

# Law of Large Numbers

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This script demonstrates the law of large numbers (LLN) along with the underlying assumptions.

Write a function to generate the sample mean given the sample size  $n$  and the distribution. We allow three distributions, namely,  $N(0, 1)$ ,  $t(2)$  and Cauchy.

```
sample_mean = function( n, distribution ){
  if (distribution == "normal"){ y = rnorm( n ) }
  else if (distribution == "t2") {y = rt(n, 2) }
  else if (distribution == "cauchy") {y = rcauchy(n) }
  return( mean(y) )
}
```

This function plots the sample mean over the path of geometrically increasing sample size.

```
LLN_plot = function(distribution){

  y_bar = rep(0, length(NN) )

  for ( i in 1:length(NN)){
    n = NN[i]
    y_bar[i] = sample_mean(n, distribution)
  }

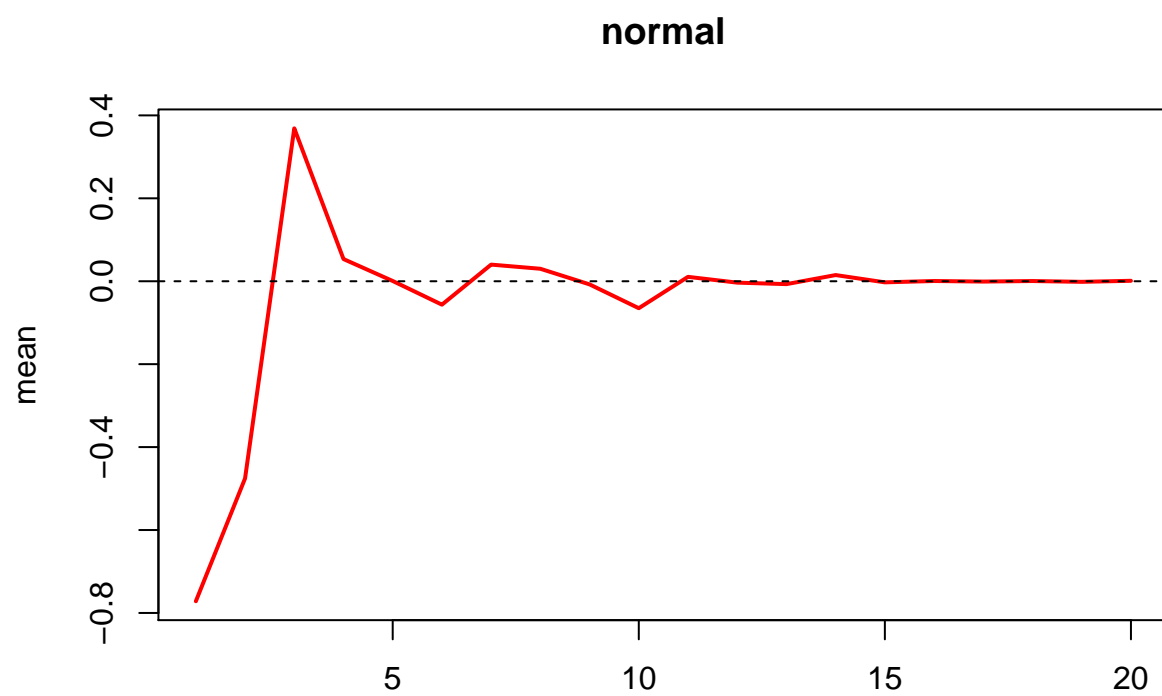
  plot(y_bar, type = "l", col = "red", ylab = "mean", xlab = "", lwd = 2, main = distribution)
  abline(h = 0, lty = 2)
  return(y_bar)
}
```

The sample size is chosen as  $2^x$ , where  $x = 1 : 20$ . We have the following observations.

- When the distribution is  $N(0, 1)$ , the Chebyshev LLN works. The sample mean converges fast.
- When the distribution is  $t(2)$ , which has zero mean but infinite variance, the Kolmogorov LLN works. The sample mean still converges, though more slowly than the  $N(0, 1)$  case.
- The Cauchy distribution has no moment at any order. The sample mean does not converge no matter how large is the sample size.

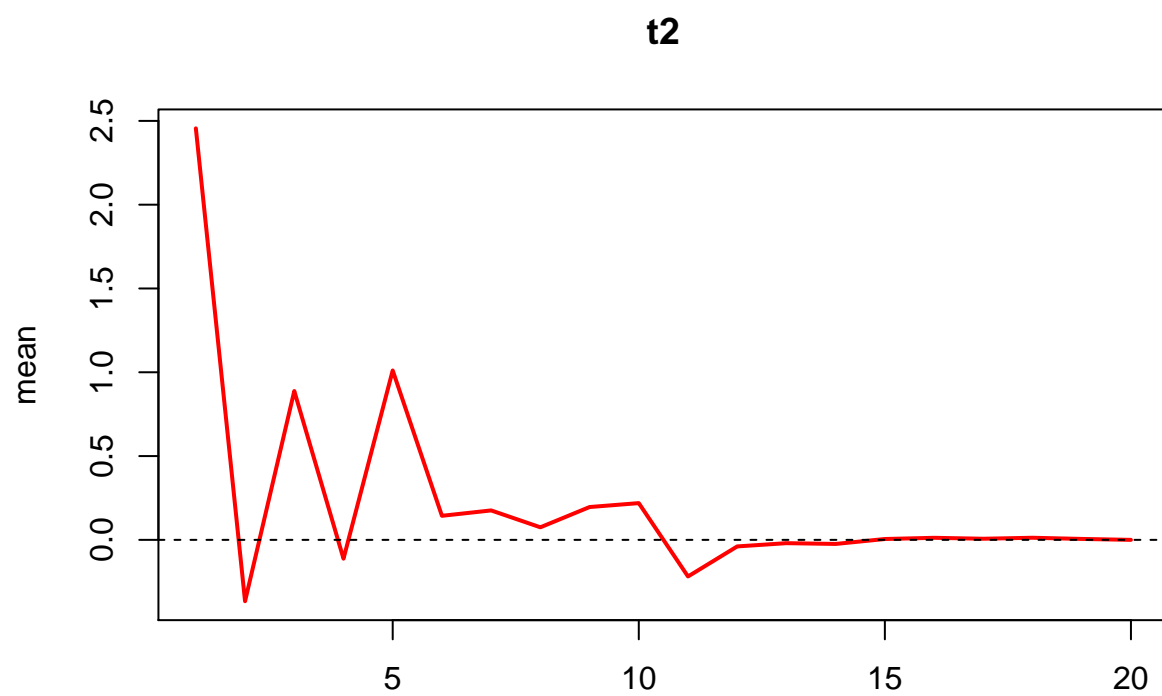
```
NN = 2^(1:20)
# set.seed(888)

LLN_plot("normal")
```



```
## [1] -0.7717494056 -0.4756059159  0.3687090773  0.0534537495  0.0005822760
## [6] -0.0565010649  0.0401188485  0.0300719151 -0.0075918164 -0.0650641832
## [11]  0.0105865936 -0.0033501961 -0.0070027534  0.0148673557 -0.0026434110
## [16]  0.0005619020 -0.0008214060  0.0005062376 -0.0014757358  0.0010627131
```

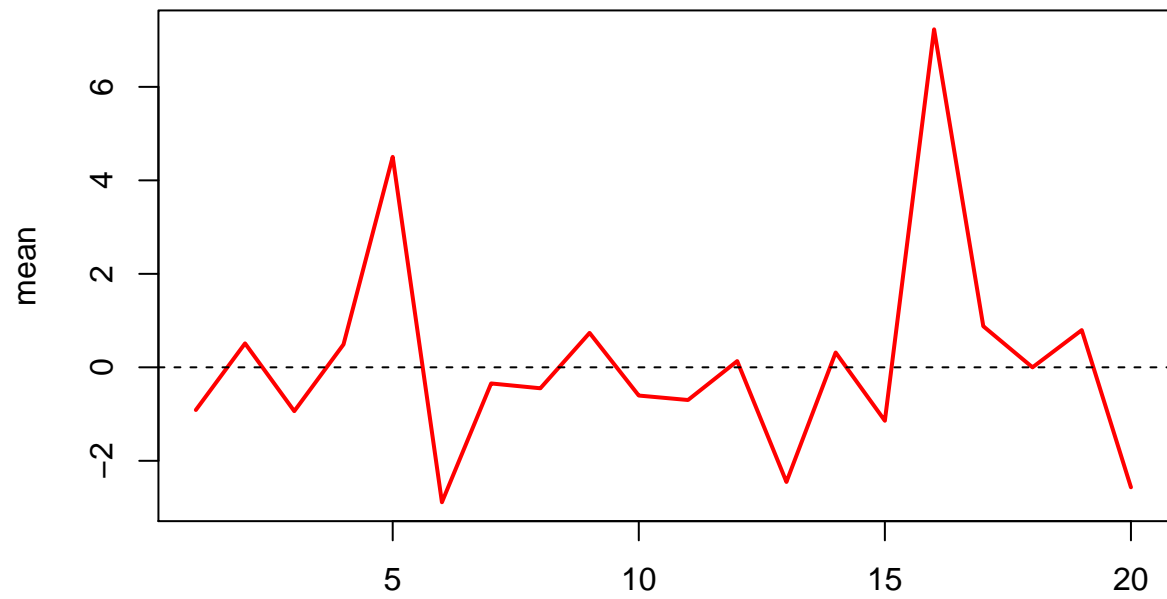
```
LLN_plot("t2")
```



```
## [1] 2.455484e+00 -3.662706e-01 8.879622e-01 -1.124155e-01 1.010695e+00
## [6] 1.434925e-01 1.755891e-01 7.489450e-02 1.958019e-01 2.192666e-01
## [11] -2.184678e-01 -3.915620e-02 -1.971898e-02 -2.439374e-02 5.144151e-03
## [16] 1.217046e-02 6.639690e-03 1.262349e-02 5.515494e-03 -9.663759e-05
```

```
LLN_plot("cauchy")
```

## cauchy



```
## [1] -0.9124382611  0.5121687524 -0.9377976405  0.4896786064  4.5017615809
## [6] -2.8861940797 -0.3451206983 -0.4483982857  0.7359634210 -0.6041106141
## [11] -0.6989490961  0.1329219062 -2.4522596315  0.3165542466 -1.1429400465
## [16]  7.2302316736  0.8794148492 -0.0004383475  0.7961802583 -2.5666203684
```