HJM

Historical → Vols → Simulated → Cap px

LMM

Cap → Caplets → Simulated curves

Work in τ

$$\frac{d}{dt} \bar{f}(t, \tau) = f(t, T - t)$$

$T = \tau + t \qquad t = T - \tau$

$v_1(t,\tau)$, $v_2(t,\tau)$, $v_3(t,\tau)$

$$v(\tau) = \sqrt{\lambda}\; e(\tau)$$

$$e^{(1)}(\tau) \;, \quad \tau = 0, 0.08, 0.5, 1, \ldots$$

fitted to τ by cubic spline

$$m(t,\tau) = v(t,\tau) \int v(t,\tau)$$

$$\frac{d}{dT}\left[r(t) \right] = 0$$

$$\frac{d}{dT}\left[\frac{1}{2}\sigma(t,T)\,\sigma(t,T) \right]$$

used product rule.

$$= \sigma(t,T)\,\frac{d}{dT}\sigma(t,T)$$

$$f(t,T) = -\frac{d}{dT}\ln Z$$

$$O(dt^2) = 0$$

$$O(dX^2) = O(dt)$$

$$\mu(t) \to r(t)$$

$$dX \to dX^{\mathbb{Q}}$$

To use SN $dX^{\mathbb{Q}}$

$$\frac{\mu - r}{\sigma}$$

$$dX_t^{\mathbb{Q}} = dX_t + \left(\frac{\mu - r}{\sigma}\right) dt$$

$$\sigma\, dX^{\mathbb{Q}} = \sigma\, dX_t + \mu\, dt - r\, dt$$

$$\sigma\, dX^{\mathbb{Q}} + r\, dt \equiv \sigma\, dX_t + \mu\, dt$$

$$\Sigma_{n\times n} = V \Lambda V$$

eigenvalues

eigenvectors in columns

$$\Lambda = \begin{bmatrix} \lambda_1 & & \cdots & 0 \\ \vdots & \lambda_2 & & \\ & & \lambda_3 & \vdots \\ 0 & & \cdots & \lambda_n \end{bmatrix}$$

$\mathrm{diag}(\Lambda)$

vector

$$x \Sigma x$$

positive definite.

$\sqrt{\lambda_1}$ – Vol. 1

$\sqrt{\lambda_2}$ – Vol. 2

$\sqrt{\lambda_3}$ – Vol 3

$$f(\tau_j) = \ldots + \underbrace{\sqrt{\lambda_1}\, e^{(1)}(\tau_j)}_{\phi_1} + \underbrace{\sqrt{\lambda_2}\, e^{(2)}(\tau_j)}_{\phi_2} + \ldots$$

Dimensions

$X \quad n \times N_{observ}$

$\uparrow$

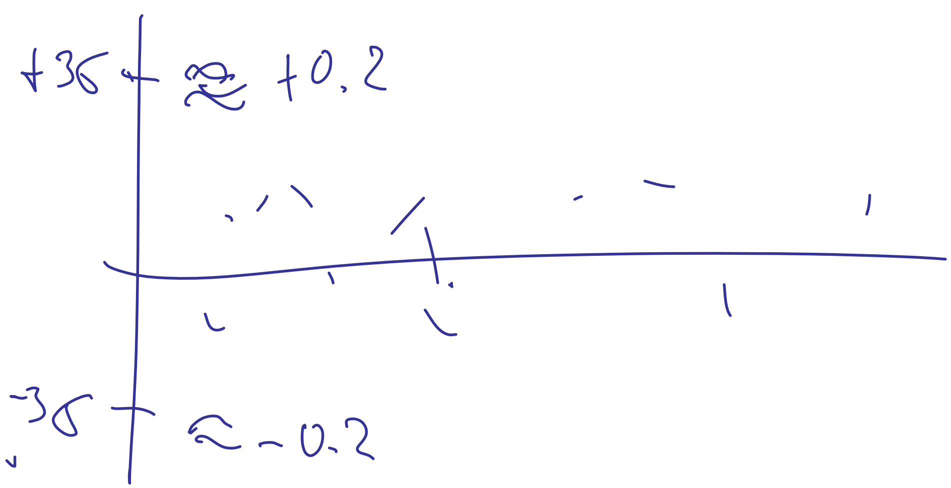
tenors

$XX' \quad n \times \cancel{N} \times \cancel{N} \times n$

$\Sigma \quad n \times n \quad 51 \times 51$

Standard Normal RV

$$Z_t = \frac{X_t - \mu}{\sigma}$$



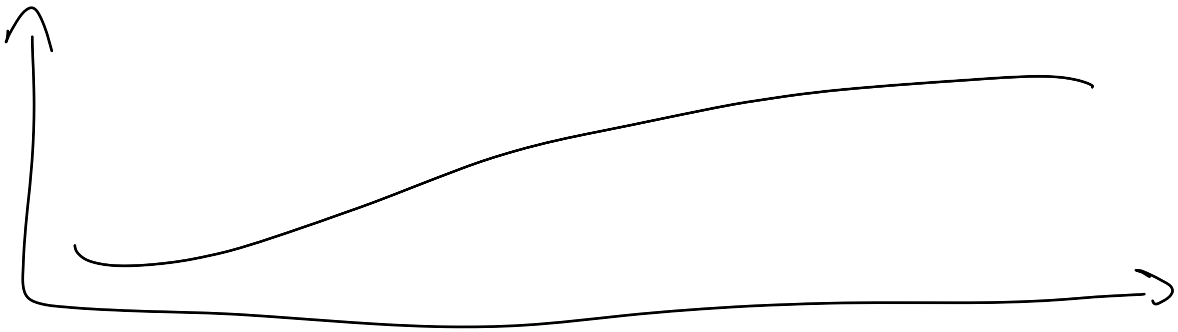
$$ZCB = \mathbb{E}^{Q}\left[\qquad \right]$$

Forward $ZCB = \mathbb{E}^{Q(m)}\left[\qquad \right]$ $\quad T_3 - T_2 = \tau$

$$F(0, T_1, T_2) \qquad F(0, T_2, T_3) \times \tau \times N$$

$$Q(m=1) \qquad Q(m=2)$$

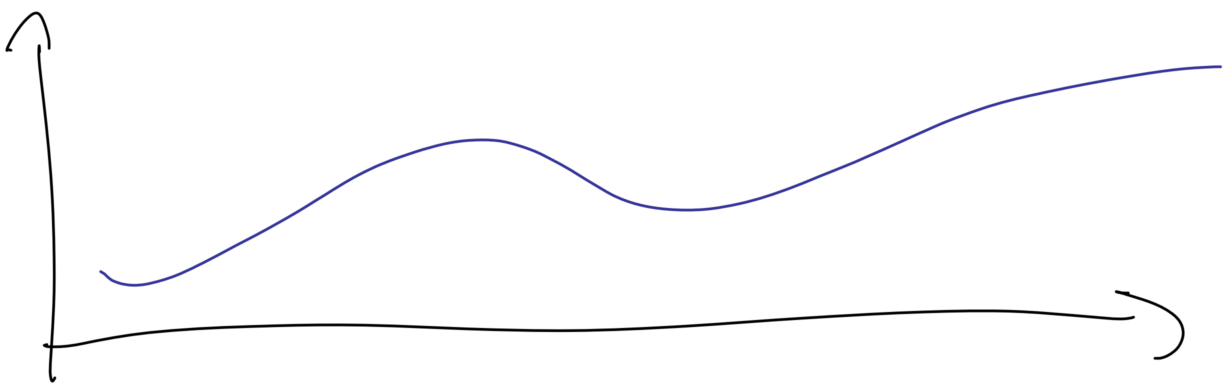
$$DF(0, T_3)$$



spot curve
ZCB yields

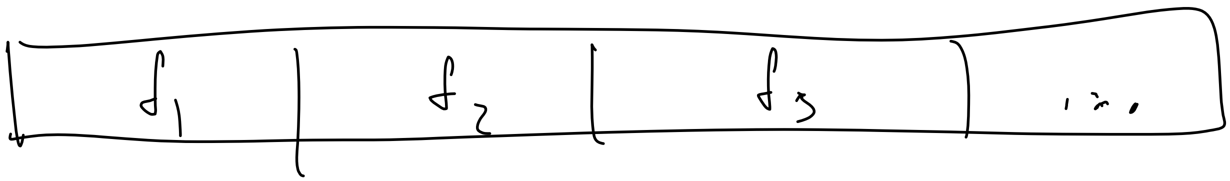
DF, $Z(0,T)$

$$Z(0,T_1) > Z(0,T_2) > \dots > Z(0,T_n)$$



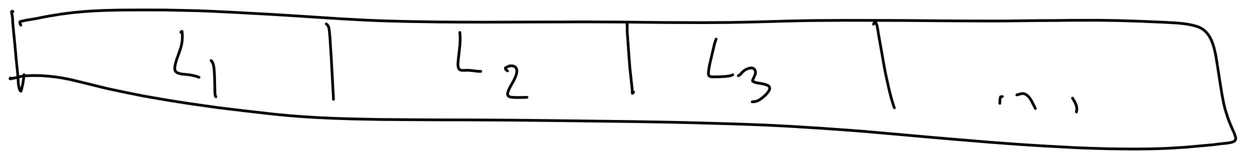
fwd curve

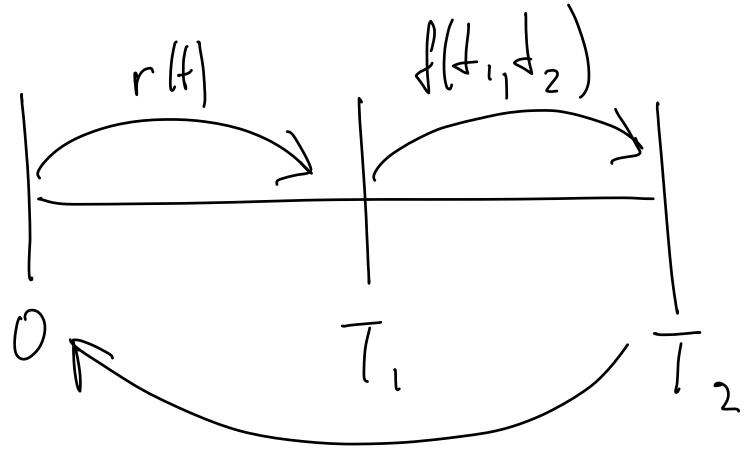
Fwd



$$L = e^{f} - 1$$

LIBOR





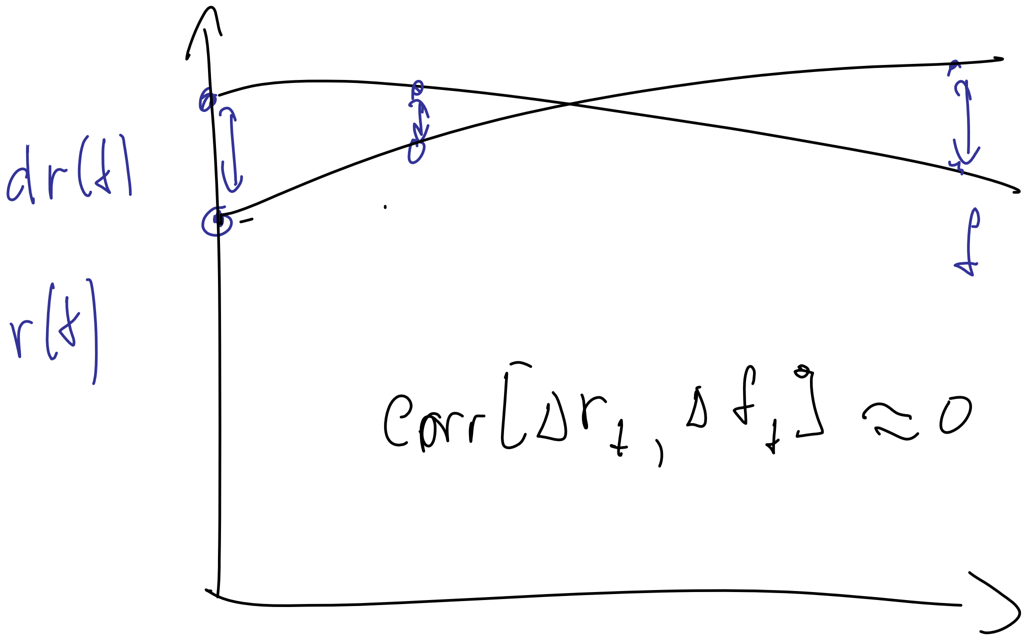
future-starting bond

$$Z(0, T_2) = Z(0, T_1) * Z(T_1, T_2)$$

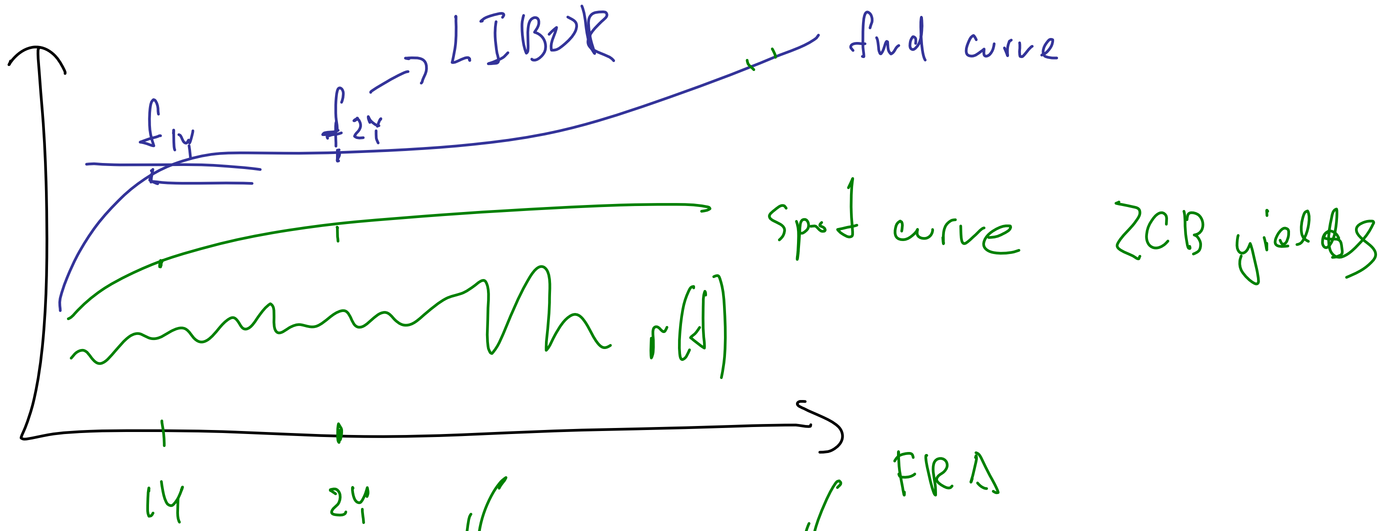
$$= e^{-rt} \quad e^{-f(t_1, t_2)\,(t_2 - t_1)}$$

$$Z(T_1, T_2) = \frac{Z_M(0, T_2)}{Z_M(0, T_1)}$$

ZCBs from market



$$\mathrm{corr}[\Delta r_t, \Delta f_t] \approx 0$$



$$E[L_j] = f(Z_j)$$

Derivative on $r(t)$

$$Z(0,T) = E^{\phi}\left[e^{-\int_0^T r(s)\,ds} \right]$$

PCA

$$dr(t) = \dots dt + \dots dX$$

v^{β}

$\omega(\dots)$