CQF Module 5 Assignment

June 2015 Cohort

Instructions

Where asked complete mathematical workings must be provided to obtain maximum credit. Each plot must have a brief explanation. Queries to Richard Diamond at *r.diamond@cqf.com*

Marking Scheme: Q1 30% **Q2** 25% **Q3** 45%

1. You are analyzing a company described by the following market data: the value of the company's equity is \$6 million and the volatility of its equity is 60%. Company's debt of \$10 million matures in 1 year, and the risk free rate is 3%. Use structural model to report:

   (a) The initial firm's assets value of V_0 and volatility σ_V.

   (b) The impact of a decrease in equity volatility σ_E on σ_V. Provide a plot with brief explanation.

   (c) The firm's probability of default at 1 year.

   (d) Use Black and Cox (1976) model with default threshold $K = \$10$ million to estimate the probability of default. Briefly explain the difference with Merton Model PD result.

   Hints: set up a system of equations in Excel/Mathematica and use Solver/alike to obtain the answers. Black and Cox PD can be calculated analytically.

2. A bivariate European binary call pays one unit of currency if both underlying assets are above the strike at maturity. Consider a simplified scenario with risk-neutral rates $r = 0.00$ (zero), time to maturity of a derivative $T = 0.5$ (6M), and two assets with the same current price $S = 110$, strike $E = 120$, correlation $\rho_K = 0.4$ and volatilities $\sigma_1 = 0.2$ and $\sigma_2 = 0.4$.

   Price **a multi-asset binary call** $B(S_1, S_2, t) = e^{-r(T-t)}C(u_1, u_2)$ using the following function for the joint probability:

   $$C(u_1, u_2, \cdots, u_n) = \frac{1}{\alpha} \ln \left[1 + \frac{\prod_{i=1}^{n}(e^{\alpha u_i} - 1)}{(e^{\alpha} - 1)^{n-1}} \right]$$

   Association parameter $-\infty < \alpha < \infty$ is related to Kendall's tau correlation ρ_K as follows:

   $$\rho_K = 1 - \frac{4}{\alpha}\left[D_1(-\alpha) - 1\right] \qquad D_1(-\alpha) = \frac{1}{\alpha}\int_0^{\alpha} \frac{x}{e^x - 1}dx + \frac{\alpha}{2}$$

   You can use Mathematica to find out the exact value of association parameter α. Alternatively, take TSE on $\frac{x}{e^x-1} \approx 1 - \frac{x}{2} + \frac{x^2}{12} - O(x^4)$ and solve analytically.

   Note: complete mathematical workings must be provided even if using software.

Credit Curve

Table 1 shows the term structure of credit spreads for two reference entities at the opposite ends of a credit spectrum: Wells Fargo (WFC) is a highly-rated institution, while Clear Channel Communications (CCMO) is highly leveraged according to Fitch Ratings (18 May 2012).

Maturity	WFC	CCMO	$Z(0;T)$
1Y	50	751	0.97
2Y	77	1164	0.94
3Y	94	1874	0.92
5Y	125	4156	0.86
7Y	133	6083	0.81

Table 1: CDS Market Data

1. Bootstrap implied survival probabilities for WFC bank with recovery rate $RR = 50\%$. Obtain the term structure of hazard rates (non-cumulative) and provide a plot fitting the Exponential PDF $f(t) = \lambda e^{-\lambda t}$ using these discrete hazard rates.

$$\lambda_m = -\frac{1}{\Delta t} \log \frac{P(0, t_m)}{P(0, t_{m-1})}$$

   where λ_m is a hazard rate for year m, $P(0, t_m)$ is a cumulative probability of survival to the end of year m (likewise for $P(0, t_{m-1})$), and $\Delta t = 1$ year.

2. Bootstrap implied survival probabilities for CCMO corporation. Assume $RR = 10\%$.
   Is there an *anomaly* for this highly-leveraged name? Describe any interesting observations.

3. In general, what is the effect of an increase in recovery rate on implied survival probabilities? To answer, provide plots of WFC survival probability term structure for different values of RR with a brief explanation.

Methodology Notes

- Bootstrapping of survival probabilities **must be coded** as a function. Excel Solver must **not** be used for this task. A spreadsheet-only solution has been provided (CDS Lecture) – its resubmission will receive a deduction in marks.

  Before bootstrapping, you have to interpolate because JPM Formulation assumes that premium and default payments are made at the year end.

- For discounting factors, the log-linear interpolation is required. For $\tau_i < \tau < \tau_{i+1}$ and discount factor $Z(0;\tau) = d(\tau)$,

$$\ln d(\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \ln d_{i+1} + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \ln d_i$$

- Credit curve can be fitted by linear interpolation directly,

$$\text{CDS}(\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \text{CDS}_{i+1} + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \text{CDS}_i$$

  Both methods retain the assumption of a piecewise constant variable which overstates the value for the concave curve and understates for the convex curve.