Math Aptitude Indicator

This is a Multiple Choice Test. For each question only one answer is correct.

Please use separate sheets of paper for calculations, and submit that along with your solutions as it will assist us in evaluating your performance if you get any questions wrong.

1. The second order derivative $\frac{d^{\,2} f}{dx^2}$ of $f(x) = x \exp(-x)$ is

   (A) $e^{-x}(x^2 - 1)$ (B) $e^{-x}(x - 1)$ (C) $f(x) + 2e^{-x}$
   $\boxed{\text{(D) } f(x) - 2e^{-x}}$ (E) $f(x) - e^{-x}$

2. Given $y = \ln|1 - x|$, find an expression for $\frac{dy}{dx} + x\frac{d^2y}{dx^2}$

   (A) $\frac{1}{(1-x)}$ $\boxed{\text{(B) } -\frac{1}{(1-x)^2}}$ (C) $\frac{x}{(1-x)^2}$ (D) $1-x$ (E) $\frac{1}{(1-x^2)^2}$

3. What is the value of the limiting problem $\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

   (A) 0 (B) ∞ (C) $-\infty$ $\boxed{\text{(D) } 3x^2}$ (E) None of the given answers

4. Calculate $\lim_{x \to 0} \frac{2x + \sin x}{x(x-1)}$

   (A) ∞ (B) 0 (C) 2 (D) 1 $\boxed{\text{(E) } -3}$

5. If

$$I = \int_0^\infty x \exp\left(-x^2\right)\, dx$$

   Then $I =$

   (A) 0 (B) ∞ (C) -2 (D) $-\frac{1}{2}$ $\boxed{\text{(E) } \frac{1}{2}}$

6. Evaluate the integral $\int_0^1 \frac{dx}{x^2 + 3x + 2}$

   (A) $\ln\frac{2}{3}$ (B) ∞ (C) 0 $\boxed{\text{(D) } -\ln\frac{3}{4}}$ (E) $\ln\frac{3}{4}$

7. What is the n^{th} order term in the Taylor series expansion of the function $f(x) = \exp x$ about $x = -3$

$\boxed{\text{(A) } \frac{e^{-3}}{n!}(x+3)^n}$ (B) $\frac{e^{-3}}{2^n}\frac{(x+3)^n}{n!}$ (C) $\frac{e}{2^{n+1}}\frac{(x-3)^{n+1}}{(n+1)!}$

(D) $\frac{e^{-3}}{2^n}\frac{(x+3)^{n-1}}{n!}$ (E) $\frac{e^{-3}}{2^n}\frac{(x-3)^{n-1}}{(n-1)!}$

8. Given the Taylor series expansion for

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + ... = \sum_{n=0}^{\infty} (-1)^n x^n$$

Obtain the series for $\ln(1+x)$

(A) $\sum_{n=0}^{\infty} n^2 (-1)^{n-1} x^{n-1}$ (B) $\sum_{n=0}^{\infty} (-1)^{n+1} (n+1) x$ $\boxed{\text{(C) } \sum_{n=0}^{\infty} (-1)^n \frac{x^{n+1}}{n+1}}$

(D) $\sum_{n=0}^{\infty} n (-1)^{n-1} \frac{x^{n-1}}{n+1}$ (E) $\sum_{n=0}^{\infty} (-1)^n \frac{x^{n-1}}{n-1}$

9. Let X be a continuous random variable with distribution

$$p(x) = \begin{cases} k\left(1-x^3\right) & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Evaluate k

(A) $\frac{3}{4}$ (B) $-\frac{4}{3}$ $\boxed{\text{(C) } \frac{4}{3}}$ (D) $\frac{3}{2}$ (E) 2

10. Consider the probability density function (pdf)

$$p(x) = \begin{cases} \frac{3}{4}\left(1-x^2\right) & \text{if } |x| \le 1 \\ 0 & \text{otherwise} \end{cases}$$

which a random variable X follows. Find the probability that $0 \le X \le \frac{1}{2}$

(A) $\frac{3}{32}$ (B) $\frac{9}{32}$ (C) $\frac{12}{88}$ (D) $\frac{1}{8}$ $\boxed{\text{(E) } \frac{11}{32}}$

11. Let X be a Normally distributed random variable with mean $(\mu) = 4$ and standard deviation $(\sigma) = 2$. If $\mathrm{E}[X]$ denotes the expectation of X, then the value of $\mathrm{E}\left[X^2\right]$ is

(A) 16 $\boxed{\text{(B) } 20}$ (C) 4 (D) 2 (E) 3

12. A random variable is uniformly distributed over $[0,1]$. What is its skew?

(A) 3 $\boxed{\text{(B) None of those listed}}$ (C) 6 (D) 2 (E) 4

13. A Normally distributed random variable with mean μ has a probability density function given by

$$\frac{\gamma}{\sqrt{2\pi\sigma}} \exp\left(-\frac{\gamma^2}{\sigma}\frac{(x-\mu)^2}{2}\right)$$

Its standard deviation is given by
(A) $\frac{\gamma^2}{\sigma}$ (B) $\frac{\sigma}{\gamma}$ $\boxed{\text{(C) } \frac{\sqrt{\sigma}}{\gamma}}$ (D) $\frac{\gamma}{\sqrt{\sigma}}$ (E) $\frac{\sqrt{\sigma}}{2\gamma}$

14. If $f(x,y) = x^2 - 4xy + y^3 + 4y$, the sum $f_{xx} + f_{yy}$ is given by
(A) $2-6y$ (B) 0 (C) $2x+4$ (D) $-4(x+y)$
$\boxed{\text{(E) None of the above}}$

15. Obtain the solution to the differential equation

$$\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

(A) $\frac{Cx}{1-Cx}$ (B) $\frac{Cx}{1+Cx}$ (C) $\frac{C-x}{1-Cx}$ (D) $\frac{1-Cx}{x+C}$
(E) $\frac{x+C}{1-Cx}$

16. The first order differential equation

$$\frac{dy}{dx} - xy = x$$

has the solution $y =$
(A) $A\exp\left(-\frac{1}{2}x^2\right)$ (B) $\exp\left(-\frac{1}{2}x^2\right) - 1$ (C) $\exp\left(\frac{1}{2}x^2\right) - A$
(D) $A\exp\left(\frac{1}{2}x^2\right) - 1$ (E) $A\exp\left(\frac{1}{4}x^2\right) - 1$

17. Solve $y'' - 4y' + 13y = 0$ to obtain $y =$
(A) $e^{2x}(A\cos 3x + B\sin 3x)$ (B)$e^{3x}\{A\cos 2x + B\sin 2x\}$ (C) $Ae^{2x}+Be^{3x}$
(D) $e^{3x}(A+Bx)$ (E) $e^{2x}(A+Bx)$

18. Solve $x^2y'' - 4xy' + 6y = 0$ to obtain $y =$
(A) $x^3(A\cos(2\ln x) + B\sin(2\ln x))$ (B) $Ax^2 + Bx^3$ (C) $x^2(A + B\ln x)$
(D) $x^3(A + B\ln x)$ (E) $x^2(A\cos(3\ln x) + B\sin(3\ln x))$

19. Determine those values of k for which $\begin{vmatrix} k & k \\ 8 & 4k \end{vmatrix} = 0$
$\boxed{\text{(A) } 0 \text{ \& } 2}$ (B) 2 & 4 (C) 0 & -2 (D) -2 & 4 (E) There is no solution

20. Consider the following problem

$$\begin{aligned} 2x \quad + y - z &= 1 \\ x \qquad\quad - 2z &= -5 \\ x - 2y + 3z &= 6 \end{aligned}$$

What are the values of x, y & z in turn?

(A) A solution does not exist (B) $\begin{bmatrix} x=0 \\ y=0 \\ z=0 \end{bmatrix}$ $\boxed{\text{(C) } \begin{bmatrix} x=1 \\ y=2 \\ z=3 \end{bmatrix}}$

(D) $\begin{bmatrix} x=-1 \\ y=0 \\ z=2 \end{bmatrix}$ (E) There are several solutions

21. Obtain the determinant of the matrix $\begin{pmatrix} 2 & 0 & 1 \\ 3 & 2 & -3 \\ -1 & -3 & 5 \end{pmatrix}$

(A) 0 $\boxed{\text{(B) } -5}$ (C) Does not exist (D) 10 (E) 24

22. Consider the vectors $\mathbf{u} = (-1,\ 0,\ k)$ and $\mathbf{v} = (6,\ -4,\ 2)$. What is the value of k for which the two vectors are orthogonal?

(A) 0 (B) 2 $\boxed{\text{(C) } 3}$ (D) $\frac{1}{2}$ (E) They can never be orthogonal

23. If $x = 2 - 3i$ is a complex number, then its size is

$\boxed{\text{(A) } \sqrt{13}}$ (B) $\sqrt{5}i$ (C) 3 (D) 2 (E) $\sqrt{\frac{2}{3}}$

24. Consider the function $f(x, y) = x^2 + y^2$ where $x = \sin 2\theta$ and $y = \cos 2\theta$. $\frac{df}{d\theta}$ is given by

(A) 1 $\boxed{\text{(B) } 0}$ (C) 2 (D) -1 (E) None of those given

25. Given the matrix $\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}$, the value of the largest eigenvalue is

$\boxed{\text{(A) } 4}$ (B) 1 (C) -1 (D) -4 (E) the eigenvalues are complex

26. Evaluate

$$\int_{-2}^{1} |x|\, dx$$

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $-\frac{5}{2}$ $\boxed{\text{(E) } \frac{5}{2}}$