

In class we looked at the Stochastic Differential Equation

$$dS = \mu S dt + \sigma S dX,$$

as a popular model for stock prices S . Using the set of prices in the Excel file, or down loading new ones test the robustness of the assumption that

$$R_i = \frac{\delta S}{S} = \mu \delta t + \sigma \sqrt{\delta t} \phi_i.$$

Here R_i represents the returns over the time-step δt , the $\phi_i \sim N(0, 1)$ and μ & σ are the constant drift and diffusion in turn.

The following experiments may assist when testing the data:

- Examine the robustness of the estimates for μ & σ to the size of δt (e.g., estimate μ & σ using all data, only the R_i for i even/odd, only every second R_i and so on).
- Construct a Quantile-Quantile plot (Q-Q plot)* of the return data (vertical axis) against a normal distribution with the same mean and variance as the data (horizontal axis).
- Compare a suitably scaled histogram of returns with a normal distribution.

***Q-Q Plot**

This plot is made up as follows:

1. Rank the empirical returns in order from smallest to largest, call these y_i with an index i going from 1 to n .
2. From the Normal distribution find the returns x_i such that the cumulative distribution at x_i has value i/n .
3. Now plot each pair (x_i, y_i) .

The better the fit between the two distributions, the closer the line is to straight.