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PREDICTION OF HEAVY OIL VISCOSITY

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ABSTRACT

The impetus for this work stems from the difficulties in obtaining reliable viscosity measurements for live oils from viscous oil reservoirs. The use of correlations to estimate fluid viscosity can provide a useful method to provide the reservoir engineer with preliminary values for reservoir calculations.

Many different correlations based on measured fluid properties have been presented in the literature to estimate fluid viscosity. Using measured data supplied through the offices of the DTI, these correlations have been assessed for their ability to predict measured viscosity data for several North Sea heavy oil reservoirs.

Arising from this assessment, prediction of dead oil viscosity, on which most of the correlations for bubble-point and thereby undersaturated oil viscosity are based, was found to be poor. Work to identify an alternative correlation was undertaken and a new correlation developed.

Particular correlations for bubble-point and undersaturated oil viscosity have been recommended as providing the best match with measured data. An engineer's flow chart has been created to summarise the findings.

INTRODUCTION

There have been difficulties in obtaining reliable viscosity measurements on live oils from viscous oil reservoirs. The initial measurements made on samples taken from the discovery well have often been suspect. It has been left until much later, when more reliable samples are taken from an appraisal well, usually after a lengthy well test, to obtain a more definitive figure for the live oil viscosity.

There are a number of correlations from which it is possible to predict live oil viscosity on the basis of the dead oil viscosity. Most of the common ones have been devised primarily for North American crudes. An assessment of the available correlations has been performed to identify an approach to estimate live oil viscosity for new discoveries.

Several oil companies have identified heavy oil reservoirs in the North Sea and obtained measured viscosity data for them. Viscosity data has been provided through the offices of the DTI for oils from four reservoirs identified as Oil 1, Oil 2, Oil 3 and Oil 4.

EXISTING CORRELATIONS

A search of the literature has identified a number of correlations for the estimation of fluid viscosity based on measured fluid properties. These correlations can be divided into three categories: dead oil viscosity (μ_{od}), bubble-point viscosity (μ_{ob}) and undersaturated oil viscosity (μ_o). Correlations which use tuned EoS parameters, such as those by Pedersen [1] and Lohrenz, Bray & Clark [2], have been omitted, though the EoS Package PVTsim [3], which just uses the reported fluid composition, without the need for any data fitting, has been included. A list of the correlations utilised is given in Table 1, with further details presented in Appendices 1, 2 and 3. Numbers in { }'s are used to identify particular correlations for use in subsequent tables and text.

In general, the correlations use the residual oil density and temperature to determine μ_{od}. μ_{ob} is obtained using μ_{od} and R_s. μ_o is typically obtained using μ_{ob} as one of the input parameters. There are obviously variations in the form of the equations. Two exceptions to this generalisation are:

- Labedi's method for μ_{ob} which uses P_b instead of R_s and
- Kahn et al's method for μ_{ob} which uses the dead oil density and gas gravity instead of μ_{od}.

Table 1 : List of Correlations Used

Correlation	Ref	μ_{od}	μ_{ob}	μ_o
Beal (Chart)	[4]	{1}	{13}	
Beal (Equation)	[5]	{2}		{24}
Beggs & Robinson	[6]	{3}	{14}	
Chew & Connally	[7]		{15}	
Egbogah-Jacks (Without Pour Point)	[8]	{4}		
Modified Egbogah-Jacks (Extra Heavy Oils)	[9]	{5}		
Modified Egbogah-Jacks (Heavy Oils)	[9]	{6}		
Glaso	[10]	{7}		
Labedi	[11]	{8}	{16}	{25}
Modified Labedi (Extra Heavy Oils)	[9]			{26}
Modified Labedi (Medium Oils)	[9]			{27}
Kahn et al (Saudi Arabian Crude Oils)	[12]		{17}	{28}
Kartoatmodjo	[13]	{9}	{18}	{29}
Modified Kartoatmodjo (Extra Heavy Oils)	[9]		{19}	
Modified Kartoatmodjo (Heavy Oils)	[9]		{20}	{30}
Modified Kartoatmodjo (Medium Oils)	[9]	{10}	{21}	
Petrosky & Farshad	[14]	{11}	{22}	{31}
PVTsim	[3]	{12}	{23}	{32}
Vazquez & Beggs	[15]			{33}

In addition to the correlations expressed as equations detailed above, the charts of Beal [4] have also been included.

MEASURED DATA

Data have been provided from four reservoirs; Oil 1, Oil 2, Oil 3 and Oil 4. Details of these data are listed in Table 2. The data supplied contained more than one set of viscosity data for each reservoir, providing additional information on the effect of temperature.

Table 2 : Summary of Data Provided

				Viscosity Data		
Reservoir	Composition	Flash Data	Differential Vaporisation Data	Dead Oil	Bubble Point Oil	Under-saturated Oil
Oil 1	✓ (C_{20}+)	✓	✓ (1 temp)	✓ (3 temp)	✓ (3 temp)	✓ (3 temp)
Oil 2	✓ (C_{36}+)	✓	✓ (1 temp)	✓ (3 temp)	✓ (3 temp)	✓ (3 temp)
Oil 3	✓ (C_{36}+)	✓	✓ (1 temp)	✓ (5 temp)	✓ (2 temp)	✓ (3 temp)
Oil 4	✓ (C_{36}+)	✓	✓ (1 temp)	✓ (5 temp)	✓ (5 temp)	✓ (5 temp)

Some of the measured viscosities were quoted in centistokes rather than centipoise. To obtain the viscosity in centipoise, the viscosity in centistokes is multiplied by the fluid density (g/cm^3). Estimates of the fluid density have been made and the viscosities adjusted accordingly.

For each dead oil viscosity measurement, it is important to know the fluid density as well as the analysis temperature. When the analysis temperature is increased, it is usual for additional gas to be liberated from the dead oil. As a consequence, the API gravity would normally be different for each analysis temperature. Densities at each temperature were only available for Oil 4, and these represented a constant API gravity for all the temperatures studied. For the other three fluids, density measurements were only available at the reservoir temperatures.

Given the usual nature of heavy oils (i.e. a general absence of components between C_3 and C_{10}), it is not unreasonable to assume there are insufficient components in the "volatile" region to be significant. The API gravity of the dead oil has therefore been assumed to remain near constant for the temperatures studied.

PREDICTIONS

Dead oil viscosities have been predicted using both forms of Beal's relationship, that is the original chart {1} and the equation {2}. For higher gravity oils, there is good agreement between the two. For lower gravity oils, the equation estimates viscosities lower than those determined from the chart. Comparison of these results with the measured data indicates the equation is more reliable than the chart. Furthermore, the chart does not cover viscosities greater than 100 cp or temperatures less than 100°F, though some extrapolation is possible.

A comparison of all the predicted viscosities, grouped by viscosity type (dead oil, bubble-point oil or undersaturated oil), are presented in Table 3, Table 4 and Table 5. The data in Table 4 has been generated using the measured dead oil viscosity. The measured bubble-point viscosity has been used to predict the undersaturated oil viscosities in Table 5.

In some cases negative viscosities were predicted. In the case of μ_o, {30}, this is due to the power form of one of the terms, in which a negative term starts to dominate once μ_{ob} becomes large. In the cases for μ_{ob}, {20} and {21}, the bubble-point viscosity is expressed as a quadratic in which the coefficient for the squared term (a function of R_s and μ_{od}) is negative. Consequently, once this function of R_s and μ_{od} starts to dominate, the predicted viscosity becomes negative.

Comparison of all the predictions with the measured dead oil viscosity data, Table 3, indicates that Beal's equation {2} is perhaps the best showing the smallest mean difference, though none of the correlations provide a reliable estimate of the dead oil viscosity.

Table 3 : Comparison of Predicted Dead Oil Viscosities

Fluid	Temp	Actual	{1}	{2}	{3}	{4}	{5}	{6}	{7}	{8}	{9}	{10}	{11}	{12}
	°F	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp
Oil 1	39	1904	*	462	#	#	#	8610	591	116	529	616	193	*
	87	272	128	125	244	76.1	728	173	94	68	93	110	54	432
	150	38	31	30	17.5	23.7	110	32.7	27.1	47	28.3	34.0	22.8	76
Oil 2	106	383	250	511	261	214	884	260	324	251	318	655	172	269
	200	29.6	48	34	13.3	52.5	97	34.2	55.7	163	67	115	55	31.8
	300	6.4	*	3.6	4.3	27.6	34.5	13.6	18.1	124	25.0	37.8	26.3	8.4
Oil 3	90	8396	*	8773	2679	1541	3971	1311	2838	983	2482	9635	990	*
	120	1349	*	1974	283	578	1030	352	1116	810	1139	3713	555	*
	150	357	*	508	77	314	421	150	541	697	623	1772	354	434
	175	143	*	179	37	217	243	89	328	628	410	1063	259	220
	200	66	104	67	21.6	162	157	59.2	213	574	286	683	198	122
Oil 4	100	3369	*	4115	978	906	2202	723	1694	808	1574	5409	689	*
	120	969	*	1423	248	465	932	312	850	658	868	2597	436	953
	150	338	*	404	69.6	256	386	135	419	566	479	1267	280	372
	200	58.1	108	61.7	20.1	135	146	54.4	169	467	223	502	159	107
	250	20.1	43	11.8	9.5	87.7	76.2	29.9	83.3	402	123	245	102	41.5
Mean Absolute Difference, %			-	33	3939	157	286	70	100	437	142	358	119	65

- Represents a large number > 10,000

* - Out of range

Provided an accurate dead oil viscosity is available together with GOR data from a differential vaporisation at the desired temperature, reasonable estimates of μ_{ob}, Table 4 ,can be obtained using the correlations of Petrosky and Farshad {22} and Kartoatmodjo {18}. In general, both these tended to overpredict viscosity so, to obtain a lower limit, it is recommended that the Beggs & Robinson correlation {14}, which consistently underpredicted, is also used. Taking an average of the three estimates yields a better mean difference (13.5%) than any of the correlations on its own, with only three instances where the difference is greater than 10%.

Table 4 : Comparison of Predicted Bubble-Point Viscosities

Fluid	Temp	Actual	{13}	{14}	{15}	{16}	{17}	{18}	{19}	{20}	{21}	{22}	{23}
	°F	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp
Oil 1	39	520	*	290	674	296	35.4	540	802	-482	-622	548	204
	87	88	*	60	120	80	23.4	93	99	70	46.2	101	59
	150	18.2	15.6	12.3	20.9	21.5	14.4	17.4	18.5	17.1	15.7	18.2	19.0
Oil 2	106	85.9	*	55.3	122	126	162	93	104	71	46	103	27.5
	200	11.8	10.9	7.8	13.8	23.0	81	11.6	13.1	11.4	10.8	12.5	8.0
	300	2.99	2.8	2.4	3.7	8.4	42.8	3.3	5.4	2.8	3.4	3.5	3.4
Oil 3	90	1685	*	890	2401	1400	#	2416	5744	-#	-#	1888	264
Oil 4	100	478.2	*	464	1128	722	5963	963	1687	-#	-#	908	163
Mean Absolute Difference, %			*	31	43	58	764	22	84	189	195	21	45

- Represents a large number > 10,000 or < -1,000

* - Out of range

The undersaturated oil viscosity at 5000 psig should be a lot easier to predict as, given the measured bubble-point viscosity, the correlations only need to estimate the effect of pressure. However, as Table 5 shows, there is a lot of variation in the predicted viscosities. The correlation of Kartoatmodjo {29} demonstrates the smallest mean difference (16%) and is generally fairly reasonable. However, even the Kartoatmodjo correlation was unable to provide a good estimate for the high viscosity Oil 3 sample (7355 cp), so care should be taken with particularly high viscosities (over 1000 cp).

On the basis of these comparisons, the weakest prediction is for the dead oil viscosity. As this is a key parameter in the other correlations for the bubble-point and undersaturated oil viscosities, there is a need to develop a new correlation for μ_{od}.

Table 5 : Comparison of Undersaturated Oil Viscosities at 5000 psig

Fluid	Temp	Actual	{24}	{25}	{26}	{27}	{28}	{29}	{30}	{31}	{32}	{33}
	°F	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp	cp
Oil 1	39	1490	2628	851	913	2546	760	1580	-1472	520	447	981
	87	209	207	140	132	316	127	195	132	88	109	157
	150	29.7	28.0	26.4	23.2	42.1	25.9	28.8	29.4	18.2	30.3	31.1
Oil 2	106	147	181	149	130	404	116	171	122	86	38.9	128
	200	18.2	15.9	17.3	14.3	30.7	15.6	16.2	16.9	11.9	10.3	16.8
	300	4	3.6	4.3	3.5	6.5	3.9	3.5	3.7	3.9	4.2	4.2
Oil 3	90	7355	#	3173	3302	10631	2349	3462	-#	1685	448	2719
Oil 4	100	1083	778	1098	1074	3701	669	1332	-942	478	275	778
Mean Absolute Difference, %			31	20	25	95	31	16	110	47	51	23

- Represents a large number > 10,000 or < -1,000

DEVELOPMENT OF NEW CORRELATION FOR μ_{od}

Sixteen data points are available on which to develop a new correlation for μ_{od}, which is rather limited. However, the data is fairly well spread, covering four different fluids with 3 or 5 temperatures per fluid. The supporting fluid property data available was not very extensive, therefore only the API gravity and temperature have been utilised.

The best two correlations for μ_{od} (Beal {2} and Modified Egbogah-Jacks (Heavy Oils) {6}) were used to generate a range of data between 10 and 20 °API and 50 to 200°F. These were plotted (Figure 1) and found to be reasonably well fitted with an equation of the form

$$\mu_{od} = a \times T^{b} \tag{1}$$

Equations of the following forms were then found to best provide the a and b values. The closeness of the two fits is shown in Figure 2.

$$Log(a) = r \times API^{2} + s \times API + t \tag{2a}$$

$$b = l \times API^{2} + m \times API + n \tag{2b}$$

Combining the results yielded the following equation for μ_{od}

$$\mu_{od} = 10^{\left(0.052 \times API^{2} - 2.2704 \times API - 5.7567\right)} \times T^{\left(-0.0222 \times API^{2} + 0.9415 \times API - 12.839\right)} \tag{3}$$

After identifying a suitable form for an equation to predict μ_{od}, a solver routine was used to tune the values of the six constants (r, s, t, l, m and n) to produce a closer fit to the measured data. This yielded the following equation.

$$\mu_{od} = 10^{\left(0.10231 \times API^{2} - 3.9464 \times API + 46.5037\right)} \times T^{\left(-0.04542 \times API^{2} + 1.70405 \times API - 19.18\right)} \tag{4}$$

A chart showing the curves generated from this correlation and overlain with the measured data can be seen in Figure 3. These curves show good agreement with the measured data. However, for temperatures above 250°F and API gravities > 20°, the curves represent behaviour that would not be expected with real fluids. In order to avoid problems in this area an alternative form of the correlation has been developed.

ALTERNATIVE FORM OF EQUATION

The form adopted for the correlation described above, Equation (4), uses two quadratics in API gravity. As can be seen in Figure 3, this produces a series of curves. To avoid the problems this causes with temperatures above 250°F and API gravities > 20°, and also given the limited amount of data, consideration has been given to the possibility of using straight lines to match the trend of API gravity with log(μ_{od}) for constant temperature. This can be achieved by replacing the two quadratics with linear functions of API gravity, Equation (5).

$$\mu_{od} = 10^{(a \times API + b)} \times T^{(c \times API + d)} \tag{5}$$

A solver has been used to fit the parameters a, b, c and d, yielding Equation (6).

$$\mu_{od} = 10^{(-0.8021 \times API + 23.8765)} \times T^{(0.31458 \times API - 9.21592)} \tag{6}$$

The resultant chart is presented as Figure 4. As can be seen the chart looks quite good and does not have the same problems with temperatures > 250°F and problems with the effect of API gravity are displaced to higher gravities.

A comparison of the predicted and actual viscosities shows that both equations yield four occurrences where the difference is greater than 20%. The mean of the absolute differences is 13% and 16%, with maximum differences of 43% and 72% for Equations (4) & (6) respectively.

ENGINEER'S FLOW CHART

In the early stages of a discovery, it can be difficult to obtain a reasonable estimate of the fluid viscosity. A flow chart has been prepared (Appendix 4), which starts from the collection of a sample of oil and then leads the engineer through a number of steps to produce estimates of the fluid viscosity.

The correlations for μ_{od} use API gravity and temperature as input parameters. It is important to recognise that the API gravity refers to the density of the fluid at 60°F after gas has been liberated from the fluid at ambient pressure and reservoir temperature. In cases where there is uncertainty about the reservoir temperature, it is suggested that a range of temperatures are considered. If it is necessary to use the same sample for these measurements, data at the lowest temperature should be gathered first and then progress to higher temperatures.

Gas-oil ratio is one of the parameters used by the correlations to predict μ_{ob}, and at the early stages of a discovery can be fairly uncertain. To handle this it is suggested that a range of values are considered to provide an indication of the significance of the uncertainty.

CONCLUSIONS

Of the existing correlations in the literature, the best estimate of the dead oil viscosity (μ_{od}) for the heavy oil fluids examined in this study was found to be provided by Beal [5]. Even so, the mean error was calculated to be 33%.

To predict the bubble point viscosity (μ_{ob}), on the basis of the dead oil viscosity (μ_{od}) and GOR, the correlations of Petrosky & Farshad [14] and Kartoatmodjo [13] were found to be the most successful, with mean differences of 21% and 22% respectively. However, they both tended to overestimate μ_{ob}. The Beggs & Robinson correlation [6], which consistently underpredicted μ_{ob} (mean error 31%), can be used to provide a lower bound. The actual viscosity should lie somewhere within the range of these estimates, in fact the average of the three values was found to yield the smallest mean difference (14%).

The prediction of undersaturated oil viscosity (μ_o), on the basis of the bubble point viscosity (μ_{ob}), was best achieved using the correlation of Kartoatmodjo [13], with a mean difference of 16%. However, when very high viscosities (>2000 cp) were involved, the correlation tended to significantly underpredict. Extra care should be taken in these circumstances.

A new correlation (Equation (4)) for the dead oil viscosity (μ_{od}) has been developed which yields a mean error of 13% for the North Sea data tested. It is recommended that this is used to estimate dead oil viscosity of other heavy oils. The correlation should not be used for API gravities > 20° or temperatures >250°F as, outside these criteria, the correlation exhibits trends that would not be expected of real fluids. A simplification of this model has also been produced (Equation (6)), which yields slightly poorer results, but is more suitable for a broader range of API gravities and temperatures. These correlations have both been developed using North Sea crudes, so should provide reasonable estimates for other fluids of a similar nature.

Starting from the collection of fluid from a well, an engineer's flow chart has been prepared as a step-by-step guide to the estimation of fluid viscosity.

This approach assumes that μ_{od} is only dependent on the oil gravity and analysis temperature. Literature reported pure component data demonstrate that this is not the case. However, given the limited data available at this stage, it is not practical to try and incorporate other parameters.

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LIST OF SYMBOLS

Symbol	Description	Units
°API	Gravity of oil at 60°F	°API
GG	Gas gravity	Air = 1.000
K_w	Watson Characterisation Factor	
P	(Reservoir) Pressure	psia
P_b	Bubble-point pressure	psia
R_s	Solution gas-oil ratio	scf/STB
T	(Reservoir) Temperature	°F
μ_o	Undersaturated oil viscosity	cp
μ_{ob}	Bubble point oil viscosity	cp
μ_{od}	Dead oil viscosity	cp

Note : In some cases, viscosity is reported in centistokes.

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Figure 1 : Dependence of Viscosity (μ_{od}) on API Gravity and Temperature (Correlations of Beal and Modified Egbogah-Jacks (Heavy Oils))

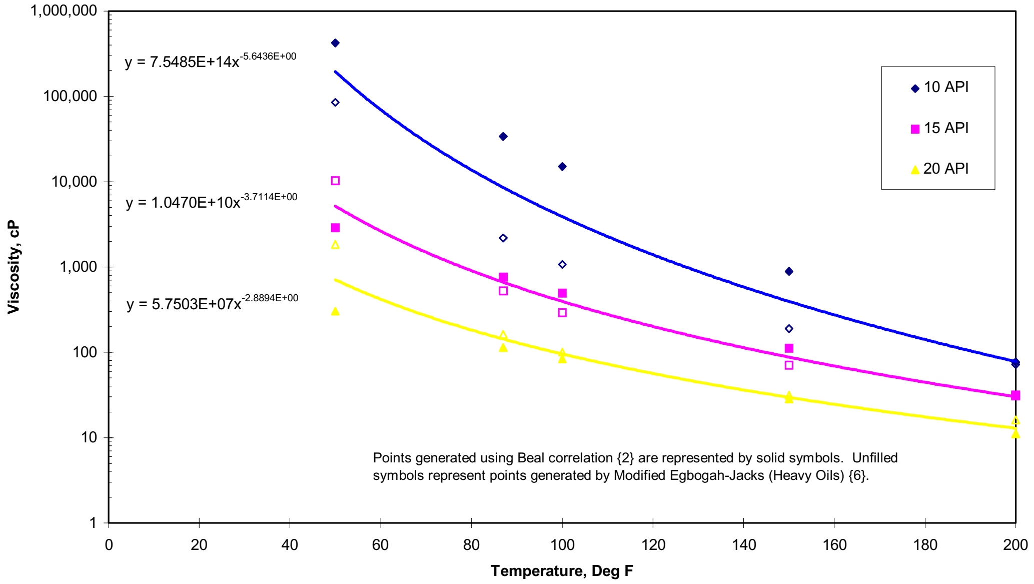


Figure 2 : Curve Fitting to Obtain Coefficients a and b

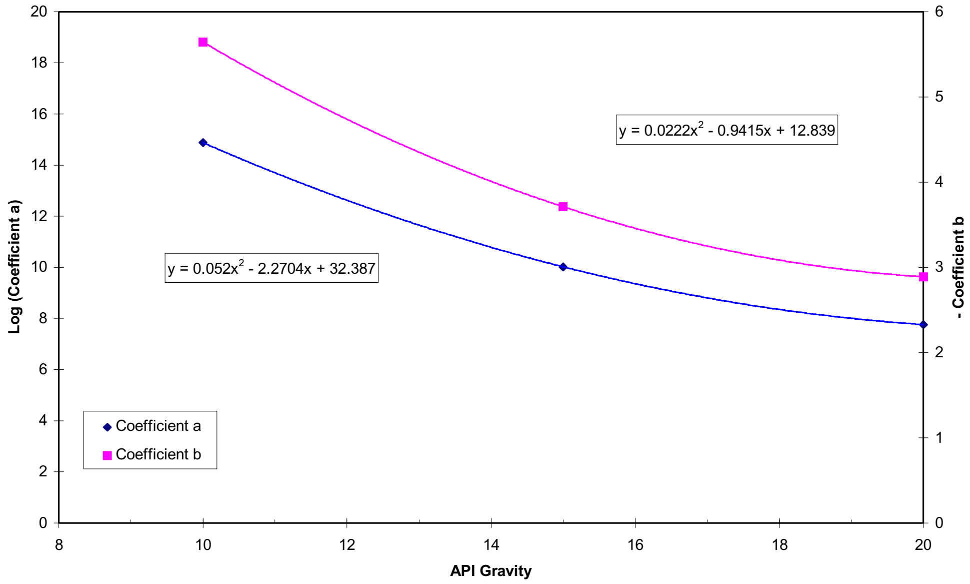


Figure 3 : New Correlation For μ_{od} (Equation (4)) Overlain With Measured Data

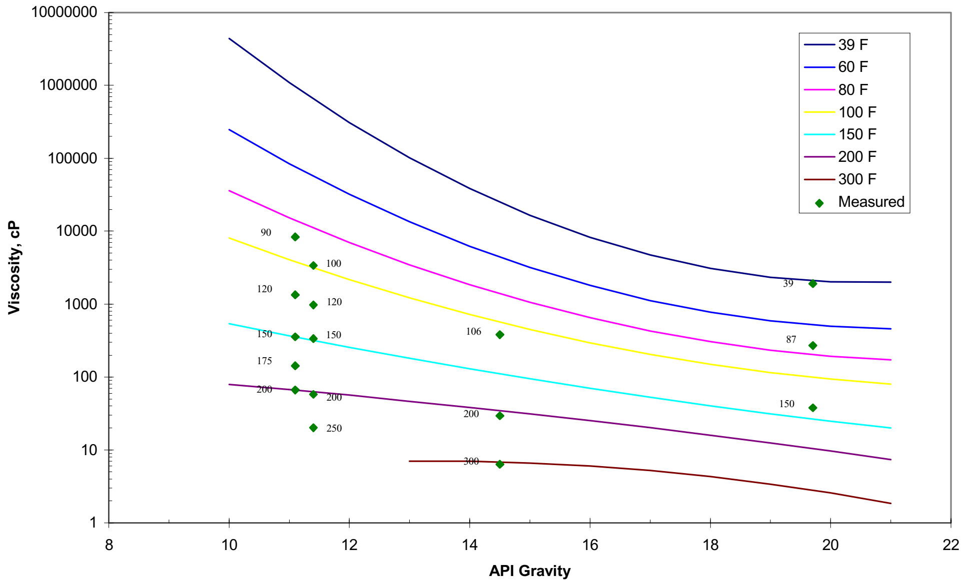
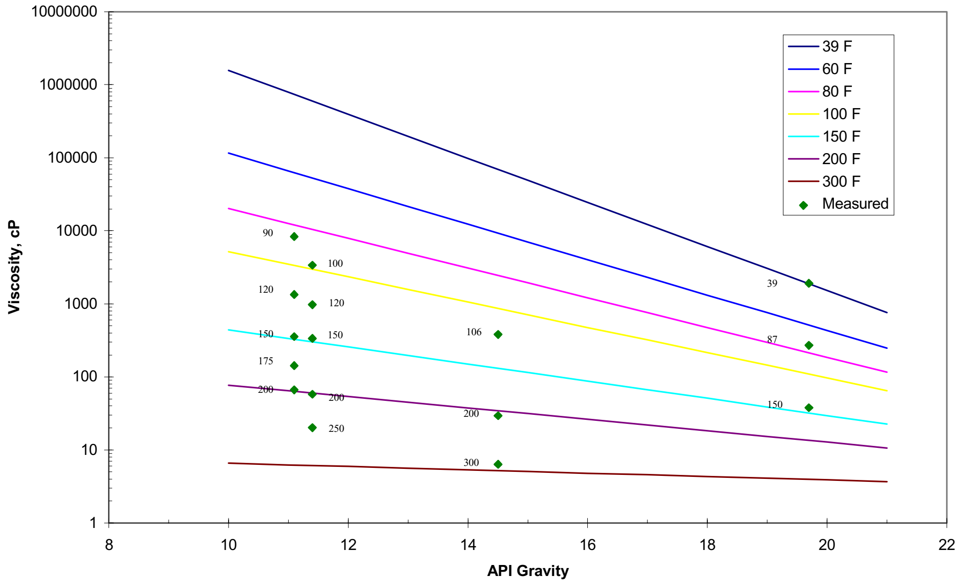


Figure 4 : Alternate Correlation For μ_{od} (Equation (6)) Overlain With Measured Data



Appendix 1 : Dead Oil Correlations

Beal **[5]**

$$\boxed{\mu_{od} = \left(0.32 + \frac{1.8 \times 10^{7}}{{}^{\circ}API^{4.53}}\right)\left(\frac{360}{T+200}\right)^{a}}$$

where

$$\boxed{a = antilog\left(0.43 + \frac{8.33}{{}^{\circ}API}\right)}$$

Beggs & Robinson **[6]**

$$\boxed{\mu_{od} = 10^{X} - 1}$$

where

$$\boxed{X = 10^{\left(3.0324 - 0.02023 \times {}^{\circ}API\right)} \times T^{-1.163}}$$

Egbogah-Jacks (Without Pour Point) **[8]**

$$\boxed{\mu_{od} = 10^{10^{\left(\left(-1.7095 + \left(389.45 / \left({}^{\circ}API + 131.5\right)\right)\right) + \left(-1.2943 + \left(135.585 / \left({}^{\circ}API + 131.5\right)\right)\right)\right) \times \log\left(\left(T-32\right) \times 5/9\right)}} - 1}$$

Modified Egbogah-Jacks (Extra-Heavy Oils) **[9]**

$$\boxed{\mu_{od} = 10^{10^{\left(1.90296 - 0.012619 \times {}^{\circ}API - 0.61748 \times \log\left(T\right)\right)}} - 1}$$

Modified Egbogah-Jacks (Heavy Oils) **[9]**

$$\boxed{\mu_{od} = 10^{10^{\left(2.06492 - 0.0179 \times {}^{\circ}API - 0.70226 \times \log\left(T\right)\right)}} - 1}$$

Glaso **[10]**

$$\mu_{od} = \left(3.141\times10^{10}\right)\times T^{-3.444}\left(\log\ {}^{\circ}API\right)^{\left[10.313\times(\log T)-36.447\right]}$$

Kartoatmodjo **[13]**

$$\mu_{od} = 16\times10^{8}\times T^{(-2.8177)}\left[\log\left({}^{\circ}API\right)\right]^{\left(5.7526\times\log(T)-26.9718\right)}$$

Modified Kartoatmodjo (Medium Oils) **[9]**

$$\mu_{od} = 220.15\times10^{9}\times T^{(-3.5560)}\left[\log\left({}^{\circ}API\right)\right]^{\left(12.5428\times\log(T)-45.7874\right)}$$

Labedi **[11]**

$$\mu_{od} = \frac{10^{9.224}}{{}^{\circ}API^{4.7013}\times T^{0.6739}}$$

Petrosky & Farshad **[14]**

$$\mu_{od} = 2.3511\times10^{7}\times T^{-2.10255}\times\left(Log{}^{\circ}API\right)^{\left(4.59388\times(LogT)-22.82792\right)}$$

Appendix 2 : Bubble-Point Oil Correlations

Beggs & Robinson **[6]**

$$\mu_{ob} = a \times \left(\mu_{od}\right)^{b}$$

where

$$a = 10.715 \times \left(R_s + 100\right)^{-0.515}$$

$$b = 5.44 \times \left(R_s + 150\right)^{-0.338}$$

Chew & Connally **[7]**

$$\mu_{ob} = a \times \left(\mu_{od}\right)^{b}$$

where

$$a = 0.20 + 0.80 \times 10^{\left(-0.00081 \times R_s\right)}$$

$$b = 0.43 + 0.57 \times 10^{\left(-0.00072 \times R_s\right)}$$

Labedi **[11]**

$$\mu_{ob} = \left(10^{2.344 - 0.03542 \times {}^{\circ}API}\right) \times \mu_{od}^{0.6447} \Big/ P_b^{0.426}$$

Kahn et al (Saudi Arabian Crude Oils) **[12]**

$$\mu_{ob} = \frac{0.09 \times \left(SG_{gas}\right)^{1/2}}{\left(R_s^{1/3} \times \left[\left(T + 459.67\right) \big/ 459.67\right]^{4.5} \times \left(1 - \left(141.5 / \left({}^{\circ}API + 131.5\right)\right)\right)^{3}\right)}$$

Kartoatmodjo [13]

$$\mu_{ob} = -0.06821 + 0.9824 \times F + 0.0004034 \times F^2$$

where

$$F = \left(0.2001 + 0.8428 \times 10^{\left(-0.000845 \times R_s\right)}\right) \times \mu_{od}^{\left(0.43 + 0.5165 \times 10^{\left(-0.00081 \times R_s\right)}\right)}$$

Modified Kartoatmodjo (Extra-Heavy Oils) [9]

$$\mu_{ob} = 2.3945 + 0.8927 \times F + 0.001567 \times F^2$$

where

$$F = \left(-0.0335 + 1.0785 \times 10^{\left(-0.000845 \times R_s\right)}\right) \times \mu_{od}^{\left(0.5798 + 0.3432 \times 10^{\left(-0.00081 \times R_s\right)}\right)}$$

Modified Kartoatmodjo (Heavy Oils) [9]

$$\mu_{ob} = -0.6311 + 1.078 \times F - 0.003653 \times F^2$$

$$F = \left(0.2478 + 0.6114 \times 10^{\left(-0.000845 \times R_s\right)}\right) \times \mu_{od}^{\left(0.4731 + 0.5158 \times 10^{\left(-0.00081 \times R_s\right)}\right)}$$

Modified Kartoatmodjo (Medium Oils) [9]

$$\mu_{ob} = 0.0132 + 0.9821 \times F - 0.005215 \times F^2$$

$$F = \left(0.2038 + 0.8591 \times 10^{\left(-0.000845 \times R_s\right)} \right) \times \mu_{od}^{\left(0.3855 + 0.5664 \times 10^{\left(-0.00081 \times R_s\right)} \right)}$$

Petrosky & Farshad [14]

$$\mu_{ob} = a \times \left(\mu_{od}\right)^b$$

where

$$a = 0.1651 + 0.6165 \times 10^{\left(-0.60866 \times 10^{-4} \times R_s\right)}$$

$$b = 0.5131 + 0.5109 \times 10^{\left(-1.1831 \times 10^{-3} \times R_s\right)}$$

Appendix 3 : Undersaturated Oil Correlations

Beal **[5]**

$$\mu_o = \mu_{ob} + 0.001 \times (P - P_b) \times \left(0.024 \times \mu_{ob}^{\ 1.6} + 0.038 \times \mu_{ob}^{\ 0.56}\right)$$

Labedi **[11]**

$$\mu_o = \mu_{ob} + \left[\frac{\left(10^{-2.488} \times \mu_{od}^{\ 0.9036} \times P_b^{\ 0.6151}\right)}{\left(10^{0.0197 \times {}^{\circ}API}\right)}\right] \times \left(\frac{P}{P_b} - 1\right)$$

Modified Labedi (Extra-Heavy Oils) **[9]**

$$\mu_o = \mu_{ob} - \left[\left(1 - \frac{P}{P_b}\right) \times \left(\frac{10^{-2.19} \times \mu_{od}^{1.055} \times P_b^{0.3132}}{10^{0.0099 \times {}^{\circ}API}}\right)\right]$$

Modified Labedi (Medium Oils) **[9]**

$$\mu_o = \mu_{ob} - \left[\left(1 - \frac{P}{P_b}\right) \times \left(\frac{10^{-3.8055} \times \mu_{od}^{1.4131} \times P_b^{0.6957}}{10^{-0.00288 \times {}^{\circ}API}}\right)\right]$$

Kahn et al (Saudi Arabian Crude Oils) **[12]**

$$\mu_o = \mu_{ob} \times e^{9.6 \times 10^{-5}(P - P_b)}$$

Kartoatmodjo **[13]**

$$\mu_o = 1.00081 \times \mu_{ob} + 0.001127 \times (P - P_b) \times \left(-0.006517 \times \mu_{ob}^{1.8148} + 0.038 \times \mu_{ob}^{1.590}\right)$$

Modified Kartoatmodjo (Heavy Oils) [9]

$$\boxed{\mu_o = 0.9886 \times \mu_{ob} + 0.002763 \times (P - P_b) \times \left(-0.01153 \times \mu_{ob}^{1.7933} + 0.0316 \times \mu_{ob}^{1.5939}\right)}$$

Petrosky & Farshad [14]

$$\boxed{\mu_o = \mu_{ob} + 1.3449 \times 10^{-3} \times (P - P_b) \times 10^{a}}$$

where

$$\boxed{a = -1.0146 + 1.3322 \times \log(\mu_{ob}) - 0.4876 \times [\log(\mu_{ob})]^2 - 1.15036 \times [\log(\mu_{ob})]^3}$$

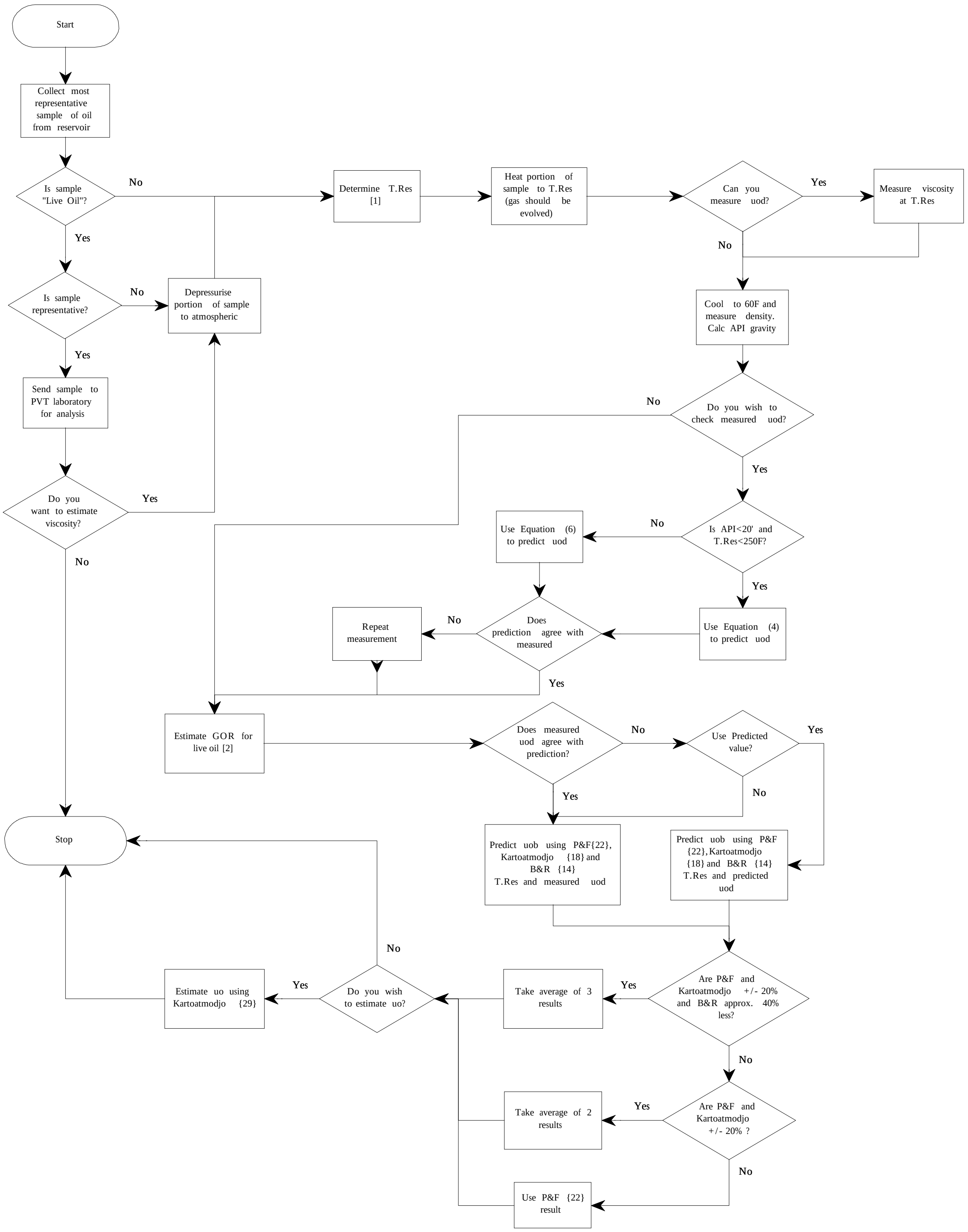
Vazquez & Beggs [15]

$$\boxed{\mu_o = \mu_{ob} \times \left(\frac{P}{P_b}\right)^m}$$

where

$$\boxed{m = 2.6 \times P^{1.187} \times antilog\left[\left(-3.9 \times 10^{-5}\right) \times P - 5.0\right]}$$

Appendix 4 : Engineer's Flow Chart for the Estimation of Heavy Oil Fluid Viscosity



[1] : If reservoir temperature is uncertain, repeat using a range of values. Use lowest temperature first.
[2] : If GOR is uncertain, repeat with a range of values.

(x) : Identifies equation to use
{x} : Represents correlations identified in Table 1