

Risks for the Long Run: A Potential Resolution of Asset Pricing Puzzles

Bansal and Yaron, JF (2004)
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Outline

- 1 Introduction
- 2 Model
 - Idea
 - Preferences and SDF
 - The Long-Run Risk Model
 - Solving the Model
 - Pricing of Short-Run, Long-Run, and Volatility-Risk
 - Volatility Feedback
- 3 Data and Model Implications
 - Data and Calibration
 - Results
- 4 Conclusion
- 5 Discussion

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The standard consumption-based asset pricing model with

- 1 Power utility
- 2 iid consumption growth

has problems:

- 1 Equity premium puzzle
- 2 Risk-free rate puzzle
- 3 Excess volatility puzzle
- 4 Cross-section of stock returns

BY propose a model that solves these puzzles, by changing the two assumptions.

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Model Components

- 1 Representative agent with Epstein-Zin preferences.

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 - Shocks to expected growth alter expectations about future growth for long-horizons: Long-Run Risk!

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- ① Representative agent with Epstein-Zin preferences.
- ② Expected consumption and dividend growth contain a small persistent component
 - Shocks to expected growth alter expectations about future growth for long-horizons: Long-Run Risk!
- ③ Time-varying economic uncertainty
 - Conditional volatility of consumption and dividend growth is time-varying, implying time-varying risk-premia.

Agents fear adverse shocks to long-run growth and volatility, and require a high risk-premium for holding risky assets.

Agents prefer early resolution of uncertainty, implying the compensation for long-run growth risk is positive.

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Risk-premia has three risk-sources:

- 1 short-run risk
- 2 long-run risk
- 3 consumption volatility risk

Preferences

A representative agent has Epstein-Zin preferences.

δ : Time-preference

γ : Risk aversion

ψ : Intertemporal elasticity of substitution

Note that when

$\gamma = \frac{1}{\psi}$ the agent is indifferent to the timing of the resolution of uncertainty

$\gamma > \frac{1}{\psi}$ the agent prefers early resolution of uncertainty

$\gamma < \frac{1}{\psi}$ the agent prefers late resolution of uncertainty

In this model, agents will prefer **early** resolution of uncertainty.

The SDF

The log of IMRS is

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}$$

where

- $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$
- $g_{t+1} = \log(C_{t+1}/C_t)$
- $r_{a,t+1}$ is the continuously compounded return on an asset having aggregate consumption as dividend (consumption claim)

Consumption Claim vs. Market Portfolio

We do not observe the return on the consumption claim, $r_{a,t}$.

Instead, we observe the return on the 'aggregate dividend claim' (the market portfolio), $r_{m,t}$.

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Consumption does not equal dividends. Difference is made up by labor income.

BY treat aggregate consumption and aggregate dividends as separate processes (not co-integrated).

Things to Model

Recall

$$m_{t+1} = \theta \log \delta - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}$$

A Campbell-Shiller approximation gives

$$\begin{aligned} r_{a,t+1} &= \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \\ r_{m,t+1} &= \kappa_0^m + \kappa_1^m z_{m,t+1} - z_{m,t} + g_{d,t+1} \end{aligned}$$

where

$$\begin{aligned} z_t &= \log(P_t/C_t) & z_{m,t} &= \log(P_t/D_t) \\ g_{t+1} &= \log(C_{t+1}/C_t) & g_{d,t+1} &= \log(D_{t+1}/D_t) \end{aligned}$$

z_t and $z_{m,t}$ are endogenous. We must model g_t and $g_{d,t}$.

Long-Run Growth and Economic Uncertainty Risks

Long-run risk modeled by a persistent growth component

$$x_{t+1} = \rho x_t + \varphi_e \sigma_t e_{t+1}$$

Model consumption and dividend growth as

$$g_{t+1} = \mu + x_t + \sigma_t \eta_{t+1}$$

$$g_{d,t+1} = \mu_d + \varphi x_t + \varphi_d \sigma_t u_{t+1}$$

$$\sigma_{t+1}^2 = \sigma^2 + \nu_1(\sigma_t^2 - \sigma_t) + \sigma_w w_{t+1}$$

where σ_t models economic uncertainty as time-varying volatility of cash flows.

ρ persistence of expected growth rate process

σ_t time-varying economic uncertainty

φ, φ_d calibrates vol of dividends and its correl with cons.

$\varphi > 1$ makes dividend growth more volatile than consumption growth

Intuition for Long-Run Risk I

Variance ratio for consumption:

$$VR_c(k) = \frac{V\left(\sum_{j=0}^k g_{t+j}\right)}{kV(g_t)} \quad (1)$$

- ① Equals 1 if consumption growth is iid.
- ② Larger than 1 when expected growth is persistent.
- ③ I.e. agents face larger consumption volatility at longer horizons.
- ④ Increasing the persistence in x_t or its vol increases long-run consumption volatility.
- ⑤ When agents prefer early resolution to uncertainty, an increase in long-run consumption vol will require a risk-premium.

Intuition for Long-Run Risk II

Consider the expected discounted cumulative consumption growth

$$E_t \left(\sum_{j=0}^{\infty} \delta^j g_{t+j} \right) = \frac{\delta x_t}{1 - \delta \rho} \quad (2)$$

Even though the volatility of x_t is low, if the persistence ρ is high, shocks to x_t (expected growth) can have huge impact on long-run growth expectations, giving volatile asset prices.

Solving the Model

Recall

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1} \quad (3)$$

$$r_{m,t+1} = \kappa_0^m + \kappa_1^m z_{m,t+1} - z_{m,t} + g_{d,t+1} \quad (4)$$

where

$$z_t = \log(P_t/C_t) \quad z_{m,t} = \log(P_t/D_t) \quad (5)$$

$$g_{t+1} = \log(C_{t+1}/C_t) \quad g_{d,t+1} = \log(D_{t+1}/D_t) \quad (6)$$

g_{t+1} and $g_{d,t+1}$ are exogenous.
 We must solve for z_t and $z_{m,t}$.

Guess and verify that

$$\log(P_t/C_t) = z_t = A_0 + A_1 x_t + A_2 \sigma_t^2 \quad (7)$$

$$\log(P_t/D_t) = z_{m,t} = A_{0,m} + A_{1,m} x_t + A_{2,m} \sigma_t^2 \quad (8)$$

with

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \quad A_{1,m} = \frac{\varphi - \frac{1}{\psi}}{1 - \kappa_{1,m} \rho} \quad (9)$$

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with

$$A_1 = \frac{1 - \frac{1}{\psi}}{1 - \kappa_1 \rho} \quad A_{1,m} = \frac{\varphi - \frac{1}{\psi}}{1 - \kappa_{1,m} \rho} \quad (9)$$

- 1 $\psi > 1$ (substitution effect dominates wealth effect) $\Rightarrow A_1 > 0$:
Expected growth $\uparrow \Rightarrow$ buy more assets \Rightarrow
wealth-to-consumption ratio, z_t , \uparrow
- 2 $\varphi > 0$, i.e. dividends are more sensitive to long-run risks \Rightarrow
 $A_{1,m} > A_1$ so changes in expected growth has larger impact
on the price of the dividend claim than on the price of the
consumption claim.

Next,

$$A_2 = \frac{0.5 \left(\left(\theta - \frac{\theta}{\phi} \right)^2 + (\theta A_1 \kappa_1 \varphi_e)^2 \right)}{\theta(1 - \kappa_1 \nu_1)} \quad (10)$$

- ① When $\gamma > 1$ and $\psi > 1$ gives $A_2 < 0$. So a rise in consumption volatility lowers price-consumption and asset values.
- ② An increase in ν_1 (the permanence of vol shocks) magnifies the effect of volatility shocks, as investors see changes in economic uncertainty as long-lasting.

Risk Premia

Can now easily show

$$m_{t+1} - E_t(m_{t+1}) = -\lambda_\eta \sigma_\eta \eta_{t+1} - \lambda_e \sigma_e e_{t+1} - \lambda_w \sigma_w w_{t+1} \quad (11)$$

where $\lambda_\eta, \lambda_e, \lambda_w$ are the market prices of short-run, long-run and volatility risks:

$$\lambda_\eta = \gamma \quad \lambda_e = \left(\gamma - \frac{1}{\psi} \right) \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right) \quad (12)$$

$$\lambda_w = \left(\gamma - \frac{1}{\psi} \right) (1 - \gamma) \left(\frac{\kappa_1 \left(1 + \left(\frac{\kappa_1 \varphi_e}{1 - \kappa_1 \rho} \right)^2 \right)}{2(1 - \kappa_1 \nu_1)} \right) \quad (13)$$

With power utility, $\lambda_e = \lambda_w = 0$.

Higher ρ increase exposure to expected growth rates, λ_e .

Equity Premium

The risk premium on any asset is

$$E_t(r_{i,t+1} - r_{f,t}) = \beta_{i,\eta} \lambda_{\eta} \sigma_t^2 + \beta_{i,e} \lambda_e \sigma_t^2 + \beta_{i,w} \lambda_w \sigma_w^2 - 0.5V(r_{i,t+1}). \quad (14)$$

- ① First beta is exposure to short-run risk
- ② Second beta is exposure to long-run risk
- ③ Third beta is exposure to vol-risk

All beta's are endogenous. The risk premium is time-varying as σ_t fluctuates, and rises in times with high economic uncertainty.

The Sharpe-ratio is time-varying.

High consumption vol. (e.g. recessions) increases the risk premium.

Volatility Feedback

The correlation btw.

- 1 Market return innovations
- 2 Market volatility innovations

is around -0.32 in the model:

$$\text{cov}\left((r_{m,t+1} - E_t r_{m,t+1}), \text{var}_{t+1}(r_{m,t+2}) - E_t(\text{var}_{t+1}(r_{m,t+2}))\right) \quad (15)$$

$$= \beta_{m,w}(\beta_{m,e}^2 - \varphi_d^2)\sigma_w^2 < 0 \quad (16)$$

The negative volatility feedback effect is generated by Epstein-Zin preferences, in which volatility is priced.

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Data

- Data from 1928 to 1998
- Consumption is measured by non-durables and services, deflated by CPI
- Dividends and market returns are from CRSP value-weighted portfolio
- BY calibrate the model to match growth rates and market data
- Calibrate the model at the monthly frequency, and aggregate to make annual growth rates match observed data
- Simulate from the model to test its ability to match the data

Parameters

- $\rho = 0.979$ ensures stationarity of expected consumption growth.
- $\mu = \mu_d = 0.0015$ to match consumption and dividend growth.
- $\sigma = 0.0078$ and $\varphi_e = 0.044$ chosen to match uncond. variance and autocorr. of consumption growth
- $\varphi = 3$ chosen to match the higher vol of dividend growth compared to consumption growth
- $\varphi_d = 4.5$ to match uncond. variance of dividend growth and its correlation with consumption

Time-Series Properties

Without fluctuating economic uncertainty ($\sigma_w = 0$)

Table I
Annualized Time-Averaged Growth Rates

Variable	Data		Model				
	Estimate	SE	Mean	95%	5%	<i>p</i> -Val	<i>Pop</i>
$\sigma(g)$	2.93	(0.69)	2.72	3.80	2.01	0.37	2.88
$AC(1)$	0.49	(0.14)	0.48	0.65	0.21	0.53	0.53
$AC(2)$	0.15	(0.22)	0.23	0.50	-0.17	0.70	0.27
$AC(5)$	-0.08	(0.10)	0.13	0.46	-0.13	0.93	0.09
$AC(10)$	0.05	(0.09)	0.01	0.32	-0.24	0.80	0.01
$VR(2)$	1.61	(0.34)	1.47	1.69	1.22	0.17	1.53
$VR(5)$	2.01	(1.23)	2.26	3.78	0.79	0.63	2.36
$VR(10)$	1.57	(2.07)	3.00	6.51	0.76	0.77	2.96
$\sigma(g_d)$	11.49	(1.98)	10.96	15.47	7.79	0.43	11.27
$AC(1)$	0.21	(0.13)	0.33	0.57	0.09	0.53	0.39
$corr(g, g_d)$	0.55	(0.34)	0.31	0.60	-0.03	0.07	0.35

Asset Pricing Implications

Without fluctuating economic uncertainty ($\sigma_w = 0$)

Table II
Asset Pricing Implications—Case I

γ	ψ	$E(R_m - R_f)$	$E(R_f)$	$\sigma(R_m)$	$\sigma(R_f)$	$\sigma(p - d)$
Panel A: $\phi = 3.0, \rho = 0.979$						
7.5	0.5	0.55	4.80	13.11	1.17	0.07
7.5	1.5	2.71	1.61	16.21	0.39	0.16
10.0	0.5	1.19	4.89	13.11	1.17	0.07
10.0	1.5	4.20	1.34	16.21	0.39	0.16
Panel B: $\phi = 3.5, \rho = 0.979$						
7.5	0.5	1.11	4.80	14.17	1.17	0.10
7.5	1.5	3.29	1.61	18.23	0.39	0.19
10.0	0.5	2.07	4.89	14.17	1.17	0.10
10.0	1.5	5.10	1.34	18.23	0.39	0.19
Panel C: $\phi = 3.0, \rho = \varphi_e = 0$						
7.5	0.5	-0.74	4.02	12.15	0.00	0.00
7.5	1.5	-0.74	1.93	12.15	0.00	0.00
10.0	0.5	-0.74	3.75	12.15	0.00	0.00
10.0	1.5	-0.74	1.78	12.15	0.00	0.00

Parameter Effects

Without fluctuating economic uncertainty

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- ① Larger risk aversion increases the equity premium, but does not change other dimensions of the model
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 - Increasing IES increases $A_{1,m}$ which increases the vol of the P/D -ratio and asset returns, and risk premia go up.

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- ③ Increasing φ increases the equity premium
 - Increasing φ makes dividends more volatile compared to consumption
- ④ When consumption growth rates are iid, the equity premium is close to zero.
- ⑤ Although the time-series dynamics of a model with small persistence in expected growth are difficult to distinguish from an iid model, the asset pricing implications are very different.

Asset Pricing Implications

With fluctuating economic uncertainty

Table IV
Asset Pricing Implications—Case II

Variable	Data		Model	
	Estimate	SE	$\gamma = 7.5$	$\gamma = 10$
Returns				
$E(r_m - r_f)$	6.33	(2.15)	4.01	6.84
$E(r_f)$	0.86	(0.42)	1.44	0.93
$\sigma(r_m)$	19.42	(3.07)	17.81	18.65
$\sigma(r_f)$	0.97	(0.28)	0.44	0.57
Price Dividend				
$E(\exp(p - d))$	26.56	(2.53)	25.02	19.98
$\sigma(p - d)$	0.29	(0.04)	0.18	0.21
$AC1(p - d)$	0.81	(0.09)	0.80	0.82
$AC2(p - d)$	0.64	(0.15)	0.65	0.67

Asset Pricing Implications

With fluctuating economic uncertainty

The following statistics match the data

- 1 The equity premium
- 2 The mean of the risk-free rate
- 3 The volatility of market returns
- 4 The volatility of the risk-free rate
- 5 The volatility and autocorrelation of the price-dividend ratio

Risk Components for Consumption Asset

Risk Components and Risk Compensation

		$\psi = 0.1$	Power Util	$\psi = 0.5$	$\psi = 1.5$
mpr_{η}	Market Price of Risk	73.60		93.60	93.60
mpr_e	Expected growth	0.00		137.23	160.05
mpr_w	Volatility	0.00		-27.05	-31.56
β_{η}	Exposure of consumption asset	1.00		1.00	1.00
β_e		-16.49		-1.83	0.61
β_w		11,026.45		1,225.16	-408.39
prm_{η}	Total Risk Compensation =	0.73		0.73	0.73
prm_e	Price of risk * beta * std of shock	0.00		-1.96	0.76
prm_w		0.00		-0.08	0.03

For $\psi < 1$, beta wrt. long-run risk is < 0 and beta wrt. vol is > 0 !!
Leads to a negative risk premium on consumption asset!

Variance of the Pricing Kernel

Max Sharpe ratio \approx vol. of pricing kernel innovation.
What determines this volatility?

Table V
Decomposing the Variance of the Pricing Kernel

Volatility of Pricing Kernel	Relative Variance of Shocks		
	Independent Consumption	Expected Growth Rate	Fluctuating Economic Uncertainty
0.73	14%	47%	39%

Max Sharpe-ratio is 0.73 (market has 0.33).

With iid growth it's 0.27.

Epstein-Zin preferences and non-iid growth generate the higher Sharpe-ratio.

Predictability of Asset Returns

Table VI
Predictability of Returns, Growth Rates,
and Price–Dividend Ratios

Variable	Panel A: Excess Returns			Panel B: Growth Rates			Panel C: Volatility		
	Data	SE	Model	Data	SE	Model	Data	SE	Model
$B(1)$	−0.08	(0.07)	−0.18	0.04	(0.03)	0.06	−8.78	(3.58)	−3.74
$B(3)$	−0.37	(0.16)	−0.47	0.03	(0.05)	0.12	−8.32	(2.81)	−2.54
$B(5)$	−0.66	(0.21)	−0.66	0.02	(0.04)	0.15	−8.65	(2.67)	−1.56
$R^2(1)$	0.02	(0.04)	0.05	0.13	(0.09)	0.10	0.12	(0.05)	0.14
$R^2(3)$	0.19	(0.13)	0.10	0.02	(0.05)	0.12	0.11	(0.04)	0.08
$R^2(5)$	0.37	(0.15)	0.16	0.01	(0.02)	0.11	0.12	(0.04)	0.05

Panel A: $r_{t+1}^e + \dots + r_{t+j}^e = \alpha(j) + B(j) \log(P_t/D_t) + \nu_{t+j}$

Panel B: $g_{t+1}^a + \dots + g_{t+j}^a = \alpha(j) + B(j) \log(P_t/D_t) + \nu_{t+j}$

Panel C:

$\log(P_{t+j}/D_{t+j}) = \alpha(j) + B(j)(\text{vol of consumption}) + \nu_{t+j}$

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Conclusion

The long-run risk model uses

- ① persistent growth
- ② time-varying vol
- ③ Epstein-Zin preferences

to produce

- ① high equity premium
- ② low risk free rate
- ③ time-varying Sharpe-ratio
- ④ predictability of asset returns

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Things that didn't get into this paper

This paper does not

- 1 Impose co-integration on aggregate consumption and dividends
- 2 Discuss the cross-section of returns
- 3 Explain the calibration procedure in detail

However, this is addressed in later papers.

Short-Run Risk

In this paper, the shocks to dividend growth and consumption are uncorrelated.

The dividend growth determines the market portfolio.
The consumption growth determines the SDF.

Hence, the market portfolio has no short-run risk in this model!

Later papers suddenly includes a short-run risk on the market portfolio (and not just the consumption asset).

Long-Run Risk vs. iid model

BY argue roughly as follows:

- 1 Statistically, it is difficult to distinguish between iid consumption growth and the long-run risk model
- 2 With iid consumption growth, the model does not match the data
- 3 The long-run risk model does match the data
- 4 Hence, BY assumes that the representative consumer believes in the long-run risk model, and discards of the iid consumption growth model!

This is reverse-engineering!

Why doesn't the representative agent assign a prior over the two models, and then try to learn about the true model?

Relation to Robust Control

The long-run risk model is **worse** for the consumer than the iid model, because an adverse shock is persistent.

In the robust-control framework, the consumer makes his decision based on the worst case model.

This justifies that the consumer acts **as if** he fully believed in the long-run risk model, even though this is not the case.