Certificate in Quantitative Finance (CQF)

Credit Default Swaps *

Problem Sheet

1 CDS: implied survival probabilities

The accompanying table (Table 1) shows the term structure of CDS premiums for two hypotetical reference entities at opposite ends of the credit spectrum: ABC Bank, which is assumed to be a highly rated institution, and XYZ Corporation, which is likely to default in the near future.

Maturity	ABC	XYZ	$Z(t,T)$
1Y	29	9100	0.9803
2Y	39	7800	0.9514
3Y	46	7400	0.9159
4Y	52	6900	0.8756
5Y	57	6500	0.8328

Table 1: Market data. Spreads for ABC and XYZ are in basis points and $Z(t,T)$ is the appropriate discount factor to be used.

For simplicity lets assume that the premium in both agreements are paid once a year. As can be seen in the table, while the five year CDS premium for ABC Bank is under 60 basis points, that for XYZ Corporation is assumed to be 6500 basis points. If you plot the premiums against maturity, you will note that there is a pronounced negative slope of the XYZ Corp. curve which is typical of companies perceived to have a high likelihood of default.

Tasks:

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(a) Compute the term-structure of the implied survival probabilities for both ABC Bank and XYZ Corporation. Use a piece-wise constant assumption. Assume recovery rates of 50% for ABC Bank and 10% for XYZ Corp.

(b) Assume now a flat CDS curve. Compute the 5Y implied survival probabilities for both ABC Bank and XYZ Corporation. Use again recovery rates of 50% for ABC Bank and 10% for XYZ Corp.

(c) Finally, compute the implied survival probabilities for 1Y, 2Y, 3Y, 4Y and 5Y for ABC Bank using recovery rates of 20%, 50% and 65%. What is the effect of changing the recovery rates on the implied survival probabilities?

2 Expected Default Times

Assuming a constant hazard rate λ the price of a credit risky zero coupon bond is given by

$$\bar{B}(0,T) = B(0,T)\exp(-\lambda T)$$

The price of \$1 paid at the time of default becomes

$$D(0,T) = \lambda \int_0^T Z(0,T)\exp(-\lambda t)\,dt$$

and the default time density is given by

$$\Pr(t < \tau \leq t + dt) = \lambda \exp(-\lambda t)\,dt.$$

Tasks:

In this context,
(a) derive the expression for the expected default time,
(b) derive the expression for the variance of the expected default time,
(c) assuming a risky bond with a hazard rate of $\lambda = 1\%$, compute the expected default time and its variance.

Hint: Remember that the expected value of a continuous random variable X is given by

$$E[X] = \int_{-\infty}^{+\infty} sf(s)ds$$

and its variance by

$$E[X^2] - E[X]^2.$$

3 The Credit Triangle

In the context of credit default swaps, the relationship

$$S = \lambda(1 - R)$$

between the credit spread (S), a constant hazard rate (λ) and the recovery rate (R) is called the *credit triangle.* This relationship states is that the required continuously-paid spread compensation for taking on a credit loss of $(1 - R)$ equals the hazard rate λ times $(1 - R)$. This expression, even though approximate, is very useful in practice and is used as a quick way to relate spreads, default probabilities and recovery rates.

Task: Derive the credit triangle formula.

Hint: Consider a simplified version of the CDS contract, in which you have a continuous stream of premium payments until maturity T or default, whichever occurs first, and also a payment of $(1 - R)$ at the time of default if default occurs before maturity T. The value of the spread S is set so that the net present value of the the contract equals zero at inception.

In this continuous context, the premium leg (PL) will be based on the fact that between time t and $t + dt$ we have a payment of $S \times dt$ provided the credit has not defaulted. Discounting this payment and integrating over the lifetime of the contract gives

$$PL(0,T) = S \int_0^T Z(0,T)Q(0,t)dt$$

where $Z(0,T)$ is a risk-free zero-coupon bond and $Q(0,t)$ is the survival probability. There is a similar continuous expression for the default leg. Use the resulting expressions to obtain your result.

4 Upfront Credit Default Swap

Traditionally most CDS are traded as a fixed running spread paid throughout the life of the contract. Recently the market has turned towards upfront CDS, where in addition to a (different) fixed running spread there is an immediate (upfront) payment when the deal is entered. This event is referred in the literature as the *Big Bang* :

> "The market for credit-default swaps on corporate and sovereign debt goes through a major overhaul on Wednesday, with changes that will standardize the terms of many of these insurance-like contracts and make them more similar to the bonds they are tied to.

> Named the "Big Bang Protocol" by the International Swaps and Derivatives Association, the raft of reforms will streamline the way the swaps are traded and how they would be settled if bonds or loans default. The adjustments are designed to help facilitate centralized clearing of the swaps, which are used by banks and money managers to hedge their portfolios or to make bets on the performance of companies and countries."

"For Credit-Default Swaps, Today Comes the Fix-It", S Ng and E Barrett, Wall Street Journal, April 8, 2009.

In this new formulation, instead of choosing the spread to equate the value of the contract legs to the protection buyer and seller, the spread is fixed at the same level for all contracts and the upfront is chosen as an add-on at the initial time to match again the legs. The recent suggestions in ISDA use just one of two running spreads, 100 bps for investment grade CDS and 500 bps for high yield CDS. The recovery is also restricted similarly to be either 40% or 20%. The upfront payment can be negative or positive, based on where the corresponding fair spread would be with respect to the fixed spread and on possible recovery differences.

Task: Develop a modified version of the standard credit default swap pricing formula to take into account the upfront spread.

Hint: Make some field research on the websites of the ISDA (www.isda.org and www.cdsmodel.com). See also: Beumee, Johan G. B., Brigo, Damiano, Schiemert, Daniel and Stoyle, Gareth,Charting a Course Through the CDS Big Bang(April 7, 2009). Available at SSRN: http://ssrn.com/abstract=1374407