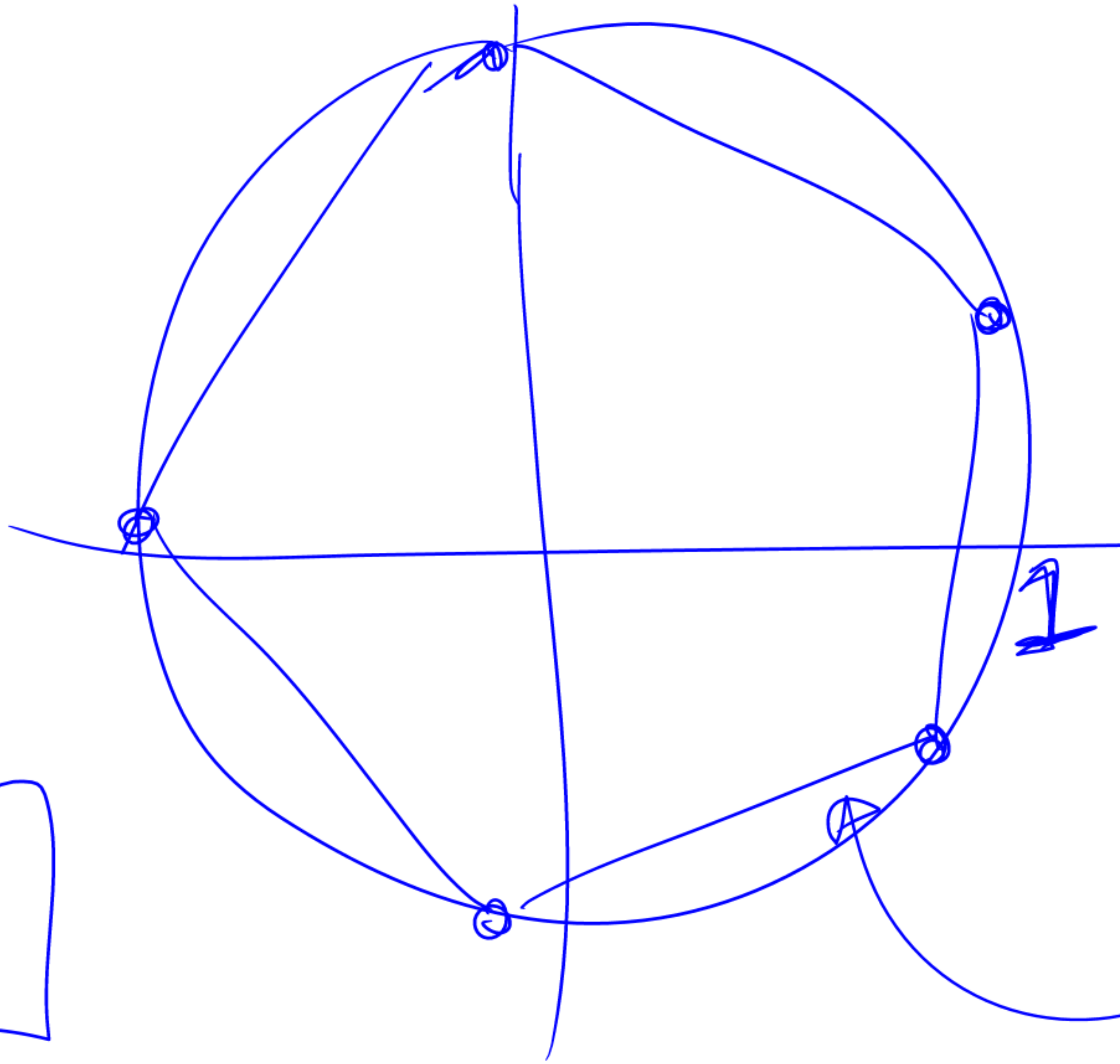
1) Fourier Transf's

2) Complex Variables

3) Stoch Vol.

4) Jump Diffusion

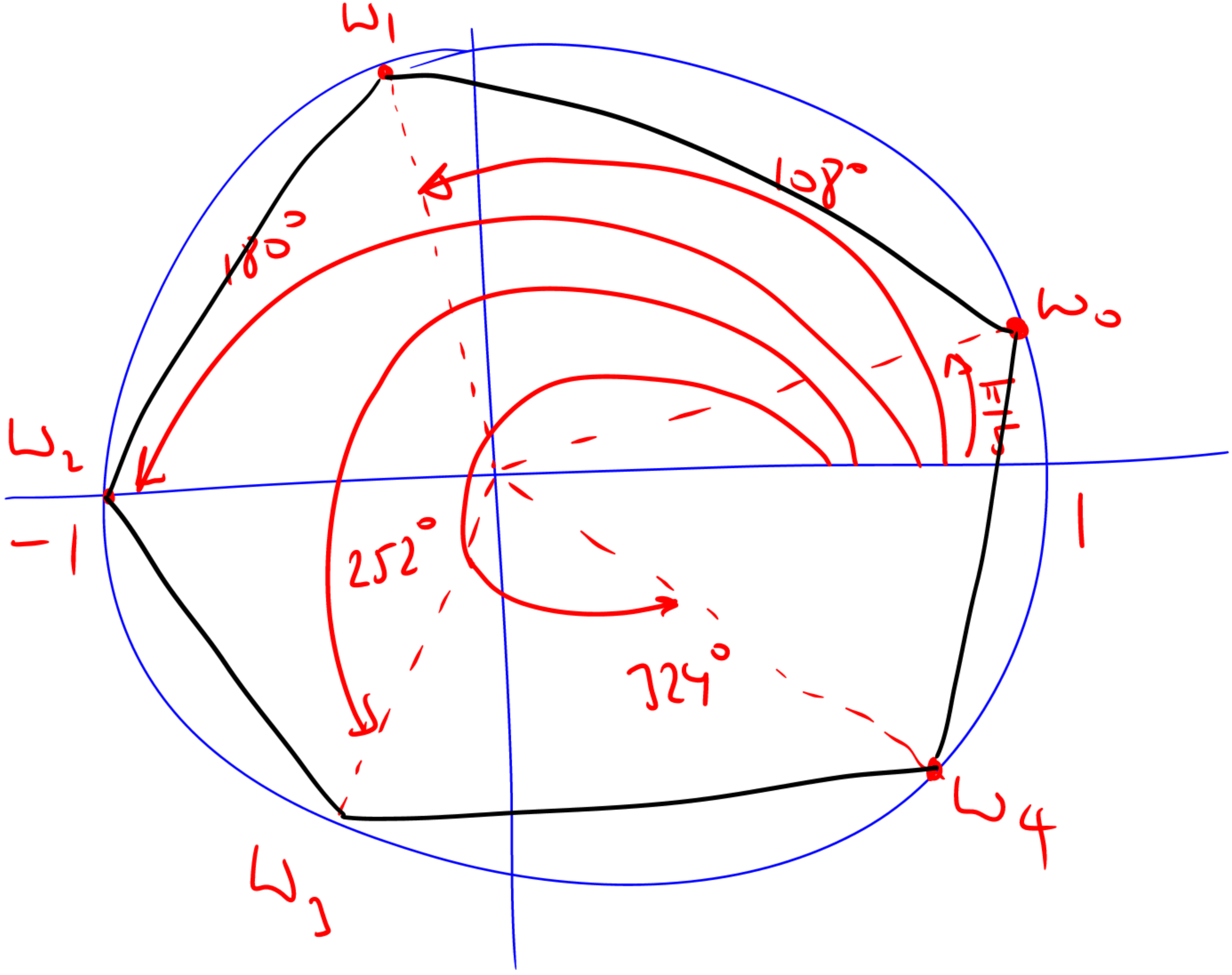
Geometric representation



$\boxed{r = 1}$

If $z = 1$ regular polygon

If any general $z \implies$ irregular polygon



w_1
w_0
w_2
w_3
w_4
-1
1
108°
180°
252°
324°
$\frac{\pi}{5}$

$$\int_{-\infty}^{\infty} \frac{x}{\underbrace{(x+1)}^{5}} \xrightarrow{\mathbb{C}}$$

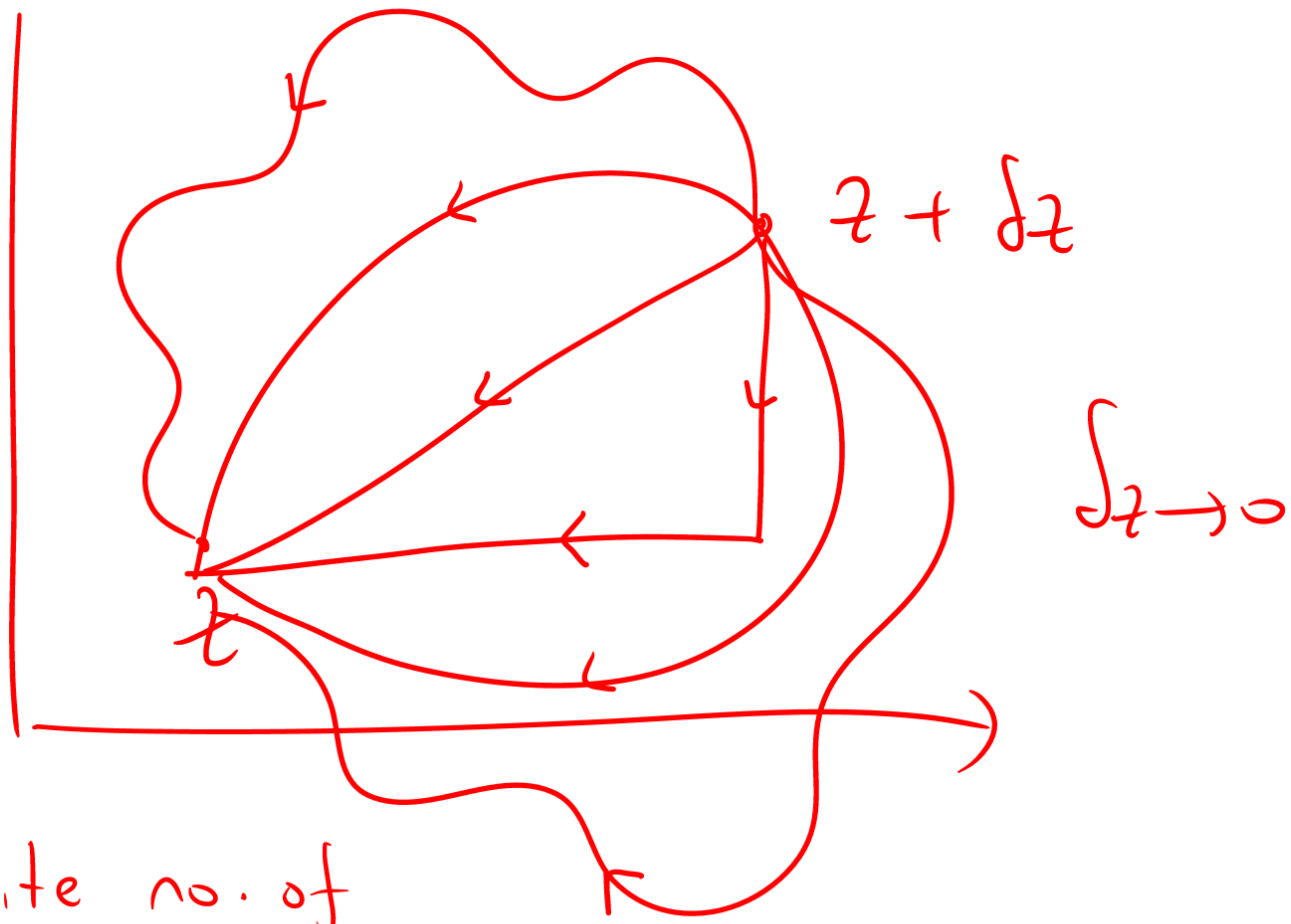
$$(z+1)^{5} \qquad z=-1 \text{ fifth root}$$

$$e^x[\cos y + i\sin y]$$

$$\underbrace{e^x\cos y}_{u(x,y)} + i\,\underbrace{e^x\sin y}_{v(x,y)} = w$$

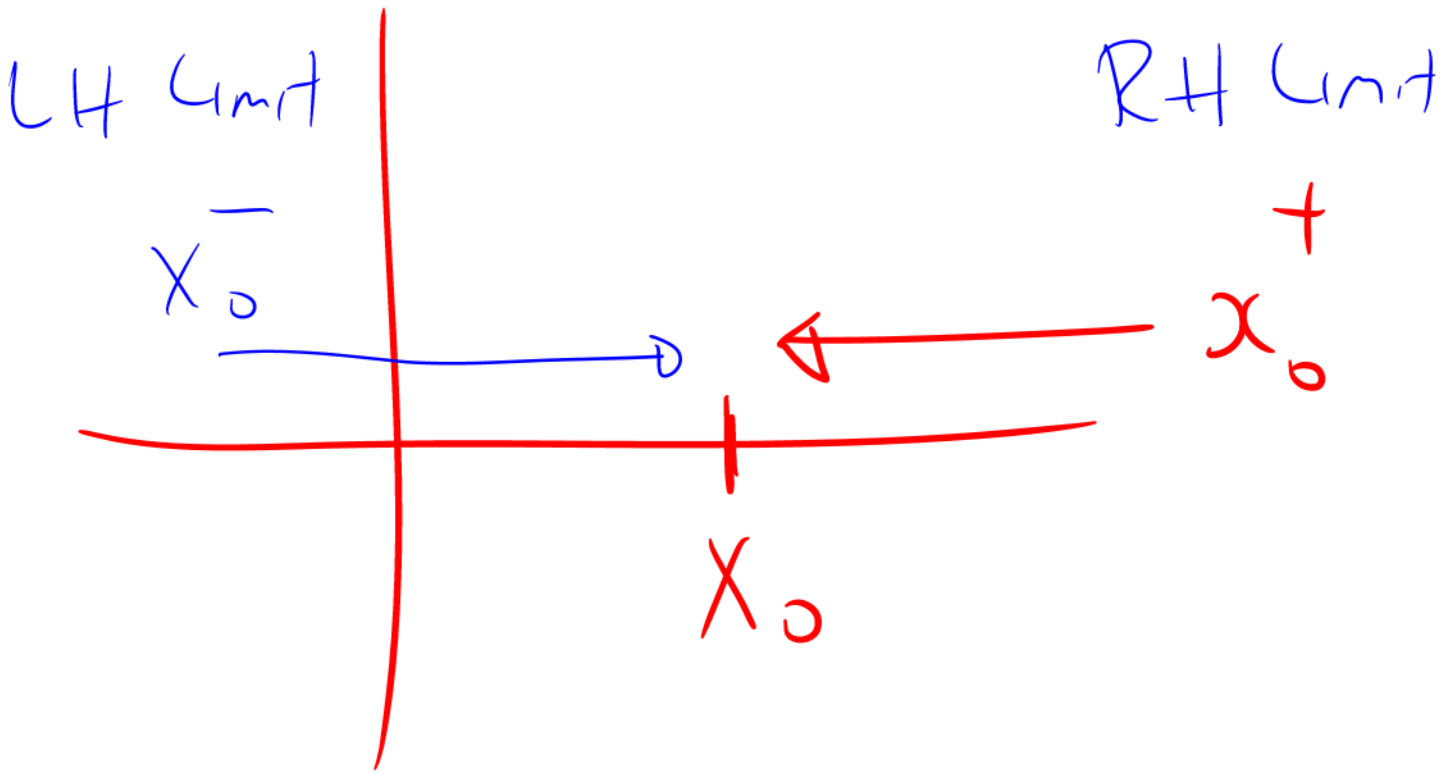
$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$$\boxed{f'(a) = \lim_{z \to a} \frac{f(z) - f(a)}{z - a}}$$



$\exists$ infinite no. of paths along which $\delta z \to 0$

$$\lim_{x \to x_0} f(x)$$

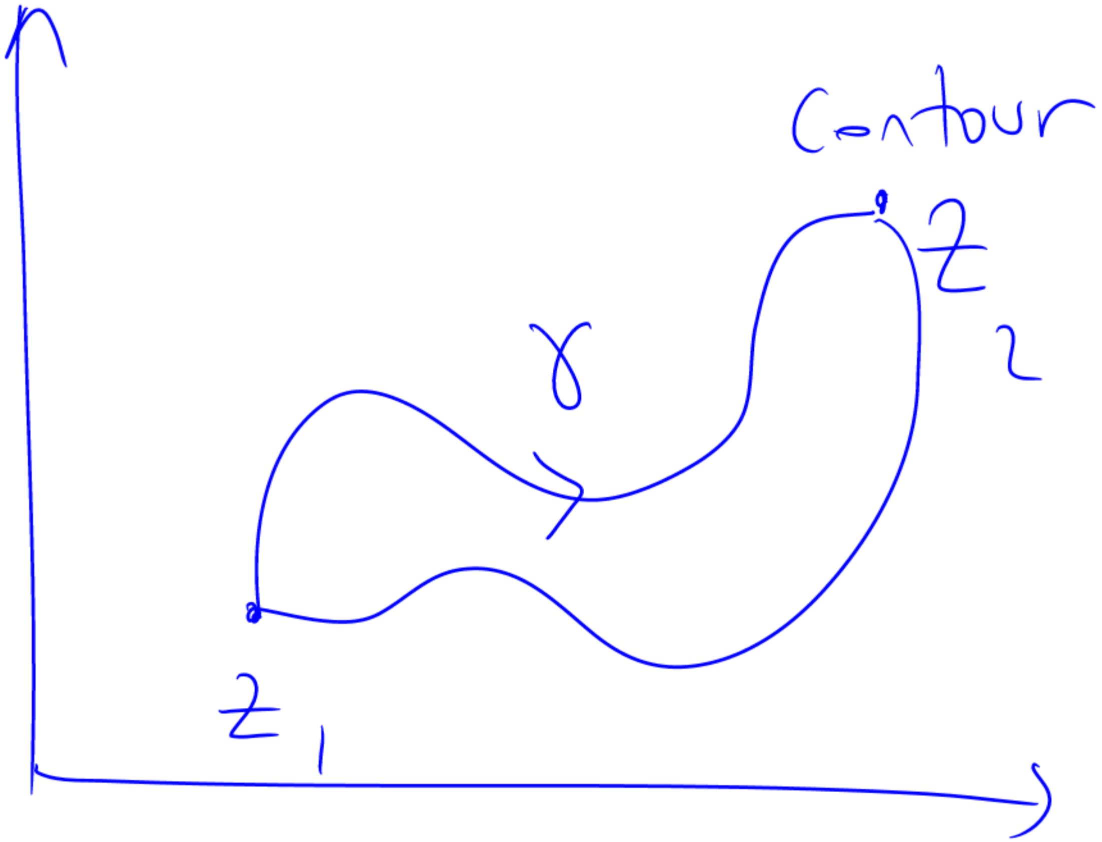


$x = f(t)$ $\quad$ t parameter

$y = g(t)$

eliminate $t \longrightarrow y = F(x)$

t	
x	
y	

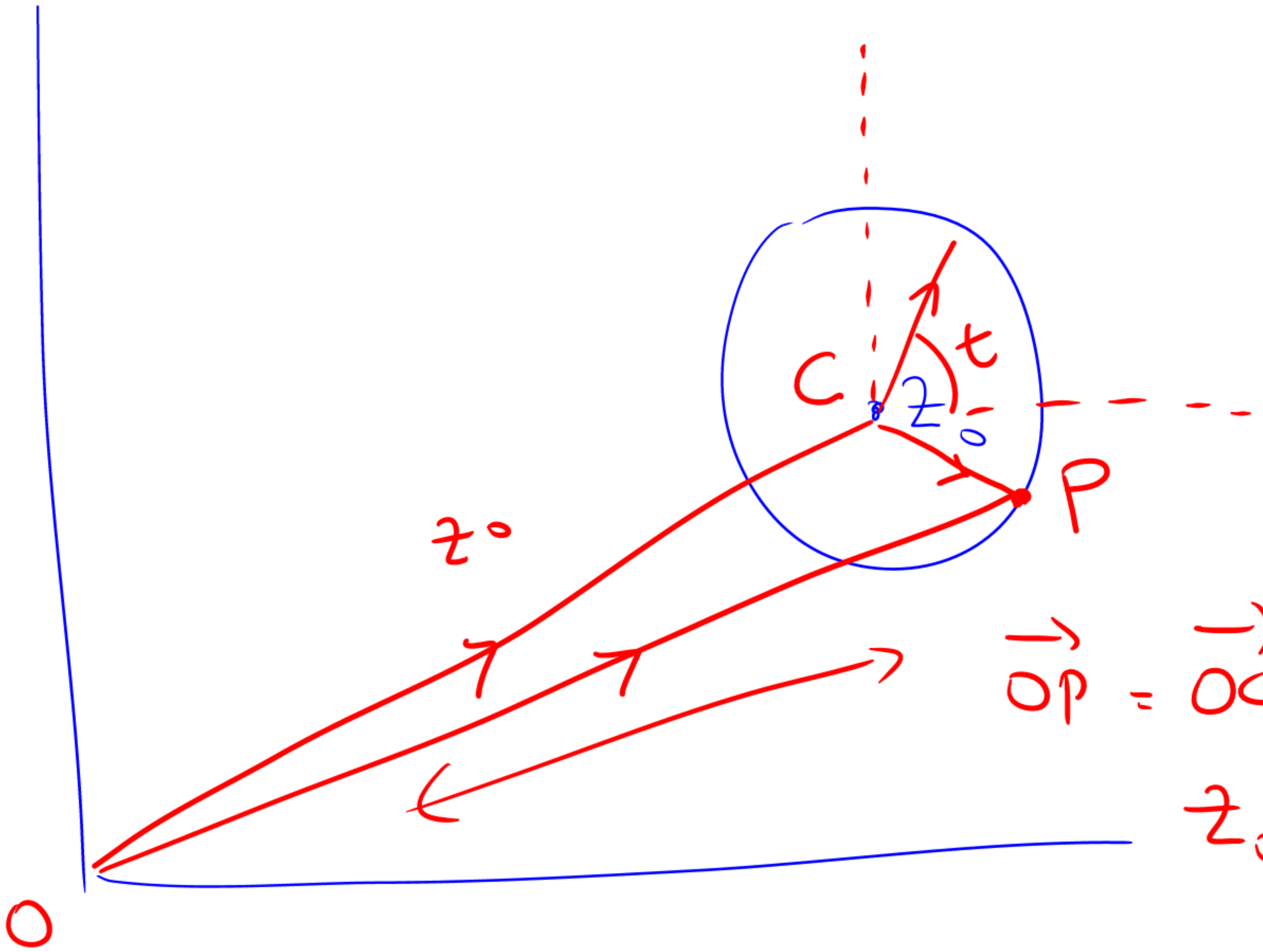


Closed contour

if

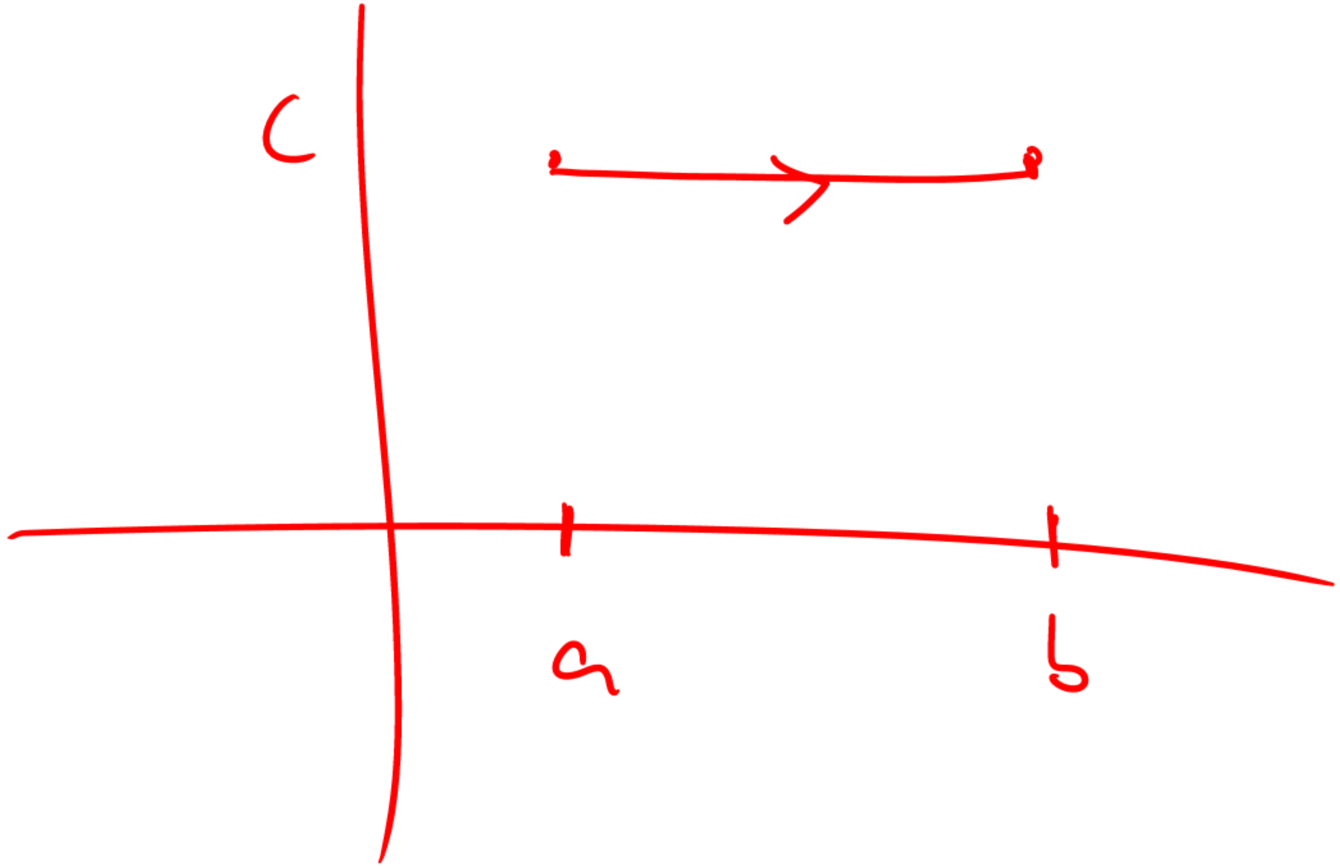
$z(a) = z(b)$

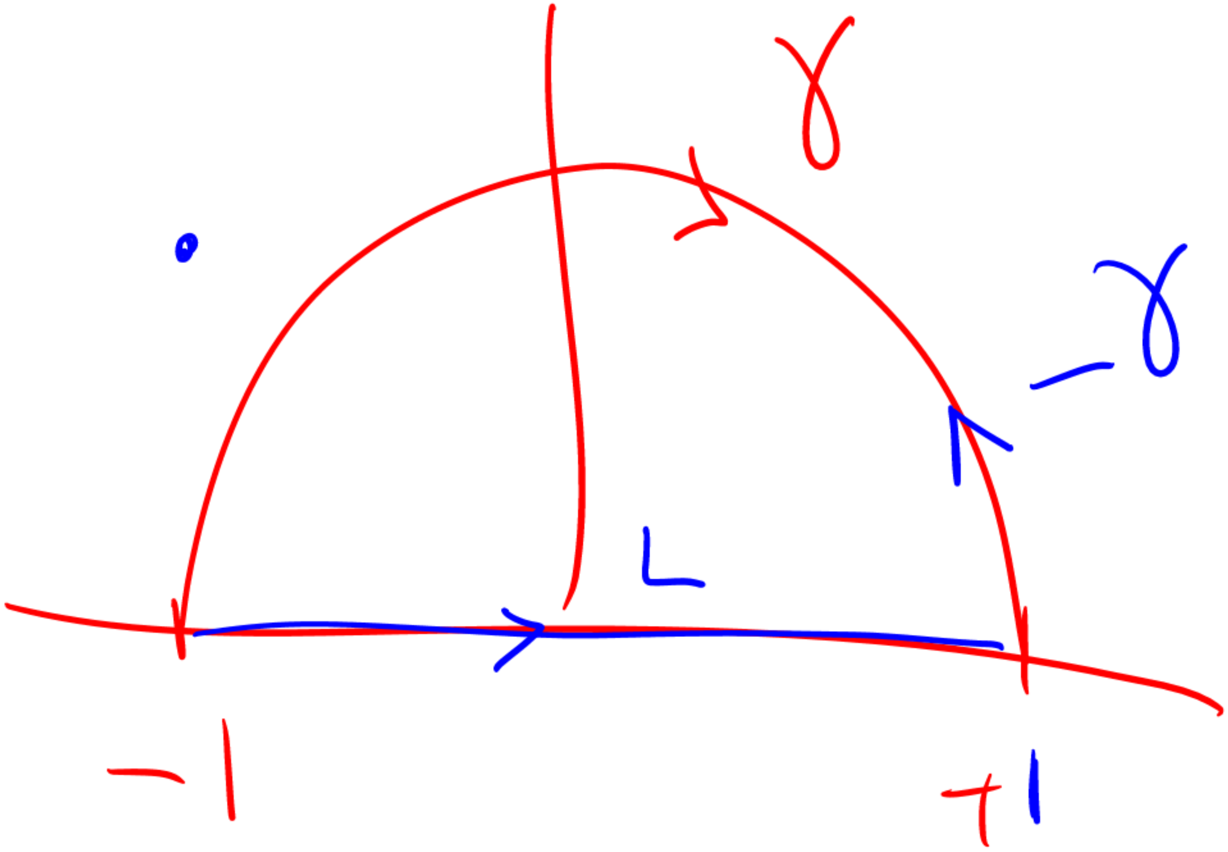
anticlockwise – +ve direction



$$\vec{OP} = \vec{OC} + \vec{CP}$$

$$z_0 + re^{it}$$


c
a
b

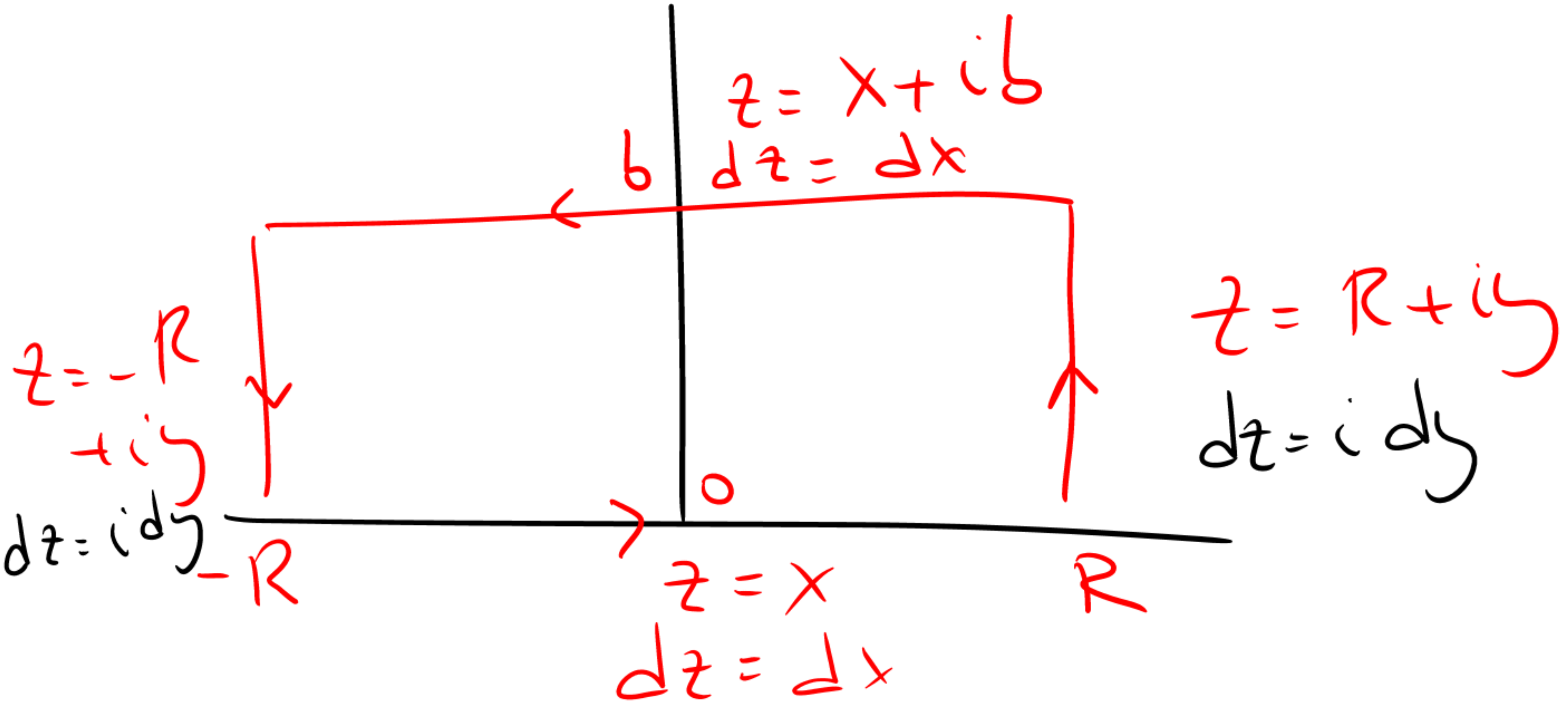


$L-\gamma$ closed.

To show $\displaystyle\int_0^\infty e^{-x^2}\cos 2bx\,dx = \frac{\sqrt{\pi}}{2}e^{-b^2}$

Note: e^{-z^2} has no singularities in the finite plane

$\therefore$ by Cauchy's Th$\underline{\text{m}}$ $\displaystyle\int e^{-z^2}\,dz = 0$



$$\int_{-R}^{R} e^{-x^2}\,dx + \int_{0}^{b} e^{-(R+iy)^2}\, i\,dy + \int_{+R}^{-R} e^{-(x+ib)^2}\,dx$$

$$+ \int_{b}^{0} e^{-(-R+iy)^2}\, i\,dy = 0$$

Now let $R \to \infty$

$$\int_{-\infty}^{\infty} e^{-x^2}\,dx + \lim_{R\to\infty} \int_{0}^{b} e^{-(R+iy)^2}\, i\,dy - \lim_{R\to\infty} \int_{0}^{b} e^{-(-R+iy)^2}\, i\,dy$$

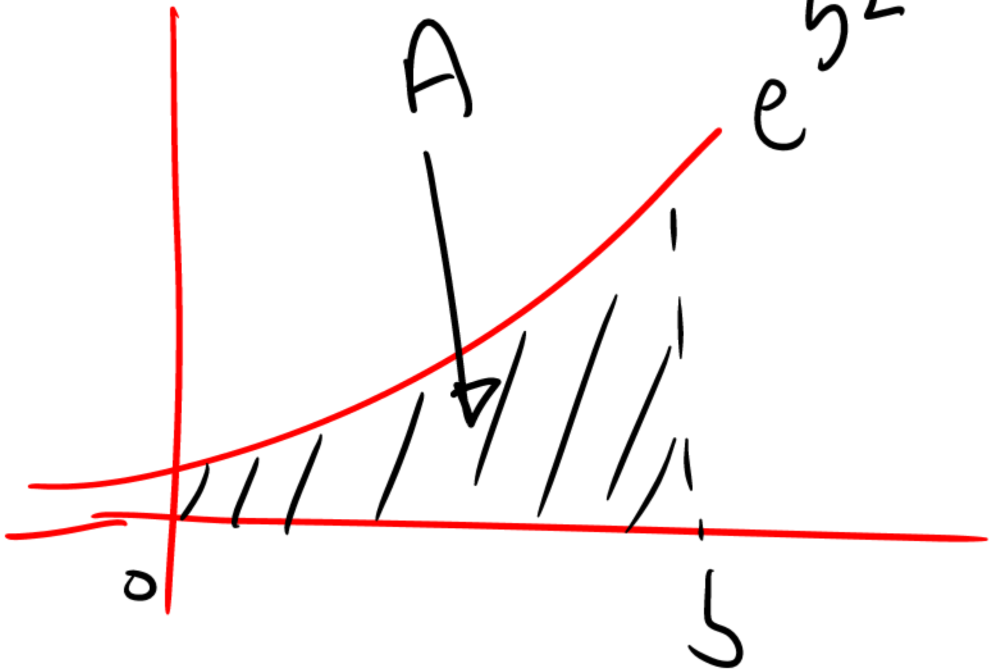
$$\int_{-\infty}^{\infty} e^{-(x+ib)^2}\,dx = 0$$

$$\left|\int_0^{\delta} e^{-(R+iy)^2}\, i\, dy\right| \le \int_0^{\delta} \left|e^{-(R+iy)^2}\, i\right| dy$$

$$= \int_0^{\delta} \left|e^{-(R^2+2iRy-y^2)}\right| dy$$

$$= \int_0^{\delta} \left|e^{-R^2}\right| \boxed{\left|e^{-2Ryi}\right|} \left|e^{y^2}\right| dy$$

$$= e^{-R^2} \int_0^{b} e^{y^2}\, dy = A e^{-R^2}$$



And $Ae^{-R^2} \to 0$ as $R \to \infty$

$$\left| \int_0^y e^{-(-R+iy)^2}\, i \, dy \right| \leq \int_0^y \left| e^{-R^2+y^2+2iRy} \right| dy$$

$$= e^{-R^2} \int_0^y e^{y^2}\, dy = Ae^{-R^2} \to 0 \text{ as } R \to \infty$$

$$\therefore \int_{-\infty}^{\infty} e^{-x^2}\,dx - \int_{-\infty}^{\infty} e^{-(x+ib)^2}\,dx = 0$$

$$-x^2+b^2-2ibx$$

$$2\underbrace{\boxed{\int_0^{\infty} e^{-x^2}\,dx}}_{}^{\sqrt{\pi}/2} - e^{b^2}\int_{-\infty}^{\infty} e^{-x^2}\left(\cos 2bx - i\sin 2bx\right)dx = 0$$

Equate Real parts

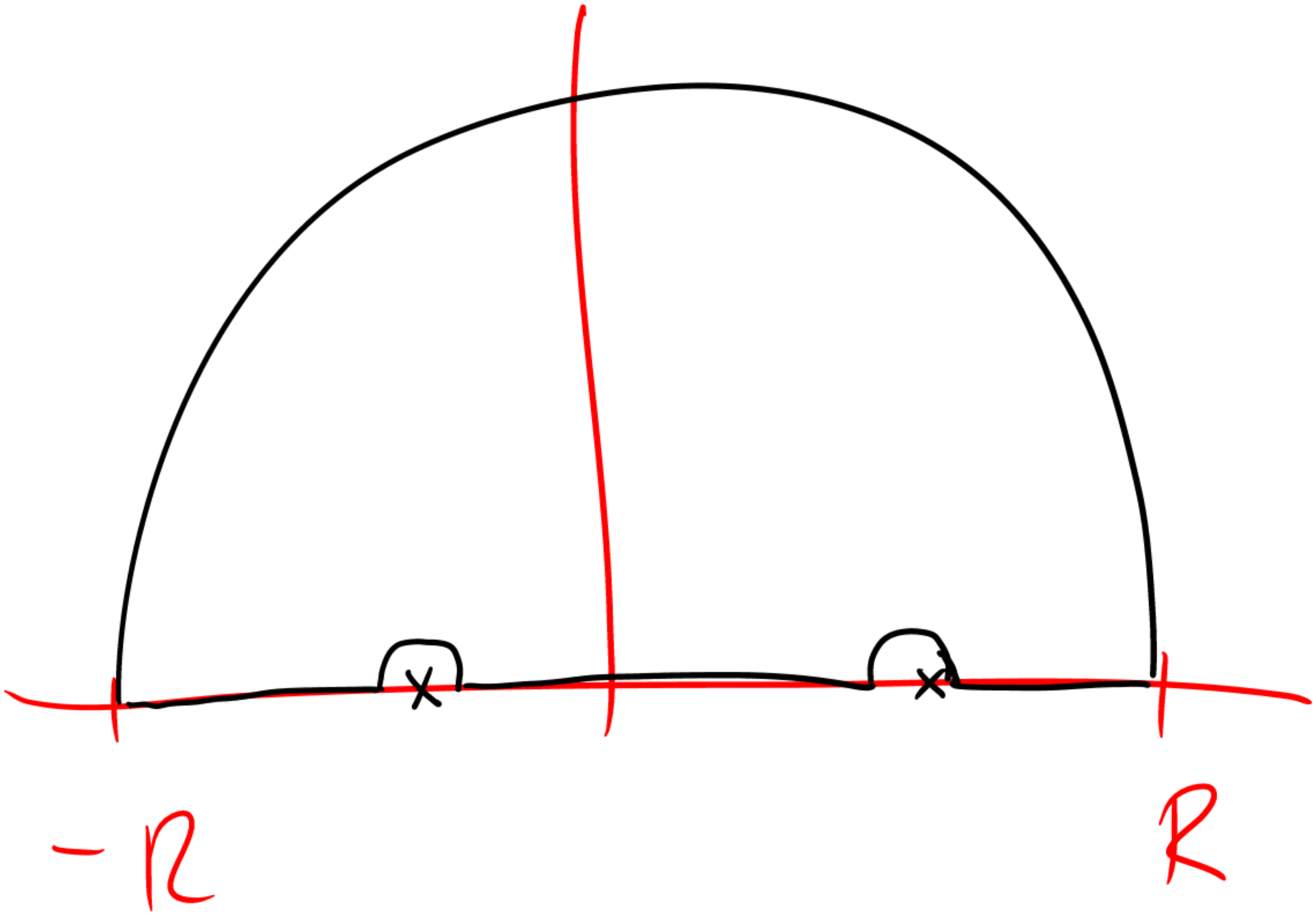
$$\sqrt{\pi} = 2e^{b^2}\int_0^{\infty} e^{-x^2}\cos 2bx\,dx$$

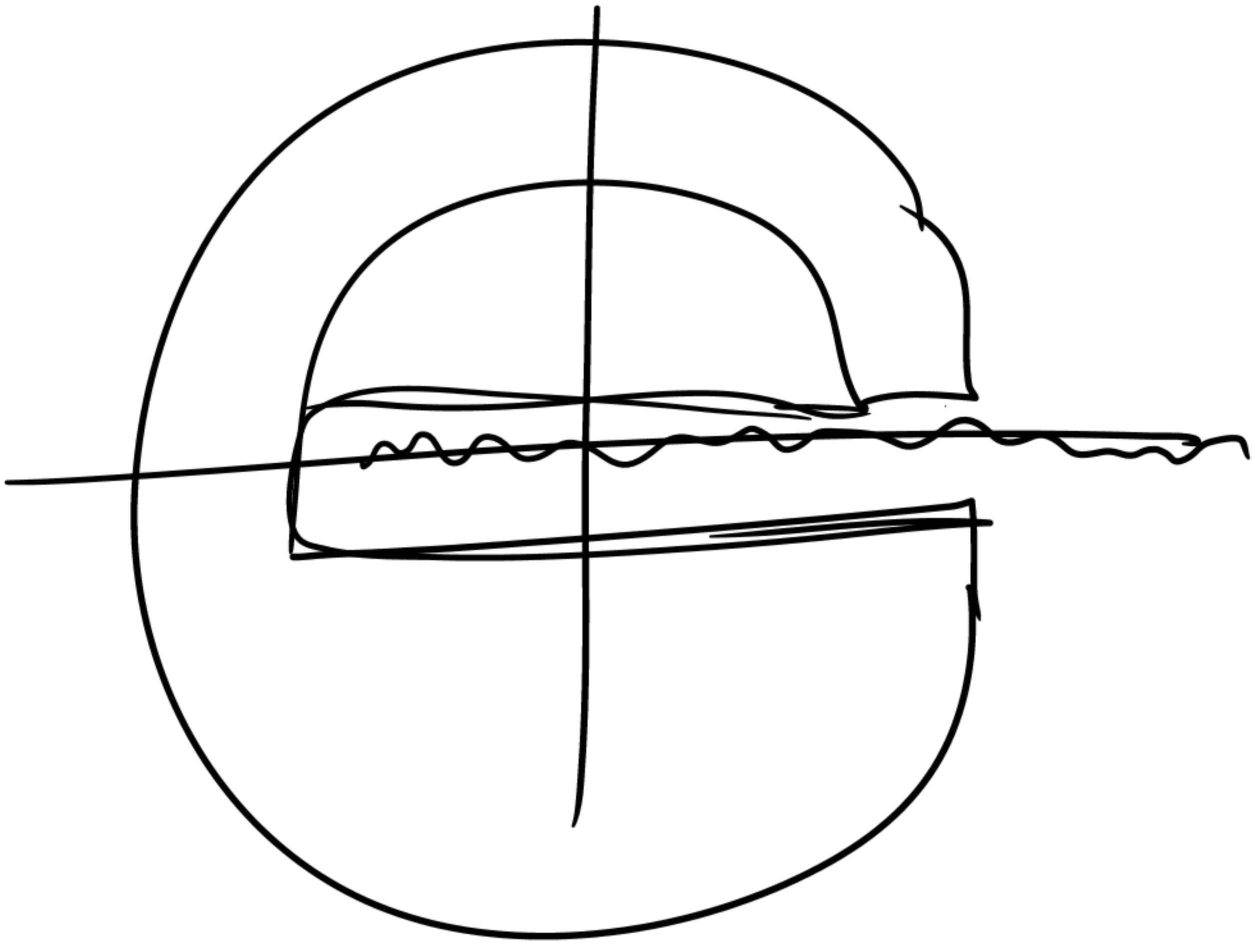
$$\boxed{\int_0^\infty e^{-x^2}\cos 2bx\,dx = \frac{\sqrt{\pi}}{2}e^{-b^2}}$$

$$\boxed{\int_{-\infty}^{\infty} e^{-x^2}\,dx = \sqrt{\pi}}$$

$\mathbb{C}$ integration

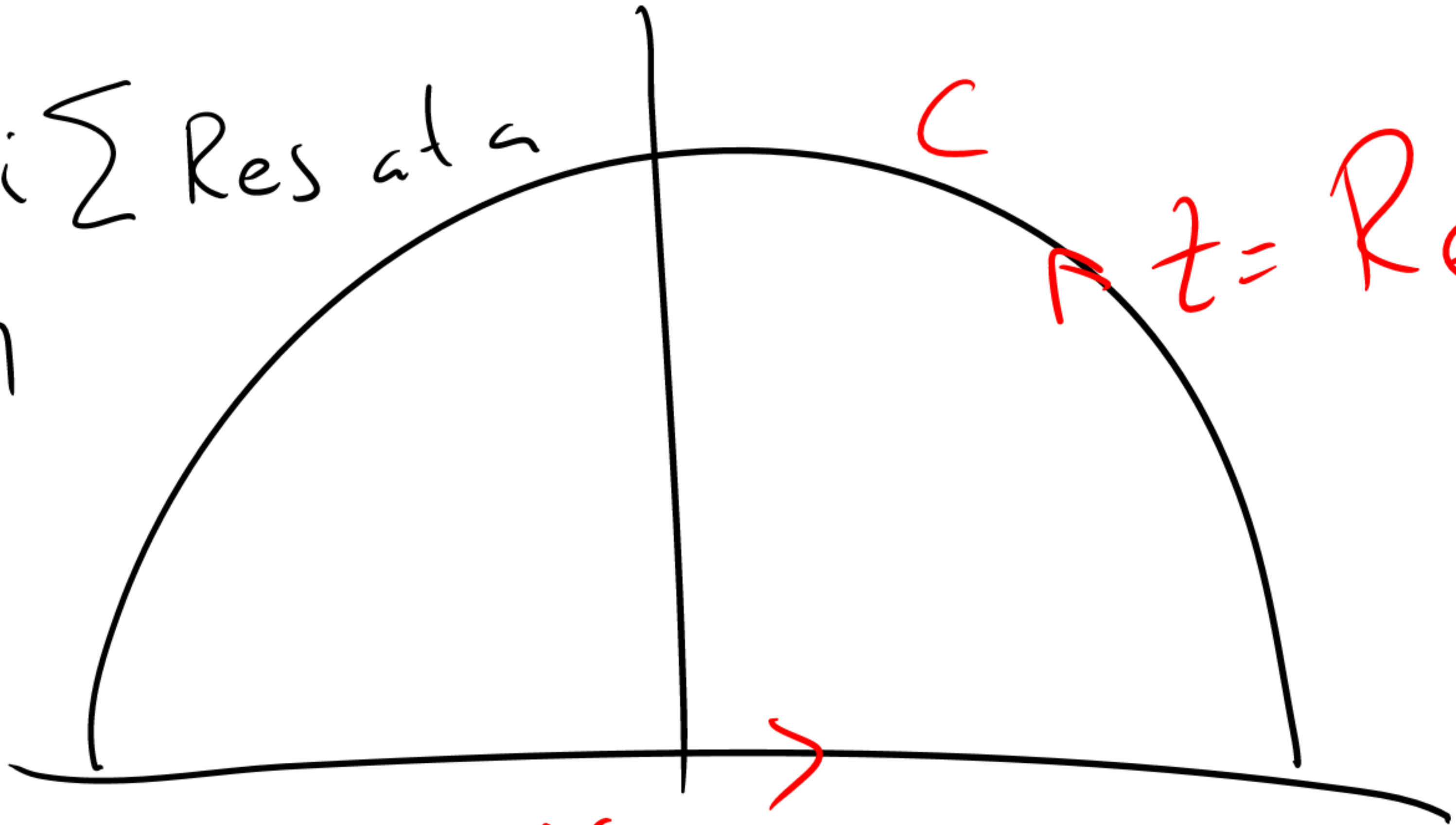
$$f(x) = \frac{1}{1-x} \qquad (1-x)^{-1}$$


-R
R



$\Gamma = C + \gamma$

$2\pi i \sum \text{Res at } a$



$z = Re^{i\theta}$

$0 \leq \theta \leq \pi$

$z = x$

$$\boxed{\int_\Gamma f(z)\,dz} = \underbrace{\int_C \cancel{f(x)}\,dx}_{\to 0} + \lim_{R\to\infty}\int_{-R}^{R} f(x)\,dx$$

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$$ $\exists$ sol^n of the form

$$u(x,t) = X(x)\,T(t) \rightarrow u_t = XT'$$

$$u_{xx} = X''T$$

$$\frac{T'}{T} = \frac{X''}{X} = \text{Function} = \text{const.}$$

depend t only   depend on x only

General Rule of Thumb

① Traded asset, replace risk-neutral ~~[illegible]~~ drift i.e r

$$\mu \longrightarrow r$$

$$\frac{dS}{S} = \mu\, dt + \sigma\, dW, \quad \mu \longrightarrow r$$

② Non-traded asset
Real drift $-$ MPOR $\times$ volatility

$$dr = u\,dt + w\,dW^{\mathbb{P}}$$

$$dr = (u - \lambda w)\,dt + w\,dW^{\mathbb{Q}}$$

real drift

MPoR

vol

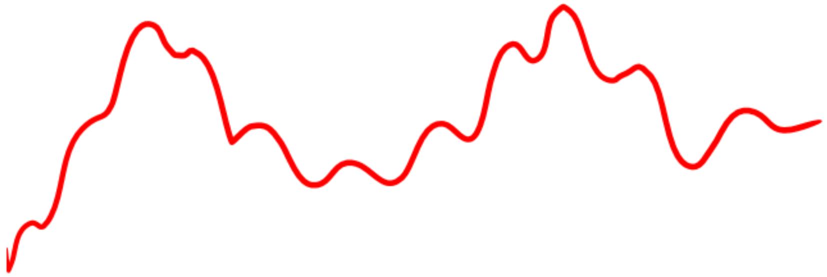
$$dV = w\frac{\partial V}{\partial r}dW + \underbrace{\left[\frac{\partial V}{\partial t} + \frac{1}{2}w^2\frac{\partial^2 V}{\partial r^2} + u\frac{\partial V}{\partial r}\right]}_{rV + \lambda w \frac{\partial V}{\partial r}} dt$$

$$\underbrace{dV}_{\text{unhedged bond}} - \overbrace{rV\,dt}^{\text{risk-free}} = \underbrace{w\frac{\partial V}{\partial r}\left[dW + \lambda dt\right]}_{}$$

$$dr = u\,dt + w\,dW \longrightarrow (u - \lambda w)\,dt + w\,dW$$

$$d\sigma = p\,dt + q\,dW_2 \longrightarrow d\sigma = \boxed{(p - \lambda q)}\,dt + q\,dW_2$$

$$(p - \lambda q)\frac{\partial V}{\partial \sigma}$$



$F = ma$

$\boxed{f = ma}$