INTEREST RATE MODELING WITH LIBOR MARKET MODEL AND CVA CALCULATION

RAN ZHAO

Certificate in Quantitative Finance

2016

INTEREST RATE MODELING WITH LIBOR MARKET MODEL AND CVA CALCULATION

by

RAN ZHAO

Final report submitted in fulfilment of the requirements for Certificate in Quantitative Finance

July 2016

ORIGINALITY DECLARATION

I, Ran Zhao, declare that the work in this final report was carried out in accordance with the requirements of Certification in Quantitative Finance and that it has not been submitted for any other academic award or publication. Except where indicated by specific reference in the text, the work is the candidate's own work. Work done in collaboration with, or with the assistance of, others, is indicated as such. Any views expressed in the final report are those of the author.

SIGNED: .... DATE: ....

ACKNOWLEDGEMENT

Many thanks to Prof. Donald Knuth for giving us TeX, and Leslie Lamport for LaTeX.

I first learned LaTeX as an undergraduate student in Computer Science at the University of Warwick. Back in Malaysia, I picked it up again while doing my M.Sc. at USM, as part of some productive procrastination (there I've admitted it!!). This coincided with Dr. Dhanesh's and Dr. Azman's efforts in raising awareness about LaTeX at USM during NaCSPC'05 – somehow one thing led to another, and I now conduct trainings and consultatons on LaTeX. ☺

Since then, many friends and fellow LaTeX users have given feedback and helped relayed important updates from IPS to me, to help improve the class and template. It has actually come to a point where you are too numerous to name! Thank you all, as well as everyone who has attended my talks and workshops, used my various templates, downloaded examples from my website (`http://liantze.penguinattack.org/latextypesetting.h`

Hope everyone graduates quickly then!

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Page

LIST OF PLATES

INTEREST RATE MODELING WITH LIBOR MARKET MODEL AND CVA CALCULATION

ABSTRACT

This is the English abstract of a USM thesis. It was prepared with the LaTeX document typesetting system.

CHAPTER 1

INTEREST RATE VOLATILITY AND DERIVATIVES

Before introduction of the market models, short rate models are widely used by practitioners for interest rate derivatives pricing. Examples of short-rate models are Vasicek (1977) model, Cox, Ingersoll, and Ross (1985) model and Hull and White (1990) model. These models establish the instantaneous spot interest rate dynamics, using single or multi-dimensional diffusion process(es). However, the interest rate dynamics from short rate models is not compatible with Black's formula for either swap or swaption. In other words, simplified and inexact assumptions are made on interest rate distribution in short rate models, in order to extrapolate the term structure of rates. This knowledge of term structure is vital to interest rate derivatives pricing. The lack of calibration to the whole forward curve is a trade-off with mimicking the Black-Scholes model for stock option in interest rate option, but brings in market inconsistency.

The LIBOR market model (LMM), instead, is based on the discretization of the yield curve into discrete forward rates. And each of these forward rate represents to the market quote of corresponds Forward Rate Agreement (FRA). More importantly, the LIBOR market model prices caps with Black's cap formula (lognormal forward-LIBOR model, LFM) and prices swaption with Black's swaption formula (lognormal forward-swap model, LSM). That is, the interest rate dynamics from the LMM are consistent with caps and swaptions, which are two most standard and basic interest-rate option on the market.

1.1 LMM Framework

1.1.1 Forward Rate

In standard LMM, we assume that the stochastic differential equation of each n spanning forward rates f_i formulates as

$$\frac{df_i}{f_i} = \mu_i(\mathbf{f},t)dt + \sigma_t(t)d\tilde{W}_i \tag{1.1}$$

where $\mathbb{E}[d\tilde{W}_i d\tilde{W}_j] = \rho_{ij}dt$. The lognormal-type model setup ensures positive forward rates. And $i, j = 1, 2, \ldots, M$. The derivative of Black's formula for caplets is detailed in Appendix

1.1.2 Numeraire and Measure

Consider the forward (adjusted) probability measure Q^i associated with numeraire $P(\cdot, T_i)$ for maturity T_i, where the price of the bond maturity coincides with the forward rate maturity. With simple compounding, it follows

$$df_i P(t,T_i) = [P(t,T_{i-1}) - P(t,T_{i-1})]/\tau_i$$

Note that $f_i P(t,T_i)$ is a tradable asset's price, where the price divides by the numeraire $P(\cdot, T_i)$ is $f_i(t)$ itself. Therefore, $f_i(t)$ follows a martingale under forward measure. Corresponding driftless dynamics for $f_i(t)$ under this measure with respect to Equation 1.1 is

$$\frac{df_i(t)}{f_i(t)} = \sigma_i(t)dW_i(t)$$

When σ is bounded and using Ito's formula, the unique strong solution of the forward rate dynamic is

$$\log f_i(T) = \log f_i(0) - \int_0^T \frac{\sigma_i(t)^2}{2} dt + \int_0^T \sigma_i(t) dW_i(t)$$

The instantaneous volatility term $\sigma_i(t)$ assumes to be piecewise-constant

$$\sigma_i(t) = \sigma_{i,\beta(t)}(t)$$

where in general $\beta(t) = m$ if $T_{m-2} < t \leq T_{m-1}, m \geq 1$.

Under this lognormal assumption, it yields that the dynamics of f_k under forward measure Q^i in three cases $i < k, i = k$ and $i > k$ are

$$\begin{aligned}
i < k, &\quad t \leq T_i : df_k(t) = \sigma_k(t) f_k(t) \textstyle\sum_{j=i+1}^{k} \frac{\rho_{k,j}\tau_j\sigma_j(t)f_j(t)}{1+\tau_j f_j(t)} dt + \sigma_k(t) f_k(t) dW_k(t) \\
i = k, &\quad t \leq T_{k-1} : df_k(t) = \sigma_k(t) f_k(t) dW_k(t) \\
i > k, &\quad t \leq T_{k-1} : df_k(t) = -\sigma_k(t) f_k(t) \textstyle\sum_{j=i+1}^{k} \frac{\rho_{k,j}\tau_j\sigma_j(t)f_j(t)}{1+\tau_j f_j(t)} dt + \sigma_k(t) f_k(t) dW_k(t)
\end{aligned}$$

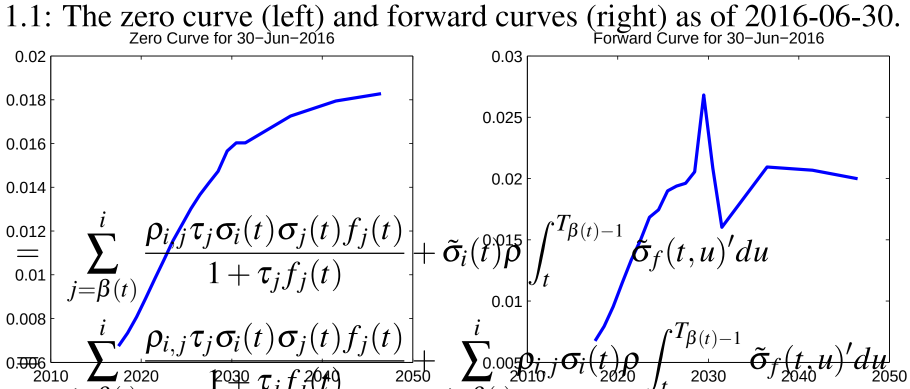
where $W = W^i$ is a Brownian motion under Q^i.

1.1.3 Risk Neutral Dynamics in LMM

According to Brigo and Mercurio (2006), the risk-neutral dynamics of forward LIBOR rates in the LMM is

$$df_i(t) = \tilde{\mu}_i(t) f_i(t) dt + \sigma_i(t) f_i(t) d\tilde{W}_i(t)$$

Figure 1.1: The zero curve (left) and forward curves (right) as of 2016-06-30.



where

$$\tilde{\mu}_i(t) = \sum_{j=\beta(t)}^{i} \frac{\rho_{i,j}\tau_j\sigma_i(t)\sigma_j(t)f_j(t)}{1+\tau_j f_j(t)} + \tilde{\sigma}_i(t)\rho\int_t^{T_{\beta(t)-1}} \tilde{\sigma}_f(t,u)'du$$

$$= \sum_{j=\beta(t)}^{i} \frac{\rho_{i,j}\tau_j\sigma_i(t)\sigma_j(t)f_j(t)}{1+\tau_j f_j(t)} + \sum_{j=\beta(t)}^{i} \rho_{i,j}\sigma_i(t)\rho\int_t^{T_{\beta(t)-1}} \tilde{\sigma}_f(t,u)'du$$

where $\tilde{\sigma}$ is the horizontal M-vector volatility coefficient for the forward rate $f_i(t)$.

1.2 Calibration of LMM to Caps Prices

Figure 1.1 plots the zero and forward curves as of market date 2016-06-30.

Figure 1.2 plots the term structure of the volatility from the calibrated LMM.

Figure 1.3 plots the simulated zero curves and forward curves in one scenario (out of 1000) from the calibrated LMM.

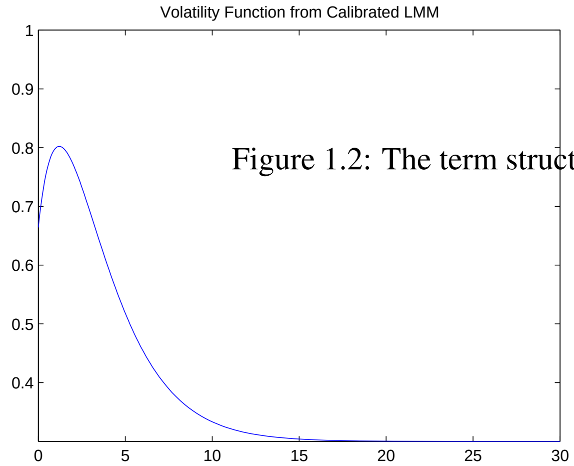


Figure 1.2: The term structure of volatility from calibrated LMM.

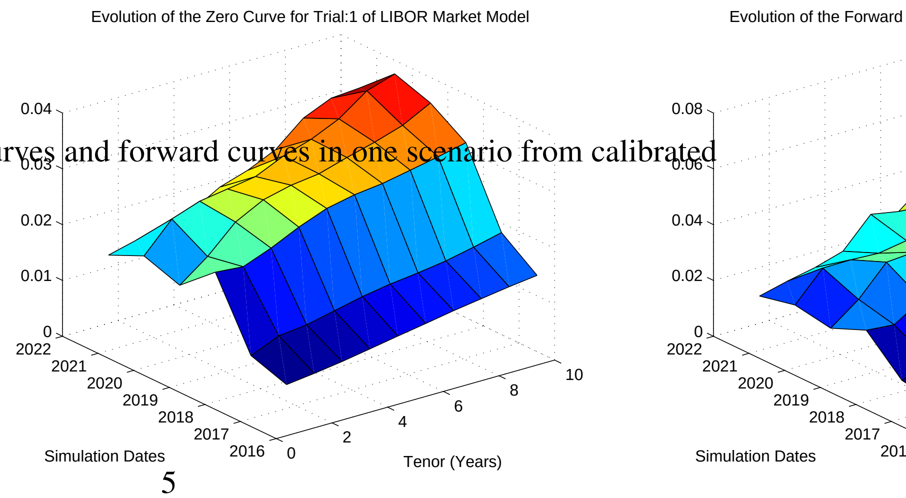


Figure 1.3: Simulated zero curves and forward curves in one scenario from calibrated LMM.

1.3 Test: Reprice Cap and Swaption

APPENDICES

APPENDIX A

DATA USED

Put some test data here.

REFERENCES

Brigo, D., & Mercurio, F. (2006). *Interest rate models - theory and practice. 2nd ed.* Springer Finance.

Cox, J., Ingersoll, J., & Ross, S. (1985). A theory of the term structure of interest rates. *Econometrica*, *53*, 385–407.

Hull, J., & White, A. (1990). Pricing interest-rate derivative securities. *The Review of Financial Studies*, *3*(4), 573–592.

Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, *5*, 177–188.