Market Skewness Risk and the Cross-Section of Stock Returns*

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Abstract

The cross-section of stock returns has substantial exposure to risk captured by higher moments in market returns. We estimate these moments from daily S&P 500 index option data. The resulting time series of factors are thus genuinely conditional and forward-looking. Stocks with high sensitivities to innovations in implied market volatility and skewness exhibit low returns on average, whereas those with high sensitivities to innovations in implied market kurtosis exhibit somewhat higher returns on average. The results on market skewness risk are robust to various permutations of the empirical setup. The estimated premium for bearing market skewness risk is between −3.72% and −5.76% annually. This market skewness risk premium is economically significant and cannot be explained by other common risk factors such as the market excess return or the size, book-to-market, momentum, and market volatility factors, or by firm characteristics. Using ICAPM intuition, the negative price of market skewness risk indicates that it is a state variable that negatively affects the future investment opportunity set.

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1 Introduction

The volatility of the stock market index return is an indicator of market-wide risk, and the CAPM predicts that it is a determinant of the market equity premium. Recent studies by Ang, Hodrick, Xing and Zhang (2006) and Adrian and Rosenberg (2008) also demonstrate that contrary to the CAPM intuition, market-wide volatility risk is priced in the cross-section of stock returns. Given these findings, and given the overwhelming evidence in the literature that market-wide skewness and kurtosis are important indicators of market-wide risk, and that those risks do not co-vary perfectly with volatility risk, an investigation of higher moments of the market return as pricing factors in the cross-section of stock returns seems worthwhile. We extend the investigation of Ang et al. (2006) and examine how market skewness and kurtosis risks affect the cross-section of stock returns within the context of the Intertemporal Capital Asset Pricing Model (ICAPM) of Merton (1973). Our hypothesis is that the market volatility, skewness, and kurtosis are all state variables reflecting the future investment opportunity set. The ICAPM then predicts that innovations in market volatility, skewness, and kurtosis must be priced risk factors in the cross-section of risky asset returns.

We empirically test this hypothesis using moments of the market return implied by S&P 500 index options, extracted using the methodology of Bakshi and Madan (2000), Carr and Madan (2001) and Bakshi, Kapadia, and Madan (2003). These moments are thus forward-looking. The moments are computed every day, so the resulting estimates are also genuinely conditional. This approach avoids the traditional trade-off problem with estimates of higher moments from historical returns data, which need to use long windows to increase precision, but short windows to obtain conditional rather than unconditional estimates.

We conduct two types of empirical exercises. First, by sorting all stocks on the NYSE, AMEX, and the NASDAQ between 1996 and 2007 in quintiles based on the sensitivities of their returns to innovations in market moments, we find evidence that the market skewness and to a lesser extent kurtosis risks are priced in the cross-section of stocks, and that stock returns have substantial exposure to higher moment risk. We find that stocks with high sensitivities to innovations in market skewness exhibit low returns on average while stocks with high sensitivities to innovations in market kurtosis exhibit somewhat higher returns on average. We extensively investigate the robustness of our empirical results and find that the effect of market skewness risk is robust, whereas that of market kurtosis risk is sensitive to variations in test methodology and data sample.

Skewness and kurtosis innovations are highly negatively correlated (-0.83) and so we rely on kurtosis innovations that have been orthogonalized by skewness. Nevertheless, the large negative correlation suggests that it might be difficult to fully separate the effects of skewness and kurtosis. We therefore construct hedge portfolios for market skewness and kurtosis risk by quadruple-sorting the stocks with respect to their sensitivities to market excess returns, innovations in market volatility, innovations in market skewness, and innovations in market kurtosis, in order to separate the effects of the market excess return and market volatility, skewness and kurtosis risks. We find that the average return on the market skewness risk hedge portfolio is -0.48% per month or -5.76% per year, and this return cannot be explained by the market beta, size factor, book-to-market factor, or the momentum factor. We also estimate the price of market skewness risk by running Fama-MacBeth regressions using different test portfolios and different beta estimation windows. We find that estimates of the price of market skewness risk are consistently negative and significant with values mostly falling between -0.31% and -0.43% per month, or between -3.72% and -5.16% per year.

Our empirical findings contribute to an existing literature that emphasizes the importance of higher-moment risk in asset pricing. Part of this literature investigates three- and four-factor CAPMs, building on the seminal contribution by Kraus and Litzenberger (1976).[1] These models are inherently static, and start from investor preferences defined over expected returns and return variance, skewness and kurtosis, to derive a cross-sectional model in which the prices of the risk factors can be signed starting from assumptions on the representative agent's utility function. We take an ICAPM approach, which is somewhat different. According to the ICAPM, higher moments matter in the cross-section of returns because they allow investors to hedge against changes in future investment opportunities. In traditional ICAPM approaches, investor preferences are defined over the first two moments, and therefore investors do not care directly about higher moments.[2] The intuition that investors dislike highly negative skewness and high kurtosis of their wealth portfolios is not relevant in this regard.

Economic theory thus provides limited guidance on the signs of the prices of risk on the higher moments of the market return in the ICAPM. The most useful alternative source of information is therefore the correlation between the moments on the one hand and the market return and the future opportunity set on the other hand. For the volatility risk factor investigated in Ang et al. (2006), it has been established empirically that increased volatility is associated with a deterioration of the investment opportunity set, leading to a negative price of risk. Unfortunately, little is known about the relation between the future investment opportunity set and higher moments. We examine the empirical relationship between the innovation in option-implied market skewness and the stock market return, and find a significant negative relationship. A positive surprise in option-implied market skewness coincides with a lower stock market return, which suggests that increased market skewness is linked to a deteriorating investment opportunity set. In this case, the expected sign of market skewness risk is negative, consistent with our cross-sectional evidence. We also find that increased skewness forecasts higher future volatility, which signals a deterioration of the future opportunity set, but it does not forecast lower future market returns.

[1] For theory and empirical results on the three- and four-factor CAPM, see for example Dittmar (2002), Friend and Westerfield (1980), Harvey and Siddique (1999, 2000a, 2000b), Hwang and Satchell (1999), Lim (1989), Kraus and Litzenberger (1976), Sears and Wei (1985, 1988), and Chabi-Yo (2008). See also Bansal, Hsieh, and Viswanathan (1993), Bansal and Viswanathan (1993), and Chapman (1997).

[2] See Chen (2003) and Chabi-Yo (2009) for notable exceptions.

Our results are related to several other studies. Kapadia (2006) finds a negative price of market skewness risk, but he uses the cross-sectional skewness across stocks at a point in time, which is very different from our approach. Adrian and Rosenberg (2008), on the other hand, document a positive price of market skewness risk when the market skewness is estimated from a historical time series of daily stock returns. Our experimentation with historical higher moments did not produce robust results, and we therefore prefer working with option-implied moments, as in Ang et al. (2006). Conrad, Dittmar, and Ghysels (2008) and Xing, Zhang, and Zhao (2008) study the cross-sectional differences in stock returns as a function of the risk-neutral skewness of individual stocks. Vanden (2004, 2006) shows that in an equilibrium setup with non-negative wealth constraints, the pricing kernel is a function of the return on a stock market index and the return on market index options as well as higher powers of these two returns. Vanden's models are related to our approach in that his pricing kernel includes information from the index option market.

Our study is most closely related to Agarwal, Bakshi, and Huij (2008). They examine a linear multifactor model with four factors: market excess return, innovation in implied market volatility, innovation in implied market skewness, and innovation in implied market kurtosis, using the same methodology that we use to extract implied moments of the market return. They test their model's ability to predict the cross section of investment fund returns and find that hedge funds are substantially exposed to the risk factors of their model while mutual funds do not exhibit significant exposure to the same risk factors. Our work differs from theirs in that we explain the expected returns of individual stocks rather than investment funds.

The paper proceeds as follows. In Section 2 we introduce the empirical model and we discuss the relationship between our findings and the existing literature. In Section 3, we discuss the data, as well as the methods used to extract higher moments from option data, and the models used to extract innovations from option-implied moments. Section 4 presents empirical results obtained by sorting the cross-section of stocks into quintiles based on sensitivities to market moments. Section 5 constructs hedge portfolios, and Section 6 estimates the price of market skewness risk. Section 7 concludes.

2 The Model

In this section we introduce the cross-sectional model and our empirical strategy. We first introduce the ICAPM setup. Subsequently we discuss alternative theoretical perspectives on the model's specification, as well as existing research that provides guidance regarding the prices of skewness and kurtosis risk.

2.1 An ICAPM Model

When the distributions of asset returns change over time, the intertemporal capital asset pricing model (ICAPM) of Merton (1973) shows that the equilibrium expected returns of risky assets in the cross-section are determined by the conditional covariances between the asset returns and the innovations in state variables that allow investors to hedge against changes in the investment opportunity set. Our hypothesis is that the moments of the stock market return, namely, volatility, skewness, and kurtosis, are such state variables.

We empirically investigate this ICAPM conditional multifactor representation of equilibrium expected returns using two empirical strategies.

The first strategy is based on univariate sorting. We use a sample of returns and moments for a time period $t = 1, ..., T$ to estimate the risky assets' sensitivities to (or loadings on) innovations in market moments through time-series regressions of the form,

$$\begin{aligned} R_{i,t} - R_{f,t} &= \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_{m,t} \\ &\quad + \beta_{\Delta SKEW}^i \Delta SKEW_{m,t} + \beta_{\Delta KURT}^i \Delta KURT_{m,t} + \varepsilon_{i,t}, \end{aligned} \tag{1}$$

where $R_{i,t}$, $R_{m,t}$, and $R_{f,t}$ are the rates of return on the ith risky asset, the market portfolio, and the risk free asset respectively. Further $\Delta\text{VOL}_{m,t+1} = \text{VOL}_{m,t+1} - \text{E}_t[\text{VOL}_{m,t+1}]$, $\Delta\text{SKEW}_{m,t+1} = \text{SKEW}_{m,t+1} - \text{E}_t[\text{SKEW}_{m,t+1}]$, and $\Delta\text{KURT}_{m,t+1} = \text{KURT}_{m,t+1} - \text{E}_t[\text{KURT}_{m,t+1}]$. The regression coefficients, β_{MKT}^i, $\beta_{\Delta VOL}^i$, $\beta_{\Delta SKEW}^i$, and $\beta_{\Delta KURT}^i$, are the measures of the ith risky asset's exposures to the market excess return, market volatility, market skewness, and market kurtosis risks. We assume that $\varepsilon_{i,t}$ is homoskedastic and independent of $R_m - R_f$, ΔVOL_m, ΔSKEW_m, or ΔKURT_m. The estimates of the regression coefficients, β_{MKT}^i, $\beta_{\Delta VOL}^i$, $\beta_{\Delta SKEW}^i$, and $\beta_{\Delta KURT}^i$ are used to sort the available assets in different portfolios, and the cross-sectional performance of these portfolios is indicative of the price of risk associated with the different factors.

The second set of results is based on multivariate sorts and cross-sectional regressions. We use regression coefficients for assets $i = 1, ..., N$ obtained from time-series regressions to estimate the prices of the market moment risks λ from the cross-sectional relationship

$$E\left[R_i\right] - R_f = \lambda_0 + \lambda_{MKT}\beta_{MKT}^i + \lambda_{\Delta VOL}\beta_{\Delta VOL}^i + \lambda_{\Delta SKEW}\beta_{\Delta SKEW}^i + \lambda_{\Delta KURT}\beta_{\Delta KURT}^i \tag{2}$$

The ICAPM logic, following Merton (1973), Campbell (1996), Ang et al. (2006), and Chen (2003), is that higher moments matter in the cross-section of returns because they allow investors to hedge against changes in future investment opportunities. The prices of risk of the factors therefore depend on whether they reflect improvements or deteriorations in the economy's (future) opportunity set. For instance, if high market volatility today is related to an unfavorable investment opportunity set tomorrow, then an asset whose return is positively related to the innovation in market volatility provides a hedge against a deterioration in the investment opportunity set. When investors are risk averse, the hedge provided by this asset is desirable, resulting in a lower expected return for such asset. The price of market volatility risk is then negative. In the opposite scenario where high market volatility is related to a favorable future investment opportunity set, the price of market volatility risk will be positive. Previous studies have found that market volatility is high when market returns are low, a phenomenon sometimes termed the leverage effect. Therefore, we can expect the sign of the price of market volatility risk, $\lambda_{\Delta VOL}$, to be negative.

2.2 Existing Empirical Evidence and Theoretical Perspectives

Following the reasoning regarding the price of market volatility risk $\lambda_{\Delta VOL}$, empirical findings on the correlation between higher market moments and market returns may provide guidance regarding the price of market skewness and kurtosis risk. Unfortunately, there is no consensus in the literature regarding these correlations, and in fact very few empirical findings are available. We investigate these correlations in Section 6.4.

Economic theory also provides little guidance on the signs of the prices of the market skewness and kurtosis risks. It is very important to note the differences between our setup and available results on the three-moment and four-moment CAPM. The three-moment and four-moment CAPM are based on the idea that investors care about not only the mean and variance of their wealth portfolios, but also the skewness and kurtosis. They are inherently static extensions of the CAPM, in which investors are no longer mean-variance optimizers. The main insight of the four-moment CAPM is that only systematic volatility, systematic skewness, and systematic kurtosis - individual assets' contribution to the market portfolio volatility, skewness, and kurtosis - rather than total volatility, skewness, and kurtosis of individual asset returns, matter in pricing. The signs of the cross-sectional prices of risk can be determined subject to certain assumptions on the utility function of the representative agent.[3]

In the traditional ICAPM setting, market moments are state variables, and investors' preferences on skewness and kurtosis of asset returns are irrelevant. Investors remain mean-variance optimizers, and higher moments of the market return affect asset prices only to the extent that they reflect changes in the mean and variance of the asset returns. While individual assets' exposures to higher moments must be reflected in their prices, the betas are constructed using different combinations of higher market moments, and it is not straightforward to relate the signs and magnitudes of our betas to those found using the four-factor CAPM.

Chabi-Yo (2009) provides theoretical guidance regarding the signs of skewness and kurtosis risk. In his model, which can be seen as an intertemporal extension of the three-moment and four-moment CAPM, market volatility, skewness and kurtosis show up as cross-sectional pricing factors.[4] The price of market volatility risk $\lambda_{\Delta VOL}$ is negative if agents' preference for skewness is smaller than one, which depends on the third derivative of the utility function. Similarly, the price of market skewness risk $\lambda_{\Delta SKEW}$ and the price of market kurtosis risk $\lambda_{\Delta KURT}$ depend on the fourth respectively the fifth derivative of the utility function, which are hard to sign. Chabi-Yo (2009) estimates the preference parameters characterizing his model and finds that the price of volatility risk and skewness risk are both negative.

An alternative interpretation of option-implied skewness is as a measure of jump risk or down-

[3] The four-moment CAPM predicts that since investors with reasonable utility functions (Kimball (1993)) prefer wealth portfolios with low volatility, high positive skewness, and low kurtosis, they need to be compensated with higher expected wealth when their portfolios exhibit returns with high volatility, low skewness, or high kurtosis.

[4] See Li (2004) for a related approach.

side risk (Bates, 2000, Pan, 2002, Doran, Peterson, and Tarrant, 2007). Under this interpretation of option-implied skewness, a positive innovation in option-implied market skewness indicates decreased jump risk in the stock market, which is likely to be related to an improved investment opportunity set. The expected relationship between ΔSKEW and the market excess return is therefore positive, and stocks with low sensitivities to innovations in market skewness provide a valuable hedge against downside risk of the stock market. Investors will require lower returns on these stocks, and we would expect the price of market skewness risk to be positive.

Yet another explanation of option-implied skewness is provided by recent studies by Bollen and Whaley (2004) and Gârleanu, Pedersen, and Poteshman (2009), who suggest that the skewness of the implied volatility curve is mainly caused by the imbalance in supply and demand for options with different strike prices. Unfortunately, because these studies take the imbalance in supply and demand in options as exogenous, it is not straightforward to interpret the relationship between innovations in option-implied market skewness and changes in the investment opportunity set using their models.

3 Data and Measurement of Innovations in Market Moments

3.1 Estimating Higher Moments of Market Returns

Several methods are available to estimate moments of the market return. The most widely used estimator is the sample moment from historical returns. However, the computation of sample moments necessitates a choice of time window. It is well known that it is difficult to estimate higher moments precisely (Kim and White, 2004), which would suggest the use of long windows, but on the other hand it is preferable to use short windows to capture the conditional nature of the factor sensitivities. In order to obtain more reliable estimates of conditional higher moments, a time-series model can be used, but then the question arises whether the empirical results are robust to the choice of time-series model.

The availability of tick-by-tick price data provides an interesting alternative. We can estimate the return variance by summing the squares of high frequency returns (e.g. 5 min.) as described in Andersen, Bollerslev, Diebold, and Labys (2003) and Barndorff-Nielsen and Shephard (2002). However, the estimation of skewness or kurtosis from high frequency data is much less customary, perhaps since the sampling properties of these estimators are as yet unknown.

In this study, we instead estimate the higher moments of the market return by extracting moments implied by daily index option prices. We use the model-free methodology proposed by Carr and Madan (2001), Bakshi and Madan (2000), Bakshi, Kapadia and Madan (2003), and Jiang and Tian (2005) to estimate the moments of market returns implied by S&P 500 index option prices, and call them VOL, SKEW, and KURT. We use implied moments over a 30-day horizon for our tests, based on options with one-month maturity. The details of the methodology and implementation are provided in the Appendix. Data on S&P 500 index options between 1996 and

2007 are obtained from OptionMetrics. Since VOL has a correlation of 0.99 with the CBOE VIX index, we use VIX instead of VOL in our tests to facilitate replication of our results and maximum comparability with the results of Ang, Hodrick, Xing and Zhang (2006).

Three remarks are in order at this point. First, in contrast with traditional historical moment estimates, option implied moments are obtained using a single day of option data, and are therefore conceptually more suited for the testing of conditional asset pricing models. Second, option implied moments have the advantage of being forward looking, which is more consistent with underlying theories of expected, i.e. future, returns. Third, option implied moments are risk-neutral moments. Note that while the nature of the pricing factors is not restricted by the ICAPM in (2) and (1), and therefore risk-neutral moments are potentially valuable candidates for pricing factors, changes in the risk-neutral pricing factors have to reflect changes in the physical distribution of returns which captures the investment opportunity set, and not only risk premia. Following Ang et al. (2006), we do not try to specify the risk premia required to convert the risk neutral moments to their physical counterparts. Doing so would require choosing one from many possible specifications of the volatility risk and jump risk premia.[5] While it is a potential disadvantage that changes in risk-neutral moments reflect both changes in the physical moment and changes in the risk premia, this has to be traded off against their advantages, notably that they can be obtained using a single day of data using the rich information available in option prices. The usefulness of these moments as pricing factors is thus largely an empirical question, and our results show that risk-neutral skewness implied from options is an important risk-factor empirically. The work of Campbell (1996) and Chen (2003) suggests that candidate ICAPM state variables ought to have forecasting power for the physical return distribution, and we investigate the forecasting performance of the option-implied moments in Section 6.4.

The daily measures of VIX, SKEW, and KURT are shown in Figure 1. Note that all three time series vary significantly through time. Consistent with available empirical evidence, the implied market skewness measures are always negative. Note also that kurtosis is always larger than three.

[Figure 1: Daily Option Implied Moments of S&P 500 Index Returns]

3.2 Measuring Innovations in Market Moments

To obtain estimates of innovations in market moments, we fit an appropriate ARMA model to the time series for each moment. The results are reported in Table 1. We report the autocorrelation functions of the original time series, AR(1) residuals, and ARMA(1,1) residuals for VIX, SKEW, and KURT in Figure 2 to demonstrate our choice of time series model for each market moment. For VIX, taking the first difference removes most of the autocorrelation in the data, whereas for SKEW and KURT, ARMA(1,1) models are needed to remove the autocorrelation. Using the first

[5] See Bates (2000), Broadie, Chernov and Johannes (2007), Pan (2002), Jones (2003), and Eraker (2004) for specifications of volatility and jump risk premia in option valuation.

difference of VIX has an additional advantage that it makes it easier to compare our empirical results to those of other related studies such as Ang, Hodrick, Xing, and Zhang (2006) since many of these studies employ the change in VIX in their analyses. To check for robustness, we repeat our tests using the ARMA(1,1) residuals of VOL as our measures of ΔVOL and find that the results are very similar. The results of this robustness test are not reported in the paper, but are available from the authors upon request.

[Figure 2: Autocorrelation Functions of Option Implied Market Moments]
[Table 1: Correlations and Averages of Daily Risk Factors]

Our main results use the entire time series to calibrate the ARMA parameters, and then use these parameters to compute ΔSKEW and ΔKURT. The use of the entire time series may seem somewhat unrealistic since investors can only observe past moments when forming their expectations on future moments. We also computed innovations recursively using only past moments, and this yields a very similar time series.

The resulting measures of innovations in market moments are obtained using the following models:

$$\Delta VOL_t = VIX_t - VIX_{t-1} \tag{3}$$

$$\Delta SKEW_t = SKEW_t - 0.9962 \times SKEW_{t-1} + 0.3618 \times \Delta SKEW_{t-1} \tag{4}$$

$$\Delta KURT_t = KURT_t - 0.9936 \times KURT_{t-1} + 0.4032 \times \Delta KURT_{t-1} \tag{5}$$

Note that the AR(1) parameters for both SKEW and KURT are very close to -1. This indicates that we can use an MA(1) model on the first differences to obtain the innovations in SKEW and KURT. The results from using this alternative assumption on the innovations are very similar to the results reported and are available from the authors upon request.

Table 1 reports the correlations between ΔVOL, ΔSKEW, and ΔKURT, as well as correlations with known pricing factors, namely the excess market return R_m-R_f, the Fama and French (1993) pricing factors SMB and HML, and the momentum factor UMD. Note that ΔVOL is highly negatively correlated (-0.79) with the market excess return and highly positively correlated (0.42) with HML. Also note that ΔSKEW and ΔKURT are highly negatively correlated (-0.83).

A negative relationship between ΔSKEW and ΔKURT is to be expected. The option implied distribution has a fat left tail and so negative skewness on average. A negative shock to skewness increases the fat left tail further and thus increases kurtosis. That is, the relationship between ΔSKEW and ΔKURT will be negative. In order to separate the effect from skewness on kurtosis from pure kurtosis dynamics, we orthogonalize ΔKURT by regressing it on the contemporaneous ΔSKEW. Throughout the paper we use the residuals from this regression as ΔKURT.

[Figure 3: Daily Innovations in Option Implied Moments of S&P 500 Index Returns]

Figure 3 plots the daily innovations for the three option implied moments. ΔVIX are the first-differences in VIX which we will henceforth refer to as ΔVOL. ΔSKEW are the ARMA residuals from (4) and ΔKURT are the ARMA residuals from (5) orthogonalized by ΔSKEW.

3.3 Return Data

We use returns on all stocks included in the CRSP NYSE/AMEX/NASDAQ Daily Stock file. The stock index return, risk-free rate, and the factor mimicking portfolio returns for size, book-to-market, and momentum factors, as well as the returns of the 25 Fama-French portfolios formed on size and book-to-market, and 49 industry portfolios are obtained from the online data library of Ken French. Since OptionMetrics option data starts in 1996, all our tests are focused on the twelve-year period between 1996 and 2007.

4 Portfolio Sorts on Exposure to Market Moment Innovations

This section reports on three empirical exercises that all follow a similar approach. In each case, we sort the cross-section of stock returns into quintiles based on the stock's sensitivity to innovations in one of the market moments. We first sort based on the sensitivity to innovations in market volatility, and subsequently we sort based on sensitivity to market skewness and kurtosis. Finally, we briefly report on an extensive robustness exercise.

4.1 Portfolios Sorted on Exposure to Innovations in Market Volatility

The main implication of the ICAPM model in (2) is that stocks with different sensitivities to innovations in market volatility, skewness, or kurtosis exhibit different returns on average. In this section, we test if this implication holds for market volatility risk by first constructing portfolios based on their sensitivities to ΔVOL, and subsequently comparing the average returns and alphas of these portfolios. In order to avoid spurious effects we consider out-of-sample returns and alphas, following the procedures in Ang et al. (2006), Agarwal et al. (2008), and Harvey and Siddique (2000a), among others.

Ang, Hodrick, Xing, and Zhang (2006) have demonstrated that stocks with high sensitivities to ΔVIX exhibit lower average returns compared to stocks with low sensitivities to ΔVIX, using all stocks in the NYSE/AMEX/NASDAQ between 1986 and 2000.[6] Their study does not consider market skewness and kurtosis risks. We investigate whether the effect of market volatility risk persists even after taking market skewness and kurtosis risks into account.

In order to capture the conditional nature of the factor exposures, we use daily return data with fairly short windows. Following Pástor and Stambaugh (2003), Ang et al. (2006) and Lewellen and Nagel (2006), we start by using a one-month window which appears to strike a good balance between

[6] For additional work on volatility and the cross-section of returns, see Adrian and Rosenberg (2008), Goyal and Santa-Clara (2003), Fu (2009), and Bali and Cakici (2007).

getting reasonably precise estimates while allowing for time-varying factor loadings. At the end of each month, we run one of the following time series regressions on daily returns of each stock during that month in order to estimate its sensitivity to ΔVOL. The first specification measures stocks' sensitivities to ΔVOL after controlling for exposure to market excess return. The second specification controls for exposure to market excess return, ΔSKEW, and ΔKURT.

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \epsilon_{i,t} \tag{6}$$

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \epsilon_{i,t} \tag{7}$$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta VOL}$, so that quintile 1 contains stocks with the lowest $\beta_{\Delta VOL}$ and quintile 5 stocks with the highest $\beta_{\Delta VOL}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After the portfolio formation, we record the daily returns of each quintile portfolio during the 1-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward one month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta VOL}$ for each quintile portfolio. To see if the effect of ΔVOL persists after controlling for other well known factors including market excess return, size, book-to-market, and momentum, we also compute Jensen's alpha of each quintile portfolio with respect to the Carhart 4-factor model (Carhart, 1997) using post-ranking daily returns over the whole sample. For each quintile portfolio, Panel A of Table 2 reports the average pre-ranking $\beta_{\Delta VOL}$, the average post-ranking monthly returns, and the Carhart 4-factor alphas. We also report the average return and Carhart 4-factor alpha of a portfolio that is long the highest quintile portfolio and short the lowest quintile portfolio, denoted as 5-1. The results for (6) are referred to as the univariate results, while the results for (7) are referred to as the multivariate results.

[Table 2: Sorting on ΔVOL Loadings]

If the innovation in market volatility is a priced risk factor, we would ideally like to see a monotonic pattern in average returns for portfolios sorted on their sensitivity to innovations in market volatility. Since high volatility is generally associated with a deterioration in the investment opportunity set, we would expect to see a decreasing pattern in average returns and alphas from quintile 1 (lowest sensitivity) to quintile 5 (highest sensitivity). We would also expect to see a negative average return and alpha for the high-low portfolio. In Panel A of Table 2, the univariate Carhart 4-factor alphas are monotonically decreasing from quintile 1 to 5, with a dispersion of −0.71%, or −8.52% per year. In the multivariate case the dispersion is −0.53% per month, or −6.36% per year, but the pattern is not entirely monotonic across quintiles. The average returns show smaller negative dispersions of −0.46% and −0.29% per month respectively.

Panels B and C of Table 2 repeat the analysis of Panel A, but now the betas are obtained using three months and six months of daily data respectively. The longer estimation period makes the estimate less genuinely conditional, but may lead to more precise estimates because of the increased sample size. The three month-betas yield negative but statistically insignificant dispersion estimates. The six-month betas yield statistically insignificant positive dispersion estimates.

Overall, we conclude that ΔVOL is a priced risk factor with a negative price of risk, which confirms the finding of Ang, Hodrick, Xing, and Zhang (2006), but the estimates are statistically significant only in the case of one-month betas, and when ΔSKEW and ΔKURT are added as factors the significance decreases. Note that the difference in significance could well be driven by the difference in sample periods. Ang, Hodrick, Xing, and Zhang (2006) uses data for 1986-2000, whereas our sample covers the 1996-2007 period.

4.2 Portfolios Sorted on Sensitivity to Innovations in Market Skewness

The portfolio formation procedure and the empirical strategy for market skewness risk are identical to that for market volatility risk, except that stocks are sorted on $\beta_{\Delta SKEW}$. Furthermore, regression (6) is replaced by

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta SKEW}^i \Delta SKEW + \epsilon_{i,t}. \tag{8}$$

The results of this procedure are reported in Table 3.

[Table 3: Sorting on ΔSKEW Loadings]

The results show that in almost all cases considered, portfolios sorted on $\beta_{\Delta SKEW}$ exhibit a monotonically decreasing pattern in average alphas and returns. The estimate of dispersion is negative in all cases. The dispersion of the alphas between quintiles 1 and 5 in Panel A is -0.80% per month in the univariate case and -1.26% per month in the multivariate case, or -9.60% respectively -15.12% per year. Both estimates are statistically significant. The estimates in Panels B and C, obtained with three-month and six-month betas, are also large and statistically significant.

Overall, there is strong evidence that ΔSKEW is a priced risk factor with a *negative* price of risk. Comparing the results in Tables 2 and 3, the exposure to skewness risk seems quantitatively larger than the exposure to volatility risk, and statistically more significant.

4.3 Portfolios Sorted on Sensitivity to Innovations in Market Kurtosis

The portfolio formation procedure for market kurtosis risk is identical to that for market volatility risk, except that stocks are sorted on $\beta_{\Delta KURT}$ and that the regression (6) is replaced by

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta KURT}^i \Delta KURT + \epsilon_{i,t} \tag{9}$$

The results of this procedure are reported in Table 4.

[Table 4: Sorting on ΔKURT Loadings]

The results in Panel A show positive dispersion estimates, but the portfolios sorted on $\beta_{\Delta KURT}$ do not exhibit a monotonically increasing pattern in average returns and alphas. The dispersion of the alphas between quintiles 1 and 5 is 0.32% per month in the univariate case and 0.43% per month in the multivariate case. However, the results obtained using three-month and six-month betas in Panels B and C are inconclusive. Overall, there is some limited evidence that ΔKURT is a priced risk factor with a *positive* price of risk.

4.4 Robustness

4.4.1 Sub-Periods

The period between 1996 and 2007 is characterized by an initial bubble-like stock market boom (until August 2000), followed by a sharp stock market decline and then recovery. In order to verify that our results are not driven by the peculiar circumstances in this sample period, we repeat our tests on two sub-periods: 1996-2000 and 2001-2007. We report results obtained using the 1-month beta estimation period and betas from regression (7), which includes innovations in all market moments.

The results are reported in Table 5. When sorting on skewness beta, we find that the patterns in average returns and in Carhart 4-factor model alphas observed in Table 3 for the period 1996-2007 are robustly present in the sub-periods, although the pattern is not always strictly monotonic. The estimates of the dispersion of the alphas are relatively robust, and are statistically significant.

The estimate of the volatility dispersion for the Carhart 4-factor model alphas is negative in both subperiods, but it is statistically significant for the 1996-2000 period only. The estimate for this period is much larger than the corresponding one in Table 2, even though the dispersion estimate based on returns is comparable. This result indicates that the effect of ΔVOL was significant during 1996-2000, but weakened during 2001-2007. When sorting on kurtosis, the 1996-2000 period does not yield conclusive evidence, but we find a positive dispersion with a monotonically increasing pattern in both alphas and returns for the 2001-2007 period.

[Table 5: Sorting on Market Moment Risk Loadings in Sub-Periods]

4.4.2 Alternative Measurement of Innovations

The results in Tables 2 through 4 are obtained using innovations in market moments resulting from fitting ARMA models to the entire time series of moments. As mentioned above, we obtain a very similar time series of innovations when estimating innovations recursively through time using only past data. We repeated our tests using the ARMA(1,1) residuals of VIX as our measure of ΔVOL and also using the model-free VOL series constructed according to Bakshi, Kapadia, and Madan (2003) instead of VIX. Finally, we repeated our tests using ARIMA(0,1,1) instead of ARMA(1,1).

In all cases, results are very similar. We do not report these results because of space constraints. They are available from the authors on request.

4.4.3 Option Maturity

The methodology of Bakshi, Kapadia, and Madan (2003) allows us to estimate option-implied moments using different option maturities. For instance, 1-month option-implied moments of the S&P 500 index reflect the investors' expectation of risk in the stock market over the next one month. We focus our analysis in this paper on the 1-month option-implied moments and their effects on the stock returns in the next month. But for other applications analyzing asset returns over a period longer than 1 month, the use of option-implied moments over a longer horizon would be more appropriate. We investigated (not reported) longer horizons using 3-month and 6-month options. For 3-month options we find again that portfolios sorted on $\beta_{\Delta SKEW}$ exhibit a decreasing pattern in average alphas and returns, but the results for the volatility and kurtosis factors are not robust. We do not find significant results when using 6-month options to estimate ΔVOL, ΔSKEW, and ΔKURT. One possible reason is that 6-month options are much less liquid, and therefore estimates of 6-month option-implied moments are more noisy.

4.4.4 Historical Moments

We repeated our procedure (not reported here) using ΔVOL, ΔSKEW, and ΔKURT estimated from moving windows of daily historical returns instead of option implied moments. In general, we were not able to arrive at robust conclusions. The signs of volatility, skewness, and kurtosis risk typically depend on the window used when estimating the moments. Moreover, the patterns across the quintile portfolios are typically not monotonic. Whereas some empirical results are consistent with the findings in Tables 2 through 4, such findings are hardly meaningful given the lack of robustness, and we therefore do not tabulate them here.

These findings are of course not necessarily surprising, nor do they invalidate the findings in Tables 2 through 4. They merely indicate that the option implied moments used in our analysis are different from historical moments. They are estimated using one day of options data with many strike prices, and they incorporate expectations about the future distribution of returns.

5 Returns on Hedge Portfolios

5.1 Constructing Hedge Portfolios

The analysis in Section 4 shows that the effect of ΔSKEW on the cross-section of stock returns is robust to variations in the empirical setup and across sample periods. The results on the effect of ΔKURT, however, change when the betas are estimated using longer windows, or for certain sub-periods.

One problem in interpreting the results in Section 4 is the correlation between different market moments. If sensitivities to different factors are correlated, then it is important to separate the pricing effects of different factors in order to identify the implication of each market moment separately. We use a four-way sort on β_{MKT}, $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$ and $\beta_{\Delta KURT}$ following the sorting approach used in Fama and French (1993), Cochrane (2004), and Liew and Vassalou (2000), among others.[7]

At the end of each month, we run a regression with market excess return, ΔVOL, ΔSKEW and ΔKURT as factors for each stock, as in equation (7). We first group the stocks into terciles based on β_{MKT} (lowest in tercile 1 and highest in tercile 3), and then group each of these 3 portfolios into terciles based on $\beta_{\Delta VOL}$ to yield $3^2 = 9$ portfolios. Subsequently, we group each of these 9 portfolios into terciles based on $\beta_{\Delta SKEW}$ to yield $3^3 = 27$ portfolios, and finally, we group each of these 27 portfolios into terciles based on $\beta_{\Delta KURT}$ to yield $3^4 = 81$ portfolios in total.

We repeat the procedure from Section 4 to obtain the time series of pre-ranking betas and post-ranking returns as well as the Carhart 4-factor alphas for each of the 81 portfolios. We do not report the details of these results because of space constraints, but they are available from the authors on request. The dispersion in alphas after controlling for the factors in the Carhart 4-factor model is substantial. The highest alpha is 1.35% per month, and the lowest −1.09% per month, or 16.20% and −13.08% per year.

The impact of each risk factor can be traced by looking at the variation in returns when varying that factor and keeping the others constant. Table 6 summarizes the results for the 81 portfolios by grouping them according to high, medium, or low exposure to each of the factors one at a time, and averaging over the 27 portfolios in each group. This grouping procedure allows us to obtain portfolios which have varying exposures to one factor, but are neutral in the other factors. The row H-L reports the average returns and alphas of the high-low portfolios that are long 27 high exposure portfolios and short 27 low exposure portfolios with respect to a given factor. The high-low portfolios capture the risk premium of being exposed to the corresponding factors and are equivalent to the SMB and HML hedge portfolios for size and book-to-market effects. We find that the average return of the $\beta_{\Delta VOL}$ high-low portfolio is −0.27% per month, that of the $\beta_{\Delta SKEW}$ high-low portfolio is −0.48% per month, and that of the $\beta_{\Delta KURT}$ high-low portfolio is 0.16% per month, but only the estimate for the $\beta_{\Delta SKEW}$ high-low portfolio is statistically significant with a t-statistic of −2.70. We also report the Carhart 4-factor alpha of the high-low portfolios to see if the return spread is captured by the Carhart 4 factors. We find that for both ΔVOL and ΔSKEW exposure portfolios, not only do the alphas decrease from low to high exposure groups as in average returns, but the magnitudes of the dispersions are even wider for the alphas. This result shows that the difference between the high and low volatility and skewness exposure portfolios cannot

[7] Agarwal, Bakshi, and Huij (2008) use a three-way sort on $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$, and $\beta_{\Delta KURT}$ when analyzing higher moments. We use a four-way sort because the large negative correlation between the market excess return and innovation in market volatility reported in Table 1 suggests that controlling for β_{MKT} is also important when separating out the pricing effects of different market moments.

be explained by the market excess return, size, book-to-market, or momentum effects. For the ΔKURT exposure portfolio, the estimate of the alpha dispersion is the same as in the case of average returns.

Thus, in summary, both ΔVOL and ΔSKEW exposure portfolios show decreasing patterns in average returns and in Carhart 4-factor alphas, consistent with our earlier results, and ΔKURT exposure portfolios show increasing patterns in average returns and in Carhart 4-factor alphas.

[Table 6: Portfolios Sorted on Sensitivities to $R_m - R_f$, ΔVOL, ΔSKEW and ΔKURT by Groups]

The returns on the volatility, skewness, and kurtosis hedge portfolios constructed above can be used as proxies for risk factors, ΔVOL, ΔSKEW and ΔKURT, in the same vein as SMB and HML. We refer to the volatility hedge portfolio as FVOL, the skewness hedge portfolio as FSKEW, and the kurtosis hedge portfolio as FKURT. They correspond to the high-low portfolios in Table 6 and are explicitly defined as

$$\begin{aligned} FVOL &= (1/27)(R\beta_{\Delta VOL,H} - R\beta_{\Delta VOL,L}) \\ FSKEW &= (1/27)(R\beta_{\Delta SKEW,H} - R\beta_{\Delta SKEW,L}) \\ FKURT &= (1/27)(R\beta_{\Delta KURT,H} - R\beta_{\Delta KURT,L}) \end{aligned}$$

where $R\beta_{\Delta VOL,H}$ and $R\beta_{\Delta VOL,L}$ denote the sum of the returns on the 27 portfolios with highest and lowest exposure to ΔVOL respectively. $R\beta_{\Delta SKEW,H}$, $R\beta_{\Delta SKEW,L}$, $R\beta_{\Delta KURT,H}$, and $R\beta_{\Delta KURT,L}$ are defined in a similar fashion.

Since FVOL, FSKEW, and FKURT are the excess returns of traded portfolios, the prices of risk corresponding to these factors are simply their average returns, which were estimated to be −0.27%, −0.48%, and 0.16% per month respectively in Table 7. To check that these estimates of prices of risk are economically significant, we compare them to the average returns of Rm-Rf, SMB, HML, and UMD in the same period. SMB and HML are Fama and French (1993) size and book-to-market factors and UMD is the momentum factor constructed by Ken French. We report the means and corresponding t-statistics of these hedge portfolio returns in Table 7.

[Table 7: Hedge Portfolio Returns]

We find that only the average returns on the skewness and momentum hedge portfolios are significantly different from zero between 1996 and 2007. The average return on UMD is the largest in magnitude, 0.83% per month, but the average return on SKEW is also large, at −0.48% per month. FSKEW also has relatively low correlations with other risk factors, a correlation of 0.29 with UMD being the largest. FVOL has somewhat higher correlations with the other risk factors, the largest being −0.40 with the market excess return, 0.31 with HML, and 0.30 with UMD. The results on the correlations of FVOL, FSKEW, and FKURT with other risk factors suggest that

FVOL might be more apt at explaining the anomalies such as book-to-market, but FSKEW and FKURT might be better than FVOL at explaining the difference in cross-sectional stock returns that cannot be explained by other risk factors.

6 Exploring the Magnitude of the Risk Premia

In the previous section, we constructed a hedge portfolio for market skewness risk, FSKEW, and estimated the price of market skewness risk to be -0.48% per month by simply computing the average monthly return of FSKEW. The prices of market volatility risk and market kurtosis risk are smaller in magnitude, at -0.27% and 0.16% per month. In this section, we check the robustness of these estimates by running a series of cross-sectional regressions using different test portfolios and different regression procedures. The 81 portfolios have the largest spread in their sensitivities to ΔVOL, ΔSKEW, and ΔKURT by construction, so that the cross-sectional regression test based on these portfolios will yield a good estimate of the price of market volatility, market skewness, and market kurtosis risk. However, because FVOL, FSKEW, and FKURT are specifically designed to capture the cross-sectional return difference of the 81 portfolios by construction, it is important to check the robustness of the cross-sectional regression test results using other sets of test portfolios.

We therefore consider three sets of test portfolios: (i) the 81 portfolios sorted on sensitivities to Rm-Rf, ΔVOL, ΔSKEW, and ΔKURT; (ii) the 25 Fama-French portfolios sorted on size and book-to-market ratio; (iii) 49 industry portfolios.

6.1 Fama-MacBeth Regressions on the 81 Hedge Portfolios

We first apply the two-pass regressions of Fama-MacBeth (1973) to the 81 portfolios. In the first stage, we regress the time series of post-ranking monthly excess returns of each of the 81 portfolios on the pricing factors to estimate the portfolio's factor betas. In the second stage, we regress the cross-section of excess returns of the 81 portfolios on their estimated factor betas to obtain the estimate of the price of risk each month. The monthly estimates of the price of risk are then averaged to yield the final estimate.

In order to run the Fama-MacBeth regression, we need to make an assumption on the form of the factor model. We consider several different specifications that control for one or more of the known pricing factors including Rm-Rf, SMB, HML, UMD, FVOL, FSKEW and FKURT. We include several benchmark models which do not include FSKEW and FKURT so that we can compare the pricing performance of the models that include these factors to those that do not. The factor models considered are: (i) CAPM, (ii) CAPM + FVOL, (iii) CAPM + FSKEW, (iv) CAPM + FKURT, (v) CAPM + FVOL + FSKEW, (vi) CAPM + FVOL + FKURT, (vii) CAPM+ FSKEW + FKURT, (viii) CAPM + FVOL + FSKEW + FKURT, (ix) Carhart 4-Factor, (x) Carhart 4-Factor + FVOL, (xi) Carhart 4-Factor +FSKEW, (xii) Carhart 4-Factor + FKURT, (xiii) Carhart 4-Factor + FVOL + FSKEW, (xiv) Carhart 4-Factor + FVOL + FKURT, (xv) Carhart 4-Factor

+FSKEW + FKURT, and (xvi) Carhart 4-Factor + FVOL + FSKEW + FKURT.

The testable prediction of model (xvi) which includes all the risk factors is

$$\begin{aligned} E[R_i] - R_f &= \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{FVOL}\beta^i_{FVOL} + \lambda_{FSKEW}\beta^i_{FSKEW} + \\ &\quad \lambda_{FKURT}\beta^i_{FKURT} + \lambda_{SMB}\beta^i_{SMB} + \lambda_{HML}\beta^i_{HML} + \lambda_{UMD}\beta^i_{UMD}. \end{aligned} \tag{10}$$

The predictions of all the other factor models can be formulated in a similar way. The results of the Fama-MacBeth regressions based on the 81 portfolios are reported in Table 8.

[Table 8: The Price of Market Volatility, Skewness, and Kurtosis Risk]

The estimate of the price of market skewness risk, λ_{FSKEW}, is -0.40% per month when we assume a factor model with Rm-Rf, FVOL, FSKEW, and FKURT as factors or when we further include SMB, HML, and UMD. All the estimates of λ_{FSKEW} are significantly different from zero and very similar in magnitude, but slightly smaller in absolute value than the estimate of -0.48% in Table 7. The estimates of λ_{FVOL} are between -0.27% and -0.33% per month. The estimates of λ_{FKURT} are between -0.05% and 0.08% per month. Note that the prices of risk of the factors commonly used in the literature, SMB, HML, and UMD, are only rarely significantly estimated when using the 81 portfolios.

Regarding the pricing performance of the different models, the adjusted R^2 statistics range from 14% for the CAPM to 28% for model (xvi) with all factors. In terms of adjusted R^2, model (viii) with three market moment factors does slightly better than the Carhart 4-factor model (ix), 23% versus 21%. Moreover, the addition of FSKEW to the Carhart 4-factor model improves the adjusted R^2 from 21% to 23%, and the addition of FVOL, FSKEW, and FKURT to the Carhart 4-factor model improves the adjusted R^2 from 21% to 28%. The performance of the market moment factor model (viii) is even better in terms of root-mean-squared pricing error, reducing the RMSE of the CAPM from 0.37% to 0.27% in monthly returns, compared to 0.32% in the Carhart 4-factor model.

The economic significance of a risk factor depends on the magnitude of its price of risk and the spread of betas among different assets. Using the β_{MKT}, β_{FVOL}, β_{FSKEW} and β_{FKURT} for the 81 portfolios estimated from the first stage time series regression in the Fama-MacBeth procedure described above, we find that the highest β_{FSKEW} is 1.05 and the lowest -1.20 (not reported). Using the estimate of -0.40% for λ_{FSKEW} from model (xvi) in Table 8, the market skewness risk premium will result in the average return on the portfolio with the lowest β_{FSKEW} to be higher than the average return on the portfolio with the highest β_{FSKEW} by $(-1.20 - 1.05) \times (-0.40) = 0.90\%$ per month or 10.80% per year. An annual risk premium of 10.80% is clearly economically significant.

We repeat the exercise using the estimated β_{FVOL} and λ_{FVOL}, and find an annual risk premium of 6.40%. Using the estimated β_{FKURT} and λ_{FKURT}, we find an annual risk premium of 0.51%.

6.2 Fama-MacBeth Regressions on Other Test Portfolios

We check the robustness of the estimate of λ_{FSKEW}, which ranged from -0.40% to -0.48% in Tables 7 and 8, by running the same Fama-MacBeth regression on other test portfolios, namely, the 25 Fama-French portfolios sorted on size and book-to-market, as well as 49 industry portfolios. We use a specification that includes all seven factors in all our robustness tests in this section. The results of the regressions are reported in Table 9. Panel A uses 1-month betas, Panel B uses 3-month betas, and Panel C uses 6-month betas.

[Table 9: Price of Market Volatility, Skewness, and Kurtosis Risk Estimated with Different Test Portfolios & Different Beta Estimation Periods]

In Panel C, the estimate of λ_{FSKEW} from the 25 size and book-to-market portfolios is -0.38%, which is exactly the same as the estimate obtained using the 81 hedge portfolios. However, the estimate is not significantly different from zero. The regression on the 49 industry portfolios gives an estimate of λ_{FSKEW} that is very similar but also not statistically significant. The estimates obtained using 1-month and 3-month betas in Panels A and B vary between -0.22% and -0.43%. Overall, we therefore conclude that the price of market skewness risk is reliably negative. The magnitude of the price of risk varies depending on the test portfolios and the regression procedure used, all but one of the estimates fall between -0.31% and -0.48% in monthly returns, or between -3.72% and -5.76% in annual returns. Given the observed spread in β_{FSKEW}, which ranges from -1.20 to 1.05, the price of risk of -3.72% to -5.76% translates into a substantial cross-sectional return difference of between 8.37% and 12.96% on an annual basis.

The estimates of λ_{FVOL} in Table 8 vary between -0.27 and -0.33, whereas in Table 7 we obtained -0.27. However, Table 9 shows substantial variation in the estimate of λ_{FVOL} dependent on the test assets and estimation windows, from -0.63 to 0.15. For λ_{FKURT}, we obtain estimates between -0.20 and 0.30 in Table 9, whereas we obtained estimates between -0.05 and 0.08 in Table 8, and 0.16 in Table 7. The estimates of the price of volatility and kurtosis risk are therefore less robust than the estimates of the price of skewness risk.

The lack of statistical significance for the moment factors for these test assets needs to be interpreted carefully in light of the fact that traditional pricing factors such as SMB, HML, and UMD are also rarely estimated significantly, even when using the 25 size and book-to-market portfolios, which should give an advantage to the SMB and HML factors. The reliably negative price of risk for the skewness factor therefore suggests that the moment factors may constitute a valuable alternative to the FF factors. These factors are moreover all the more attractive because they can be clearly motivated from an economic perspective.

6.3 Fama-MacBeth Regressions using Size and Book-to-Market Characteristics

We investigate the robustness of the estimates of λ_{FSKEW}, λ_{FVOL}, and λ_{FKURT} to the use of characteristics rather than size and book-to-market factor exposure in the cross-sectional regression

(10), as in Daniel and Titman (1997). Specifically, we replace the SMB and HML factors by firm size, measured by market equity value ME, and book-to-market ratio BM respectively, and the resulting specification is

$$E[R_i] - R_f = \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{FVOL}\beta^i_{FVOL} + \lambda_{FSKEW}\beta^i_{FSKEW} + \tag{11}$$

$$\lambda_{FKURT}\beta^i_{FKURT} + \lambda_{ME}ME + \lambda_{BM}BM + \lambda_{UMD}\beta^i_{UMD}. \tag{12}$$

The prices of risk on the SMB and HML factors are not statistically significant in Tables 8 and 9. This may favor the performance of the moment factors, or more seriously, the inclusion of SMB and HML may lead to problems associated with useless factors (see Kan and Zhang, 1999). Table 10 presents results for the 81 hedge portfolios. As in Table 9, we use 1-month, 3-month, and 6-month betas.

[Table 10: SIZE and B/M instead of SMB and HML Betas]

ME and BM are the averages of the stocks in each portfolio. The market equity value of each stock is computed by multiplying the daily stock price by the number of shares outstanding. The book value of a stock for each quarter is computed as the COMPUSTAT book value of stockholders' equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock, following Fama and French (1993). The same book value is used for the entire quarter to compute daily book-to-equity ratios.

Table 10 indicates that the estimates of λ_{FSKEW} obtained using the 81 portfolios are significant and very similar to the corresponding estimates in Tables 8 and 9. Estimates of λ_{FVOL} and λ_{FKURT} in Table 10 are also similar to those in Table 8 and 9.

6.4 Interpreting the Negative Price of Market Skewness Risk

Our interpretation of the sign on the estimated price of market skewness risk closely follows the explanation for the sign on market volatility risk. The negative price of market volatility risk suggests that increased stock market volatility is an indication of a deteriorating investment opportunity set. Since investors want to hedge themselves against the risk of a deteriorating investment opportunity set, they will want to hold stocks that have high returns when the market volatility is higher than expected. High demand for stocks whose returns are highly correlated with innovations in market volatility leads to lower required returns for these stocks according to the ICAPM. In other words, the price of market volatility risk must be negative, consistent with the empirical results.

The argument in the previous paragraph is based on the assumption that increased stock market volatility is an indication of a deteriorating investment opportunity set. It is important to point out that what makes this assumption reasonable is the well documented empirical fact that the innovation in stock market volatility is negatively correlated with the stock market return, referred to as the leverage effect (Black, 1976). To see why the leverage effect is consistent with the fact that increased stock market volatility is related to a deteriorating investment opportunity set, consider the following scenario. Suppose that there is an upward surprise (or positive innovation) in stock market volatility. If high stock market volatility is considered bad news for the economy, then a positive innovation in volatility will lead to an immediate drop in the stock market index and vice versa. Therefore, the negative relationship between volatility and the return of the stock market index implies that increased stock market volatility is considered bad news for the economy, i.e., an indication of a deteriorating investment opportunity set.

A similar argument can be made to interpret the negative price of market skewness risk that we find in the data. Recall that the correlation between ΔSKEW and the market excess return is -0.20, as reported in Table 1. This negative correlation is consistent with a negative price of market skewness risk based on the same argument used to explain the negative price of market volatility risk. In order to check that the negative relationship between ΔSKEW and the market excess return is significant, we regress the market excess return on ΔSKEW and compute the t-statistics of the slope coefficients. The results are reported in Table 11. We include the lagged market excess return for robustness, but this does not affect the results much. The slope coefficient is -0.013 with a significant t-statistic of -6.33. We also add ΔVOL and ΔKURT as regressors; the point estimates and t-statistics change somewhat when ΔVOL is included, but the estimate stays negative and statistically significant.

[Table 11: Time-Series Relationship between ΔSKEW, ΔKURT, and Market Excess Return]

The negative relationship between ΔSKEW and market excess return lends support to our finding of a negative price of market skewness risk. The negative price of market skewness risk is also consistent with the findings of Chabi-Yo (2009). Notice also that all point estimates for ΔVOL are negative and statistically significant, and all point estimates for ΔKURT are positive and statistically significant. These findings are consistent with Table 1, and also with our negative respective positive estimates of the price of volatility and kurtosis risk in Tables 2 through 10, although these estimates are not always robust.

Campbell (1996), Chen (2003), and Ang et al. (2006) argue that candidate pricing factors in the ICAPM ought to forecast the future investment opportunity set. Chen (2003) extends Campbell's (1996) model to allow for stochastic second moments and shows that risk-averse investors want to hedge directly against changes in future market volatility in this environment. We therefore investigate the forecasting power of ΔVOL, ΔSKEW, and ΔKURT in Table 12. To benchmark our results to the available literature on equity premium forecasting, see for instance Bollerslev, Tauchen and Zhou (2009) and Goyal and Welch (2008), we use monthly data. In the equity premium literature, successful candidate forecasts such as the price-earnings ratio, price-dividend ratio, market volatility, or the volatility risk premium typically yield adjusted R-squares in the 1%-2% range. Table 12 reports results on forecasting the excess market return and market volatility using lagged moments of the market return for our 1996-2007 sample. Monthly market volatility, MVOL, is estimated using the daily excess market returns within the month. Panel B indicates that changes in skewness are positively related to higher future volatility, and therefore to a deterioration of the investment opportunity set, and the estimate is statistically significant. We obtain similar results (not reported) when using daily option-implied skewness to forecast daily realized volatility estimated from intraday returns.

Panel A indicates that lagged market skewness is not a significant predictor of future market returns; moreover, the estimated sign is positive. Interestingly though, the t-statistics and adjusted R-squares are higher than for market volatility. Lagged kurtosis is estimated with the expected positive sign, and the estimates are statistically significant. However, forecasting market returns is notoriously difficult and sensitive to outliers, and given the limited size of our sample these results ought to be interpreted cautiously.

[Table 12: Forecast Regression for Market Return and Volatility]

7 Conclusion

We investigate the pricing implications of volatility, skewness, and kurtosis of the market return in the cross-section of stock returns, using estimates of the moments of the market return extracted from index options. We find that stocks with high sensitivities to innovations in implied market volatility and skewness exhibit low returns on average, whereas those with high sensitivities to innovations in implied market kurtosis exhibit somewhat higher returns on average. While the results on market skewness risk are robust, the results on market kurtosis risk are sensitive to variations in the empirical setup and across sample periods. The estimated premium for bearing market skewness risk is between -3.72% and -5.76% annualized. This market skewness risk premium is economically significant and cannot be explained away by known risk factors such as the market excess return, the size, book-to-market, momentum, and market volatility factors, or by firm characteristics such as firm and book-to-market.

Using intuition from the ICAPM, the negative price of market skewness risk indicates that market skewness is a state variable whose values are negatively related to the future investment opportunity set. To verify this we examine the empirical relationship between the innovation in option-implied market skewness and the stock market return, and find a significant negative relationship. A positive surprise in option-implied market skewness coincides with a lower stock market return, which suggests that increased market skewness is linked to a deteriorating investment opportunity set. In this case, the expected sign of market skewness risk is negative, consistent with our cross-sectional evidence. We also find that increased skewness forecasts higher future volatility, which signals a deterioration of the future opportunity set, but it does not forecast lower future market returns.

A number of extensions of our investigation may prove worthwhile. First, we use the cross-section of stock returns for 1996-2007 because we are constrained by the availability of index option data from OptionMetrics. Using other sources of option data, we may be able to obtain a longer time series of option-implied moments, which would allow us to enlarge the sample by going further back in time. Second, the appeal of volatility as a pricing factor in the ICAPM, as documented in Ang et al. (2006), is partly motivated by the well-documented negative correlation between market returns and market volatility. We have provided evidence that market skewness is also negatively related to the stock market return, but additional empirical evidence with plausible economic explanations for the observed relationship is warranted. Finally, the three- and four-factor CAPM differ from our ICAPM approach in the sense that the signs on some of the factors can be motivated by utility theory. Investigating the three-factor CAPM using option-implied moments may therefore be worthwhile.

8 Appendix

8.1 Estimation of option implied moments

Let $R(t,\tau) = \ln S(t+\tau) - \ln S(t)$. We want to extract the following moments from option prices:

$$VOL(t,\tau) = \left\{ E_t^q \left[\left(R(t,\tau) - E_t^q \left[R(t,\tau) \right] \right)^2 \right] \right\}^{1/2} \tag{13}$$

$$SKEW(t,\tau) = \frac{E_t^q \left[\left(R(t,\tau) - E_t^q \left[R(t,\tau) \right] \right)^3 \right]}{\left\{ E_t^q \left[\left(R(t,\tau) - E_t^q \left[R(t,\tau) \right] \right)^2 \right] \right\}^{3/2}} \tag{14}$$

$$KURT(t,\tau) = \frac{E_t^q \left[\left(R(t,\tau) - E_t^q \left[R(t,\tau) \right] \right)^4 \right]}{\left\{ E_t^q \left[\left(R(t,\tau) - E_t^q \left[R(t,\tau) \right] \right)^2 \right] \right\}^{2}} \tag{15}$$

where $E_t^q[\cdot]$ is the expected value under the risk-neutral measure. If we expand the powers inside the expectations, we can see that these moments are functions of

$$E_t^q \left[R(t,\tau) \right], \; E_t^q \left[R^2(t,\tau) \right], \; E_t^q \left[R^3(t,\tau) \right], \; E_t^q \left[R^4(t,\tau) \right] \tag{16}$$

or

$$E_t^q \left[e^{-r\tau} R(t,\tau) \right], \; E_t^q \left[e^{-r\tau} R^2(t,\tau) \right], \; E_t^q \left[e^{-r\tau} R^3(t,\tau) \right], \; E_t^q \left[e^{-r\tau} R^4(t,\tau) \right]. \tag{17}$$

where r is the constant risk free rate. Note that the quantities in (17) can be interpreted as prices of the contracts whose payoffs, $H[S]$ ($S = S(t+\tau)$ for notational convenience), are

$$H[S] = \begin{cases} R(t,\tau) \\ R^2(t,\tau) \\ R^3(t,\tau) \\ R^4(t,\tau) \end{cases} \tag{18}$$

Bakshi and Madan (2000) have shown that any twice-continuously differentiable payoff function, $H[S]$, can be spanned algebraically as

$$\begin{aligned} H\left[S\right] &= H\left[\overline{S}\right] + \left(S - \overline{S}\right) H_S\left[\overline{S}\right] + \int_{\overline{S}}^{\infty} H_{SS}\left[K\right]\left(S-K\right)^{+} dK \\ &\quad + \int_{0}^{\overline{S}} H_{SS}\left[K\right]\left(K-S\right)^{+} dK. \end{aligned} \tag{19}$$

The prices of these contracts are then

$$\begin{aligned} E_t^q\left\{e^{-r\tau} H\left[S\right]\right\} &= \left(H\left[\overline{S}\right] - \overline{S} H_S\left[\overline{S}\right]\right) e^{-r\tau} + H_S\left[\overline{S}\right] S\left(t\right) \\ &\quad + \int_{\overline{S}}^{\infty} H_{SS}\left[K\right] C\left(t,\tau;K\right) dK \\ &\quad + \int_{0}^{\overline{S}} H_{SS}\left[K\right] P\left(t,\tau;K\right) dK. \end{aligned} \tag{20}$$

where $C\left(t,\tau;K\right)$ and $P\left(t,\tau;K\right)$ are prices of the European call and put options with the strike price of K. As a result, we can calculate the prices of derivatives whose payoffs only depend on S, given the prices of (i) zero coupon bond, (ii) underlying stock and (iii) a series of OTM calls and puts. We can use this methodology to first calculate the quantities in (17), and then use these quantities to calculate the option implied moments in (13), (14), and (15).

We use the data on S&P 500 index options between 1996 and 2007 available on the OptionMetrics Ivy DB. We use the average of the bid and ask quotes for each option contract and filter out options with zeros bids as well as those whose average quotes are less than \$3/8. We also filter out quotes that do not satisfy standard no-arbitrage conditions. Finally, we eliminate in-the-money options because they are less liquid than out-of-the-money and at-the-money options. We eliminate put options with strike prices of more than 103 % of the underlying asset price (K/S > 1.03) and call options with strike prices of less than 97 % of the underlying asset price (K/S < 0.97).

Since we do not have a continuity of strike prices, we calculate the integrals using cubic splines. We only estimate the moments for days that have at least two OTM call prices and two OTM put prices available. For each maturity (e.g. 15, 43, 71, 169, 260, 351, 533 and 715 on a given day), we interpolate implied volatilities using a cubic spline across moneyness levels (K/S) to obtain a continuum of implied volatilities. For moneyness levels below or above the available moneyness level in the market, we use the implied volatility of the lowest or highest available strike price. After implementing this interpolation-extrapolation technique, we obtain a fine grid of 1000 implied volatilities for moneyness levels between 0.01 % and 300 %. We then convert these implied volatilities into call and put prices using the following rule: moneyness levels smaller than 100 % (K/S < 1) are used to generate put prices and moneyness levels larger than 100 % (K/S > 1) are used to generate call prices using trapezoidal numerical integration. Linear interpolation is used to calculate the moments at a given future horizon (e.g. 30-day, 180-day).

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Figure 1
Daily Option Implied Moments of S&P 500 Index Returns

We plot daily option implied volatility, skewness, and kurtosis of the S&P 500 index return between 1996 and 2007. The model-free methodology developed in Carr and Madan (2001), Bakshi and Madan (2000), and Bakshi, Kapadia, and Madan (2003) is applied to extract the option implied moments using option data available from OptionMetrics Ivy DB. See the Appendix for details of the methodology and implementation.

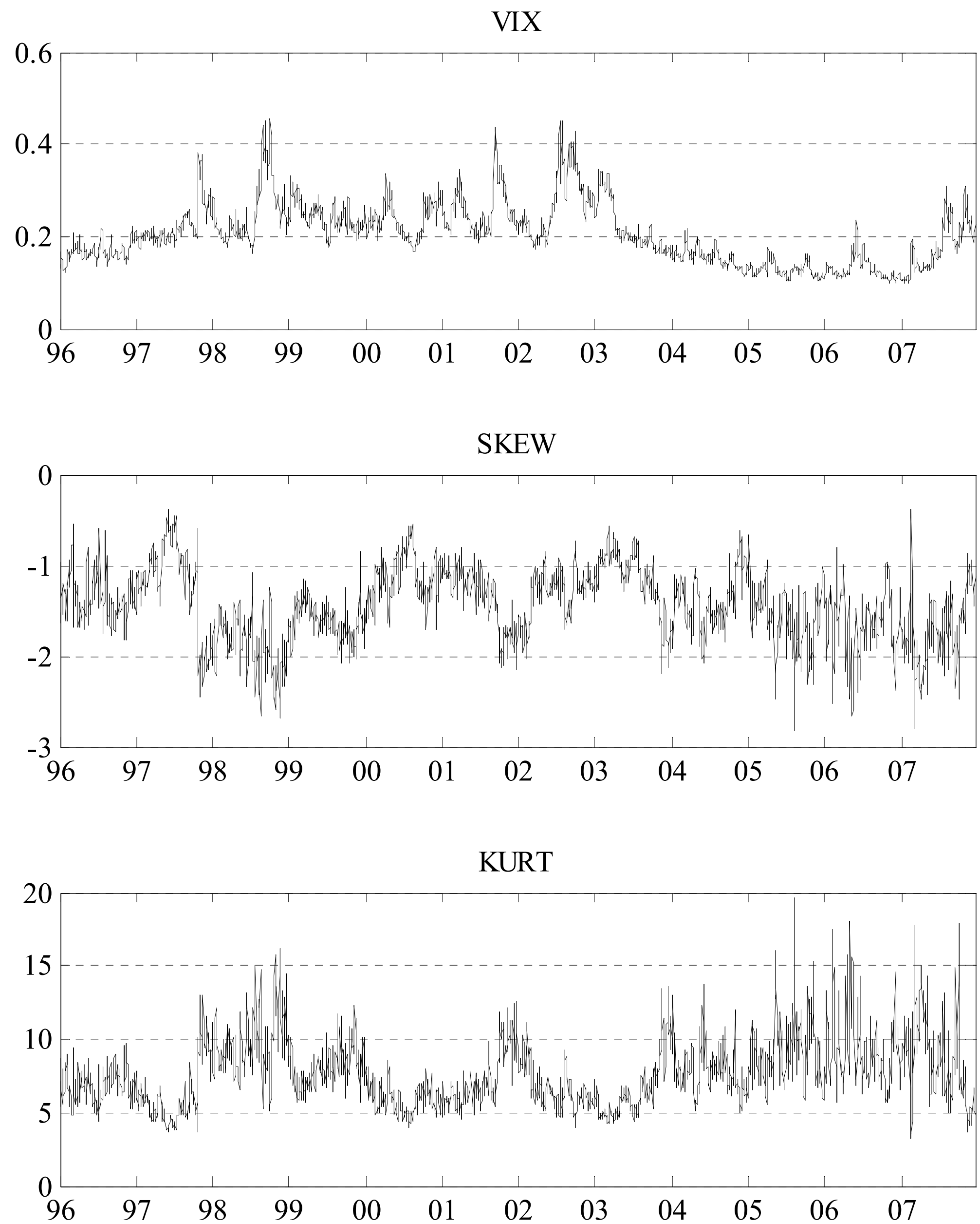

Figure 2
Autocorrelation Functions of Option Implied Market Moments

We plot sample autocorrelations of the original time series, AR(1) residuals, and ARMA(1,1) residuals for VIX, SKEW, and KURT. The horizontal dashes around 0 indicate 95% confidence intervals.

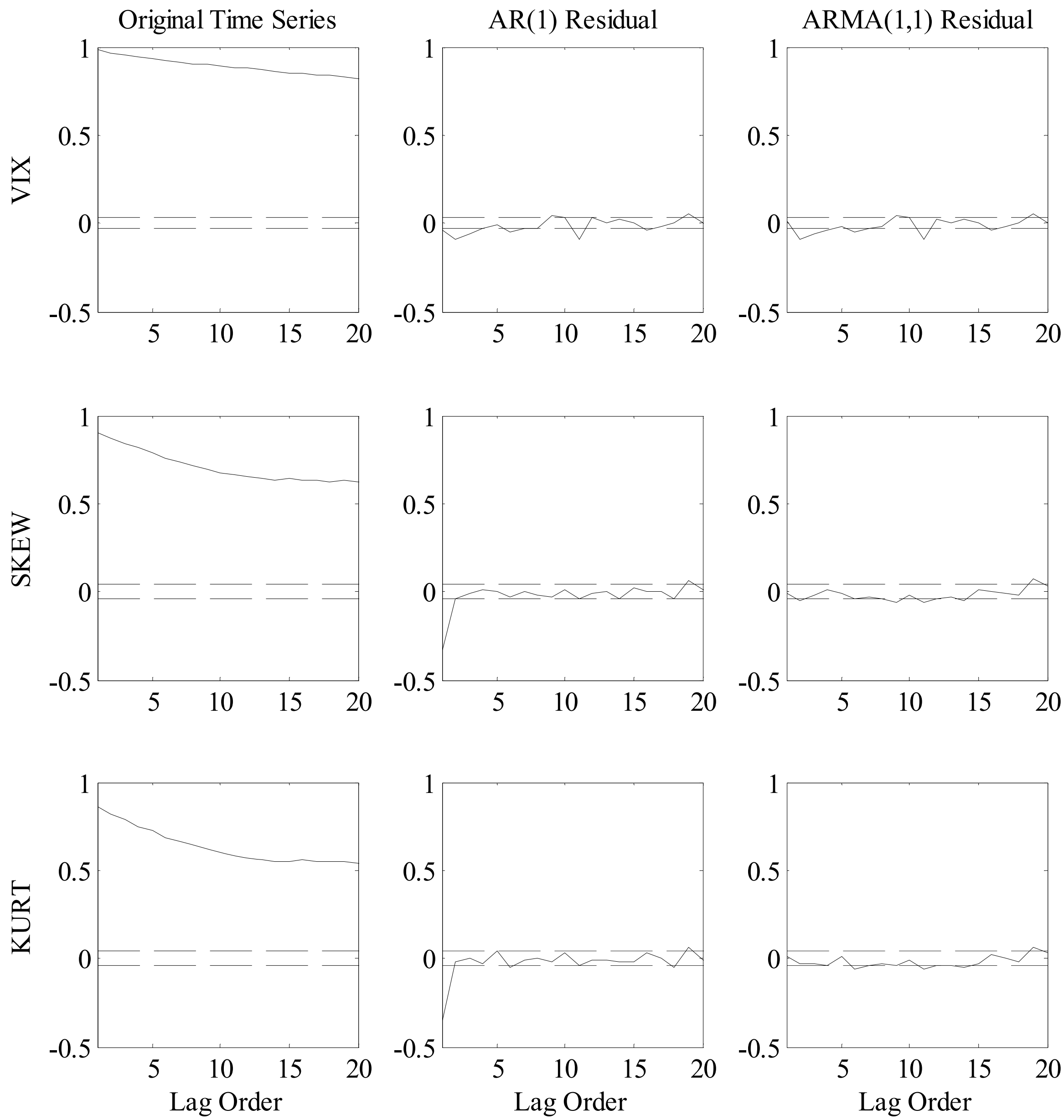

Figure 3
Daily Innovations in Option Implied Moments of S&P 500 Index Returns

We plot daily innovations in option implied volatility, skewness, and kurtosis of the S&P 500 index between 1996 and 2007. For ΔSKEW and ΔKURT the innovations are the residuals obtained after fitting the entire time series of the moments to the appropriate ARMA model. ΔKURT is further orthogonalized by ΔSKEW. For ΔVIX we simply use first differences of the VIX series.

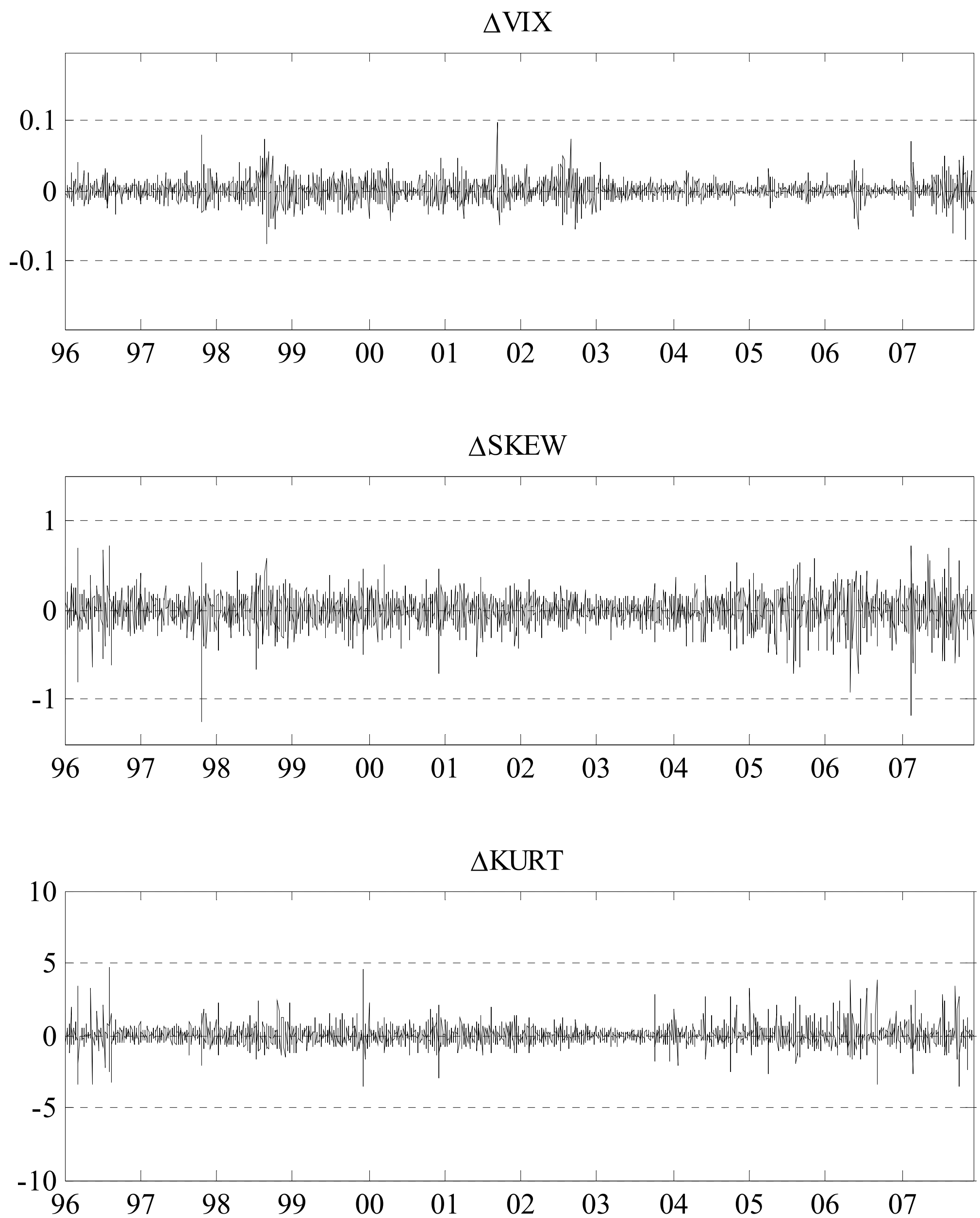

Table 1
Correlations and Averages of Daily Risk Factors

We report the correlations between daily innovations in implied moments, ΔVOL, ΔSKEW, and ΔKURT and the standard pricing factors, R_m-R_f, SMB, HML, and UMD. $\Delta VOL_t = VIX_t - VIX_{t-1}$ whereas ΔSKEW and ΔKURT are the residuals from fitting an ARMA(1,1) to the time series of corresponding moments using the entire sample. SMB, HML, and UMD are the returns on the hedge portfolios for size, book-to-market, and momentum risks. We also report the average of each factor as well as the AR(1) and MA(1) parameters used to construct the ΔSKEW and ΔKURT residuals.

	AR(1) Parameter	MA(1) Parameter	Average	Correlation ΔVOL	ΔSKEW	ΔKURT
ΔVOL	-1.0000	0.0000	2.9E-05		0.17	-0.25
ΔSKEW	-0.9962	0.3618	-8.4E-03			-0.83
ΔKURT	-0.9936	0.4032	8.0E-02			
Rm-Rf			2.7E-04	-0.79	-0.20	0.28
SMB			3.9E-05	0.10	0.01	-0.04
HML			1.8E-04	0.42	0.09	-0.13
UMD			4.5E-04	0.01	-0.01	0.01

Table 2
Sorting on ΔVOL Loadings

At the end of each rolling 1-month (Panel A), 3-month (Panel B), or 6-month (Panel C) period, we run the following regression on the daily returns of each stock.

(Univariate) $R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \varepsilon_{i,t}$

(Multivariate) $R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \varepsilon_{i,t}$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta VOL}$, so that quintile 1 contains stocks with the lowest $\beta_{\Delta VOL}$ and quintile 5 those with the highest $\beta_{\Delta VOL}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After the portfolio formation, we record the daily returns of each quintile portfolio during the 1-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward by 1 month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta VOL}$ for each quintile portfolio. This table reports the average pre-ranking beta and post-ranking return (monthly return in %) for each quintile portfolio. We also compute the Jensen's alpha of each quintile portfolio with respect to the Carhart 4-factor model by running a time-series regression of the post-ranking daily returns on daily Rm-Rf, SMB, HML, and UMD. We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in %. t-statistics for the Carhart 4-factor alpha estimates are reported in parentheses. t-statistics that are significant with 95% confidence are boldfaced.

FACTOR : ΔVOL	QUINTILE PORTFOLIOS					
	1	2	3	4	5	5-1
Panel A : 1-Month Beta Estimation Period						
Volatility Beta (Univariate)	-1.3015	-0.4415	-0.0028	0.4447	1.4034	2.7049
Average Return	1.05	0.90	0.98	1.08	0.59	-0.46
Carhart 4-Factor Alpha	0.34	0.07	0.08	0.12	-0.37	-0.71
	(1.58)	(0.68)	(0.85)	(1.05)	(-1.60)	**(-1.92)**
Volatility Beta (Multivariate)	-1.4382	-0.4789	-0.0006	0.4872	1.5447	2.9829
Average Return	0.95	0.86	1.12	1.01	0.66	-0.29
Carhart 4-Factor Alpha	0.22	0.02	0.27	0.03	-0.31	-0.53
	(1.03)	(0.22)	**(2.80)**	(0.26)	(-1.37)	(-1.49)
Panel B : 3-Month Beta Estimation Period						
Volatility Beta (Univariate)	-0.6876	-0.2413	0.0023	0.2531	0.7799	1.4675
Average Return	0.81	0.87	0.95	1.03	0.89	0.08
Carhart 4-Factor Alpha	0.15	0.01	0.03	0.06	0.08	-0.07
	(0.70)	(0.09)	(0.32)	(0.47)	(0.33)	(-0.17)
Volatility Beta (Multivariate)	-0.7080	-0.2462	0.0044	0.2615	0.8023	1.5103
Average Return	0.93	0.89	0.96	0.99	0.83	-0.10
Carhart 4-Factor Alpha	0.26	0.01	0.04	0.03	0.03	-0.24
	(1.25)	(0.05)	(0.40)	(0.28)	(0.11)	(-0.61)
Panel C : 6-Month Beta Estimation Period						
Volatility Beta (Univariate)	-0.4829	-0.1685	0.0033	0.1809	0.5590	1.0419
Average Return	0.95	0.80	0.83	1.01	1.06	0.11
Carhart 4-Factor Alpha	0.25	-0.10	-0.09	0.06	0.40	0.15
	(1.20)	(-0.92)	(-0.79)	(0.49)	(1.47)	(0.37)
Volatility Beta (Multivariate)	-0.4895	-0.1721	0.0030	0.1839	0.5695	1.0590
Average Return	1.01	0.82	0.79	0.95	1.00	-0.01
Carhart 4-Factor Alpha	0.32	-0.10	-0.18	0.05	0.37	0.05
	(1.55)	(-0.89)	(-1.60)	(0.37)	(1.37)	(0.13)

Table 3
Sorting on ΔSKEW Loadings

At the end of each rolling 1-month (Panel A), 3-month (Panel B), or 6-month (Panel C) period, we run the following regression on the daily returns of each stock.

(Univariate) $R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta SKEW}^i \Delta SKEW_t + \varepsilon_{i,t}$

(Multivariate) $R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \varepsilon_{i,t}$

We then sort the stocks into quintiles based on their regression coefficients, $\beta_{\Delta SKEW}$, so that quintile 1 contains stocks with the lowest $\beta_{\Delta SKEW}$ and quintile 5 those with the highest $\beta_{\Delta SKEW}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After the portfolio formation, we record the daily returns of each quintile portfolio during the 1-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward by 1 month at a time. At the end of the procedure, we have a time series of daily post-ranking returns as well as a time series of monthly pre-ranking $\beta_{\Delta SKEW}$ for each quintile portfolio. This table reports the average pre-ranking beta and post-ranking return (monthly return in %) for each quintile portfolio. We also compute the Jensen's alpha of each quintile portfolio with respect to the Carhart 4-factor model by running a time-series regression of the post-ranking daily returns on daily Rm-Rf, SMB, HML, and UMD. We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in %. t-statistics for the Carhart 4-factor alpha estimates are reported in parentheses. t-statistics that are significant with 95% confidence are boldfaced.

FACTOR : ΔSKEW	QUINTILE PORTFOLIOS					
	1	2	3	4	5	5-1
Panel A : 1-Month Beta Estimation Period						
Skewness Beta (Univariate)	-0.0627	-0.0203	0.0008	0.0216	0.0648	0.1275
Average Return	1.22	1.12	0.89	0.84	0.63	-0.59
Carhart 4-Factor Alpha	0.52	0.27	0.02	-0.13	-0.28	-0.80
	(2.53)	**(2.58)**	(0.16)	(-1.18)	(-1.37)	**(-2.42)**
Skewness Beta (Multivariate)	-0.0844	-0.0265	0.0015	0.0295	0.0874	0.1718
Average Return	1.44	1.02	0.96	0.91	0.53	-0.91
Carhart 4-Factor Alpha	0.84	0.19	0.04	-0.05	-0.43	-1.26
	(3.89)	**(1.83)**	(0.47)	(-0.47)	**(-2.03)**	**(-3.66)**
Panel B : 3-Month Beta Estimation Period						
Skewness Beta (Univariate)	-0.0334	-0.0110	0.0006	0.0122	0.0359	0.0693
Average Return	1.10	1.09	0.92	0.80	0.60	-0.51
Carhart 4-Factor Alpha	0.53	0.26	0.01	-0.15	-0.34	-0.87
	(2.49)	**(2.60)**	(0.12)	(-1.34)	(-1.62)	**(-2.55)**
Skewness Beta (Multivariate)	-0.0406	-0.0128	0.0013	0.0154	0.0439	0.0844
Average Return	1.31	1.01	0.85	0.87	0.62	-0.68
Carhart 4-Factor Alpha	0.80	0.22	-0.08	-0.09	-0.36	-1.16
	(3.52)	**(1.97)**	(-0.93)	(-0.80)	**(-1.69)**	**(-3.24)**
Panel C : 6-Month Beta Estimation Period						
Skewness Beta (Univariate)	-0.0230	-0.0078	0.0005	0.0088	0.0254	0.0484
Average Return	1.08	0.98	0.89	0.91	0.81	-0.27
Carhart 4-Factor Alpha	0.52	0.15	0.05	-0.04	-0.12	-0.64
	(2.45)	(1.44)	(0.57)	(-0.32)	(-0.58)	**(-1.88)**
Skewness Beta (Multivariate)	-0.0266	-0.0087	0.0011	0.0108	0.0297	0.0564
Average Return	1.08	0.95	0.85	0.96	0.85	-0.23
Carhart 4-Factor Alpha	0.62	0.21	-0.06	0.03	-0.15	-0.77
	(2.56)	**(1.82)**	(-0.65)	(0.24)	(-0.67)	**(-1.98)**

Table 4
Sorting on ΔKURT Loadings

At the end of each rolling 1-month (Panel A), 3-month (Panel B), or 6-month (Panel C) period, we run the following regression on the daily returns of each stock.

(Univariate) $R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta SKEW}^i \Delta KURT_t + \varepsilon_{i,t}$

(Multivariate) $R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \varepsilon_{i,t}$

We then sort stocks into quintiles based on their regression coefficients, $\beta_{\Delta KURT}$, so that quintile 1 contains stocks with the lowest $\beta_{\Delta KURT}$ and quintile 5 those with the highest $\beta_{\Delta KURT}$. We form value-weighted portfolios by weighing each stock in the quintile by its relative market value within the quintile at the end of the beta estimation period. After the portfolio formation, we record the daily returns of each quintile portfolio during the 1-month period following the estimation period and refer to these returns as post-ranking returns. We repeat the procedure by rolling the beta estimation window forward by 1 month at a time. At the end of the procedure, we have time series of daily post-ranking returns as well as time series of monthly pre-ranking $\beta_{\Delta KURT}$ for each quintile portfolio. This table reports the average pre-ranking beta and post-ranking return (monthly return in %) for each quintile portfolio. We also compute the Jensen's alpha of each quintile portfolio with respect to the Carhart 4-factor model by running a time-series regression of the post-ranking daily returns on daily Rm-Rf, SMB, HML, and UMD. We multiply daily alphas by 21 to obtain monthly alphas and report the monthly alphas in %. t-statistics for the Carhart 4-factor alpha estimates are reported in parentheses. t-statistics that are significant with 95% confidence are boldfaced.

FACTOR : ΔKURT	QUINTILE PORTFOLIOS					
	1	2	3	4	5	5-1
Panel A : 1-Month Beta Estimation Period						
Kurtosis Beta (Univariate)	-0.0191	-0.0064	-0.0003	0.0058	0.0181	0.0372
Average Return	0.80	0.87	0.99	0.95	1.10	0.30
Carhart 4-Factor Alpha	-0.01	0.02	0.07	0.00	0.31	0.32
	(-0.06)	(0.14)	(0.75)	(0.04)	(1.47)	(0.93)
Kurtosis Beta (Multivariate)	-0.0247	-0.0084	-0.0004	0.0075	0.0237	0.0484
Average Return	0.79	0.94	0.97	1.06	0.96	0.17
Carhart 4-Factor Alpha	-0.15	0.01	0.08	0.19	0.28	0.43
	(-0.71)	(0.09)	(0.85)	**(1.76)**	(1.28)	(1.22)
Panel B : 3-Month Beta Estimation Period						
Kurtosis Beta (Univariate)	-0.0103	-0.0037	-0.0004	0.0029	0.0093	0.0196
Average Return	0.80	0.80	1.01	1.07	0.80	0.00
Carhart 4-Factor Alpha	0.02	-0.05	0.13	0.14	0.02	0.00
	(0.10)	(-0.53)	(1.34)	(1.28)	(0.09)	(-0.01)
Kurtosis Beta (Multivariate)	-0.0124	-0.0046	-0.0005	0.0035	0.0114	0.0238
Average Return	0.81	0.88	0.93	1.06	0.90	0.08
Carhart 4-Factor Alpha	-0.12	-0.04	0.02	0.21	0.28	0.40
	(-0.63)	(-0.39)	(0.24)	**(2.02)**	(1.20)	(1.19)
Panel C : 6-Month Beta Estimation Period						
Kurtosis Beta (Univariate)	-0.0074	-0.0027	-0.0004	0.0019	0.0062	0.0136
Average Return	1.04	0.89	0.98	1.02	0.62	-0.42
Carhart 4-Factor Alpha	0.31	0.03	0.10	0.10	-0.10	-0.41
	(1.53)	(0.27)	(1.02)	(1.01)	(-0.56)	(-1.39)
Kurtosis Beta (Multivariate)	-0.0084	-0.0032	-0.0005	0.0022	0.0073	0.0158
Average Return	1.11	0.97	0.94	0.96	0.61	-0.50
Carhart 4-Factor Alpha	0.19	0.10	0.02	0.10	0.09	-0.10
	(0.97)	(0.94)	(0.21)	(1.01)	(0.39)	(-0.29)

Table 5
Sorting on Market Moment Risk Loadings in Sub-Periods

At the end of each month, we run the following regression on daily returns of each stock:

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \varepsilon_{i,t}$$

The rest of the procedure is same as in Tables 2-4. t-statistics for the Carhart 4-factor alpha estimates are reported in parentheses. t-statistics that are significant with 95% confidence are boldfaced.

	QUINTILE PORTFOLIOS					
	1	2	3	4	5	5-1
Panel A : 1996 - 2000						
Volatility Beta	-1.5106	-0.5171	-0.0173	0.4958	1.5925	3.1031
Average Return	1.47	1.36	1.75	1.62	1.06	-0.41
Carhart 4-Factor Alpha	0.50	0.00	0.48	0.09	-0.64	-1.13
	(1.45)	(-0.01)	**(2.69)**	(0.38)	**(-1.67)**	**(-1.92)**
Skewness Beta	-0.0993	-0.0317	0.0010	0.0340	0.0993	0.1986
Average Return	1.94	1.57	1.41	1.54	0.99	-0.94
Carhart 4-Factor Alpha	0.87	0.35	-0.01	0.05	-0.59	-1.46
	(2.71)	**(1.93)**	(-0.07)	(0.28)	**(-1.82)**	**(-2.84)**
Kurtosis Beta	-0.0299	-0.0102	-0.0007	0.0088	0.0276	0.0575
Average Return	1.38	1.61	1.48	1.55	1.08	-0.30
Carhart 4-Factor Alpha	-0.08	0.23	0.01	0.21	-0.11	-0.03
	(-0.21)	(1.22)	(0.08)	(1.09)	(-0.31)	(-0.05)
Panel B : 2001 - 2007						
Volatility Beta	-1.3826	-0.4504	0.0125	0.4823	1.5120	2.8947
Average Return	0.49	0.51	0.60	0.58	0.24	-0.26
Carhart 4-Factor Alpha	0.12	0.10	0.11	0.01	-0.27	-0.39
	(0.45)	(0.95)	(1.16)	(0.11)	(-1.04)	(-0.92)
Skewness Beta	-0.0718	-0.0222	0.0020	0.0261	0.0784	0.1502
Average Return	0.79	0.54	0.60	0.48	0.18	-0.61
Carhart 4-Factor Alpha	0.56	0.05	0.07	-0.03	-0.30	-0.86
	(2.02)	(0.45)	(0.68)	(-0.24)	(-1.12)	**(-1.91)**
Kurtosis Beta	-0.0210	-0.0071	-0.0003	0.0065	0.0206	0.0416
Average Return	0.36	0.47	0.60	0.60	0.56	0.20
Carhart 4-Factor Alpha	-0.14	-0.04	0.13	0.09	0.24	0.38
	(-0.60)	(-0.39)	(1.35)	(0.85)	(0.92)	(0.96)

Table 6
Portfolios Sorted on Sensitivities to Rm-Rf, ΔVOL, ΔSKEW, and ΔKURT by Groups

At the end of each month, we run the following regression on daily returns of each stock:

$$R_{i,t} - R_{f,t} = \beta_0^i + \beta_{MKT}^i \left(R_{m,t} - R_{f,t}\right) + \beta_{\Delta VOL}^i \Delta VOL_t + \beta_{\Delta SKEW}^i \Delta SKEW_t + \beta_{\Delta KURT}^i \Delta KURT_t + \varepsilon_{i,t}$$

We form 81 portfolios with varying sensitivities to R_m- R_f, ΔVOL, ΔSKEW, and ΔKURT by sequentially grouping the stocks into terciles sorted on β_{MKT}, $\beta_{\Delta VOL}$, $\beta_{\Delta SKEW}$, and $\beta_{\Delta KURT}$ (lowest in tercile 1 and highest in tercile 3). We repeat the previous procedure to obtain a time series of pre-ranking betas and post-ranking returns as well as post-ranking Jensen's alphas with respect to the Carhart 4-factor model for each of the 81 portfolios. We then group the 81 portfolios into groups in such a way that each group contains stocks with low, medium or high exposures to only one of the factors. We report the average betas, average monthly returns, and the average Carhart 4-factor alphas of these portfolios for each group. We also report the Newey and West (1987) t-statistics with 12 lags for the difference in average returns and alphas between the high and low exposure groups. t-statistics that are significant with 95% confidence are boldfaced.

Portfolios		Average Return (% month)	Carhart 4-Factor Alpha (% month)	Pre-ranking Exposure			
				β_{MKT}	$\beta_{\Delta VOL}$	$\beta_{\Delta SKEW}$ (10^{-1})	$\beta_{\Delta KURT}$ (10^{-1})
β_{MKT}	L	1.10	0.23	-0.6767	-0.8980	-0.1062	0.0358
	M	1.00	0.09	0.6592	-0.0383	0.0157	-0.0035
	H	0.75	-0.10	2.5458	0.9091	0.1279	-0.0307
	H-L	-0.35	-0.33				
		(-0.95)	(-1.32)				
$\beta_{\Delta VOL}$	L	1.10	0.30	0.3848	-1.0153	0.0563	-0.0313
	M	0.93	0.03	0.7996	0.0232	0.0154	-0.0029
	H	0.83	-0.11	1.3439	0.9650	-0.0343	0.0258
	H-L	-0.27	-0.41				
		(-1.21)	**(-2.02)**				
$\beta_{\Delta SKEW}$	L	1.18	0.38	0.7872	-0.0010	-0.6328	0.0496
	M	0.97	0.08	0.8208	0.0009	0.0165	-0.0044
	H	0.70	-0.23	0.9203	-0.0269	0.6537	-0.0536
	H-L	-0.48	-0.61				
		(-2.70)	**(-2.25)**				
$\beta_{\Delta KURT}$	L	0.87	0.00	0.9581	-0.0487	0.0807	-0.1561
	M	0.96	0.06	0.8248	-0.0023	0.0126	-0.0046
	H	1.03	0.16	0.7454	0.0239	-0.0558	0.1523
	H-L	0.16	0.16				
		(0.97)	(0.77)				

Table 7
Hedge Portfolio Returns

We report the average monthly returns of the hedge portfolios, FVOL, FSKEW, and FKURT, corresponding to the factors, ΔVOL, ΔSKEW, and ΔKURT. Using 81 portfolios sorted on sensitivities to Rm-Rf, ΔVOL, ΔSKEW, and ΔKURT, we form hedging portfolios by going long stocks in the tercile with the highest sensitivities to the corresponding risk factor and taking short positions in stocks in the tercile with the lowest sensitivities to the same risk factor. We also report the correlations of the hedging portfolio returns with R_m-R_f, SMB, HML, and UMD. SMB and HML are Fama and French (1993) size and book-to-market factors respectively, and UMD is the momentum factor constructed by Kenneth French. The period covered is 1996 to 2007. We report the Newey-West t-statistics with 12 lags for the average returns. Significant t-statistics at the 95% level are boldfaced.

	Mean (% mth)	t-stat	Correlation						
			FVOL	FSKEW	FKURT	Rm-Rf	SMB	HML	UMD
FVOL	-0.27	(-1.21)	1.00	0.21	0.15	-0.40	0.08	0.31	0.30
FSKEW	-0.48	**(-2.70)**		1.00	-0.19	-0.17	0.10	0.13	0.29
FKURT	0.16	(0.97)			1.00	0.14	0.03	-0.13	-0.03
Rm-Rf	0.55	(1.42)				1.00	0.22	-0.53	-0.21
SMB	0.22	(0.72)					1.00	-0.49	0.17
HML	0.40	(1.00)						1.00	-0.06
UMD	0.83	**(2.24)**							1.00

Table 8
The Price of Market Volatility, Skewness, and Kurtosis Risk

We report the estimated prices of risk for various multifactor models with Rm-Rf, FVOL, FSKEW, FKURT, SMB, HML, and UMD as factors. For each model considered, we estimate the prices of risk λ by applying the two-pass regression procedure of Fama-MacBeth (1973) to the post-ranking monthly returns of the 81 portfolios sorted on sensitivities to Rm-Rf, ΔVOL, ΔSKEW, and ΔKURT. We estimate the β's by running a time series regression on the full-sample post-ranking returns, then estimate λ's by running a cross-sectional regression every month. The Newey-West t-statistics with 12 lags are reported in the parentheses. Significant t-statistics at the 95% confidence level are boldfaced.

Benchmark	CAPM								Carhart 4-factor							
Model	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	(xii)	(xiii)	(xiv)	(xv)	(xvi)
Constant	0.80	0.91	0.90	0.80	0.98	0.99	0.88	0.99	0.92	1.00	0.88	0.92	1.01	1.03	0.89	1.02
	(2.61)	**(3.19)**	**(2.81)**	**(2.37)**	**(3.27)**	**(2.97)**	**(2.55)**	**(2.97)**	**(2.92)**	**(3.01)**	**(2.73)**	**(2.84)**	**(3.04)**	**(3.29)**	**(2.72)**	**(3.22)**
Rm-Rf	-0.15	-0.26	-0.24	-0.16	-0.31	-0.33	-0.22	-0.32	-0.28	-0.37	-0.22	-0.27	-0.36	-0.40	-0.23	-0.37
	(-0.32)	(-0.54)	(-0.47)	(-0.30)	(-0.63)	(-0.63)	(-0.42)	(-0.62)	(-0.64)	(-0.86)	(-0.53)	(-0.64)	(-0.85)	(-0.93)	(-0.54)	(-0.87)
FVOL		-0.31			-0.27	-0.33		-0.28		-0.28			-0.27	-0.30		-0.28
		(-1.39)			(-1.21)	(-1.59)		(-1.30)		(-1.18)			(-1.13)	(-1.33)		(-1.22)
FSKEW			-0.43		-0.40		-0.44	-0.40			-0.41		-0.40		-0.42	-0.40
			(-2.45)		**(-2.29)**		**(-2.53)**	**(-2.29)**			**(-2.21)**		**(-2.16)**		**(-2.28)**	**(-2.18)**
FKURT				0.00		0.08	-0.05	0.02				0.02		0.06	-0.02	0.02
				(-0.03)		(0.51)	(-0.30)	(0.12)				(0.13)		(0.34)	(-0.13)	(0.09)
SMB									-0.48	-0.35	-0.40	-0.48	-0.18	-0.30	-0.38	-0.17
									(-1.54)	(-0.97)	(-1.33)	(-1.53)	(-0.53)	(-0.87)	(-1.26)	(-0.50)
HML									0.05	0.03	0.11	0.06	0.09	0.07	0.09	0.09
									(0.10)	(0.07)	(0.20)	(0.11)	(0.16)	(0.12)	(0.16)	(0.17)
UMD									-1.28	-0.98	-0.85	-1.29	-0.32	-0.88	-0.79	-0.31
									(-2.21)	(-1.36)	(-1.55)	**(-2.13)**	(-0.45)	(-1.31)	(-1.32)	(-0.44)
Average adj. R^2	0.14	0.18	0.17	0.17	0.20	0.21	0.20	0.23	0.21	0.23	0.23	0.24	0.25	0.26	0.26	0.28
RMSE (% mth)	0.37	0.33	0.30	0.37	0.27	0.32	0.29	0.27	0.32	0.32	0.29	0.32	0.27	0.31	0.28	0.27

Table 9
Price of Market Volatility, Skewness, and Kurtosis Risk Estimated with Different Test Portfolios & Different Beta Estimation Periods

We report the estimated prices of risk in a multifactor model of the form,

$$E[R_i]-R_f = \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{FVOL}\beta^i_{FVOL} + \lambda_{FSKEW}\beta^i_{FSKEW} + \lambda_{FKURT}\beta^i_{FKURT} + \lambda_{SMB}\beta^i_{SMB} + \lambda_{HML}\beta^i_{HML} + \lambda_{UMD}\beta^i_{UMD}.$$

We estimate the prices of risk λ by applying the two-pass regression procedure of Fama-MacBeth (1973) to the post-ranking returns of the 81 portfolios sorted on sensitivities to Rm-Rf, ΔVOL, ΔSKEW , and ΔKURT, as well as the 25 Fama-French portfolios sorted on size and book-to-market and 49 industry portfolios. In Panel A, we estimate the β's by running a time series regression on the returns over non-overlapping 1-month periods, then we estimate λ's by running cross-sectional regressions over the same 1-month periods. In Panel B and C, the procedure is the same as in Panel A, but uses rolling 3-month and 6-month periods. The Newey-West t-statistics with 12 lags are reported in parentheses. Significant t-statistics at the 95% confidence level are boldfaced.

Test Portfolios	Price of risk (λ, %month)						
	Rm-Rf	FVOL	FSKEW	FKURT	SMB	HML	UMD
Panel A: Rolling 1-Month Beta							
81 portfolios	-0.14	-0.12	-0.31	0.08	-0.03	0.11	0.15
	(-0.34)	(-0.55)	**(-1.94)**	(0.62)	(-0.15)	(0.40)	(0.62)
25 size & B/M portfolios	-0.98	0.15	-0.41	-0.03	0.15	0.36	0.39
	(-2.19)	(0.51)	(-1.57)	(-0.23)	(0.53)	(0.95)	(1.28)
49 industry portfolios	0.15	-0.14	-0.22	-0.04	-0.21	-0.03	0.37
	(0.30)	(-0.57)	**(-1.95)**	(-0.29)	(-0.96)	(-0.12)	(1.56)
Panel B: Rolling 3-Month Beta							
81 portfolios	-0.25	-0.13	-0.40	0.09	0.06	0.14	0.08
	(-0.68)	(-0.57)	**(-2.58)**	(0.76)	(0.26)	(0.51)	(0.30)
25 size & B/M portfolios	-0.77	-0.49	-0.43	-0.09	0.14	0.39	1.31
	(-1.40)	(-1.14)	(-1.39)	(-0.33)	(0.49)	(0.97)	**(2.49)**
49 industry portfolios	0.54	0.00	-0.31	0.30	-0.37	0.00	1.09
	(1.10)	(-0.00)	**(-1.69)**	(1.34)	(-1.51)	(0.02)	**(3.78)**
Panel C: Rolling 6-Month Beta							
81 portfolios	-0.32	-0.20	-0.38	0.06	0.20	0.13	0.12
	(-0.82)	(-0.87)	**(-2.77)**	(0.45)	(0.72)	(0.50)	(0.45)
25 size & B/M portfolios	-1.02	-0.63	-0.38	-0.20	0.09	0.49	1.81
	(-1.88)	(-1.60)	(-0.93)	(-0.78)	(0.32)	(1.24)	**(3.25)**
49 industry portfolios	0.63	-0.03	-0.35	0.10	-0.33	0.05	1.72
	(1.27)	(-0.10)	(-1.30)	(0.47)	(-1.29)	(0.16)	**(5.29)**

Table 10
SIZE and B/M instead of SMB and HML Betas

We report the estimated prices of risk in a multifactor model of the form,

$$E[R_i] - R_f = \lambda_0 + \lambda_{MKT}\beta^i_{MKT} + \lambda_{FVOL}\beta^i_{FVOL} + \lambda_{FSKEW}\beta^i_{FSKEW} + \lambda_{FKURT}\beta^i_{FKURT} + \lambda_{ME}ME + \lambda_{BM}BM + \lambda_{UMD}\beta^i_{UMD}.$$

The procedure is same as in Table 9 except that the market equity value (ME) and the book-to-market ratio (BM) of the portfolios are used instead of β_{SMB} and β_{HML} in the second-stage cross-sectional regression. We use the 81 portfolios sorted on sensitivities to Rm-Rf, ΔVOL, ΔSKEW, and ΔKURT as test assets. ME and BM are the averages for the companies in each portfolio. The market equity value of each stock is computed by multiplying the daily stock price by the number of shares outstanding. The book value of a stock for each quarter is computed as the COMPUSTAT book value of stockholders' equity, plus deferred taxes and investment tax credit, minus the book value of preferred stock, following Fama and French (1993). The same book value is used for the entire quarter to compute daily book-to-equity ratios.

Price of risk (λ, %month)						
Rm-Rf	FVOL	FSKEW	FKURT	SIZE ($B)	B/M	UMD
Panel A: Rolling 1-Month Beta						
-0.10	-0.08	-0.40	0.07	-0.07	0.01	0.14
(-0.24)	(-0.32)	**(-2.34)**	(0.66)	(-1.61)	(0.99)	(0.56)
Panel B: Rolling 3-Month Beta						
-0.22	-0.13	-0.41	0.08	-0.02	0.02	0.09
(-0.44)	(-0.57)	**(-2.52)**	(0.80)	(-0.45)	(1.14)	(0.35)
Panel C: Rolling 6-Month Beta						
-0.27	-0.18	-0.37	0.07	-0.01	0.00	0.22
(-0.51)	(-0.81)	**(-2.44)**	(0.63)	(-0.09)	(0.32)	(0.97)

Table 11
Time-Series Relationship between ΔSKEW, ΔKURT, and Market Excess Return

This table reports the estimates and Newey-West adjusted t-statistics of the slope coefficients from regressing the daily excess returns of the S&P 500 index on the lagged excess returns, ΔVOL, ΔSKEW, and ΔKURT. Significant t-statistics at the 95% confidence level are boldfaced.

	Dependent Variable: Rm - Rf (t)							
ΔVOL(t)		-0.661			-0.651	-0.654		-0.644
		(-36.50)			**(-35.05)**	**(-35.24)**		**(-34.57)**
ΔSKEW(t)			-0.013		-0.005		-0.013	-0.005
			(-6.33)		**(-3.67)**		**(-6.02)**	**(-3.69)**
ΔKURT(t)				0.004		0.001	0.004	0.001
				(7.65)		**(3.00)**	**(7.59)**	**(3.26)**
Rm-Rf (t-1)	-0.003	0.057	0.005	-0.014	0.059	0.055	-0.007	0.056
	(-0.12)	**(3.62)**	(0.26)	(-0.72)	**(3.87)**	**(3.48)**	(-0.37)	**(3.71)**
Adj. R^2 (%)	-0.03	62.51	4.05	3.92	63.00	62.63	7.98	63.14

Table 12
Forecast Regression for Market Return and Volatility

We report the estimates and Newey-West adjusted t-statistics of the slope coefficients from regressing the monthly excess return of the S&P 500 index (Panel A) and the monthly MVOL (Panel B), which is defined as the volatility of the daily market excess return, on innovations in VOL, SKEW, and KURT in the previous month. Significant t-statistics at the 95% confidence level are boldfaced.

Panel A: Rm - Rf (t)								
ΔVOL (t-1)		-0.073			-0.045	-0.012		0.021
		(-0.74)			(-0.43)	(-0.11)		(0.20)
ΔSKEW (t-1)			0.011		0.011		0.011	0.012
			(1.57)		(1.56)		(1.53)	(1.55)
ΔKURT (t-1)				0.004		0.004	0.004	0.004
				(1.95)		**(1.75)**	**(2.06)**	**(1.99)**
Rm-Rf (t-1)	0.022	-0.026	0.034	-0.011	0.004	-0.018	0.001	0.013
	(0.40)	(-0.26)	(0.64)	(-0.19)	(0.04)	(-0.19)	(0.01)	(0.14)
Adj. R^2 (%)	-0.66	-1.17	0.50	0.07	-0.13	-0.65	1.25	0.55
Panel B: MVOL (t)								
ΔVOL (t-1)		0.371			0.373	0.339		0.344
		(3.07)			**(3.34)**	**(2.64)**		**(2.84)**
ΔSKEW (t-1)			0.012		0.013		0.012	0.012
			(1.93)		**(2.05)**		**(1.73)**	**(1.95)**
ΔKURT (t-1)				-0.006		-0.002	-0.006	-0.002
				(-2.43)		(-0.95)	**(-2.45)**	(-0.87)
MVOL (t-1)	0.619	0.589	0.624	0.607	0.594	0.588	0.613	0.593
	(12.41)	**(11.13)**	**(12.50)**	**(12.03)**	**(11.25)**	**(11.19)**	**(12.09)**	**(11.27)**
Adj. R^2 (%)	37.90	41.64	38.26	38.88	42.08	41.39	39.18	41.80