$$y = f(x)$$ explicit

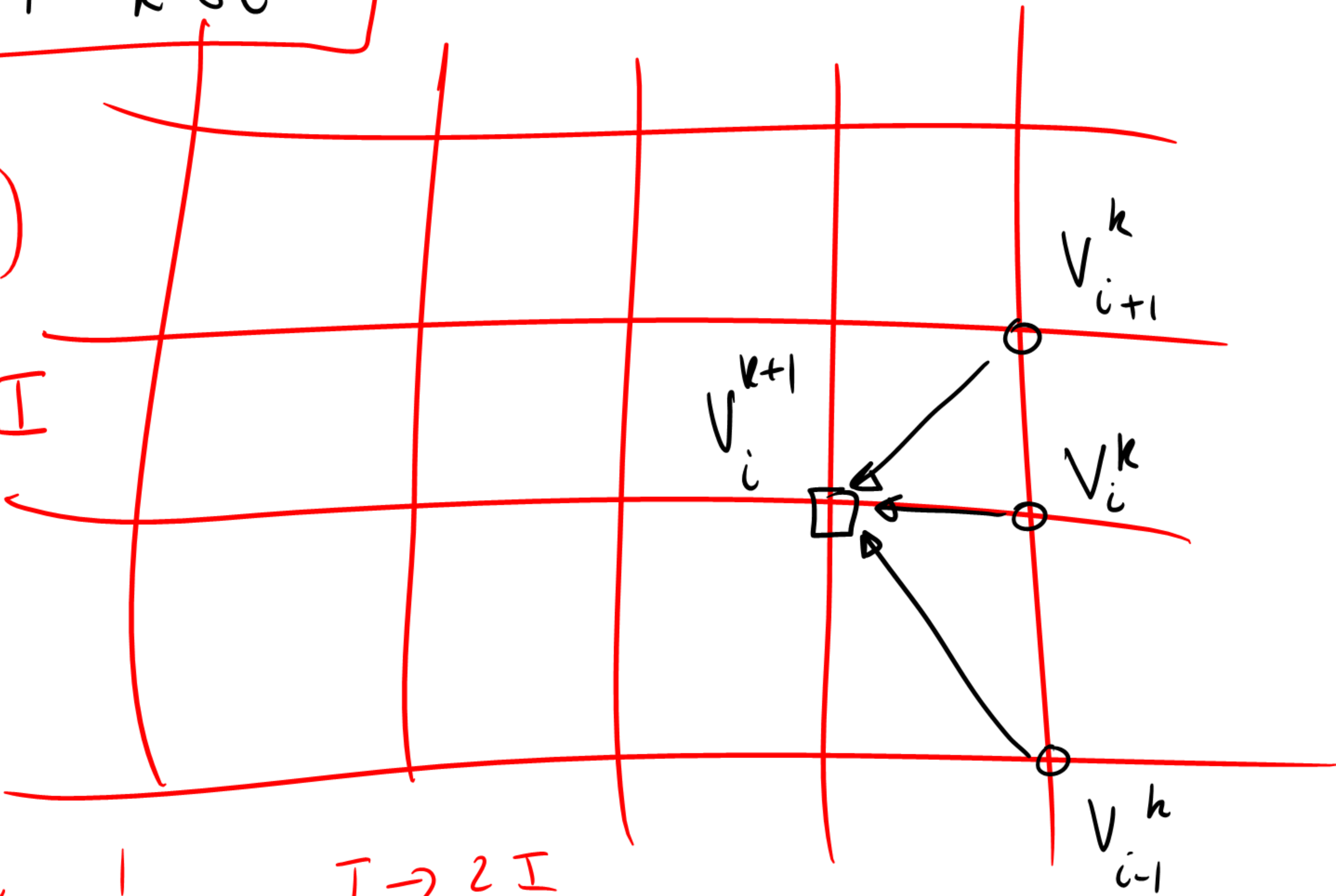
$$f(x, y) = 0$$ implicit

$$\underbrace{V_i^{k+1}}_{\text{unknown}} = F\left(\overbrace{V_{i-1}^{k},\ V_i^{k},\ V_{i+1}^{k}}^{\text{known}}\right)$$

$$\boxed{t = T - k\,\delta t}$$

$O(\delta t, \delta S^2)$

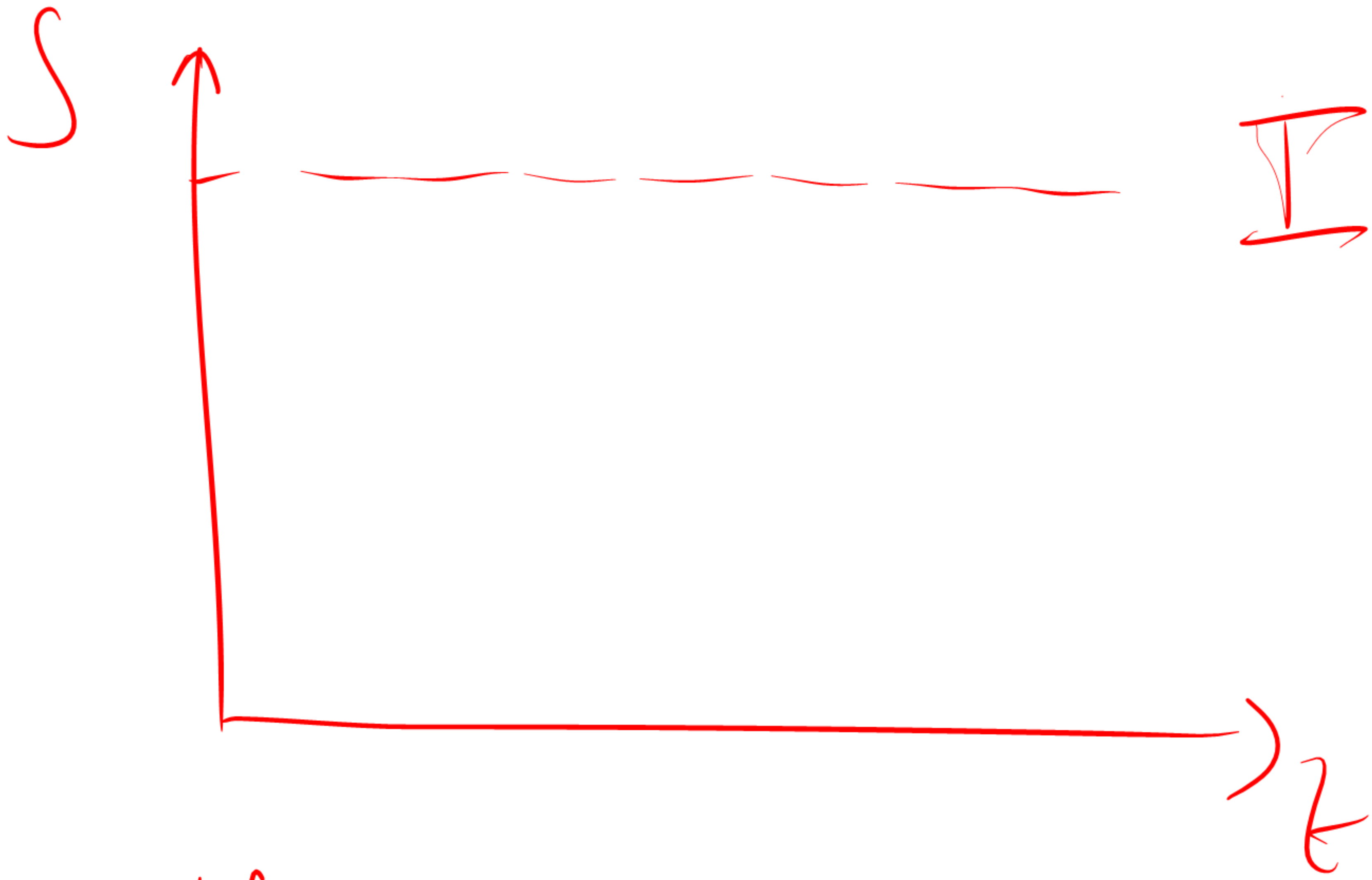
$0 \leq i \leq I$



$$\delta t \leq \frac{1}{\sigma^2 I^2}$$

$I \to 2I$

$$4\,\delta t \leq \frac{1}{\sigma^2 I^2}$$



V_{I+1}

$$M\underline{v} = \underline{g}$$

$$L(U\underline{v}) = \underline{g}$$

$$\overset{②}{U\underline{v} = \underline{w}} \longrightarrow L\underline{w} = \underline{g} \quad ①$$

Solve ① $\longrightarrow \underline{w}$ $\quad \begin{pmatrix} & 0 \\ & \end{pmatrix}\underline{w} = \underline{g}$

$\longrightarrow$ iteration value 1

i.e. $\underline{v}^{(1)}$

Subst back into (*)

$\underline{v}^{(2)}$

Strictly diagonally dominant

$$\begin{pmatrix} a & b & c & d \\ A & B & C & D \\ \alpha & \beta & \gamma & \delta \\ \boxed{1} & \boxed{2} & \boxed{3} & \boxed{4} \end{pmatrix}$$

✓ $|a| > |b| + |c| + |d|$

✓ $|B| > |A| + |C| + |D|$

✓ $|\gamma| > |\alpha| + |\beta| + |\delta|$

✓ $|\boxed{4}| > |\boxed{1}| + |\boxed{2}| + |\boxed{3}|$

$\underline{x} \in \mathbb{R}^n \qquad \underline{x} = (x_1, x_2, \ldots, x_n)^T$

$$\|\underline{x}\|_p = \left(\sum_{i=1}^{n} x_i^p\right)^{1/p}$$

norm

$$\|\underline{x}\|_2 = \sqrt{x_1^2 + \cdots + x_n^2}$$

l_2-norm

l_∞ norm or $\|\underline{x}\|_\infty$

$$\|\underline{x}\|_\infty = \max_{1 \leq i \leq n} |x_i|$$

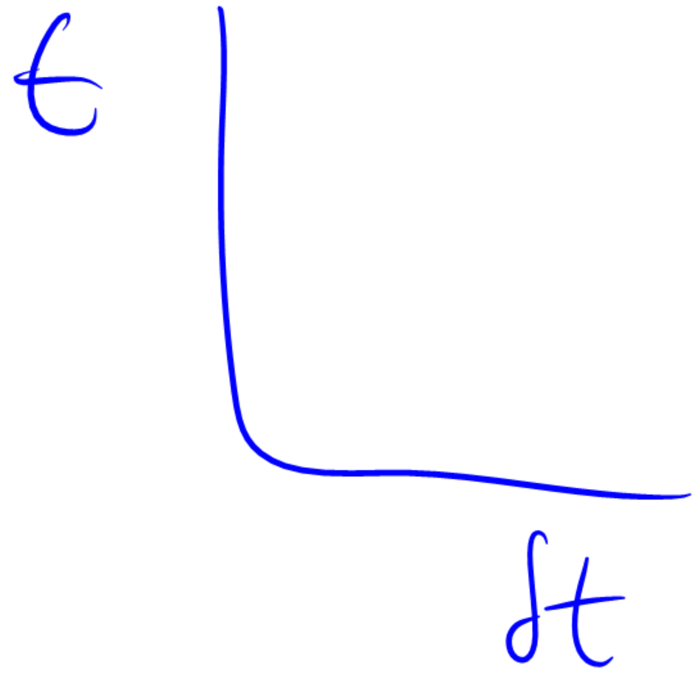
$$\underline{x} = (-10, \pi, 6, 7, e)$$

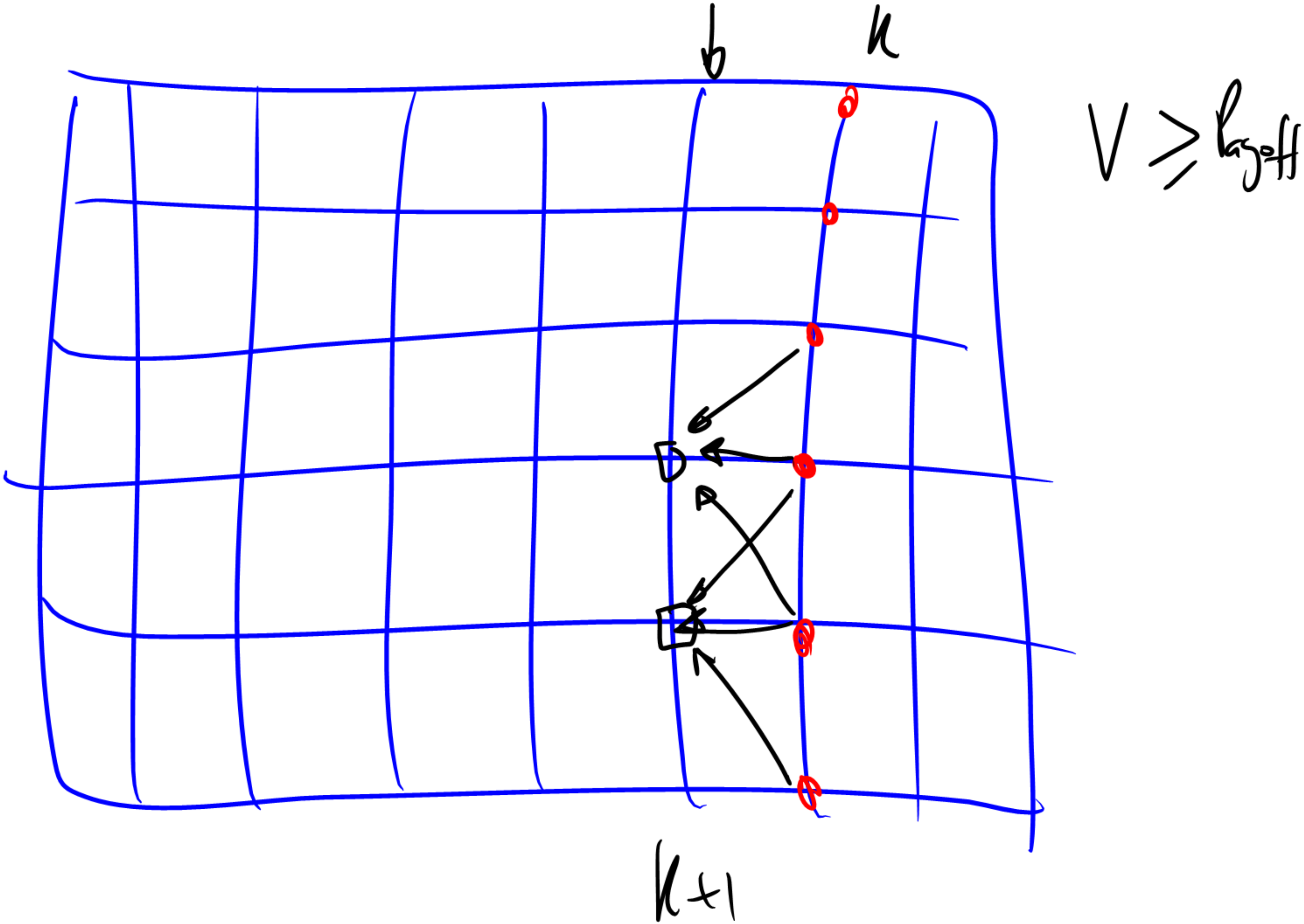
$$\|\underline{x}\|_\infty = 10$$

$$\frac{\left\| \underline{x}^{(n+1)} - \underline{x}^{(n)} \right\|_\infty}{\left\| \underline{x}^{(n)} \right\|_\infty} < \varepsilon$$

δJ δt

$$E = |\text{Exact} - \text{Approx}|$$





k
k+1
$V \geq V_{\text{cutoff}}$

$$V_{ij}^{k}$$

$$V(s, r, t)$$

$0 \le i \le I$

$0 \le j \le J$

$0 \le k \le K$

$$\frac{\partial^2 V}{\partial s \partial r} = \frac{\partial}{\partial s}\left[\frac{\partial V}{\partial r} \right]$$

$$\frac{\partial^2 V}{\partial J \partial r} = \frac{\partial}{\partial J}\left(\frac{\partial V}{\partial r} \right)$$

$$= \frac{1}{2\delta r} \frac{\partial}{\partial J}\left[V_{i,j+1}^{k} - V_{i,j-1}^{k} \right]$$

$$= \frac{\frac{1}{2\delta r}\left[\left(V_{i+1,j+1}^{k} - V_{i+1,j-1}^{k}\right) - \left(V_{i-1,j+1}^{k} - V_{i-1,j-1}^{k}\right)\right]}{2\delta J}$$

$$\frac{\partial V}{\partial t} + a(S,I,t)\frac{\partial^2 V}{\partial S^2} + b(S,I,t)\frac{\partial V}{\partial S} + f(S,I,t)\frac{\partial V}{\partial I} + c(S,I,t)V = 0$$

Upwind differencing

1 If $f(S, I, t) > 0$ then $\boxed{\begin{aligned} S &= i\,\delta S \\ t &= k\,\delta t \\ I &= j\,\delta I \end{aligned}}$

$$f \frac{\partial V}{\partial I} = f^{k}_{i,\,j+\frac{1}{2}} \frac{V^{k}_{i\,j+1} - V^{k}_{i\,j}}{\delta I} \qquad I = j\delta I$$

But if

2 if $f(S, I, t) < 0$ then

$$f \frac{\partial V}{\partial I} = f^{k}_{i,\,j-\frac{1}{2}} \frac{V^{k}_{i\,j} - V^{k}_{i\,j-1}}{\delta I}$$