Experiments

Early 1970's

100 —60%→ 101

100 —40%→ 99

 I calculate 0.5

 He thinks 0.6

0.55

Profit 0.05

Expected return $\frac{0.6-0.55}{0.55} \simeq 9\%$

0.54 0.53 0.52 ...... 0.5

cash

$$
\begin{tikzcd}[row sep=small]
 & -\frac{99}{2} \\
-\frac{99}{2} \arrow[ur, no head] \arrow[dr, no head] & \\
 & -\frac{99}{2}
\end{tikzcd}
$$

stock

$$
\begin{tikzcd}[row sep=small]
 & \frac{101}{2} \\
\frac{100}{2} \arrow[ur, no head] \arrow[dr, no head] & \\
 & \frac{99}{2}
\end{tikzcd}
$$

$$
\begin{tikzcd}[row sep=small]
 & 1 \\
0.5 \arrow[ur, no head] \arrow[dr, no head] & \\
 & 0
\end{tikzcd}
$$

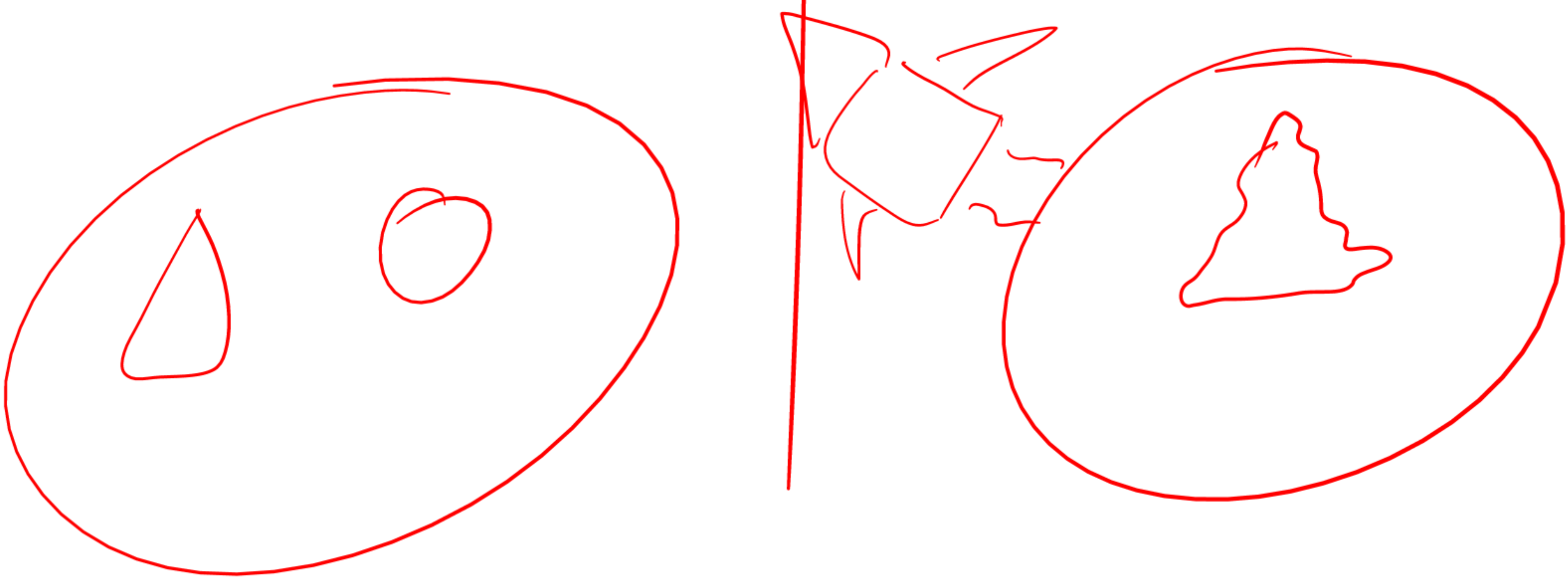
Replication

risk neutral world

- No Data
- abstract probability
- no concept of risk

Real world

- Risk averse
- Data



$$\begin{tikzcd}[column sep=large]
 & 101 \\
100 \arrow[ur, "p'"] \arrow[dr, "1-p'"'] & \\
 & 99
\end{tikzcd}$$

$$101p' + (1-p')99 = 100$$

$$\boxed{p' = 0.5}$$

$$\begin{tikzcd}[column sep=large]
 & 1 \\
V \arrow[ur, "p'"] \arrow[dr, "1-p'"'] & \\
 & 0
\end{tikzcd}$$

$$V = 0.5 \times 1 + 0.5 \times 0$$

$$\boxed{V = 0.5}$$

risk neutral pricing

$$\begin{array}{ccc} & & 103 \\ & \nearrow & \\ 100 & & \\ & \searrow & \\ & & 98 \end{array}$$

$p' =$ risk neutral probability

$$\begin{array}{ccc} & & 103 \\ & \overset{p'}{\nearrow} & \\ 100 & & \\ & \underset{1-p'}{\searrow} & \\ & & 98 \end{array}$$

$$103p' + 98(1-p') = 100$$

$$p' = 0.4$$

$$\begin{array}{ccc} & & 3 \\ & \overset{p'}{\nearrow} & \\ V & & \\ & \underset{1-p'}{\searrow} & \\ & & 0 \end{array}$$

$$3p' + 0(1-p') = V$$

$$3 \times 0.4 + 0 \times 0.6 = V$$

$$\boxed{v = 1.2}$$

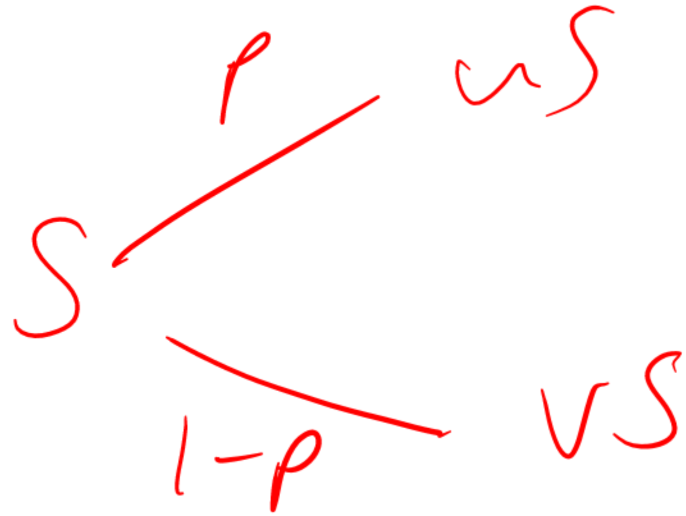
Delta Hedging

Risk Neutral pricing.

Lognormal

Binomial

$$dS = \mu S\,dt + \sigma S\,dX$$



mean change in asset price

$$E[dS] = \mu S\,dt = p\,uS + (1-p)\,vS - S$$

variance

$$\sigma^2 S^2\,dt = E[X^2] - [E(X)]^2$$

$$
\begin{tikzcd}[column sep=large, row sep=small]
 & uS \\
S \arrow[ur, dash] \arrow[dr, dash] & \\
 & vS
\end{tikzcd}
\qquad
\begin{tikzcd}[column sep=large, row sep=small]
 & V^+ - \Delta uS \\
V - \Delta S \arrow[ur, dash] \arrow[dr, dash] & \\
 & V^- - \Delta vS
\end{tikzcd}
$$

*

1 Hedging

$$V^+ - \Delta uS = V^- - \Delta vS$$

$$\Delta = \frac{V^+ - V^-}{(u-v)S}$$

② value at expiry

$$v^+ - \frac{v^+ - v^-}{(u-v)\cancel{S}} \cdot u\cancel{S}$$

$$= v^+ - u\frac{(v^+ - v^-)}{u-v}$$

③ Present value and equate

$$V - \Delta S = \frac{1}{1+r\delta t}\left(V^+ - \frac{u(V^+ - V^-)}{u-v}\right)$$

$$V = \Delta S + $$

$$\boxed{V = \frac{V^+ - V^-}{u-v} + \frac{1}{1+r\delta t}\left(\frac{uV^- - vV^+}{u-v}\right)}$$