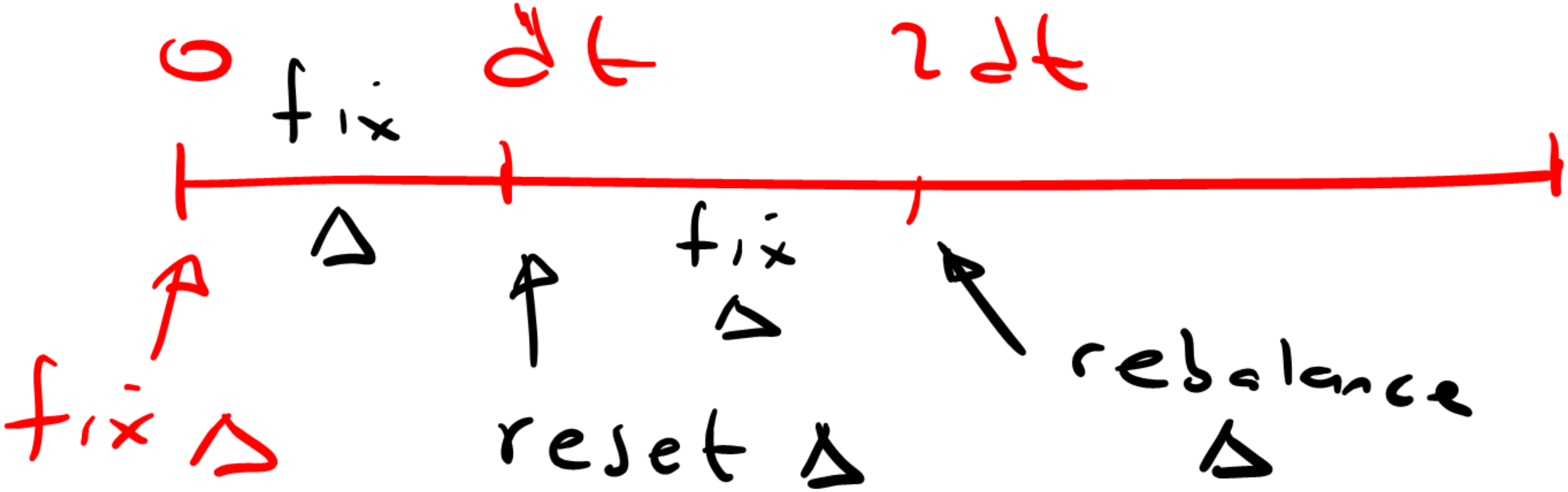
$$\sigma_t^2 \longrightarrow \frac{1}{T-t}\int_t^T \sigma_u^2 \, du$$

$$r_t \longrightarrow \frac{1}{T-t}\int_t^T r_u \, du$$



Call option   +ve   Correlatio

$C(S,t)$ $S\uparrow$ $C\uparrow$

$S\downarrow$ $C\downarrow$

Put option

-ve correl

$P(S,t)$ $\uparrow$ as $S\downarrow$

$\downarrow$ as S P

Call

1) Final Condition (Payoff)

$$C(S,T) = \max(S - E, 0)$$

2) B.C.s

a) $S \to 0 \qquad C \to 0$

b) $S \to \infty \qquad C \sim S$

Put

1) Payoff

$$P(S,T) = \max(E - S, 0)$$

2) B.C.s

a) $S \to \infty \qquad P \to 0$

b) $S \to 0$

$$\underbrace{\cancel{C}}_{\to 0} - P = \underbrace{\cancel{S}}_{\to 0} - E e^{-r(T-t)}$$

$$P = E e^{-r(T-t)}$$

Solving the B.S.E

1. Use transf^ns B.S.E ⟶ 1d heat eq^n

2. Use similarity reduction to solve the heat eq^n

3. unwind steps performed in 1

$$\frac{\partial w}{\partial t} = c^2 \frac{\partial^2 w}{\partial x^2}$$

$$\frac{\partial w}{\partial t} = c^2 \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)$$

$$= \cdots + \frac{\partial^2 w}{\partial t^2}$$

$$= c^2 \nabla^2 w$$

$$V = S e^{-D\tau} N(d_1) - E e^{-r\tau} N(d_2)$$

$$\rightarrow E = S e^{(r-D)\tau}$$

$$V = S e^{-D\tau} \left(N(d_1) - N(d_2) \right)$$

$$N(x) = \frac{1}{2} + \frac{1}{\sqrt{2\pi}} \left(x - \frac{x^3}{6} + \frac{x^5}{40} + \cdots \right)$$

$$C = S\,N(d_1) - E\,e^{-r(T-t)}\,N(d_2)$$

$$\frac{\partial C}{\partial S} = N(d_1) + S\,\frac{\partial N(d_1)}{\partial S}$$

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}s^2}\,ds \qquad S\,\frac{\partial}{\partial d_1}\;\frac{\partial d_1}{\partial S}$$

$$\frac{dN}{dx} = \frac{1}{\sqrt{2\pi}}\,e^{-\frac{1}{2}x^2}$$

$$C - P = S - e^{-r(T-t)}$$

$$\frac{\partial}{\partial S}: \quad \frac{\partial C}{\partial S} - \frac{\partial P}{\partial S} = 1$$

$$\Delta_c - \Delta_p = 1$$

$$\frac{\partial}{\partial S}: \quad \Gamma_c = \Gamma_p$$

$$\Theta + \frac{1}{2}\sigma^2 S^2 \Gamma + rS\Delta = rV$$

$\upsilon = \text{vega}$

$$\frac{\partial}{\partial \sigma}\left[\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV\right] = 0$$

$$\frac{\partial}{\partial t}\underbrace{\left(\frac{\partial V}{\partial \sigma}\right)}_{\upsilon} + \sigma S^2 \Gamma + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2}{\partial S^2}\frac{\partial V}{\partial \sigma}$$

$$+ rS\frac{\partial}{\partial S}\frac{\partial V}{\partial \sigma} - r\frac{\partial V}{\partial \sigma} = 0$$

$$\frac{\partial \upsilon}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 \upsilon}{\partial S^2} + rS\frac{\partial \upsilon}{\partial S} - r\upsilon = -\sigma S^2 \Gamma$$