



```
System.out.println("hello, world!");
```

Practice Mode

[anthony.das@gmail.com](mailto:anthony.das@gmail.com) | [Contest scoreboard](#) | [Sign out](#)

Qualification Round 2016

[A. Counting Sheep](#)[B. Revenge of the Pancakes](#)[C. Coin Jam](#)[D. Fractiles](#)**Contest Analysis**[Questions asked](#) 1**Submissions****Counting Sheep**

7pt	Not attempted 26558/29356 users correct (90%)
8pt	Not attempted 25729/26216 users correct (98%)

**Revenge of the Pancakes**

10pt	Not attempted 22527/23686 users correct (95%)
10pt	Not attempted 21383/22147 users correct (97%)

**Coin Jam**

10pt	Not attempted 13361/15342 users correct (87%)
20pt	Not attempted 6297/9111 users correct (69%)

**Fractiles**

10pt	Not attempted 8250/9708 users correct (85%)
25pt	Not attempted 2356/4955 users correct (48%)

**Top Scores**

Lewin	100
Endagorion	100
xiaowuc1	100
xyz111	100
HellKitsune123	100
seanwentzel	100
ivan.popelyshev	100
burunduk3	100
Nicolas16	100
ctunoku	100

**Contest Analysis**
[Overview](#) | [Problem A](#) | [Problem B](#) | [Problem C](#) | [Problem D](#)
**Coin Jam: Analysis**

One approach here is to enumerate strings of length  $N$  and see if they are jamcoins. To iterate over potential jamcoins, we can iterate over the base-2 interpretations (odd numbers between  $2^{N-1}+1$  and  $2^N-1$ , inclusive) and use a recursive method to convert these to different bases.

For the Small dataset, it is sufficient to use [trial division](#) to find factors of potential jamcoins. Concretely, we can look for a nontrivial divisor of an integer  $k$  by testing divisibility of every integer in increasing order from 2 to the square root of  $k$ , inclusive and stopping if we find one. Searching past the square root of the number is not needed as if  $k$  has some nontrivial divisor  $d$ , then  $k/d$  is also a non-trivial divisor and the smaller of  $d$  and  $k/d$  is at most the square root of  $k$ . A sample implementation of this in C++ looks like this:

```
long long convertBinaryToBase(int x, int base) {
    // Some languages have built-ins which make this easy.
    // For example, in Python, we can avoid recursion and
    // just return int(bin(x)[2:], base)
    if (x == 0)
        return 0;
    return base * convertBinaryToBase(x / 2, base) + (x % 2);
}

long long findFactor(long long k) {
    for (long long d = 2; d * d <= k; d++)
        if (k % d == 0)
            return d;
    return 0;
}

void printCoins(int N, int X) {
    for (long long i = (1 << N-1) + 1; X > 0; i += 2) {
        vector<long long> factors;
        for (int base = 2; base <= 10; base++) {
            long long x = convertBinaryToBase(i, base);
            long long factor = findFactor(x);
            if (!factor)
                break;
            factors.push_back(factor);
        }
        if (factors.size() < 9)
            continue;

        cout << convertBinaryToBase(i, 10);
        for (long long factor : factors)
            cout << " " << factor;
        cout << endl;
        X -= 1;
    }
}
```

Solving the large test case may present unique challenges, depending on the programming language used. We're given that  $N = 32$ , which means that in base 10 we'll have 32-digit numbers, which we can't store in a 64-bit integer. These numbers are also large enough that running the trial division algorithm on a single prime would take a huge amount of time, potentially longer than the duration of this contest! While we can solve these issues by stopping the trial division early (say, after checking up to 1000) and using arbitrary precision integers, there's actually a much nicer approach!

We can make some observations about jamcoins if we look at the output to the Small dataset from our program above. Considering the first 50 jamcoins of length 16, we find that 18 of them have divisors "3 2 3 2 7 2 3 2 3" and 11 of them have divisors "3 2 5 2 7 2 3 2 11". What's the pattern in these numbers? The numbers 5, 7, 11 from the second list provide a useful hint: they're one more than their respective bases. Giving this some thought, we can notice that the second list of divisors is formed by taking the smallest prime factor of  $b+1$  for each base  $b$ . This suggests that  $b+1$  is always a divisor for each of these 11 jamcoins, and we can easily verify that this is true. Understanding the other common divisor list is left as an exercise for the reader.

Note that  $b+1$  in base  $b$  is  $11_b$ . A simple test exists to determine divisibility of 11 in base-10: a number is divisible by 11 if the sum of its digits in odd positions and the sum of its digits in even positions differ by a multiple of 11. This [divisibility rule](#) for 11 can be extended to a simple rule: a string of 0s and 1s which starts and ends with a 1 and has the same number of 1s at odd and even indices is divisible by  $b+1$  when interpreted as a base- $b$  number. Such a string is therefore a jamcoin, but not every jamcoin necessarily matches this condition. A stricter condition that may be easier to notice is that a string of 0s and 1s which starts and ends with a 1 is a jamcoin if all 1s are paired, i.e. it matches the regular expression  $11(0|11)^*11$ . For example, in any base  $b$ ,  $11011_b = 1001_b \times 11_b$ .

The last example here also suggests a more general rule. Consider any string  $p$  of 0s and 1s with at least two characters, which starts and ends with a 1. Any string which is  $p$  repeated multiple times with any number of 0s between repetitions is a jamcoin. In the previous example we have  $p = 11$  and this general rule can be expressed as the regular

expression  $(1[\emptyset 1]^*1)(\emptyset^*\backslash 1)^+$ . For example, in any base  $b$ ,  $1110100011101011101_b = 11101_b \times 10000001000001_b$ .

We can use any of these rules to mine jamcoins easily. The Python 2 code below mines jamcoins with exactly 5 pairs of 1s, which finds enough for both the Small and Large datasets.

```
def printCoins(N, X):
    # N digits, 10 1s, N-10 0s
    for i in range(N-10):
        for j in range(N-10-i):
            for k in range(N-10-i-j):
                l = N-10-i-j-k
                assert l >= 0
                template = "11{}11{}11{}11{}11"
                output = template.format("{}*i, "{}*j, "{}*k, "{}*1)
                factors = "3 2 5 2 7 2 3 2 11"
                print output, factors
                X -= 1
            if X == 0:
                return
    # If we get here, we didn't mine enough jamcoins!
    assert False
```

Another interesting observation (from contestant StupidCorn) is that if you take a jamcoin from the Small dataset and append it to itself, the new string is a valid jamcoin for the Large dataset! This is because appending a coin of length  $L$  to itself ensures that  $b^L + 1$  is a divisor for any base  $b \geq 2$ , and we know the new jamcoin starts and ends with 1 because the smaller jamcoin did. So, if you can extend your code from the Small dataset to find 500 jamcoins of length 16, you can easily solve the Large.

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