Decays $K^{\pm}\to\pi^{\pm}l^{+}l^{-}$ and limits on the mass of the neutral Higgs boson

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It is shown that the limits on the weak decays $K^{\pm}\to\pi^{\pm}l^{+}l^{-}$ imply a lower bound of about 325 MeV for the mass of the neutral Higgs boson.

The question of the existence of the physical neutral Higgs boson of the standard $SU(2)\times U(1)$ gauge theory of the weak and electromagnetic interactions is of considerable interest. The question is complicated by the absence of any reliable estimate of its mass. There is a theoretical argument,[1] based on the stability of the broken-symmetry vacuum, that m_H should be greater than 6 or 7 GeV, but the argument breaks down[2] if there exist fermions with mass greater than the mass of the massive gauge vector bosons, of if the Higgs-boson self-coupling is strong; so it is important to consider possible empirical limits on the existence of a lighter Higgs boson. In a comprehensive article on the phenomenology of the Higgs boson, Ellis, Gaillard, and Nanopoulos[3] (EGN) quoted a lower limit of only about 15 MeV from nuclear physics.[4]

The branching ratio for the decay $K^+\to\pi^+e^+e^-$ is[5] $(2.6\pm0.5)\times10^{-7}$ and for $K^+\to\pi^+\mu^+\mu^-$ is $<2.4\times10^{-6}$. These decays can be simulated by the decay sequence $K^+\to\pi^+H$ and $H\to l^+l^-$. If one can calculate the branching ratios for these processes, one may be able to rule out the existence of a Higgs boson of $m_H<m_K-m_\pi\approx350$ MeV. EGN considered these decays but found too much uncertainty in the calculation to draw any conclusion.[6] More recently, Vainshtein, Zakharov, and Shifman[7] (VZS) have presented a calculation of the branching ratio for $K^{\pm}\to\pi^{\pm}H$ and claim that the result rules out the existence of a Higgs boson of $m_H<m_K-m_\pi$. Their calculation is based on repeated application of the low-energy theorems of broken scale invariance and broken chiral invariance, and is strictly correct only in the limit of all external four-momenta vanishing, which is a considerable extrapolation from the physical region. In this circumstance it seems useful to present the result of another calculation which has an entirely different basis.

The calculation is a quark-model calculation, analogous to the quark-model calculation[8] of the purely weak part of the decay $K_L\to\mu^+\mu^-$. First one computes the free-quark transition amplitude for $s\to d+H$, and then one interprets the corresponding quark transition operator as the quark-model representation of the (divergence of the) physical vector and axial-vector currents, whose K-to-π matrix elements are know from ordinary semileptonic K decays. The free-quark transition amplitude $s\to d+H$ occurs in the one-loop order of perturbation theory in the standard model and has already been calculated by the present authors[9] in a different context (inclusive decays of heavy flavored mesons), and we simply quote the result here.

$$M(s\to d+H)\approx\frac{3g^3}{256\pi^2}\frac{1}{m_W{}^3}(C_{ts}C_{td}^*m_t{}^2)\times m_s\bar{d}(p')(1+\gamma_5)s(p)\ . \quad (1)$$

In this expression, m_s and m_t are quark masses; m_W is the W-boson mass; C_{ts}, C_{td} are elements of the Kobayashi-Maskawa (KM) quark mixing matrix; and $g^2/8m_W{}^2=G_F/\sqrt{2}$. In this calculation the approximations

$$m_W,m_t>>m_s>>m_d$$

have been made. No approximation on the ratio m_t/m_W has been made.[9] The second step is the replacement

$$\begin{aligned}m_s\bar{d}(p')(1+\gamma_5)s(p)&=-(p'-p)^\mu\bar{d}(p')\gamma_\mu(1-\gamma_5)s(p)+O(m_d)\\&\to i\partial^\mu(V_\mu-A_\mu)\ . \end{aligned}\quad (2)$$

The K^+-to-π^+ matrix element of (2) is

$$\langle\pi^+(p')|i\partial^\mu V_\mu|K^+(p)\rangle\approx-m_K{}^2\sqrt{2}f_+(0)\ , \quad (3)$$

where $f_+(q^2)$ is the form factor measured in the $K^+\to\pi^0l^+\nu$ decays and is well approximated by its SU(3) value $f_+(0)\approx1/\sqrt{2}$. Then, taking the experimental[5] K^+ lifetime, $T(K^+)=1.24\times10^{-8}$ sec, our computed branching ratio for $K^+\to\pi^+H$ is

$$\begin{aligned}B(K^+\to\pi^+H)&=\frac{\Gamma(K^+\to\pi^+H)}{\Gamma(K^+\to\text{all})}\\&\approx1.2\times10^3\left|C_{ts}C_{td}^*\frac{m_t{}^2}{m_W{}^2}\right|^2\phi\ ,\end{aligned}\quad (4)$$

where ϕ is a phase-space factor, normalized to one for $m_H^2 << m_K^2$,

$$\phi = \frac{2p'}{m_K} = \left[1 - \frac{(m_H + m_\pi)^2}{m_K^2}\right]^{1/2} \left[1 - \frac{(m_H - m_\pi)^2}{m_K^2}\right]^{1/2} . \tag{5}$$

The elements of the KM matrix involving the t quark, and the t-quark mass are not known, but the combination

$$|\bar{g}_t|^2 = |C_{ts} C_{td}^* m_t^2 / m_W^2|^2 \tag{6}$$

appearing in (4) is closely related to the quantity

$$|g_t|^2 = |C_{ts} C_{td}^* G(x_t)|^2 , \quad x_t = m_t^2/m_W^2 , \tag{7}$$

$$G(x) = \frac{x}{1-x} - \frac{1}{4}\frac{x^2}{1-x} - \frac{3}{4}\frac{x^2}{(1-x)^2} \ln\frac{1}{x} , \tag{8}$$

which enters into the quark calculation[10] of the purely weak-interaction contribution to the observed decay $K_L \to \mu\bar{\mu}$:

$$\frac{\Gamma_{\rm wk}(K_L \to \mu\bar{\mu})}{\Gamma(K^+ \to \mu\nu)} = \frac{G_F^2 m_W^4}{2\pi^4} \frac{(1 - 4m_\mu^2/m_K^2)^{1/2}}{(1 - m_\mu^2/m_K^2)^2} \times \frac{|C_{ts} C_{td}^* G(x_t)|^2}{|C_{us}|^2} , \tag{9}$$

$$C_{us} = \sin\theta_1 \cos\theta_3 = 0.22 .$$

Then

$$\frac{\Gamma_{\rm wk}(K_L \to \mu\bar{\mu})}{\Gamma(K_L \to {\rm all})} = 1.34 \times 10^{-3} |g_t|^2 . \tag{10}$$

We cannot directly obtain $|g_t|$ from comparison of (10) with the experimental[5] branching ratio $(9.1 \pm 1.9) \times 10^{-9}$ because there is a combined weak and electromagnetic contribution which is at the same order of magnitude. The absorptive part of this contribution is reliably calculated[11] in terms of the experimental branching ratio for $K_L \to \gamma\gamma$, and contributes $(5.9 \pm 0.6) \times 10^{-9}$. The remaining $(3.2 \pm 2.5) \times 10^{-9}$ is attributed to the combination of the dispersive part of the weak-electromagnetic contribution and the purely weak contribution (10). The dispersive part of the weak-electromagnetic contribution is not well known but has been estimated[12] to be small compared to the absorptive part. If it is negligible then $|g_t|^2$ is determined to be between 5.2×10^{-7} and 4.3×10^{-6}. Comparison of (6) and (7), and (8) shows that $|\bar{g}_t| > |g_t|$. So a conservative bound is

$$|\bar{g}_t|^2 > 10^{-7} . \tag{11}$$

[If $|\bar{g}_t|^2$ is as small as this lower bound, then $\Gamma_{\rm wk}$, Eq. (10), provides less than 10% of the difference between the observed branching ratio and the computed absorptive contribution.] Substitution of (11) into (4) gives

$$B(K^+ \to \pi^+ H) > 1.2 \times 10^{-4} \phi . \tag{12}$$

We remark that this is consistent with the low-energy-theorem result of VZS,[7] which is about $2 \times 10^{-4}\phi$.

To compare with the experimental value, and limit, of the branching ratios for $K^+ \to \pi^+ e^+ e^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$, Eq. (12) has still to be multiplied by the branching ratio for $H \to l^+ l^-$. We require

$$B(K^+ \to \pi^+ H) B(H \to e^+ e^-) < B(K^+ \to \pi^+ e^+ e^-) = (2.6 \pm 0.5) \times 10^{-7} , \tag{13}$$

$$B(K^+ \to \pi^+ H) B(H \to \mu^+ \mu^-) < B(K^+ \to \pi^+ \mu^+ \mu^-) \lesssim 2.4 \times 10^{-6} . \tag{14}$$

For $m_H \leq 2m_\mu$, the branching ratio for $H \to e^+ e^-$ is of order unity,[3] and for $2m_\mu < m_H \leq 2m_\pi$, the branching ratio for $H \to \mu^+ \mu^-$ is of order unity; so $m_H < 2m_\pi$ is ruled out. For $m_H > 2m_\pi$, the mode $H \to \pi\pi$ competes strongly with $H \to \mu^+ \mu^-$. The $H \to \pi\pi$ rate is difficult to compute. VZS claim that for $m_H < 1$ GeV, the mode $H \to \mu^+ \mu^-$ still dominates, while EGN expect it to be small, but still greater than a few percent for $m_H < m_K - m_\pi$. Even with the EGN lowest estimate, (12) is still in conflict with (14), so we conclude that $m_H \gtrsim 325$ MeV $[\phi(325) \approx 0.3]$.

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