LIMITS ON A LIGHT HIGGS BOSON

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We reexamine the bounds on a very light Higgs boson (ϕ) coming from limits on the decays $K\to\pi+\phi$ and $B\to\phi+X$. We show that, if there are only three families, $m_\phi>2m_\tau$, and that regardless of the number of families $m_\phi>360$ MeV.

1. Introduction

The effective potential [1] for the Higgs particle (ϕ) (which is the neutral scalar component of an $SU(2)_{weak}$ doublet) may be written as

$$V(\phi)=-\mu^2\phi^2+C\phi^4\ln(\phi^2/M^2)\,, \tag{1.1}$$

where μ and M are constants determined by the Higgs particle mass and by the fact that the potential must have a minimum at $\phi=v/\sqrt{2}$, where $v=246$ GeV. At one-loop order, C is given by

$$C=\frac{1}{16\pi^2v^4}\left(3\sum_V m_V^4+m_\phi^4-4\sum_f m_f^4\right), \tag{1.2}$$

where V runs over the three weak vector bosons and f runs over the fermions (each color counted separately). If we require that the electroweak symmetry breaking vacuum be an absolute minimum of the potential (1.1), we find [2] that

$$m_\phi^2\geqslant Cv^2\,. \tag{1.3}$$

If there are no heavy fermions, this implies [2] that the Higgs must be heavier than 7 GeV. However, if there are heavy fermions (e.g. an 80 GeV quark or a 105 GeV lepton) it is possible for the Higgs to be arbitrarily light. The heavy fermion could be the top quark, or it could be a fermion in a fourth family. In this note we will consider the experimental limits on a light Higgs boson coming from limits on the decays $K\to\pi+\phi$ and $b\to s+\phi$.

2. Flavor-changing Higgs couplings

We begin by constructing an effective lagrangian to describe the flavor-changing couplings of a light Higgs to quarks. In this section we will describe the $\Delta S=1$ couplings of a light Higgs. The analogous $\Delta B=1$ couplings are discussed in the next section.

It is easy to write all the Higgs couplings obtained from tree diagrams. At tree level, the only scale in the standard model lagrangian is the vacuum expectation value of the scalar field, so one can obtain the tree-level Higgs couplings (aside from the Higgs self couplings) by rescaling all mass dimensions by a factor of $1+\phi/v$. The flavor diagonal Higgs couplings are the Yukawa couplings

$$\mathscr{L}_{\text{mass}} = -(1+\phi/v)\bar{\psi}M\psi\,, \tag{2.1}$$

where M is the fermion mass matrix. The flavor-changing $\Delta S=1$ couplings induced at tree level at the weak scale after integrating out the W are (fig. 1)

$$\mathscr{L}_{\text{tree}} = -(G_F/\sqrt{2})V_{us}V_{ud}^*(1+\phi/v)^{-2}[\bar{d}\gamma_\mu(1-\gamma_5)u][\bar{u}\gamma^\mu(1-\gamma_5)s]+\text{h.c.}\,, \tag{2.2}$$

where the V_{ij} are elements of the KM matrix, with similar expressions for the other flavor-changing couplings.

In addition, there are flavor-changing two-quark interactions induced at one loop. Evaluating the diagrams in fig. 2, we find [#1]

$$\mathscr{L}_{\text{one-loop}} = +\frac{3\alpha}{32\pi\sin^2\theta_W}\left(\sum_i V_{is}\frac{m_i^2}{M_W^2}V_{id}^*\right)\frac{\phi}{v}\,[m_s\bar{d}(1+\gamma_5)s+m_d\bar{d}(1-\gamma_5)s]+\text{h.c.}\,, \tag{2.3}$$

in a basis where the quark kinetic energy terms are diagonal and properly normalized and the quark mass terms are diagonal. Here i runs over the charge $\frac{2}{3}$ quarks fields and we have used the equations of motion. This result is in agreement with Willey and Yu [4].

Eqs. (2.2) and (2.3) give the effective lagrangian for the $\Delta S=1$ Higgs couplings renormalized at a scale equal to the W mass – all the couplings and masses should be understood to be the effective masses and couplings at the scale M_W. In order to predict the Higgs couplings at the scales relevant to experiments, m_K for kaon decay and m_b for b decay, we need to consider the evolution of these coupling constants in the effective theory below M_W. The Higgs four-fermion operator (2.2) evolves in the same way as the usual $\Delta S=1$ operator [5]. Thus the effective lagrangian renormalized at a scale of 1 GeV is the Gilman–Wise $\Delta S=1$ lagrangian multiplied by $(1+\phi/v)^{-2}$. At first sight it would appear that the $\Delta S=1$ four-quark operators mix with the two-quark Higgs flavor-changing operator (2.3) through the diagram shown in fig. 3. Explicit evaluation of this diagram, however, shows that it is finite and hence there is no such mixing. Consequently, the only renormalization of (2.3) is due to gluon exchange. Since the anomalous dimension of (2.3) is the same as that of a mass operator, QCD scaling can be taken into account by replacing $m_{s,d}(M_W)$ by $m_{s,d}(\mu)$, the "running" quark masses. Similarly,

[#1] It has been asserted [3] that the coefficient of the $\bar{d}s\phi$ operator is proportional to m_s^3. Explicit evaluation of the diagrams in fig. 2 shows that this assertion is incorrect.

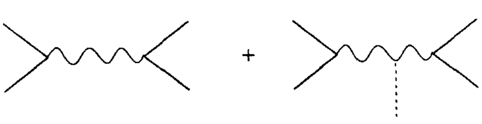

Fig. 1. Flavor-changing tree diagrams.

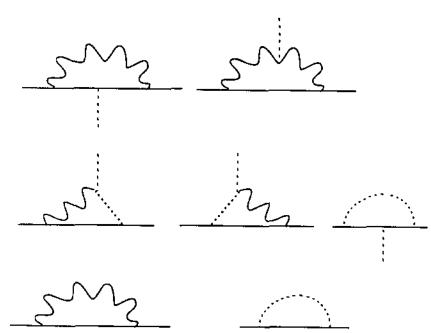

Fig. 2. Loop diagrams in 't Hooft–Feynman gauge for flavor-changing Higgs couplings. The diagram proportional to m_ϕ^2 is negligible for a light Higgs boson.

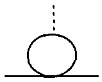

Fig. 3. Operator mixing graph for flavor-changing Higgs boson couplings. The infinite part of this graph vanishes.

the QCD renormalization of the flavor diagonal term (2.1) also converts the mass matrix M into the running mass matrix $M(\mu)$.

3. B meson decay

The $\Delta B=1$ effective lagrangian is derived in a similar manner. The four-Fermi operators analogous to (2.2) are numerically unimportant, and will be neglected. The one loop contribution analogous to (2.3) is

$$\mathscr{L} = + \frac{3\alpha}{32\pi \sin^2\theta_{\mathrm{W}}} V_{\mathrm{tb}} \frac{m_{\mathrm{t}}^2}{M_{\mathrm{W}}^2} V_{\mathrm{ts}}^* \frac{\phi}{v} [m_{\mathrm{b}} \bar{\mathrm{s}}(1+\gamma_5)\mathrm{b}] + \mathrm{h.c.}, \tag{3.1}$$

where we have neglected terms suppressed by m_{s} and have included only the dominant top quark piece in the sum. If there were additional families, the contributions of the other heavy charge $\frac{2}{3}$ quarks might also be important. We can use this operator to calculate the decay of B mesons. The operator $m_{\mathrm{b}}\phi\bar{\mathrm{s}}(1+\gamma_5)\mathrm{b}$ can also be written in the form $\mathrm{i}\partial_\mu\phi\bar{\mathrm{s}}\gamma^\mu(1-\gamma_5)\mathrm{b}$ (again neglecting terms suppressed by m_{s}). The hadronic part of the operator (3.1), $\bar{\mathrm{s}}(1+\gamma_5)\mathrm{b}$, is therefore identical to the operator which occurs in semileptonic B decay. We therefore expect that uncertainties in the matrix element because of strong interaction effects will be minimized if we calculate the ratio $\mathrm{B}\to\phi\mathrm{X}$ to $\mathrm{B}\to\mathrm{e}\nu\mathrm{X}$, and compare with experiment. The theoretical ratio of decay rates is calculated using the free quark model to be

$$\frac{\Gamma(\mathrm{B}\to\phi\mathrm{X})}{\Gamma(\mathrm{B}\to\mathrm{e}\nu\mathrm{X})} = \frac{27\alpha^2}{128\sin^4\theta_{\mathrm{W}} G_{\mathrm{F}}^2 m_{\mathrm{b}}^2 v^2 f(m_{\mathrm{c}}/m_{\mathrm{b}})} \frac{|V_{\mathrm{ts}}V_{\mathrm{tb}}^*|^2}{|V_{\mathrm{cb}}|^2} \left(\frac{m_{\mathrm{t}}}{M_{\mathrm{W}}}\right)^4 \left(1-\frac{m_\phi^2}{m_{\mathrm{b}}^2}\right)^2$$

$$= (2.85) \frac{|V_{\mathrm{ts}}V_{\mathrm{tb}}^*|^2}{|V_{\mathrm{cb}}|^2} \left(\frac{m_{\mathrm{t}}}{M_{\mathrm{W}}}\right)^4 \left(1-\frac{m_\phi^2}{m_{\mathrm{b}}^2}\right)^2,$$

where $f(m_{\mathrm{c}}/m_{\mathrm{b}})=0.51$ is the phase space factor in semileptonic B decay, and we have used $m_{\mathrm{b}}=4.5$ GeV. Thus

$$\mathrm{BR}(\mathrm{B}\to\phi\mathrm{X}) = (0.35) \frac{|V_{\mathrm{ts}}V_{\mathrm{tb}}^*|^2}{|V_{\mathrm{cb}}|^2} \left(\frac{m_{\mathrm{t}}}{M_{\mathrm{W}}}\right)^4 \left(1-\frac{m_\phi^2}{m_{\mathrm{b}}^2}\right)^2, \tag{3.2}$$

where we have used the experimental branching ratio for $\mathrm{B}\to\mathrm{e}\nu\mathrm{X}$ of 12.3%.

If there are only three families, unitarity of the KM matrix implies that $|V_{\mathrm{tb}}|\approx 1$ and $|V_{\mathrm{ts}}|\approx|V_{\mathrm{cb}}|$. Moreover, the argument given in the first section implies that the top quark must weigh at least 80 GeV if there is a light Higgs boson. Therefore combining (3.2) with the branching ratio for the Higgs to decay into $\mu^+\mu^-$ or $\mathrm{e}^+\mathrm{e}^-$ gives us the branching ratio for $\mathrm{B}\to\ell^+\ell^-\mathrm{X}$. The experimental upper bound for this branching ratio is 0.31% [6]. This is sufficient to rule out all Higgs bosons with a mass less than $2m_\tau$ and greater than $2m_{\mathrm{e}}$, even if one uses the estimate of the $\mu^+\mu^-$ branching ratio of the Higgs given by Voloshin [7].

4. K meson decays

If there are more than three generations there is no limit on the Higgs mass from B decay because of our ignorance of the top quark mass and mixing angles. Therefore we consider bounds on a light Higgs boson coming from limits on the decays $\mathrm{K}\to\phi+\pi$. It is convenient to consider these decays in the context of an effective chiral lagrangian. The effective quark lagrangian renormalized at a scale of 1 GeV is (keeping only terms to first order in ϕ/v.)

$$\mathscr{L}=\mathrm{i}\bar{\psi}\partial\!\!\!/\psi-(1+\phi/v)\bar{\psi}M\psi+(1-2\phi/v)\mathscr{L}_{\Delta S=1}+\mathscr{L}_{\text{one-loop}}\,, \tag{4.1}$$

where $\mathscr{L}_{\Delta S=1}$ is the Gilman–Wise $\Delta S=1$ lagrangian [5] renormalized at 1 GeV, and $\mathscr{L}_{\text{one-loop}}$ is given in eq. (2.3). The operators in (4.1) can now be matched to corresponding operators in chiral perturbation theory. The chiral lagrangian to second order in derivatives, and first order in the mass-matrix and ϕ/v is (using the notation of ref. [8])

$$\begin{aligned}\mathscr{L}=&\tfrac{1}{4}f^2\,\mathrm{Tr}\,\partial^\mu\Sigma\partial_\mu\Sigma^\dagger+\tfrac{1}{2}f^2(1+\phi/v)\mathrm{Tr}\,\mu M\Sigma^\dagger+\text{h.c.}-\tfrac{1}{2}f^2(\phi/v)\,\mathrm{Tr}\,\mu N\Sigma^\dagger+\text{h.c.}\\&+\tfrac{1}{4}\lambda f^2(1-2\phi/v)\mathrm{Tr}(h+h^\dagger)\partial^\mu\Sigma\partial_\mu\Sigma^\dagger+\tfrac{1}{4}B\lambda f^2[\partial^\mu(1-2\phi/v)]\mathrm{Tr}(h+h^\dagger)\Sigma\partial_\mu\Sigma^\dagger\\&+\tfrac{1}{2}Af^2(1-2\phi/v)\mathrm{Tr}[h\mu M(1+\phi/v)\Sigma^\dagger]+\text{h.c.}+\tfrac{1}{4}af^2(1-2\phi/v)T^{ij}_{kl}(\Sigma\partial_\mu\Sigma^\dagger)_{ij}(\Sigma\partial_\mu\Sigma^\dagger)^{kl}+\text{h.c.}\,,\end{aligned} \tag{4.2}$$

where $\Sigma=\exp(2\mathrm{i}\pi/f)$, $f=93$ MeV, A, B, λ, μ and a are constants,

$$h=\begin{pmatrix}0&0&0\\0&0&1\\0&0&0\end{pmatrix},\quad M=\begin{pmatrix}m_\mathrm{u}&0&0\\0&m_\mathrm{d}&0\\0&0&m_\mathrm{s}\end{pmatrix},\quad T^{12}_{13}=T^{21}_{13}=T^{12}_{31}=T^{21}_{31}=-T^{22}_{23}=-T^{22}_{32}=\tfrac{1}{2}\,,$$

and

$$N=\begin{pmatrix}0&0&0\\0&0&\zeta m_\mathrm{s}\\0&\zeta^* m_\mathrm{d}&0\end{pmatrix},\quad \zeta=\frac{3\alpha}{16\pi\sin^2\theta_\mathrm{W}}\sum_i V_{is}\frac{m_i^2}{M_\mathrm{W}^2}V^*_{id}\,.$$

It is possible to do a global $\mathrm{SU}(3)_\mathrm{L}$ rotation to diagonalize the Goldstone boson mass matrix. This rotation eliminates the $A\,\mathrm{Tr}hM\Sigma^\dagger$ term, but modifies the corresponding Higgs coupling to $-2A(\phi/v)\mathrm{Tr}\,hM\Sigma^\dagger$. Using the equations of motion (or alternatively, by making a ϕ dependent $\mathrm{SU}(3)_\mathrm{L}$ rotation) we can eliminate A by redefining the value of B. In this basis, fitting to the observed values for the $K\to\pi\pi$ decays gives $\lambda=3.2\times10^{-7}$ and $a=-1.0\times10^{-8}$ [9]. The $\Delta S=1$, $\Delta I=\frac{3}{2}$ term with coefficient a is much smaller than the $\Delta I=\frac{1}{2}$ piece and thus can be neglected [#2].

While the sign of the flavor-changing mass N term in (4.2) is determined by the sign of the pion mass term, the absolute sign of the $\lambda(\Delta I=\frac{1}{2})$ term is not known. In what follows, we will assume that the relative *signs* are correctly given by the "vacuum insertion" approximation. This assumption is conservative since, as we shall see, this implies that the two terms interfere destructively and tend to decrease the $\mathrm{K}\to\pi\phi$ amplitude [#3]. Using (4.2) we find

$$\begin{aligned}&\mathscr{A}(\mathrm{K_L}\to\pi^0\phi)=\mathscr{A}(\mathrm{K}^+\to\pi^+\phi)\\&=\left[-(1.5\times10^{-10})\left(1+\frac{m_\pi^2-m_\phi^2}{m_\mathrm{K}^2}\right)+(0.72\times10^{-10})+\sum_{i\neq\mathrm{c}}\eta_i^*+B(0.68\times10^{-10})\right]\mathrm{GeV}\end{aligned} \tag{4.3}$$

for the $\mathrm{K}\to\pi\phi$ amplitude, where

$$\eta_i=\frac{m_\mathrm{K}^2}{2v}\frac{3\alpha}{16\pi\sin^2\theta_\mathrm{W}}V_{is}\frac{m_i^2}{M_\mathrm{W}^2}V^*_{id}\,. \tag{4.4}$$

The first term in (4.3) is the contribution of the $\Delta I=\frac{1}{2}$ term, the second is η_c. We see that the $\mathrm{K}\to\pi\phi$ amplitude is of order 10^{-10} GeV.

[#2] The $\Delta I=\frac{1}{2}$ enhancement was ignored in ref. [4].

[#3] This is the opposite of the sign found by Willey [4].

A Higgs produced in K decay will decay predominantly into two muons if $m_\phi > 2m_\mu$, into two electrons if $2m_\mu > m_\phi > 2m_e$, and will most likely escape undetected if $m_\phi < 2m_e$. The limits on the branching ratios, and the corresponding limits on the $K \to \pi\phi$ amplitudes are:

$\mathrm{BR}(K_L \to \pi\mu^+\mu^-) < 1.2 \times 10^{-6}$, $|\mathscr{A}(K_L \to \pi\phi)| < 0.20 \times 10^{-10} (m_K/2p_\phi)^{1/2}\,\mathrm{GeV}$ [10] ,
$\mathrm{BR}(K_L \to \pi e^+e^-) < 2.3 \times 10^{-6}$, $|\mathscr{A}(K_L \to \pi\phi)| < 0.28 \times 10^{-10}\,\mathrm{GeV}$ [10] ,
$\mathrm{BR}(K^+ \to \pi + \text{nothing}) < 3.8 \times 10^{-8}$, $|\mathscr{A}(K^+ \to \pi\phi)| < 0.036 \times 10^{-10}\,\mathrm{GeV}$ [11] ,

where the p_ϕ is the momentum of the Higgs particle and the $(m_K/2p_\phi)^{1/2}$ incorporates the phase space suppression for a "heavy" Higgs. For any $m_\phi \leqslant 360$ MeV, $(m_K/2p_\phi) \leqslant 25$. Hence, barring any accidental cancellations in the amplitude (4.3), we see that the Higgs must be heavier than 360 MeV.

It is interesting to note that there is a decay which can be exactly calculated to lowest order in chiral perturbation theory [12]: $K \to e\nu\phi$. Here the lowest order contribution comes from the semileptonic analog of the graph shown in fig. 1 and to lowest order we find

$$\mathrm{BR}(K \to e\nu\phi) = 4 \times 10^{-8} f(x) , \tag{4.5}$$

where $x = m_\phi^2 / m_K^2$ and

$$f(x) = (1 - 8x + x^2)(1 - x^2) - 12x^2 \ln x . \tag{4.6}$$

Unfortunately, current limits on $K \to e\nu\mu^+\mu^-$ and $K \to e\nu e^+e^-$ are not strong enough to provide interesting constraints.

5. Conclusions

In this note we have considered the experimental limits on the existence of a very light Higgs boson. We have shown that if there are only three families, the Higgs must be heavier than $2m_\tau$, and that regardless of the number of families, the Higgs must be heavier than 360 MeV (barring an accidental cancellation in the amplitude).

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