

# 1 Review of Probability

## 1.1 Probability

- Measure space  $(\Omega, \mathcal{F})$ .
- Probability space  $(\Omega, \mathcal{F}, P)$ .
- Random variable  $X : \Omega \mapsto \mathbb{R}$  is a measurable function from  $(\Omega, \mathcal{F})$  and  $(\mathbb{R}, \mathcal{B})$ .
- Cumulative distribution function
- Probability density function
  - Univariate random variable
  - Bivariate random variable
  - Multivariate random variable
- Conditional distribution

## 1.2 Expectation

- Expectation is an average of random variable with respect to a measure.
- Conditional expectation
- Law of iterated expectation:  $E[E[Y|X]] = E[Y]$ .
- Properties of conditional expectations
  1.  $E[E[Y|X_1, X_2] | X_1] = E[Y|X_1]$
  2.  $E[E[Y|X_1] | X_1, X_2] = E[Y|X_1]$
  3.  $E[h(X) Y | X] = h(X) E[Y | X]$