CQF Module 2 Examination (Exam 1)

June 2019 Cohort

Instructions

Answers to all questions **are required**. Complete mathematical and computational workings must be provided. ALL ANSWERS MUST BE IN THE PDF FILE INCLUDING GRAPHS. Excel and code files, if more than one file, have to be uploaded as ZIP. Please use LASTNAME_ to start file name of PDF and ZIP. Portal, upload and extension questions to Orinta.Juknaite@fitchlearning.com.

All tasks on this exam are possible to solve in Excel. Portfolio computation to use array functions *MMULT(), MINV()*. VaR backtesting uses *IF()*. Computation in Python, Matlab or R is encouraged.

Marking Scheme: Q1 30% **Q2** 20% **Q3** 40% **Q4** 10%

Optimal Portfolio Allocation

Consider an investment universe of the following risky assets with a dependence structure (correlation). Use the ready appropriate formulae from Portfolio Optimisation Lecture/Webex and data below.

Asset	$\boldsymbol{\mu}$	$\boldsymbol{\sigma}$	$\boldsymbol{w}$
A	0.02	0.05	w_1
B	0.07	0.12	w_2
C	0.15	0.17	w_3
D	0.20	0.25	w_4

$$\boldsymbol{R} = \begin{pmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.3 & 1 & 0.6 & 0.6 \\ 0.3 & 0.6 & 1 & 0.6 \\ 0.3 & 0.6 & 0.6 & 1 \end{pmatrix}$$

Question 1. Consider the optimization for a target return m.

$$\underset{\boldsymbol{w}}{\operatorname{argmin}} \frac{1}{2}\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w} \qquad \text{s.t. } \boldsymbol{w}'\mathbf{1} = 1, \qquad \mu_\Pi = \boldsymbol{w}'\boldsymbol{\mu} = m$$

- Formulate the Lagrangian function and give its partial derivatives. No further derivation required.
- Compute the allocations $\boldsymbol{w}^*$ and portfolio risk $\sigma_\Pi = \sqrt{\boldsymbol{w}'\boldsymbol{\Sigma}\boldsymbol{w}}$, for $m = 4.5\%$.
- Stress correlation matrix (multiply all correlations by $\times 1.25$ and $\times 1.5$) and respectively compute the allocations and risk for the same $m = 4.5\%$.
- Inverse optimisation: randomly generate $> 1,500$ allocations $\boldsymbol{w}$, compute σ_Π and μ_Π, and plot. Most points will not be on the Efficient Frontier but the plot will reveal Markowitz bullet.

Question 2. Consider Tangency Portfolio problem.

- Formulate optimisation, the Lagrangian function and give its partial derivatives only.
- For the range of tangency portfolios given by $r_f = 50bps, 75bps, 100bps, 150bps, 175bps$ compute allocations and σ_Π. Present results in a table.

VaR Backtesting

Question 3. As a market risk analyst, each day you calculate VaR from the available prior data. Then, you wait ten days to compare your prediction value VaR_{t-10} to the realised return and check if the prediction about the worst loss was breached. You are given a dataset with *Closing Prices*.

- Implement VaR backtesting by computing 99%/10day Value at Risk *on rolling basis* using standard deviation from 22 returns and appropriate Factor value. (a) Report the percentage of VaR breaches and (b) number of consecutive breaches. (c) Provide a plot which clearly identifies breaches.

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- For comparison, implement backtesting on rolling basis using EWMA on variance. EWMA recursive calculation is done in the same way as GARCH, and σ_{t+1} is effectively a predicted volatility.

$$\sigma_{t+1}^2 = \lambda \sigma_t^2 + (1-\lambda) r_{t-1}^2$$

  with the value $\lambda = 0.72$ which is smaller than suggested by original RiskMetrics methodology but minimises out of sample forecasting errors. Provide the same deliverables (a), (b), and (c).

- Extreme Value Theory was applied, so that tail index $\xi = 0.4$ and scale parameter $\beta = 0.00002$ were fitted from the tail losses. Compute *one-off* EVT VaR for the entire dataset.

$$\text{VaR}_c = u + \frac{\beta}{\xi}\left[\left(\frac{N}{N_u}(1-c)\right)^{-\xi} - 1\right]$$

  where $c = 95\%$ confidence, $N_u = 62$ is the number of exceedances among $N = 1249$ observations (returns), and $u = -0.012617$ is chosen threshold for loss.

Question 4. Liquidity-Adjusted VaR (LVaR) Express your results in terms of the proportion (percentage) attributed to the classical VaR and proportion attributed to the effect of the liquidity adjustment. Formula below takes positive value of the Standard Normal Factor.

$$\begin{aligned}
\text{LVaR} &= \text{LVaR} + \Delta_{Liquidity} \\
&= \text{Portfolio Value} \times \left[-\mu + \text{Factor} \times \sigma + \frac{1}{2}(\mu_{Spread} + \text{Factor} \times \sigma_{Spread})\right]
\end{aligned}$$

(a) Consider a portfolio of USD 16 million composed of shares in a technology company. Daily mean and volatility of its returns are 1% and 3%, respectively. Bid-ask spread also varies with time, its daily mean and volatility are 35 bps and 150 bps. Compute 99%/1D LVaR and attributions to it,

(b) Now consider GBP 40 million invested in UK gilts. Take the daily volatility of portfolio returns as 3% and bid-ask spread is 15 bps (no spread volatility). Compute 99%/1D LVaR and attributions. What would happen if the bid-ask spread increases to 125 bps?

Further Instructions

Clarifying only questions on tasks are welcome to Richard.Diamond@fitchlearning.com. Please make good use of lecture notes and particularly, Solutions. The tutor is unable to confirm numerical answers and methods of calculation/spreadsheets. WEBEX sessions were run on VaR Statistical Essentials and Portfolio Optimisation (Matrix Algebra), you will find them relevant.

To compute the 99%/10day Value at Risk for an investment in the market index on the rolling basis. We drop the expected return (mean) from the VaR formula

$$\text{VaR}_{10D,t} = \text{Factor} \times \sigma_t \times \sqrt{10}$$

- Appropriate Factor value to be used (Standard Normal Percentile), the tutor will not confirm the numerical value. It is also your task to identify the eligible number of observations for which VaR is available and can be backtested: N_{obs} will not be confirmed.
- Compute a column of rolling standard deviation over log-returns for observations $1-21, 2-22, \ldots$. Compute VaR for each day t, after the initial period. This is your worst loss prediction.
- **Regardless** of how many observations there are in a sample (10, 21, 100, etc.), variance is *an average of squared daily differences* $\frac{\sum(r_t-\mu)^2}{(N-1)}$ and so, timescale remains 'daily'.
- VaR is fixed at time t and compared to the return realised from t to $t+10$. A breach occurs when that forward realised 10-day return $\ln(S_{t+10}/S_t)$ is below the VaR_t quantity.

  $r_{10D,t+10} < \text{VaR}_{10D,t}$ means breach, given both numbers are negative.

  In Excel, you will have a column for VaR_t series, a column of $r_{10D,t+10}$ series, and indicator column $\{0,1\}$ for a breach using $IF()$ function.
- To obtain the conditional probability of breach $N_{conseq}/N_{breaches}$, identify consecutive breaches. For example, the sequence $1,1,1$ means two consecutive breaches occurred.
- As an extra (**not a requirement**), you can apply statistical tests to the issue of independence of breaches in VaR (eg, conditional coverage, Christoffersen's exceedance independence).

  Alternatively simply provide Q-Q plots for 1D returns and conclude if Normally distributed returns was a reasonable assumption. Lecture One (Module One) solutions give instruction on Q-Q plots.

END OF EXAM