Hypothesis testing. Lecture 7

$\theta = r(\beta)$. r is ~~the~~ restriction.

e.g. $r\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix} = \beta_1$ $\quad r\left(\begin{pmatrix}\beta_1\\ \beta_2\\ \beta_3\end{pmatrix}\right) = \beta_1/\beta_2$.

null hypothesis. $\theta = \theta_0$.

alternative $\theta \neq \theta_0$

T is small with large prob when Ho is true.

T is big with large prob when Ho is true.

test statistic. $T = T_n$ is a function of the data

e.g. t-statistic. $\dfrac{\hat\theta - \theta_0}{s(\hat\theta)}$.

	Accept	reject
Ho true	C	type I
False	type II	C

~~truth~~	~~Ho true~~	~~Ho false~~
~~decision Accept~~	~~Accept correct~~	~~Type I error~~
~~reject~~	~~reject~~ ~~Type II error~~	~~correct~~

Critical value c.

if $T_n > c$, reject.

Type I: false rejection

pre-specify a significance level 0.05.

How to decide c?

find the asy distribution ~~taken~~ under the null

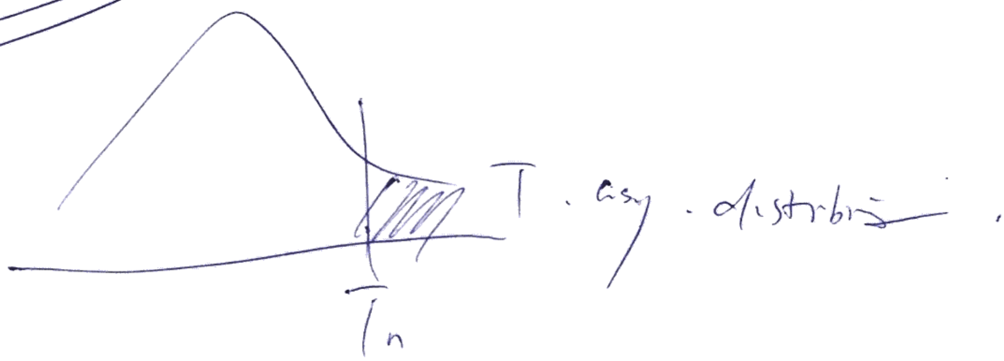
better to say "Do not reject"

Do minimize practice. under given prob of Type I error, min. Type II

Reject prob under the alternative is "Power".

$\pi_n(\theta) = \Pr(\text{reject } H_0 \mid H_1 \text{ true})$

p-value: the prob that we observe a ~~stat~~ test statistic $\geq T_n$.



reject the null under α-significance level if

p-value $\leq \alpha$.

Interpretation:

Cannot say H_1 or H_1 is true with some prob.

For frequent, either H_1 or H_0 is true with certainty.

probability is only about the decision is right or wrong.

Wald test:

a generalize vector vector of T-test. delta method

$$W_n = (\hat{\theta} - \theta_0)' \hat{V}_{\hat{\theta}}^{-1} (\hat{\theta} - \theta_0)$$

if the restriction is linear $R\beta = r$.

$$W_n = (R\hat{\beta} - r)' (R \hat{V}_{\hat{\beta}} R')^{-1} (R\hat{\beta} - r)$$

y β_1 β_2 β_3

H_0: $\beta_1 = 0$

$\beta_2 + \beta_3 = 1.$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ & & \end{bmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Thm: $W_n \xrightarrow{d} \chi^2_q$.

$\mathrm{rank}(V) = q$

Lemma: for k dimension $Z \sim N(0, V)$

$$Z' V^{-1} Z \sim \cancel{\chi^2(\mathrm{rank}(V))}\ \chi^2_q.$$

~~because~~ because $V^{-\frac{1}{2}} Z \sim N(0, I_k)$

$$(V^{-\frac{1}{2}} Z)' V^{-\frac{1}{2}} Z \sim \cancel{N(0)}\ \chi^2(k)$$

An alternative criterion: the descrepancy between the criterion function with or without the restriction.

① minimum distance Test

② likelihood ~~ratio~~ ratio test.

minimum distance.

MD estimator

$$J_n(\beta) = n(\hat{\beta} - \tilde{\beta}_\beta)' W_n (\hat{\beta} - \tilde{\beta})$$

check the distance of two estimators

with or without the restriction.

Efficient weighting matrix: $W_n = \hat{V}_\beta^{-1}$.

β is under the true value. Under the true value,

what is the weight for each element.

Otherwise, the asy. distribution is unknown.

Likelihood ratio test.

$$LR_n = 2(\log L_n(\hat{\theta}) - \log L_n(\tilde{\theta}))$$

restricted vs unrestricted θ.