Module 3 Assignment

This is a Computational Finance task on the use of the Explicit Finite Difference Method and Monte Carlo scheme to price Binary options.

Part 1

Consider the problem for the price of a digital call option $V(S,t)$

$$\begin{aligned} \frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV &= 0 \\ V(S,T) &= \mathcal{H}(S_T - E) \\ &= \begin{cases} 1 & S_T > E \\ 0 & S_T \leq E \end{cases} \end{aligned} \tag{1}$$

where $\mathcal{H}(\cdot)$ is the heaviside function. S is the spot price of the underlying financial asset, t is the time, $E > 0$ is the strike price, T the expiry date, $r \geq 0$ the interest rate and σ is the volatility of S.

(1) has a closed form solution given by

$$V(S,t) = e^{-r(T-t)} N(d_2) \tag{2}$$

Solve (1) by the *Explicit finite difference method* using a backward marching scheme, i.e. of the form

$$V_n^{m-1} = F\left(V_{n-1}^m, V_n^m, V_{n+1}^m\right)$$

where $V(S,t)$ can be expressed in discrete form as V_n^m.

Part 2

Now use the expected value of the discounted payoff under the risk-neutral density $\mathbb{Q}$

$$V(S,t) = \mathbb{E}^{\mathbb{Q}}\left[e^{-\int_t^T r_\tau d\tau} \mathbf{Payoff}(S_T)\right] \tag{3}$$

to obtain an approximation to the Black-Scholes price given by (2), where the underlying should be simulated using the Milstein scheme.

For Part 1 and 2 compare your approximations to the exact solution (2). Consider varying step sizes and numbers of sample paths.

You may use Paul's data as in the practical part of the Introduction to Numerical Methods lecture.

You should submit a brief report (and code separately) to include:

- Outline of the numerical procedure used
- Results - appropriate tables and error graphs
- Any interesting observations and problems encountered

Queries to riaz.ahmad@fitchlearning.com