Algebra of OLS

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We demonstrate the OLS estimator and its algebraic properties

Generate the data.

```
n = 20 # sample size
K = 3  # number of paramters

b0 = as.matrix( c(0.5, 1, -1) ) # the true coefficient

X = cbind(1, matrix( rnorm(n * (K-1)), nrow = n ) )  # the regressor matrix
e = rnorm(n,1) # the error term

Y = X %*% b0 + e # generate the dependent variable
```

OLS estimator

```
bhat = solve(t(X)%*%X, t(X) %*% Y )
```

Calculate the estimate as $\hat{\beta} = (X'X)^{-1}X'Y =$ 1.7446292, 1.1389028, -1.0903429.

Residual

The residual $\hat{e} = Y - X'\beta$. Verify $X'\hat{e} = 0$.

```
ehat = Y - X %*% bhat

print( t(X) %*% ehat )
```

```
##               [,1]
## [1,] -5.329071e-15
## [2,]  1.942890e-15
## [3,]  4.773959e-15
```

Notice that

- $\sum_{i=1}^{n} e_i =$ 24.4225307,
- $\sum_{i=1}^{n} \hat{e}_i =$ -5.21805e-15.

Define P_X and M_X, and show $\hat{e} = M_X Y = M_X e$.

```r
PX = X %*% solve( t(X) %*% X) %*% t(X)
MX = diag(rep(1,n)) - PX
print( cbind( ehat, MX %*% Y, MX %*% e) )
```

```
##              [,1]         [,2]         [,3]
##  [1,]  0.187653869  0.187653869  0.187653869
##  [2,]  0.304539430  0.304539430  0.304539430
##  [3,] -2.076542407 -2.076542407 -2.076542407
##  [4,]  0.510331369  0.510331369  0.510331369
##  [5,] -0.142729755 -0.142729755 -0.142729755
##  [6,] -1.402221819 -1.402221819 -1.402221819
##  [7,] -0.613570376 -0.613570376 -0.613570376
##  [8,] -1.738474619 -1.738474619 -1.738474619
##  [9,] -1.451860378 -1.451860378 -1.451860378
## [10,]  0.899525924  0.899525924  0.899525924
## [11,]  1.378111735  1.378111735  1.378111735
## [12,]  0.260899210  0.260899210  0.260899210
## [13,]  1.055177120  1.055177120  1.055177120
## [14,]  2.027768795  2.027768795  2.027768795
## [15,] -0.839224006 -0.839224006 -0.839224006
## [16,] -0.004038138 -0.004038138 -0.004038138
## [17,]  0.299225513  0.299225513  0.299225513
## [18,]  0.735293883  0.735293883  0.735293883
## [19,]  0.079347597  0.079347597  0.079347597
## [20,]  0.530787053  0.530787053  0.530787053
```