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PHYS 239: Modern Collider Physics
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Problem Set 5

Before you begin this problem set, keep in mind the following:

- **Due date is Monday, May 15, 5pm for the draft.**
- **Due date is Thursday, May 18, 5pm for the corrections.**
- You should try each problem to the best of your ability. You can work with your class peers and consult internet resources in discussing a problem, but when writing/coding up your solution, you should not be consulting any other source specific to the problem.
- Leave space for your corrections. Do not try to cram as many solutions into as small a space as possible.

1. **Hadronization: Rapidity plateau** [20 points]. When quarks and gluons fragment into jets, the soft particles created during the fragmentation process are produced in colored flux tubes. This means they have limited transverse momentum with respect to the *jet axis*[1] and that they are produced uniformly in longitudinal phase space. A consequence of this is that production is uniform in rapidity

$$y = \frac{1}{2} \ln \left(\frac{E + p_{\parallel}}{E - p_{\parallel}} \right) , \tag{1}$$

where $p_{\parallel}$ is the particle's momentum with respect to the jet axis.

(a) Show that a particle's rapidity is related to its velocity along the jet axis $\beta_{\parallel}$ by the expression

$$y = \operatorname{arctanh}(\beta_{\parallel}) . \tag{2}$$

(b) Show that the rapidity difference between two particles in a jet is invariant with respect to Lorentz boosts along the jet direction.

(c) Show that in the limit where particle masses can be neglected the rapidity y can be approximated by the expression

$$y \approx -\ln(\tan(\theta/2)) , \tag{3}$$

where θ is the angle the particle makes with respect to the jet axis.

[1] For this problem, the jet axis refers to the initial quark or gluon direction.

(d) Consider $e^+e^- \to$ hadrons in the center-of-mass frame where the energies of the initial e^+ and e^- beams are $E_{\text{beam}} = E_{\text{CM}}/2$. The distribution of particles will be approximately uniform in y between a minimum value y_{min} and a maximum value y_{max} where $y_{\text{min}} = -y_{\text{max}}$. Using the definition of rapidity above, find an approximate value for y_{max} for hadrons of species h and mass m_h as a function of E_{beam}.

(e) Using this result, show that the average multiplicity of final state hadrons h of mass m_h is

$$n_h \propto \log\left(\frac{E_{\text{CM}}}{m_h}\right) \tag{4}$$

In other words, the multiplicity of hadrons grows logarithmically with the center-of-mass energy.

2. **Hadronization: Fragmentation functions** [20 points]. The fragmentation function $D_q^h(z)$ is defined as the probability that a quark q will hadronize to produce a hadron of species h with energy fraction between z and $z + dz$. These fragmentation functions must satisfy conservation of momentum and unitarity so that

$$\sum_h \int_0^1 z D_q^h(z) dz = 1 \tag{5}$$

$$\sum_h \int_{z_{\text{min}}}^1 D_q^h(z) dz = \sum_h n_h \,, \tag{6}$$

where the sum is over all hadron species, $z_{\text{min}} = m_h/E_q$ with m_h the hadron mass and E_q the quark energy, and n_h is the average number of hadrons of type h produced by the fragmentation of the quark.

Fragmentation functions are often parameterized as

$$D_q^h(z) = \mathcal{N}\frac{(1-z)^\alpha}{z} \,, \tag{7}$$

where α and $\mathcal{N}$ are constants.

(a) Show that

$$\mathcal{N} = (\alpha + 1)\langle z \rangle \,, \tag{8}$$

where $\langle z \rangle$ is the average fraction of the quark momentum carried by hadrons of type h after fragmentation.

(b) Show that this formalism reproduces the previous result

$$n_h \propto \log\left(\frac{E_{\text{CM}}}{2m_h}\right) \tag{9}$$

for the process $e^+e^- \to 2$ jets.

3. **Hadronization in** $e^+e^- \to$ hadrons **with PYTHIA8.3** [30 points].

   (a) Generate 10,000 events in PYTHIA8.3 $e^+e^- \to$ hadrons events on the Z pole. *Hint:* In Python, this should look like the following

```
import pythia8

# Set up Pythia instance
pythia = pythia8.Pythia()

# Configure Pythia to generate e+e- to hadrons
pythia.readString("PDF:lepton = off")
pythia.readString("WeakSingleBoson:ffbar2gmZ = on")
pythia.readString("23:onMode = off")
pythia.readString("23:onIfAny = 1 2 3 4 5")
pythia.readString("Beams:idA =  11")
pythia.readString("Beams:idB = -11")
mZ = pythia.particleData.m0(23)
pythia.settings.parm("Beams:eCM", mZ)
```

   (b) For each final-state hadron in each event, compute the rapidity y as follows. Find the closest parton from the hard scattering process (status -23) (e.g. the parton with the minimum opening angle

$$\theta = \arccos\left(\frac{\boldsymbol{p}_{\text{hadron}} \cdot \boldsymbol{p}_{\text{parton}}}{|\boldsymbol{p}_{\text{parton}}||\boldsymbol{p}_{\text{hadron}}|}\right) \tag{10}$$

   Compute the component of the hadron momentum parallel to the intial parton momentum as $p_{\|} = \boldsymbol{p}_{\text{hadron}} \cdot \boldsymbol{p}_{\text{parton}} / |\boldsymbol{p}_{\text{parton}}|$

   (c) Make a histogram showing the distribution of hadron rapidity over all events from $y = 0$ to 5. Your plot should look something like data from the TASSO Collaboration [1] shown in Fig. 1.

References

[1] TASSO Collaboration, "Jet Fragmentation and QCD Models in e^+e^- Annihilation at C.M. Energies Between 12-GeV and 41.5-GeV", *Z. Phys. C* **41** (1988) 359, doi:10.1007/BF01585620.

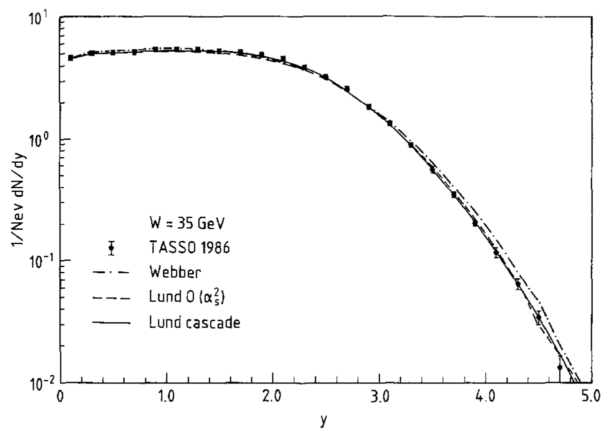

Figure 1: Rapidity y distribution for $E_{\rm CM} = 35\,\mathrm{GeV}$ from the TASSO Collaboration [1].