Empirical Asset Pricing

Part 6: Credit risk

UCLA | Fall 2016

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This version: November 28, 2016

24. Credit facts and puzzles

Credit spreads and expected losses

	Maturity					
	1 - 3 years		3 - 5 years		7 - 10 years	
Rating	Spread	Expected Loss	Spread	Expected Loss	Spread	Expected Loss
AAA	49.5	0.06	63.9	0.18	74.0	0.61
AA	59.0	1.24	71.2	1.44	88.6	2.70
A	88.8	1.12	102.9	2.78	117.5	7.32
BBB	169.0	12.5	170.9	20.12	179.6	34.56
BB	421.2	103.1	364.6	126.7	322.3	148.1
B	760.8	426.2	691.8	400.5	512.4	329.4

Table 14.1: Spreads and expected default losses by rating. Source: Amato and Remolona [2003].

Source: Singleton (2006)

Defaults and recovery rates

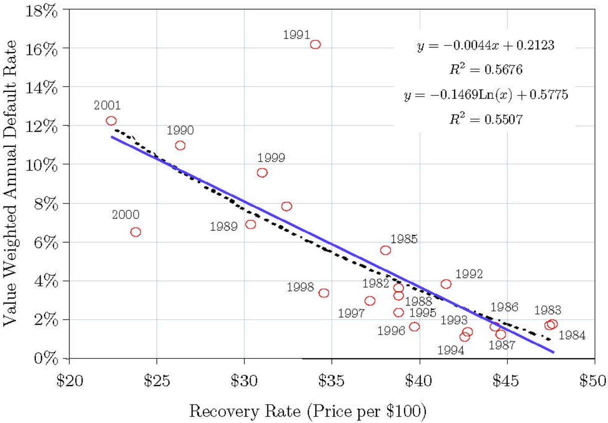


Figure 14.2: Correlation of Speculative Grade Default and Recovery Rates. Source: Moodys Default and Recovery Report [2003].

Source: Singleton (2006)

Defaults, leverage and ratings

Panel A: Summary statistics

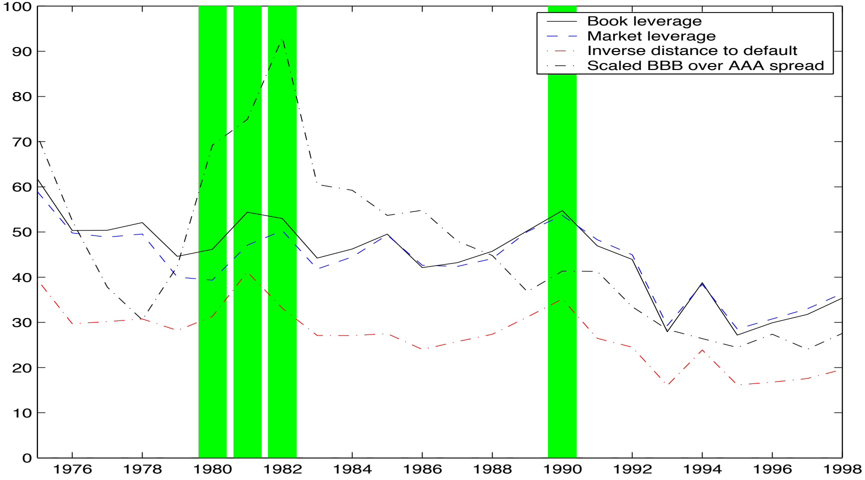
Variable	Mean	Std.	Min	Max
P/D ratio	27.77	14.54	10.12	85.42
Baa–Aaa spread (%)	1.09	0.41	0.60	2.33
Four-year default probability (%)	1.55	1.04	0.00	3.88
Book leverage of Baa	0.45	0.09	0.27	0.62
Market leverage of Baa	0.44	0.08	0.29	0.59
Inverse of the DD of Baa	0.28	0.07	0.16	0.42

Panel B: Correlation matrix of some benchmark variables

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
P/D ratio (1)	1.00						
Consumption growth (2)	0.14	1.00					
	0.22						
Baa–Aaa spread (3)	−0.37	−0.32	1.00				
	0.00	0.00					
Four-year default probability (4)	0.19	0.21	0.34	1.00			
	0.30	0.24	0.05				
Book leverage of Baa (5)	−0.70	−0.26	0.57	0.10	1.00		
	0.00	0.20	0.00	0.63			
Market leverage of Baa (6)	−0.61	−0.16	0.49	0.06	0.97	1.00	
	0.00	0.45	0.01	0.79	0.00		
Inverse of the DD of Baa (7)	−0.71	−0.40	0.60	0.14	0.96	0.87	1.00
	0.00	0.05	0.00	0.52	0.00	0.00	

Source: Chen, Collin-Dufresne, and Goldstein (2009)

Time series of leverage for BBB rated firms



Source: Chen, Collin-Dufresne, and Goldstein (2009)

Time series of credit spreads

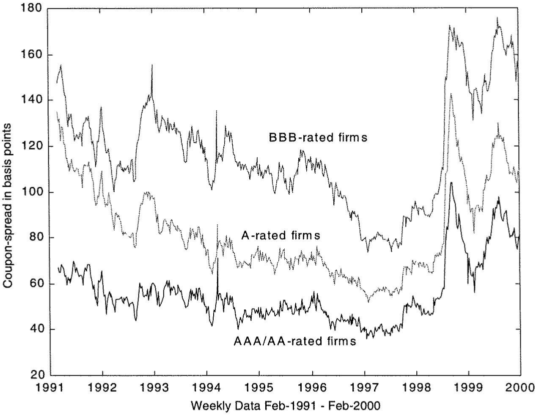


Figure 1
Time series of the coupon spreads
For each week, coupon spreads of 592 bonds of 104 firms are averaged within each rating category. The graph depicts the resulting time series.

Source: Driessen (2005)

25. Reduced-form evidence

Hazard rates

- Need concepts to characterize defaults
- Time is discrete. Let T denote the time of exit (default)
- Probability of exit at time t is $f_t = P(T = t)$
- The survivor function is $S_t = P(T > t) = \sum_{u=t+1}^{\infty} f_u$
- The hazard rate is

$$
\begin{aligned}
h_t &= P(T = t | T \geq t) = \frac{P(T = t, T \geq t)}{P(T \geq t)} \\
&= \frac{P(T \geq t | T = t) P(T = t)}{P(T \geq t)} = \frac{f_t}{S_{t-1}}
\end{aligned}
$$

- Can also write $S_t = S_0 \cdot \Pi_{u=1}^{t} (1 - h_u)$

Risky bond pricing

- Consider an $n-$period zero-coupon bond with uncertain recovery in the case of default

$$\widehat{P}_t^n = h_t e^{-r_t} E_t^{\mathbb{Q}} rec_{t+1} + (1-h_t) e^{-r_t} E_t^{\mathbb{Q}} \widehat{P}_{t+1}^{n-1}$$

- Assume recovery of market value: $E_t^{\mathbb{Q}} rec_{t+1} = (1-L_t) E_t^{\mathbb{Q}} \widehat{P}_{t+1}^{n-1}$:

$$\begin{aligned}
\widehat{P}_t^n &= h_t e^{-r_t}(1-L_t) E_t^{\mathbb{Q}} \widehat{P}_{t+1}^{n-1} + (1-h_t) e^{-r_t} E_t^{\mathbb{Q}} \widehat{P}_{t+1}^{n-1} \\
&= (h_t(1-L_t) + (1-h_t)) e^{-r_t} E_t^{\mathbb{Q}} \widehat{P}_{t+1}^{n-1} \equiv e^{-(r_t+s_t)} E_t^{\mathbb{Q}} \widehat{P}_{t+1}^{n-1}
\end{aligned}$$

with $e^{-s_t} \equiv h_t(1-L_t) + (1-h_t) = 1 - h_t L_t \approx e^{-h_t L_t}$

- Substitute recursively:

$$\begin{aligned}
\widehat{P}_t^n &= e^{-(r_t+s_t)} E_t^{\mathbb{Q}} \left(e^{-(r_{t+1}+s_{t+1})} \widehat{P}_{t+2}^{n-2} \right) = E_t^{\mathbb{Q}} e^{-\sum_{j=0}^{n-1}(r_{t+j}+s_{t+j})} \\
&= E_t \left(M_{t,t+n} e^{-\sum_{j=0}^{n-1} s_{t+j}} \right)
\end{aligned}$$

- See Duffie and Singleton (1999); Lando (1998); Monfort and Renne (2014)

Functional forms of hazard rates

- Logistic regression function:

$$h_t = (1 + e^{-(\alpha + \beta^\top x_t)})^{-1}$$

- Proportional hazard model:

$$h_t = 1 - e^{-\exp(\alpha + \beta^\top x_t)} \approx e^{\alpha + \beta^\top x_t}$$

- Affine model:

$$h_t = 1 - e^{\alpha + \beta^\top x_t} \approx \alpha + \beta^\top x_t$$

so is not guaranteed to be non-negative

An affine model

Table 3
Summary of extended Kalman filter estimates of 161 firms' default risk processes implied by corporate bond yields

Variable	1st quartile	Median	3rd quartile
$100 \cdot \alpha_j$	0.396	0.749	1.175
κ_j	0.023	0.238	0.600
$100 \cdot \theta_j$	0.072	0.559	2.814
λ_j	−0.485	−0.235	−0.050
σ_j	0.051	0.074	0.104
$\beta_{1,j}$	−0.242	−0.096	0.000
$\beta_{2,j}$	−0.080	−0.009	0.062
$100 \cdot \kappa_j \theta_j$	0.020	0.103	0.306
$\kappa_j + \lambda_j$	−0.150	−0.033	0.118
$100 \cdot \sqrt{\Sigma_j}$	0.082	0.101	0.121
Mean fitted $h_{j,t} \cdot 100$	1.086	1.359	1.876
Mean fitted $h^*_{j,t} \cdot 100$	0.238	0.573	1.079
RMSE of yield to maturity (basis points)[a]	7.39	9.83	11.05

The instantaneous default-free interest rate is given by $r_t = \alpha_r + s_{1,t} + s_{2,t}$, where $s_{1,t}$ and $s_{2,t}$ are independent square-root processes. Firm j's instantaneous default risk is given by

$$h_{j,t} = \alpha_j + h^*_{j,t} + \beta_{1,j}(s_{1,t} - \overline{s_{1,t}}) + \beta_{2,j}(s_{2,t} - \overline{s_{2,t}}),$$

where $h^*_{j,t}$ follows a square-root process that is independent of the processes for $s_{i,t}$, $i = 1, 2$:

$$dh^*_{j,t} = \kappa_j(\theta_j - h^*_{j,t})dt + \sigma_j \sqrt{h^*_{j,t}}\, dZ_{j,t} \qquad \text{(true measure)}$$

$$dh^*_{j,t} = (\kappa_j \theta_j - (\kappa_j + \lambda_j) h^*_{j,t})dt + \sigma_j \sqrt{h^*_{j,t}}\, d\hat{Z}_{j,t} \qquad \text{(martingale measure).}$$

Month-end yields on firm j's noncallable coupon bonds are all observed with i.i.d., normally distributed measurement error with mean zero and variance Σ_j. The estimation period is January 1985–December 1995, although most of the firms do not have data over this entire period.

[a] The square root of the mean of the squared differences between the actual and fitted yields to maturity on firm j's bonds.

Source: Duffee (1999)

Implied term structure of credit spreads

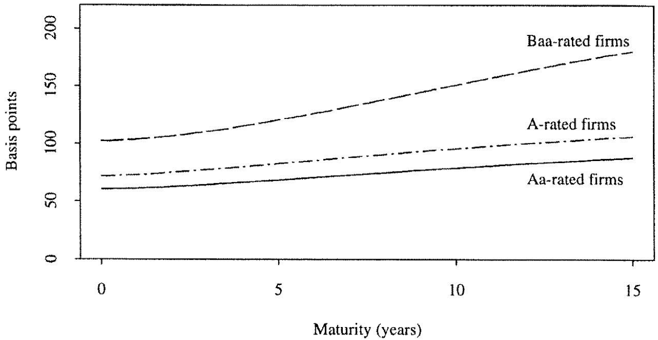


Figure 1
Term structures of yield spreads
This figure displays hypothetical term structures of zero-coupon corporate bond yields less zero-coupon Treasury bond yields implied by a two-factor translated square-root model of Treasury yields and a translated square-root model of instantaneous default risk. The default-risk parameters for each credit rating are the median estimates across those firms with the given rating. The two default-free factors are set equal to their mean values over the sample period 1985–1995.

Source: Duffee (1999)

Which fraction of credit spreads is due to default risk?

- Driessen (2005) estimates an affine model to decompose spreads into default, liquidity, and tax factors

- Elements of his model:
  - Risk-free rate: $r_t^{\$} = \delta_0 + \delta_1 X_{1t} + \delta_2 X_{2t}$
  - Risk-adjusted expected loss:
    $h_{jt}^{\mathbb{Q}} L = \gamma_{0j} + \sum_{i=1}^{2} \gamma_{ij} F_{it} + G_{jt} + \sum_{i=1}^{2} \alpha_{ij} X_{it}$
  - Default risk premium: $\mu = h_{jt}^{\mathbb{Q}} / h_{jt}^{\mathbb{P}}$
  - Liquidity via “instantaneous” credit spread:
    $s_{jt}^{\mathbb{Q}} = h_{jt}^{\mathbb{Q}} L + (\beta_{0j}^{l} + \beta_{1j}^{l} l_t)$
  - Taxes: scale coupon, C, and loss, L, by $1 - t_{state}(1 - t_{federal})$

- How does one separate l_t from F_{it} in the data?
  - Driessen (2005) uses bond’s age as a proxy for liquidity, forms LAge-M-HAge portfolio to proxy for l_t

Time series of liquidity spreads

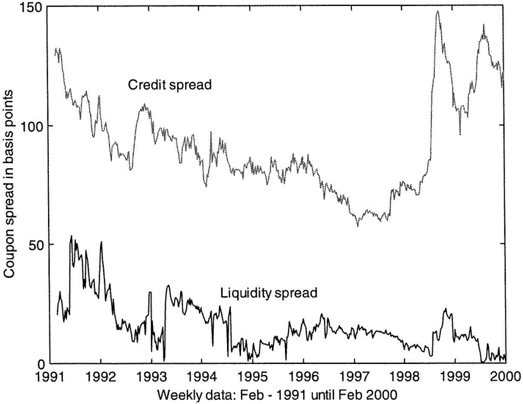


Figure 2
Time series of the liquidity spreads
Using Kalman filter QML, a one-factor square-root model is estimated for the spread between high-age and low-age bonds (see Section 4.2). The graph depicts the filtered factor values for this liquidity spread and the average coupon spread of all bonds for each week.

Source: Driessen (2005)

Conditional default probabilities

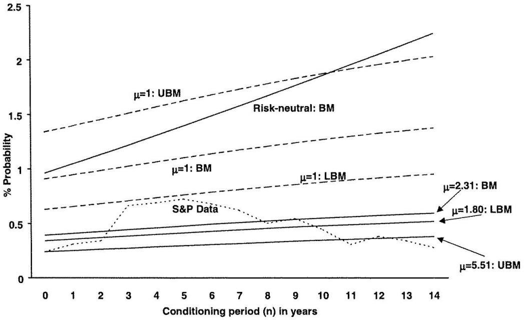


Figure 3
Conditional default probabilities
The graph contains yearly conditional default probabilities for BBB-rated firms. The yearly conditional default probability is defined as the probability of default in the next year, given that no default has occurred before. The line "S&P Data" gives the historically estimated default probabilities, obtained from S&P data. The other lines all are model-implied default probabilities:

- "Risk-neutral: BM" refers to the risk-neutral probabilities for the benchmark model (BM).
- "$\mu = 1$: BM" refers to the actual probabilities in case $\mu = 1$ for the BM.
- "$\mu = 2.31$: BM" refers to the actual probabilities at the estimated value for μ (2.31) for the BM.

Furthermore, the actual probabilities for the upper bound and lower bound models (UBM and LBM) are given, both for the case where $\mu = 1$ and at the estimated values (5.51 and 1.80, respectively).

UBM excludes tax and liquidity effects; LBM also uses AA curve instead of Treasuries as a benchmark

Decomposed bond risk premium

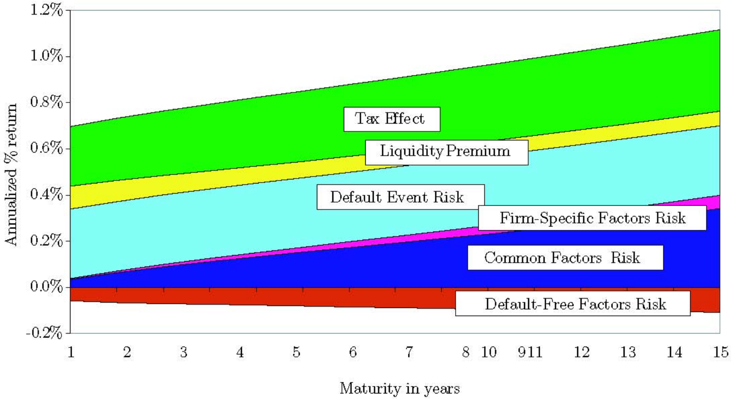


Figure 14.3: Decomposition of Expected Excess Returns on BBB-rated Bonds. Source: Driessen (2004)

Source: Driessen (2005)

26. Lessons from contingent claims appraoch

The Merton (1974) model

- Debt and equity are viewed as options on the value of unlevered assets, V_t, with a growth rate G_t^V:

$$g_t^V = (\mu - \delta) + \sigma \varepsilon_t$$

- Then, credit spread of maturity T is

$$y - r = -T^{-1} \log(1 - L\Phi(\Phi^{-1}(\pi) + \theta T^{1/2})),$$

where

$$\begin{aligned} L \quad & \text{is loss given default} \\ \theta = (\mu - r)/\sigma \quad & \text{is the (log) Sharpe ratio} \\ \Phi(\cdot) \quad & \text{is the normal c.d.f.} \\ \pi \quad & \text{is expected default rate:} \end{aligned}$$

$$\pi = \Phi\left(-\frac{\log(V_0/B) + (\mu - \delta)T}{\sigma T^{1/2}}\right)$$

with B as default boundary (in Merton it is face value of debt)

The determinants of credit spread changes

- Collin-Dufresne, Goldstein, and Martin (2001) ask if any of these inputs affect changes in credit spreads

$$\Delta CS_t^i = \alpha + \beta_1^i \,\Delta lev_t^i + \beta_2^i \,\Delta r_t^{10} + \beta_3^i \,(\Delta r_t^{10})^2 + \beta_4^i \,\Delta slope_t + \beta_5^i \,\Delta VIX_t$$
$$+ \beta_6^i \,S\&P_t + \beta_7^i \,\Delta jump_t + \epsilon_t^i.$$

Variable	Description	Predicted Sign
Δlev_t^i	Change in firm leverage ratio	+
Δr_t^{10}	Change in yield on 10-year Treasury	−
$\Delta slope_t$	Change in 10-year minus 2-year Treasury yields	−
ΔVIX_t	Change in implied volatility of S&P 500	+
$S\&P_t$	Return on S&P 500	−
$\Delta jump$	Change in slope of Volatility Smirk	+

- Items 2, 4, 5 and 6 are significant, but explain 25% of variation
- The first PC of the residuals explains 75% of the remaining variation – outside the model

Extensions

- The Merton model was extended to
  - allow for default decision at each coupon date (coupon as a compound option) (Geske, 1977)
  - accomodate ability to default at any time (Leland, 1994 and Leland and Toft, 1996)
  - time-varying default boundary (Longstaff and Schwartz, 1995 and Collin-Dufresne and Goldstein, 2001)

The Black and Cox model (I)

- The Black and Cox (1976) model allows for a possibility of default at any time
- The zero-coupon bond price is:

$$\begin{aligned} D_t(T) &= E_t^{\mathbb{Q}}\left(Be^{-r(T-t)}\mathbf{1}_{\{\tau \geq T, V_T \geq B\}}\right) \\ &+ \beta_1 E_t^{\mathbb{Q}}\left(V_T e^{-r(T-t)}\mathbf{1}_{\{\tau \geq T, V_T < B\}}\right) \\ &+ \beta_2 K E_t^{\mathbb{Q}}\left(e^{-\gamma(T-\tau)} e^{-r(\tau-t)}\mathbf{1}_{\{t < \tau < T\}}\right), \end{aligned}$$

  where

$$v_t = \begin{cases} Ke^{-\gamma(T-t)}, & t < T \\ B, & t = T \end{cases} \qquad \text{(Default boundary)}$$

$$\tau = \inf\{t \in [0, T[: V_t \leq v_t\} \qquad \text{(Default time)}$$

$$1 - L_t = \begin{cases} \beta_2 V_t, & t < T \\ \beta_1 V_t, & t = T \end{cases} \qquad \text{(Recovery)}$$

- Zero-coupon bond is not very interesting because there is no economic reason to default prior to maturity

The Black and Cox model (II)

- Suppose a coupon is paid out continuously at the rate c
- The value of coupons is:

$$A_t(T) = E_t^{\mathbb{Q}}\left(\int_t^T ce^{-r(s-t)}\mathbf{1}_{\{\tau>s\}}ds\right) = ce^{rt}\int_t^T e^{-rs}\mathbb{Q}_t(\tau > s)ds$$

- Fascinating first-passage math which we will skip (look up Bielecki and Rutkowski)
- Assume $\gamma = 0$, $K \geq B$ then consol's bond ($T = \infty$) price is:

$$\begin{aligned} D_t &= \frac{c}{r}\left(1-\left(\frac{K}{V_t}\right)^{a+\zeta}\right)+\beta_2 K\left(\frac{K}{V_t}\right)^{a+\zeta}, \\ \zeta &= \sigma^{-2}(\nu^2+2\sigma^2 r)^{1/2}, \quad a = \nu\sigma^{-2}, \quad \nu = r-\delta-\sigma^2/2 \end{aligned}$$

Optimality

- Optimal default time (barrier) is chosen by stockholders to maximize the value of equity – a perpetual American option problem ($\delta = 0$):

$$
\begin{aligned}
D_t &= \frac{c}{r}\left(1 - \left(\frac{K}{V_t}\right)^{2r/\sigma^2}\right) + K\left(\frac{K}{V_t}\right)^{2r/\sigma^2}, \\
K^* &= c/(r + \sigma^2/2)
\end{aligned}
$$

- Now can construct optimal capital structure by selecting c to maximize the overall firm value (Leland, 1994): $c \propto V_0$

Dynamic capital structure

- Goldstein, Ju, and Leland (2001) propose upward changes in debt levels during the lifetime of a firm

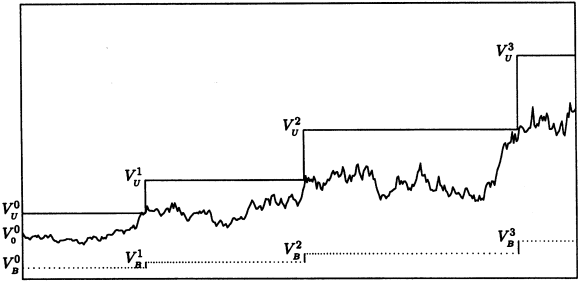


FIG. 1.—A typical sample path of firm value with log-normal dynamics. Initially, firm value is V_0^0. Period 0 ends either by firm value reaching V_B^0, at which point the firm declares bankruptcy, or by firm value reaching V_U^0, at which point the debt is recalled and the firm again chooses an optimal capital structure. Note that the initial firm value at the beginning of period 1 is $V_0^1 = V_U^0 \equiv \gamma V_0^0$. Due to log-normal firm dynamics, it will be optimal to choose $V_U^n = \gamma^n V_U^0, V_B^n = \gamma^n V_B^0$.

“Dynamic” capital structure

- $K^* \propto c^*$, $c^* \propto V_0$
  - A and B are identical firms except for $V_0^B = \gamma V_0^A$
  - Everything scales by γ
  - Assume call at par and then issue new (larger) debt
  - Probability of default is unchanged

- Because time to maturity is infinite, the valuation is the same at each refi point

Implications

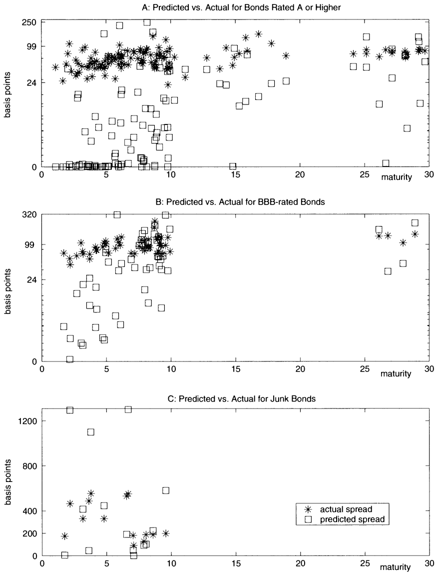
- Management will choose to issue a smaller amount of debt initially
- This explains why most static models predict optimal leverage ratios that are well above what is observed in practice
- For a given level of initial debt, bonds issued from such a firm are riskier, since the bankruptcy threshold rises with the level of outstanding debt
- This might explain why most static models predict yield spreads that are too low

Calibrated parameters

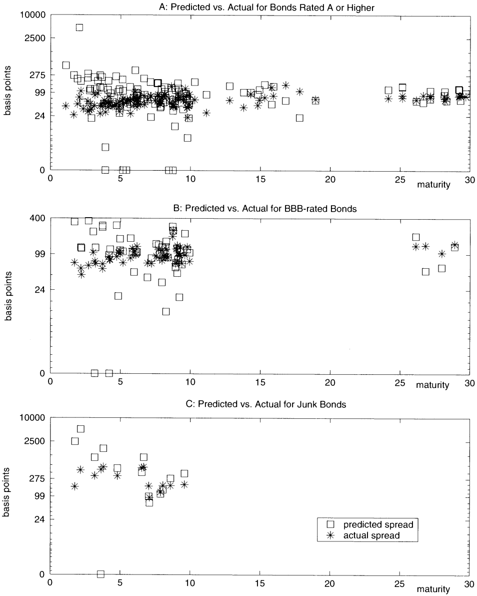
- Eom, Helwege, and Huang (2004) attempt to test these models:
  - They calibrate parameters
  - They compute pricing errors for each bond in their sample

Parameter	Description	Estimated as	Data source
Bond features			
	c Coupon	Given	FID
	T Maturity	Given	FID
	F Face value	Total liabilities	Compustat
	w Recovery rate	Given	Moody's
Firm characteristics			
	V Firm value	Total liabilities plus market value of equity	Compustat and CRSP
	μ_v Asset returns	Average monthly change in V	Compustat and CRSP
	σ_v Asset volatility	Historical equity volatility adjusted for leverage	Compustat and CRSP
	δ Payout ratio	Weighted average of c and the share repurchase-adjusted dividend yield	Compustat, CRSP and FID
	κ_ℓ Speed of adjustment to target leverage	Coefficient from a regression of changes in log leverage against lagged leverage and r	CRSP, CMT and Compustat
	ϕ Sensitivity of target leverage to interest rates	Coefficient from a regression of changes in log leverage against lagged leverage and r	CRSP, CMT and Compustat
	τ Tax rate	Assumed at 0.35	—
Interest rates			
	r Risk-free rate	NS or Vasicek models	CMT
	ρ Correlation between V and r	Correlation between equity returns and r	CRSP and CMT
	σ_r Interest rate volatility	Vasicek model	CMT

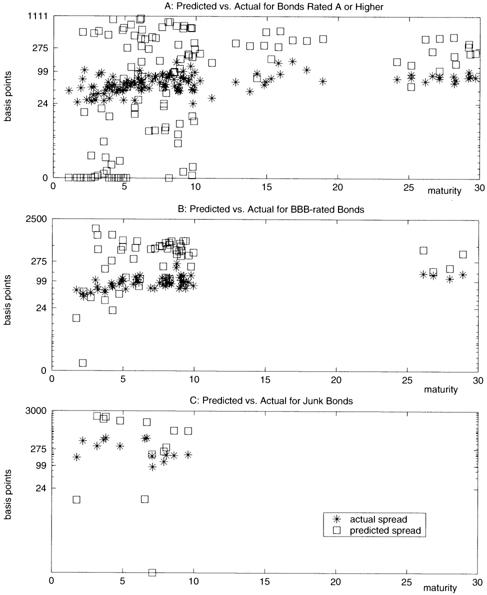
The Merton model



The Leland-Toft model



The CDG model



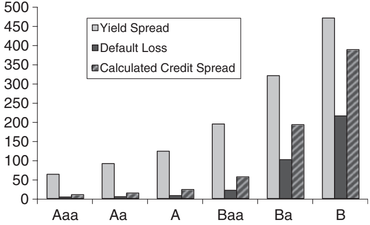
Alternative calibration approach

- Huang and Huang (2012) claim that all the different models have remarkably consistent implications for pricing of credit risk if one calibrates to match historical bond default rates and equity risk premium

Credit rating	Target: Leverage ratio (%)	Target: Equity premium (%)	Target: Cumulative default prob. (%) 1 yr.	4 yrs.	10 yrs.	Recovery rate (%)	Avg. yld. spreads (%) 4 yrs.	10 yrs.
Aaa	13.08	5.38	0.00	0.04	0.77	51.31	0.55	0.63
Aa	21.18	5.60	0.03	0.23	0.99	51.31	0.65	0.91
A	31.98	5.99	0.01	0.35	1.55	51.31	0.96	1.23
Baa	43.28	6.55	0.12	1.24	4.39	51.31	1.58	1.94
Ba	53.53	7.30	1.29	8.51	20.63	51.31	3.20	3.20
B	65.70	8.76	6.47	23.32	43.91	51.31	4.70	4.70

- Choose V_0, the asset risk premium (derived from a model), and σ to match the targets
- Choose r, δ, B to match historical data
- Recovery $1 - L$=51.31%

Result



- The fraction of spreads due to credit risk is 20%, on average
- Similar flavor to Collin-Dufresne, Goldstein, and Martin (2001)

The "convexity" effect

- The issue that comes up in many papers is that of a credit spread of "representative" firm vs average credit spreads
- Consider the Merton model with $r = 0.05$, $\mu = 0.075$, $\delta = 0.06$, Sharpe ratio=0.22, $T = 4$, $L = 0.551$, and $B = 0.356$
- Suppose half of the firms are with $V_0 = 120$ (large distance to default) and the other half is $V_0 = 80$. The average credit spread is:

$$\begin{aligned} \overline{CS} &= 0.5 \cdot CS(V_0 = 120) + 0.5 \cdot CS(V_0 = 80) \\ &= 0.5 \cdot 12.69 + 0.5 \cdot 100.23 = 56.46 \text{ bps} \end{aligned}$$

- Now, construct a representative firm with $\bar{V}_0 = 0.5 \cdot 120 + 0.5 \cdot 80 = 100$
  - Match historical default rates, recovery rates, and Sharpes: $B = 0.397$ implies $\pi(V_0 = 100) = 1.55\%$
  - $CS(V_0 = 100, B = 0.397) = 59.9$ bps

Revisit the calibration ...

- Chen, Collin-Dufresne, and Goldstein (2009) focus on the Baa-Aaa spread because it should reflect credit risk only
- According to CCDG, HH effectively calibrate to π and L
- CCDG calibrate to π, L, θ

Baa–Aaa spreads as a function of Sharpe ratio for the benchmark model

	T = 4 years			T = 10 years		
Sharpe	Baa	Aaa	Baa–Aaa	Baa	Aaa	Baa–Aaa
0.15	44.0	1.6	42.4	67.7	12.0	55.7
0.20	54.9	2.2	52.7	88.1	17.4	70.7
0.25	68.1	3.0	65.1	112.8	24.6	88.2
0.30	83.7	4.1	79.6	141.7	34.2	107.5
0.35	102.0	5.5	96.5	175.1	46.6	128.5
0.40	123.4	7.4	116.0	212.9	62.2	150.7

The four-year Baa (Aaa) default rate is 1.55% (0.04%). The 10-year Baa (Aaa) default rate is 4.89% (0.63%). The recovery rate is 0.449.

- In the data, Sharpe $\approx$ 0.22 and the Baa-Aaa spread is 94-102 bps

27. Credit risk and business cycles

Revisit the theory ...

- Bond price is

$$
\begin{aligned}
P_t &= E_t(M_{t,t+T}(1 - \mathbf{1}_{\{\tau \le T\}} L_{t+\tau})) \\
&= E_t M_{t,t+T} E_t(1 - \mathbf{1}_{\{\tau \le T\}} L_{t+\tau})) + cov_t(M_{t,t+T}, 1 - \mathbf{1}_{\{\tau \le T\}} L_{t+\tau})) \\
&= Q_t^1 (1 - E_t(\mathbf{1}_{\{\tau \le T\}} L_{t+\tau})) - cov_t(M_{t,t+T}, \mathbf{1}_{\{\tau \le T\}} L_{t+\tau}))
\end{aligned}
$$

- HH force all models to agree on the 1st term, therefore, there is not much difference in the 2nd term across the different models

- Can generate low prices (high spreads) via countercyclical cashflows, specifically
  - A large covariance of M and $\mathbf{1}$. Because $\tau = inf\{t : V_t \le B_t\}$
    - (A) A large negative covariance between M and V (value of assets)
    - (B) A large covariance between M and B (default boundary)
  - (C) A large covariance of M and L

- Channel (A) is used frequently to explain the equity premium

The CC model

- Price-consumption ratio

$$\frac{P_t^C}{C_t} = E_t \sum_{i=1}^{\infty} M_{t,t+i} G_{t+i}$$

- Price-dividend ratio for a claim on dividends plus interest (output) with growth rate $G_t^Y = Y_t / Y_{t-1}$

$$\begin{aligned} g_t^Y &= g^Y + \gamma_0^Y [\rho w_{gt} + (1-\rho^2)^{1/2} w_{Yt}] \\ \frac{P_t^Y}{Y_t} &= E_t \sum_{i=1}^{\infty} M_{t,t+i} G_{t+i}^Y \end{aligned}$$

- Cannot compute analytically. CCDG show that CC compute these wrong and offer an alternative:

$$\frac{P_t^C}{C_t} = \sum_{i=1}^{\infty} E_t^{\mathbb{Q}} \left(\prod_{j=0}^{i-1} R_{t+j,t+j+1} \right)^{-1} G_{t+i}$$

Implications

- One can show that dynamics of aggregate firm value V are:

$$g_t^V = \mu(s_{t-1}) - Y_{t-1}/P_{t-1}^Y + \sigma(s_{t-1})w_{Vt}$$

- For a "representative" individual firm assume:

$$g_t^{V_i} = g_t^V + \sigma_i w_{it}$$

  where i stands for idiosyncratic

- What is the difference from regular equity valuation that we've seen in earlier classes?

- Here there is a default if V_{it} breaches the boundary B

Constant boundary

Model-generated four-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the constant default boundary case

		Baa			Aaa			
$s(0)$	Population Distribution (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Baa–Aaa Spread (bp)
−2.96	0.06	91.3	6.61	1.22	6.0	0.40	0.03	85.3
−2.86	0.07	90.1	6.50	1.36	5.9	0.39	0.04	84.2
−2.76	0.09	88.0	6.30	1.43	5.8	0.37	0.04	82.2
−2.66	0.11	85.5	6.12	1.54	5.3	0.34	0.04	80.2
−2.56	0.13	80.5	5.77	1.75	5.0	0.32	0.05	75.5
−2.46	0.15	76.0	5.46	1.89	4.8	0.31	0.05	71.2
−2.36	0.14	66.2	4.76	2.08	4.0	0.25	0.06	62.2
−2.27	0.04	52.9	3.82	2.20	3.0	0.17	0.06	49.9
	Average	82.3	5.90	1.55	5.2	0.34	0.04	77.1
	Std. Dev.	12.7	0.90	0.41	1.0	0.07	0.01	11.7

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be independent of $s(0)$ and constant over time ($B_{\mathrm{Baa}} = 0.356$, $B_{\mathrm{Baa}} = 0.208$). Population averages over the steady-state distribution are then determined.

- Improves upon Merton by 20 bps
- Std.dev. is 12 bps (vs 41 bps in the data)
- Negative correlation between 4-yr default prob [increases in s_0 (default rates are procyclical)] and Baa-Aaa spread [declines in s_0]
- The only margin for improving is the correlation between M and cashflows: channel (A) does not work, let's try (B)

Time-varying boundary (I)

- Plan (B)
- Distance to default is procyclical:

$$\frac{V_0}{B} = \frac{V_0}{F} \times \frac{F}{B} \propto \frac{V_0}{F}$$

- Define market leverage *MLV* as the ratio of market debt to market debt + market equity

$$MLV_{\text{Baa}} = 0.52 - 0.61 S_0$$

- Assume default boundary to be a constant fraction of the initial leverage:

$$B_{it} = \Psi_i MLV_{Baa}, \quad \Psi_{Baa} = 0.750, \quad \Psi_{Aaa} = 0.435$$

- Volatility of spreads has increased
- But spread magnitude and cyclicality of default probabilities are still off

Time-varying boundary (II)

- Add countercyclical time variation in the default boundary:

$$B_{it} = \Psi_i(0.52 - 0.61S_0)(1 - 4(S_t - S)),$$

with $\Psi_{Baa} = 0.696$, and $\Psi_{Aaa} = 0.394$

Model-generated four-year Baa and Aaa credit spreads as a function of initial log-surplus consumption ratio $s(0)$ for the countercyclical default boundary case

		Baa			Aaa			
$s(0)$	Population Distribution (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Spread over Treasury (bp)	Q-Default Rate (%)	P-Default Rate (%)	Baa–Aaa Spread (bp)
−2.96	0.05657	151.1	10.65	1.74	11.4	0.80	0.06	139.7
−2.86	0.07159	140.4	9.92	1.72	10.4	0.73	0.05	130.0
−2.76	0.08983	127.6	9.05	1.65	9.4	0.65	0.05	118.2
−2.66	0.10887	112.4	8.00	1.57	7.9	0.54	0.05	104.5
−2.56	0.12837	94.6	6.77	1.48	6.8	0.46	0.04	87.8
−2.46	0.14735	78.2	5.62	1.34	5.5	0.36	0.02	72.7
−2.36	0.14429	56.1	4.05	1.21	3.9	0.24	0.04	52.2
−2.27	0.03753	33.6	2.43	0.98	2.1	0.11	0.02	31.5
	Average	115.6	8.18	1.55	8.5	0.59	0.04	107.1
	Std. Dev.	50.3	3.45	0.29	4.0	0.30	0.01	46.3

Without loss of generality, initial firm value is normalized to one. Default boundary is specified to be a function of both initial state of the economy and current state of the economy: $B_{\text{Baa}}(S(t), S(0)) = \psi^*_{\text{Baa}}[0.52 - 0.61S(0)][1 - \text{slope} * (S(t) - \overline{S})]$. The parameters $\{\psi_{\text{Baa}}, \psi_{\text{Baa}}\}$ are chosen to match historical default rates. *Slope* is chosen to closely capture both the historical variation in spreads and the regression coefficient of spreads on future default rates. Population averages over the steady-state distribution are then determined.

Time-varying recovery rate

- Plan (C)

- Relate recovery to S in two steps:
  1. Relate the recovery rate to the cycle dummy [1982-2004]
     $1 - L_\tau = 0.47 - 0.15$ cycle
  2. Relate the cycle dummy to surplus [1919-2004]
     cycle $= 0.51 - 3.80 S_\tau$

- Get procyclical recovery $1 - L_\tau = const + 0.57 S_\tau$

- Goes in the right direction but not enough on its own

Recursive preferences

- The setup is the same as in BY2

- Price-dividend ratio for a claim on dividends plus interest (output) with growth rate $G_t^Y = Y_t / Y_{t-1}$

$$
\begin{aligned}
g_t^Y &= g^d + \eta^d x_t + q^Y v_t^{1/2} u_{t+1}, \\
q^Y &= q^d \cdot (\text{std.dev. of } Y \text{ in the data}) / (\text{std.dev. of } d \text{ in the data})
\end{aligned}
$$

- Solve for the value, V, of the claim on the aggregate output and assume that a "representative firm's" value, V_i, deviates from this by idiosyncratic noise (calibrated to match the 4-year default probability under a constant default bound)

- Consider 3 cases (we will review 2):
    - (I) Growth rate risk: $v_t = v$ (and risk premiums are constant)
    - (III) Full model

Results for 4-year Baa

CASE I Stochastic growth - Constant volatility and risk-premia							
P def prob	P-DP Std Dev	Q def prob	Q-DP Std Dev	Average Spread	Std Dev. Spread	Reg Coef	σ_{idio}^{Baa}
0.0155	0.0031	0.036	0.0066	0.0045	0.00082	4.04	0.245
(0.0005)	(0.0009)						
CASE III Stochastic growth and volatility - Time varying risk-premia							
P def prob	P-DP Std Dev	Q def prob	Q-DP Std Dev	Average Spread	Std Dev. Spread	Reg Coef	σ_{idio}^{Baa}
0.0155	0.0049	0.0492	0.0204	0.0058	0.0025	1.853	0.241
(0.0004)	(0.0007)						

Table 6: Estimated values of P and Q default probabilities as well as the unconditional mean and variance of the credit spread for four year to maturity Baa firms. Standard errors of estimates are in parenthesis. Parameters of the typical Baa firm are as defined above for the output claim. The spread is simulated within a structural model which assumes a constant nominal default boundary at 60% of the average Baa leverage ratio ($K = 0.6 * 0.4328$. Upon default bond recover constant fraction of face value corresponding to average historical Baa recovery rate 51.31%. Simulations are run with 100000 runs for each price estimation (conditional on state), with standard antithetic variance reduction.

- High vol states are more expensive
- Defaults occur more frequently when vol is high (via the cash-flow channel)
- The combined effect (countercyclical risk premium) increases credit spreads
- Does not have the CC problem of procyclical default rates

Results for other terms and credit quality

	4-year Baa			4-year Aaa			
	Spread over Treasury	Q-Default Rate	P-Default Rate	Spread over Treasury	Q-Default Rate	P-Default Rate	(Baa - Aaa) Spread
Average	0.0058	0.0492	0.0155	0.0006	0.0052	0.0004	0.0052
Std Dev.	0.0025	0.0204	0.0049	0.0007	0.0001	0.0000	0.0018
	10-year Baa			**10-year Aaa**			
	Spread over Treasury	Q-Default Rate	P-Default Rate	Spread over Treasury	Q-Default Rate	P-Default Rate	(Baa - Aaa) Spread
Average	0.0118	0.240	0.0487	0.0051	0.1154	0.0063	0.0055
Std Dev.	0.0036	0.0636	0.0123	0.0028	0.0588	0.0044	

Table 7: Model generated 4-year and 10-year Baa and Aaa credit spreads. The idiosyncratic risk needed to match the 4-year historical default rate for Baa (Aaa) of 1.55% (0.04%) is $\sigma_{idio}^{Baa} = 0.241$ ($\sigma_{idio}^{Baa} = 0.110$). The idiosyncratic risk needed to match the 10-year historical default rate for Baa (Aaa) of 4.89% (0.63%) is $\sigma_{idio}^{Baa} = 0.188$ ($\sigma_{idio}^{Baa} = 0.078$).

- Baa spreads over Treasuries are similar to CC
- Aaa spreads are much larger and more realistic
- But, as a result, the Baa-Aaa spread is much more flat
- Introducing a countercyclical default boundary will help raise spreads, but will further increase the covariation between default probabilities and credit spreads

Extensions

- After that, it is mostly a mop-up operation (still ongoing)
- Chen (2010) and Bhamra, Kuehn, and Strebulaev (2010) endogenize the default boundary in a world with recursive preferences
- Overlay the CCDG framework with the dynamic capital structure of GJL
- Recessions are times of high marginal utilities, which means that default losses that occur during such times will affect investors more
- Recessions are also times when firm cash flows are expected to grow more slowly, and to become both more volatile and more correlated with the market
- These two effects, combined with higher risk prices at such times, lower equity holders' continuation values, making defaults more likely in recessions
- Because many firms experience poor performances in recessions, liquidating assets during such times can be particularly costly, which results in higher default losses
- Taken together, the countercyclical variation in risk prices, default probabilities, and default losses raises the present value of expected default losses for bond holders, which leads to high credit spreads and low leverage ratios

Some details

- Expected consumption growth and volatility of consumption change with regimes

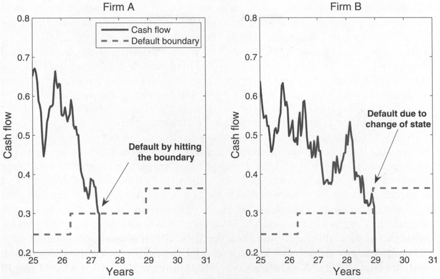


Figure 2. Illustration of two types of defaults. In the left panel, default occurs when the cash flow drops below a default boundary; in the right panel, default occurs when the default boundary jumps up, which is triggered by a change of aggregate state.

- The second type of default generates default waves: firms with cash flows between two default boundaries can default at the same time when a large shock arrives.

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