$0<p_1, \dots, p_m$ are prices. N: # of customers.

g_i: prob. of customer i buy the good. R: # of goods.

$p_i g_i$: expected revenue from customer i.

$g_i \downarrow$ if $p_i \uparrow$.

Want if $p_i g_i < p_j g_j$ $i<j$

Proof. We can model this as an MDP.

States: $S=\{0, 1, \dots, R\}$. Epoches: per customer. N horizons.

Actions: $i \in \{1, \dots, m\} = A$

Reward: $r(s,i) = p_i g_i \cdot \mathbb{1}(s>0)$.

Transition: $s=0$. $P_{00}(i) = 1$.

$s>0$. $P_{s,s}(i) = 1-g_i$

$P_{s,s-1}(i) = g_i$

$$V_n(s) = \max_{i\in A}\left\{ r(s,i) + \sum_{s'\in S} P_{ss'}(i)\, V_{n-1}(s') \right\}. \qquad V_1(s) = r(s,i) = \begin{cases} p_i g_i & \text{if } s=R. \\ 0 & \text{otherwise.} \end{cases}$$

$$= \max_{i\in A}\left\{ p_i g_i \cdot \mathbb{1}(s>0) + P_{s,s-1}(i)\, V_{n-1}(s-1) + P_{ss}(i)\, V_{n-1}(s) \right\}.$$

if $s=0$ $\quad V_n(0) = \max_{i\in A}\{V_{n-1}(0)\} = V_{n-1}(0) = \cdots V_1(0) = 0.$

if $s>0$ $\quad V_n(s) = \max_{i\in A}\left\{ p_i g_i + g_i V_{n-1}(s-1) + (1-g_i) V_{n-1}(s) \right\}.$

$$= \max_{i\in A}\left\{ \underline{p_i g_i} + V_{n-1}(s) + \underline{g_i}\,(\underbrace{V_{n-1}(s-1) - V_{n-1}(s)}_{\le 0?}) \right\}.$$

$i^* = \arg\max\{\ \}$ $\quad = p_m g_m + V_{n-1}(s) + g_m(V_{n-1}(s-1) - V_{n-1}(s)).$

$\Rightarrow i^* = m.$

induction.

Want $V_n(s-1) - V_n(s) \overset{?}{\le} 0.$ $\quad V_n(0) - V_n(1) \le 0$ ✓.

Suppose $V_n(s) - V_n(s-1) \ge 0.$ $\quad V_{n-1}(s)$

$$V_n(s+1) - V_n(s) = \max_{i\in A}\left\{ p_i g_i + g_i V_{n-1}(s) + (1-g_i) V_{n-1}(s+1) \right\}$$

$$- \max\left\{ p_i g_i + g_i V_{n-1}(s-1) + (1-g_i) V_{n-1}(s) \right\}.$$

$$\ge \max\left\{ g_i(\underbrace{V_{n-1}(s) - V_{n-1}(s-1)}_{\ge 0}) + (1-g_i)(V_{n-1}(s+1) - V_{n-1}(s)) \right\}.$$

$$\ge (1-g_i)(V_{n-1}(s+1) - V_{n-1}(s)).$$

$$\vdots$$

$$\ge (1-g_i)^{n-1}(\underbrace{V_1(s+1) - V_1(s)}_{①}) \ge 0.$$

$$\begin{cases} \text{if } s+1=R. & ① = (1-g_i)^{n-1} g_i p_i \ge 0. \\ \text{otherwise} & ① = 0. \end{cases}$$

QED.