Law of Large Numbers

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This script demonstrates the law of large numbers (LLN) along with the underlying assumptions.

Write a function to generate the sample mean given the sample size n and the distribution. We allow three distributions, namely, $N(0,1)$, $t(2)$ and Cauchy.

```
sample_mean = function( n, distribution ){
  if (distribution == "normal"){ y = rnorm( n ) }
  else if (distribution == "t2") {y = rt(n, 2) }
  else if (distribution == "cauchy") {y = rcauchy(n) }
  return( mean(y) )
}
```

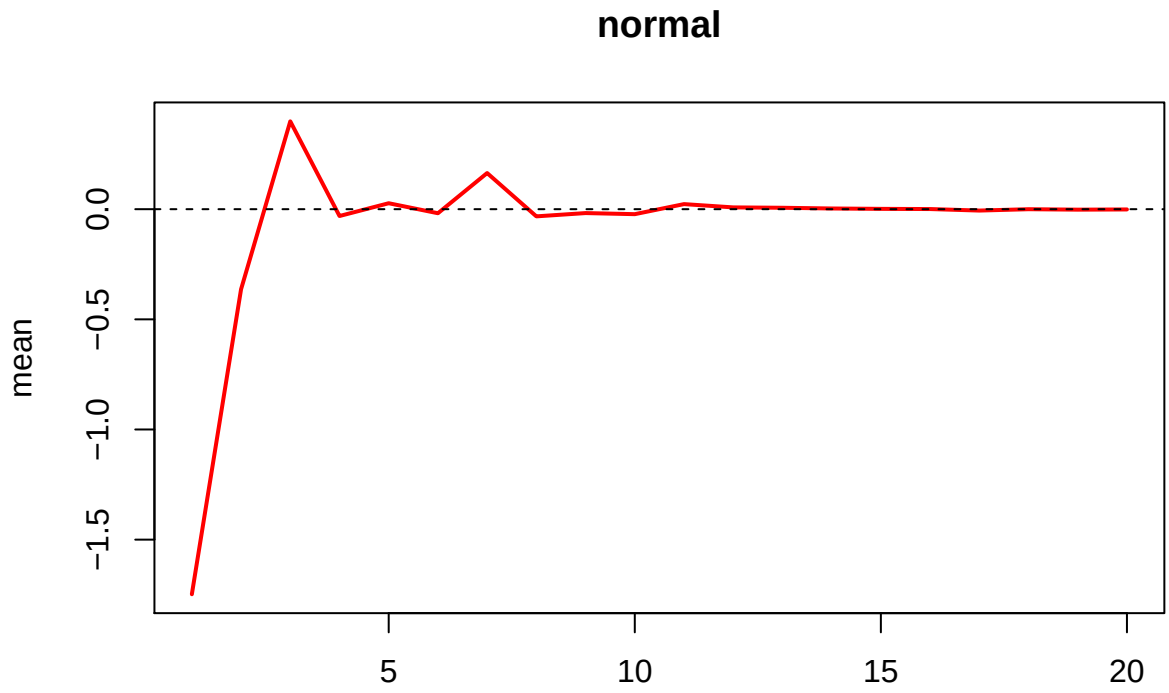
This function plots the sample mean over the path of geometrically increasing sample size.

```
LLN_plot = function(distribution){

  y_bar = rep(0, length(NN) )

  for ( i in 1:length(NN)){
    n = NN[i]
    y_bar[i] = sample_mean(n, distribution)
  }

  plot(y_bar, type = "l", col = "red", ylab = "mean", xlab = "", lwd = 2, main = distribution)
  abline(h = 0, lty = 2)
  return(y_bar)
}
```

The sample size is chosen as 2^x, where $x = 1:20$. We have the following observations.

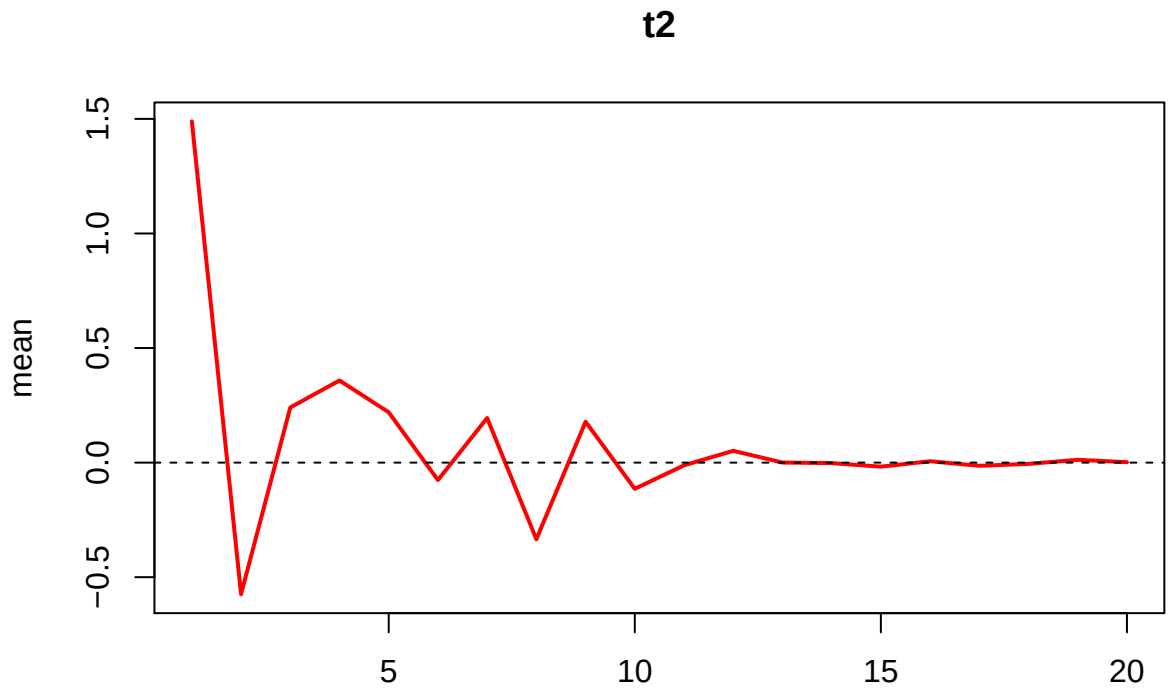
- When the distribution is $N(0,1)$, the Chebyshev LLN works. The sample mean converges fast.
- When the distribution is $t(2)$, which has zero mean but infinite variance, the Kolmogorov LLN works. The sample mean still converges, though more slowly than the $N(0,1)$ case.
- The Cauchy distribution has no moment at any order. The sample mean does not converge no matter how large is the sample size.

```
NN = 2^(1:20)
set.seed(888)

LLN_plot("normal")
```



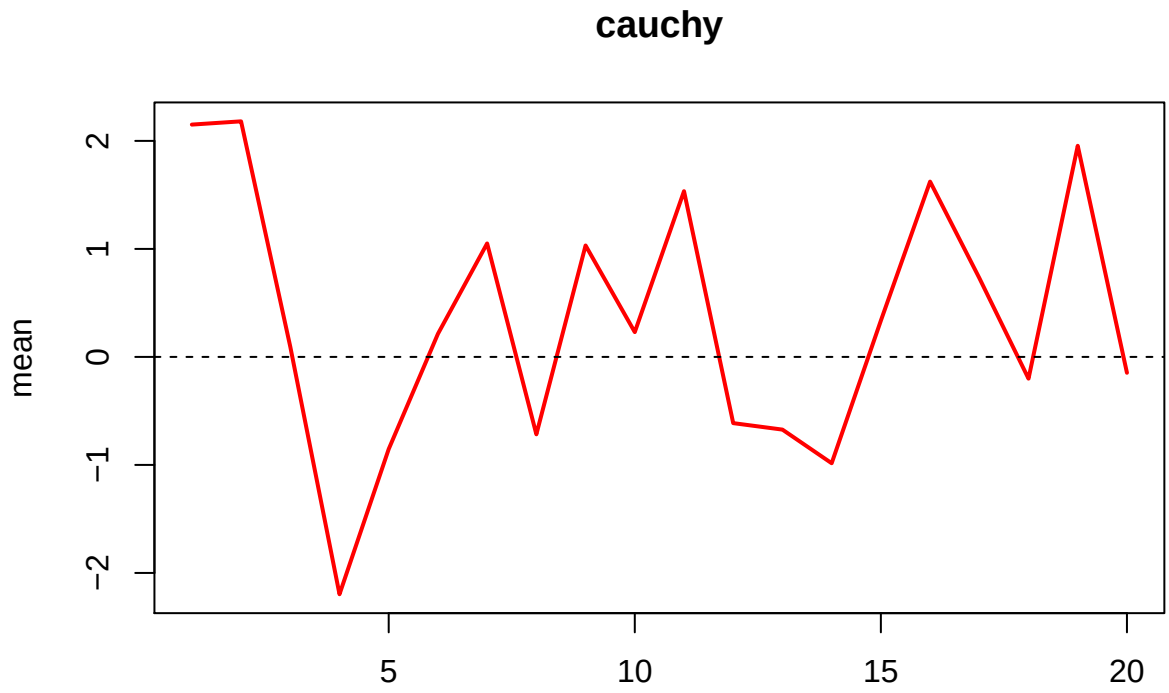

```
##  [1] -1.747855e+00 -3.637642e-01  3.986830e-01 -3.103209e-02  2.703446e-02
##  [6] -1.837923e-02  1.640634e-01 -3.261440e-02 -1.767885e-02 -2.265788e-02
## [11]  2.299199e-02  8.170195e-03  6.455310e-03  3.170345e-03  1.402089e-03
## [16]  6.296362e-04 -6.252846e-03 -8.619267e-05 -2.130231e-03 -1.176901e-03
```

```
LLN_plot("t2")
```




```
##  [1]  1.4892673417 -0.5744131768  0.2405682944  0.3585470349  0.2196349165
##  [6] -0.0762972932  0.1947510596 -0.3347942419  0.1784276509 -0.1144624890
## [11] -0.0118276309  0.0514328599  0.0002773015 -0.0021086253 -0.0177449848
## [16]  0.0059950669 -0.0139132184 -0.0056334633  0.0122718764  0.0019012487
```

```
LLN_plot("cauchy")
```




```
##  [1]  2.15081817  2.18096276  0.09466019 -2.19728118 -0.85354178
##  [6]  0.21363641  1.05059724 -0.71674645  1.03164250  0.22961519
## [11]  1.53445750 -0.61294086 -0.67305109 -0.98443164  0.33575069
## [16]  1.62310464  0.73089010 -0.20130535  1.95462813 -0.14691187
```