IMPLIED VOLATILITY SURFACE AND MODEL OF VIX

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ABSTRACT. In this paper, we first back out the Black-Scholes implied volatility based on real dated VIX options. An obvious observation is that the VIX implied volatilities are upward-sloping, in opposite to the implied volatility skew for SPX index options. Also there are at least two factors that drive the dynamic of VIX implied volatilities, since the volatility smile varies with short-term and long-term VIX levels. Then we derive that any twice-differentiable payoff at time T may be statically hedged using a portfolio of European options expiring at time T. And VIX^2 is the fair strike of a variance swap, and the fair value of VIX^2 can be calculated from the strip of European options. Finally, we propose double stochastic volatility model with diffusion underlying dynamic for jointly SPX and VIX options modeling.

1. INTRODUCTION

Black [2] first introduced the European futures options pricing based on Black-Scholes framework [3]. The common assumption for the process followed by future price F in risk-neutral framework is

$$dF = \sigma F dW, \tag{1}$$

where σ is a constant and W is the Wiener process. Similar to a non-dividend-paying stock, the differential equation satisfied by a derivative dependent on a futures price is

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf.$$

Then the European call price c and the European put price p for the futures option are given by following equations,

$$\begin{aligned} c &= e^{-rT}[F_0 N(d_1) - KN(d_2)], && (2)\\ p &= e^{-rT}[KN(-d_2) - F_0 N(-d_1)], && (3) \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\log(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}},\\ d_2 &= \frac{\log(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}. \end{aligned}$$

Following the Black-Scholes framework, more recent researches come up with stochastic volatility (SV) model [6, 1, 5], where the volatility surface (w.r.t. different terms and strikes) are fitted better to the empirical observations. The SV modelings are motivated by the intermittency and clustering feature of volatility from underlying dynamics. A common stochastic volatility process satisfies the SDE:

$$dS_t = \mu_t S_t dt + \sqrt{v_t} S_t dW_1, dv_t = \alpha(S_t, v_t, t)dt + \eta\beta(S_t, v_t, t)\sqrt{v_t} dW_2,$$

where $\mathbb{E}[dW_1 dW_2] = \rho dt$. μ_t is the (deterministic) instantaneous drift of stock price returns, η is the volatility of volatility and ρ is the correlation between random stock price returns and changes in v_t. $W1$ and W_2 are Wiener processes.

The volatility drift and vol of vol term are in most generic functional forms and can determine by time (t), instantaneous volatility level (v_t) and the spot price (S_t). Note that the SV setup ensures the standard time-dependent volatility version of the Black-Scholes formula may be retrieved in the limit $\eta \to 0$.

Particularly, S&P500 index returns, shown in Figure 1, reinforce the stochastic volatility and volatility mean-reverting characteristics. More importantly, the "volatility" of the S&P500 index is tradable. The VIX index represents the volatility of SPX in a precise way, and CBOE has listed futures on VIX index since 2004 and VIX option in 2006.

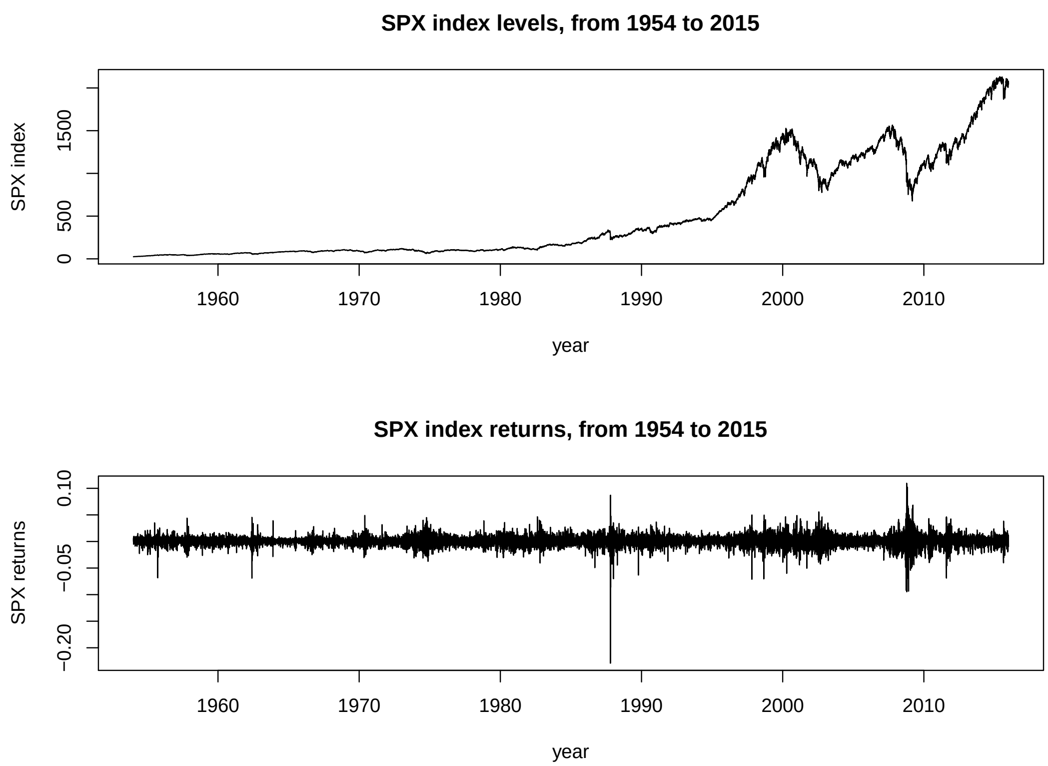

Figure 1 The SPX index levels and return on daily basis. Time period is from 1954 to 2015.

The rest of the paper lays out as following: Section 2 presents the observations and findings on implied volatility from VIX options. Section 3 conjectures a two-factor VIX model and discusses the joint VIX model with SPX dynamics.

2. VIX Implied Volatility

The option data from OptionMetrics contains daily standardized VIX option bid/offer price, Delta of the option and other information including expiration and strike. The forward price, here the underlying future price, is interpolated from OptionMetrics' standard volatility surface using log-linear interpolation. The reasoning of selecting log-linear interpolation is that the future price

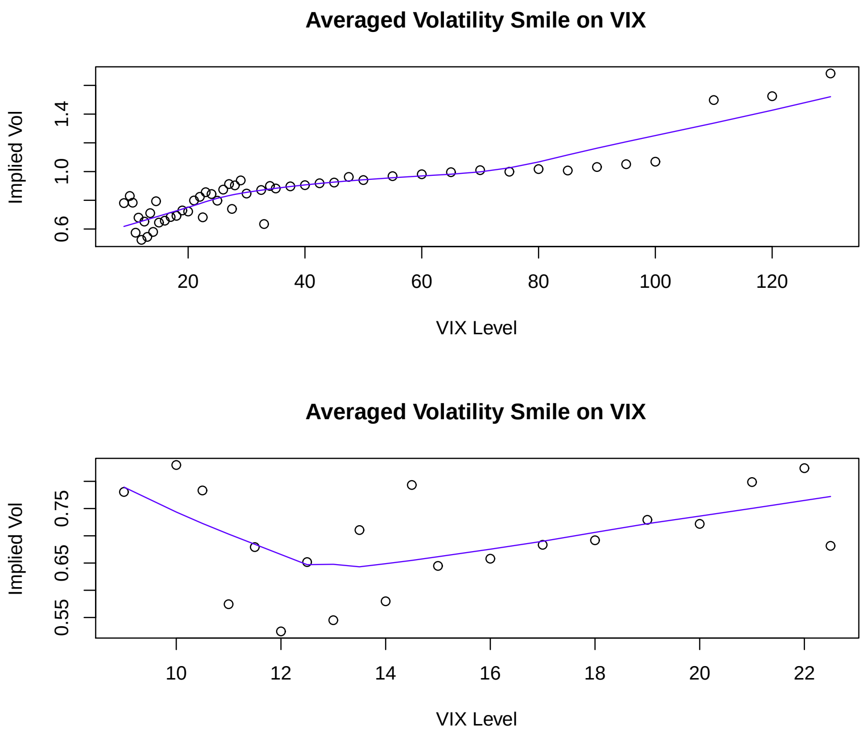

Figure 2 The averaged VIX option implied volatility over different terms. The graph plots the implied volatility dynamic on various strikes. The blue line is the fitted implied volatility smile by lowess regression. The graph on the top has strikes (of VIX) from 5 to 130. The graph on the bottom has strikes from 5 to 22.

is positively correlated with discount factor, and we assume a flat-forward interpolation keeps most of properties from term structure. The zero yield curve is also from OptionMetrics database.

The VIX future has multiplier of 1000, so the strike price of options are in magnitude of 1000 times VIX. To back-out the implied volatility of VIX options, we use the Black model as specified in 2. The market premium is calculated from the average of best bid and best offer price. The underlying future price is interpolated log-linearly from standard option volatility, and the interest rate is interpolated linearly from the term structure of zero yield.

According call-put parity, the implied volatility from call option and put option with same strike will be the same. Figure 2 plots the averaged implied volatility over the terms. The graph on the top shows the implied volatility dynamics of VIX option with strike price ranging from 5% to 130%. Unlike the volatility skew shape as in equity option, the VIX option has higher implied vol (SPX's vol of vol). The strike-zoom-in graph on the bottom from Figure 2 show the volatility smile at relatively low VIX strike levels, ranging from 5% to 23%.

Figure 3 separates the average on implied volatility when the short term and long term are high/low respectively. From the graphs, it is obvious that the implied volatility levels have different shapes, in terms of both magnitude and the slope. That is, the implied volatility smile of VIX options are not only driven by the VIX level, but also term structure and skew. Therefore, empirical observations conflict one-factor stochastic volatility model, which captures only the level of volatility but not sufficient to fit term structure additionally.

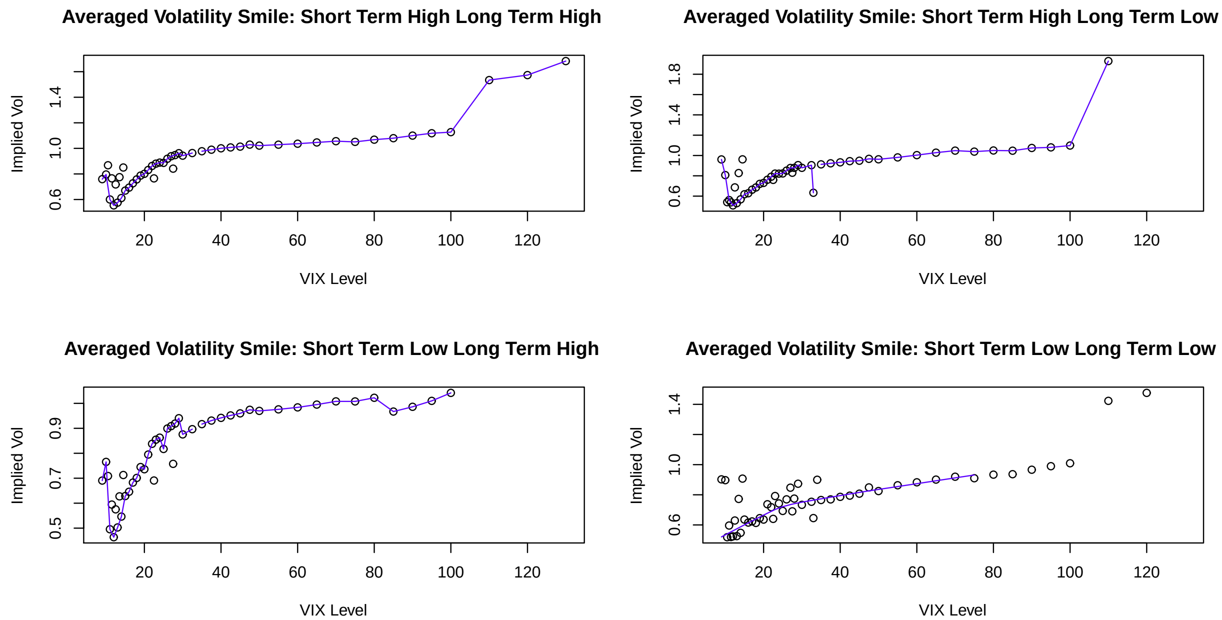

Figure 3 The averaged VIX option implied volatility over different terms, when the short/long term implied volatilities are high/low. Four graphs display the combination of the short- and long-term VIX. Each graph plots the implied volatility dynamic on various strikes.

Then the idea of two-factor stochastic volatility modeling becomes natural. In Section 3, we will discuss the rationale of the double-stochastic-variance (DSV) model.

3. Modeling VIX and SPX Jointly

Before proceeding to the conjecture of the VIX model, we firstly show that VIX represents the volatility of SPX in a precise way. Therefore one can model VIX using variance swap instead of the implied volatilities. And also the relationship between the prices of options on SPX and options on VIX can be established. In particular, the fair values of the variance swaps can be expressed in terms of the market prices of European options, which provides an model-independent way of linking SPX and VIX.

Assume there are European options with all possible strikes K and maturity are traded on the market, then any twice-differentiable payoff at time T may be statically hedged using a portfolio of European options expiring at time T. To prove this, consider the value of a claim with payoff $g(S_T)$ at time T as

$$\begin{aligned} g(S_T) &= \int_0^\infty g(K)\delta(S_T-K)dK \\ &= \int_0^F g(K)\delta(S_T-K)dK + \int_F^\infty g(K)\delta(S_T-K)dK. \end{aligned}$$

Follow the derivation in [4], and integrate by parts twice, we have

$$\begin{aligned}
g(S_T) &= g(F) - \int_0^F g'(K)(K - S_T)^+ dK \\
&\quad + \int_F^\infty g'(K)(S_T - K)^+ dK, \\
g(S_T) &= \int_0^F g''(K)(K - S_T)^+ dK + \int_F^\infty g''(K)(S_T - K)^+ dK \\
&\quad + g(F) + g'(F)[(F - S_T)^+ - (S_T - F)^+] \\
&= \int_0^F g''(K)(K - S_T)^+ dK + \int_F^\infty g''(K)(S_T - K)^+ dK + g(F) + g'(F)(F - S_T) \\
&\quad + g(F) + g'(F)(F - S_T). \qquad (4)
\end{aligned}$$

With $F = \mathbb{E}[S_T]$,

$$\mathbb{E}[g(S_T)] = g(F) + \int_0^F dK\tilde{P}(K)g''(K) + \int_F^\infty dK\tilde{C}(K)g''(K).$$

The Equation 4 shows that any European type twice-differentiable payoff can be replicated by a portfolio of European options (puts and calls) with strike from 0 to ∞. To be specified, the weight of each option in the portfolio is the second derivative of the payoff at the strike price of that options. Additionally, the weight of each option with certain strike depends only on the strike price and the payoff function, but not on time or S_t. Thus the replication is model-independent.

The variance swap has log contract payoff at time T, $\log(S_T/F)$. Then we have $g''(K) = -1/S_T^2\big|_{S_T=K}$,

$$\mathbb{E}\left[\log\left(\frac{S_T}{F}\right)\right] = -\int_0^F \frac{dK}{K^2}\tilde{P}(K) - \int_F^\infty \frac{dK}{K^2}\tilde{C}(K).$$

Express the log-strike variable $k = \log(K/F)$, we have

$$\mathbb{E}\left[\log\left(\frac{S_T}{F}\right)\right] = -\int_\infty^0 dk p(k) - \int_0^\infty dk c(k), \qquad (5)$$

with

$$c(k) = \frac{\tilde{C}(Fe^k)}{Fe^k}, \quad c(k) = \frac{\tilde{C}(Fe^k)}{Fe^k},$$

representing option prices expressed in terms of percentage of the strike price.

Assume $r = 0$ and there is no dividend applied, we yield $F = S_0$. By Ito's Lemma,

$$\begin{aligned}
\log\left(\frac{S_T}{F}\right) &= \log\left(\frac{S_T}{S_0}\right) \\
&= \int_0^T d\log(S_t) \\
&= \int_0^T \frac{dS_t}{S_t} - \int_0^T \frac{\sigma_t^2}{2}dt. \qquad (6)
\end{aligned}$$

Hence we connect the log-contract payoff with total variance in Equation 6. The first in Equation 6 represents the payoff of a hedging strategy which involves maintaining a constant dollar amount in stock. The log-contract payoff can be hedged by a portfolio of European options as shown in 5. Then the total variance $\int_0^T \sigma_t^2 dt$ can also be replicated by a model-independent approach, but requires the stock price dynamic as a diffusion (no jumps). With risk-neutral expectation on both sides of Equation 6, we yield

$$\mathbb{E}\left[\int_0^T \sigma_t^2 dt\right] = -2\mathbb{E}\left[\log\left(\frac{S_T}{F}\right)\right] = 2\left[\int_{-\infty}^0 dk p(k) + \int_0^\infty dk c(k)\right].$$

That is, the fair price of the total variance is equal to the value of an infinite strip (ATM call and ATM put) of European options in model-independent approach.

Per CBOE VIX white paper, "CBOE is changing VIX to provide a more precise and robust measure of expected market volatility and to create a viable underlying index for tradable volatility products." The definition is

$$VIX^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2, \tag{7}$$

where Q_i is the price of the out-of-the-money option with strike K_i and K_0 is the highest strike below the forward price F.

To be specifically,

$$\begin{aligned}
\frac{VIX^2 T}{2} &= \int_0^F \frac{dK}{K^2} P(K) + \frac{F}{\infty}\frac{dK}{K^2} C(k) \\
&= \int_0^{K_0} \frac{dK}{K^2} P(K) + \int_{K_0}^{\infty} \frac{dK}{K^2} C(k) + \int_{K_0}^{F} \frac{dK}{K^2}(P(K) - C(K)) \\
&= \int_0^{\infty} \frac{dK}{K^2} Q(K) + \int_{K_0}^{F} \frac{dK}{K^2}(K - F) \\
&\approx \int_0^{\infty} \frac{dK}{K^2} Q(K) + \frac{1}{K_0^2}\int_{K_0}^{F} dK(K - F) \\
&= \int_0^{\infty} \frac{dK}{K^2} Q(K) - \frac{1}{K_0^2}\frac{(K_0 - F)^2}{2},
\end{aligned}$$

which yields the discretization of the last expression as

$$VIX^2 = \frac{2}{T}\sum_i \frac{\Delta K_i}{K_i^2} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2.$$

Therefore we show that VIX^2 is the fair strike of a variance swap, and the fair value of VIX^2 can be calculated from the strip of European options. In practice, we may need to interpolate and extrapolate option prices for each expiration.

From the observations from Section 2, we know that the VIX smiles are upward. Also when the SPX index decreases, volatility (hereby VIX) usually increases. In 2004, CBOE introduced VIX future, whose value at time t is

$$\mathbb{E}_t\left[\sqrt{\mathbb{E}_T\left[\int_T^{T+\Delta} v_s ds\right]}\right].$$

In 2006, CBOE listed VIX option. The VIX option expiring at time T with strike K_{VIX} at time t is

$$\mathbb{E}_t\left[\left(\sqrt{\mathbb{E}_T\left[\int_T^{T+\Delta} v_s ds\right]} - K_{VIX}\right)^+\right].$$

For the VIX option, $g(x) = x^2$. From Equation 4, we have

$$\mathbb{E}_t\left[\int_T^{T+\Delta} v_s ds\right] = F_{VIX}^2 + 2\int_0^{F_{VIX}} \tilde{P}(K)dK + 2\int_{F_{VIX}}^{\infty} \tilde{C}(K)dK.$$

To this point, it becomes natural to model the SPX and VIX option jointly. Firstly, we observed that there are at least two factors that drive the move of volatilities. Secondly, the fair value of variance of SPX index, VIX^2, can be calculated from European option when the underlying price dynamic is a diffusion. Hence we come up with the double stochastic variance model (DSV) has dynamics of

$$
\begin{aligned}
dS_t &= \sqrt{v_t} S_t dW_t^1, \\
dv_t &= \kappa_1 (v_t' - v_t) dt + \xi_1 v_t dW_t^2, \qquad (8)\\
dv_t' &= \kappa_2 (\theta - v_t') dt + \xi_2 v_t' dW_t^3,
\end{aligned}
$$

(9)

where the Brownian motions W_i could all be correlated with $\mathbb{E}[dW_t^i dW_t^j] = \rho_{ij} dt$. Instantaneous variance v mean-reverts to a level v' that itself moves slowly over time and mean-reverts to the long-term mean level θ. To conclude, the fair strike of a variance swap is

$$
\begin{aligned}
\mathbb{E}\left[\int_t^T v_s ds \middle| \mathscr{F}_t \right] &= \theta\tau + (v_t - \theta) \frac{1 - e^{-\kappa_1 \tau}}{\kappa_1} \\
&+ (v_t' - \theta) \frac{\kappa_1}{\kappa_1 - \kappa_2} \left[\frac{1 - e^{-\kappa_2 \tau}}{\kappa_2} - \frac{1 - e^{-\kappa_1 \tau}}{\kappa_1} \right]
\end{aligned}
$$

References

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[5] Duffie, Darrell, Jun Pun, and Kenneth Singleton. Transform analysis and asset pricing for affine jump-diffusions. *Econometrica*, 68:1343–1376, 2000.

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Appendix: Code

```
setwd('C:\\Users\\ranzhao\\Documents\\Empirical-Asset-Pricing\\Assignment 7')
setwd('D:\\PhD FE\\Empirical-Asset-Pricing\\Assignment 7')
setwd('D:\\Empirical-Asset-Pricing\\Assignment 7')

spx_index_values = read.csv('spx_index_values.csv', header = TRUE)
par(mfrow=c(2,1))
plot(as.Date(as.character(spx_index_values$Date), "%m/%d/%Y"), spx_index_values$SPX.Index, type='l',
     main='SPX index levels, from 1954 to 2015',
     xlab='year', ylab='SPX index')

# calculate the return series
spx_index_values$Return = rep(0, dim(spx_index_values)[1])
spx_index_values$Return[2:length(spx_index_values$Return)] =
  log(spx_index_values$SPX.Index[2:length(spx_index_values$SPX.Index)] /
        spx_index_values$SPX.Index[1:(length(spx_index_values$SPX.Index)-1)])
data.length = length(spx_index_values$Return)

plot(as.Date(as.character(spx_index_values$Date), "%m/%d/%Y"), spx_index_values$Return, type='l',
     main='SPX index returns, from 1954 to 2015',
     xlab='year', ylab='SPX returns')

require(data.table)
option.data = fread('VIXoptions.csv', header = T, sep = ',')
implied.data = fread('VIXoptionsStd.csv', header = T, sep = ',')
ir.data = fread('zeroyieldcurve.csv', header = T)

option.data = as.data.frame(option.data)
implied.data = as.data.frame(implied.data)
ir.data = as.data.frame(ir.data)

option.data = option.data[,c("date","exdate","cp_flag","strike_price","best_bid","best_offer","delta")]
option.data$days = as.numeric(as.Date(as.character(option.data$exdate), format="%Y%m%d")-as.Date(as.character(option.data$date), format="%Y%m%d"))
option.data$T = option.data$days/360

option.data = option.data[!is.na(option.data$delta),]
implied.data = implied.data[implied.data$cp_flag=="C",c("date","days","forward_price")]

# interpolation future prices and interest rate
option.data$F0 = 0
option.data$r = 0

all.dates = unique(option.data$date)
for (i in all.dates){ # slow
  all.days = unique(option.data$days[option.data$date==i])
  for (j in all.days){
    ix0 = implied.data$days[implied.data$date==i]
    iy0 = implied.data$forward_price[implied.data$date==i]
    option.data$F0[option.data$date==i&option.data$days==j] = linear.inter(ix0, iy0, j, "log")
    # handle missing data
    ir.ix0 = ir.data$days[ir.data$date==i]
```

```
    ir.iy0 = ir.data$rate[ir.data$date==i]/100
    if (length(ir.ix0)==0){
      ir.ix0 = ir.data$days[ir.data$date==i.last]
      ir.iy0 = ir.data$rate[ir.data$date==i.last]/100
    }
    option.data$r[option.data$date==i&option.data$days==j] = linear.inter(ir.ix0, ir.iy0, j, "linear")
  }
  i.last = i
}

# calculate the BS implied volatility
option.data$BS_iv = 0
for (k in 1:length(option.data$date)){ # very slow
  option.data$BS_iv[k] = bs.iv(option.data$F0[k],
                               option.data$strike_price[k]/1000,
                               option.data$T[k],
                               option.data$r[k],
                               0.5*(option.data$best_bid[k]+option.data$best_offer[k]),
                               option.data$cp_flag[k])
}
# option.data$BS_iv = lapply(cbind(option.data$F0, option.data$strike_price/1000, option.data$T, option.data$r, 0.5*(option.data$best_bid+option.data$best_offer), option.data$cp_flag), bs.iv)

option.data = option.data[!is.na(option.data$BS_iv),]
all.strikes = sort(unique(option.data$strike_price))
vix.vol.smile = rep(0, length(all.strikes))
for (s in 1:length(all.strikes)){
  vix.vol.smile[s] = mean(option.data$BS_iv[option.data$strike_price==all.strikes[s]])
}

par(mfrow=c(2,1))
plot(all.strikes/1000, vix.vol.smile, type='p', main='Averaged Volatility Smile on VIX',xlab='VIX Level',ylab='Implied Vol')
lines(lowess(all.strikes/1000,vix.vol.smile), col="blue") # lowess line (x,y)

plot(all.strikes[1:20]/1000, vix.vol.smile[1:20], type='p', main='Averaged Volatility Smile on VIX',xlab='VIX Level',ylab='Implied Vol')
lines(lowess(all.strikes[1:20]/1000,vix.vol.smile[1:20]), col="blue") # lowess line (x,y)

short.term.vol.thres = mean(option.data$BS_iv[option.data$days<=60])
long.term.vol.thres = mean(option.data$BS_iv[option.data$days>=120])

short.term.high.long.term.high = c()
short.term.high.long.term.low = c()
short.term.low.long.term.high = c()
short.term.low.long.term.low = c()

for (i in all.dates){
  short.term.vol = mean(option.data$BS_iv[option.data$date == i & option.data$days<=60])
  long.term.vol = mean(option.data$BS_iv[option.data$date == i & option.data$days>=120])
  if (short.term.vol >= short.term.vol.thres & long.term.vol >= long.term.vol.thres){
```

```
    short.term.high.long.term.high = rbind(short.term.high.long.term.high, option.data[option.data$date == i,])
  } else if (short.term.vol >= short.term.vol.thres & long.term.vol < long.term.vol.thres){
    short.term.high.long.term.low = rbind(short.term.high.long.term.low, option.data[option.data$date == i,])
  } else if (short.term.vol < short.term.vol.thres & long.term.vol >= long.term.vol.thres){
    short.term.low.long.term.high = rbind(short.term.low.long.term.high, option.data[option.data$date == i,])
  } else {
    short.term.low.long.term.low = rbind(short.term.low.long.term.low, option.data[option.data$date == i,])
  }
}

vix.vol.smile.short.term.high.long.term.high = rep(0, length(all.strikes))
vix.vol.smile.short.term.high.long.term.low = rep(0, length(all.strikes))
vix.vol.smile.short.term.low.long.term.high = rep(0, length(all.strikes))
vix.vol.smile.short.term.low.long.term.low = rep(0, length(all.strikes))

for (s in 1:length(all.strikes)){
  vix.vol.smile.short.term.high.long.term.high[s] = mean(short.term.high.long.term.high$BS_iv[short.term.high.long.term.high$strike_price==all.strikes[s]])
  vix.vol.smile.short.term.high.long.term.low[s] = mean(short.term.high.long.term.low$BS_iv[short.term.high.long.term.low$strike_price==all.strikes[s]])
  vix.vol.smile.short.term.low.long.term.high[s] = mean(short.term.low.long.term.high$BS_iv[short.term.low.long.term.high$strike_price==all.strikes[s]])
  vix.vol.smile.short.term.low.long.term.low[s] = mean(short.term.low.long.term.low$BS_iv[short.term.low.long.term.low$strike_price==all.strikes[s]])
}
par(mfrow=c(2,2))
plot(all.strikes/1000, vix.vol.smile.short.term.high.long.term.high, type='p', main='Averaged Volatility Smile: Short Term High Long Term High',xlab='VIX Level',ylab='Implied Vol')
lines(lowess(all.strikes/1000,vix.vol.smile.short.term.high.long.term.high), col="blue") # lowess line (x,y)

plot(all.strikes/1000, vix.vol.smile.short.term.high.long.term.low, type='p', main='Averaged Volatility Smile: Short Term High Long Term Low',xlab='VIX Level',ylab='Implied Vol')
lines(lowess(all.strikes/1000,vix.vol.smile.short.term.high.long.term.low), col="blue") # lowess line (x,y)

plot(all.strikes/1000, vix.vol.smile.short.term.low.long.term.high, type='p', main='Averaged Volatility Smile: Short Term Low Long Term High',xlab='VIX Level',ylab='Implied Vol')
lines(lowess(all.strikes/1000,vix.vol.smile.short.term.low.long.term.high), col="blue") # lowess line (x,y)

plot(all.strikes/1000, vix.vol.smile.short.term.low.long.term.low, type='p', main='Averaged Volatility Smile: Short Term Low Long Term Low',xlab='VIX Level',ylab='Implied Vol')
lines(lowess(all.strikes/1000,vix.vol.smile.short.term.low.long.term.low), col="blue") # lowess line (x,y)

linear.inter = function(ix0,iy0,ix,inter.method="log"){
# log-linear and linear interpolation function
```

```
    order = sort(ix0, index.return=TRUE)
  x0 = order$x
  y0 = iy0[order$ix]
  n = length(ix0)
  if (ix < min(ix0)){
    ix1 = ix0[1]
    ix2 = ix0[2]
    iy1 = iy0[1]
    iy2 = iy0[2]
  } else if (ix>max(ix0)){
    ix1 = ix0[n-1]
    ix2 = ix0[n]
    iy1 = iy0[n-1]
    iy2 = iy0[n]
  } else {
    ix1 = ix0[max(which(ix>=ix0))]
    ix2 = ix0[min(which(ix<=ix0))]
    iy1 = iy0[max(which(ix>=ix0))]
    iy2 = iy0[min(which(ix<=ix0))]
  }
  if (ix1 == ix2){
    return(iy1)
  }
  else {
    if (inter.method == "log"){
      iy = (ix-ix1)/(ix2-ix1)*log(iy2) + (ix2-ix)/(ix2-ix1)*log(iy1)
      return(exp(iy))
    } else if (inter.method == "linear") {
      iy = (ix-ix1)/(ix2-ix1)*iy2 + (ix2-ix)/(ix2-ix1)*iy1
      return(iy)
    }
  }
}


bs.iv = function(S, K, T, r, market, type){
# calculate Black-Scholes implied volatility
  sig <- 0.20
  sig.up <- 2
  sig.down <- 0.001
  count <- 0
  err <- BS(S, K, T, r, sig, type) - market

  ## repeat until error is sufficiently small or counter hits 1000
  while(abs(err) > 0.0001 && count<3000){
    if(err < 0){
      sig.down <- sig
      sig <- (sig.up + sig)/2
    }else{
      sig.up <- sig
      sig <- (sig.down + sig)/2
    }
    err <- BS(S, K, T, r, sig, type) - market
    count <- count + 1
  }

  ## return NA if counter hit 1000
  if(count==3000){
```

```r
      return (NA)
    } else {
      return ( sig )
    }
}

BS = function(S, K, T, r, sig, type="C"){
# calculation option price using Black-Scholes model
    d1 <- (log(S/K) + (r + sig^2/2)*T) / (sig*sqrt(T))
    d2 <- d1 - sig*sqrt(T)
    if(type=="C"){
      value <- S*exp(-r*T)*pnorm(d1) - K*exp(-r*T)*pnorm(d2)
    }
    if(type=="P"){
      value <- K*exp(-r*T)*pnorm(-d2) - S*exp(-r*T)*pnorm(-d1)
    }
    return(value)
  }
```

assignment7.R