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I explain why at-the-money implied volatility is a biased and inefficient forecast of future realized volatility using the insights from the empirical option-pricing literature. First, I explain how the risk premia, which manifest themselves through disparity between objective and risk-neutral probability measures, lead to the disparity between realized and implied volatilities. Second, I show that this disparity is a function of the latent spot volatility, which I estimate using the historical volatility and high–low range. An empirical exercise that is based on at-the-money implied volatility series of foreign currencies and stock market indexes, is supportive of my risk premia-based explanation of the bias.

KEY WORDS: Error-in-variables problem; Implied volatility; Jump–diffusion processes; Quadratic variation; Range; Realized volatility.

1. INTRODUCTION

A great deal of work in empirical finance has been dedicated to establishing the informational content of implied volatility. Generally, tests concentrate on the following regression:

$$RV_{t+\tau,\tau} = a + b \cdot \sigma^2_{t,\tau} + c \cdot PV_{t,\tau} + \epsilon_{t+\tau}, \qquad (1.1)$$

where $\sigma^2_{t,\tau}$ is the Black–Scholes variance implied at time t from an at-the-money (ATM) option contract with time to maturity τ, RV is the realized variance over the period t to $t+\tau$, and PV is an additional predictive variable typically measured by the historical variance over the preceding period $t-\tau$ to t. The objective of the tests is to establish whether implied volatility is the most efficient (i.e., unbiased, $a=0$, $b=1$) and subsuming of all relevant information ($c=0$) forecast of future volatility. A number of robust conclusions have emerged: ATM implied volatility is (1) informative about future volatility, (2) superior to other measures of volatility, and (3) an upward-biased predictor (see, e.g., Day and Lewis 1992; Canina and Figlewski 1993; Lamoureux and Lastrapes 1993; Jorion 1995; Blair, Poon, and Taylor 2001; among others). However, it is still unclear why implied volatility is biased (the “unbiasedness puzzle”) and whether it is informationally efficient.

This article explains the unbiasedness puzzle by bridging the apparent gap between the volatility forecasting and empirical option pricing literatures. Although most existing work treats the two as somewhat separate, there is much to be learned about the findings of the former based on the modeling techniques and empirical results of the latter. Taking an option pricing stance allows one to reinterpret the well-studied forecasting regressions in a more economically coherent way.

More specifically, I use affine jump–diffusion models, which are widely accepted as plausible descriptions of asset returns and their options, as an illustrative framework for understanding the volatility forecasting results. These models yield closed-form expressions of expected future volatility and option prices and approximate expressions for implied volatility, which allow for the investigation of the structural features of the data-generating process responsible for the findings reported in the forecasting literature.

First, I explain how the disparity between objective and risk-neutral probability measures leads to the disparity between the realized and ATM implied volatilities. I explicitly establish the link between the expectation of future volatility under the objective probability measure as a dependent variable and ATM implied volatility as a predictive variable via the volatility risk premia terms. Second, I show that this disparity (for the class of models considered) is related to the stochastic spot volatility. Using spot volatility as an additional predictive variable removes the documented implied volatility bias. Hence, I am able to justify, from a theoretical perspective, the regressions that have been put forward in the past. This demonstration, in turn, proves the informational inefficiency of ATM implied volatility.

The emergence of latent spot volatility as a required predictive variable underscores a new problem—it has to be estimated. The historical volatility frequently used in the literature as an additional predictive variable could be used as a possible estimator of the spot volatility. However, I show that this particular estimator is not optimal because the regression estimates are so imprecise that it is difficult to conduct statistical inference. I use a much more efficient range-based estimator of the spot volatility (Parkinson 1980) to remedy this problem. Estimated volatility introduces an error-in-variables (EIV) problem that has a much more dramatic impact on the results than errors in the measurement of implied volatilities accounted for in prior studies. I incorporate instrumental variables in the estimation procedure to reduce the impact of these errors.

Because I focus on the contribution of volatility risk premia and the new econometric issues mentioned previously, I intentionally select a dataset free of implied volatility measurement errors and the telescoping maturity problem in my empirical study. Specifically, I use daily returns and implied volatilities of two U.S. equity indices (S&P 100 and Nasdaq 100) and three foreign currency exchange rates, the British pound (GBP), Japanese yen (JPY), and Swiss franc (CHF). Each implied volatility series is constructed such that it corresponds to daily ATM contracts with 1 month to maturity.

I find that the implied volatility bias disappears in my sample of two U.S. equity indices and three currency rates once I (1) account for time-varying diffusive and constant jump volatility

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risk premia, (2) use a range-based estimator of latent volatility, and (3) tackle the error-in-variables problem via instrumental variables. Both diffusive and jump volatility risks contribute to the bias found in previous studies. Interestingly, with the exception of the S&P 100, the data provide evidence of the dominant role that is played by the volatility jump risk premium. It appears that the stochastic discount factor has a relatively higher correlation with rapid moves in volatility in the foreign exchange rates and Nasdaq market than with the diffusive changes in volatility.

The article is organized into six sections. In the next section I briefly review the recent related literature. In Section 3 I introduce an illustrative jump–diffusion model and derive the impact of volatility risk premia on volatility forecasting regressions. In the fourth section I discuss ways of implementing these regressions and focus on spot volatility estimation via historical volatility and high–low range. Also in Section 4 a simulation study assesses the relative efficiency of range versus historical volatility estimators and verifies the forecasting regression outcome for the simulated data. In Section 5 I introduce the details of my dataset and report the regression results and in the last section I conclude. The Appendix provides the technical details.

2. LITERATURE REVIEW

Lamoureux and Lastrapes (1993) were the first to build an explicit link between the future volatility and ATM implied volatility based on the Hull–White (1987) option pricing model. One of the critical assumptions of this model is that volatility risk is diversifiable, and, therefore, volatility does not command a risk premium. The findings of Lamoureux and Lastrapes suggest that the omitted volatility risk premium could be responsible for the bias in the volatility forecasts. This hypothesis should be pursued further.

Recent work in the option pricing literature explicitly models and estimates the volatility risk premium and finds it to be significant based on the information contained in the option prices and the respective returns on the underlying assets (see Benzoni 1998; Chernov and Ghysels 2000; Pan 2002; Jones 2003; Eraker 2004; among others). In contemporaneous and independent work, Poteshman (2000) also argued for the importance of the volatility risk premium in explaining biases. His approach is different from the one presented here. Among other things, he used volatility implied from the Heston (1993) stochastic volatility model rather than from the Black–Scholes model.

The results of recent works, which emphasize the econometric aspects of the investigated regressions, do not yield uniform results. Christensen and Prabhala (1998) concluded that "implied volatility outperforms past volatility in forecasting future volatility and even subsumes the information content of past volatility in some... specifications" and reported the evidence of the implied volatility's unbiasedness. They implemented the forecasting regressions in logs of the volatilities. Poteshman (2000) pointed out that such a transformation leads to an upward bias in the regression coefficient and concluded that this could be responsible for their findings. Bandi and Perron (2001) explored the possibility of the fractionally cointegrating relationship between realized and implied volatility (Christensen and Nielsen 2007 performed a similar analysis). Although they found that "a long-run one-to-one correspondence between implied and realized volatility series is rather strong," they concluded that "little can be said about... short-term unbiasedness."

More recently, an approach similar to mine was applied to study other issues related to volatility. Bollerslev and Zhou (2006) derived the volatility–return relationships in addition to the volatility forecasting equation in the framework of the Heston model. Carr and Wu (2003) studied the forecasting relationship under different angles, using both specifications nested in my model and nonparametric replication of expected volatilities via a range of options. They applied their analysis to estimate volatility risk premia in individual stock returns. Jiang and Tian (2005) used a similar replication technique developed by Britten-Jones and Neuberger (2000) to study the information content of model-free implied volatility.

3. OPTION PRICING FOUNDATIONS OF THE FORECASTING REGRESSIONS

This section describes the connection between realized and ATM implied volatilities in the no-arbitrage framework that is based on an illustrative model specification, which is further based on empirical findings in option pricing literature. Section 3.1 introduces such a model. Section 3.2 discusses the theoretical underpinnings of realized volatility and prepares the groundwork for forecasting regressions by deriving expected values of the continuous-time counterpart of the realized volatility (i.e., quadratic variation). Section 3.3 develops the link between expected quadratic variation and ATM implied volatility. Section 3.4 combines these developments to derive a no-arbitrage relationship between the implied volatility and the expected future volatility, which is the main focus of this study. Finally, Section 3.5 concludes the theoretical developments by illustrating how various risk premia could affect the regression coefficients.

3.1 An Illustrative Jump–Diffusion Model

In this section I derive implications for the informational content of implied volatility in the framework of jump–diffusion models of asset returns. I rely on a specification that provides a good framework for capturing many empirically relevant features of return dynamics. This specification will, thus, be able to describe and illustrate the mechanisms generating the typical results of the implied volatility-based regressions.

I assume that an asset price or exchange rate, S_t, and its instantaneous variance, V_t, jointly solve the system of stochastic differential equations:

$$\frac{dS_t}{S_t} = \mu_t\,dt + \sqrt{V_t}\,dW_{st}(P) + \left(e^{J_{st}(P)} - 1\right)dN_{st}(P), \tag{3.1}$$

$$dV_t = \left(\theta^P - \kappa^P V_t\right)dt + \sigma(V_t)\,dW_{vt}(P) + J_{vt}(P)\,dN_{vt}(P) \tag{3.2}$$

on the probability space $(\Omega, \mathcal{F}, P)$. The Brownian motions W_s and W_v are assumed to be correlated: $\text{cov}(W_{st}, W_{vt}) = \rho t$. The coefficient ρ is often referred to as the leverage effect. N_s (N_v) is the Poisson counting process with intensity λ_s^P (λ_v^P). This specification nests well-known models such as stochastic volatility (SV), when $\lambda_s^P = \lambda_v^P = 0$, and stochastic volatility with jumps in returns (SVJ), when $\lambda_v^P = 0$. The full model will be referred to as the stochastic volatility with independent jumps in returns and volatility (SVIJ) model.

By a slight abuse of notation, I symbolize discontinuity in the corresponding process with "$f(J_{\cdot t})\,dN_{\cdot t}$." I define a jump–size distribution whenever there is a jump (i.e., Poisson process N is incremented by 1). Jump in the asset price is lognormally distributed (i.e., J_s is distributed normally with mean μ_s^P and variance σ_s^2). Such a jump specification implies normally distributed jumps in log returns. The jump in volatility, J_v, is distributed exponentially with mean μ_v^P.

The affine version of this model was proposed by Duffie, Pan, and Singleton (2000). It is capable of capturing most features of equity and foreign exchange returns as documented in the empirical research (see, for instance, Eraker, Johannes, and Polson 2003 for evidence based on the S&P and Nasdaq returns; and Bates 1996a for the Deutsche mark exchange rate). Some studies emphasize the importance of two stochastic volatility factors (see Alizadeh, Brandt, and Diebold 2007 and Bollerslev and Zhou 2002 for foreign exchange rates and Chernov, Gallant, Ghysels, and Tauchen 2003 for equity indices). I focus on a single volatility factor specification because the presence of the jump components alleviates, at least for illustrative purposes, the need for inclusion of additional volatility factors.

I also assume that the linearity of the volatility drift is preserved under the change of probability measure. Linearity of the volatility drift under both measures is the key feature that I rely on in my derivations. Namely, under the risk-neutral measure Q, the dynamics of asset prices are described by

$$\begin{aligned}\frac{dS_t}{S_t} &= \left(\mu_t^Q - \lambda_s^Q\left(e^{\mu_s^Q + .5\sigma_s^2} - 1\right)\right)dt + \sqrt{V_t}\,dW_{st}(Q)\\ &\quad + \left(e^{J_{st}(Q)} - 1\right)dN_{st}(Q), \qquad (3.3)\\ dV_t &= (\theta^Q - \kappa^Q V_t)\,dt + \sigma(V_t)\,dW_{vt}(Q)\\ &\quad + J_{vt}(Q)\,dN_{vt}(Q), \qquad (3.4)\end{aligned}$$

where μ_t^Q is equal to either r_t^d (the risk-free rate) in the case of stocks or $r_t^d - r_t^f$ (the difference between the domestic and foreign risk-free rates) in the case of foreign exchange (FX) rates. I assume that the jump components belong to the same distribution families as under the P measure, but with different parameter values (i.e., the volatility jump intensity is λ_v^Q).

One should be careful in interpreting the FX risk premia because their specification depends on the choice of domestic versus foreign perspective (I am grateful to the associate editor for raising this issue). Because of the asymmetry of the FX rates (S_t for the domestic investor, $1/S_t$ for the foreign investor), the magnitude of the risk premia seems to be different, a so-called "directional bias." Therefore, the risk premia will have to be specified in terms of the foreign currency (see Bates 1996a). However, the chosen perspective does not affect pricing. Indeed, Dumas, Jennergren, and Näslund (1995) showed that as long as risks cannot be diversified away, that is, they require premia, the no-arbitrage pricing argument establishes a unique option price regardless of the perspective. Finally, Bardhan (1995) mentioned that if investors operate in terms of log-exchange rates, as in my empirical exercise, the directional bias disappears.

3.2 Expected Quadratic Variation

The purpose of implied volatility-based regressions is to forecast realized volatility, which is closely linked to the concept of quadratic variation. Assume prices are sampled n times per day. Then the realized volatility of returns

$$r_t(n) = \log\left(S_t/S_{t-1/n}\right) \qquad (3.5)$$

is measured as

$$RV_{t+\tau,\tau}^{(n)} \equiv \sum_{i=1}^{\tau n} r_{t+i/n}^2(n) \qquad (3.6)$$

(see, e.g., French, Schwert, and Stambaugh 1987; Andersen, Bollerslev, Diebold, and Labys 2001).

The quadratic variation of returns is computed as

$$QV_{t+\tau,\tau} = \operatorname*{plim}_{n\to\infty} RV_{t+\tau,\tau}^{(n)} \qquad (3.7)$$

and the average quadratic variation is defined as

$$\overline{QV}_{t+\tau,\tau} = \frac{1}{\tau} QV_{t+\tau,\tau}, \qquad (3.8)$$

where the expression on the right side of (3.7) is the property, not the definition, of the quadratic variation (the details are provided in Protter 1990). For example, in the particular case of the SVIJ model, the average quadratic variation is equal to

$$\overline{QV}_{t+\tau,\tau} = \frac{1}{\tau}\int_t^{t+\tau} V_u\,du + \frac{1}{\tau}\sum_{t<u\leq t+\tau} J_{su}^2 \Delta N_{su}. \qquad (3.9)$$

I rely on the linear structure of the variance drift under both probability measures in (3.2) and (3.4), and the Markov property of the variance, to derive the expressions for the expected value of the average quadratic variation (as in Feller 1951; Cox, Ingersoll, and Ross 1985; Meddahi and Renault 2004; among others). As shown in Appendix A,

$$E_t^M(\overline{QV}_{t+\tau,\tau}) = A_\tau^M V_t + B_\tau^M + \lambda_s^M \sigma_s^2, \qquad (3.10)$$

where

$$A_\tau^M = -\frac{1}{\kappa^M \tau}\left(e^{-\kappa^M\tau} - 1\right), \qquad (3.11)$$

$$B_\tau^M = \frac{\theta^M + \lambda_v^M \mu_v^M}{\kappa^M}(1 - A_\tau^M), \qquad (3.12)$$

and M denotes a particular probability measure (P or Q).

3.3 At-the-Money Implied Volatility and Quadratic Variation

I must link, theoretically, the expected quadratic variation to the ATM implied volatility in order to interpret the implied volatility-based regressions through the lens of my model. I rely on the insight of Hull and White (1987), who showed that a call option price can be represented as an expected value of the Black–Scholes formula, evaluated at the average quadratic variation

$$C^{\mathrm{HW}}(S_t, V_t, r, K, \tau) = E_t^Q\{C^{\mathrm{BS}}(S_t, \overline{QV}_{t+\tau,\tau}, r, K, \tau)\}. \quad (3.13)$$

This result is valid for an SV model without the leverage effect (i.e., $\rho = 0$). Because the Black–Scholes formula is a linear function of volatility for approximately ATM options, this result implies that

$$E_t^Q\left(\sqrt{\overline{QV}_{t+\tau,\tau}}\right) = \sigma_{t,\tau}, \quad (3.14)$$

where $\sigma_{t,\tau}$ is the Black–Scholes volatility implied at time t from an at-the-money (ATM) option contract with time to maturity τ. According to Bates (1996b), if

$$E_t^Q(\overline{QV}_{t+\tau,\tau}) \approx \sigma_{t,\tau}^2 \quad (3.15)$$

is used instead of (3.14), the Jensen inequality bias is less than .5% for 1- to 12-month ATM options. For this reason, I use (3.15) because it is more convenient for theoretical developments. In general, computing option prices as if there is no leverage effect could lead to nontrivial errors along the implied volatility smile. I argue in Appendix B that, based on a range of empirically plausible parameter values and option contract specifications, the expression (3.15) is an accurate approximation for ATM short-maturity options.

Extending the approximation formula to the full SVIJ model requires an additional assumption that the risk-neutral mean jump in returns, μ_s^Q, is equal to 0 (the derivation is provided in App. C). Although empirical results in some studies contradict such an assumption, I believe that it remains sensible for illustrative purposes. I provide sensitivity analysis for this assumption in Appendix C. The approximation (3.15) continues to perform well for the SVIJ model with a nonzero leverage effect.

3.4 The Role of the Volatility Risk Premium in Forecasting Regressions

I emphasize the presence of a volatility risk premium by rewriting the expression (3.15) as

$$\sigma_{t,\tau}^2 = E_t^P(\overline{QV}_{t+\tau,\tau}) + \underbrace{E_t^Q(\overline{QV}_{t+\tau,\tau}) - E_t^P(\overline{QV}_{t+\tau,\tau})}_{\text{volatility risk premium}} \quad (3.16)$$

and use the theoretical values of the expected average quadratic variation under both probability measures (3.10)–(3.12) to establish the contribution of the volatility risk premium explicitly as

$$\begin{aligned} &E_t^P(\overline{QV}_{t+\tau,\tau}) \\ &\quad = \sigma_{t,\tau}^2 - [A_\tau^Q - A_\tau^P]V_t - [B_\tau^Q - B_\tau^P] - \sigma_s^2[\lambda_s^Q - \lambda_s^P] \\ &\quad \equiv c^J + c_\tau^B + \sigma_{t,\tau}^2 + c_\tau^A V_t, \end{aligned} \quad (3.17)$$

with obvious notations for constants c^A, c^B, and c^J, which depend only on the option's time to maturity τ and the model parameters.

Note [see (3.11) and (3.12)] that jump risk premia affect the expectation of the future quadratic variation only through the terms c^J (jump in returns contribution) and c^B (jump in volatility contribution). Therefore, empirical values of the regression coefficients should allow one to gauge whether diffusive or jump risk is more prominent by contrasting the values of the intercept with the slope of the spot volatility.

The decomposition (3.17) implies the following regression equation:

$$\widehat{\overline{QV}}_{t+\tau,\tau} + \nu_{t+\tau} = a + b \cdot \sigma_{t,\tau}^2 + c \cdot (\hat{V}_t + \omega_t) + \varepsilon_{t+\tau}, \quad (3.18)$$

where the error terms ν and ω are an explicit recognition that neither realized nor spot volatilities are observable and, hence, are measured with an error. According to my theory, the regression slope b should be equal to 1. I can detect two potential sources of bias when the regression is implemented.

First, because V is correlated with $\sigma_{t,\tau}^2$, omitting V from the regression (3.18) would bias an estimate of b. The bias can be computed explicitly. Combine (3.10), (3.15), and (3.17) to obtain

$$\begin{aligned} E_t^P(\overline{QV}_{t+\tau,\tau}) &= c^J + c_\tau^B + \sigma_{t,\tau}^2 + c_\tau^A \frac{\sigma_{t,\tau}^2 - B_\tau^Q - \lambda_s^Q \sigma_s^2}{A_\tau^Q} \\ &= \text{intercept} + \frac{A_\tau^P}{A_\tau^Q} \sigma_{t,\tau}^2, \end{aligned} \quad (3.19)$$

which might explain the biases reported in the previous work. I will explore the size of the bias $(1 - A_\tau^P / A_\tau^Q)$ that one could expect in Section 3.5.

Second, the fact that the spot volatility is not observable complicates the estimation because it introduces the error-in-variables (EIV) problem. The EIV problem in the explanatory variable V leads to the attenuation effect (i.e., the coefficient c will be biased toward 0). Both the EIV problem and the nonzero correlation between the regressors lead to the bias in b as well. This problem will have to be taken into account when implementing regressions based on real data.

3.5 Model-Based Regression Coefficients

I conclude this section by discussing the empirically gauged, model-specific values of the coefficients in the decomposition (3.17). To illustrate what the numerical values of these coefficients could be, I compute them for a set of known structural model parameters. I use the results of Eraker et al. (2003), who estimated physical measure parameters of the affine models, using the returns on the S&P 500 and Nasdaq 100 indices from 1980 to 1999 and from 1985 to 1999, respectively. I rely on the estimates reported in Bates (2000), Eraker (2004), and Pan (2002) to come up with the risk-neutral parameter values.

Given the numerical values of the parameters, expressions (3.11), (3.12), and (3.17), I compute the values of the coefficients c^J, c_τ^B, and c_τ^A for $\tau = 22$ business days. Table 1 reports the selected parameter values for each model and each dataset, the values of the regression coefficients, and the bias $(1 - A_\tau^P / A_\tau^Q)$ resulting from the omission of V_t from the regression [see (3.19)]. As noted during the discussion of the decomposition (3.17), different types of risk premia contribute to the regression through different channels. To highlight this effect,

Table 1. Model-based regression coefficients and bias

Panel A. S&P

Model	SV			SVJ			SVIJ		
P parameters	$\theta^P = .02, \kappa^P = .02$			$\theta^P = .01, \kappa^P = .01$			$\theta^P = .01, \kappa^P = .03$ $\lambda_v^P = .0055, \mu_v^P = 1.79$		
Q parameters	$\kappa^Q = .006$			$\kappa^Q = .005$			$\kappa^Q = .005, \lambda_v^Q = .011$		
Risk premia	c^B	c^A	Bias	c^B	c^A	Bias	c^B	c^A	Bias
$V0J0$	0	0	0	0	0	0	0	0	0
$V1J0$	−.0247	−.1518	−.1622	−.0059	−.0673	−.0717	−.0401	−.1778	−.1877
$V0J1$							−.0913	0	0
$V1J1$							−.1481	−.1778	−.1877

Panel B. Nasdaq

Model	SV			SVJ			SVIJ		
P parameters	$\theta^P = .05, \kappa^P = .03$			$\theta^P = .03, \kappa^P = .02$			$\theta^P = .04, \kappa^P = .04$ $\lambda_v^P = .0140, \mu_v^P = 2.52$		
Q parameters	$\kappa^Q = .006$			$\kappa^Q = .007$			$\kappa^Q = .03, \lambda_v^Q = .0560$		
Risk premia	c^B	c^A	Bias	c^B	c^A	Bias	c^B	c^A	Bias
$V0J0$	0	0	0	0	0	0	0	0	0
$V1J0$	−.0739	−.1743	−.1868	−.0251	−.1117	−.1187	−.0299	−.0485	−.0663
$V0J1$							−.9029	0	0
$V1J1$							−.9753	−.0485	−.0663

NOTE: This table shows the impact of risk premia on the theoretical values of the regression coefficients in

$$E_t(\overline{QV}_{t+\tau,\tau}) = c^J + c^B_\tau + \sigma^2_{t,\tau} + c^A_\tau V_t$$

if the data were generated by one of the discussed models. I also compute the bias in the at-the-money implied volatility slope if V_t is omitted from the regression. The values of the parameters corresponding to the physical probability measure are obtained from Eraker et al. (2003) and are reported in the tables in appropriate places. I rely on the estimates reported in Bates (2000), Eraker (2004), and Pan (2002) to come up with the risk-neutral parameter values. Given the realistic values of σ_s, the constant $c^J = (\lambda_s^P - \lambda_s^Q)\sigma_s^2$ is very close to 0, even for very large values of return jump intensity risk premia. Therefore, for simplicity, I disregard the premium in this example. The risk premium notation $VxJy$ means that if x or y equals 0, the volatility (V) or volatility jump (J) risk premia correspondingly were assumed to be equal to 0.

I consider cases where only one type of risk is compensated in the market. Although I concentrate on the SVIJ model, I report the value of regression coefficients for the nested SV and SVJ models as well.

If there is no premium for volatility or for jumps (case $V0J0$), then, of course, all the coefficients are equal to 0, and one obtains the Hull–White case: $a = 0$, $b = 1$, and $c = 0$ in (1.1). Allowing for a modest jump risk premium (case $V0J1$) changes things slightly. Coefficient c^A is still 0, implying a unity slope of implied volatility in (1.1). The intercept coefficient c^B, however, is quite large in magnitude, indicating an important impact of a jump risk premium. On the other hand, a modest diffusive volatility risk premium is reflected in a small intercept in the case $V1J0$. A major part of the diffusive volatility premium channels through the large absolute value of the slope coefficient c^A. Combining the two risk premia in the case $V1J1$ preserves the same trade-off between the two types of volatility risk.

To better understand the interplay between the volatility and volatility jump risk premia, I intentionally assumed a very small volatility risk premium and a very large volatility jump risk premium in the case of the SVIJ model for Nasdaq. One can immediately notice the relative decrease in the absolute values of c^A and c^B because of the smaller volatility risk premium (case $V1J0$). At the same time, there is an increase in the absolute value of c^B because of the higher volatility jump risk premium (cases $V0J1$ and $V1J1$). Summarizing, a smaller volatility risk premium leads to a smaller bias in the slope of the implied volatility and a larger volatility jump risk premium increases the absolute value of the intercept.

Finally, the bias, resulting from omission of V, ranges from 15% to 20% for the leading examples SV and SVIJ. This value is on the lower end of the bias values reported in the literature (see Poteshman 2000 for a detailed comparison of the studies). Various factors, such as considered assets, sample time period, or data frequency, could account for the difference.

4. VOLATILITY ESTIMATION FOR THE PREDICTIVE REGRESSION

4.1 Empirical Implementation Design

The previous section establishes the important role played by the volatility risk premium in volatility forecasting. The decomposition of the forecast into the Black–Scholes implied volatility and the volatility risk premium is quite constructive. I can use the relationship in (3.18) to implement appropriate tests of the unbiasedness hypothesis via linear regression. As discussed previously, I need to estimate the unobservable quadratic variation and spot volatility in order to implement this regression.

A natural way to estimate QV is to rely on its definition in (3.7) and estimate it consistently by the realized volatility

RV based on daily returns [$n = 1$ in (3.6)]. One could use exactly the same construct as an, albeit more coarse, estimator of the spot variance V, that is,

$$\hat{V}_t = HV_{t,\tau} = \frac{1}{\tau}\sum_{i=0}^{\tau-1} r_{t-i}^2(1). \tag{4.1}$$

As Andersen, Bollerslev, and Diebold (2007) pointed out, this measure of spot variance is justified by the Foster–Nelson (1996) theory of rolling sample variance estimators within the diffusion setting. This theory implies that the realized volatility computed from increasingly many return observations (τ approaching ∞) over increasingly finer time intervals [n in the definition of $r_t(n)$ in (3.5) approaching ∞] is consistent for the corresponding instantaneous volatility.

Many researchers use historical volatility HV as the second predictive variable PV in the regression (1.1). Given our observation that PV must be an estimator of V, it is not necessary to limit oneself to HV. As will be shown, it is a relatively noisy estimator of the spot volatility. I propose another estimator of the spot volatility, which was largely unexplored in the predictive volatility regression literature. The high–low range-based volatility estimator

$$\hat{V}_t = R_t = \delta\Big(\max_{1\le i\le n} r_{t+i/n}(n) - \min_{1\le i\le n} r_{t+i/n}(n)\Big)^2 \tag{4.2}$$

(where n is the number of observations per day and δ is a scaling constant that ensures the unbiasedness of the estimator) is known to be very efficient in the constant volatility case from the works of Parkinson (1980) and Garman and Klaas (1980), among others. Because stochastic volatility and jumps are present in my setup and the data, the efficiency will likely be lost. This estimator requires the knowledge of prices within a day and, therefore, is formally a high-frequency estimator (I thank a referee for this point). Thus, R is expected to perform better. Indeed, Andersen and Bollerslev (1998) provided simulation evidence that, in terms of efficiency, R is comparable to an HV estimator based on a sampling frequency of between 2 and 3 hours.

Both spot volatility estimators HV and R could be biased upward because of the jumps in returns. The impact of this potential bias could be large. However, in the context of my models and empirically gauged parameter values, which imply relatively rare and state-independent jumps, the bias is likely to be small. I verify this claim by conducting a simulation study based on the SVIJ model in Section 4.2. In particular, I find that the potential upward bias in R might affect the estimate of c in (3.18), but not the estimate of the implied volatility loading b. I also implement both estimators of V based on the real data in Section 4.2.

Because I am estimating both the dependent and independent variables, I correct for the EIV problem by incorporating instrumental variables Z_t. I do this in the GMM framework with the Hansen–Hodrick (1980) error correction procedure for overlapping data. I pick a just-identified set of moment conditions based on the regression in (3.18):

$$E\big[\big(\widehat{QV}_{t+\tau,\tau} - a - b\cdot\sigma_{t,\tau}^2 - c\cdot\hat{V}_t\big)\otimes Z_t\big] = 0. \tag{4.3}$$

There are certain issues associated with the use of instrumental variables. Even if one carefully selects instrumental variables that remove the bias associated with the EIV problem, the variance of the estimators will increase. Moreover, the Monte Carlo study in Hall, Rudebusch, and Wilcox (1996) indicates that the typical procedures proposed to select a good set of instruments can actually exacerbate problems associated with potentially irrelevant instruments.

I select my instruments keeping these potential shortcomings in mind and follow the generic recommendation of trying to select the instruments Z, such that they are highly correlated with the spot volatility V, but have low, preferably zero, correlation with the measurement error ω. Besides being used as the estimators of V, HV and R could serve as instruments based on these two requirements. I gain further confidence in this selection from the simulation study conducted in Section 4.2. These instruments deliver the results expected from the theory when $\hat{V} = R$. Unfortunately, the estimation results are unstable when $\hat{V} = HV$ (I elaborate on this further in Sec. 4.2). I believe that this outcome can be attributed to the fact that HV is a very noisy estimator, as I demonstrate in my simulation study.

Another potential issue with my approach is the reliability of the GMM standard errors. Because I use daily data, there is a high degree of implied volatility autocorrelation. Asymptotically, the Hansen–Hodrick procedure is appropriate; however, as Hodrick (1992) explained, errors may be downward biased in small samples. To mitigate this issue, I use very large datasets in my study, which implies that asymptotic standard errors are likely to be appropriate.

4.2 Simulation Study

Before I proceed with the empirical study, I perform a simulation analysis (I am grateful to a referee for suggesting this study). There are two purposes for this. First, I want to show that HV is a much noisier estimator of the spot volatility than R. Second, I want to verify that by implementing the regression in (4.3), I indeed obtain the relationships prescribed by my theoretical analysis.

The simulation is based on the illustrative SVIJ model. The parameter values are provided in Table 2. The discretization interval, Δt, was selected to be equal to 1/10 of 1 day to mitigate the discretization error and to compute the range, R. The simulation is implemented using standard techniques (i.e., the Euler scheme is used for the diffusion part, and the jump component is simulated based on the Poisson distribution with parameter $\lambda\Delta t$; see Platen and Rebolledo 1985 for further details). I simulate a long series, corresponding to 4,000 or 5,000 daily observations (depending on the particular exercise) of the underlying returns and their corresponding spot volatilities.

To compare the efficiency of HV and R, I use 5,000 simulated spot volatilities and construct the historical volatility and range based on the simulated returns and formulas (4.1) and (4.2), respectively. The root mean squared error (RMSE) for HV is equal to 8.18×10^{-5}, and for R it is 3.75×10^{-5} (these numbers are very small because the regressions are implemented for daily variances). Using range as an estimator of the spot volatility results in a 54% efficiency improvement over historical volatility. As noted earlier, the reason for the efficiency

Table 2. Simulation study

Regression	$\hat{a}$	$\hat{b}$	$\hat{c}$
Full	−.071	.995	−.085
	(.007)	(.055)	(.092)
Restricted	−.053	.883	0
	(.002)	(.016)	—

NOTE: I replicate the regression results in a simulation study. My base "true" SVIJ model has the following dynamics under the P measure:

$$\frac{dS_t}{S_t} = \mu\, dt + \sqrt{V_t}\, dW_{st}(P) + \left(e^{J_{st}(P)} - 1\right) dN_{st}(P),$$

$$dV_t = (\theta^P - \kappa^P V_t)\, dt + \sigma_v \sqrt{V_t}\, dW_{vt}(P) + J_{vt}(P)\, dN_{vt}(P),$$

with the following daily parameter values based on the studies by Eraker et al. (2003) and Eraker (2004): $\mu = .04\%$ (12% per year), $r = .01\%$ (3% per year), $\theta^P = .01$, $\kappa^P = .03$, $\sigma_v = .08$, $\rho = -.5$, $\lambda_s^P = .0046$, $\mu_s^P = -3\%$, $\sigma_s = 3\%$, $\lambda_v^P = .0055$, and $\mu_v^P = 1.79$. The risk-neutral values are $\kappa^Q = .006$, $\lambda_s^Q = .0046$, $\mu_s^Q = 0\%$, $\lambda_v^Q = .011$, and $\mu_v^Q = 1.79$. I report the estimation results for the following regression using the simulated data:

$$E\left[\left(RV_{t+\tau,\tau}^{(1)} - a - b \cdot \sigma_{t,\tau}^2 - c \cdot R_t\right) \otimes (1, HV_{t,22}, R_t)'\right] = 0.$$

I also estimate a restricted version of the regression by using the implied volatility only. Because I omit the estimate of spot volatility here, there is no need to use instrumental variables for this regression.

improvement is that the estimator R relies on the intraday information.

To check the regression results, I implement the analysis described in Section 4.1 500 times. Each time, I simulate 4,000 daily observations, which correspond to the length of my typical dataset. For each simulated day, I compute the ATM option price according to the results developed in Duffie et al. (2000). Then assuming that the computed price is the market price in my simulation implies the Black–Scholes volatility. Computation of other regression terms is more straightforward. I simply compute the estimates of variance and quadratic variation from the simulated returns.

I implement two types of regressions for each simulated path. The first one is the full regression described in (4.3) with $\hat{V}_t = R_t$, and $Z_t = (1, HV_{t,22}, R_t)'$. The second regression omits the spot volatility (i.e., ignores the volatility risk premium). Because I omit the spot volatility V here, there is no need to use instrumental variables for this regression. I collect the estimates of the loading on the Black–Scholes implied volatility, $\hat{b}$, from both regressions. Because I repeat this procedure 500 times, I am able to compute the average regression coefficient and its standard deviation. Table 2 reports the results.

For the full regression (4.3), the average $\hat{b}$ is equal to .995 with a standard deviation of .055 and the average $\hat{c}$ is equal to −.085 with a standard deviation of .092. For the reduced regression, the average $\hat{b}$ is equal to .883 with a standard deviation of .016. Thus, I confirm both the bias introduced by the omission of the volatility risk component and the absence of bias, if the risk premium is properly accounted for. I simulated prices with the leverage effect and computed implied volatility without it. Hence, my results verify that in addition to other sensitivity checks reported throughout the article, the estimate of $\hat{b}$ is not driven by the leverage effect. Possible upward bias in the estimate of V acknowledged earlier does not seem to have an impact either, at least within my simulation design. Note that the difference between the two regression coefficients is only marginally significant. The reason for this outcome is that the estimate of b is much noisier in the full regression due to the use of the instrumental variables. Finally, I observe that the absolute value of $\hat{c}$ is smaller than the theoretical value reported in Table 1, though the two values cannot be distinguished statistically. Most likely, the upward bias in R is responsible for this outcome.

5. EMPIRICAL ILLUSTRATION

5.1 Data

I illustrate the theoretical results of the article by implementing the forecasting regressions for several assets prominent in the literature: volatilities implied from U.S. equity indices and FX rates. I want to concentrate on the contribution of the volatility risk premium and problems associated with the unobservability of the spot volatility to the biases in the forecasting regression. For this reason, I construct a dataset that does not have other issues affecting the unbiasedness results, such as errors in computing implied volatilities, telescoping time to maturity, and varying moneyness [Christensen and Prabhala (1998), Fleming (1998), Poteshman (2000), and Christensen, Hansen, and Prabhala (2001) pointed out these problems and suggested econometric techniques to resolve them].

In particular, I use the old VIX (VXO after September 22, 2003) and VXN, which are CBOE market volatility indices. They are constructed so that the level of the volatility index is equal to the implied volatility from a synthetic option contract on either S&P 100 (OEX) or Nasdaq 100 (NDX) that always has $\tau = 1$ month to maturity and is exactly at the money. Whaley (2000) provided additional details. Mixon (2001) points out that the two indices constructed by CBOE do not have an immediate interpretation as either volatility per trading day or volatility per calendar day. I rescale the data to obtain volatility per calendar day. I also use 1-month ATM implied volatilities for the U.S. dollar exchange rate with the GBP, JPY, and CHF, which were provided by a major New York investment bank. I do not use the new VIX index, which is a model-free volatility concept (see, for instance, Britten-Jones and Neuberger 2000), because this article focuses on the traditional implied volatility concept and because the new measure is not available for all the different markets that we study.

The final piece of the dataset is the daily high–low prices, which are required to construct the range estimate of the spot volatility. The equity indices data come from Yahoo!Finance. The FX spot high–low data are hard to obtain. Because the difference in ranges should be negligible, I use futures rather than the spot data. I obtained the data from the Institute for Financial Markets and used contracts closest to maturity with at least 2 weeks to delivery to mitigate the difference between the spot range and futures range.

I use a set of series, each of which starts at different points in time and ends on the same date, June 29, 2001. Because the frequency of the data is daily, I have a substantial number of observations even for the relatively small NDX sample. Figure 1 plots the time series of the underlying and implied volatilities. Table 3 reports the sample information. Both the underlying and volatility series suggest the presence of jumps in both returns and volatility. The measures of skewness and kurtosis support this observation.

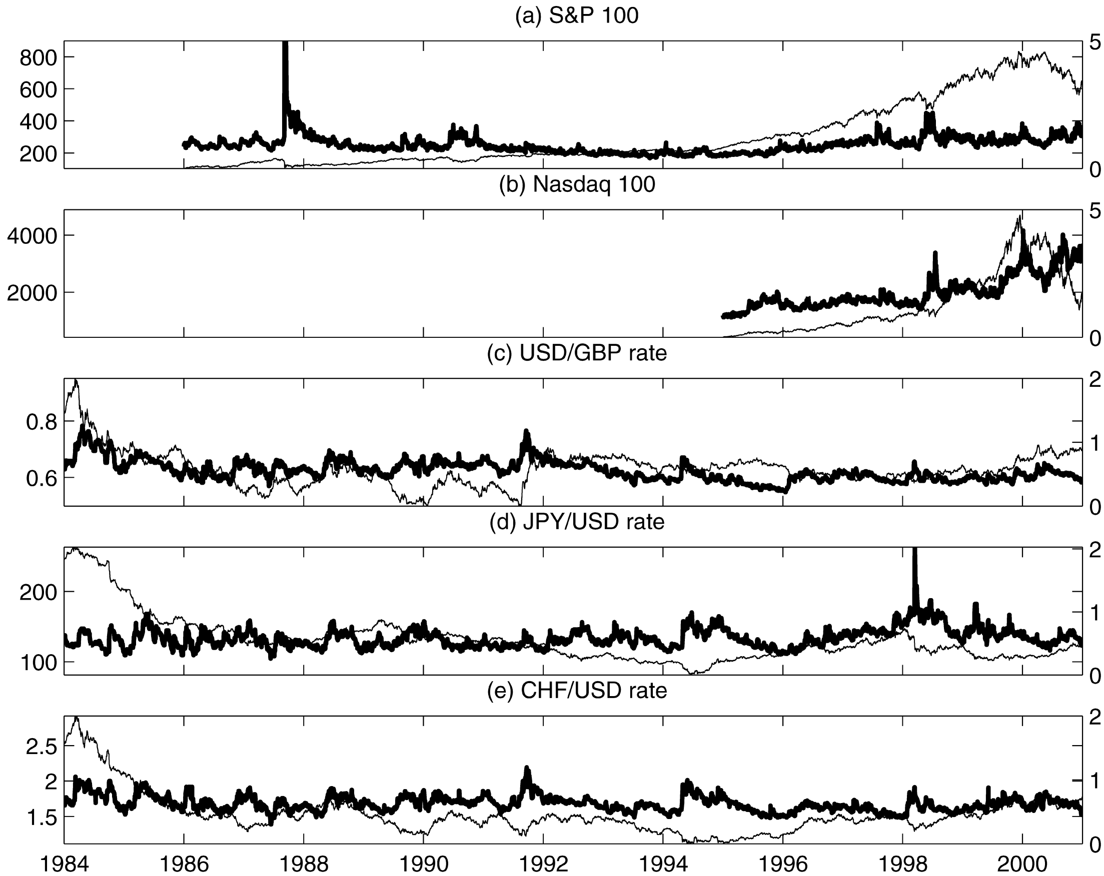

Figure 1. Data series used in the empirical example. The thin line depicts the underlying asset series with the scale on the left. The thick line depicts the implied volatility, expressed in percentages per day, with the scale on the right.

5.2 Results

Table 4 reports the estimates of the regression based on (3.18). Columns 2 and 3 report the ordinary least squares (OLS) estimates with the coefficient c constrained to be equal to 0 (i.e., assuming informational efficiency of the implied volatility) and unconstrained, respectively. The theoretical developments show that such OLS estimates are not appropriate. However, I report them to benchmark my results against traditional studies (that is why I limit OLS to the case $\hat{V}_t = HV_{t,22}$ only). Columns 4 and 5 report GMM estimates corresponding to two different estimators of the spot volatility. The GMM results are accompanied by the Wald test of the joint restriction implied by the volatility unbiasedness hypothesis, $a = 0$ and $b = 1$.

The OLS results are consistent with the literature; the unbiasedness and informational efficiency hypotheses are rejected in most cases. The exceptions are NDX and GBP. In these cases, the implied volatility loading is roughly equal to 1 when the state-varying part of the volatility risk premium is assumed to be equal to 0. This result, combined with the decomposition (3.17), suggests that NDX and GBP reflect only the constant volatility jump risk premium. This conjecture will be verified in a more rigorous GMM setting.

For the GMM results with $\hat{V} = HV$, I get mixed results. On the one hand, the implied volatility slope coefficient estimate $\hat{b}$ is almost exactly equal to 1, as predicted by (3.17). On the other hand, the estimates are very imprecise. This aspect is not surprising because the unobservability of both QV and V adds

Table 3. Sample statistics

Series	Dates	Sample size	Changes in underlying: Mean	Std. dev.	Skewness	Kurtosis	Changes in volatility: Mean	Std. dev.	Skewness	Kurtosis
OEX	01/02/86–06/29/01	3,906	.04	1.14	−2.57	56.23	.0023	.07	16.72	644.97
NDX	01/03/95–06/29/01	1,634	.09	2.38	.09	6.69	.0016	.04	.45	5.42
GBP	10/05/84–06/29/01	4,014	−.00	.66	−.10	8.72	.0007	.04	1.85	15.99
JPY	10/05/84–06/29/01	4,014	−.01	.72	−.54	7.43	.0014	.05	1.86	15.15
CHF	10/05/84–06/29/01	4,014	−.00	.76	−.22	5.95	.0007	.04	1.61	15.85

NOTE: I report the sample period and sample statistics for the five time series used in the article. Because my illustrative model describes the dynamics of the relative changes in prices and level changes in volatilities, I compute the statistics for the logarithmic changes in the underlying assets and for the simple relative changes in the respective implied volatilities.

Table 4. Estimation of the volatility forecasting regression

Parameter	OLS, $c = 0$	OLS, $\hat{V}_t = HV_{t,22}$	GMM, $\hat{V}_t = HV_{t,22}$	GMM, $\hat{V}_t = R_t$
OEX			$Z_t = (1, HV_{t-24,22}, \sigma^2_{t-46,22})$	$Z_t = (1, HV_{t,22}, R_t)$
$\hat{a}$	.6117	.6177	.1035	−.3513
s.e.	(.0457)	(.0458)	(.1839)	(.1070)
$\hat{b}$	.3258	.2823	.9907	1.2307
s.e.	(.0267)	(.0369)	(1.0813)	(.1242)
$\hat{c}$	0	.0388	−.1384	−.8790
s.e.	—	(.0227)	(1.2082)	(.1884)
Wald test			.65	14.01
NDX			$Z_t = (1, r^2_{t-16}(1), HV_{t-26,22})$	$Z_t = (1, HV_{t,22}, R_t)$
$\hat{a}$	−.7400	−.5591	−.0909	−.1679
s.e.	(.0485)	(.0455)	(.5769)	(.2734)
$\hat{b}$	1.0598	.7757	1.0381	1.2081
s.e.	(.0105)	(.0190)	(.3503)	(.0898)
$\hat{c}$	0	.2059	.1495	.1034
s.e.	—	(.0119)	(.1837)	(.1011)
Wald test			.02	6.05
GBP			$Z_t = (1, HV_{t-22,22}, HV_{t-44,22})$	$Z_t = (1, HV_{t,22}, R_t)$
$\hat{a}$	.0056	.0055	.0277	−.1095
s.e.	(.0076)	(.0081)	(.0472)	(.0225)
$\hat{b}$	.9481	.9496	1.1121	1.0401
s.e.	(.0197)	(.0367)	(.4127)	(.0840)
$\hat{c}$	0	−.0012	−.2359	.0109
s.e.	—	(.0230)	(.3545)	(.0114)
Wald test			1.77	165.87
JPY			$Z_t = (1, r^2_{t-25}(1), \sigma^2_{t-46,22})$	$Z_t = (1, HV_{t,22}, R_t)$
$\hat{a}$	.1020	.1137	.1099	−.1452
s.e.	(.0096)	(.0096)	(.0919)	(.0337)
$\hat{b}$	.7644	.5523	1.0141	1.1910
s.e.	(.0227)	(.0343)	(1.1154)	(.1110)
$\hat{c}$	0	.1735	−.2608	−.0414
s.e.	—	(.0212)	(.8025)	(.0284)
Wald test			9.19	47.55
CHF			$Z_t = (1, \sigma^2_{t-38,22}, HV_{t-42,22})$	$Z_t = (1, HV_{t,22}, R_t)$
$\hat{a}$	.0985	.1233	.2372	−.2841
s.e.	(.0116)	(.0121)	(.1497)	(.1158)
$\hat{b}$	.8088	.5849	1.0464	1.5387
s.e.	(.0264)	(.0424)	(1.2423)	(.3331)
$\hat{c}$	0	.1542	−.5467	−.0353
s.e.	—	(.0230)	(1.0303)	(.0296)
Wald test			3.36	65.38

NOTE: I report the estimation results for the regression:

$$E[(\widehat{QV}_{t+\tau,\tau} - a - b \cdot \sigma^2_{t,\tau} - c \cdot \hat{V}_t) \otimes Z_t] = 0.$$

Columns 2 and 3 report the standard OLS estimation with $c = 0$ and unconstrained c, respectively. In the last two columns, the regression is estimated via GMM using the Hansen–Hodrick standard errors. The two columns correspond to two different estimators of the spot volatility V_t: historical volatility ($HV_{t,\tau}$) and range (R_t). Instrumental variables Z_t correct for the EIV problem. I also report a Wald test of the joint efficiency hypothesis that $a = 0$ and $b = 1$ (the 5% critical value is 5.99).

a lot of noise [recall the error terms ν and ω in the regression (3.18)]. Their variability will naturally affect the regression standard errors. In particular, all the Wald tests, with the exception of the JPY, fail to reject the unbiasedness hypothesis due to this effect.

Moreover, my results indicate that it is very hard to establish a consistent set of instruments in this case. They were selected by trial and error: Other choices, which were based on various combinations of r^2, σ^2, HV, and R with different lags, produced unusual results with either negative slopes or huge standard errors. It is difficult to develop a systematic approach for testing the forecasting ability of the implied volatility when HV is used as an additional predictive variable.

The use of the range-based estimator R of V resolves this problem. Indeed, the standard errors of the estimates reported in the last column decrease by a dramatic amount. This happens because R is a more efficient estimator due to its intraday nature. The instrumental variables are the same for all assets in this case. This feature gives additional confidence in the reliability of the results. Now all implied volatility loadings are significantly different from 0, are not significantly different from 1; and are biased upward. The Wald tests reject the unbiasedness hypothesis consistent with these findings.

However, the standard errors for the loading on implied volatility, b, are quite large. I can think of two possible explanations for this outcome. First, extra noise introduced by the instrumental variables inflates the standard errors. This observation is consistent with the results of my simulation study. It appears that one must face a trade-off between the bias in the forecasting of future volatility and the precision of the forecast. Second, it could be hard to identify a diffusive volatility risk premium from a short-maturity at-the-money implied volatility only.

For OEX, the slope coefficient $\hat{c}$, which reflects the volatility risk premium, is negative. This aspect is consistent with the sign of the theoretical values of this coefficient reported in Table 1. The magnitude of the regression coefficients is qualitatively similar to those computed based on the S&P gauged parameter values in the SVIJ model in Table 1, panel A.

For NDX, the value of $\hat{c}$ is positive and not significantly different from 0, indicating a potentially zero volatility risk premium. The small magnitude of the risk premium could be explained by the different nature of the Nasdaq data-generating process documented by Eraker et al. (2003): The jumps in NDX are much larger in magnitude and are more frequent. Schwert (2002) provided additional highlights of the VXN "jumpiness" by comparing VXN, VIX, and RV's computed based on NDX and OEX. Therefore, in the case of Nasdaq, a jump in the volatility risk premium may play a much more important role than a diffusive volatility risk premium. This interpretation is consistent with empirically gauged coefficient values in the SVIJ model, case $V0J1$ in Table 1, panel B.

Moving on to the FX results, I find that the coefficient c is negative for JPY and CHF and positive for GBP, but all of these values are insignificant. There is a difference between the currencies, however. If there is no volatility risk premium, the loading on the implied volatility should be equal to 1, even if we implement a simple OLS. The GBP, similarly to NDX, has this property. I interpret this as evidence that only the risk of a volatility jump is compensated for the GBP market. The JPY and CHF have large biases in the OLS case, so I would interpret the results as a lack of precision caused by instrumental variables. Under this view, the JPY and CHF are compensated for diffusive and jump risks in volatility; however, the respective risk premia are quite modest.

What could explain the differences in the risk premia for these currencies? The CHF is known to be a very stable currency. Nothing in the Swiss economy, its government policy, or its history would suggest dramatic changes in volatility, or in the exchange rate itself. The same can be said about the JPY, but for a different reason: The Japanese government is known to intervene on a regular basis, so that a stable JPY would help maintain the trade balance. Therefore, it is natural to expect that the implicit stochastic discount factor has a low correlation with the volatilities of these currencies. As a result, modest diffusive volatility risk premia are to be expected for these two currencies.

The situation with the GBP is more complicated because the start of my sample coincides with the beginning of a political movement to promote the euro. This movement led to the setup of the exchange rate mechanism and later to its crisis in 1992. The GBP devaluation had the same impact on the FX markets as the October 1987 crash had on the equity markets. Hence, it is not surprising that options prices reflect risk premia for extreme events.

Overall, my empirical findings conform to my theoretical results. Interestingly, with the exception of the S&P 100, the volatility jump risk seems to play a much greater role than the diffusive volatility risk. This finding indicates that further exploration of the jump risk premia is a promising avenue of research.

6. CONCLUSION

This article unifies two strands of the options literature. One strand attempts to forecast future volatility based on the at-the-money implied volatility. The other strand develops and estimates option pricing models. I use flexible jump–diffusion models and the Hull–White (1987) option valuation approach to derive the relationship between the future quadratic variation of returns and the Black–Scholes implied volatility.

Following the conjecture of Lamoureux and Lastrapes (1993) I explicitly account for the volatility risk premia. I find that these premia explain previously documented biases in volatility forecasts. If investors demand premia for these risks, the standard volatility forecasting regression obtains an additional term that is a linear function of the latent spot volatility. Omitting this term biases the slope of the implied volatility downward. Thus, I provide an economic justification for the additional predictive variable often used in forecasting regressions in an ad hoc fashion.

I estimate the spot volatility in two different ways: via historical volatility and the high–low range. The range estimator proves to be much more efficient both in simulations and in real data because of its reliance on the intraday information. The need to estimate the additional predictor introduces errors in the variables. If this problem is not appropriately accounted for, it can generate additional biases and decrease the precision of the regression estimates. I address the issue by using instrumental variables in the GMM estimation framework.

I illustrate my theoretical discussion using volatilities implied from 1-month at-the-money options on two U.S. equity indices (S&P 100 and Nasdaq 100) and three foreign currency exchange rates (British pound, Japanese yen, and Swiss franc). This special feature of the datasets removes multiple econometric problems advanced in earlier studies as possible causes for forecasting biases and allows one to focus on the contribution of the volatility risk premia. The implied volatility bias disappears after appropriate implementation of the regressions. Interestingly, the obtained regression coefficient values suggest the prominence of the volatility jump risk premium in all asset prices with the exception of the S&P 100, where the diffusive volatility risk premium plays an important role as well.

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APPENDIX A: EXPECTED VALUE OF THE AVERAGE INTEGRATED VOLATILITY

I am interested in deriving the formula (3.10) for the expected average quadratic variation. This result hinges on the expression for $E_t^M(\bar{V}_{t+\tau,\tau})$, where V_t follows a jump–diffusion process with linear drift, such as (3.2) or (3.4) under some probability measure M. In the course of the derivation, the superscript M is dropped to avoid cumbersome notation.

Following the standard approach, I first apply Itô's lemma to $\exp(\kappa t)V_t$:

$$\begin{aligned} d(e^{\kappa t}V_t) &= \kappa e^{\kappa t} V_t\,dt + e^{\kappa t}\,dV_t \\ &= \theta e^{\kappa t}\,dt + e^{\kappa t}\sigma(V_t)\,dW_{vt} + e^{\kappa t}J_{vt}\,dN_{vt}. \end{aligned} \tag{A.1}$$

Now, computing V_s for $s \geq t$,

$$\begin{aligned} V_s &= e^{-\kappa(s-t)}V_t + \theta e^{-\kappa s}\int_t^s e^{\kappa u}\,du \\ &\quad + e^{-\kappa s}\int_t^s e^{\kappa u}\sigma(V_u)\,dW_{vu} + e^{-\kappa s}\int_t^s e^{\kappa u}J_{vu}\,dN_{vu}. \end{aligned} \tag{A.2}$$

Therefore,

$$\begin{aligned} E_t(V_s) &= e^{-\kappa(s-t)}V_t + \theta e^{-\kappa s}\int_t^s e^{\kappa u}\,du \\ &\quad + e^{-\kappa s}\int_t^s e^{\kappa u}E_t(J_{vu}\,dN_{vu}) \\ &= e^{-\kappa(s-t)}V_t + \frac{\theta}{\kappa}\big(1-e^{-\kappa(s-t)}\big) + \frac{\lambda_v\mu_v}{\kappa}\big(1-e^{-\kappa(s-t)}\big). \end{aligned} \tag{A.3}$$

After computing the conditional expectation, the diffusion term drops out. Therefore, its functional form is irrelevant for my results.

Now, using Fubini's formula, I can compute the expected value of integrated volatility:

$$E_t(\bar{V}_{t+\tau,\tau}) = \frac{1}{\tau}\int_t^{t+\tau} E_t(V_u)\,du. \tag{A.4}$$

I then compute the expected average quadratic variation:

$$\begin{aligned} E_t(\overline{QV}_{t+\tau,\tau}) &= E_t(\bar{V}_{t+\tau,\tau}) + \frac{1}{\tau}E_t\bigg(\int_{\mathbb{R}\backslash 0} x^2 N_\tau(dx)\bigg) \\ &= E_t(\bar{V}_{t+\tau,\tau}) + \int_{-\infty}^{\infty} x^2\lambda_s\frac{1}{\sqrt{2\pi\sigma_s^2}}e^{-x^2/2\sigma_s^2}\,dx \\ &= E_t(\bar{V}_{t+\tau,\tau}) + \lambda_s\sigma_s^2. \end{aligned} \tag{A.5}$$

Finally, (A.5) and (A.3) imply the formulas (3.10)–(3.12).

APPENDIX B: THE LEVERAGE EFFECT AND AT–THE–MONEY OPTIONS IN THE SV MODEL

Romano and Touzi (1997) and Willard (1997) generalized the Hull–White formula for the case of nonzero correlation to

$$\begin{aligned} &C^{\mathrm{W}}(S_t, V_t, r, K, \tau) \\ &\quad = E_t^Q\big\{C^{\mathrm{BS}}\big(S_t\xi_t, (1-\rho^2)\overline{QV}_{t+\tau,\tau}, r, K, \tau\big)\big\}, \end{aligned} \tag{B.1}$$

where

$$\xi_t = \exp\bigg(\rho\int_t^{t+\tau}\sqrt{V_u}\,dW_{vu}(Q) - \frac{1}{2}\rho^2 QV_{t+\tau,\tau}\bigg). \tag{B.2}$$

Garcia, Ghysels, and Renault (2007) reviewed this result in detail and linked it to the expectations taken under the P measure. Because ξ_t is a random variable, this option pricing formula cannot be inverted easily as was done in the zero-correlation case. Two approaches can be used to establish the relationship between implied volatility and quadratic variation, either numerical examples based on the empirically plausible parameter values or analytical approximation.

I first illustrate this issue by using numerical examples. Figure B.1 reports the implied volatility smiles for the SV and SVIJ models. For each model, I depict the smiles based on two sets of parameters, which differ only in the value of the leverage effect. The parameters for the SV model are obtained from Duffie (2001). The values are extreme in the sense that they are obtained by calibrating the model to a cross section of option prices and by completely ignoring the underlying asset dynamics. The parameters from the SVIJ model are obtained from Eraker et al. (2003), who obtained the values based exclusively on the time series of the underlying returns. It is clear from the picture that the overall impact of the leverage effect on the smile could be quite dramatic; however, there is no change in the value of the at-the-money option.

Two analytical approximations of (B.1) are available. The first is from appendix A.2.2 in Garcia, Lewis, and Renault (2001), in which they constructed an approximation around the zero value of the volatility of volatility coefficient σ_v (approximation I hereafter). This approximation implies that proceeding as if there is no leverage effect [i.e., using (3.15)], is first-order accurate for short-maturity at-the-money options when the value of σ_v is sufficiently small. The second is from equation 5.18 in Comte, Coutin, and Renault (2003), in which they constructed an approximation around the zero value of the correlation coefficient ρ (approximation II hereafter), which implies that the scaling of quadratic variation in the formula (3.15) by $(1-\rho^2)$ [i.e., ignoring the random variable ξ_t

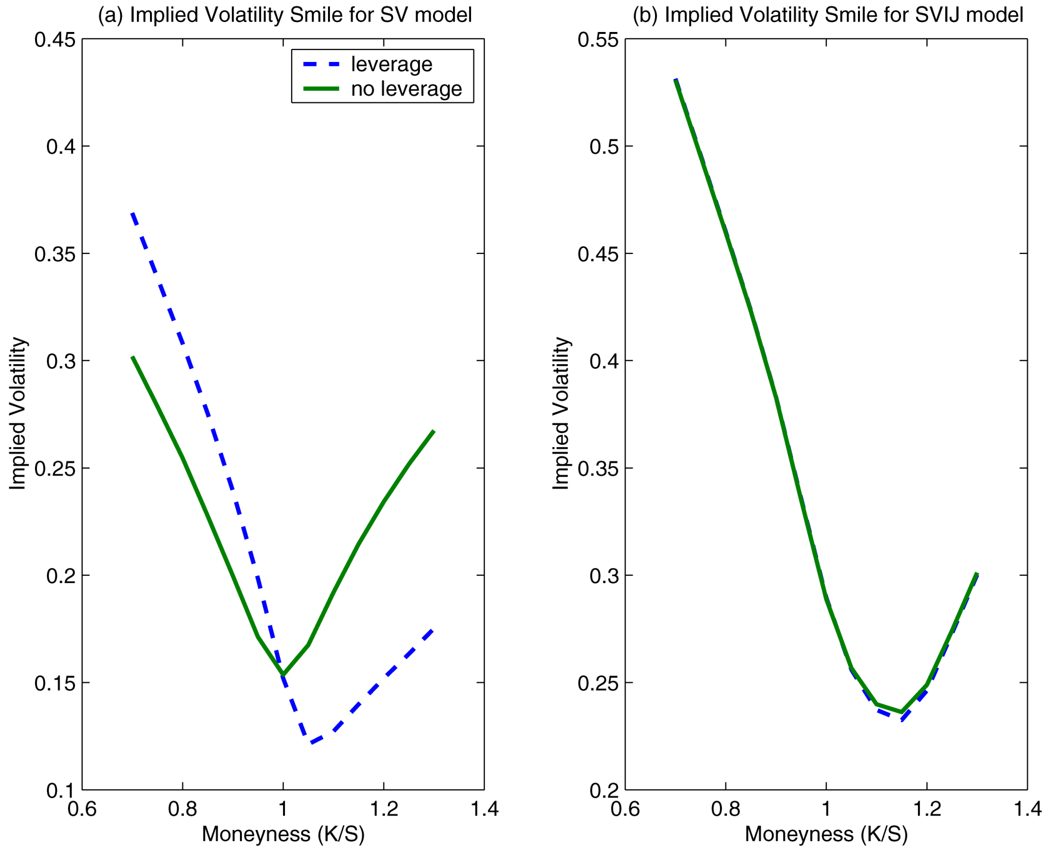

Figure B.1. Impact of the leverage effect on at-the-money option pricing. This figure depicts the impact of the leverage effect omission on implied volatilities of the at-the-money options. Panel (a) shows implied volatility smiles for an affine stochastic volatility model (SV). I use parameter values from Duffie (2001), which were obtained by calibrating the model to the entire cross section of S&P 500 option prices on one day (11/02/1993). The parameter values are $\kappa^Q = .09$, $\theta = .05$, and $\sigma_v = 1.52$, and the leverage effect is equal to $-.66$. Panel (b) shows implied volatility smiles for an affine stochastic volatility with jumps in returns and volatility (SVIJ). I use parameter values from Eraker et al. (2003), who estimated the model based on a time series of the underlying S&P 500 index from 1980 to 1999. The parameter values are $\kappa = .03$, $\theta = .04$, $\sigma_v = .09$, $\lambda_s = .0046$, $\mu_s = -.03$, $\sigma_s = .03$, $\lambda_v = .0055$, and $\mu_v = 1.79$, and the leverage effect is equal to $-.50$. Note that whether or not the leverage is switched off, the implied volatilities for at-the-money options are identical.

and inverting the formula (B.1)] is first-order accurate for options with short maturity and any strike when the value of ρ is sufficiently small (I am grateful to a referee for bringing this approximation to my attention). Both approximations hinge on the undesirable assumptions of a model being very close to either the constant volatility Black–Scholes case or the zero-leverage Hull–White case. Because the approximations are asymptotic in particular parameter values, the practical performance is what ultimately establishes the more accurate approximation.

Normally, in order to evaluate either of the two approximations, one has to compute the correct value of the implied volatility based on a model and compare it to the theoretical value of the expected average quadratic variation (scaled appropriately by correlation if necessary). Because the value of expected average quadratic variation does not depend on the magnitude of the leverage effect, I can make a slightly stronger statement in the case of approximation I. I will check whether for sufficiently small values of σ_v approximately at-the-money options have the same implied volatility, regardless of the magnitude of the leverage effect. This condition, if true, will imply effectiveness of the approximation (3.15).

Table B.1 reports the comparison results based on empirically plausible parameter values taken from the studies by Eraker et al. (2003) and Eraker (2004). The magnitude of implied volatility, and possibly the approximation error, will depend on the initial value of the instantaneous variance V_t. I consider three scenarios: V_t is below the long-run mean, at the long-run mean, or above the long-run mean. I also consider two maturities: 1 month and 3 months.

In the case of approximation I in panel A, I start with a value of $\sigma_v = .08$, which corresponds to its estimate based on daily S&P 500 returns from 1980 to 1999. As indicated by the relative error, there are minimal differences between the implied volatilities of options with no leverage effect and the empirically plausible leverage effect. The results do not change much as I start increasing σ_v up to 10 times its estimated value, although there is some deterioration in the quality of approximation for larger values of σ_v, longer maturities, and low starting values of the instantaneous variance. I also took the computations to another extreme and computed the approximation bias in the worst case, according to panel A, that is, the largest value of σ_v and the lowest initial volatility value. In addition, I assumed a very high leverage effect, $\rho = -.95$. The resulting relative error is -1.08% and 1.06% for the 1-month and 3-month maturities, respectively. I conclude that approximation I is of

Table B.1. Leverage effect in the SV model

Panel A. Approximation based on small values of σ_v

		Implied volatility				Relative error	
		$\rho=-.5$		$\rho=0$			
σ_v	$\sqrt{V_t}$	1 month	3 months	1 month	3 months	1 month	3 months
.08	$.75\sqrt{\bar V}$	12.12%	12.51%	12.12%	12.51%	−.01%	−.01%
	$\sqrt{\bar V}$	15.86%	15.84%	15.86%	15.84%	.00%	−.01%
	$1.25\sqrt{\bar V}$	19.65%	19.30%	19.65%	19.30%	.00%	−.01%
.24	$.75\sqrt{\bar V}$	12.09%	12.44%	12.04%	12.33%	−.42%	−.92%
	$\sqrt{\bar V}$	15.79%	15.67%	15.76%	15.59%	−.21%	−.52%
	$1.25\sqrt{\bar V}$	23.34%	22.49%	23.33%	22.47%	−.03%	−.10%
.40	$.75\sqrt{\bar V}$	11.87%	11.97%	11.78%	11.81%	−.73%	−1.30%
	$\sqrt{\bar V}$	15.62%	15.24%	15.56%	15.12%	−.38%	−.79%
	$1.25\sqrt{\bar V}$	23.21%	22.14%	23.19%	22.10%	−.07%	−.19%
.80	$.75\sqrt{\bar V}$	10.88%	10.40%	10.80%	10.29%	−.79%	−1.07%
	$\sqrt{\bar V}$	14.76%	13.58%	14.68%	13.52%	−.53%	−.44%
	$1.25\sqrt{\bar V}$	22.58%	20.58%	22.54%	20.60%	−.18%	.10%

Panel B. Approximation based on small values of ρ

		Implied volatility		$\sqrt{(1-\rho^2)E_t^Q(\overline{QV})}$		Relative error	
ρ	$\sqrt{V_t}$	1 month	3 months	1 month	3 months	1 month	3 months
−.95	$.75\sqrt{\bar V}$	12.12%	12.51%	10.51%	10.87%	−68.74%	−68.69%
	$\sqrt{\bar V}$	15.86%	15.84%	13.75%	13.75%	−68.75%	−68.72%
	$1.25\sqrt{\bar V}$	19.65%	19.30%	17.03%	16.74%	−68.76%	−68.74%
−.25	$.75\sqrt{\bar V}$	12.12%	12.51%	11.75%	12.15%	−3.04%	−2.87%
	$\sqrt{\bar V}$	15.86%	15.84%	15.37%	15.37%	−3.10%	−2.97%
	$1.25\sqrt{\bar V}$	19.65%	19.30%	19.04%	18.71%	−3.12%	−3.04%
−.10	$.75\sqrt{\bar V}$	12.12%	12.51%	12.08%	12.48%	−.37%	−.18%
	$\sqrt{\bar V}$	15.86%	15.84%	15.79%	15.79%	−.42%	−.29%
	$1.25\sqrt{\bar V}$	19.65%	19.30%	19.56%	19.23%	−.45%	−.36%
.00	$.75\sqrt{\bar V}$	12.12%	12.51%	12.14%	12.55%	.14%	.32%
	$\sqrt{\bar V}$	15.86%	15.84%	15.87%	15.87%	.08%	.21%
	$1.25\sqrt{\bar V}$	19.65%	19.30%	19.66%	19.33%	.05%	.15%

NOTE: I evaluate the quality of at-the-money option price approximations in the affine SV model in the presence of the leverage effect, $\rho \neq 0$. The first approximation implies that, for sufficiently small values of the volatility of volatility coefficient σ_v, the at-the-money option implied volatility could be approximated as

$$\sigma_{t,\tau}^2 \approx E_t^Q(\overline{QV}_{t+\tau,\tau}).$$

Panel A evaluates the quality of this approximation for a range of values of σ_v. The second approximation implies that, for sufficiently small values of the correlation coefficient ρ,

$$\sigma_{t,\tau}^2 \approx (1-\rho^2)E_t^Q(\overline{QV}_{t+\tau,\tau})$$

for any option's strike. Panel B evaluates the quality of this approximation for a range of values of ρ. The base "true" SV model has the following dynamics under the Q measure:

$$\frac{dS_t}{S_t} = r\,dt + \sqrt{V_t}\,dW_{st}(Q),$$

$$dV_t = (\theta^P - \kappa^Q V_t)\,dt + \sigma_v\sqrt{V_t}\,dW_{vt}(Q),$$

with the following parameter values based on the studies by Eraker et al. (2003) and Eraker (2004): $r = .01\%$ (3% per year), $\theta^P = .02$, $\kappa^Q = .006$, and $\sigma_v = .08$. In this case, the long-run variance mean $\bar V$ is equal to θ^P/κ^P, so I assume that $\kappa^P = .02$.

high quality and is robust to substantive deviations from the theoretical prescriptions of short maturity and small σ_v.

Panel B of the table evaluates the quality of approximation II. I directly compare theoretical implied variance to expected average quadratic variation scaled by $1-\rho^2$ (note that in order to relate the comparison to the results for approximation I, I report the implied volatility and the square root of the scaled expected average quadratic variation). I see that the quality of this approximation deteriorates quite quickly as the absolute values of ρ increase. When the correlation is equal to $-.25$, the errors are triple those of the largest ones encountered for approximation I. When I reach the empirically plausible value of $-.5$ (unreported for brevity), the relative errors exceed 10%. As in the case of approximation I, I take the leverage effect to an extreme and report the relative error for $\rho = -.95$. In contrast to the former approximation, the error is huge, at -68%.

Regardless of the value of ρ, the implied volatility values are the same, providing another illustration that the leverage effect does not affect the at-the-money options. The relative error increases proportionately to the inverse of the square root of $1 - \rho^2$. Thus, approximation I appears to be more appropriate for at-the-money options. At-the-money implied volatilities determine the level of the implied volatility smile and, as such, are related to the corresponding risk premia and the volatility level. Hence, they will not be significantly affected by changes in the leverage in contrast to the rest of the smile. Therefore, the approximation will work well for assets with both pronounced leverage, such as equity indices, and minimal leverage, such as currency rates. Approximation II promises to deliver accurate estimates for the whole range of strikes, but with a small leverage effect.

Approximation I continues to perform well for the SVIJ model with a nonzero leverage effect. The comparison of the two approximations for this model is available on the Web (*www.gsb.columbia.edu/faculty/mchernov/tableB2.pdf*).

APPENDIX C: EXTENSION OF THE HULL–WHITE FORMULA TO JUMPS

I want to show that the approximation (3.15) holds for models with jumps. I start the discussion with the Merton (1976) constant volatility model. This model is nested in the SVIJ model (3.1)–(3.2).

Merton (1976) expressed the call option price as

$$\begin{aligned}
&C^{\mathrm{M}}(S_t, V, r, K, \tau) \\
&= \sum_{n=0}^{\infty} \frac{e^{-\lambda_s^Q e^{\mu_s^Q} \tau} (\lambda_s^Q e^{\mu_s^Q} \tau)^n}{n!} \\
&\quad \times C^{\mathrm{BS}}\left(S_t, V + \frac{n\sigma_s^2}{\tau}, r - \lambda_s^Q \mu_s^Q + \frac{n \log(1+\mu_s^Q)}{\tau}, K, \tau\right) \\
&= E_t^Q\left\{C^{\mathrm{BS}}\left(S_t, V + \frac{n\sigma_s^2}{\tau}, r, K, \tau\right)\right\}, \qquad \text{(C.1)}
\end{aligned}$$

where expectation is taken with respect to the distribution of the number of jumps, n, over the interval τ. The second equality holds if μ_s^Q is equal to 0 (Merton 1976, footnote 13). This formula implies that, for at-the-money options,

$$E_t^Q\left(V + \frac{n\sigma_s^2}{\tau}\right) = \sigma_{t,\tau}^2. \qquad \text{(C.2)}$$

The expression in the left side of the formula is equal to the expected average quadratic variation. This can be shown in three steps. First, note that

$$E_t^Q\left(V + \frac{n\sigma_s^2}{\tau}\right) = V + \frac{\sigma_s^2}{\tau} E_t^Q(n) = V + \lambda_s^Q \sigma_s^2. \qquad \text{(C.3)}$$

Second,

$$QV_{t+\tau,\tau} = V\tau + \sum_{t<u\le t+\tau} J_{su}^2 \Delta N_{su}. \qquad \text{(C.4)}$$

Third,

$$\begin{aligned}
E_t^Q(\overline{QV}_{t+\tau,\tau}) &= V + \frac{1}{\tau} E_t^Q\left(\int_{\mathbb{R}\backslash 0} x^2 N_\tau(dx)\right) \\
&= V + \int_{-\infty}^{\infty} x^2 \lambda_s^Q \frac{1}{\sqrt{2\pi\sigma_s^2}} e^{-x^2/2\sigma_s^2}\, dx \\
&= V + \lambda_s^Q \sigma_s^2. \qquad \text{(C.5)}
\end{aligned}$$

Furthermore, the last relationship, combined with (C.2) and (C.3), yields the relationship (3.15).

I follow the derivation of Hull and White (1987) to extend the formula to the case of the full SVIJ model and show that, conditional on the averaged quadratic variation $\overline{QV}_{t+\tau,\tau}$ in (3.8), $r_{t+\tau}(1/\tau) = \log(S_{t+\tau}/S_t)$ has normal distribution with variance $QV_{t+\tau,\tau}$. Combining this argument with (3.9) and (C.5), I obtain

$$C^{\mathrm{HW-M}}(S_t, V_t, r, K, \tau) = E_t^Q\{C^{\mathrm{BS}}(S_t, \overline{QV}_{t+\tau,\tau}, r, K, \tau)\}. \qquad \text{(C.6)}$$

This formula then yields (3.15).

Because the results hinge on the assumption that $\mu_s^Q = 0$, I check the sensitivity of the implied volatilities to this parameter (I thank a referee for suggesting this exercise). Table C.1 reports the relative error in implied volatilities resulting from the assumption $\mu_s^Q = 0$, whereas, in fact, this parameter is different from 0. It appears, based on empirically plausible parameter values, that the introduced downward bias is modest.

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Table C.1. Risk-neutral mean jump in the SVIJ model

		Implied volatility					
		$\mu_s^Q \neq 0$		$\mu_s^Q = 0$		Relative error	
μ_s^Q	$\sqrt{V_t}$	1 month	3 months	1 month	3 months	1 month	3 months
−.50%	$.75\sqrt{\bar{V}}$	13.44%	14.69%	13.43%	14.68%	−.09%	−.11%
	$\sqrt{\bar{V}}$	17.18%	17.85%	17.17%	17.84%	−.05%	−.06%
	$1.25\sqrt{\bar{V}}$	21.01%	21.19%	21.01%	21.18%	−.03%	−.04%
−1.00%	$.75\sqrt{\bar{V}}$	13.46%	14.73%	13.43%	14.68%	−.28%	−.32%
	$\sqrt{\bar{V}}$	17.20%	17.87%	17.17%	17.84%	−.17%	−.20%
	$1.25\sqrt{\bar{V}}$	21.03%	21.21%	21.01%	21.18%	−.12%	−.13%
−2.00%	$.75\sqrt{\bar{V}}$	13.56%	14.84%	13.43%	14.68%	−1.00%	−1.07%
	$\sqrt{\bar{V}}$	17.29%	17.97%	17.17%	17.84%	−.64%	−.70%
	$1.25\sqrt{\bar{V}}$	21.10%	21.29%	21.01%	21.18%	−.44%	−.49%
−3.00%	$.75\sqrt{\bar{V}}$	13.72%	15.02%	13.43%	14.68%	−2.12%	−2.24%
	$\sqrt{\bar{V}}$	17.42%	18.11%	17.17%	17.84%	−1.39%	−1.51%
	$1.25\sqrt{\bar{V}}$	21.21%	21.41%	21.01%	21.18%	−.98%	−1.07%

NOTE: I evaluate the quality of option price approximation in the affine SVIJ model when the risk-neutral mean jump in the asset price, μ_s^Q, is not equal to 0. The base "true" SVIJ model has the following dynamics under the Q measure:

$$\frac{dS_t}{S_t} = (r - \lambda_s^Q(e^{\mu_s^Q + .5\sigma_s^2} - 1))\,dt + \sqrt{V_t}\,dW_{st}(Q) + (e^{J_{st}}(Q) - 1)\,dN_{st}(Q),$$

$$dV_t = (\theta^P - \kappa^Q V_t)\,dt + \sigma_v\sqrt{V_t}\,dW_{vt}(Q) + J_{vt}(Q)\,dN_{vt}(Q),$$

with the following parameter values based on the studies by Eraker et al. (2003) and Eraker (2004): $r = 3\%$, $\theta^P = .01$, $\kappa^Q = .006$, $\sigma_v = .08$, $\rho = -.5$, $\lambda_s^Q = .0046$, $\sigma_s = 3\%$, $\lambda_v^Q = .011$, and $\mu_v^Q = 1.79$. In this case the long run variance mean $\bar{V}$ is equal to $(\theta^P + \lambda_v^P \mu_v^P)/\kappa^P$, so I assume that $\kappa_v^P = .03$, $\lambda_v^P = .0055$, and $\mu_v^P = 1.79$.

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