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P1)

A. CMPT333 is a variable
B. cmpt 333 is a constant
C. 333 is a constant
D. “cmpt 333” is a constant
E. p(X,x) is a non-ground atomic formula
F. p(3,4,5) is a ground atomic formula
G. “p(3,4,5)” is a constant

P2) (csg(“CMPT220”, S, G) AND snap(S, “L. Van Pelt”, A, P)) → answer(G)

P3) a) (∀X)(∃Y)NOT(p(X) OR p(Y) AND q(X))

b) (∃X)(NOTp(X) AND ((∃Y)p(Y) OR (∃X)q(X, Z)))

P4)

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A)      (∀X)                         B)     AND
          |                                  /   \
        (∃X)                           (∃X)     OR
          |                              |       /  \
         NOT                            NOT   (∃Y)  (∃X)
          |                              |      |      |
          OR                            p(X)   p(Y)   q(X,Z)
         /  \
     p(Y)    AND
            /   \
         p(Y)    q(X)
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P5) Change the variable Z, so it's not the same

P6) (∀C) (csg(C,S, “A”) AND snap(S, “C.Brown), A, P) → answer(“A”))

(∃C) (NOT csg(C,S, “A”) AND snap(S, “C.Brown), A, P) → answer(“A”))

P7) A) (∀X)(∃Y)(loves(X, Y))

loves(a, b) Domain- a,b,c

loves(c, b)

TRUE when b is Y

FALSE when b is X

B) p(X) → NOTp(X)

p(X)= False

p(X)= True

C) (∃X)p(X) → (∀X)p(X)

P8) (p(X) OR q(Y)) ≡ (q(Y) OR p(X)) This is a tautology because it is identical to the commutative law for or, just with p(X) or p(Y) inserted where p and q are. (p + q) ≡ (q + p).

(p(X, Y) AND p(X, Y)) ≡ p(X, Y) This is the tautology pp ≡ p idempotence of AND. just with p(X,Y) substituted for p.

(p(X) → FALSE) ≡ NOT p(X) This is the tautology implication of AND and OR

P9)

A. (∃X)(NOT p(X)) AND((∃Y)p(Y))) OR ((∃X)q(X, Z)))) ≡ (∃X')(NOT p(X')) AND((∃Y')p(Y'))) OR ((∃X')q(X', Z'))))

B. (∃X)(∃~~X~~Y)p(X) OR (X)q(X) OR r(X)) ≡ (∃Z)(∃Y)p(X') OR (Z)q(Y) OR r(X')) -1

P10) -2

A. p(X, Y) AND (∃Y)q(Y) - p(X, Y) AND (∃Z)q(Z)

B. (∃X)(p(X, Y) OR (∃X)p(Y, X)) - (∃X)(p(X, Y) OR (∃Z)p(Y, Z))

P11) Does law (E AND (QX)F) → (QX)(E AND F) imply that p(X, Y) AND (∃X)q(X) is equivalent to (∃X)(p(X, Y) AND q(X))?

That law does not imply that you could pull the (∃X) and put it in front of the expression (p(X, Y) AND q(X)) because in the law, it shows the (QX) was able to be pulled from F, and be applied to the whole expression. BUt you can't change a variable's occurrence. This only works becaus e it is the same variable, and the occurrence is the same. The (∃X) was only applying to the q(X), not p(X,Y), that's why the law can't be used to justify their equivalence.

P12)

A. (∃X)(NOT p(X)) AND((∃Y)p(Y))) OR ((∃X)q(X, Z)))) in prenex form is:
(∃X)(∃Y)(NOT p(X)) ANDp(Y)) OR q(X, Z))) -2
∃Z

(∃X)(∃X)p(X) OR (X)q(X) OR r(X)) in prenex form is the same expression.

P13)

((Q1X)E) → (Q2Y)F) would turn into ((Q1X)(Q2Y)(E→F)) -2

P14)

NOT ((∃X)(∃Y)p(X, Y)) = ((~~∃~~∀X)(~~∃~~∀Y)(NOT p(X, Y)))
NOT ((∃X)p(X) OR (∃Y)q(X, Y))) = ((~~∃~~∀X)(NOT p(X)) OR (~~∃~~∀Y) (NOT q(X, Y)))) -3

P15)

When (∃X)E, it is true that E is a tautology, because the existential quantifier, is not quantifying E, it's quantifying X. And E is equivalent to E, therefore it's a tautology.

-4

Turn the following into closed expressions by universally quantifying each of the free variables. If necessary, rename variables so that no two quantifier occurrences use the same variable. a) p(X, Y) AND (∃Y)q(Y) b) (∃X)(p(X, Y) OR (∃X)p(Y, X)) Problem 11 Does law (E AND

(QX)F) → (QX)(E AND F) imply that p(X, Y) AND (∃ X)q(X) is equivalent to (∃ X)(p(X, Y) AND q(X)) explain your answer. Problem 12 Transform the expressions of Problem 9 into prenex form. 2 Problem 13 Show how to move quantifiers through an → operator. That is, turn the expression ((Q1X)E) → (Q2Y)F) into a prenex form expression. What constraints on free variables in E and F do you need? Problem 14 (NOT ((∀X)E)) ≡ ((∃ X)(NOT E)) (NOT ((∃ X)E)) ≡ ((∀X)(NOT E)) We can use the two tautologies above to move NOT's inside quantifiers as well as to move them outside. Using these laws, plus DeMorgan's laws, we can move all NOT's so they apply directly to atomic formulas. Apply this transformation to the following expressions. 1. NOT ((∃ X)(∃ Y)p(X, Y)) 2. NOT ((∃ X)p(X) OR (∃ Y)q(X, Y))) Problem 15 Is it true that E is a tautology whenever (∃ X)E is a tautology?