Certificate in Quantitative Finance (CQF)
Credit Default Swaps *
Solutions

1 CDS: implied survival probabilities

1.1 Part a: implied survival probabilities with term-structure hazard rates

See Table 1.

Maturity	ABC	XYZ
1Y	99.42	49.72
2Y	98.45	30.60
3Y	97.26	18.87
4Y	95.88	14.10
5Y	94.37	11.52

Table 1: Survival probabilities for ABC and XYZ, in percent.

1.2 Part b: implied survival probabilities with flat hazard rates

See Table 2.

1.3 Part c: implied survival probabilities for various recovery rates

See Table 3.

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Maturity	ABC	XYZ
1Y	98.87	58.06
2Y	97.76	33.71
3Y	96.66	19.58
4Y	95.57	11.37
5Y	94.49	6.60

Table 2: Survival probabilities for ABC and XYZ, in percent.

Maturity	$R = 20\%$	$R = 50\%$	$R = 65\%$
1Y	99.64	99.42	99.18
2Y	99.03	98.45	97.80
3Y	98.28	97.26	96.12
4Y	97.40	95.88	94.17
5Y	96.44	94.37	92.06

Table 3: Survival probabilities for ABC for various recovery rate assumptions.

2 Expected Default Times

2.1 Part a

The expected default time can be calculated using integration by parts

$$E[\tau] = \lambda \int_0^\infty s \exp(-\lambda s)\, ds = \left[s e^{-\lambda s} \right]_0^\infty + \int_0^\infty e^{-\lambda s} ds = \frac{1}{\lambda}$$

2.2 Part b

The expected variance of the default time is

$$E[\tau^2] - E[\tau]^2 = \int_0^\infty s^2 \lambda \exp(-\lambda s)\, ds - \frac{1}{\lambda^2} = \left(\frac{2}{\lambda^2} - \frac{1}{\lambda^2} \right) = \frac{1}{\lambda^2}$$

2.3 Part c

With $\lambda = 1\%$ the expected default time is 100 years and its variance is 10,000 years.

3 The Credit Triangle

This problem is solved by assuming a continuous approximation to the pricing of a CDS.

The premium leg (PL) is

$$PL(0,T) = S\int_0^T Z(0,T)P(0,t)dt$$

where $(0,t)$ is the survival probability aqs seen from time zero.
The default leg (DL) is

$$DL(0,T) = (1-R)\int_0^T D(0,T)(-dP(0,t)dt)$$

with $D(0,T)$ the discount factor for time T.
Since $dP(0,t) = -\lambda(t)P(0,t)dt$ this can be written as

$$DL(0,T) = (1-R)\int_0^T D(0,t)\lambda(t)P(0,t)dt$$

and with a constant hazard rate this becomes

$$DL(0,T) = (1-R)\lambda\int_0^T D(0,t)P(0,t)dt$$

The value of the spread S which makes the protection and the premium legs equal is given by

$$S = (1-R)\lambda$$

4 Upfront Credit Default Swap

The upfront CDS replaces the premium leg of a CDS with a single payment of $U(0)$ at the initiation of the contract. The two legs of the contract are therefore the payment of the upfront value and the protection (default) leg. We can therefore determine $U(0)$ by setting the net value of the CDS contract equal to zero at initiation. The value of the contract, from the point of view of a protection buyer who has paid the upfront at time $t = 0$ to buy protection to time T, is

$$(1-R)\int_0^T D(0,s)(-dP(0,s)) - U(0) = 0$$

such that

$$U(0) = (1-R)\int_0^T D(0,s)(-dP(0,s))$$

Once the upfront payment has been made, it goes into the cash account of the protection seller. The mark-to-market value of the contract for the protection buyer at a later time t is given by

$$MTM(t) = (1 - R)\int_0^T D(0,s)(-dP(0,s))$$

which is simply the value of the protection (default) leg of the standard CDS.