$V(S_t)$

$\hookrightarrow$ GBM $\longrightarrow$ B.S.E

$V(r, t)$

$\hookrightarrow$ SDE for $\longrightarrow$ B.P.E
just

Vasicek 7P

$$dr = (\eta - \gamma r)\,dt + \sigma\,dX$$

$$= -\gamma(r_t - \bar{r})\,dt + \sigma\,dX$$

-ve I.R

CIR 8J

$$dr = (\eta - \gamma r)\,dt + \sigma\sqrt{r}\,dX$$

$$= -\gamma(0 - \bar{r})\,dt + \sigma\sqrt{\cancel{r_t}}\,dX$$

$$r_t \to 0$$

$$r \to 0$$

$$V = V(r, t; T)$$

$$dV_i = \left(\frac{\partial V_i}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V_i}{\partial r^2} \right) dt + \frac{\partial V}{\partial r} dr$$

for $i = 1, 2$

subst in A

$u\,dt + w\,dX$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$

$$u(x,t) = X(x)\,T(t)$$

$$\frac{\partial u}{\partial t} = X T'$$

$$\frac{\partial^2 u}{\partial x^2} = X'' T$$

$$X T' = X'' T$$

$$\underbrace{\frac{T'}{T}}_{\text{indep of } x} = \underbrace{\frac{X''}{X}}_{\text{indep. of } t} = \text{const.} \rightarrow \text{func}^{n} \text{ indep of } x, t$$

$$\frac{dS}{S} = \mu\, dt + \sigma\, dX \qquad \mu \longrightarrow r$$

$$dr = u\, dt + w\, dX \qquad u \longrightarrow (u - \lambda w)$$

Eq 6

$$\frac{\partial v}{\partial t} + \frac{1}{2}\omega^2 \frac{\partial^2 v}{\partial r^2} + u\frac{\partial v}{\partial r} = \underbrace{2\omega \frac{\partial v}{\partial r} + rV}_{(1a)}$$

$$a(r,t) = 2\omega - u$$

$$\boxed{a(r,t) = \lambda w - u}$$

$$\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} - a(r,t)\frac{\partial V}{\partial r} - rV = 0$$

$$\frac{\partial V}{\partial t} + \frac{1}{2} w^2 \frac{\partial^2 V}{\partial r^2} + u\frac{\partial V}{\partial r} = a\frac{\partial V}{\partial r} + rV + u\frac{\partial V}{\partial r}$$

$$dV = w\frac{\partial V}{\partial r}\, dX + \left(a\frac{\partial V}{\partial r} + u\frac{\partial V}{\partial r} + rV\right) dt$$

$$dV - rV\,dt = w\frac{\partial V}{\partial r}\, dX + (a+u)\frac{\partial V}{\partial r}\, dt$$

$$= w\,\partial V/\partial r \left[dX + \boxed{\left(\frac{a+u}{w}\right)} dt \right]$$

λ

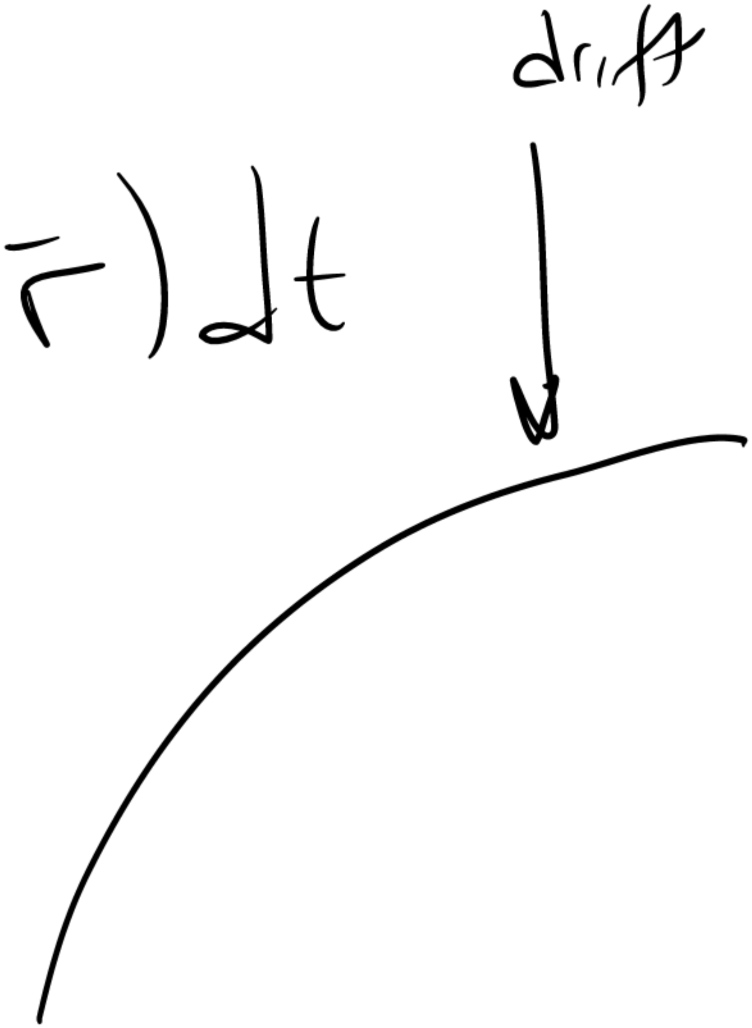
real drift MPoR × diffusion/vol.

$$dr = \underbrace{(u - \lambda w)}_{\text{risk-adjusted drift}} dt + w\, dX$$

Vasicek

$$dr = -\gamma(r - \bar{r})\,dt$$

$$r_t = Ae^{-\gamma t}$$



$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS\frac{\partial V}{\partial S} - rV = 0$$

$$\mu - \lambda\sigma$$

Sharpe Ratio

$$\boxed{\lambda = \frac{\mu - r}{\sigma}}$$

$V = S$ asset

~~$V = S_0 e^{rt}$ cash~~

$$\frac{\partial V}{\partial t} = 0 \qquad \frac{\partial V}{\partial S} = 1 \qquad \frac{\partial^2 V}{\partial S^2} = 0$$

$$0 + 0 + (\mu - \lambda\sigma)S - rS = 0$$

$$\boxed{r = \mu - \lambda\sigma}$$

$$\frac{d\beta}{dt} = \gamma\beta - 1$$

$$\int_t^T \frac{d\beta}{\gamma\beta - 1} = \int_t^T ds$$

$\beta(t)$?

$\beta(T) = 0$

① $\frac{\partial^2}{\partial r^2}(w^2) = 0$

$\int ① \, dr$ $\qquad \frac{\partial}{\partial r}(w^2) = \alpha(t)$

$\int$ again $\qquad w^2 = \alpha(t) r + \beta(t)$

$\boxed{\text{slide 54}}$

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial l}dl + \frac{1}{2}\frac{\partial^2 V}{\partial r^2}\underset{w^2 dt}{dr^2} + \frac{1}{2}\frac{\partial^2 V}{\partial l^2}\underset{q^2 dt}{dl^2} + \frac{\partial^2 V}{\partial r \partial l}\underset{\rho w q \, dt}{dr\,dl}$$

$$= \underbrace{\left(\frac{\partial V}{\partial t} + \frac{1}{2}w^2\frac{\partial^2 V}{\partial r^2} + \frac{1}{2}q^2\frac{\partial^2 V}{\partial l^2} + \rho w q \frac{\partial^2 V}{\partial r \partial l}\right)}_{\mathcal{L}(V)}dt + \underbrace{\frac{\partial V}{\partial r}dr + \frac{\partial V}{\partial l}dl}$$

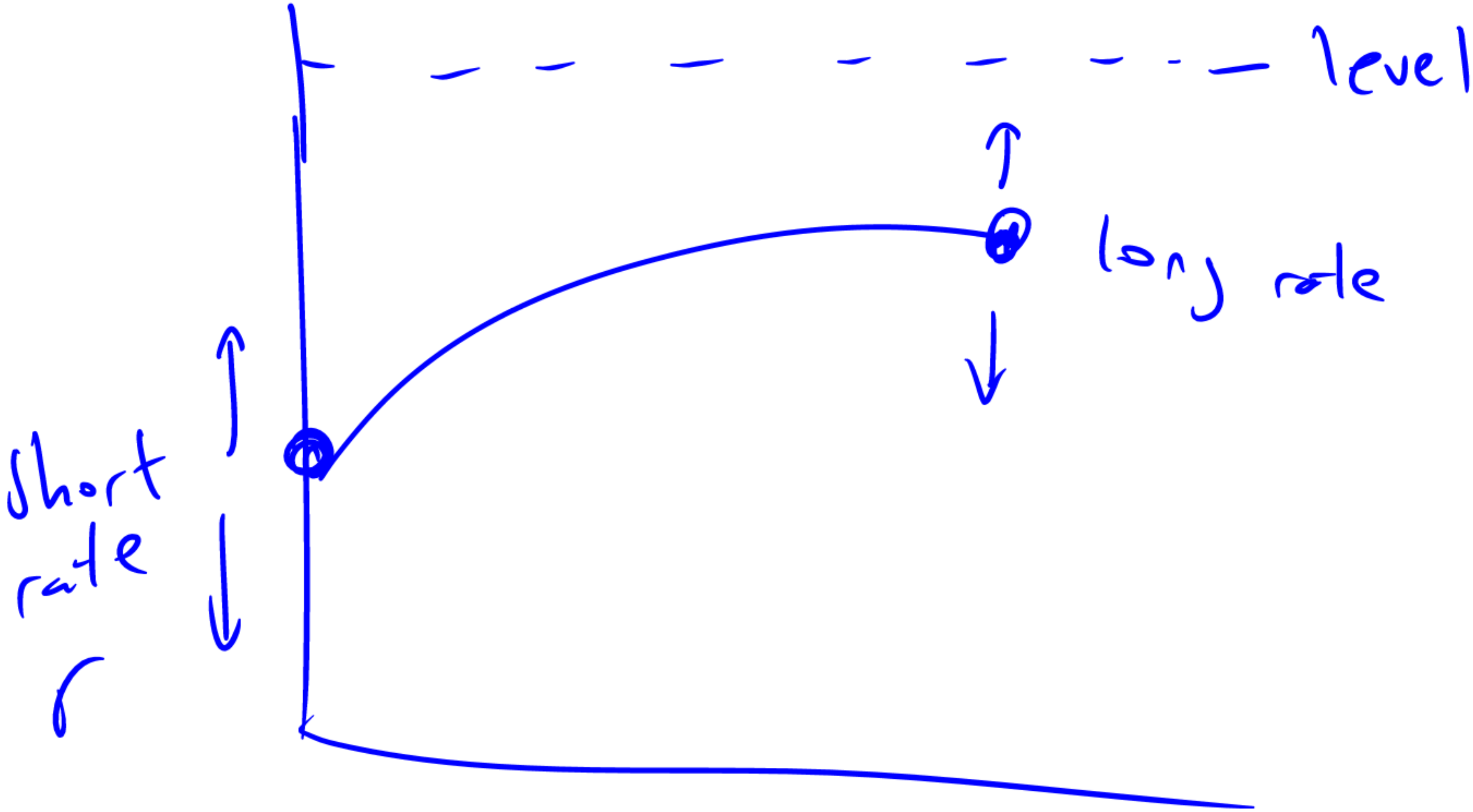
$$dV = \mathcal{L}(V)\,dt + \frac{\partial V}{\partial r}\,dr + \frac{\partial V}{\partial \lambda}\,d\lambda$$

$$dV_1 = - \quad \_\_\_$$

$$dV_2 = - \quad \_$$

$$\frac{\partial p}{\partial t} = \frac{1}{2}\sigma^2\left[\frac{\partial^2 p}{\partial y^2} + \frac{\partial^2 p}{\partial z^2}\right]$$

r λ



level
long rate
short rate
r

$$\boxed{dy = A\,dt + B\,dx}$$

$$\phi^{+}(s,t)$$

$$\phi^{-}(s,t)$$