CQF Value at Risk

Exercises

1. Consider a position of £5 million in a single asset X with daily volatility of 1%. What are the annualised and 10-day standard deviations? Using the Normal factor calculate 99%/10day VaR in money terms.

2. Now, consider a portfolio of two assets X and Y, £100,000 investment each. The daily volatilities of both assets are 1% and correlation between their returns is $\rho_{XY} = 0.3$. Calculate 99%/5day Analytical VaR (in money terms) for this portfolio.

3. Assume that P&L of an investment portfolio is a random variable that follows Normal distribution $X \sim N(\mu, \sigma^2)$. Use the definition of *VaR as a percentile* to derive analytical expression for VaR calculation.

4. Assume 'elliptical markets': asset returns are Normally distributed or close. What percentage of returns are outside two standard deviations from the mean? Consider the left tail.

   Within that tail, what is the mean of standardised returns – that is, what is an average tail loss? Provide analytical solutions for abstract μ, σ using a simplifying assumption of Standard Normal Distribution.

   PDF for Normal Distribution $N(\mu, \sigma^2)$ is $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$.

5. Recall the example of three bonds A,B and C from the Market Risk Measurement (Value at Risk) lecture: each bond has a face value of £1,000 payable at maturity and the independent probability of default 0.5%, when the loss is the full face value.

   (a) Calculate the Expected Shortfall (within the 1% tail) of bonds A, B and C.

   (b) Calculate the Expected Shortfall (within the 1% tail) of a portfolio equally invested in bonds A, B and C.

   (c) Compare results from (a) and (b) to conclude whether ES is *sub-additive* in this case.

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6. VaR calculation for a portfolio of derivatives often breaks down the contribution to P&L from each Delta, Gamma and Vega Greek. Consider a formula used in Analytical VaR calculation to determine the contribution of cross-asset movements (correlation)

$$\text{Factor} \times \sqrt{\delta t} \sqrt{\sum_{j=1}^{N} \sum_{i=1}^{N} \rho_{ij}\, \sigma_i \sigma_j\, \Delta_i S_i\, \Delta_j S_j}$$

where Delta is used to approximate the change in value over δt. What are the key assumptions of this calculation?

7. What are the two main numerical methods used for the Empirical VaR estimation? What are their drawbacks?