Law of Large Numbers

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This script demonstrates the law of large numbers (LLN) along with the underlying assumptions.

Write a function to generate the sample mean given the sample size n and the distribution. We allow three distributions, namely, $N(0,1)$, $t(2)$ and Cauchy.

```
sample_mean = function( n, distribution ){
  if (distribution == "normal"){ y = rnorm( n ) }
  else if (distribution == "t2") {y = rt(n, 2) }
  else if (distribution == "cauchy") {y = rcauchy(n) }
  return( mean(y) )
}
```

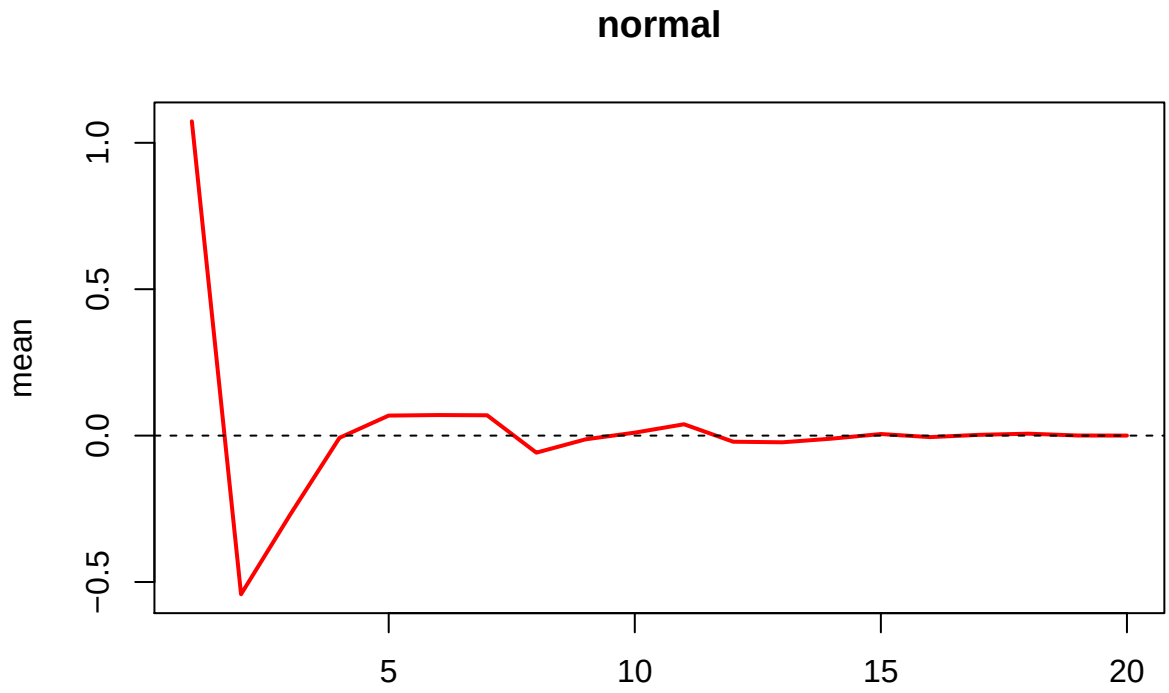
This function plots the sample mean over the path of geometrically increasing sample size.

```
LLN_plot = function(distribution){

  y_bar = rep(0, length(NN) )

  for ( i in 1:length(NN)){
    n = NN[i]
    y_bar[i] = sample_mean(n, distribution)
  }

  plot(y_bar, type = "l", col = "red", ylab = "mean", xlab = "", lwd = 2, main = distribution)
  abline(h = 0, lty = 2)
  return(y_bar)
}
```

The sample size is chosen as 2^x, where $x = 1:20$. We have the following observations.

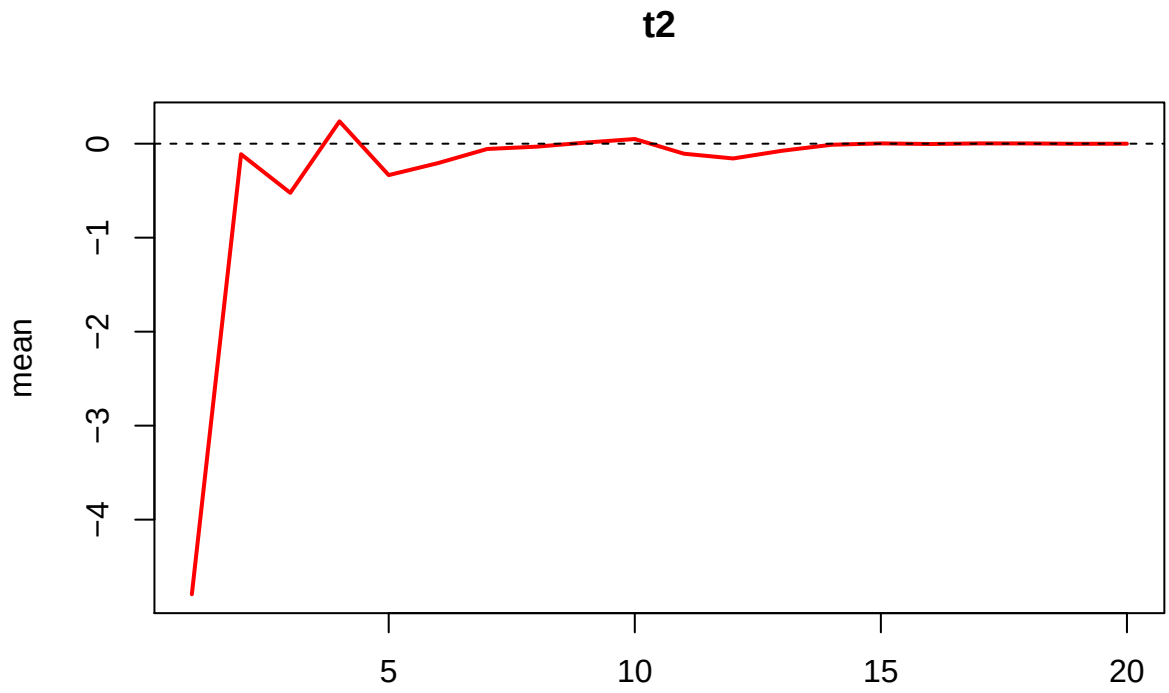
- When the distribution is $N(0,1)$, the Chebyshev LLN works. The sample mean converges fast.
- When the distribution is $t(2)$, which has zero mean but infinite variance, the Kolmogorov LLN works. The sample mean still converges, though more slowly than the $N(0,1)$ case.
- The Cauchy distribution has no moment at any order. The sample mean does not converge no matter how large is the sample size.

```
NN = 2^(1:20)
# set.seed(888)

LLN_plot("normal")
```



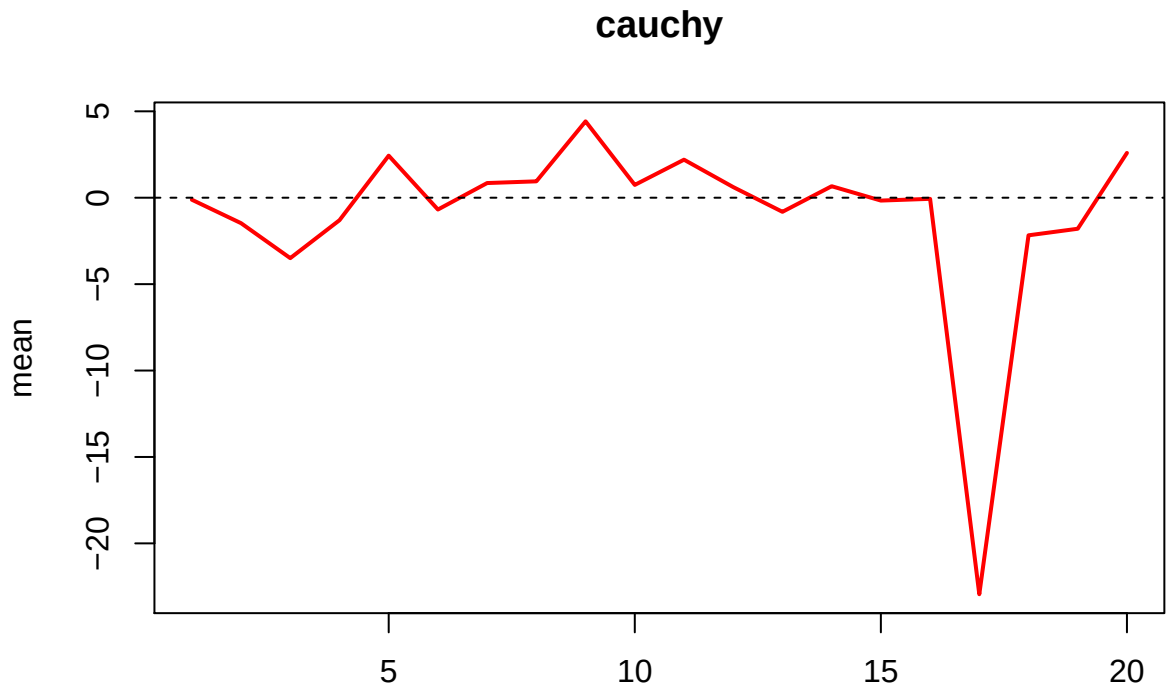

```
##  [1]  1.073250e+00 -5.417357e-01 -2.694184e-01 -7.486344e-03  6.839306e-02
##  [6]  7.019400e-02  6.945264e-02 -5.813986e-02 -1.270306e-02  1.011478e-02
## [11]  3.873818e-02 -2.088460e-02 -2.281571e-02 -1.036759e-02  5.376242e-03
## [16] -5.059390e-03  2.624790e-03  6.543929e-03  4.141748e-04  6.416357e-05
```

```
LLN_plot("t2")
```




```
##  [1] -4.7939309727 -0.1124550284 -0.5229967530  0.2376937778 -0.3342702544
##  [6] -0.2066910420 -0.0563865189 -0.0317164987  0.0116880986  0.0495973870
## [11] -0.1065343779 -0.1570533986 -0.0747409463 -0.0112520907  0.0031048498
## [16] -0.0035180191  0.0032218490  0.0024735646 -0.0012460367 -0.0002738821
```

```
LLN_plot("cauchy")
```




```
##  [1]  -0.11540499  -1.47297488  -3.49620150  -1.29996829   2.43442207
##  [6]  -0.68359027   0.84893986   0.94748657   4.41917687   0.74134461
## [11]   2.20129306   0.61667266  -0.81563831   0.66509952  -0.16374608
## [16]  -0.06356284 -22.94307437  -2.17455128  -1.79599427   2.59079040
```