Best linear predictor.

Lecture 2 (Shenzhen) 2nd.

$m(X) = E(y|X)$ in general is unknown, and may not be linear. The linear CEF is just an approximation.

~~Assumption~~: $Q_{XX} = E(XX')$ is positive def and finite.

~~Def~~: target: minimize $S(\beta) = E(y - X'\beta)^2$

the mean square error:

$y - X'\beta$ is the error is a r.v.

$(y - X'\beta)$ is a distance

$E(\quad \downarrow \quad)$ is ~~the~~ a constant.

The BLP of y given X $P(y|X) = X'\beta$.

Derivation: ~~$S(\beta) = E(y)^2 - E((y \cdot X'\beta)) + \beta E(XX)\beta$~~,

$\frac{\partial S}{\partial \beta} =$ ~~$E(y - X\beta)$~~ $2E((y - X'\beta)X') = 0$.

$\beta = (E(X'X))^{-1} E(X'y)$.

$e = y - x\beta$ is defined as the regression error.

by def. $E(xe) = 0$.

because $E[x(y - xE(x'x)^{-1}E(x'y))]$

$= E(xy) - E(xx)^{-1}E(x'x)^{-1}E(x'y)) = 0$,

when the first regressor is a constant,

it implies $E(e) = 0$.

When no constant, not necessary the case. It is desirable to have a constant.

The linear model is general ~~It doesn't have to be~~

CEF doesn't have to be linear. Linear Projection still exists.

The next page: sub vector, omit variable bias.

Regression coefficient (constant)

separate the constant from other regressor.

① $y = X'\beta + \alpha + e$, X has no constant.

~~take expecte~~

② $Ey = EX'\beta + E\alpha + Ee = EX'\beta + \alpha + 0$.

~~so. $\alpha = Ey - (EX)\beta$~~

~~Substitute α back.~~

~~$y - Ey = (X - EX)\beta + 0 + e$~~

Substract ② from ① so that α is cancelled.

$$y - Ey = (X - EX)\beta + e.$$

$X - \mu_x$ and e are uncorrelated

by the linear projection formula

$$\beta = \left(E\left[(X - EX)(X - EX)'\right]\right)^{-1} E\left[(X - EX)(y - Ey)\right]$$

$$= \left[\mathrm{var}(X)\right]^{-1} \mathrm{cov}(X, y)$$