

# HW\_3

## 2. MadGraph Bhabha scattering $e^+e^- \rightarrow e^+e^-$ (20 points).

The differential cross section for Bhabha scattering in QED in the high-energy limit can be written in terms of the Mandelstam variables  $s = (p_1 + p_2)^2$ ,  $t = (p_1 - p_3)^2$ , and  $u = (p_1 - p_4)^2$ ,

$$\frac{d\sigma}{d\Omega} = \frac{\pi\alpha^2}{s} \left[ u^2 \left( \frac{1}{s} + \frac{1}{t} \right)^2 + \left( \frac{t}{s} \right)^2 + \left( \frac{s}{t} \right)^2 \right].$$

Note that if we ignore the electron mass,  $s + t + u = 0$ .

### a. (5 points)

Rewrite this formula in terms of  $s$  and  $\cos \theta$ .

We ought to know that at high energies,

$$\begin{aligned} t &= (p_1 - p_1')^2 \\ &\approx -2p_1 \cdot p_1' \\ &= -2|\vec{p}_1||\vec{p}_1'|(1 - \cos \theta) \end{aligned}$$

Since we're in the center of mass frame we can reasonably define

$$\vec{p}_1 = E[1, 0, 0, 1] \rightarrow |\vec{p}_1| = E.$$

And since we're doing  $e^+e^- \rightarrow e^+e^-$ , we can also assume  $|\vec{p}_1| = |\vec{p}_1'| = E$ . We also ought to know that  $s \approx 4E^2$  at high energy. Altogether this gets us

$$t = -\frac{s}{2}(1 - \cos \theta).$$

Jamming this stuff into  $s + t + u = 0$  we get an expression for  $u$ :

$$u = -\frac{s}{2}(1 + \cos \theta).$$

One thoroughly jammed, we can move onto plugging into the differential cross-section, which according to Mathematica turns out to be,

$$\frac{d\sigma}{d\Omega} = \frac{\pi\alpha^2(\cos(2\theta) + 7)^2}{8s(\cos(\theta) - 1)^2}.$$

## b. (5 points)

What feature of the diagrams causes the differential cross section to diverge as  $\theta \rightarrow 0$ ?

As  $\theta \rightarrow 0$ ,  $t \rightarrow 0$ , and so the momentum transfer approaches zero.

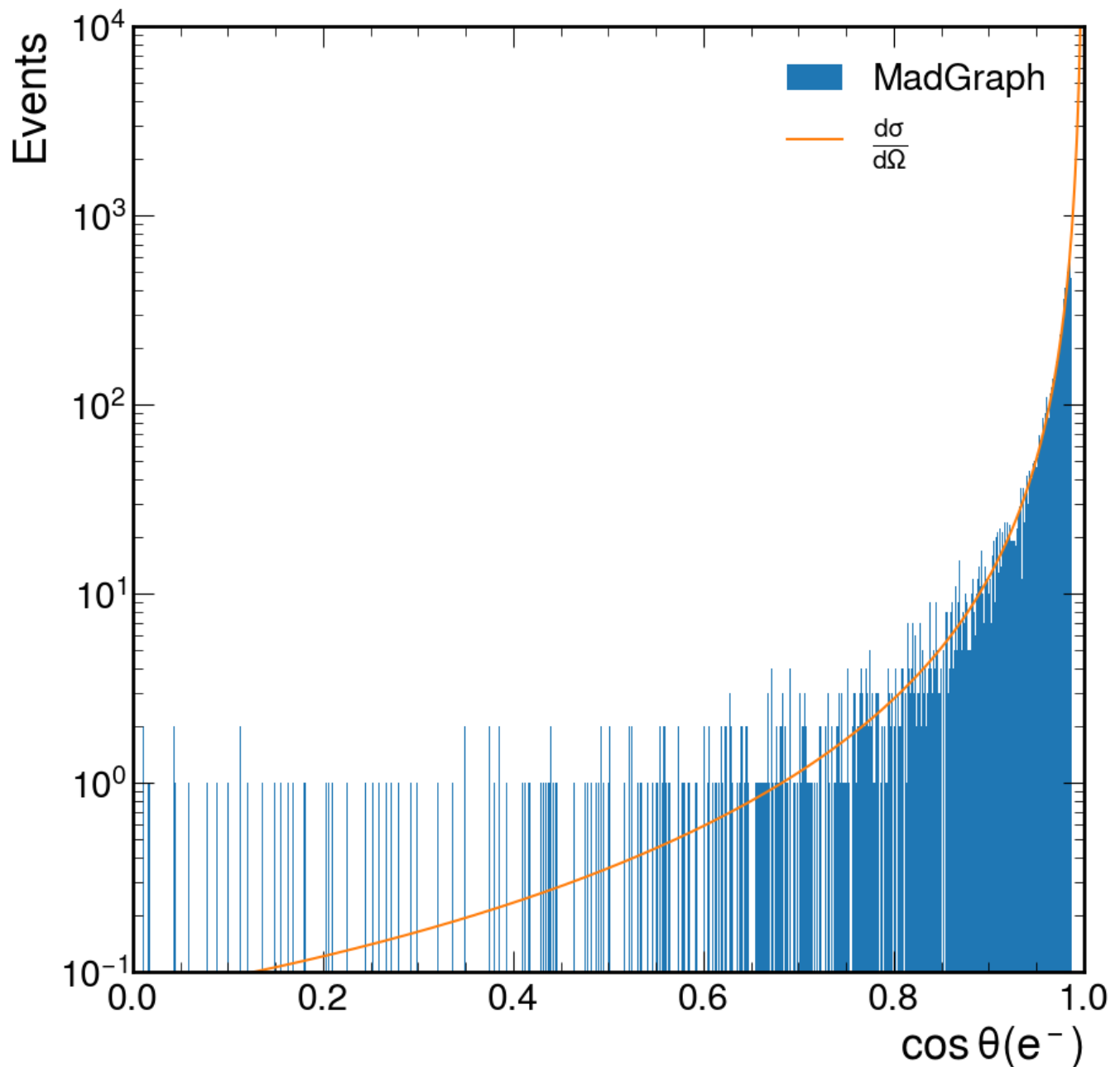
Why didn't we see this for  $e^+e^- \rightarrow \mu^+\mu^-$ ?

My guess is that the momentum transfer does not approach zero in this case.

## c. (10 points)

Generate 10,000 events using MadGraph (excluding the  $Z$  boson exchange diagram) at  $\sqrt{s} = 1\text{TeV}$ .

Plot the resulting distribution as a function of  $\cos\theta$  and compare to the theoretical expectation.



What difference(s) do you observe?

There's no divergence in the MadGraph, probably because there's some sort of cutoff.

### 3. MadGraph vs. ALEPH experimental results (20 points).

Using MadGraph, reproduce the experimental results from the ALEPH Collaboration, i.e. the total (inclusive) cross section  $\sigma$  and forward-backward asymmetry  $A_{\text{FB}}$  of the muons as a function of  $\sqrt{s}$  in the process  $e^+e^- \rightarrow \mu^+\mu^-$ .

You will need to run MadGraph at a series of  $\sqrt{s}$  values, so you will need to edit the `runcard.dat` directly. Of course, both  $Z$  boson and  $\gamma$  exchange diagrams need to be included.

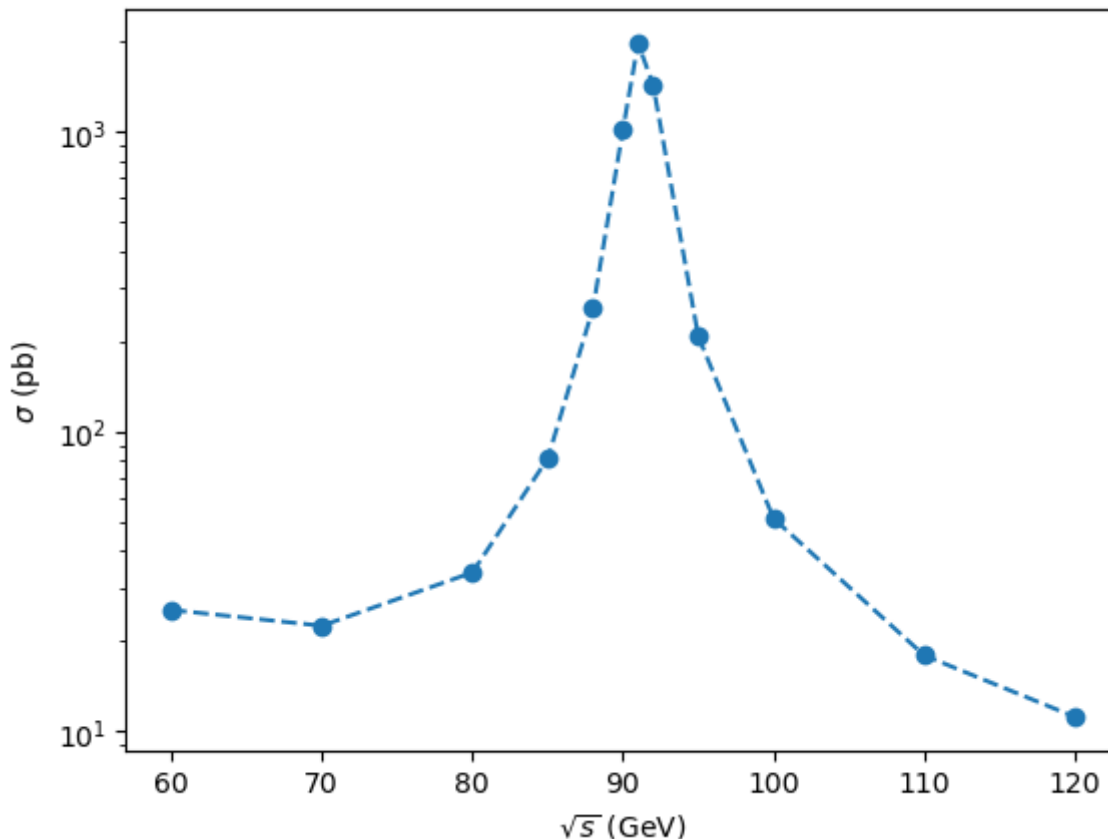
The forward-backward asymmetry is a measure of how many the imbalance between the forward and the backward directions:

$$A_{\text{FB}} = \frac{\sigma(\cos \theta > 0) - \sigma(\cos \theta < 0)}{\sigma(\cos \theta > 0) + \sigma(\cos \theta < 0)}$$

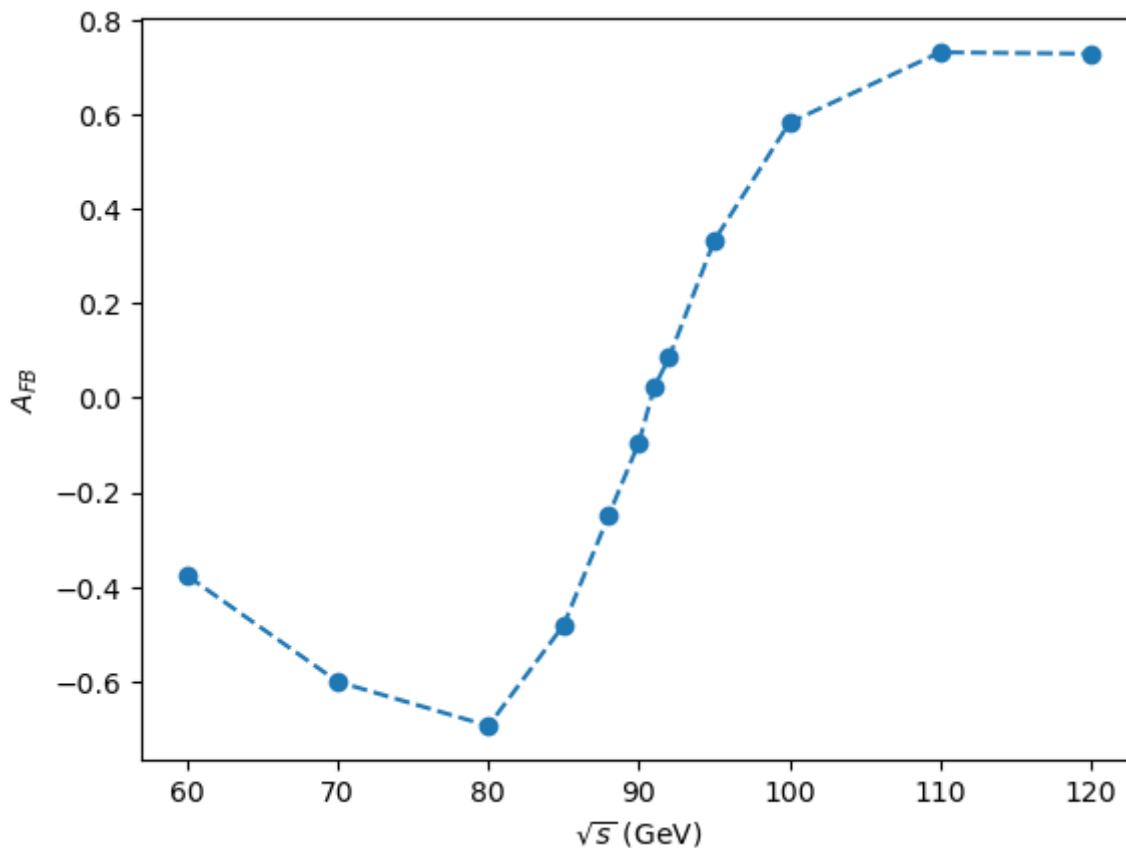
For  $e^+e^- \rightarrow \mu^+\mu^-$ , this quantity is nonzero in the standard model because of the chiral couplings of the  $Z$  boson.

In particular, generate 1,000 events at  $\sqrt{s} = 60, 70, 80, 85, 88, 90, 91, 92, 95, 100, 110, \text{ and } 120, \text{ GeV}$ . Plot  $\sigma$  and  $A_{\text{FB}}$  versus  $\sqrt{s}$  and compare to the data.

The peak  $\sigma$  at 90 GeV matches with Aleph, which is a good sign.



The "zeroness" of the  $A_{\text{FB}}$  at 90 GeV matching that of Aleph is also a good sign.



#### 4. $e^+e^- \rightarrow e^+e^-$ at NLO in QED (10 points).

a. Draw all the LO and NLO in QED diagrams for  $e^+e^- \rightarrow e^+e^-$ . How many are there in total?

The simplest thing I can do to the s-channel is replace an electron/positron edge with an  $e^\pm \rightarrow e^\pm \gamma \rightarrow e^\pm$  diagram, or I can replace the  $\gamma$  edge with a  $\gamma \rightarrow e^+e^- \rightarrow \gamma$  diagram.

So I can make a single change on one of the 5 edges in the  $e^+e^- \rightarrow e^+e^-$ , leaving us with 5 NLO diagrams for the s-channel.

Similarly for the t-channel, leaving us with a total of 10 NLO diagrams and 2 LO diagrams.

b. Use MadGraph to generate the diagrams up to NLO order in QED.

The MadGraph syntax for this is:

```
generate <process> / <excluded particles> [QED]
```

The syntax that I ended up doing after downloading and extracting [loop\\_qcd\\_qed\\_sm](#):

```
convert model ./loop_qcd_qed_sm
import ./loop_qcd_qed_sm/
generate e+ e- > e+ e- / g ghg ghg~ u c d s b u~ c~ d~ s~ b~ ghg~
```

However, this get's me a bunch of explicitly, non-loop diagrams, with an added absorbed or emitted photon.

Makes me feel like I did not properly exclude everything.

Maybe I didn't properly exclude ghosts?

Anyways, there were 24 NLO diagrams generated in total.