$$\begin{cases} \dfrac{dS_1}{S_1} = \mu_1 dt + \sigma_1 dX_1 \\ \dfrac{dS_2}{S_2} = \mu_2 dt + \sigma_2 dX_2 \end{cases}$$

$$dX_1\, dX_2 = \rho\, dt$$

dt

$$\int (\ )\, dt + (\ ) dx =$$

Dis$^{\underline{n}}$ of $L(t)$

$$L = Y(1-\theta)$$

$$\Pr(L \le l) = \Pr(Y(1-\theta) \le l)$$

$$= \Pr\left(Y \le \frac{l}{1-\theta}\right)$$

$$= G\left(\frac{l}{1-\theta}\right)$$

pdf of $Z(t)$

$$\frac{G'(y)}{1-\theta}$$

$$G'(y) = \frac{\partial}{\partial y} \Phi\Bigg(\underbrace{\frac{\sqrt{1-\rho}\,\Phi^{-1}(y) - d}{\sqrt{\rho}}}_{a(y)} \Bigg)$$

$$= \frac{\partial}{\partial a} \Phi(a) \, \frac{\partial a}{\partial y}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{a^2}{2}} \sqrt{\frac{1-\rho}{\rho}} \, \frac{\partial}{\partial y} \Phi^{-1}(y)$$

$$x = \Phi^{-1}(y) \Rightarrow y = \Phi(x)$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\left[\Phi^{-1}(y)\right]^2}$$

$$= \cancel{\frac{1}{\sqrt{2\pi}}} e^{-\frac{a^2}{2}} \sqrt{\frac{1-\rho}{\rho}} \cancel{\sqrt{2\pi}} e^{\frac{1}{2}\left[\Phi^{-1}(y)\right]^2}$$

$$= \sqrt{\frac{1-\rho}{\rho}} e^{-\frac{a^2}{2} + \frac{1}{2}\left[\Phi^{-1}(y)\right]^2}$$

$$E\left\{ L(t_j, \ell_1, \ell_2) \right]$$

$$= E\left\{ L(t_j, 0, \ell_2) \right] - E\left\{ L(t_j, 0, \ell_1) \right)$$

Expected Value of Base Tranche Loss

$$E\left[L(t;0,\ell) \right]$$

$$= E\left[\ell \, \mathbb{I}\{L(t) > \ell\} \right] + \qquad 1$$

$$E\left[L(t) \, \mathbb{I}\{L(t) < \ell\} \right] \qquad 2$$

$$① = \ell \, E\left[\mathbb{I}\{ L(t) > \ell \} \right]$$

$$= \ell \Pr\left(L(t) > \ell \right)$$

$$= \ell \left(1 - \Pr\left(L(t) < \ell \right) \right)$$

$$= \ell \left[1 - G\left(\frac{\ell}{1-\theta} \right) \right]$$

$$= \ell \left[1 - \Phi\left(\underbrace{\frac{\sqrt{1-\rho}\, \Phi^{-1}\left(\frac{\ell}{1-\theta}\right) - d}{\sqrt{\rho}}}_{a(\ell)} \right) \right]$$

$$= \ell \, \Phi(-a)$$

$$② = E\left\{ F(t|z)(1-\theta)\, \mathbb{I}\{ F(t|z)(1-\theta) < \ell \} \right\}$$

$$= (1-\theta)\, E\left[\Phi\left(\frac{d-\sqrt{\rho} z}{\sqrt{1-\rho}} \right) \mathbb{I}\{ z > -a \} \right]$$

$$F(t|z) < \frac{\ell}{1-\theta}$$

$$\Phi\left(\frac{d-\sqrt{\rho} z}{\sqrt{1-\rho}} \right) < \frac{\ell}{1-\theta}$$

$$z <$$

$$= (1-\theta) \int_{-a}^{\infty} \Phi\left(\frac{d-\sqrt{\rho} z}{\sqrt{1-\rho}} \right) d\Phi(z)$$

$$= (1-\theta) \int_{-a}^{\infty} \Pr(A < d \mid z)\, d\Phi(z)$$

$$= (1-\theta) \int_{-a}^{\infty} \Pr(A<d, Z=z)\, dz$$

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$d\Phi(z)$$

$$= (1-\theta) \Pr(A<d, Z>-a)$$

$$= f(z)\, dz$$

$$\frac{\Pr(A<d, Z=z)}{f(z)}.$$

$$A = \sqrt{\rho}\, Z + \sqrt{1-\rho}\, \varepsilon$$

$$\tilde{Z} = -Z$$

$$A = -\sqrt{\rho}\, \tilde{Z} + \sqrt{1-\rho}\, \varepsilon$$

$$= (1-\theta) \Pr(A<d, \tilde{Z}<a)$$

$$= (1-\theta)\, \Phi_2(d, a, -\sqrt{\rho})$$