$$dS = \mu S\,dt + \sigma S\,dW$$

diffusion process

$$[t, \; t + dt]$$

$$X : \omega \in \Omega \longrightarrow \mathbb{R}$$

$$R_i \longrightarrow \begin{cases} +1 & H \\ -1 & T \end{cases}$$

$$R_0 = 0$$

$$\mathbb{E}[x] = \sum x_i$$

$$\mathbb{E}[X] = \sum x_i p_i$$

$\log$

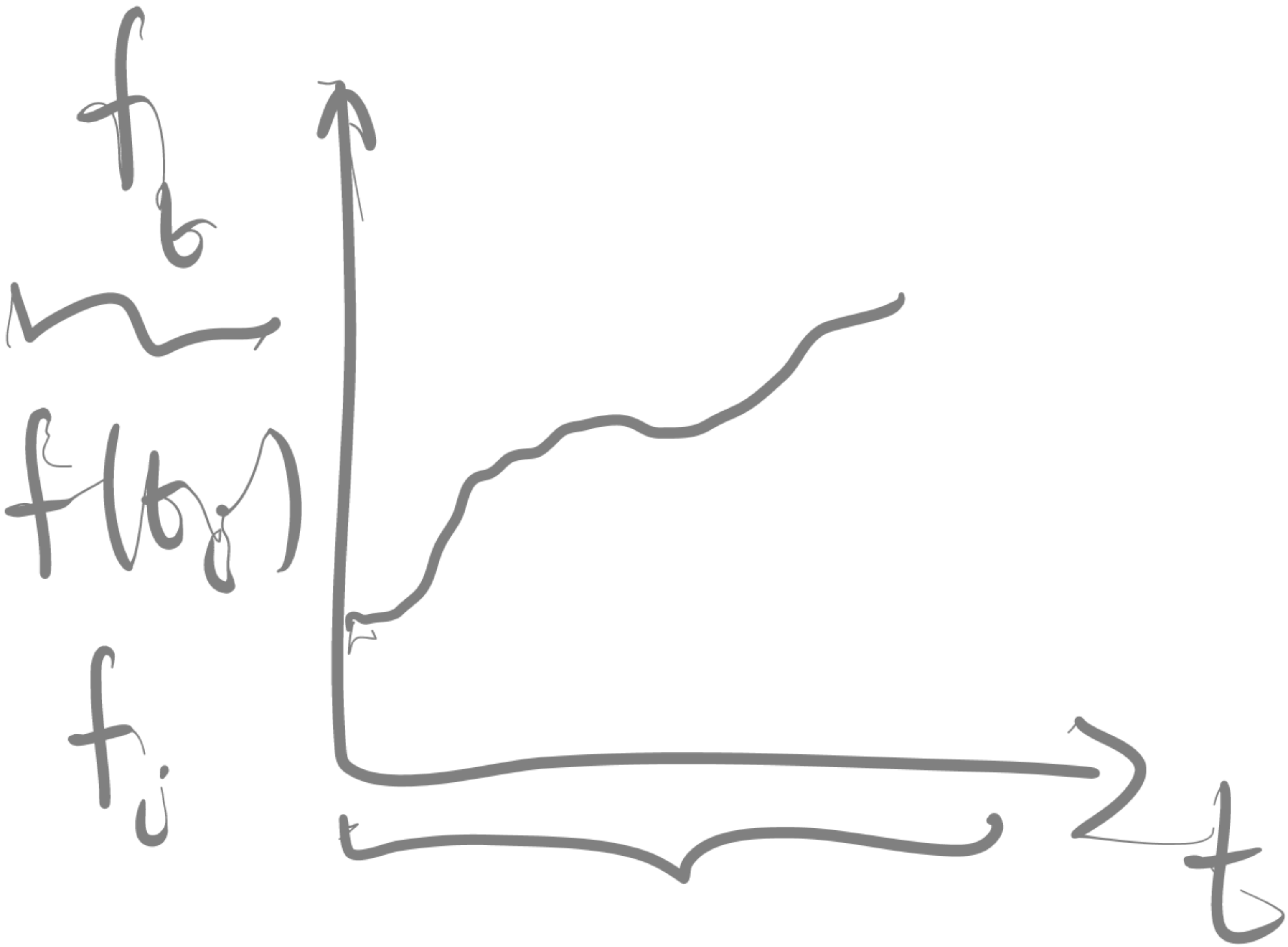
$$\mathbb{E}[X] = \sum x_i f_i$$

$$= +1\left(\tfrac{1}{2}\right) + (-1)\left(\tfrac{1}{2}\right) = 0$$

$$\mathbb{V}[X] = \sum x_i^2 f_i - \mu^2$$

$$= 1$$

$$\mathbb{V}[W_n] = \mathbb{E}\left[\left(\sum R_j\right)^2\right]$$

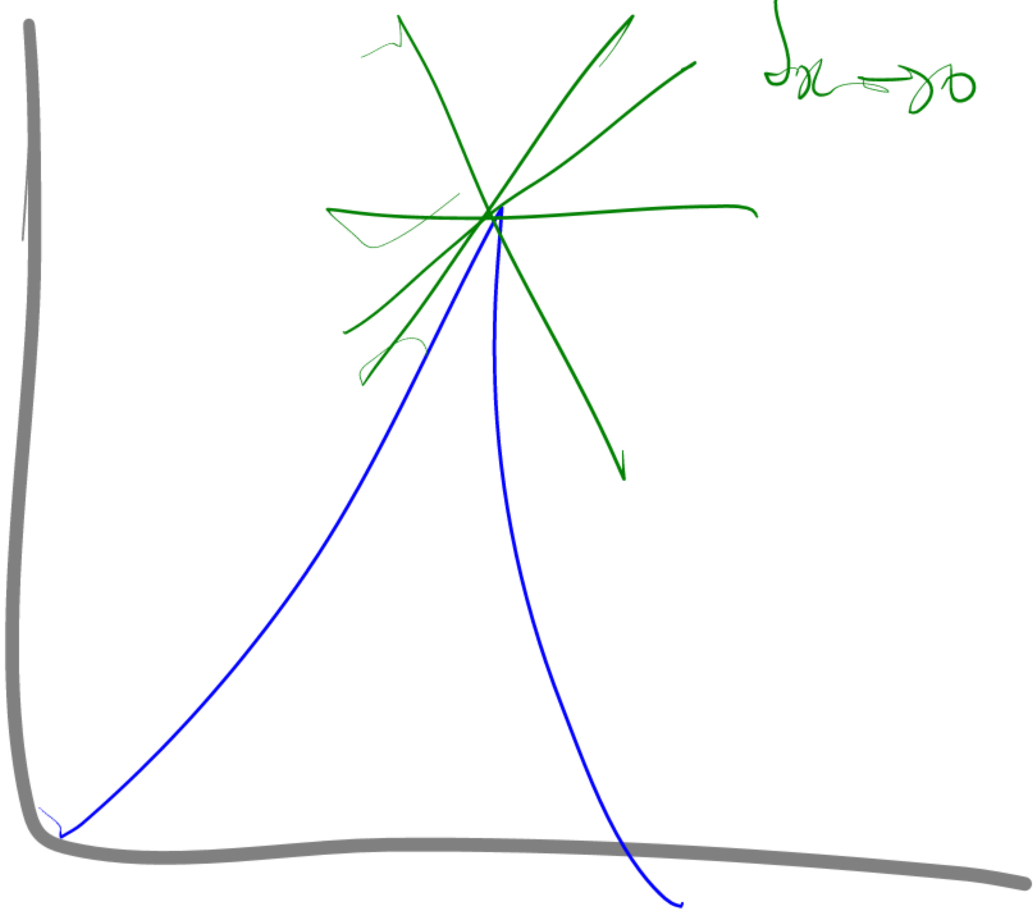


$t = i\,\delta t \qquad t/n = \delta t$

$$\frac{\delta y^2}{\delta t}$$

$$\delta y^2 \sim O(\delta t)$$

$$\delta y \sim O(\underbrace{\sqrt{\delta t}}_{\sqrt{\frac{t}{n}}})$$



$$\lim_{\delta x \to 0} \frac{f(x+\delta x) - f(x)}{\delta x}$$

$$W_t \sim N(0, t)$$

$$dW_t \sim N(0, dt)$$

$f(x) = x^2$

$F(w) = w^2$

$df = 2x\,dx$

$\frac{dF}{dw} = 2w$?

$F = F(W)$ Itô's

$$dF = \frac{dF}{dW}\,dW + \frac{1}{2}\frac{d^2F}{dW^2}\,dt$$

$$O(\sqrt{dt})$$

$$dS = \mu S \, dt + \sigma S \, dW$$

drift

diffusion

$F = W^2$ $\quad F' = 2W$ $\quad F'' = 2$

$$dF = F'\,dW + \frac{1}{2}\frac{d^2F}{dW^2}\,dt$$

$$\boxed{dF = 2W\,dW} + dt$$

Correction term

$F_\varepsilon \sin\omega \qquad F'_\varepsilon \cos\omega$

$F''_\varepsilon \sin\omega$

differentiate

$$dF_\varepsilon = \underbrace{F'\,d\omega}_{\cos\omega\,d\omega} + \underbrace{\frac{1}{2}F''\,dt}_{-\frac{1}{2}\sin\omega}$$

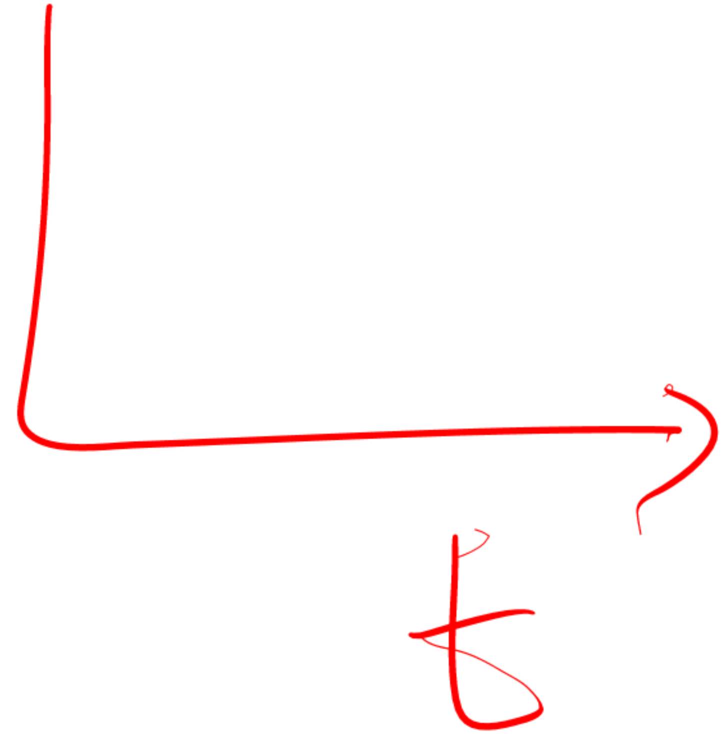
$$t^2 + W = F(t, W)$$

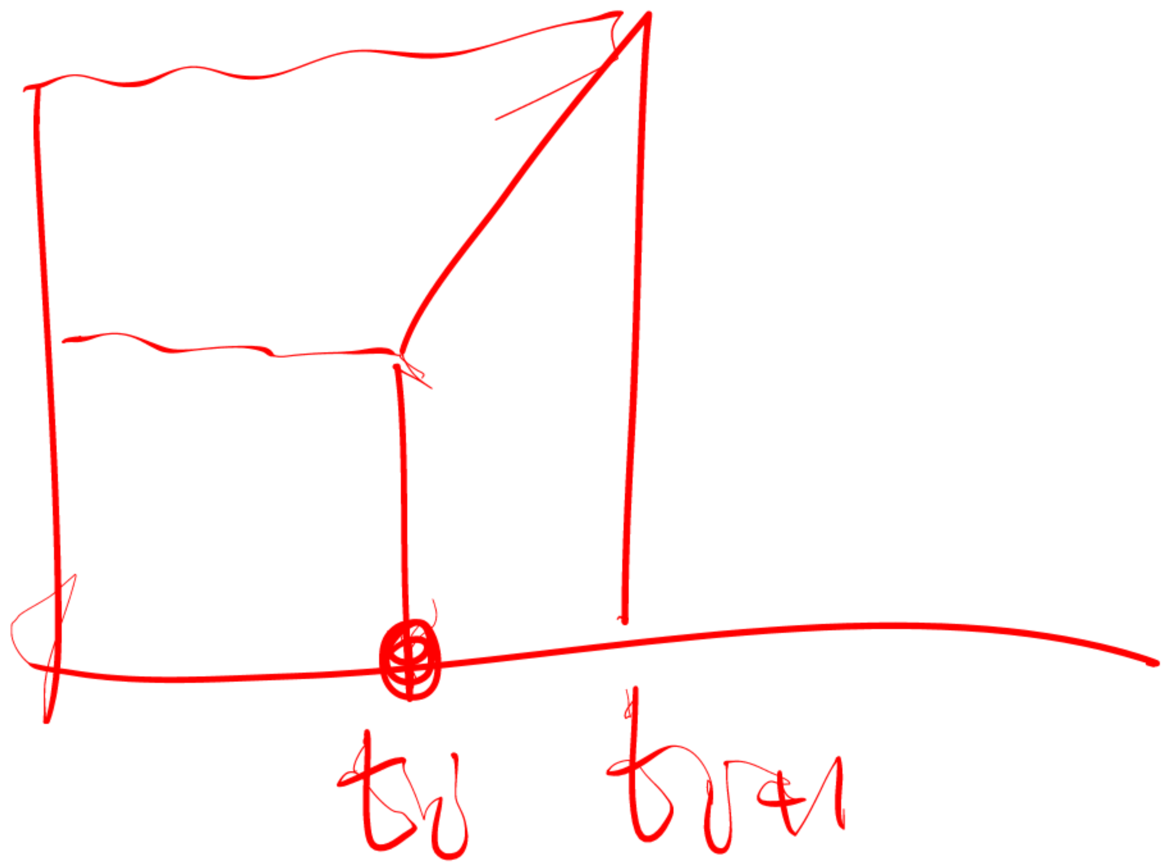
$$t^2 e^W = F(t, W)$$

$$W \longrightarrow dW + W$$

$$t \longrightarrow t + dt$$

$$df = \frac{\partial f}{\partial t} \quad f(W)$$





$$dG = A\,dt + B\,dW$$

$$\underbrace{\int_0^T dG} = \int_0^T A\,dt + \int_0^T B\,dW$$

$$G(T) = G(0) + \int_0^T A\,dt + \underbrace{\int_0^T B\,dW}$$

$$dG = A\,dt + B\,dW$$

$$dG^2 = (dG)^2 \qquad dW^2 \sim O(dt)$$

$$= A^2 dt^2 + 2AB\,dt\,dW + B^2 dt$$

$$\to 0 \qquad O(dt^{3/2})$$

$$\int \frac{dS}{S} = \mu \int dt$$

$$\mu \to r$$

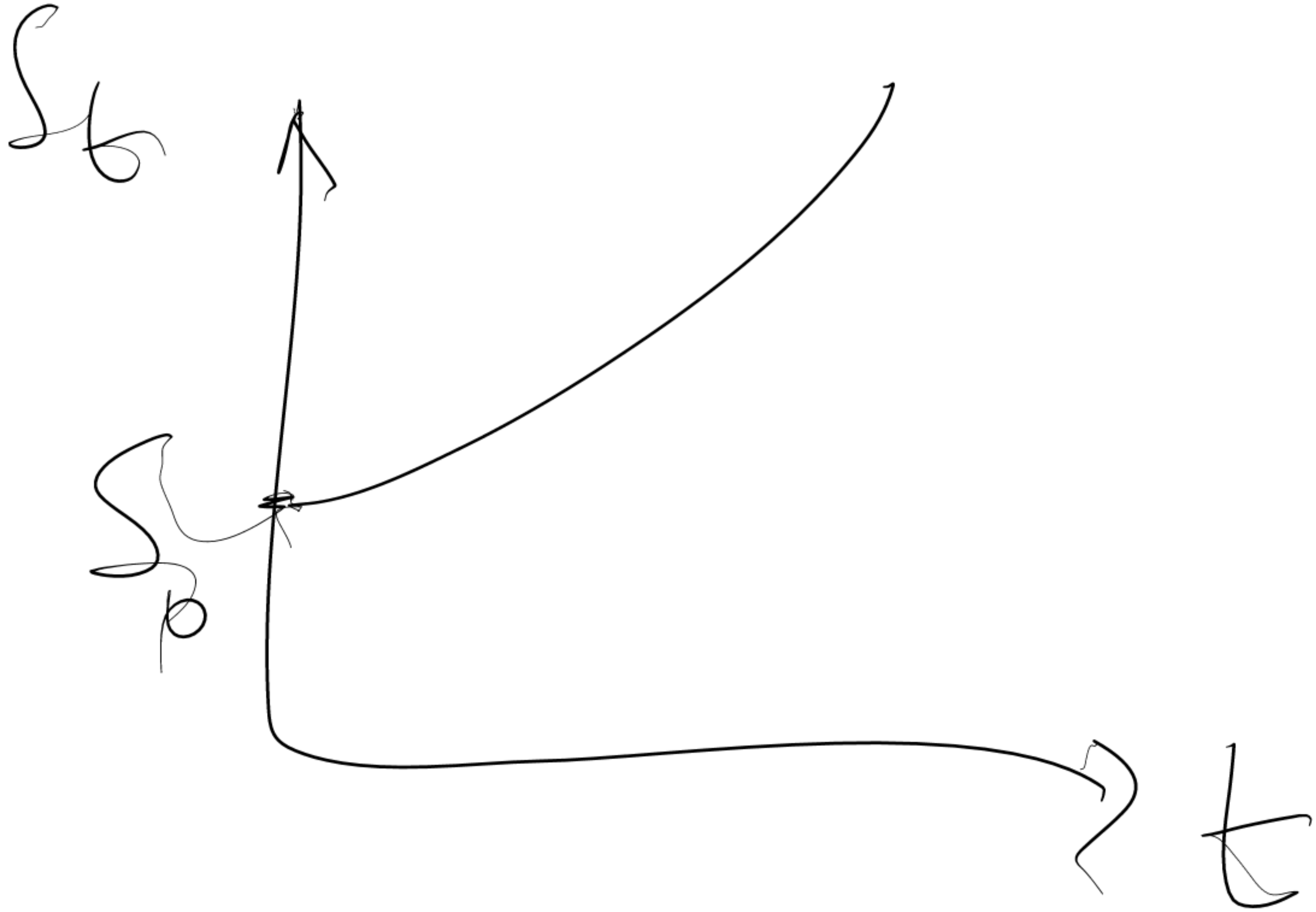
$$\log S = \mu t + C$$

At $t = 0$

$$S(0) = S_0$$

$$S(t) = A e^{\mu t} \Rightarrow A = S_0$$

$$S_t = S_0 e^{rt}$$



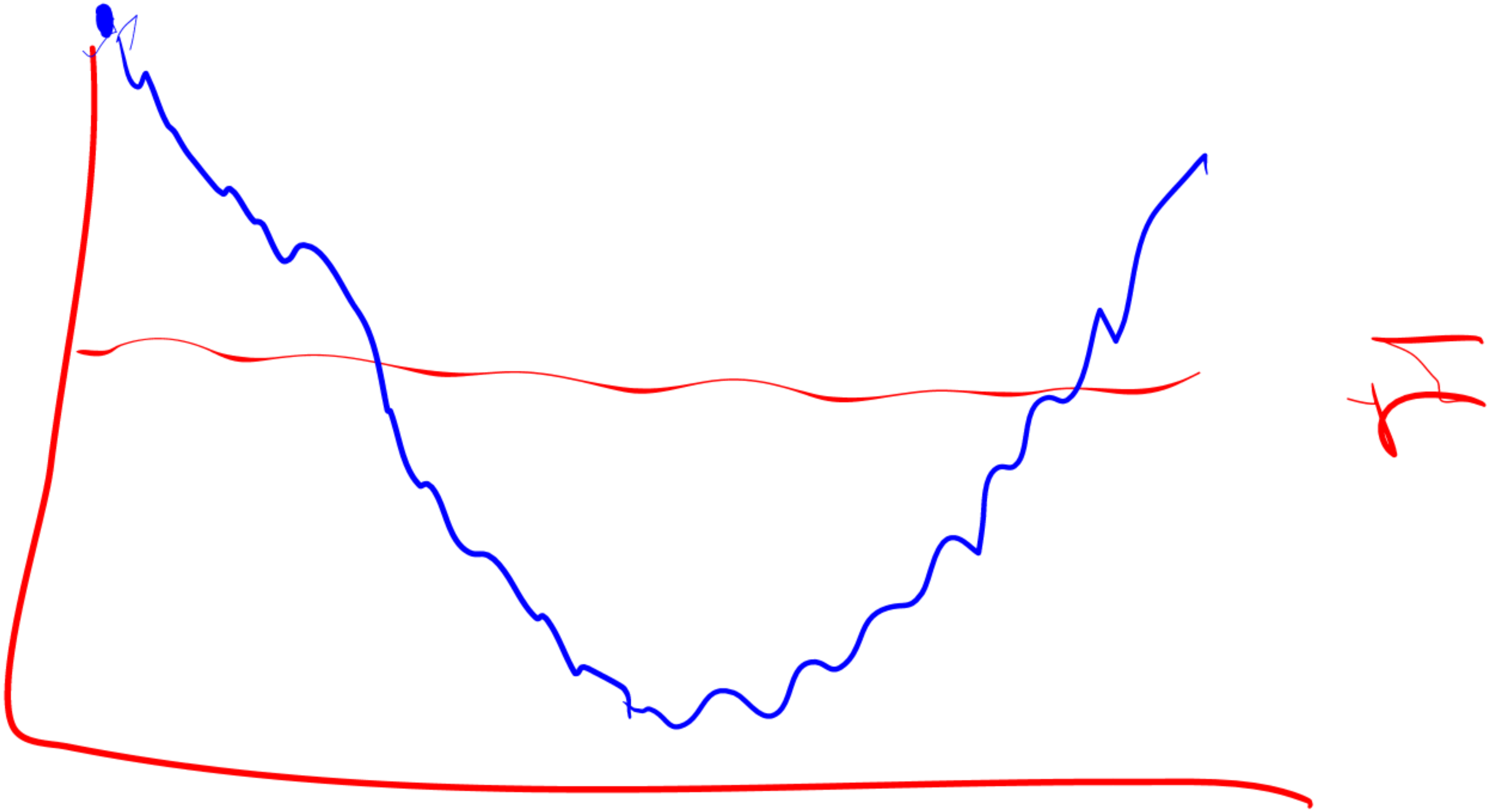
S_b
S_p
t

Vasicek

$$dr = (\eta - \gamma r)\,dt + \sigma\,dW$$

$$= \gamma\left(\underbrace{\eta/\gamma}_{\bar{r}} - r\right)dt + \sigma\,dW$$

$$= -\gamma(r - \bar{r})\,dt + \sigma\,dW$$



$$-\gamma(r-\bar{r})\,dt$$

$r > \bar{r}$

$r < \bar{r}$

CIR $\quad 0.01 \quad \boxed{10^{-2}}$

$$dr = -\gamma(r - \bar{r})\,dt$$

$$+ \sigma\sqrt{r}\,dW$$

$$+ \beta r^{\nu}\,dW \qquad r$$