X-Valuation Adjustment (CVA, DVA, FVA) - Implementation

Abstract

The aim of this final lecture on credit derivatives is to introduce the simple implementation of counterparty credit valuation adjustment (CVA). We use Microsoft Excel. We will review the methodologies currently used to quantify CVA in terms of exposure and Monte Carlo simulation. We give various examples to illustrate this methodology as well as DVA, FVA and others.

Instructor:

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SDA Bocconi Business School
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"During the financial crisis, roughly two-thirds of losses attributed to counterparty credit risk were due to CVA losses and only about one-third were due to actual defaults."

Bank for International Settlements
Basel, Switzerland

http://www.bis.org/press/p110601.htm

Credit Valuation Adjustment (CVA) is a form of "adjustment" or "correction" that must be applied to the price of financial instruments to take into account counterparty risk.

$$\pi^* = \pi - CVA$$

corrected price

price

correction

Credit Valuation Adjustment

1 Credit

involves credit risk, particularly default

2 Valuation

related to the value or price of the instrument

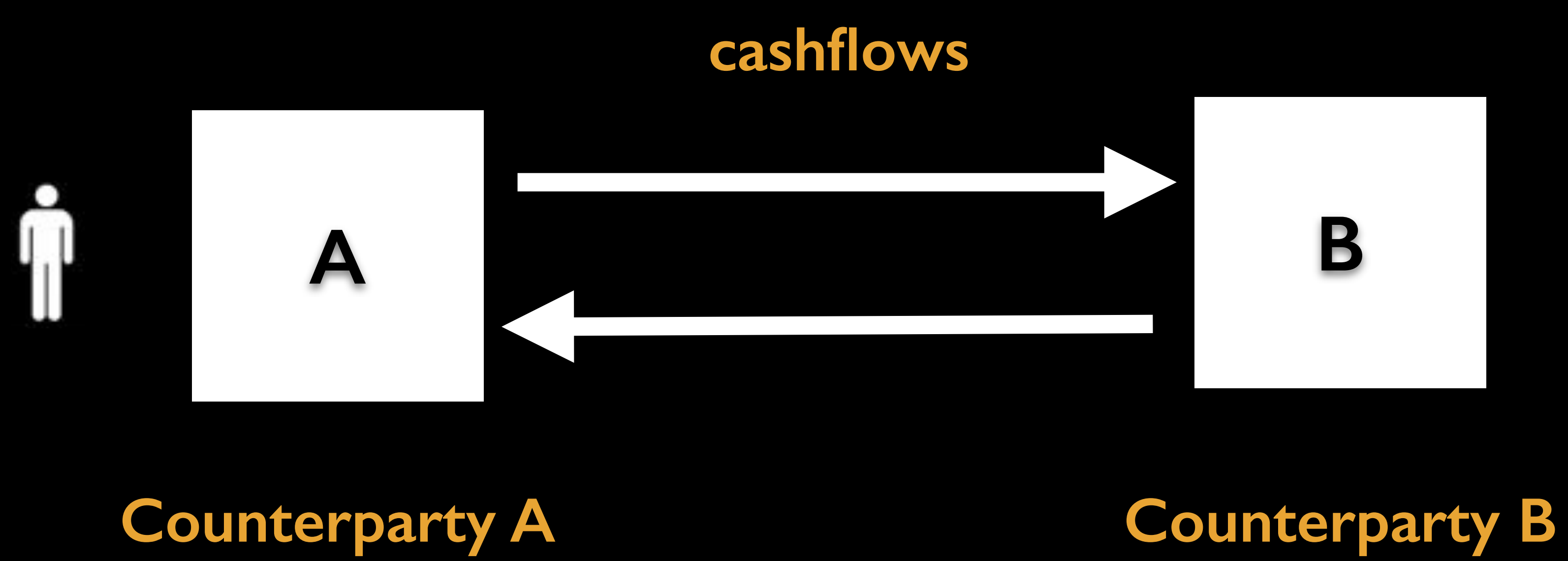
3 Adjustment

associated with a correction to the price of the instrument

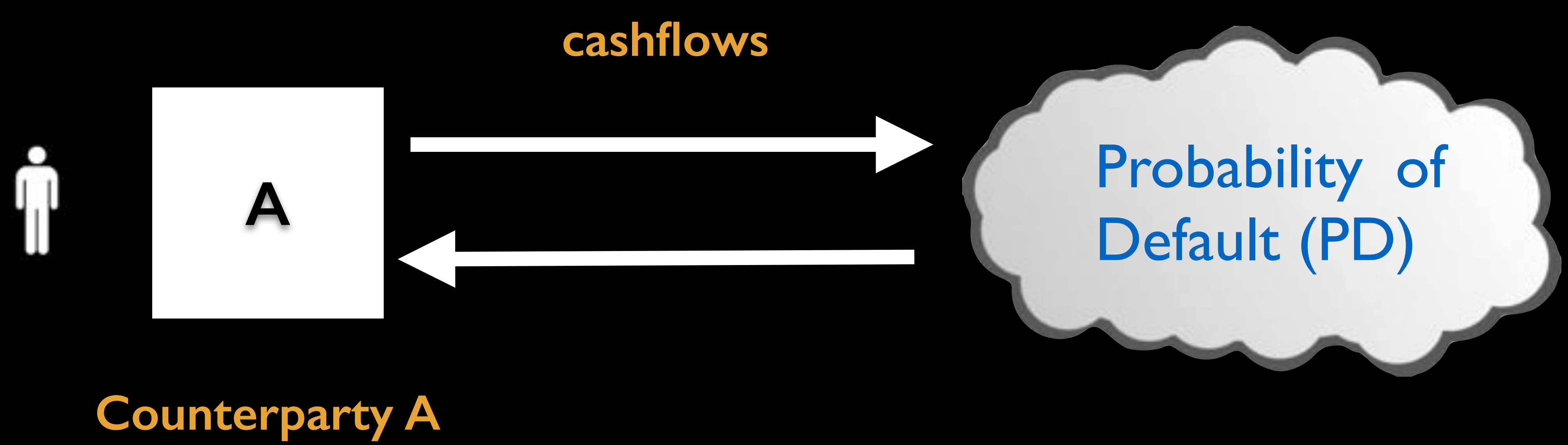
Part 1 Credit

1 Credit

Consider a plain vanilla interest rate swap…



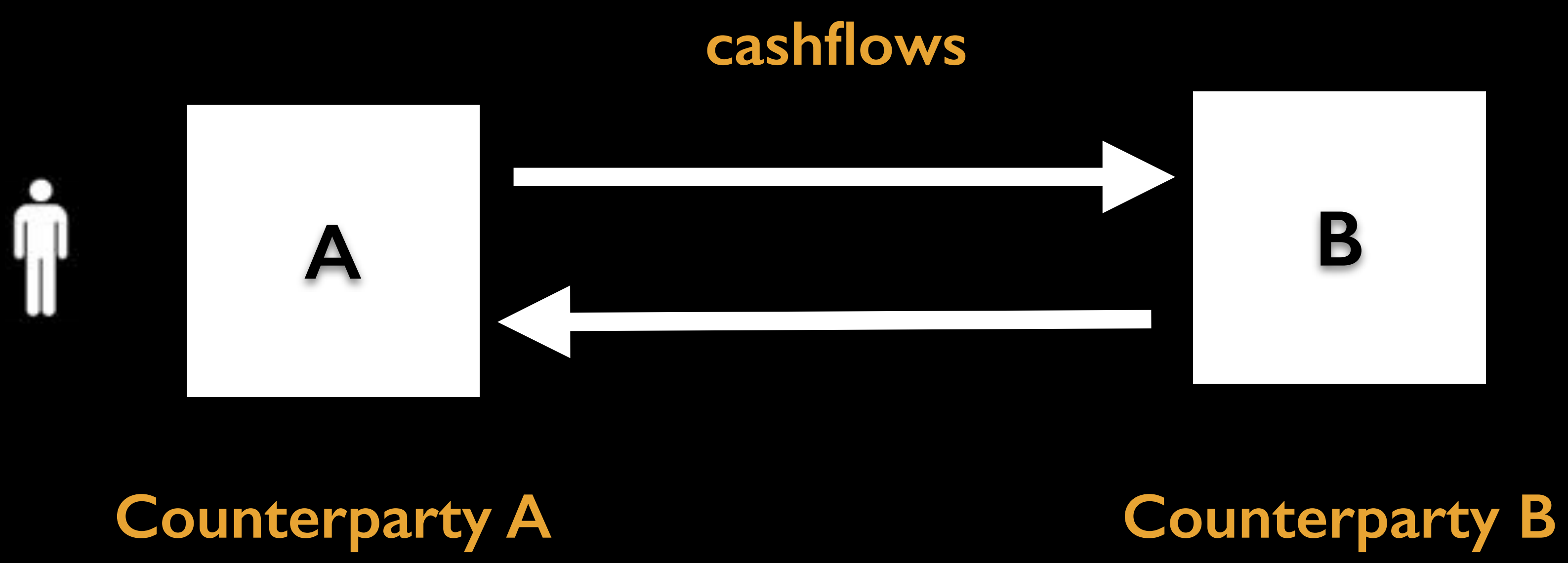
Consider a plain vanilla interest rate swap…



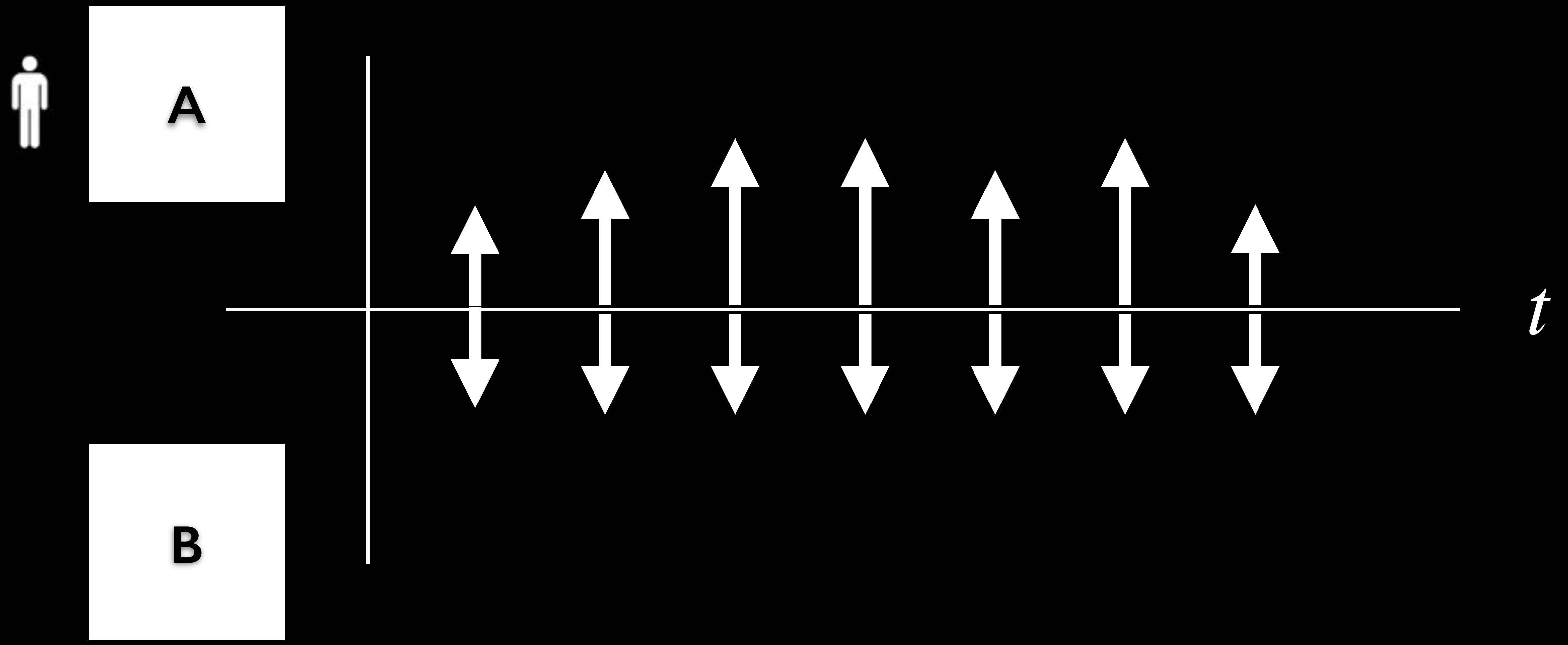
the probability of default quantifies the likelihood of the counterparty "disappearing" (and thus not paying anymore)…

Part 2 Valuation

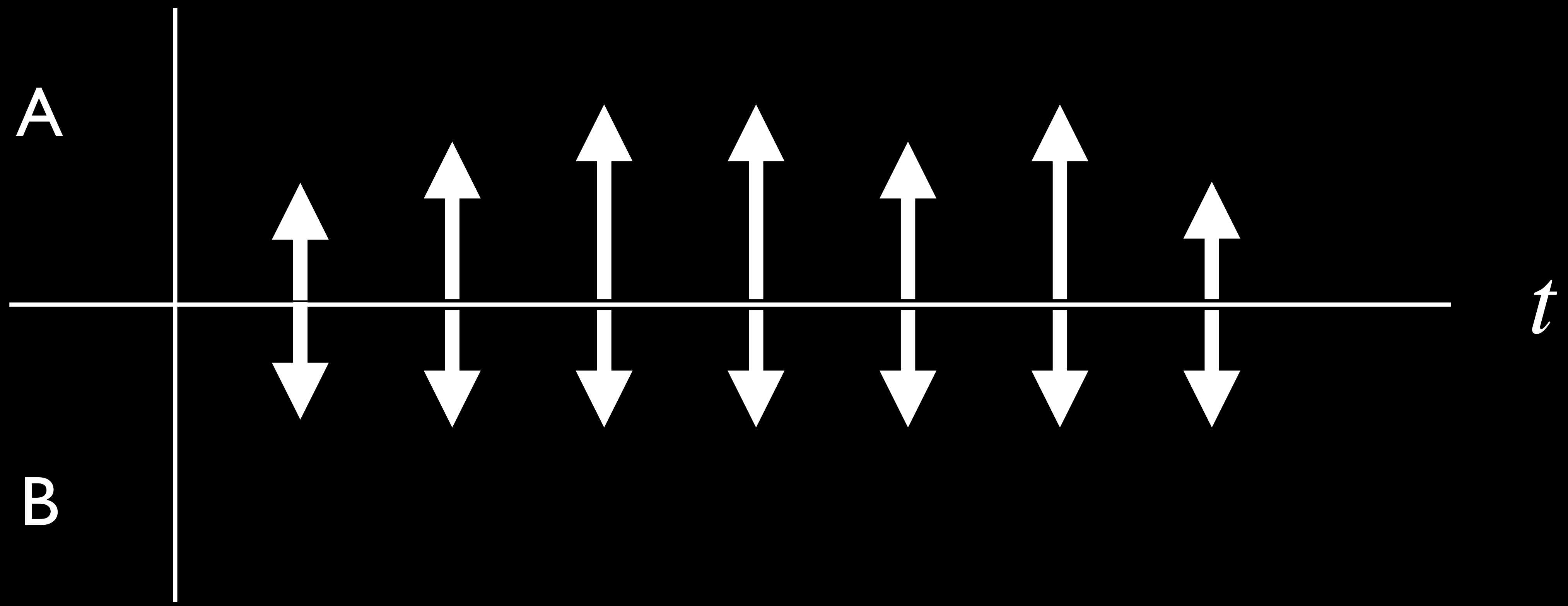
What is the value (price) of interest rate swap?



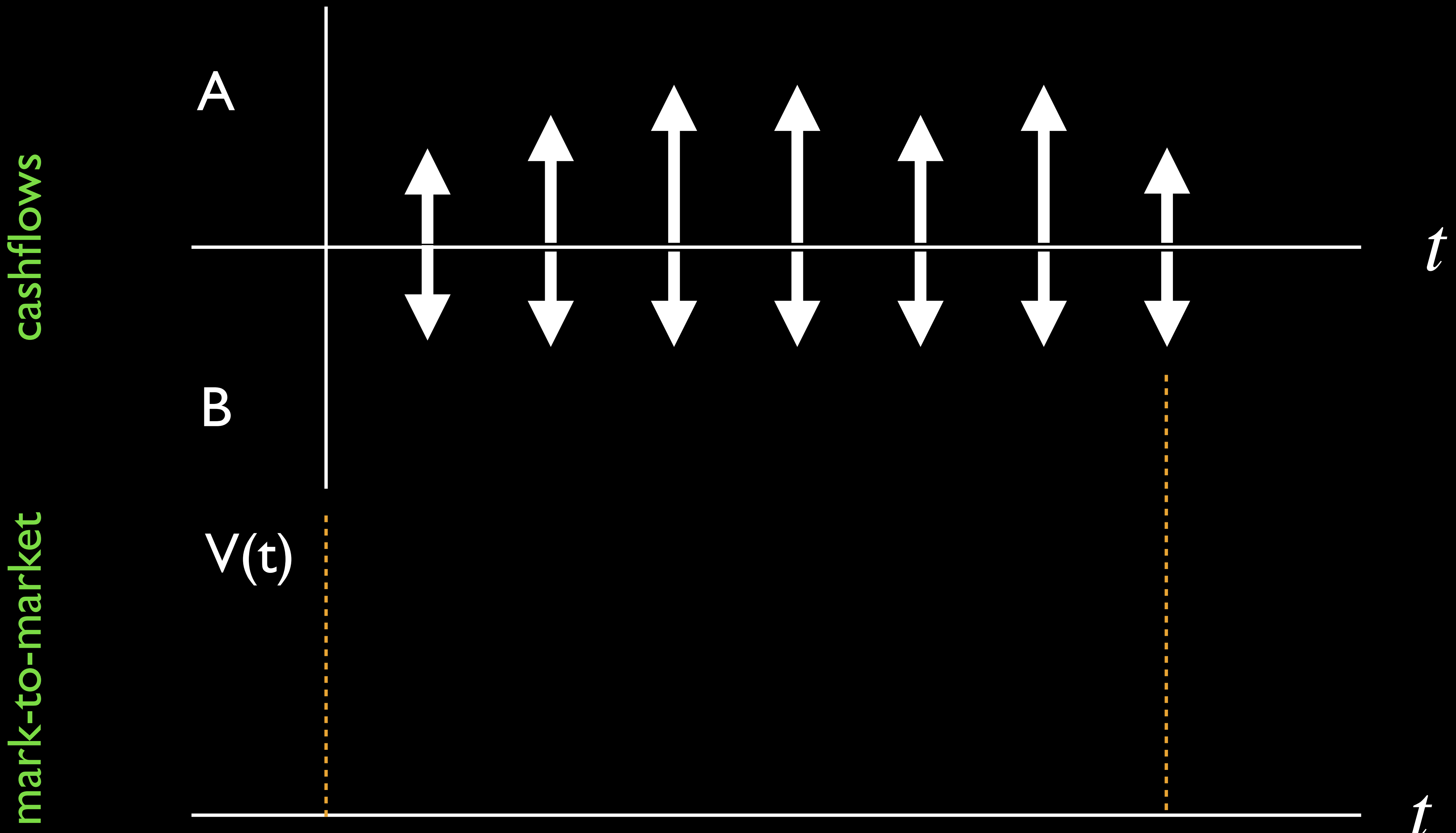
What is the value (price) of interest rate swap?

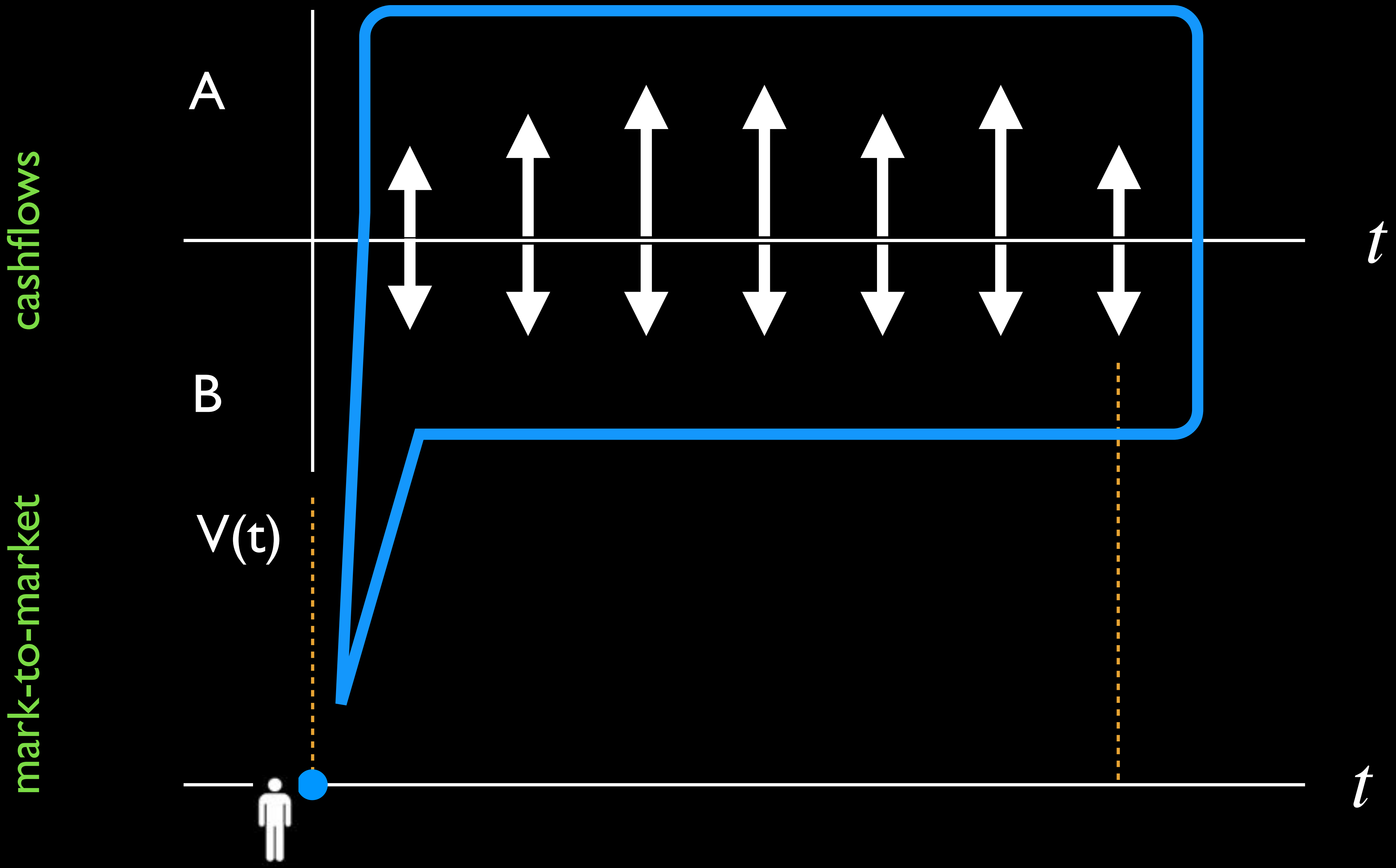


from the point of view of A, the value of the IRS is the sum of the expected cashflows paid (fixed) and received (floating)…

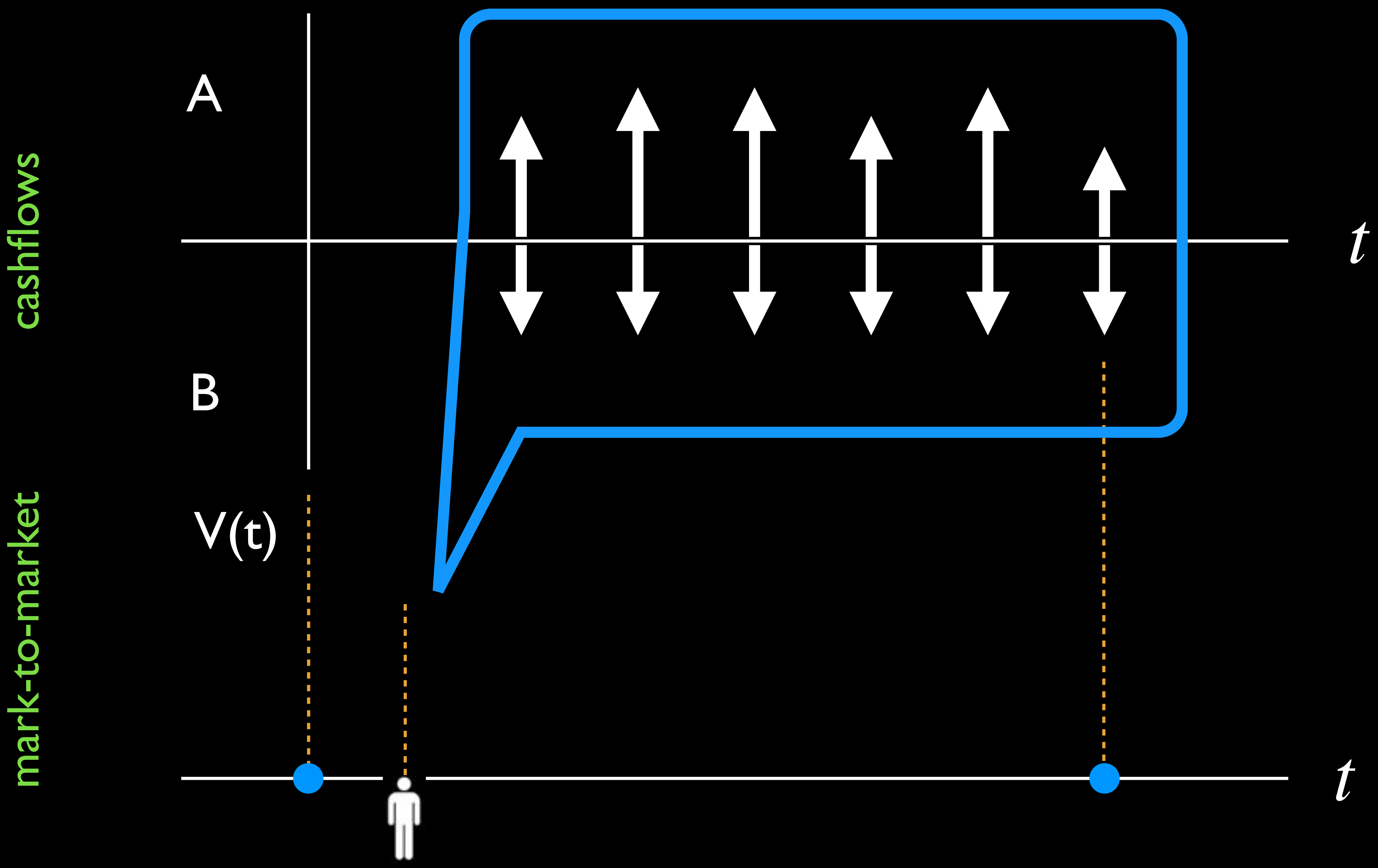


from the point of view of A, the value of the IRS is the sum of the expected cashflows paid (fixed) and received (floating)…which cashflows to consider depends on the valuation time





cashflows
mark-to-market
A
B
V(t)
t
t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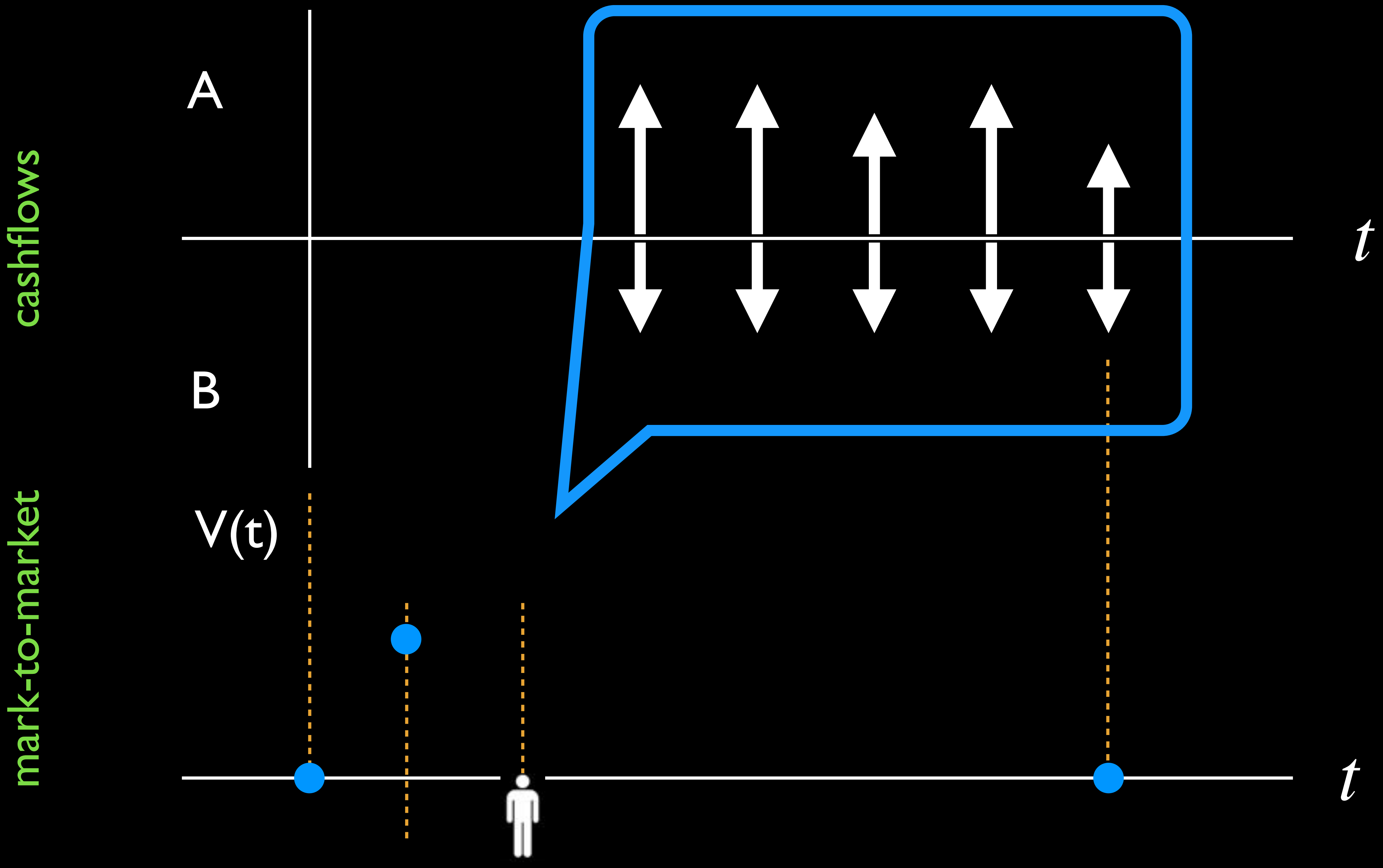
B
t
mark-to-market
V(t)
t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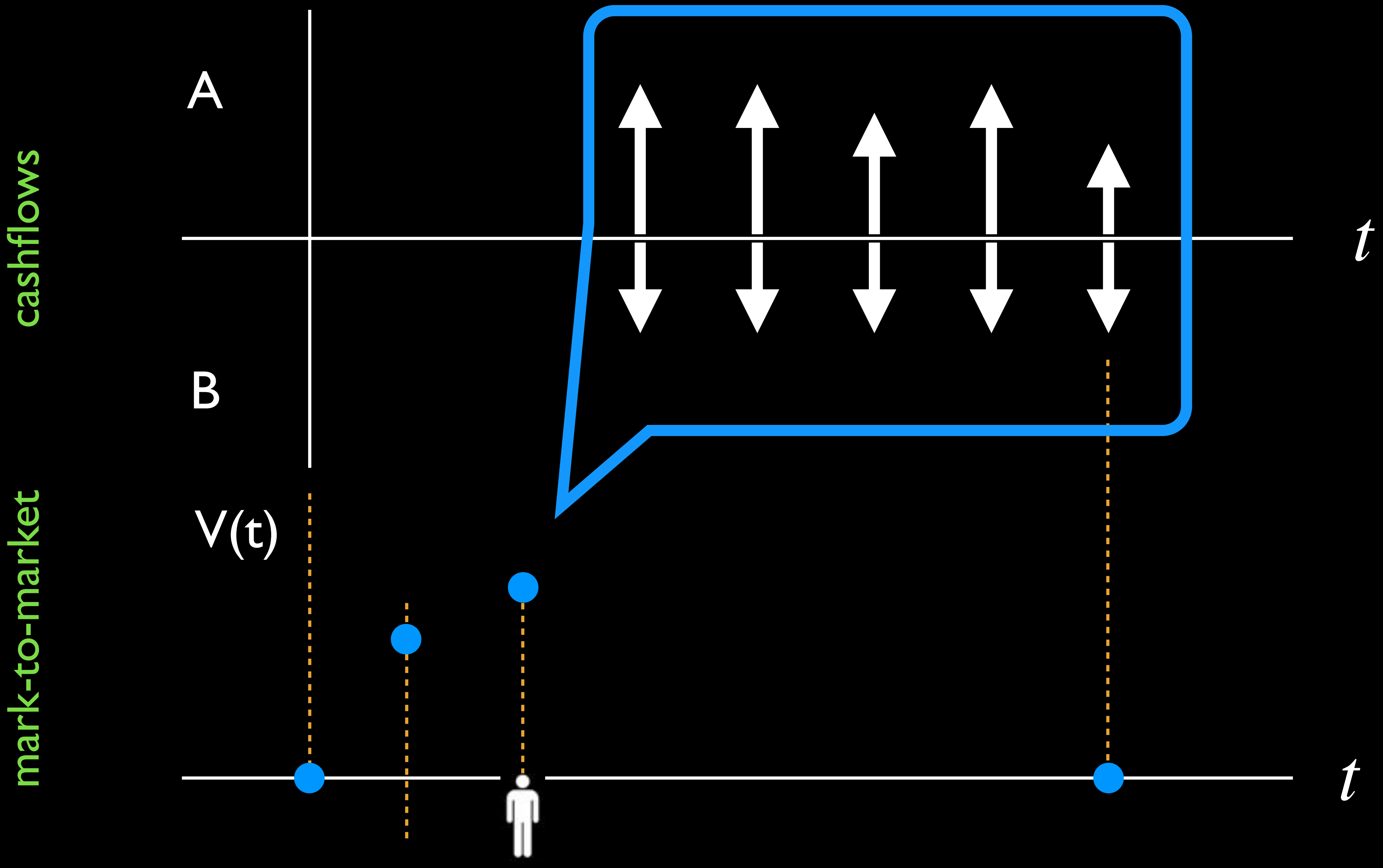
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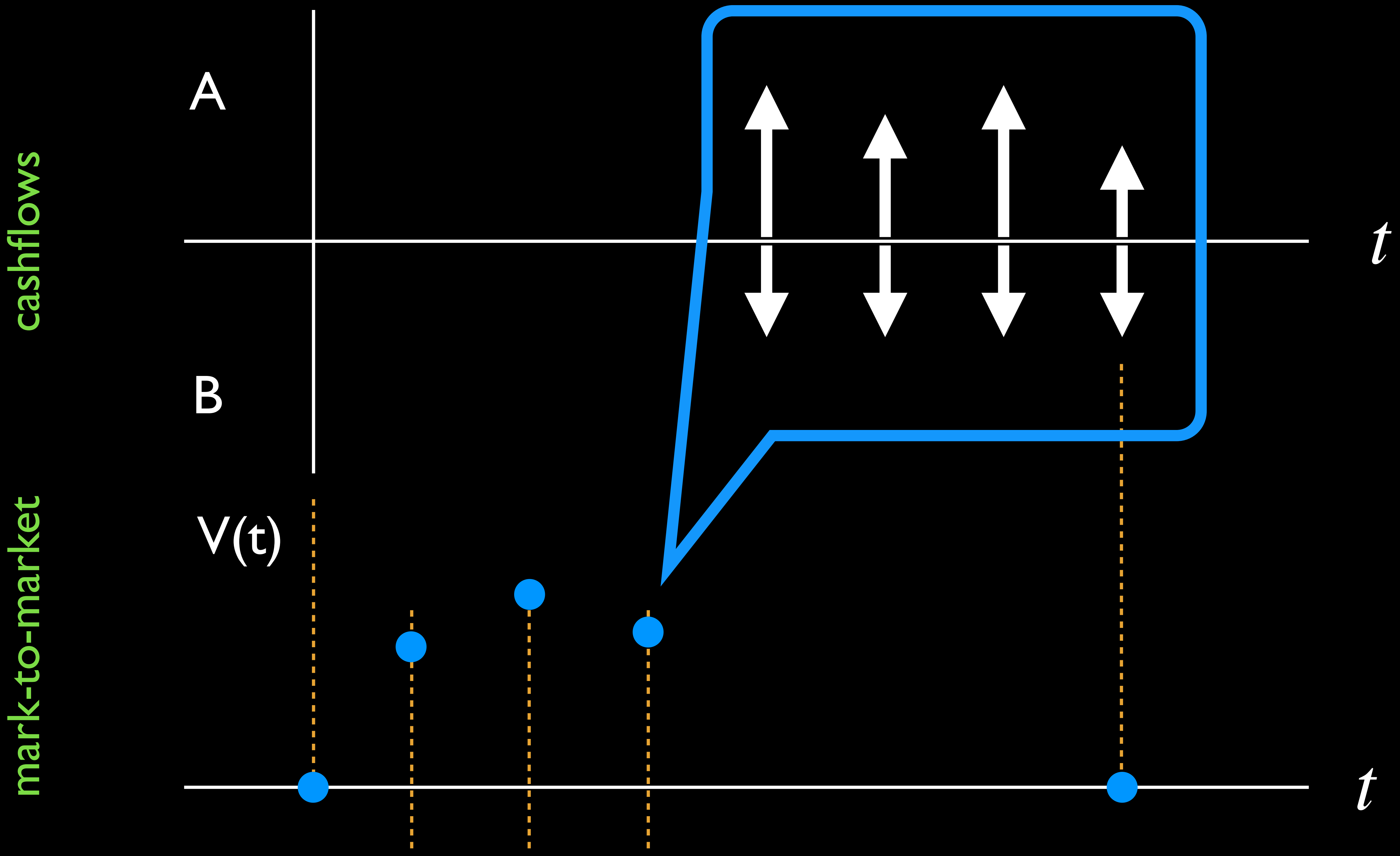
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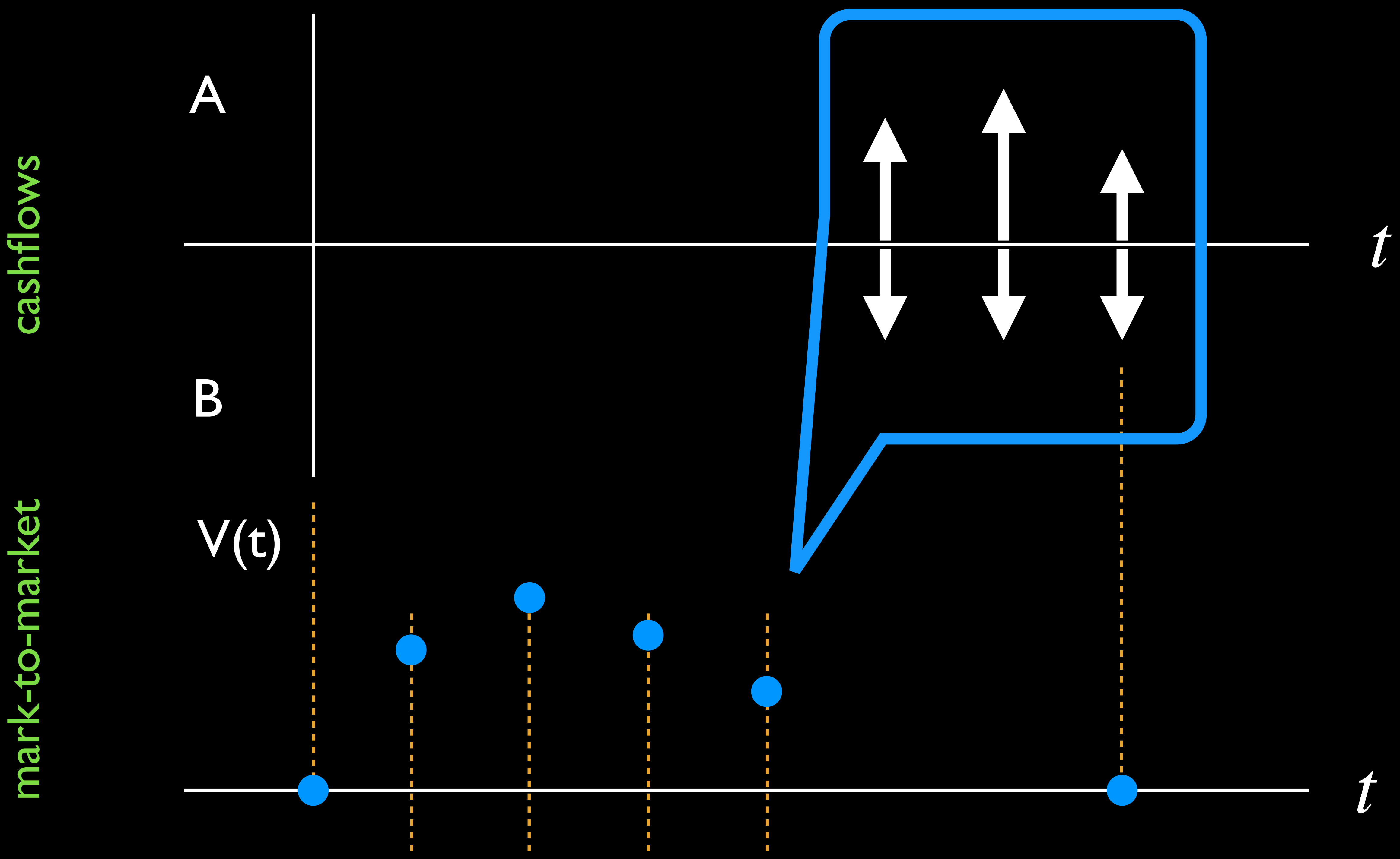
cashflows
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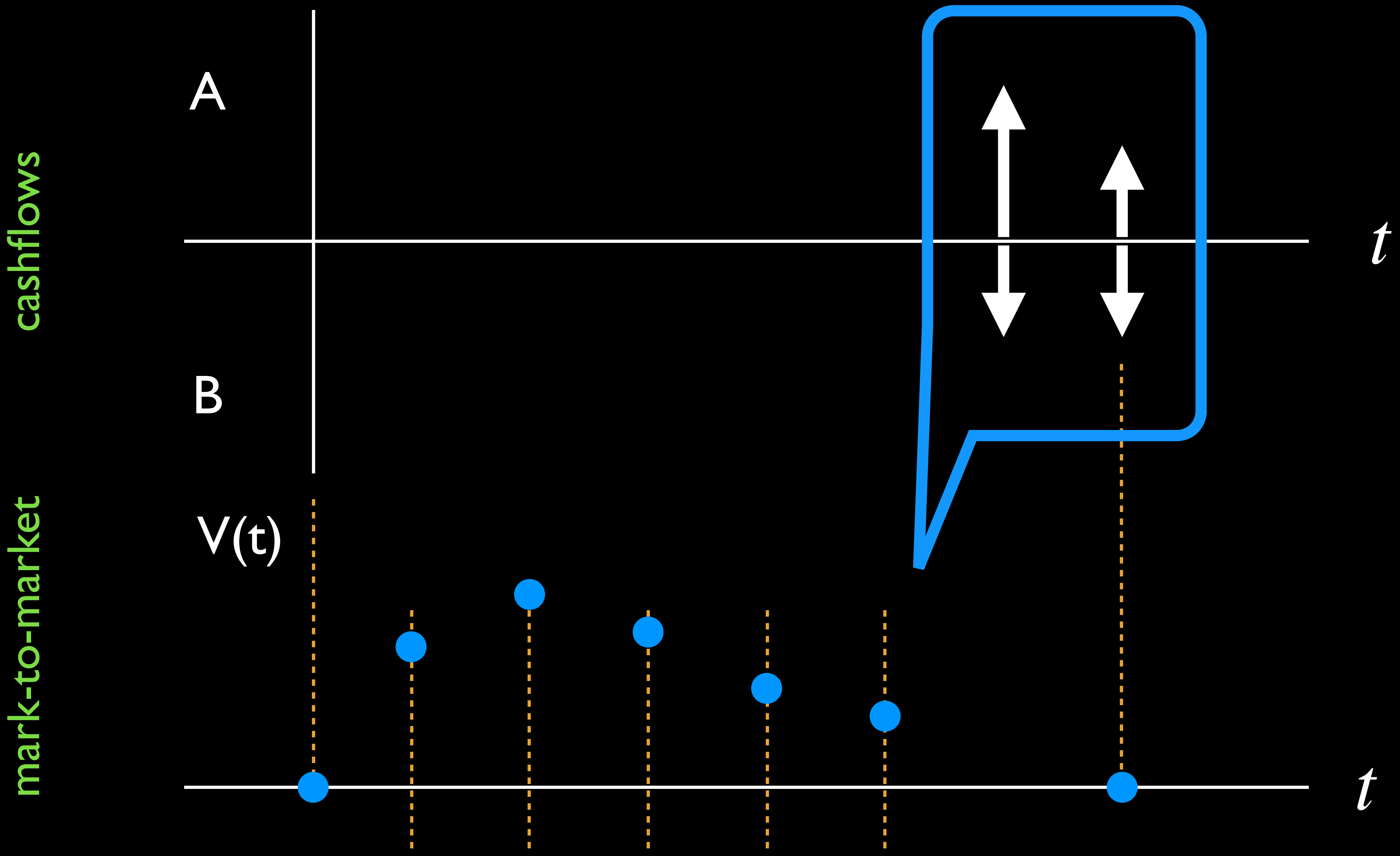
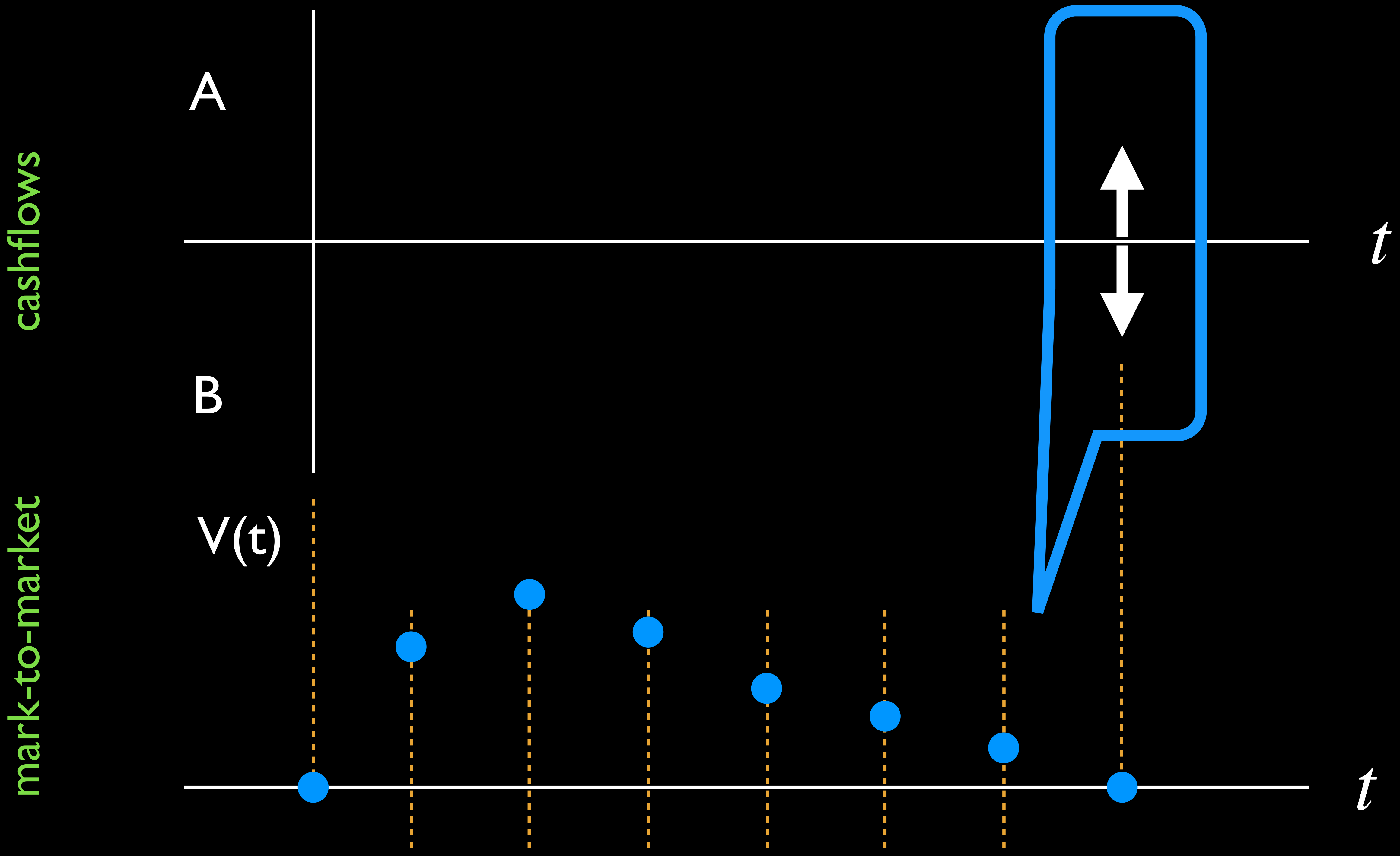
B
t
mark-to-market
V(t)
t


cashflows
A
B
t
mark-to-market
V(t)
t


cashflows
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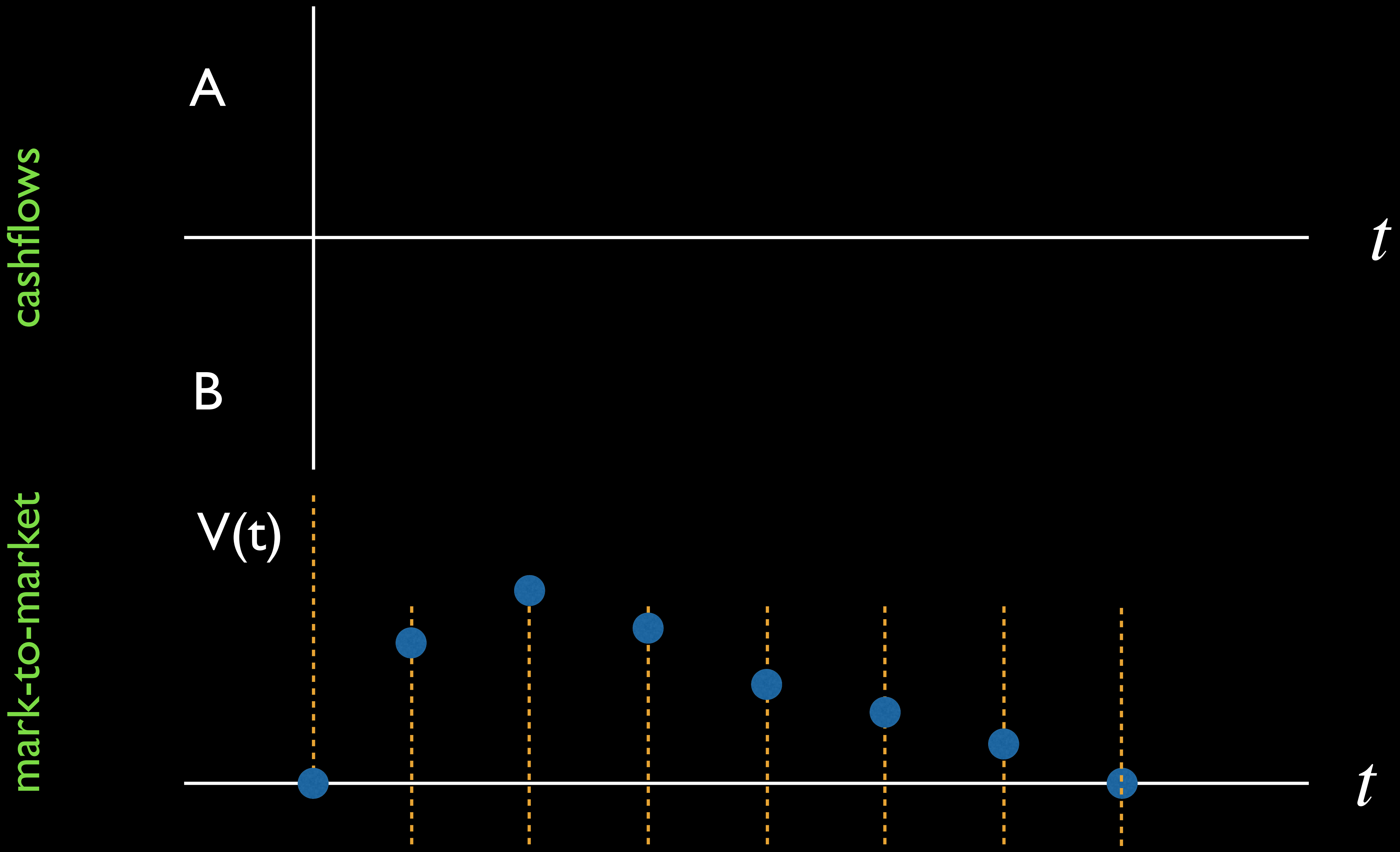
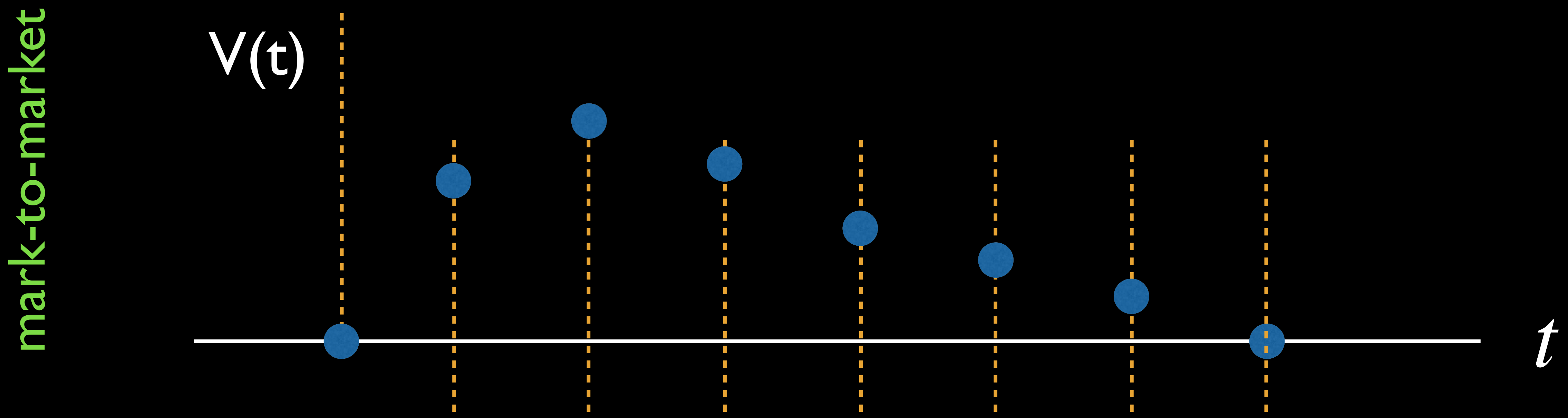
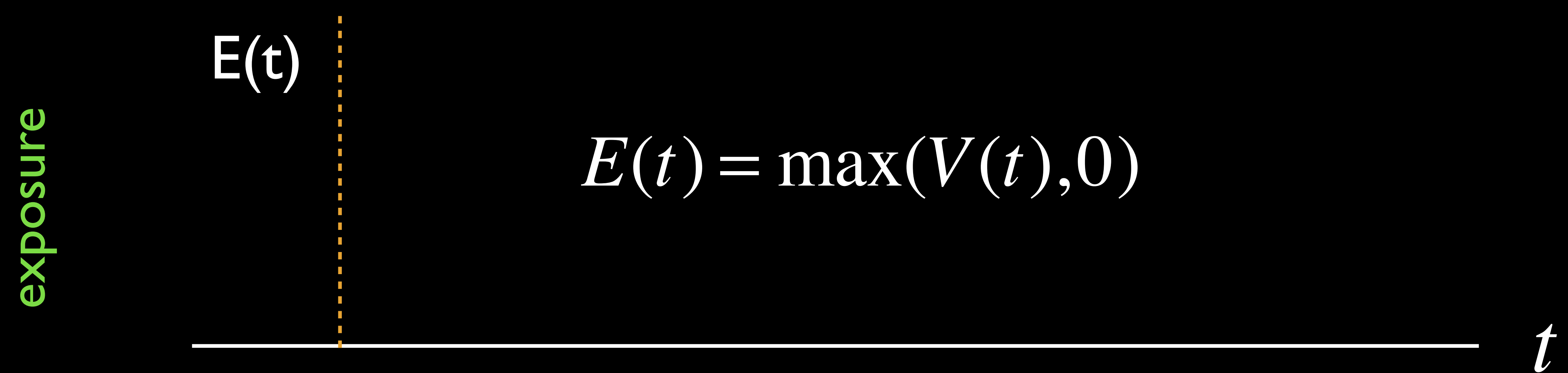
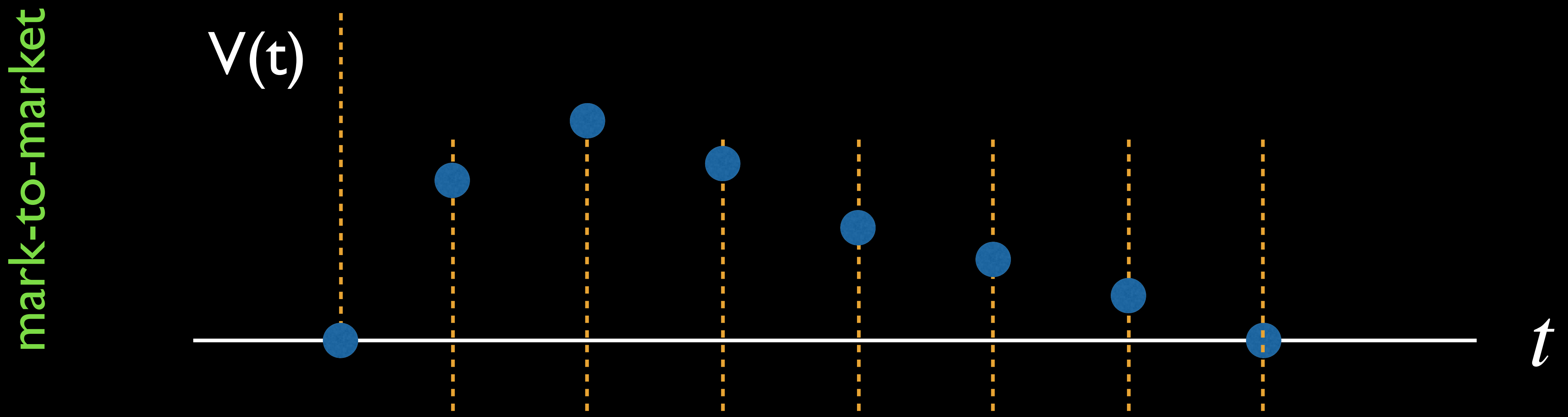
B
t
mark-to-market
V(t)
t

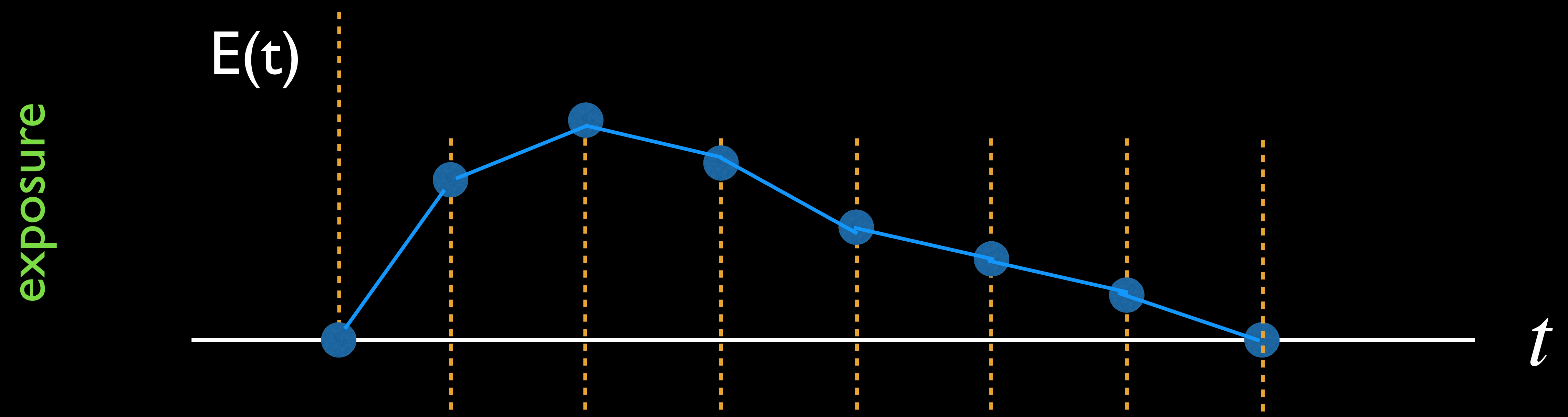

cashflows
A
B
t
mark-to-market
V(t)
t

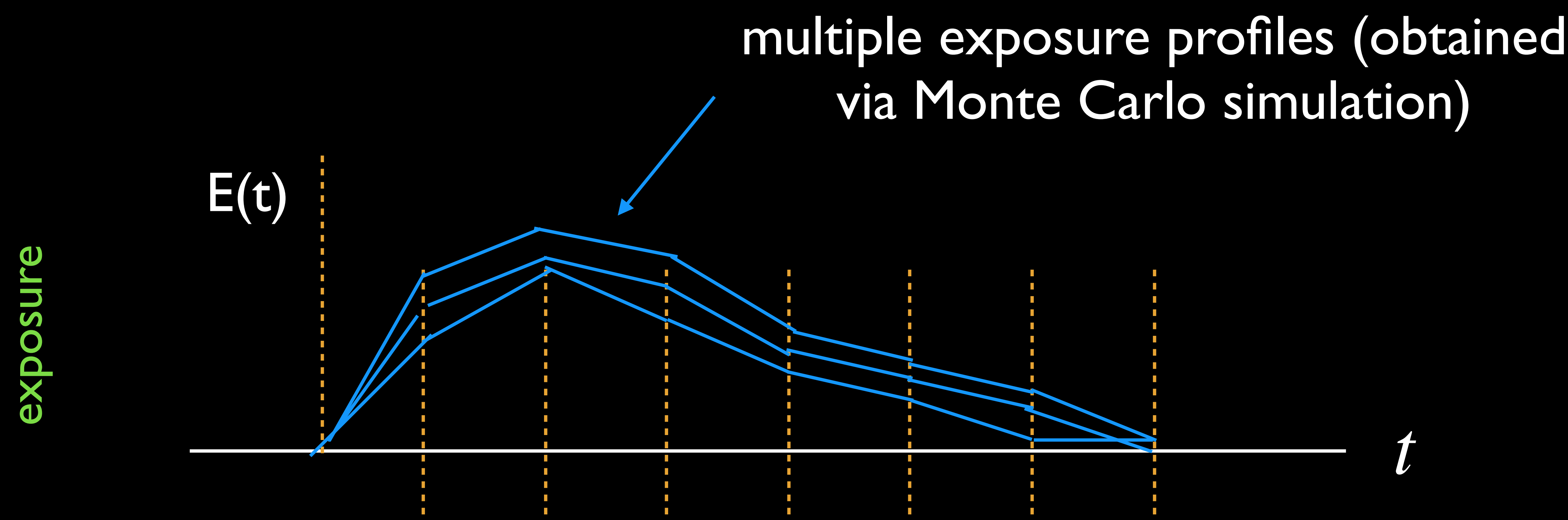


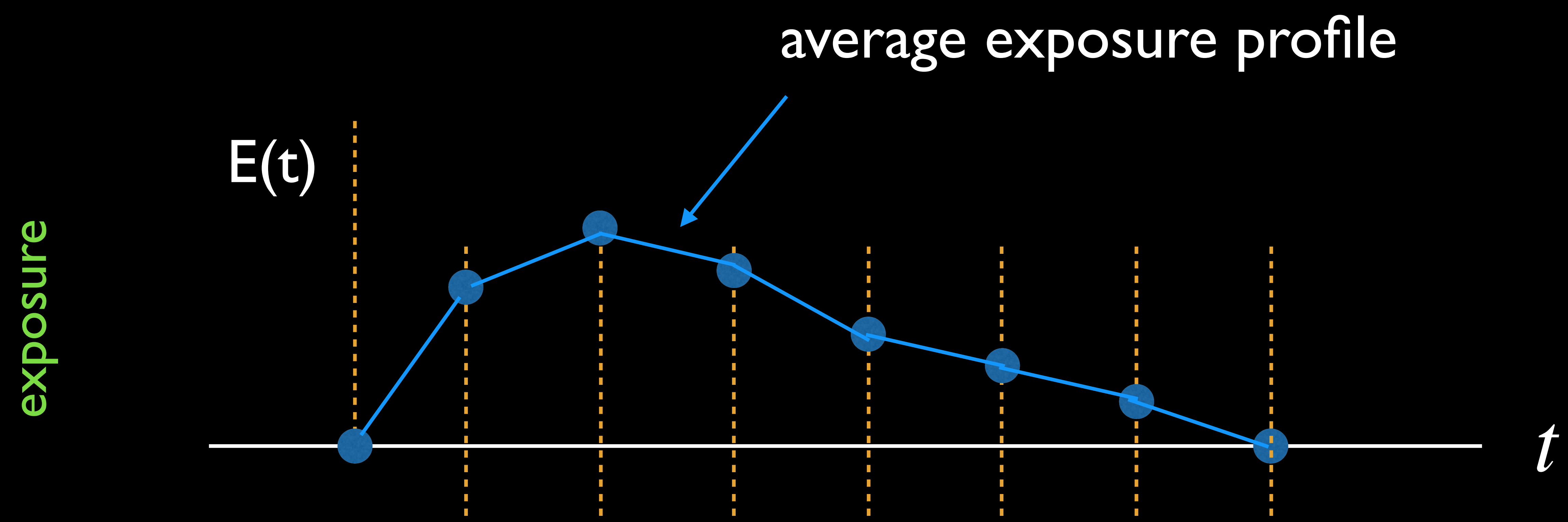


$$E(t) = \max(V(t), 0)$$


mark-to-market
V(t)
t


exposure
E(t)
t


multiple exposure profiles (obtained via Monte Carlo simulation)
E(t)
exposure
t



the average exposure profile quantifies the average value of the losses as a function of time during the life of the instrument …

Part 3 Adjustment

What is the value of the adjustment ?

$CVA =$ a monetary value representing the reduction in value of the instrument due to the default of the counterparty

What is the value of the adjustment ?

expected loss …

$$CVA = E\left[L\right]$$

expected loss …

$$CVA = E\left[L\right]$$

$$L = \boxed{\text{AMOUNT LOST}} \times \boxed{\text{PROBABILITY OF DEFAULT}} \times \boxed{\text{DISCOUNT FACTOR}}$$

present value of probable loss…

future value that could be lost …

with some amount of probability…

discounted to the present …

$$L(\tau) = \boxed{(1-R)E(\tau)} \times \boxed{PD(\tau)} \times \boxed{\mathrm{DF}(\tau)}$$

expected loss …

$$CVA = E\left[L\right]$$

$$L(\tau) = \boxed{(1-R)E(\tau)} \times \boxed{PD(\tau)} \times \boxed{\mathrm{DF}(\tau)}$$

In consequence …

$$CVA = E\left[(1-R)E(t) \times PD(t) \times \mathrm{DF(t)}\right]$$

And to compute the expectation we integrate …

$$CVA = \int_0^T (1 - R)E(t) \times \mathrm{DF(t)} \times dPD(t)$$

the present valued monetary adjustment…

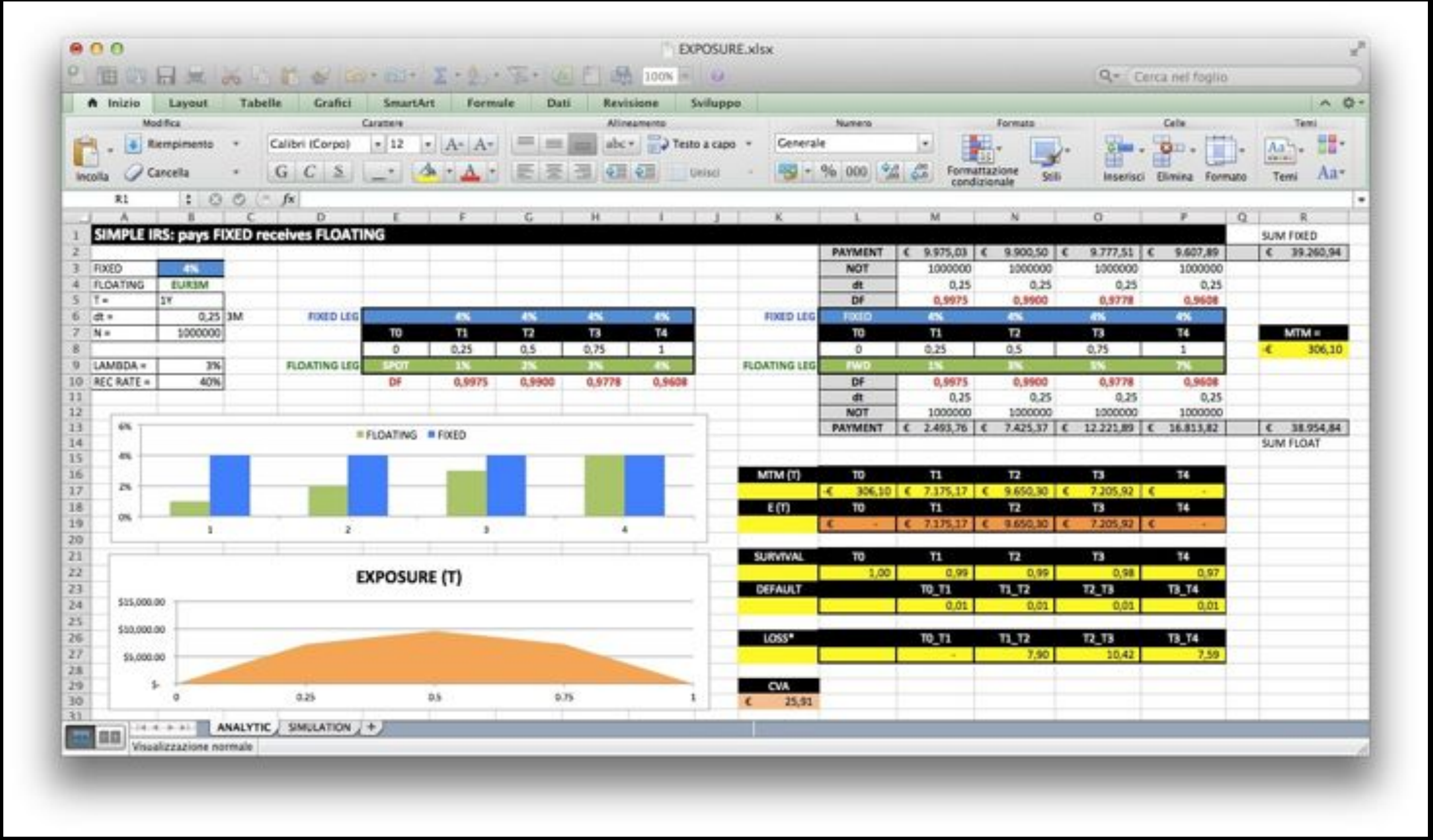
averaged over the life of the instrument …

of the potential losses, considering exposure and recovery…

discounted to the present …

considering the default probability of the counterparty

$$CVA = \int_0^T (1-R)E(t) \times \mathrm{DF(t)} \times dPD(t)$$



Essential Bibliography:

*Jon Gregory, Counterparty Credit Risk and Credit Value Adjustment: A Continuing Challenge for Global Financial Markets, 2nd Edition, Wiley, 2012.

*Zhu, Steven H. and Pykhtin, Michael, A Guide to Modeling Counterparty Credit Risk. GARP Risk Review, July/August 2007. Available at SSRN: http://ssrn.com/abstract=1032522

*Damiano Brigo, Massimo Morini, Andrea Pallavicini, Counterparty Credit Risk, Collateral and Funding: With Pricing Cases For All Asset Classes, Wiley 2013.

*Cesari G et al, Modelling, Pricing, and Hedging Counterparty Credit Exposure, A Technical Guide, Springer Finance, 2010.

Laboratory: CVA of an IRS (Static)

Credit Valuation Adjustment (CVA)

$$CVA = \int_0^T \underbrace{(1-R)}_{LGD} \; \underbrace{EE_t}_{\substack{Expected\\Exposure}} \; \underbrace{DF_t}_{\substack{Discount\\Factor}} \; \underbrace{dPD_t}_{\substack{Default\\\mathrm{Pr}\,obability}}$$

$$EE_t = E^Q\left[\max(MTM_t, 0)\right]$$

$$dPD_t = \Delta PD_t = PD(t_{i-1}, t_i) = P(0, t_{i-1}) - P(0, t_i)$$

STEP 1: INPUTS of the contract

Plain vanilla IRS, pays fixed, receives floating, with quarterly payments. Notional is 1,000,000 USD. The fixed (swap) rate is 39 basis points, i.e. 0.39% pa. The curve of US treasuries (US Treasury Yield curve) is today: 0.04% at 3M, 0.25% at 6M, 0.32% at 9M and 0.40% at 12M.

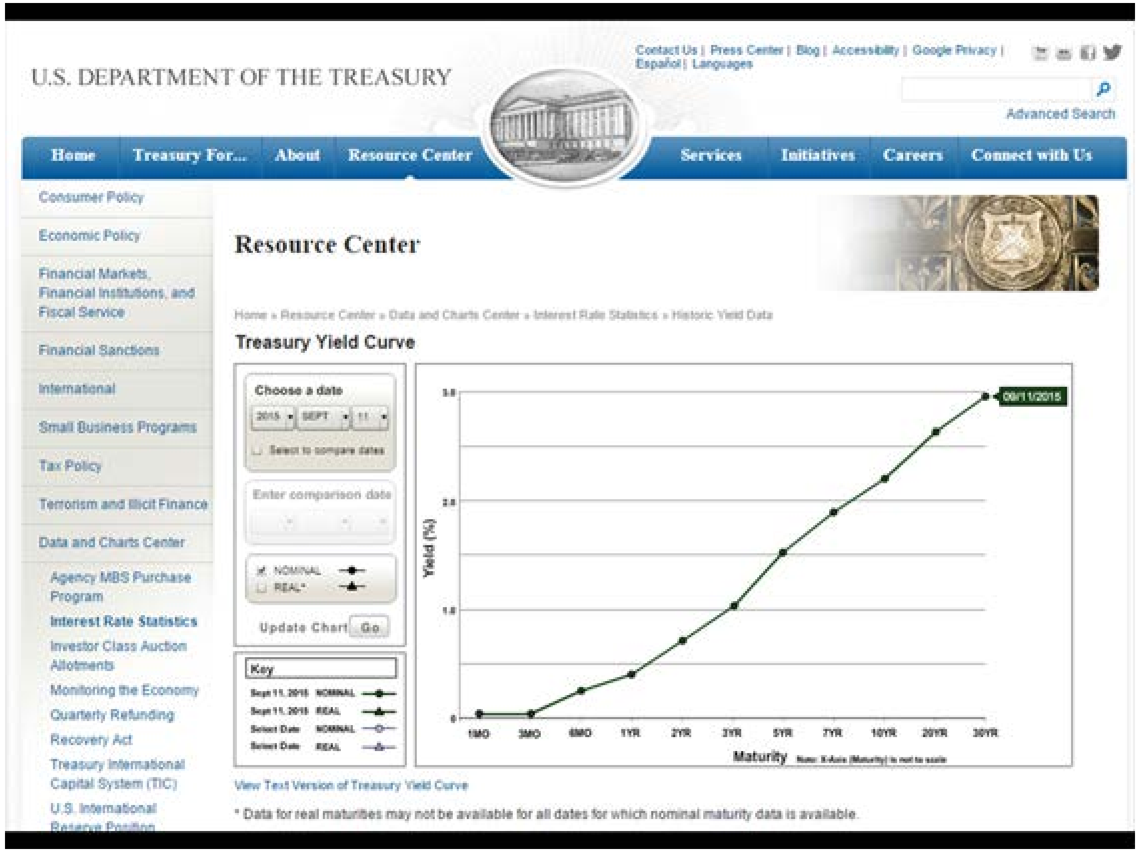
The credit characteristics of the counterparty are:

$$\lambda = 3\% \, pa$$

$$R = 40\%$$

PASO 1: INPUTS	
FIJO	0,39%
VARIABLE	T-YIELD3M
T =	12M
dt =	0,25
N =	$ 1.000.000,00
LAMBDA =	3%
REC RATE =	40%

STEP 2: FORWARD RATES & DISCOUNT FACTORS



Forward rates:

$$L_1 = S_1$$

$$L_i = \frac{S_i T_i - S_{i-1} T_{i-1}}{T_i - T_{i-1}} \qquad \forall i = 2,3,4$$

Discount factors:

$$DF_i = \exp\left(-S_i T_i\right) \qquad \forall i = 1,2,3,4$$

Default probabilities:

$$PD_i = \exp\left(-\lambda T_{i-1}\right) - \exp\left(-\lambda T_i\right) \qquad \forall i = 1,2,3,4$$

http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/Historic-Yield-Data-Visualization.aspx

STEP 2: FORWARD RATES & DISCOUNT FACTORS

PASO 1: INPUTS	
FIJO	0,39%
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LAMBDA =	3%
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PASO 2: FORWARDS + FACTORES DESCUENTO				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
SPOT	0,04%	0,25%	0,32%	0,40%
FWD	0,04%	0,46%	0,46%	0,64%
DF	0,9999	0,9988	0,9976	0,9960
PD	0,75%	0,74%	0,74%	0,73%

Exposure IRS

IRS | Foglio1 (2) | Foglio1 | STATIC | SIMULATION

STEP 3: MARK TO MARKET PRICE OF THE SWAP

Each swap payment:

$$\boxed{N \times \Delta t \times DF_i \times (\mathrm{L}_i - K)}$$

N : notional

Δt : day count fraction

DF_i : discount factor

L_i : forward rate

K : swap rate

MTM as a function of time:

$$MTM(\mathrm{T}_0) = \sum_{i=1}^{4} N \times \Delta t \times DF_i \times (\mathrm{L}_i - K)$$

$$MTM(\mathrm{T}_1) = \sum_{i=2}^{4} N \times \Delta t \times DF_i \times (\mathrm{L}_i - K)$$

$$MTM(\mathrm{T}_2) = \sum_{i=3}^{4} N \times \Delta t \times DF_i \times (\mathrm{L}_i - K)$$

$$MTM(\mathrm{T}_3) = \sum_{i=4}^{4} N \times \Delta t \times DF_i \times (\mathrm{L}_i - K)$$

$$MTM(\mathrm{T}_4) = 0$$

STEP 3: MARK TO MARKET PRICE OF THE SWAP

PASO 1: INPUTS	
FIJO	0,39%
VARIABLE	T-YIELD3M
T =	12M
dt =	0,25
N =	$ 1.000.000,00
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PASO 2: FORWARDS + FACTORES DESCUENTO

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PASO 3: PRECIO MARK-TO-MARKET DEL SWAP

T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
N=	$1.000.000,00	$ 1.000.000,00	$ 1.000.000,00	$1.000.000,00
FWD=	0,04%	0,46%	0,46%	0,64%
FIJO	0,39%	0,39%	0,39%	0,39%
DF=	0,9999	0,9988	0,9976	0,9960
dt=	0,25	0,25	0,25	0,25
PAGO	$ -874,91	$ 174,78	$ 174,58	$ 622,50

MTM(T0)	MTM(T1)	MTM(T2)	MTM(T3)	MTM(T4)
$ 96,95	$ 971,87	$ 797,09	$ 622,50	$0,00

IRS | Foglio1 (2) | Foglio1 | STATIC | SIMULATION

STEP 4: COMPUTE THE EXPOSURE

$$E(\mathrm{T}_i) = \max\left(MTM(\mathrm{T}_i), 0\right) \qquad \forall i = 1, 2, 3, 4$$

PASO 1: INPUTS	
FIJO	0,39%
VARIABLE	T-YIELD3M
T =	12M
dt =	0,25
N =	$ 1.000.000,00
LAMBDA =	3%
REC RATE =	40%

PASO 2: FORWARDS + FACTORES DESCUENTO				
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PASO 3: PRECIO MARK-TO-MARKET DEL SWAP				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
N=	$1.000.000,00	$ 1.000.000,00	$ 1.000.000,00	$1.000.000,00
FWD=	0,04%	0,46%	0,46%	0,64%
FIJO	0,39%	0,39%	0,39%	0,39%
DF=	0,9999	0,9988	0,9976	0,9960
dt=	0,25	0,25	0,25	0,25
PAGO	$ -874,91	$ 174,78	$ 174,58	$ 622,50

MTM(T0)				
$ 96,95	MTM(T1)			
	$ 971,87	MTM(T2)		
		$ 797,09	MTM(T3)	
			$ 622,50	MTM(T4)
				$0,00

PASO 4: CALCULAR EXPOSICION (EXPOSURE)				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
$ 96,95	$ 971,87	$ 797,09	$ 622,50	$ -

IRS | Foglio1 (2) | Foglio1 | STATIC | SIMULATION

STEP 5: COMPUTE THE CVA

$$CVA \approx \sum_{i=1}^{N}(1-R)E\left(\frac{T_{i-1}-T_i}{2}\right)DF\left(\frac{T_{i-1}-T_i}{2}\right)PD\left(\frac{T_{i-1}-T_i}{2}\right)$$

PASO 1: INPUTS

FIJO	0,39%
VARIABLE	T-YIELD3M
T =	12M
dt =	0,25
N =	$ 1.000.000,00
LAMBDA =	3%
REC RATE =	40%

PASO 2: FORWARDS + FACTORES DESCUENTO

T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
SPOT	0,04%	0,25%	0,32%	0,40%
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PASO 3: PRECIO MARK-TO-MARKET DEL SWAP

T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
N=	$1.000.000,00	$ 1.000.000,00	$ 1.000.000,00	$1.000.000,00
FWD=	0,04%	0,46%	0,46%	0,64%
FIJO	0,39%	0,39%	0,39%	0,39%
DF=	0,9999	0,9988	0,9976	0,9960
dt=	0,25	0,25	0,25	0,25
PAGO	$ -874,91	$ 174,78	$ 174,58	$ 622,50

MTM(T0)	MTM(T1)	MTM(T2)	MTM(T3)	MTM(T4)
$ 96,95	$ 971,87	$ 797,09	$ 622,50	$0,00

PASO 4: CALCULAR EXPOSICION (EXPOSURE)

T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
$ 96,95	$ 971,87	$ 797,09	$ 622,50	$ -

PASO 5: CALCULAR CVA

	T0-T1	T1-T2	T2-T3	T3-T4
EXPOSURE	$ 534,41	$ 884,48	$ 709,80	$ 311,25
DF	0,9999	0,9988	0,9976	0,9960
PD	0,75%	0,74%	0,74%	0,73%
(1-R)	60%	60%	60%	60%
CVA_PERIODO	€ 2,40	€ 3,93	€ 3,13	€ 1,36
CVA	€ 10,81			

CVA_PERIODO

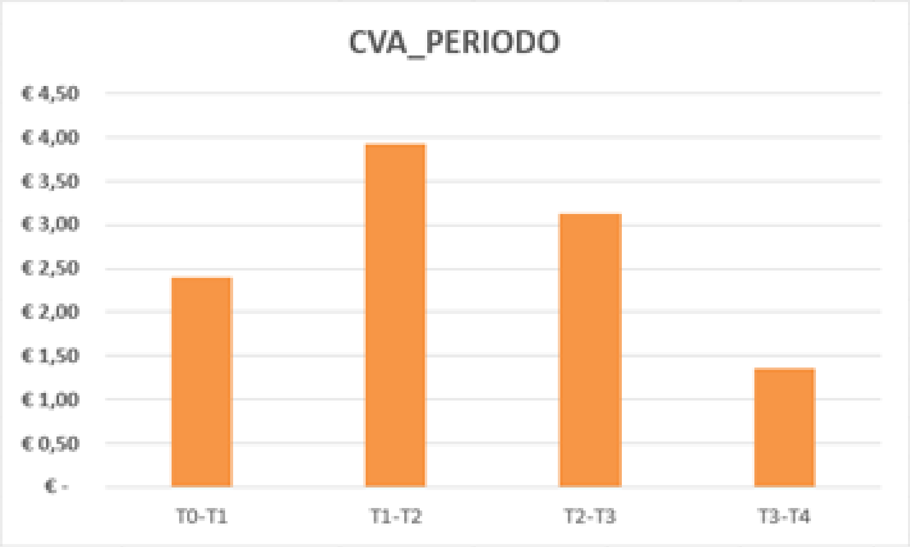
IRS | Foglio1 (2) | Foglio1 | STATIC | SIMULATION

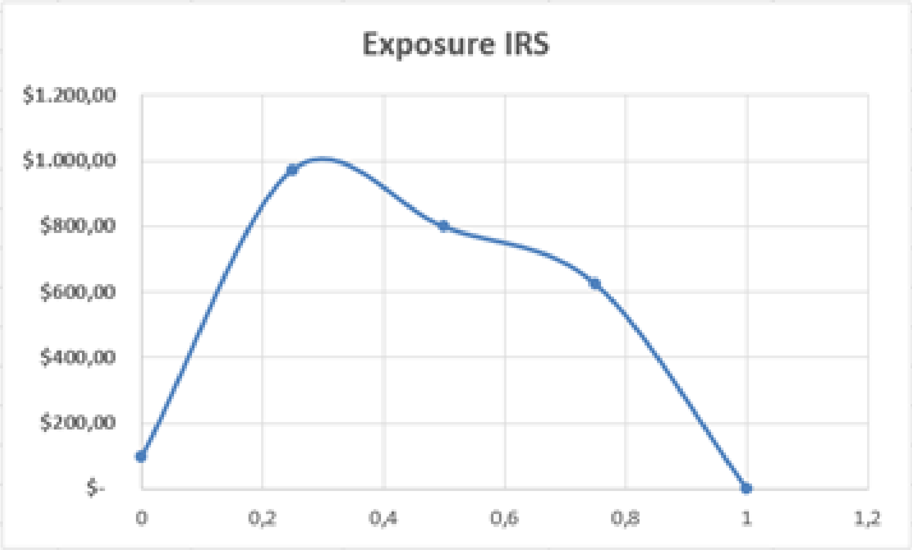
FINAL RESULT

PASO 5: CALCULAR CVA				
	T0-T1	T1-T2	T2-T3	T3-T4
EXPOSURE	$ 534,41	$ 884,48	$ 709,80	$ 311,25
DF	0,9999	0,9988	0,9976	0,9960
PD	0,75%	0,74%	0,74%	0,73%
(1-R)	60%	60%	60%	60%
CVA_PERIODO	€ 2,40	€ 3,93	€ 3,13	€ 1,36
CVA	€ 10,81			

$ 622,50	MTM(T4)
	$0,00

PASO 4: CALCULAR EXPOSICION (EXPOSURE)				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
$ 96,95	$ 971,87	$ 797,09	$ 622,50	$ -





IRS | Foglio1 (2) | Foglio1 | STATIC | SIMULATION

Laboratory: CVA of an IRS (Dynamic)

MONTE CARLO SIMULATION:
GENERATE SIMULATED FORWARD RATES,
SUBSTITUTE HERE SIMULATION 1…

PASO 1: INPUTS	
FIJO	0,39%
VARIABLE	T-YIELD3M
T =	12M
dt =	0,25
N =	$ 1.000.000,00
LAMBDA =	3%
REC RATE =	40%

PASO 2: FORWARDS + FACTORES DESCUENTO				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
SPOT	0,04%	0,25%	0,32%	0,40%
FWD	0,04%	0,46%	0,46%	0,64%
DF	[illegible]	[illegible]	[illegible]	0,9960
PD	0,75%	0,74%	0,74%	0,73%

PASO 3: PRECIO MARK-TO-MARKET DEL SWAP				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
N=	$1.000.000,00	$ 1.000.000,00	$ 1.000.000,00	$1.000.000,00
FWD=	0,04%	0,46%	0,46%	0,64%
FIJO	0,39%	0,39%	0,39%	0,39%
DF=	0,9999	0,9988	0,9976	0,9960
dt=	0,25	0,25	0,25	0,25
PAGO	$ -874,91	$ 174,78	$ 174,58	$ 622,50

MTM(T0)	MTM(T1)	MTM(T2)	MTM(T3)	MTM(T4)
$ 96,95	$ 971,87	$ 797,09	$ 622,50	$0,00

Be careful with discounting…

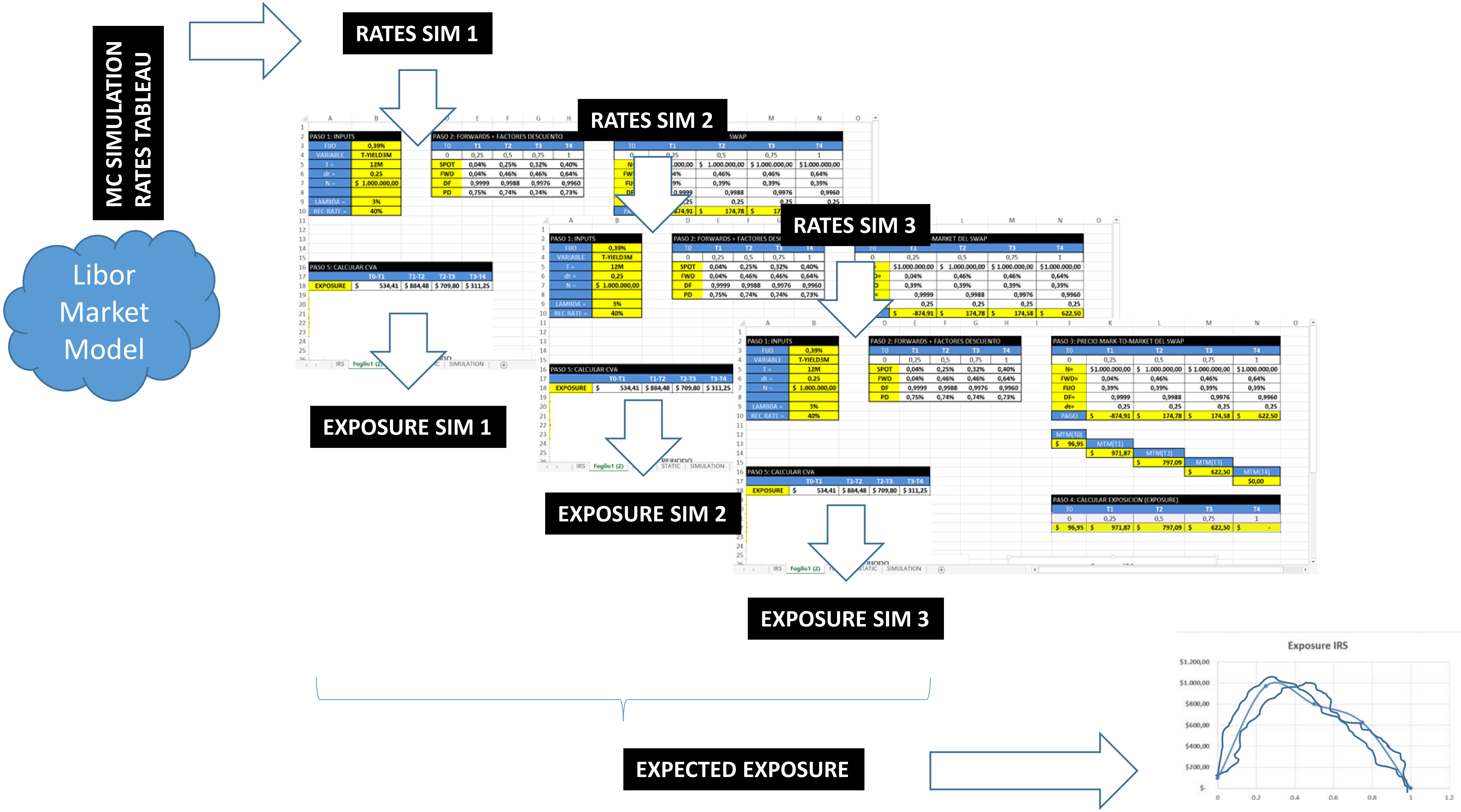
PASO 5: CALCULAR CVA				
	T0-T1	T1-T2	T2-T3	T3-T4
EXPOSURE	$ 534,41	$ 884,48	$ 709,80	$ 311,25

PASO 4: CALCULAR EXPOSICION (EXPOSURE)				
T0	T1	T2	T3	T4
0	0,25	0,5	0,75	1
$ [illegible]	$ [illegible]1,87	$ 797,09	$ 622,50	$ -

OBTAIN EXPOSURE PROFILE
FOR SIMULATION 1…

CVA PERIODO

IRS | Foglio1 (2) | Foglio1 | STATIC | SIMULATION


MC SIMULATION
RATES TABLEAU
Libor
Market
Model
RATES SIM 1
RATES SIM 2
RATES SIM 3
EXPOSURE SIM 1
EXPOSURE SIM 2
EXPOSURE SIM 3
EXPECTED EXPOSURE
Exposure IRS
$1.200,00
$1.000,00
$800,00
$600,00
$400,00
$200,00
$-
0
0.2
0.4
0.6
0.8
1
1.2

The average of all the exposure profiles...

$$EE_t = E^Q\left[E_t\right]$$

EXPECTED EXPOSURE

- **EXPOSURE SIM 1**
- **EXPOSURE SIM 2**
- **EXPOSURE SIM 3**

	A	B	C	D	E
16	PASO 5: CALCULAR CVA				
17		T0-T1	T1-T2	T2-T3	T3-T4
18	EXPOSURE				
19	DF	0,9999	0,9988	0,9976	0,9960
20	PD	0,75%	0,74%	0,74%	0,73%
21	(1-R)	60%	60%	60%	60%
22	CVA_PERIODO	€ 2,40	€ 3,93	€ 3,13	€ 1,36
23	CVA	€ 10,81			
24					

AGGREGATED CVA

$$CVA = \int_0^T \underbrace{(1-R)}_{LGD}\ \underbrace{EE_t}_{\substack{Expected\\Exposure}}\ \underbrace{DF_t}_{\substack{Discount\\Factor}}\ \underbrace{dPD_t}_{\substack{Default\\Probability}}$$

Laboratory: XVA

Bilateral CVA Formula

$$BCVA = CVA + DVA$$

$$CVA: \int_0^T \underbrace{(1-R_C)}_{LGD} \underbrace{EE_t}_{\substack{\text{Expected} \\ \text{Exposure}}} \underbrace{DF_t}_{\substack{\text{Discount} \\ \text{Factor}}} \underbrace{P_I}_{\substack{\text{Survival of} \\ \text{Counterparty } I}} \underbrace{dPD_C}_{\substack{\text{Default of} \\ \text{Counterparty } C}}$$

$$DVA: \int_0^T \underbrace{(1-R_I)}_{LGD} \underbrace{NEE_t}_{\substack{\text{Negative} \\ \text{Expected} \\ \text{Exposure}}} \underbrace{DF_t}_{\substack{\text{Discount} \\ \text{Factor}}} \underbrace{P_C}_{\substack{\text{Survival of} \\ \text{Counterparty } C}} \underbrace{dPD_I}_{\substack{\text{Default of} \\ \text{Counterparty } I}}$$

$$EE_t = E^Q\left[\max(MTM_t, 0)\right] \quad NEE_t = E^Q\left[\min(MTM_t, 0)\right]$$

FVA Formula

$$FVA = -\int_0^T \underbrace{EFV_t}_{\substack{Funding\\ \text{Pr}ofile}} \ \underbrace{FS_t}_{\substack{Forward\\ Funding\\ Spread}} \ \underbrace{P_t}_{\substack{Survival\\ \text{Pr}obability}} \ dt$$

$$EE_t = E^Q\left[\max(MTM_t, 0)\right]$$

$$NEE_t = E^Q\left[\min(MTM_t, 0)\right]$$

$$EFV_t = E^Q\left[MTM_t\right] = EE_t + NEE_t$$

ColVA and KVA Formula

$$ColVA = -\int_0^T \underbrace{ECB_t}_{\substack{Expected\\ Collateral\\ Balance}} \; \underbrace{CS_t}_{\substack{Collateral\\ Spread}} \; \underbrace{P_t}_{\substack{Survival\\ \Pr obability}} \; dt$$

$$KVA = -\int_0^T \underbrace{ECP_t}_{\substack{Discoounted\\ \exp ected\\ capital\\ profile}} \; \underbrace{CC_t}_{\substack{\text{Cos}t\ of\\ Capital}} \; \underbrace{P_t}_{\substack{Survival\\ \Pr obability}} \; dt$$

Appendix: The LMM 1F