MODEL STOCK RETURNS WITH CHANGE OF NUMERAIRE

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ABSTRACT.

1. CHANGE OF NUMERAIRE

Define the stock price dynamics of three stocks (IBM, Apple, Facebook) under physical measure as

$$\begin{aligned} dS_I(t) &= (\mu_I + \frac{1}{2}\sigma_I^2)dt + \sigma_I dW_I(t) \\ dS_A(t) &= (\mu_A + \frac{1}{2}\sigma_A^2)dt + \sigma_A dW_A(t) \\ dS_F(t) &= (\mu_F + \frac{1}{2}\sigma_F^2)dt + \sigma_F dW_F(t) \end{aligned}$$

If we use the Facebook stock as numeraire, we obtain

$$\begin{aligned} dS_I^{(F)}(t) &= (\mu_I - \mu_F)dt + (\sigma_I - \sigma_F)d\tilde{W}_I^{(F)}(t) \\ dS_A^{(F)}(t) &= (\mu_A - \mu_F)dt + (\sigma_A - \sigma_F)d\tilde{W}_A^{(F)}(t) \end{aligned}$$

where $\tilde{W}_I^{(F)}$ and $\tilde{W}_A^{(F)}$ are Brownian motions under measure of Facebook stock return. This way of modeling cannot tell the relative volatility because the Brownian motion is symmetric. But the drift term would interpret as the excess return of stock return over the numeraire stock return.

If the original model is not iid, there is a compensation term for covariance between numeraire and underlying stocks. That is,

$$\begin{aligned} dS_I^{(F)}(t) &= (\mu_I - \mu_F + \rho\sigma_I\sigma_F)dt + (\sigma_I - \sigma_F)d\tilde{W}_I^{(F)}(t) \\ dS_A^{(F)}(t) &= (\mu_A - \mu_F + \rho\sigma_A\sigma_F)dt + (\sigma_A - \sigma_F)d\tilde{W}_A^{(F)}(t) \end{aligned}$$

The drift return is more than just the return-premium, where ρ could be estimated jointly with the return series.

REFERENCES

Appendix: Code