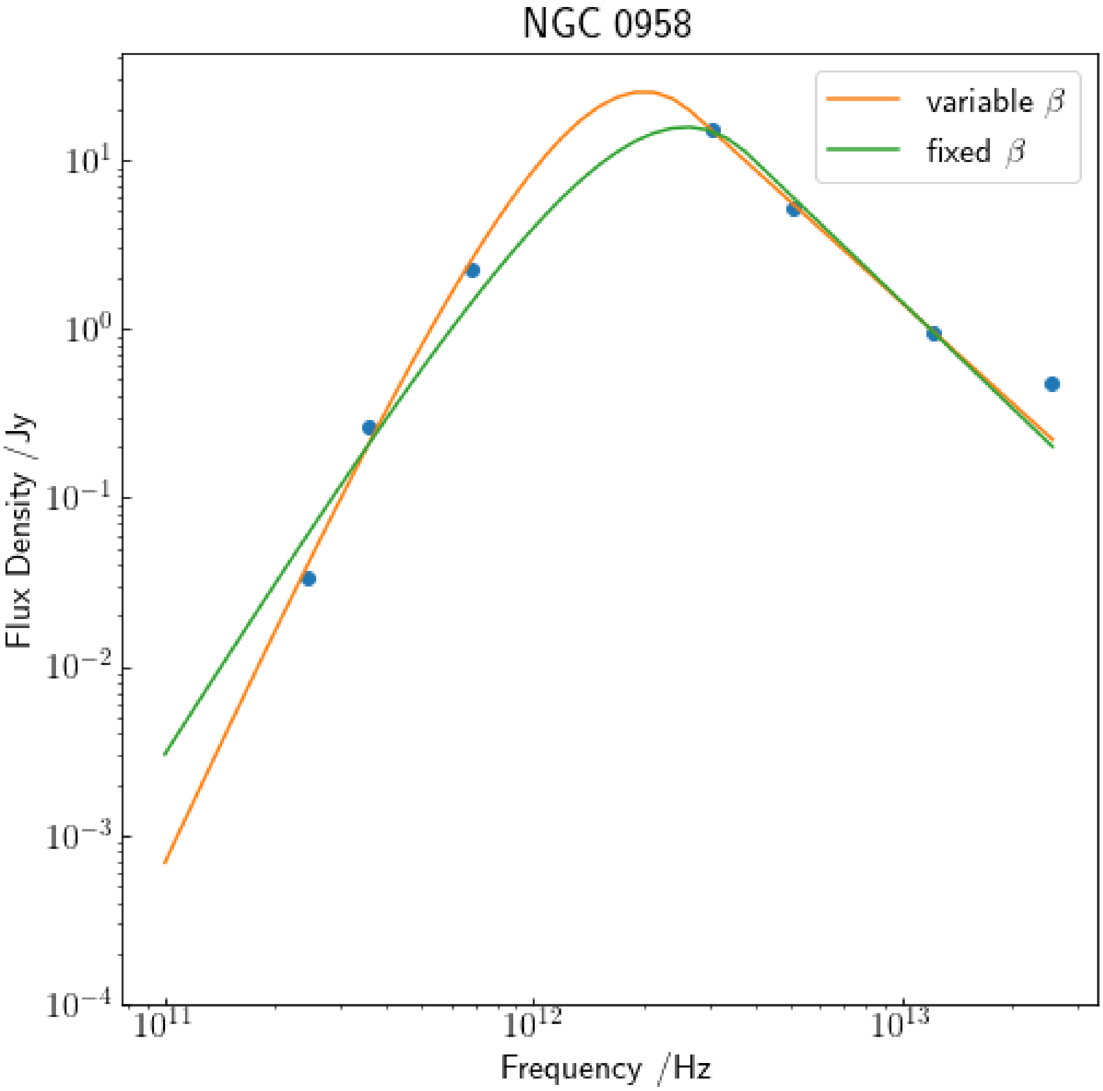
[17]:

```
fit_sub = optimize.fmin(penalty2, [-32, 28.8, 2.02], args=(frequencies_NGC,␣
 ↪flux_densities_NGC, errors_NGC))
ngc_fit_sub = model2(frequencies_NGC, fit_sub)
print(fit_sub)

#plot fit for NGC 09958
x = np.logspace(11,13.41000)

f, ax = plt.subplots(1, figsize=(8,8))
ax.plot(frequencies_NGC, flux_densities_NGC, 'o')
ax.plot(x, model1(x,fit), label=r'variable $\beta$')
ax.plot(x, model2(x,fit_sub), label=r'fixed $\beta$')
ax.set_xscale('log')
ax.set_xticks([1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.0001, 0.001, 0.01, 0.1, 1, 10])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('NGC 0958')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
ax.legend()
f.show()
```

```
Optimization terminated successfully.
         Current function value: 48.202585
         Iterations: 103
         Function evaluations: 184
[-26.74880419  28.08899738   2.09593489]
```

```
<ipython-input-17-4c9fa50d16fc>:22: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
```

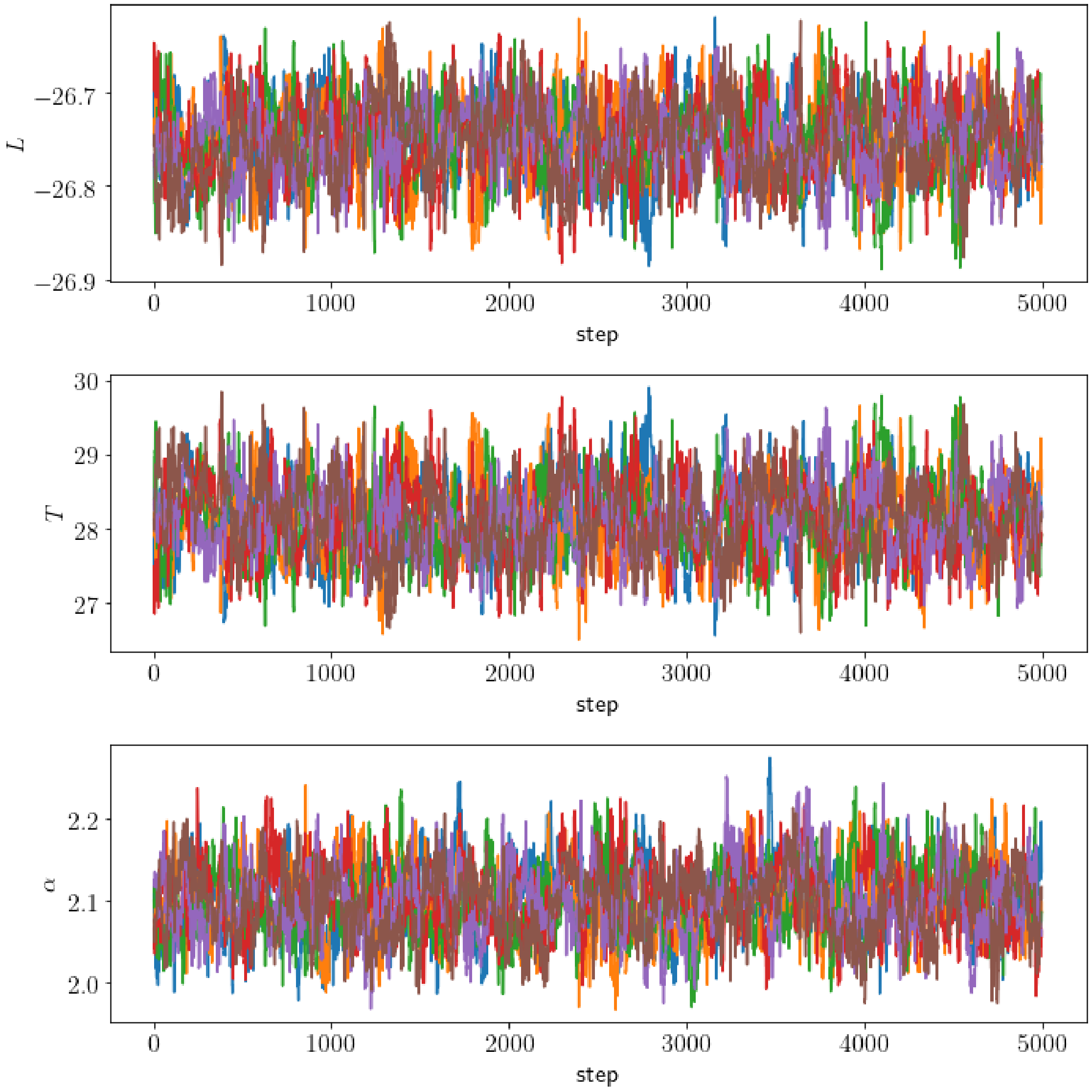
```
f.show()
```

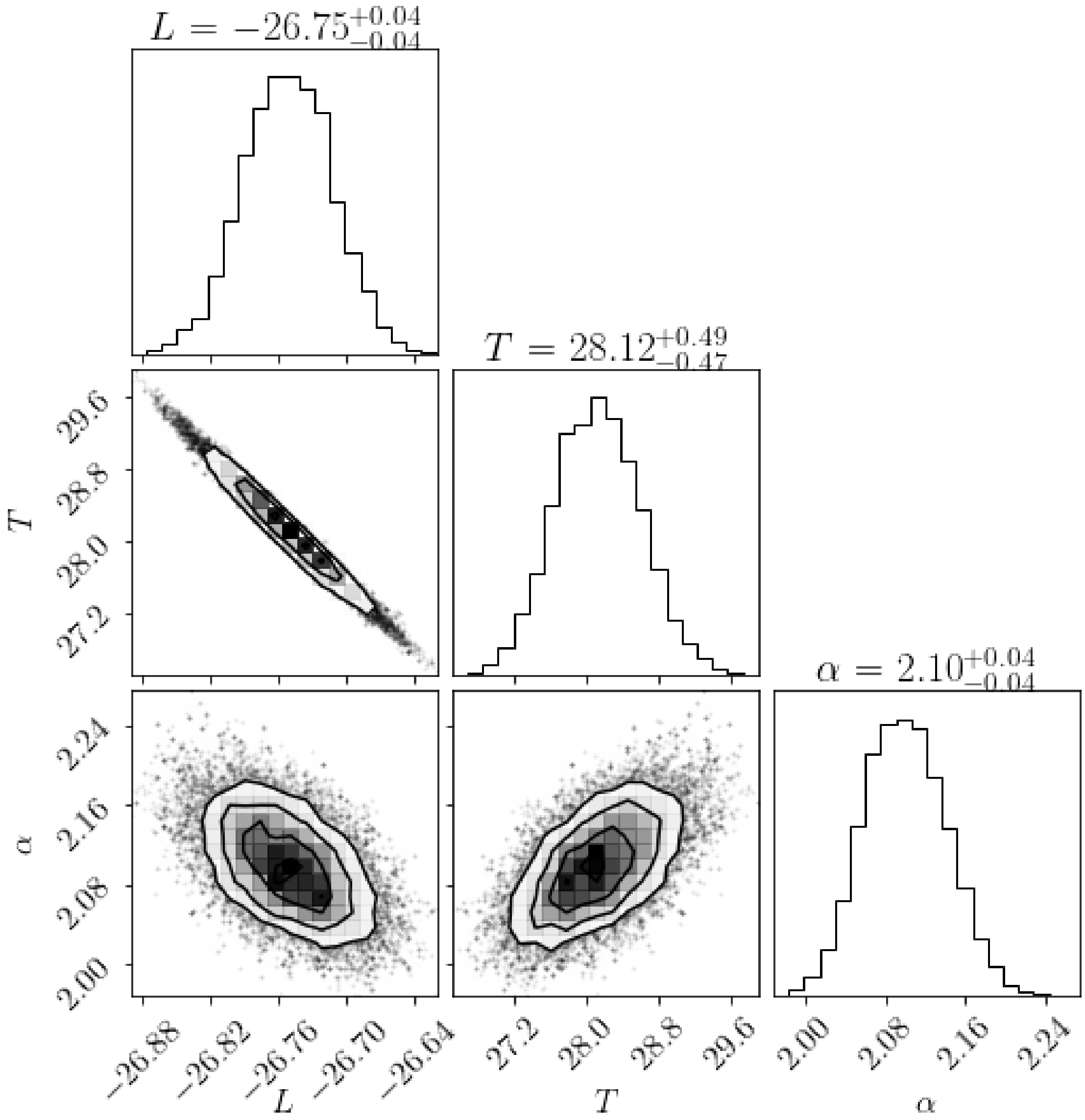


[18]:
```
# Emcee run for NGC 0958 model

ndim = 3
nwalk = ndim*2
nburn = 2000
nmain = 5000

# Random starting points
p0 = np.zeros((nwalk, ndim))
for i in range(nwalk):
    p0[i] = fit_sub + np.random.uniform(low=-0.05, high=0.05, size=3)
```

```
sampler = emcee.EnsembleSampler(nwalk, ndim, lnprob2, args=(frequencies_NGC,␣
 ↪flux_densities_NGC, errors_NGC))

# Burn-in run
pos,prob,state = sampler.run_mcmc(p0, nburn)

sampler.reset()

# Main run
res = sampler.run_mcmc(pos, nmain)
samples = sampler.chain.reshape((-1,ndim))

# plot the individual parameters for model
f, ax = plt.subplots(ndim, 1, figsize=(10, 10))
for idim in range(ndim):
    for iwalk in range(nwalk):
        ax[idim].plot(sampler.chain[iwalk,:,idim])
    ax[idim].set_xlabel('step')
ax[0].set_ylabel(r'$L$')
ax[1].set_ylabel(r'$T$')
ax[2].set_ylabel(r'$\alpha$')
f.tight_layout()
f.show()

# Plot corner plot
f = corner.corner(samples, show_titles=True, labels=(r'$L$', r'$T$',␣
 ↪r'$\alpha$'))
f.show()
```

```
<ipython-input-18-56f912b94e74>:35: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
<ipython-input-18-56f912b94e74>:39: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```


L
-26.7
-26.8
-26.9
0
1000
2000
3000
4000
5000
step
T
30
29
28
27
0
1000
2000
3000
4000
5000
step
α
2.2
2.1
2.0
0
1000
2000
3000
4000
5000
step



5.0.2 Arp 220

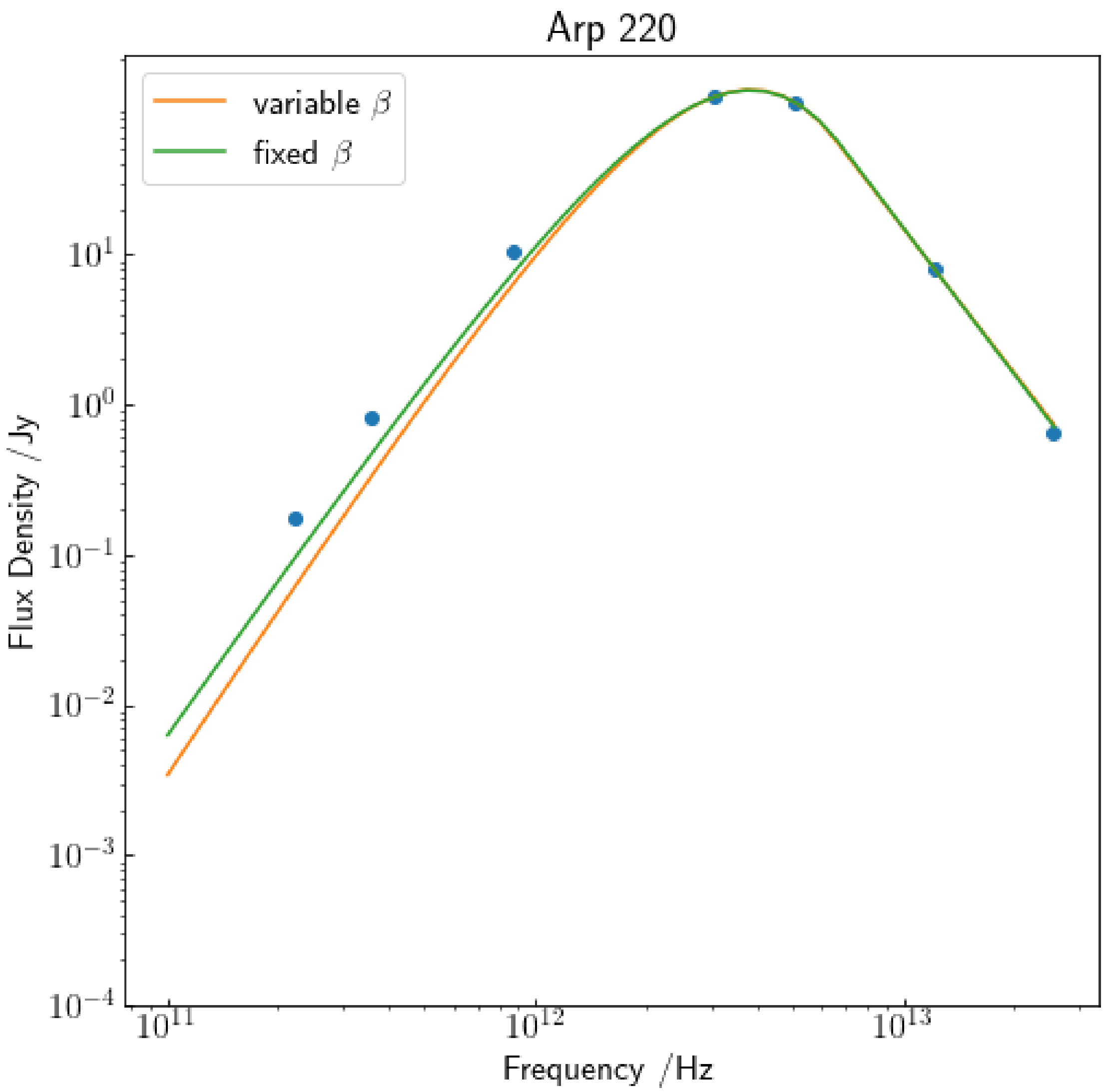
[19]:

```
fit_sub2 = optimize.fmin(penalty2, [-32, 28.8, 2.02], args=(frequencies_Arp,␣
 ↪flux_densities_Arp, errors_Arp))
arp_fit_sub = model2(frequencies_Arp, fit_sub2)
print(fit_sub2)

#plot fit for Arp 220
x = np.logspace(11,13.41000)

f, ax = plt.subplots(1, figsize=(8,8))
```

```
ax.plot(frequencies_Arp, flux_densities_Arp, 'o')
ax.plot(x, model1(x,fit2), label=r'variable $\beta$')
ax.plot(x, model2(x,fit_sub2), label=r'fixed $\beta$')
ax.set_xscale('log')
ax.set_xticks([1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.0001, 0.001, 0.01, 0.1, 1, 10])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('Arp 220')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
ax.legend()
f.show()
```

```
Optimization terminated successfully.
         Current function value: 36.494131
         Iterations: 145
         Function evaluations: 253
[-26.61136047  41.47725442   3.23307213]
```

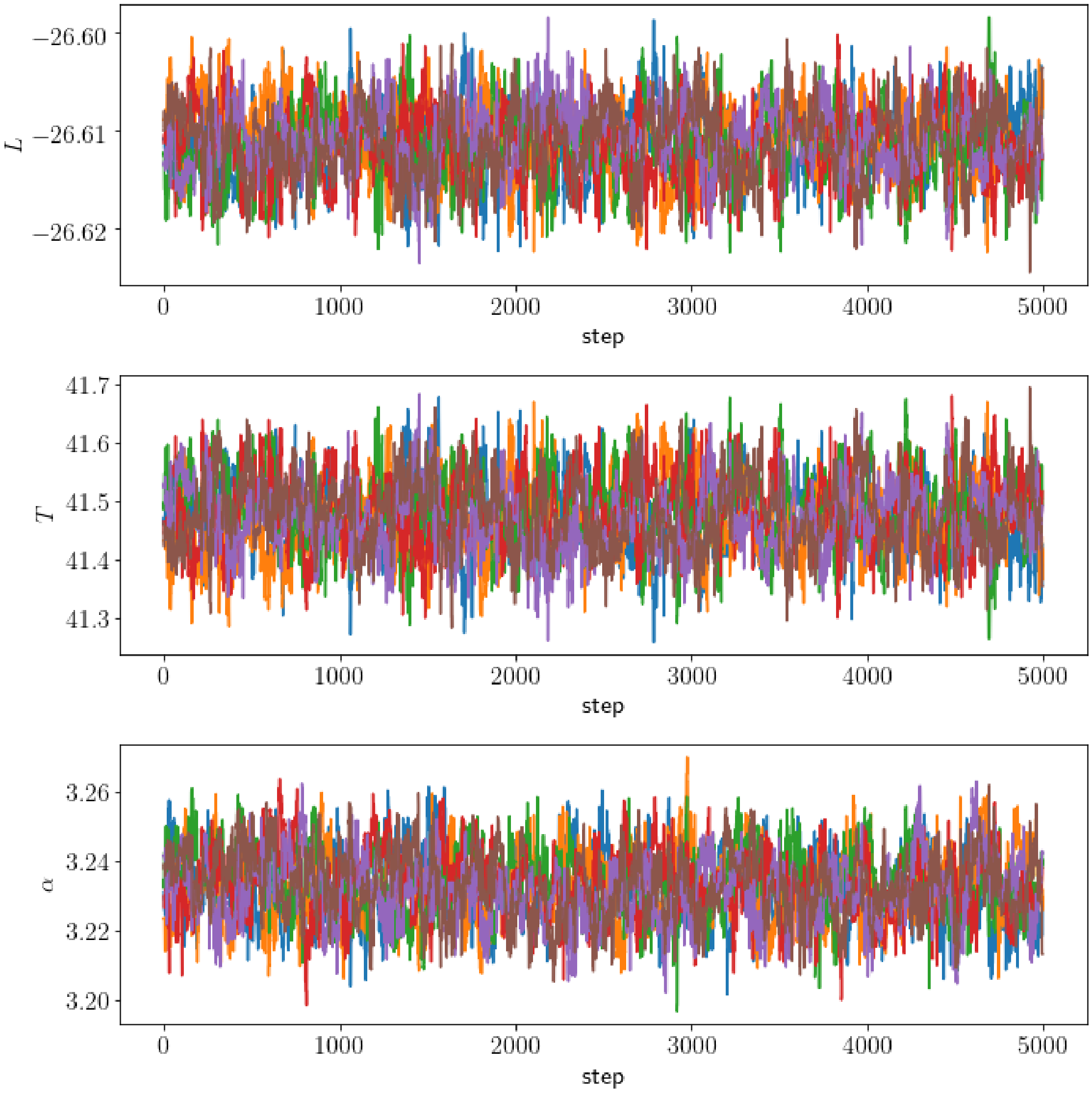
```
<ipython-input-19-d3d671a4cc63>:22: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

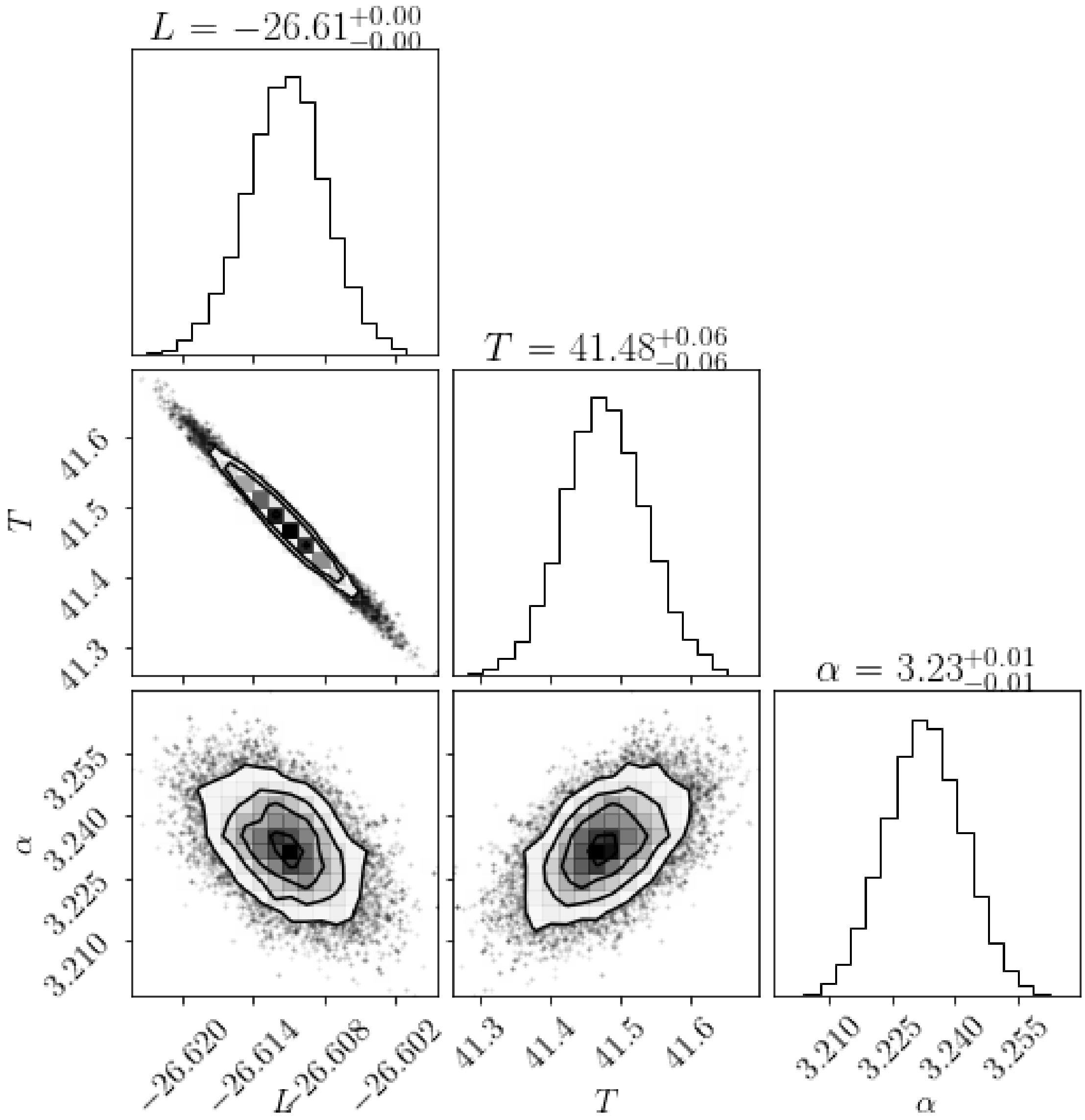


[20]:
```
# Emcee run for Arp 220 model

ndim = 3
nwalk = ndim*2
nburn = 2000
nmain = 5000

# Random starting points
p0 = np.zeros((nwalk, ndim))
for i in range(nwalk):
    p0[i] = fit_sub2 + np.random.uniform(low=-0.05, high=0.05, size=3)
```

```
sampler = emcee.EnsembleSampler(nwalk, ndim, lnprob2, args=(frequencies_Arp,␣
 ↪flux_densities_Arp, errors_Arp))

# Burn-in run
pos,prob,state = sampler.run_mcmc(p0, nburn)

sampler.reset()

# Main run
res = sampler.run_mcmc(pos, nmain)
samples = sampler.chain.reshape((-1,ndim))

# plot the individual parameters for model
f, ax = plt.subplots(ndim, 1, figsize=(10, 10))
for idim in range(ndim):
    for iwalk in range(nwalk):
        ax[idim].plot(sampler.chain[iwalk,:,idim])
    ax[idim].set_xlabel('step')
ax[0].set_ylabel(r'$L$')
ax[1].set_ylabel(r'$T$')
ax[2].set_ylabel(r'$\alpha$')
f.tight_layout()
f.show()

# Plot corner plot
f = corner.corner(samples, show_titles=True, labels=(r'$L$', r'$T$',␣
 ↪r'$\alpha$'))
f.show()
```

```
<ipython-input-20-43f8d303f4d2>:35: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
<ipython-input-20-43f8d303f4d2>:39: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```


−26.60
−26.61
−26.62
L
0
1000
2000
3000
4000
5000
step
41.7
41.6
41.5
41.4
41.3
T
0
1000
2000
3000
4000
5000
step
3.26
3.24
3.22
3.20
α
0
1000
2000
3000
4000
5000
step



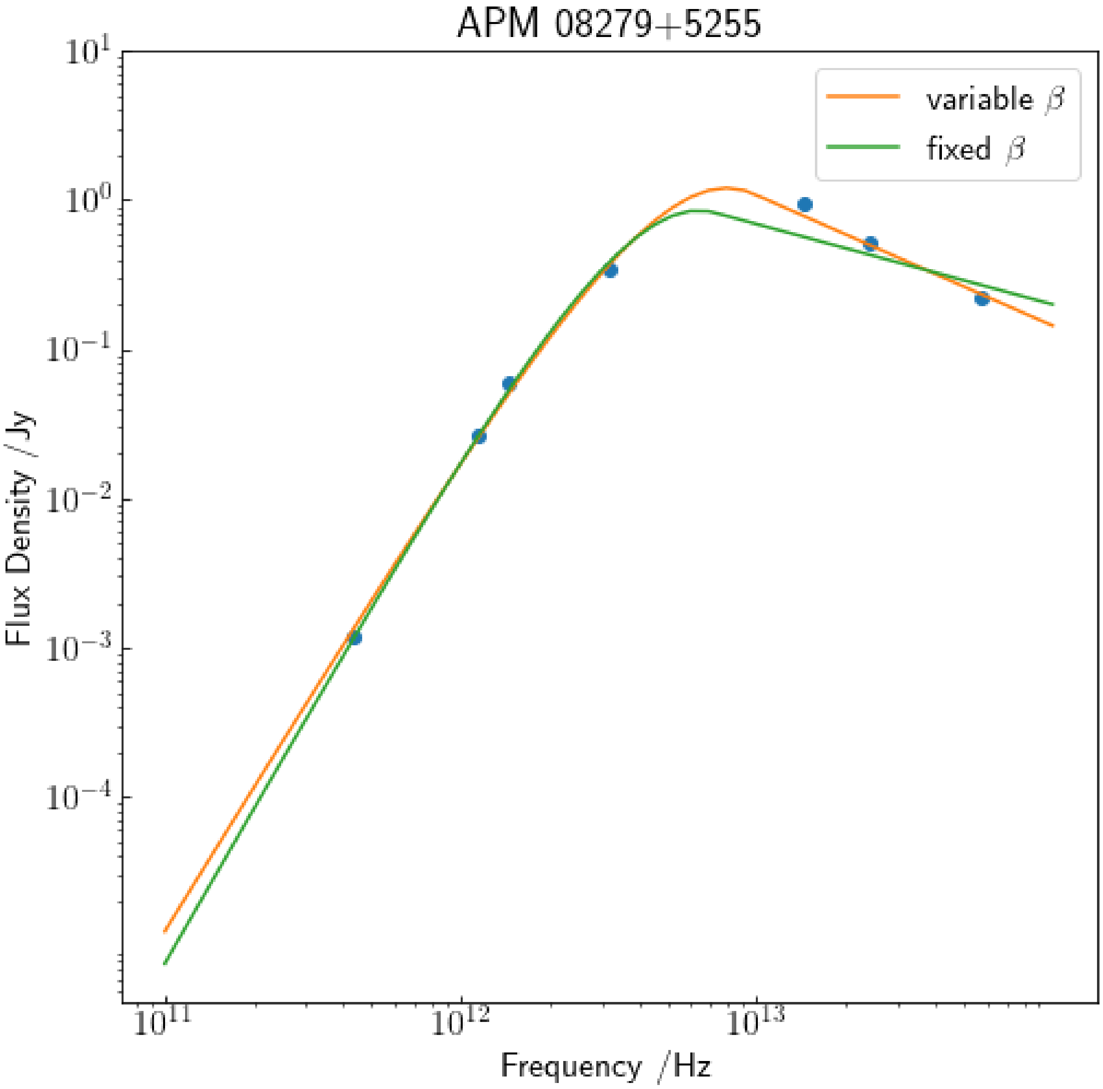
5.0.3 APM 08279+5255

[21]:
```
fit_sub3 = optimize.fmin(penalty2, [-32, 28.8, 2.02], args=(frequencies_APM,␣
 ↪flux_densities_APM, errors_APM))
apm_fit_sub = model2(frequencies_APM, fit_sub3)
print(fit_sub3)

#plot fit for APM 08279+5255
x = np.logspace(11,14)

f, ax = plt.subplots(1, figsize=(8,8))
```

```
ax.plot(frequencies_APM, flux_densities_APM, 'o')
ax.plot(x, model1(x,fit3), label=r'variable $\beta$')
ax.plot(x, model2(x,fit_sub3), label=r'fixed $\beta$')
ax.set_xscale('log')
ax.set_xticks([1e+11, 1e+12, 1e+13])
ax.set_yscale('log')
ax.set_yticks([0.0001, 0.001, 0.01, 0.1, 1, 10])
ax.tick_params(axis="y",direction="in")
ax.tick_params(axis="x",direction="in")
ax.set_title('APM 08279+5255')
ax.set_ylabel('Flux Density /Jy')
ax.set_xlabel('Frequency /Hz')
ax.legend()
f.show()
```

```
Optimization terminated successfully.
         Current function value: 11.086767
         Iterations: 155
         Function evaluations: 272
[-29.75877803  68.65791851   0.5384134 ]
```

```
<ipython-input-21-453536120d0d>:22: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```

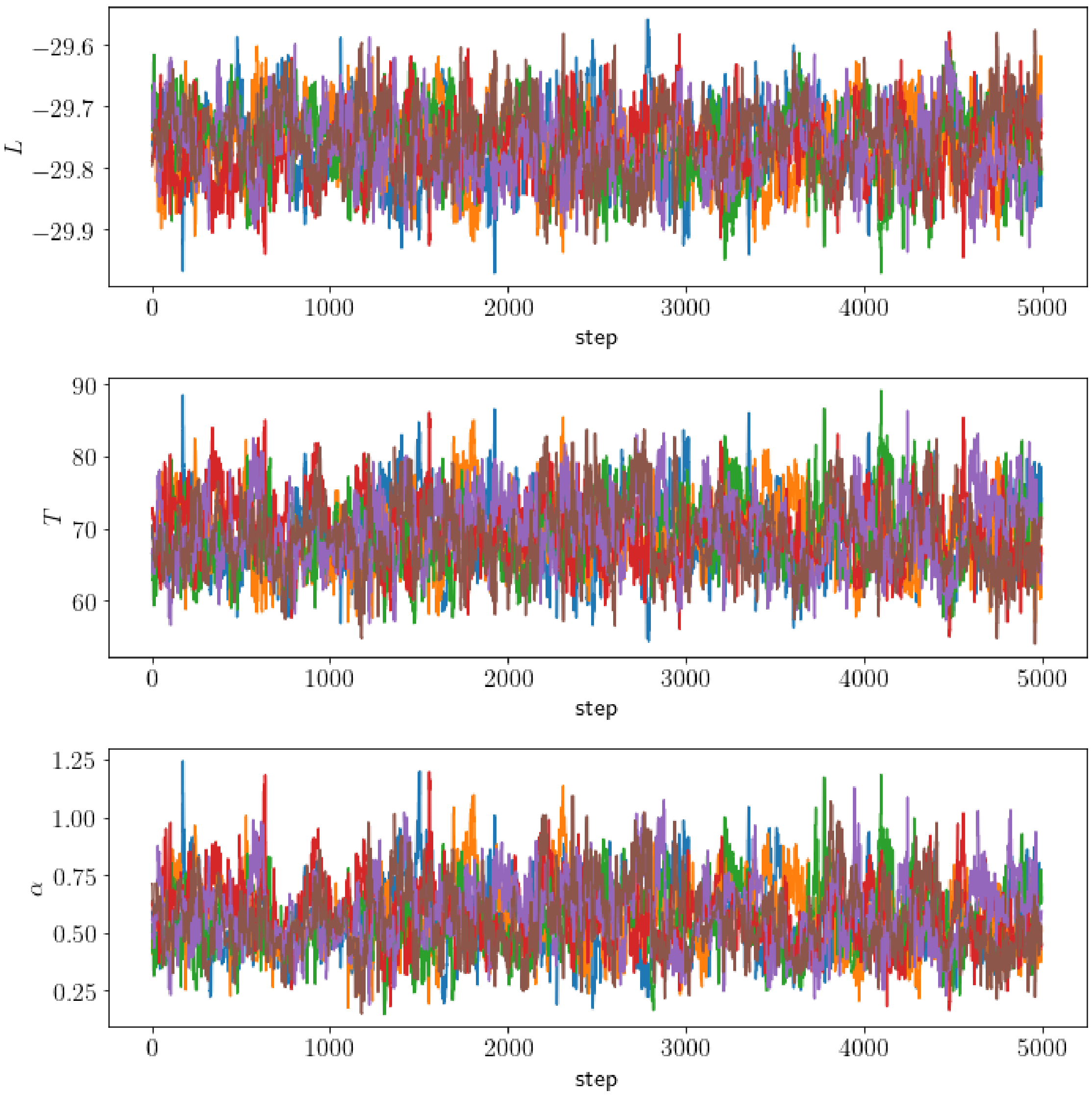


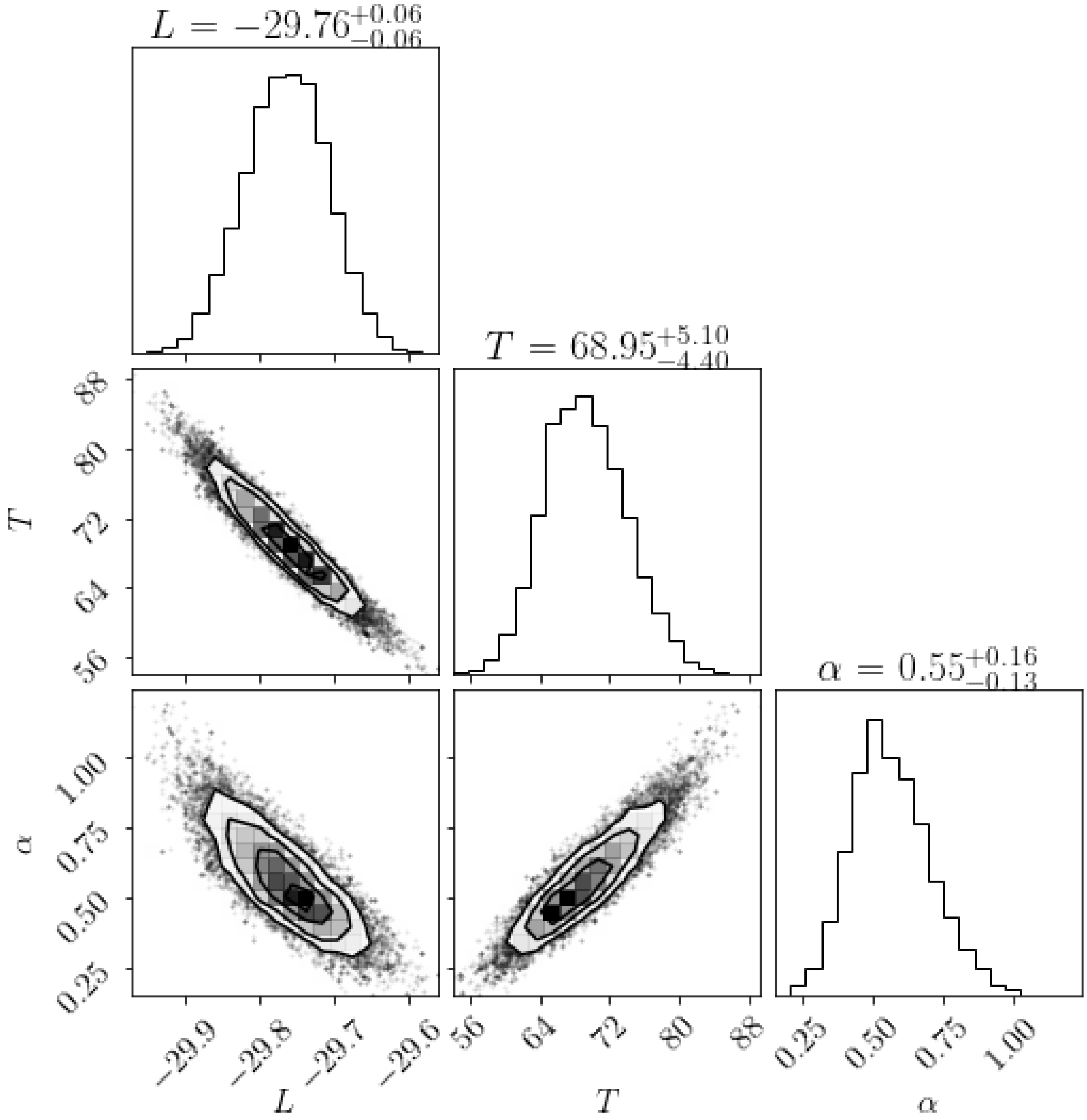
[22]:

```
# Emcee run for Arp 220 model

ndim = 3
nwalk = ndim*2
nburn = 2000
nmain = 5000

# Random starting points
p0 = np.zeros((nwalk, ndim))
for i in range(nwalk):
    p0[i] = fit_sub3 + np.random.uniform(low=-0.05, high=0.05, size=3)
```

```python
sampler = emcee.EnsembleSampler(nwalk, ndim, lnprob2, args=(frequencies_APM, flux_densities_APM, errors_APM))

# Burn-in run
pos,prob,state = sampler.run_mcmc(p0, nburn)

sampler.reset()

# Main run
res = sampler.run_mcmc(pos, nmain)
samples = sampler.chain.reshape((-1,ndim))

# plot the individual parameters for model
f, ax = plt.subplots(ndim, 1, figsize=(10, 10))
for idim in range(ndim):
    for iwalk in range(nwalk):
        ax[idim].plot(sampler.chain[iwalk,:,idim])
    ax[idim].set_xlabel('step')
ax[0].set_ylabel(r'$L$')
ax[1].set_ylabel(r'$T$')
ax[2].set_ylabel(r'$\alpha$')
f.tight_layout()
f.show()

# Plot corner plot
f = corner.corner(samples, show_titles=True, labels=(r'$L$', r'$T$', r'$\alpha$'))
f.show()
```

```
<ipython-input-22-e02e4a717b80>:35: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
<ipython-input-22-e02e4a717b80>:39: UserWarning: Matplotlib is currently using
module://ipykernel.pylab.backend_inline, which is a non-GUI backend, so cannot
show the figure.
  f.show()
```


−29.6
−29.7
−29.8
−29.9
L
0
1000
2000
3000
4000
5000
step
90
80
70
60
T
0
1000
2000
3000
4000
5000
step
1.25
1.00
0.75
0.50
0.25
α
0
1000
2000
3000
4000
5000
step


$L = -29.76^{+0.06}_{-0.06}$
$T = 68.95^{+5.10}_{-4.40}$
$\alpha = 0.55^{+0.16}_{-0.13}$
T
88
80
72
64
56
α
1.00
0.75
0.50
0.25
−29.9
−29.8
−29.7
−29.6
56
64
72
80
88
0.25
0.50
0.75
1.00
L
T
α

[ ]: