Intermediate Financial Economics
Homework II

CEMA, CUFE

Due in class on Monday, March 25, 2013

1. Cochrane Chapter 1 Problem 1 (page 33).
2. Prove that the logarithmic utility is the special case of the power utility when the coefficient of the relative risk aversion is one. That is, $\lim_{\rho \to 1} \frac{x^{1-\rho}-1}{1-\rho} = \ln x$
3. Prove: Suppose X and Y are both normally distributed. Then Y is riskier than X if and only if $Var[X] \leq Var[Y]$. (Hint: use Proposition II of Lecture 3.)
4. Consider a risky consumption plan z. The payoffs of z is $(2, 0, -2)$ with equal probabilities. Another risky consumption plan y has payoffs $(3, 0, -1, -2)$ with equal probabilities.
   (a) Which has a higher variance, y or z?
   (b) Is y riskier than z? Prove or disprove.
5. Consider a one-period model where a risk averse person maximizes the expected utility of future wealth. There are one risky stock and one riskless bond for investment at date 0. At date 1, the risky stock will have payoffs $x_S = 4$ or 0 with equal probabilities. Suppose the price of the stock at date 0 is $P_S = 1.667$. The riskless bond will pay 1 unit of consumption at date 1 for sure. Its price at date 0 is $P_f = 0.909$. Suppose the utility function is
$$u(W) = -W^{-2}/2$$
   Let the current wealth W_0 be 10.
   (a) What is the absolute risk aversion for this person? Is it CARA, DARA, or IARA?
   (b) What is the relative risk aversion for this person? Is it CRRA, DRRA, or IRRA?
   (c) What is the random gross return of the stock? What is the expected gross return of the stock? What is the riskless gross return of the bond?
   (d) Write down the optimization problem of the risk averse person. What is the first order condition?
   (e) What is the optimal investment in the risky stock? How much percent of the initial wealth is invested in the stock?
   (f) Suppose now the initial wealth doubles into W_0'=20. Will the investment in the stock increase? Will the stock investment as a percentage of the initial wealth increase? Why? Explain.