CQF Module 5 Assignment

January 2016 Cohort

Instructions

Where asked complete mathematical workings must be provided to obtain maximum credit. Each plot must have a brief explanation. Queries to Richard Diamond at *r.diamond@cqf.com*

Marking Scheme: Q1 30% **Q2** 25% **Q3** 45%

1. You are analyzing a company with the equity of \$3 million and its volatility σ_E is 50%. Company's debt of \$5 million notional matures in 1 year, and the risk-free rate is 2%. Build a structural model to report:

   (a) Computation of the firm's assets value of V_0 and volatility σ_V.
   Note: set up a system of equations in Excel/Mathematica and use Solver/alike for numerical root-finding.

   (b) Sensitivity of the 1Y PD to the equity volatility input σ_E on the plot.

   Compute the sensitivity for both, Merton PD and Black-Cox PD, for which set $K = \$5$ million and use analytical results from the Lecture.
   Briefly explain the difference in sensitivity *wrt* volatility levels above 60%.

2. A bivariate European binary call pays one unit of currency if both underlying assets are above the strike at maturity, $u_i = \Pr^{\mathbb{Q}}(S_{i,T} > K)$. Assume strike $K = 120$, $T = 0.5$ (6M) risk-neutral rates $r = 0.00$ (zero) for all assets. Consider a simplified scenario: the current prices $S_1 = 90$, $S_2 = 110$, volatilities $\sigma_1 = 0.3$ and $\sigma_2 = 0.5$ and correlation $\rho_K = 0.35$.

   Let's use the Frank Copula, from Archimedean family with known function for the joint cumulative probability, to price **a multi-asset binary call** $B(S_1, S_2, t) = e^{-r(T-t)}C(u_1, u_2)$.

$$C(u_1, u_2, \cdots, u_n) = \frac{1}{\alpha} \ln \left[1 + \frac{\prod_{i=1}^n (e^{\alpha u_i} - 1)}{(e^\alpha - 1)^{n-1}} \right]$$

   Instead of correlation, the copula requires the association parameter $-\infty < \alpha < \infty$, related to Kendall's tau value via the following transformation:

$$\rho_K = 1 - \frac{4}{\alpha} [D_1(-\alpha) - 1] \qquad D_1(-\alpha) = \frac{1}{\alpha} \int_0^\alpha \frac{x}{e^x - 1} dx + \frac{\alpha}{2}$$

   You can use Mathematica to find out the exact value of the parameter α.
   Alternatively, apply TSE on $\frac{x}{e^x-1} \approx 1 - \frac{x}{2} + \frac{x^2}{12} - O(x^4)$ and solve analytically.

   Note: complete mathematical workings must be provided, if using Mathematica the full script must be provided.

Maturity, T	λ not cum.	DF $Z(0;T)$
1Y	0.00995	0.97
2Y	0.02087	0.94
3Y	0.02579	0.92

Maturity	CDS DB EUR
1Y	141.76
2Y	165.36
3Y	188.56
4Y	207.32
5Y	218.38

Table 1: CDS Data (hazard rates on left, market data on right)

Credit Curve

Table 1 shows two small datasets from the credit markets. The set of hazard rates is typical for a highly-rated bank in stable times. The spreads for DB are currently heightened (3 July 2016).

1. Given the term structure of hazard rates and discounting factors, interpolate to quarterly increments $\Delta t = 0.25$ and price the CDS with accruals on the assumption of flat spread for all tenors. $RR = 40\%$. You will have to create PL and DL computation for each quarterly period (on a spreadsheet or as code output) and use Solver to find the spread.

2. Bootstrap implied survival probabilities for DB bank with recovery rate $RR = 40\%$ on assumption that the premium paid annually in arrears (no accruals), default payment made at the end of one year period (no need for quarterly interpolation here). Use continuous DF consistent with the risk-free rate of 0.8%.
   PrSurv bootstrapping from CDS quotes must be coded as a function by you – resubmission of CDS lecture spreadsheet or non-original code will receive a sizeable deduction in marks.

3. Obtain the term structure of hazard rates for DB and plot Exponential *pdf* $f(t) = \lambda e^{-\lambda t}$ as appropriate for piecewise constant lambda. Describe the instability you observe.

$$P(0,T) = \exp\left(-\sum_{t=1}^{T} \lambda_t \Delta t\right) \qquad \lambda_m = -\frac{1}{\Delta t}\log\frac{P(0,t_m)}{P(0,t_{m-1})}$$

where $P(0,T)$ is a cumulative PrSurv to the end of period T, λ_m is a hazard rate for period m.

Interpolation

- For discounting factors, the log-linear interpolation is required. For $\tau_i < \tau < \tau_{i+1}$

  $$\ln \mathrm{DF}(0,\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \ln \mathrm{DF}(0,\tau_{i+1}) + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \ln \mathrm{DF}(0,\tau_i)$$

  the way to read: as $\tau \to \tau_{i+1}$ the weight for $\ln \mathrm{DF}(0,\tau_i)$ goes to zero.

- Credit spreads or equally hazard rates fitted linearly or by the method of your choice.

  $$\mathrm{CDS}(\tau) = \frac{\tau - \tau_i}{\tau_{i+1} - \tau_i} \mathrm{CDS}_{i+1} + \frac{\tau_{i+1} - \tau}{\tau_{i+1} - \tau_i} \mathrm{CDS}_i$$

  The assumption of a piecewise constant variable overstates the value for a concave curve and understates for the convex one.