$\Pi \to \mu_\Pi$

$\to \sigma^2_\Pi$

$w_A \to$ asset A

$(1-w_A) \to$ RFA

$\mathbb{E}[ax+by]$

$= a\,\mathbb{E}[x] + b\,\mathbb{E}[y]$

$$\mu_\Pi = \mathbb{E}\left[w_A R_A + (1-w_A) R_F\right]$$

$$= w_A \mathbb{E}[R_A] + (1-w_A) R_F$$

$$= (1-w_A) R_F + w_A \mu_A$$

$$\boxed{\mu_\Pi = R_F + w_A \underbrace{(\mu_A - R_F)}_{}}$$

Risk premium

$\mathbb{V}[c] = 0$

$\mathbb{V}(ax+b)$

$= a^2\,\mathbb{V}[x]$

$$\sigma^2_\Pi = \mathbb{V}\left[w_A R_A + (1-w_A) R_F\right] = w_A^2\,\mathbb{V}[R_A] = w_A^2 \sigma_A^2$$

$$\to \boxed{\sigma_\Pi = w_A \sigma_A}$$

$$\mu_\pi = R_F + w_A(\mu_A - R_F)$$

$$\sigma_\pi = w_A \sigma_A$$

$$\begin{cases} w_A = 0 \\ \rightarrow \mu_\pi = R_F \\ \sigma_\pi = 0 \end{cases}$$

$$\begin{cases} w_A = 100\% \\ \mu_\pi = \mu_A \\ \sigma_\pi = \sigma_A \end{cases}$$

μ_Π, σ^2_Π

$w_A \rightarrow$ % of wealth in A

$w_B = 1 - w_A \rightarrow$ % of wealth in B

$$\mu_\Pi = \mathbb{E}\left[w_A R_A + (1-w_A) R_B\right]$$

$$= w_A \mathbb{E}[R_A] + (1-w_A)\mathbb{E}[R_B]$$

$$\boxed{\begin{aligned}\mu_\Pi &= w_A \mu_A + (1-w_A)\mu_B \\ &= \mu_B + w_A(\mu_A - \mu_B)\end{aligned}}$$

Remember : $\mathbb{V}[aX + bY]$

$$= a^2\, \mathbb{V}[X] + b^2\, \mathbb{V}[Y]$$

$$+ 2ab\, \mathrm{Cov}[X\, Y] = 2ab\, \rho\, \sigma_A \sigma_B$$

$$\sigma_\Pi^2 = \mathbb{V}\left[w_A R_A + (1-w_A) R_B\right]$$

$$= w_A^2\, \mathbb{V}[R_A] + (1-w_A)^2\, \mathbb{V}[R_B] + 2 w_A (1-w_A)\, \mathrm{Cov}(R_A, R_B)$$

$$\sigma_\Pi^2 = w_A^2 \sigma_A^2 + (1-w_A)^2 \sigma_B^2 + 2 w_A (1-w_A) \sigma_A \sigma_B \rho$$

$$\sigma_A = \sqrt{w_A^2 \sigma_A^2 + (1-w_A)^2 \sigma_B^2 + 2 w_A (1-w_A) \sigma_A \sigma_B \rho}$$

Because $-1 \le \rho \le 1$

$$w_A^2\sigma_A^2 + (1-w_A)^2\sigma_B^2 - 2w_A(1-w_A)\sigma_A\sigma_B \le \sigma_\pi^2 \le w_A^2\sigma_A^2 + (1-w_A)^2\sigma_B^2 + 2w_A(1-w_A)\sigma_A\sigma_B$$

$$0 \le (w_A\sigma_A - w_B\sigma_B)^2 \le \sigma_\pi^2 \le (w_A\sigma_A + w_B\sigma_B)^2$$

$$\sigma_\pi \le w_A\sigma_A + (1-w_A)\sigma_B$$

$$\mu_\pi = w_A\mu_A + (1-w_A)\mu_B$$

$\uparrow$ linear

Torion

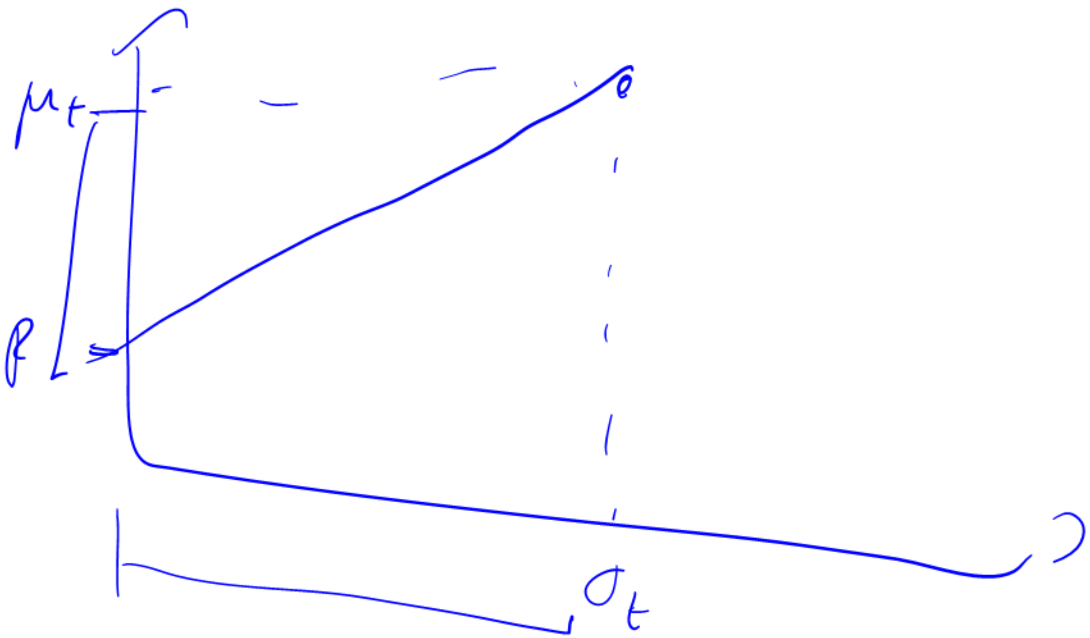
y-variable

$$\mu_\Pi = R + w_t(\mu_t - R) = R + \frac{\sigma_\Pi}{\sigma_t}(\mu_t - R) = R + \sigma_\Pi \times \frac{\mu_t - R}{\sigma_t}$$

x-variable

$$\sigma_\Pi = w_t \sigma_t \longrightarrow w_t = \frac{\sigma_\Pi}{\sigma_t}$$

$$\frac{\mu_t - R}{\sigma_t} = \text{Slope of the CML} = SR$$



$$\sum_{i=1}^{N} w_i = 1 \iff w_1 + w_2 + \dots + w_N = 1$$

$$w = \begin{pmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_N \end{pmatrix} \qquad \mathbb{1} = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$$

N elements

$$w^T \mathbb{1} = \begin{pmatrix} w_1 & w_2 & \dots & w_N \end{pmatrix} \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} = \mathbb{1}^T w$$

$$= w_1 + w_2 + w_3 + \dots + w_N$$

$$= 1$$

$$\mu_\pi = \sum_{i=1}^{N} w_i \mu_i$$

$$\boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix}$$

$$\mu_\pi = \mathbf{w}^T \boldsymbol{\mu}$$

$$= \begin{pmatrix} w_1 & w_2 & \cdots & w_N \end{pmatrix} \begin{pmatrix} \mu_1 \\ \mu_2 \\ \vdots \\ \mu_N \end{pmatrix}$$

$$= w_1 \mu_1 + w_2 \mu_2 + \cdots$$

$$+ w_N \mu_N$$

$$= \sum_{i=1}^{N} w_i \mu_i$$

$$\Sigma_1 = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \qquad \begin{pmatrix} \sigma_1^2 & \rho_{21}\sigma_2\sigma_1 & & \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & & \\ & & \ddots & \\ & \rho_{ij}\sigma_i\sigma_j & & \sigma_N^2 \end{pmatrix}$$

$$w = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$1 \times \cancel{2 \qquad 2} \times \cancel{2 \qquad 2} \times 1$$

$$w^T \Sigma_1 w = \begin{pmatrix} w_1 & w_2 \end{pmatrix} \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$= w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + 2w_1w_2\rho\sigma_1\sigma_2$$

$\pounds 1 \rightarrow$ 1.01 FTSE S_1

$\pounds 1 \rightarrow$ IR S_0

0.99

$\ln \frac{S_1}{S_0}$

$= \ln S_1 - \ln S_0$