CQF Module 2 Exercise Solution

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A. Optimal Portfolio Allocations

1

$$\begin{aligned} \underset{X}{\text{minimize}} \quad & \text{trace}(X) \\ \text{subject to} \quad & X_{ij} = M_{ij},\ (i,j) \in \Omega, \\ & X \succeq 0. \end{aligned}$$

2.a
2.b
3.a
3.b

B. Value at Risk on FTSE 100

1

2.a
2.b
2.c
2.d
3.a
3.b
4
5.a
5.b
5.c

C. Stochastic Calculus

1

2
3
4.a Starting with the lower triangular matrix A, we have

$$\Sigma = AA' = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1-\rho^2}\sigma_2 \end{pmatrix} \begin{pmatrix} \sigma_1 & \rho\sigma_2 \\ 0 & \sqrt{1-\rho^2}\sigma_2 \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

which yields the original covariance matrix.

4.b Given $Y = AX$,

$$\begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix} = \begin{pmatrix} \sigma_1 & 0 \\ \rho\sigma_2 & \sqrt{1-\rho^2}\sigma_2 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \begin{pmatrix} \sigma_1 X_1 \\ \rho\sigma_2 X_1 + \sqrt{1-\rho^2}\sigma_2 X_2 \end{pmatrix}$$

4.c