

Appendix: Why is VaR not necessarily subadditive?

Imagine three bonds, A, B and C whose returns are independent. Each bond has a face value of £1,000, payable at maturity and a likelihood of default, where the loss is the full face value of the bond, of 0.5%. The 99% VaR of each bond, therefore, is £0.

How about the VaR of a portfolio containing A, B and C?

There are three scenarios:

Scenario	Loss	Likelihood
No default	£0	$0.995 \times 0.995 \times 0.995 \approx 98.5\%$
1 default	£1,000	$3 \times 0.005 \times 0.995 \times 0.995 \approx 1.5\%$
2 defaults	£2,000	$3 \times 0.005 \times 0.005 \times 0.995 \approx 0.0075\%$
3 defaults	£3,000	$3 \times 0.005 \times 0.005 \times 0.995 \approx 0.00001\%$

The portfolio VaR at 99% is £1,000, which is greater than the VaR of each bond multiplied by 3.