CQF Module 5 Exercise Solution

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1 a To compute the firm's asset value and volatility, set up the Merton type structural model as

$$\begin{aligned} E_0 &= V_0 N(d_1) - D\exp(-rT)N(d_2) \\ d_1 &= \frac{1}{\sigma_V}\left[\log\left(\frac{V_0}{D}\right) + \left(r + \frac{1}{2}\sigma_V^2\right)T\right] \\ d_2 &= d_1 - \sigma_V\sqrt{T} \\ \sigma_E &= \sigma_V N(d_1)\frac{V_0}{E_0} \end{aligned}$$

To solve the simultaneous equations numerically, I use MATLAB to find the minimum of the penalty function, where the deviations of E_0 and σ_E between what are given in the context and computed results are calculated. The optimization results yield

$$\begin{cases} V_0 = & 7.9088 \\ \sigma_V = & 19.12\% \end{cases}$$

Substitute the solutions into the simultaneous equations above, we yield back the equity value and equity volatility. The codes solving the equations are provided in the Appendix.

1 b The probability of the default for Merton model is

$$\mathbb{P}[V_t < D] = N(-d_2)$$

whereas in the Black-Cox PD is calculated as

$$\mathbb{P}[\tau \le T|\tau > t] = N(h_1) + \exp\left\{2\left(r - \frac{\sigma_V^2}{2}\right)\log\left(\frac{K}{V_0}\right)\frac{1}{\sigma_V^2}\right\}N(h_2)$$

Using the simultaneous equations in (1a) to solve for V_0 and σ_V, we have the following sensitivity between σ_E and the probability of default.

As shown in Figure 1, the

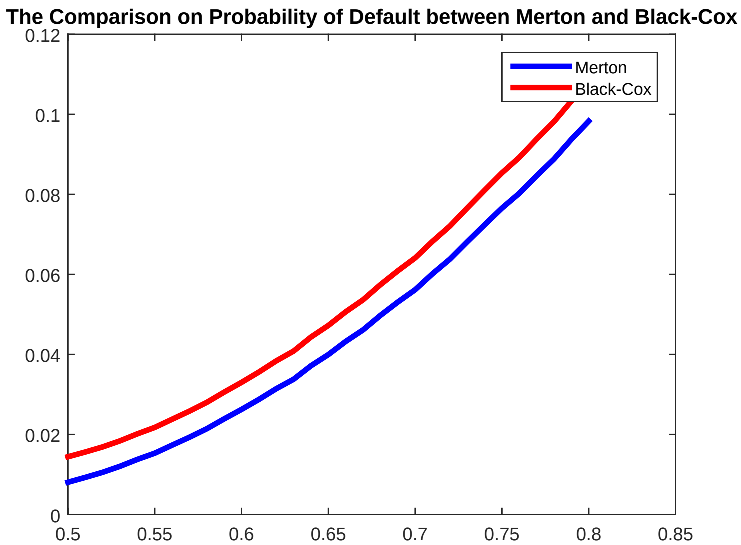

Figure 1: The comparison of probability of default between Merton and Black-Cox Models

Appendix

```
function diff_mse = compute_E0(V0, sigmaV)
%COMPUTE the difference of equity value
% between calculated initial equity value and 3M
%
%INPUTS
%  V0:      the initial asset value
%  sigmaV:  the volaility of assets
%  inputM:  include r, T, D
%
%OUTPUT
%  diff_mse: calculated mse using INPUTS - context

r = 0.02;
D = 5;
T = 1;

d_1 = (1/(sigmaV*sqrt(T))) * ...
    ( log(V0/D) + (r+0.5*sigmaV^2)*T );
d_2 = d_1 - sigmaV*sqrt(T);

E0 = V0*normcdf(d_1,0,1) - D*exp(-r*T)*normcdf(d_2,0,1);
sigmaE = sigmaV*normcdf(d_1,0,1)*V0/E0;

diff_mse = 10*(E0-3)^3 + (sigmaE-0.5)^2;

end
```

compute_E0.m

```
% solve V0 and sigma_V
[results,fval] = fsolve(@(x) compute_E0(x(1),x(2)),[9;0.25], ...
```

```
    optimoptions('fmincon','MaxFunEvals',10000,'MaxIter',10000));
% check answer
compute_E0(results(1), results(2))
```

compute_value_vol_1a.m