Report: ERGM model

Anthony Ebert

22/08/2019

ERGM: $p(y|\theta) = \frac{h(y)\exp[\theta \cdot s(y)]}{z(\theta)}$ Mechanistic model: $q(y|\theta^q)$

$$
\begin{aligned}
KL[q(\cdot|\theta^q)||p(\cdot|\theta)] &= \sum_{y\in\mathcal{Y}} q(y|\theta^q)\log\frac{q(y|\theta^q)}{p(y|\theta)} \\
&= \log[z(\theta)] - \sum_{y\in\mathcal{Y}} q(y|\theta^q)h(y) - \sum_{y\in\mathcal{Y}} q(y|\theta^q)\left[\sum_i \theta_i s_i(y)\right] + \sum_{y\in\mathcal{Y}} q(y|\theta^q)\log q(y|\theta^q)
\end{aligned}
$$

We can ignore $z(\theta)$ since θ is fixed. We want to find θ^q associated with q which most closely replicates p.

The specific ERGM model which is used as the statistical (and mechanistic) model has two summary statistics: number of edges and number of triangles. The true value for θ_1 is zero and the true value of θ_2 is 0.05. The plots below show the KL divergence when θ_2^q is changed.

The reference measure h is the binomial likelihood. The number of nodes is 10, Monte Carlo estimates (n = 50) are used for all the summation terms over the distribution q.

The relative importance of the likelihood and entropy

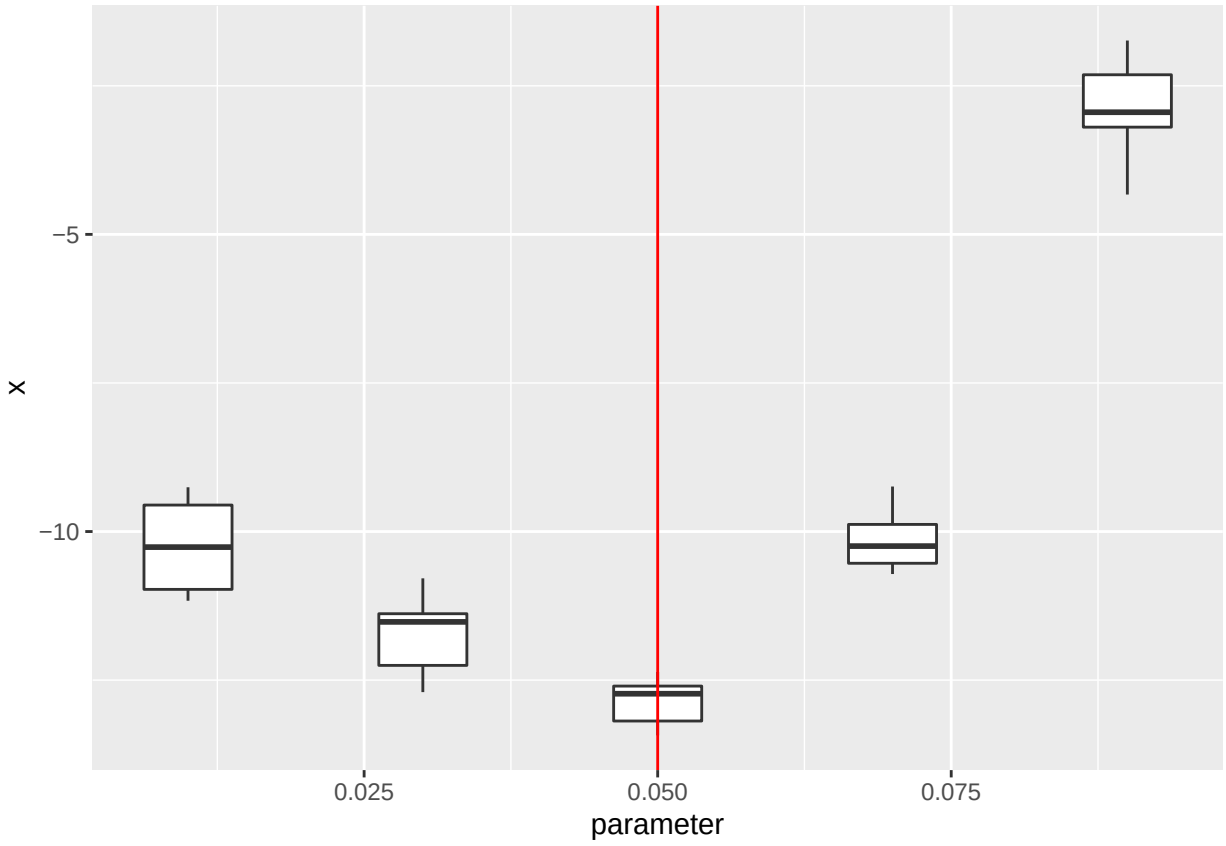

Figure 1: KL divergence calculation with entropy

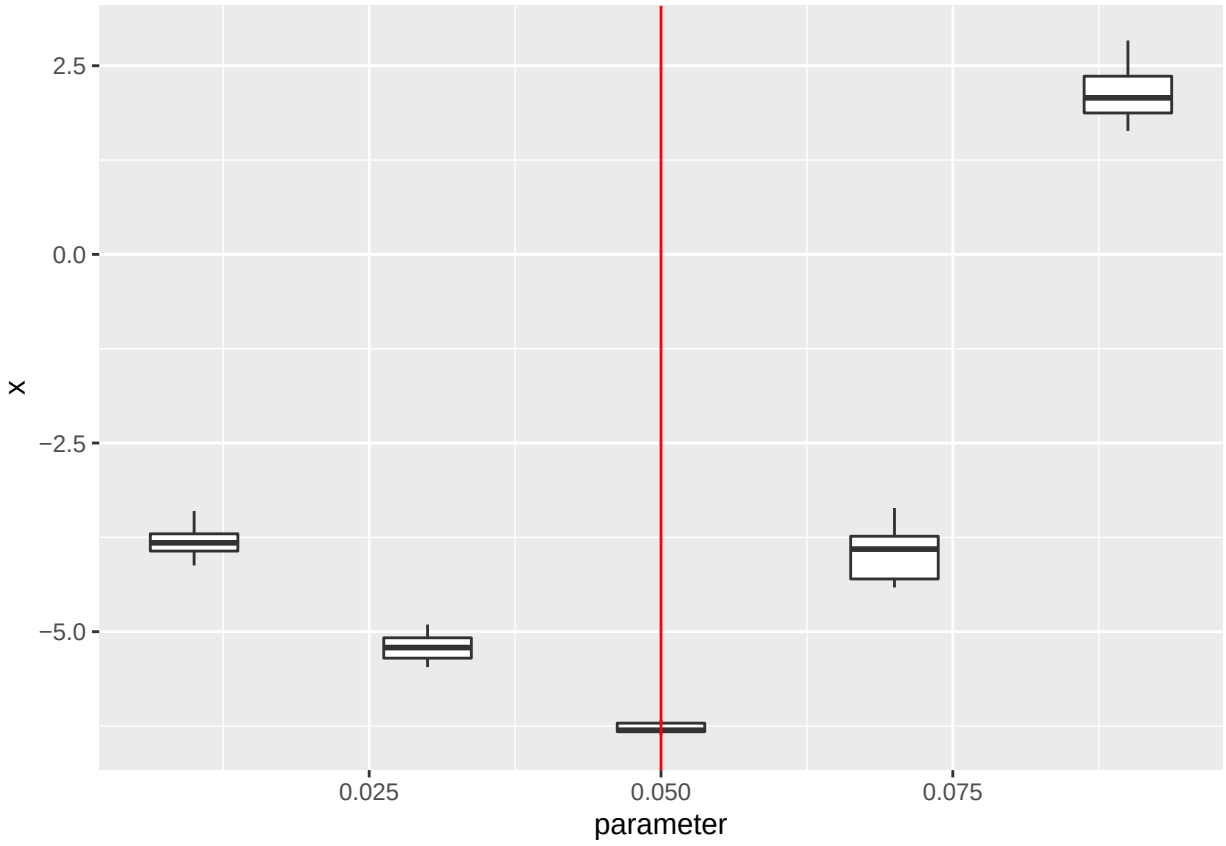

Figure 2: KL divergence calculation without entropy

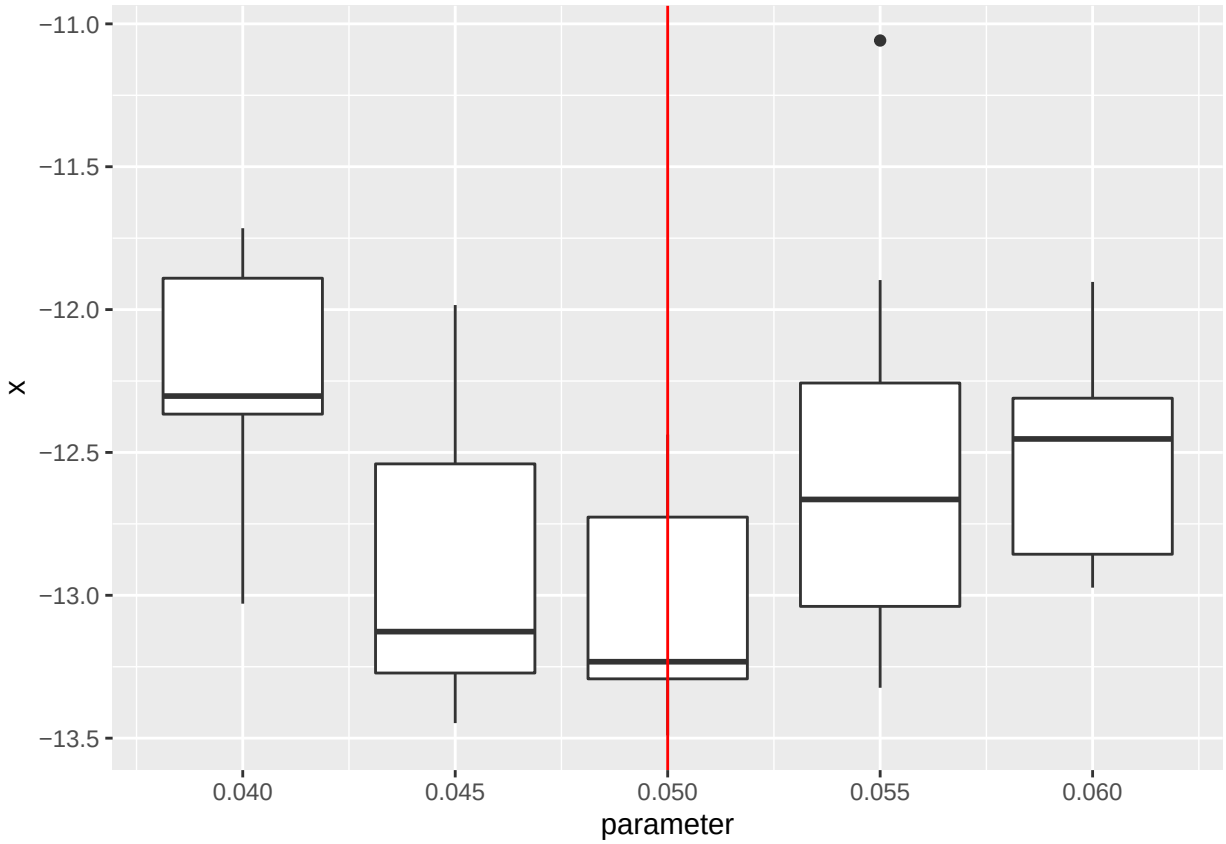

Figure 3: KL divergence calculation with entropy

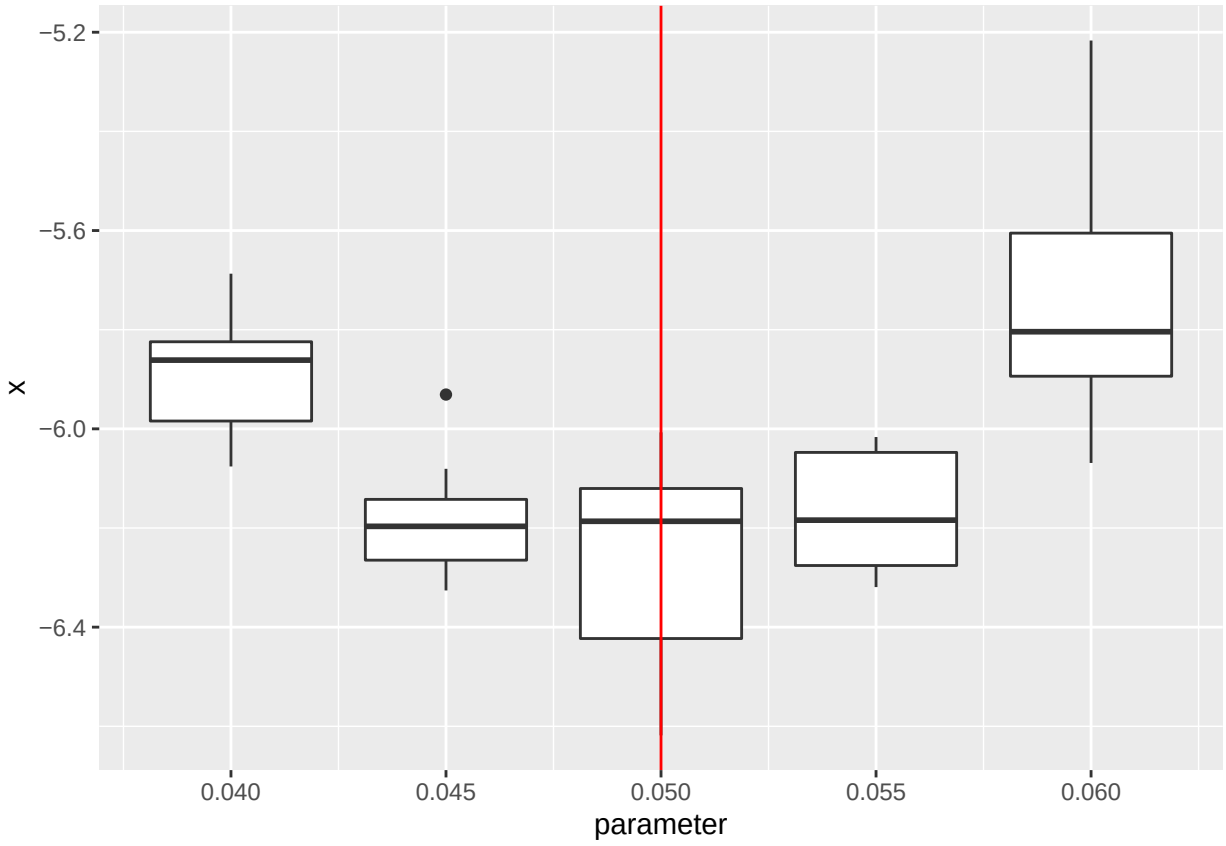

Figure 4: KL divergence calculation without entropy