HW_3_corrected

A repo on this hw can be found here.

2. MadGraph Bhabha scattering $e^+e^- \to e^+e^-$ (20 points).

The differential cross section for Bhabha scattering in QED in the high-energy limit can be written in terms of the Mandelstam variables $s = (p_1 + p_2)^2$, $t = (p_1 - p_3)^2$, and $u = (p_1 - p_4)^2$,

$$\frac{d\sigma}{d\Omega} = \frac{\pi\alpha^2}{s}\left[u^2\left(\frac{1}{s} + \frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right].$$

Note that if we ignore the electron mass, $s + t + u = 0$.

a. (5 points) ✅

Rewrite this formula in terms of s and $\cos\theta$.

We ought to know that at high energies,

$$\begin{aligned} t &= (p_1 - p_1')^2 \\ &\approx -2p_1 \cdot p_1' \\ &= -2|\vec{p_1}||\vec{p_1}'|(1 - \cos\theta) \end{aligned}$$

Since we're in the center of mass frame we can reasonably define

$$\vec{p_1} = E[1, 0, 0, 1] \to |\vec{p_1}| = E.$$

And since we're doing $e^+e^- \to e^+e^-$, we can also assume $|\vec{p_1}| = |\vec{p_1}'| = E$. We also ought to know that $s \approx 4E^2$ at high energy. Altogether this gets us

$$t = -\frac{s}{2}(1 - \cos\theta).$$

Jamming this stuff into $s + t + u = 0$ we get an expression for u:

$$u = -\frac{s}{2}(1 + \cos\theta).$$

One thoroughly jammed, we can move onto plugging into the differential cross-section, which according to Mathematica turns out to be,

$$\frac{d\sigma}{d\Omega} = \frac{\pi\alpha^2(\cos(2\theta)+7)^2}{8s(\cos(\theta)-1)^2}.$$

b. (5 points) ❌

What feature of the diagrams causes the differential cross section to diverge as $\theta \to 0$?

As $\theta \to 0$, $t \to 0$, and so the momentum transfer approaches zero.

I was sorta right about the t-channel thing. The inclusion of the t-channel is why there's a divergence. The reason turns out that $t \to 0$ corresponds to the intermediate photon approaching on-shell.

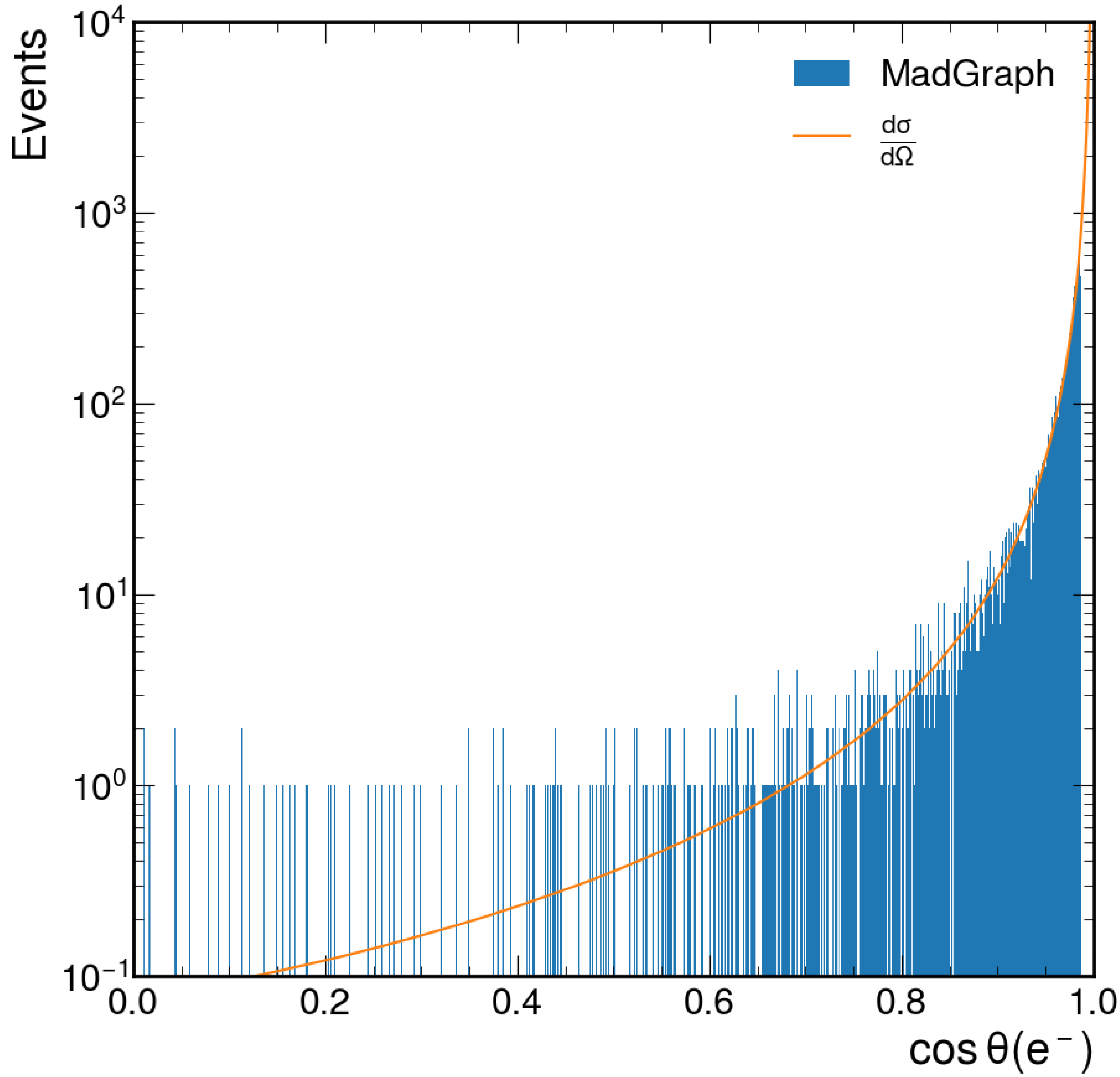
Why didn't we see this for $e^+e^- \to \mu^+\mu^-$?

My guess is that the momentum transfer does not approach zero in this case.

To elaborate, it's because there is no t-channel to diverge in this process.

c. (10 points) ✅

Generate 10,000 events using MadGraph (excluding the Z boson exchange diagram) at $\sqrt{s} = 1$TeV.
Plot the resulting distribution as a function of $\cos\theta$ and compare to the theoretical expectation.



What difference(s) do you observe?

There's no divergence in the MadGraph, probably because there's some sort of cutoff.

3. MadGraph vs. ALEPH experimental results (20 points). ✅

Using MadGraph, reproduce the experimental results from the ALEPH Collaboration, i.e. the total (inclusive) cross section σ and forward-backward asymmetry A_{FB} of the muons as a function of $\sqrt{s}$ in the process $e^+e^- \to \mu^+\mu^-$.

You will need to run MadGraph at a series of $\sqrt{s}$ values, so you will need to edit the `runcard.dat` directly. Of course, both Z boson and γ exchange diagrams need to be included.

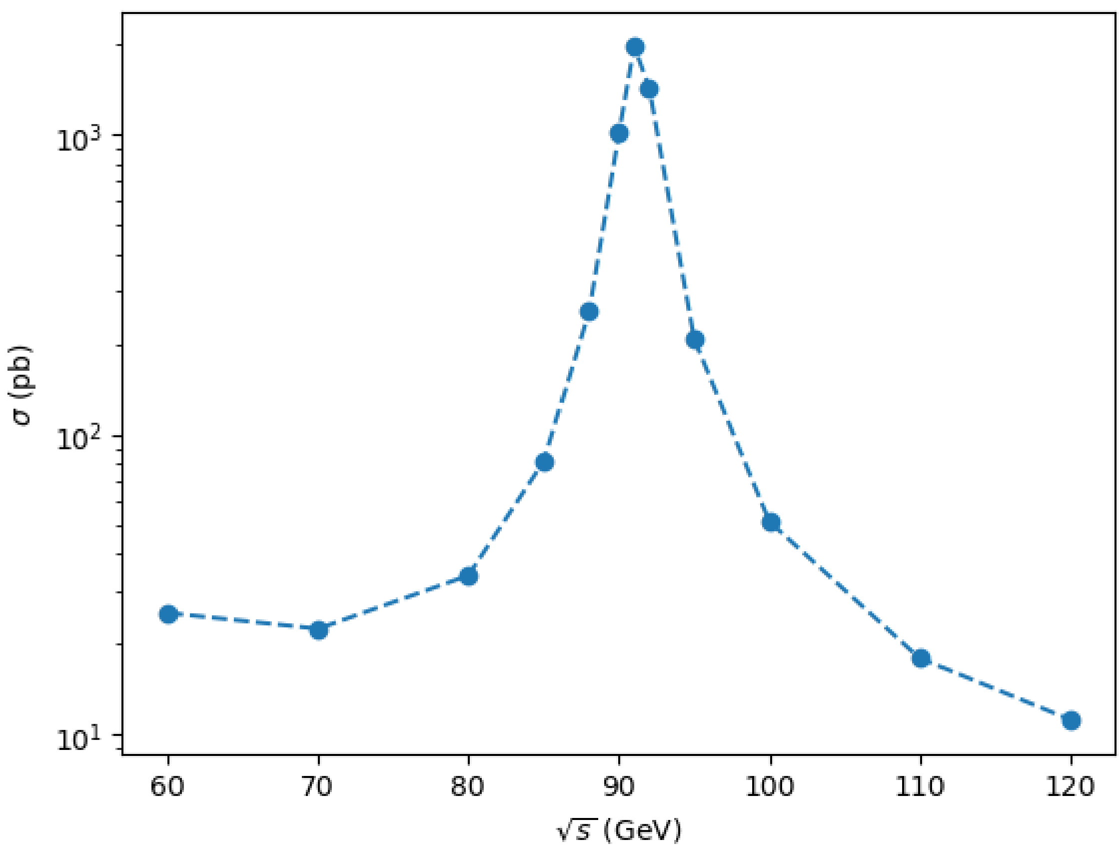
The forward-backward asymmetry is a measure of how many the imbalance between the forward and the backward directions:

$$A_{\text{FB}} = \frac{\sigma(\cos\theta > 0) - \sigma(\cos\theta < 0)}{\sigma(\cos\theta > 0) + \sigma(\cos\theta < 0)}$$

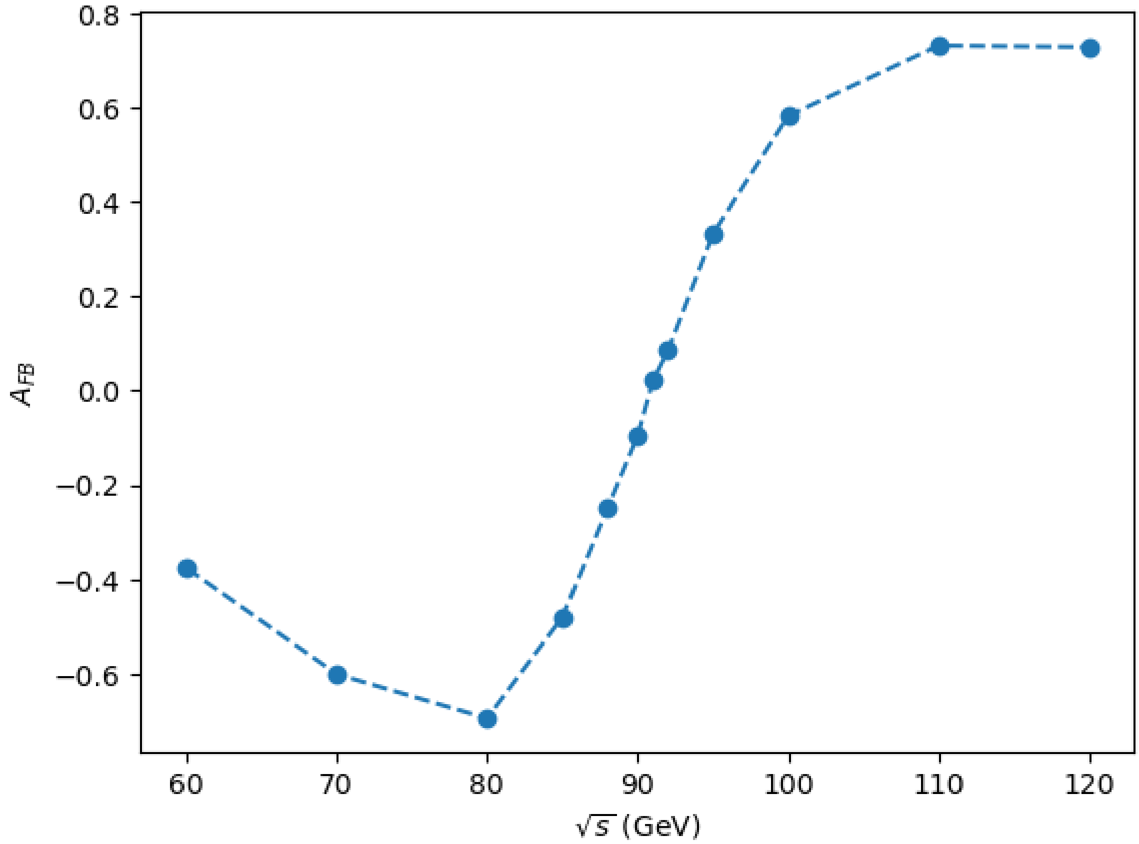
For $e^+e^- \to \mu^+\mu^-$, this quantity is nonzero in the standard model because of the chiral couplings of the Z boson.

In particular, generate 1,000 events at $\sqrt{s} = 60$, 70, 80, 85, 88, 90, 91, 92, 95, 100, 110, and 120,GeV. Plot σ and A_{FB} versus $\sqrt{s}$ and compare to the data.

The peak σ at 90 GeV matches with Aleph, which is a good sign.



The "zeroness" of the A_{FB} at 90 GeV matching that of Aleph is also a good sign.



4. $e^+e^- \to e^+e^-$ at NLO in QED (10 points).

a. Draw all the LO and NLO in QED diagrams for $e^+e^- \to e^+e^-$. How many are there in total? ✅

The simplest thing I can do to the s-channel is replace an electron/positron edge with an $e^\pm \to e^\pm\gamma \to e^\pm$ diagram, or I can replace the γ edge with a $\gamma \to e^+e^- \to \gamma$ diagram.
So I can make a single change on one of the 5 edges in the $e^+e^- \to e^+e^-$, leaving us with 5 NLO diagrams for the s-channel.
Similarly for the t-channel, leaving us with a total of 10 NLO diagrams and 2 LO diagrams.

b. Use MadGraph to generate the diagrams up to NLO order in QED. ✅

The MadGraph syntax for this is:

```
generate <process> / <excluded particles> [QED]
```

The syntax that I ended up doing after downloading and extracting loop_qcd_qed_sm:

```
convert model ./loop_qcd_qed_sm
import ./loop_qcd_qed_sm/
generate e+ e- > e+ e- / g ghg ghg~ u c d s b u~ c~ d~ s~ b~ gh
```

However, this get's me a bunch of explicitly, non-loop diagrams, with an added absorbed or emitted photon.
Makes me feel like I did not properly exclude everything.
Maybe I didn't properly exclude ghosts?

Anyways, there were 24 NLO diagrams generated in total. Their postscripts can be found here.