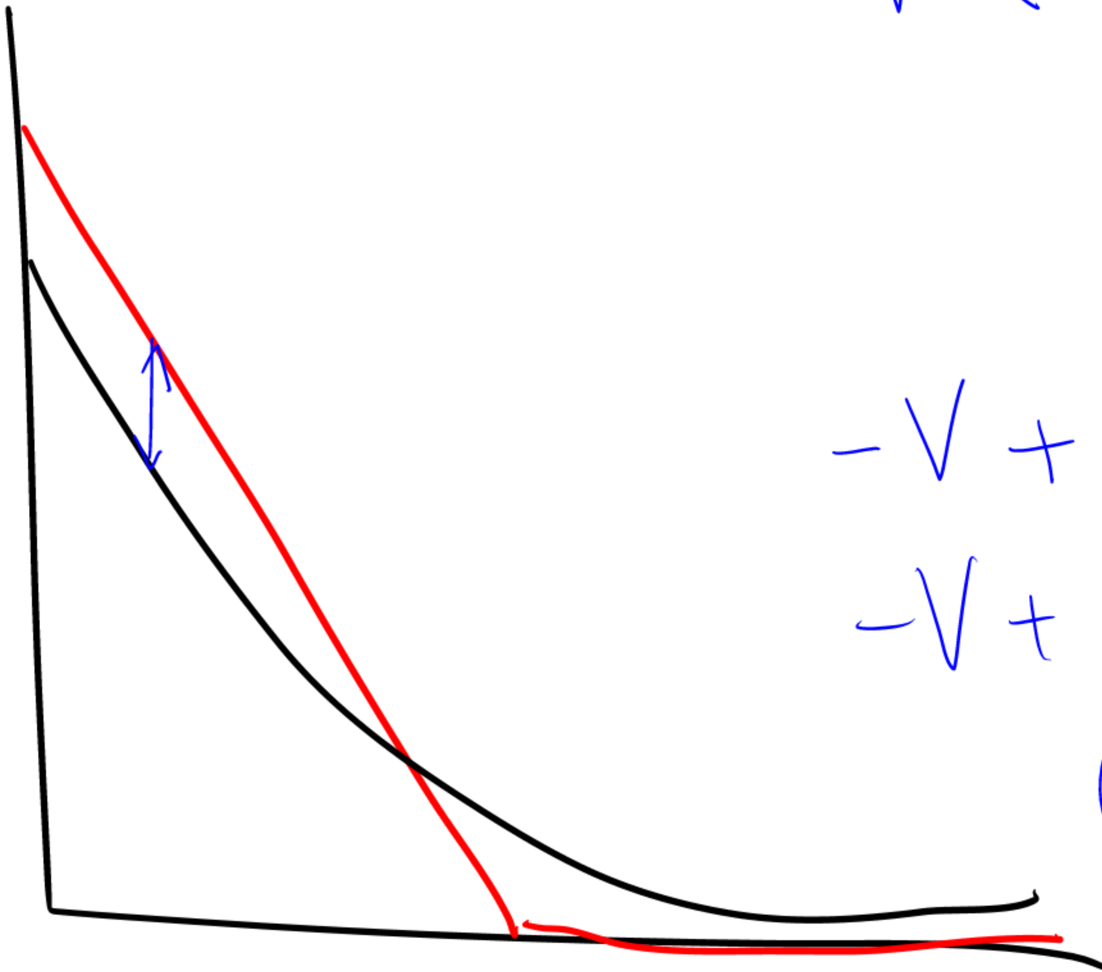


$V <$ Payoff

$\epsilon - s$

$-V + \epsilon - s$

$-V + (\epsilon - s)$

$(\epsilon - s) - V > 0$

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} = r\left(V - S\frac{\partial V}{\partial S}\right)$$

$$1 \qquad\qquad\qquad\qquad 2$$

$$1 > 2 \implies \text{Arb}$$

$$1 < 2$$

Perpetual American Option

$V = V(S)$ $\quad ax^2y'' + bxy' + cy = 0$

B.S.E $\quad \frac{1}{2}\sigma^2 S^2 \frac{d^2V}{dS^2} + rS\frac{dV}{dS} - rV = 0$

$\exists$ sol^n $V = S^\lambda$

A.E. $\quad \frac{1}{2}\sigma^2\lambda^2 + (r - \frac{1}{2}\sigma^2)\lambda - r = 0$

$$\lambda^2 + \left(\frac{2r}{\sigma^2} - 1\right)\lambda - \frac{2r}{\sigma^2} = 0$$

$$(\lambda - 1)\left(\lambda + \frac{2r}{\sigma^2}\right) = 0$$

$$\lambda = 1,\ -\frac{2r}{\sigma^2}$$

$$V(s) = As + Bs^{-2r/\sigma^2}$$

$$\lim_{s \to \infty} V(s) = 0$$

$V(S^*) = E - S^*$ $\quad S^*$ - const

① $V \to 0$ as $S \to \infty$

Gs $V(S) = AS + BS^{-2r/\sigma^2}$

$A = 0 \Rightarrow V(S) = BS^{-2r/\sigma^2}$

② $V(S^*) = E - S^* = BS^{*-2r/\sigma^2}$

$$D = \frac{E - S^*}{S^{*^{-2r/\sigma^2}}}$$

$$\therefore \quad V(S) = \left(E - S^*\right)\left(\frac{S}{S^*}\right)^{-2r/\sigma^2}$$

where $\bar{\alpha} = \frac{-2r}{\sigma^2}$

Summary

Put ① $\Pi \cdot s \cdot \epsilon$

$+$ ② $V(s^*) = \epsilon - s^*$

③ $\left.\frac{dV}{ds}\right|_{s=s^*} = -1$

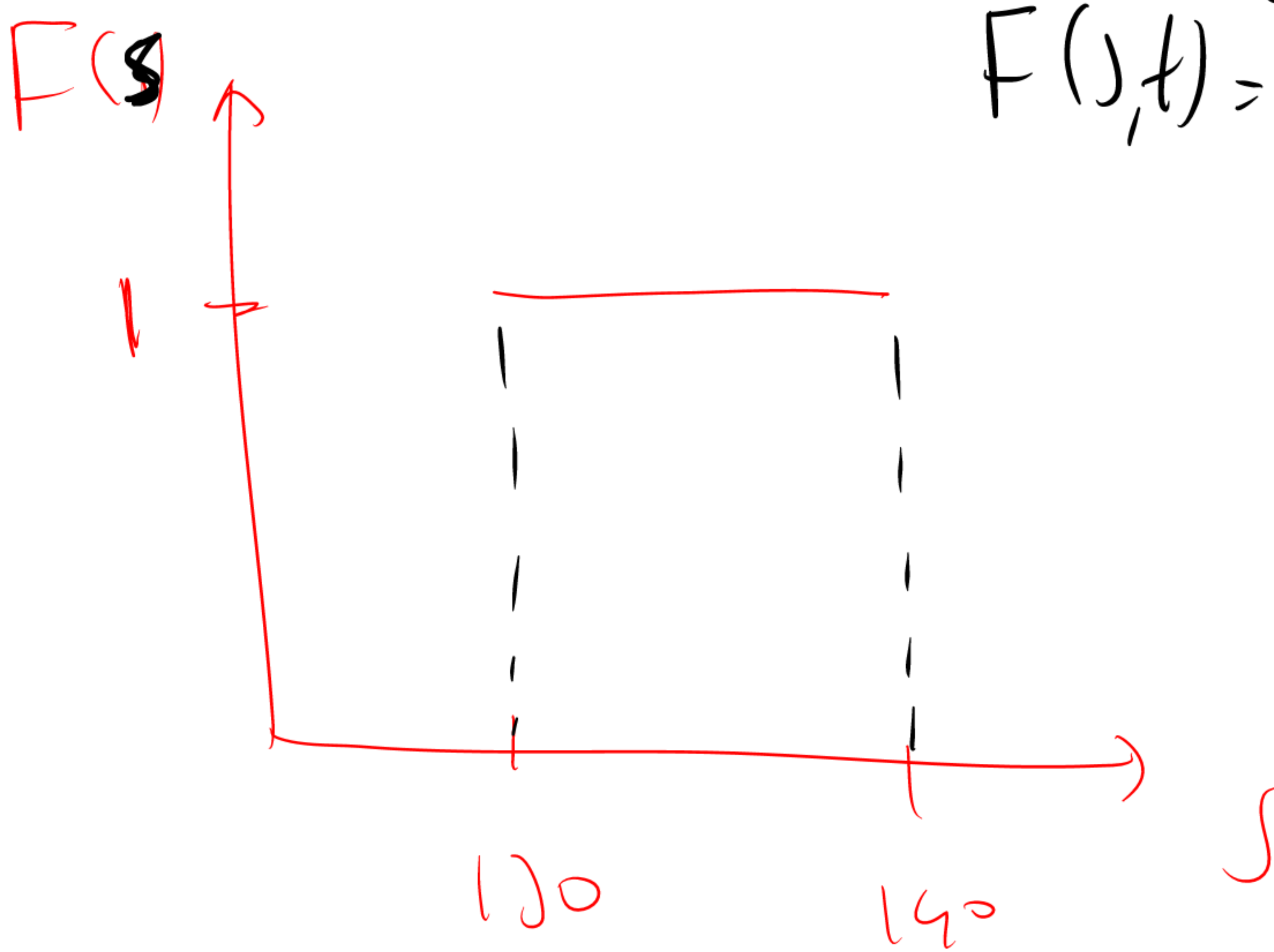
Smooth pasting condition

$A = 0 \quad \Pi = \ldots$

④ $\lim_{s \to \infty} V(s) = 0$

Call ① $\Pi \cdot s \cdot \epsilon$ ② $V(s^*) = s^* - \epsilon$ ③ $\left.\frac{dV}{ds}\right|_{s^*} = +1$

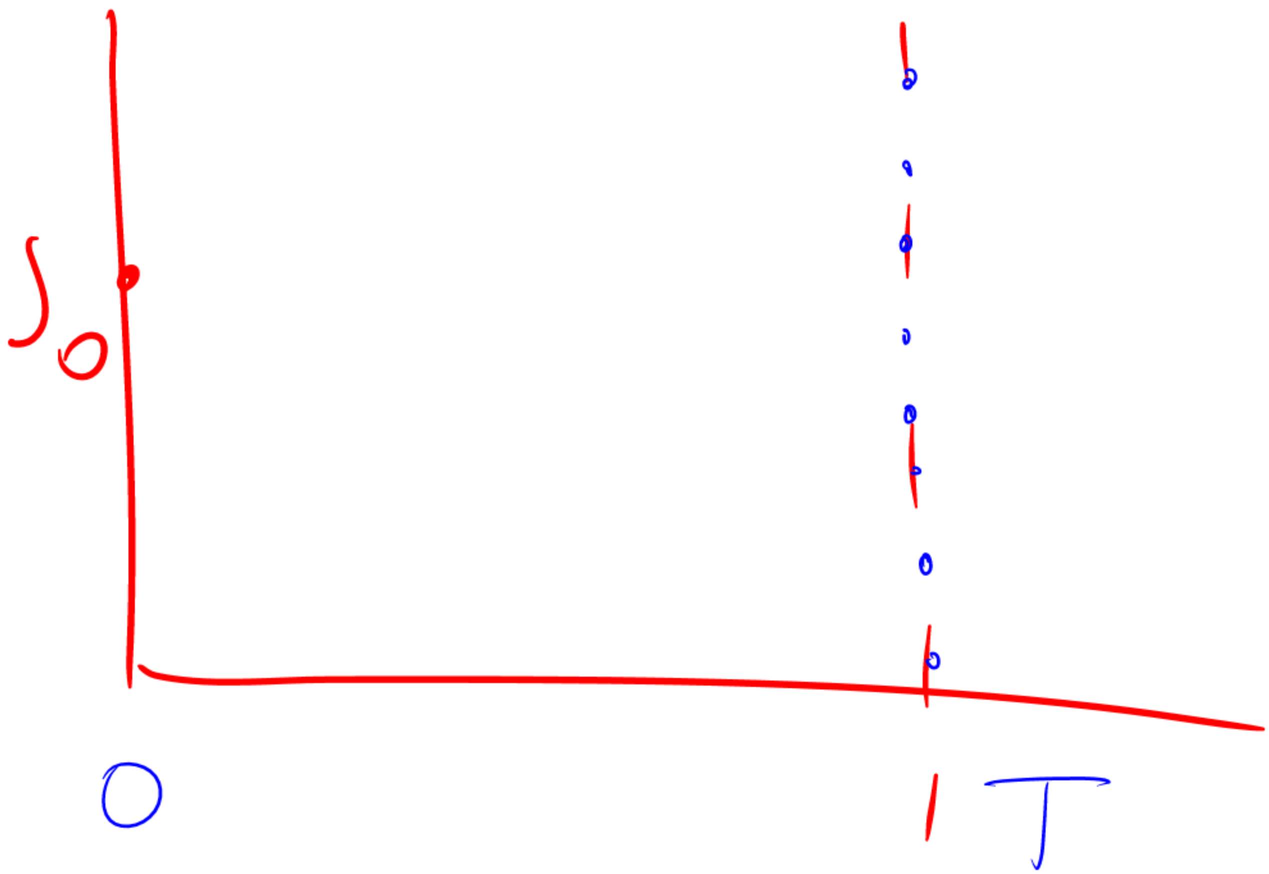
$V(0) = 0$



$$F(S,t) = \begin{cases} 1 & 130 < S < 140 \\ 0 & \text{otherwise} \end{cases}$$

forcing term

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV + F(S,t) = 0$$


S_0
0
T

$$A = \frac{1}{N} \sum_{i=1}^{N} S(t_i)$$

$$A_i = \frac{1}{i} \sum_{k=1}^{i} S(t_k)$$ Running ave.

$$A = \frac{1}{T}\int_0^T S \, dt$$

$$A(t) = \frac{1}{t}\int_0^t S \, d\tau$$

$$A_g = \left(\prod_{i=1}^{N} S_i \right)^{1/N}$$

$$\log A_g = \frac{1}{N} \log \prod_{i=1}^{N} S_i$$

$$\log A_g = \frac{1}{N} \sum_{i=1}^{N} \log S_i$$

$$A_g = e^{\frac{1}{N} \sum_{j=1}^{N} \log S_j}$$

$$e^{\frac{1}{T} \int_0^T \log S(t)\,dt}$$

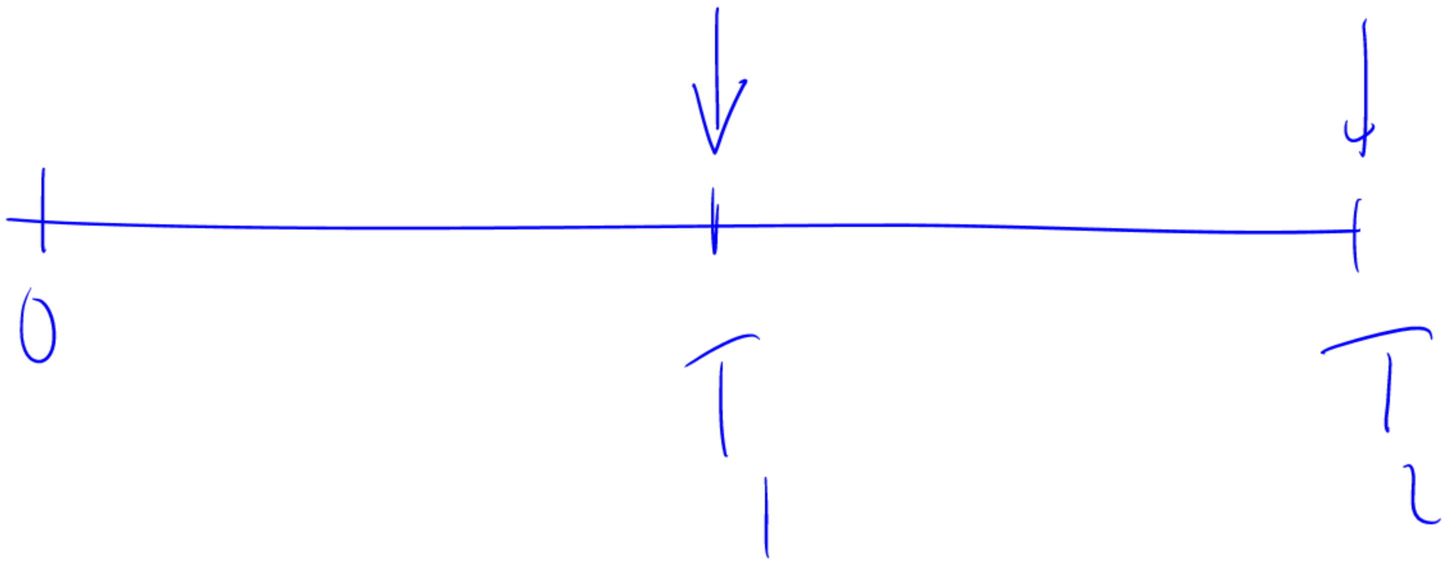
$$\frac{dS_i}{S_i} = \mu_i \, dt + \sigma_i \, dW_i$$

$$\mathbb{E}\left[dW_i \, dW_j\right] = \rho_{ij} \, dt$$

C(P)

right to Buy a put

right to sell



Option (Option (. . .))

1. Simulate underlying state variable under the risk-neutral measure / random walk

$$\frac{dS}{S} = r\,dt + \sigma\,dW^{Q}$$

2. Discount the payoff according to the security of interest

3. Take the average

Barrier Option

$$\Pi = V - \Delta S$$

$$d\Pi = dV - \Delta \, dS \qquad \text{Itô}$$

$$= \frac{\partial V}{\partial t} dt + \frac{\partial V}{\partial S} dS + \frac{1}{2} \frac{\partial^2 V}{\partial S^2} dS^2$$

$$dS^2 \to \sigma^2 S^2 \, dt$$

$$+ \frac{\partial V}{\partial I} dI$$

$$- \Delta \, dS$$

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2\eta^2\frac{\partial^2 V}{\partial \eta^2}+rS\frac{\partial V}{\partial S}+S\frac{\partial V}{\partial I}-rV=0$$

$$V=I\,W(R,t)$$

$$\frac{\partial V}{\partial t}=$$

$$\frac{\partial V}{\partial S}=$$

$$\frac{\partial^2 V}{\partial \eta^2}$$

$$\frac{\partial V}{\partial I}=$$