MAXIMUM LIKELIHOOD ESTIMATION OF VASICEK MODEL AND CALIBRATION ON MERTON JUMP DIFFUSION MODEL USING METHOD OF MOMENTS

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ABSTRACT.

1. MAXIMUM LIKELIHOOD AND VASICEK MODEL ESTIMATION

1.1. **Introduction.** The Vasicek model [1] is a mathematical finance model that describes the dynamics of interest rates. This model allows the short-term interest rate, the spot rate, to follow a random walk, which leads to a parabolic partial differential equation for the prices of bonds and other interest rate derivative products. The Vasicek model referred in this paper belongs to one factor interest rate model, where there is only one source of randomness, the spot interest rate.

The spot rate evolution is modeled in the following stochastic differential equation:

$$dr_t = a(b - r_t)dt + \sigma dW_t \tag{1}$$

where W_t is the Wiener process. a can be interpreted as the speed of mean-reverting, and b is the long term level of the spot rate. σ, here as a constant, is the instantaneous volatility.

In this paper, I use the 3-month Treasury Bills rates to approximate the spot rate. The data source is Federal Reserve Bank of St. Louis website [1].

1.2. **Discretization of Vasicek Model.** In discrete time, use ϵ_t to represent the white noise with expected value of 0 and variance of 1, evolving at time t. Then the discrete version of Vasicek model expresses as

$$\begin{aligned}
\Delta r_t &= a(b - r_t)\Delta t + \sigma \Delta W_t \\
r_{t+1} - r_t &= a(b - r_t)\Delta t + \sigma(W_{t+1} - W_t) \\
&= a(b - r_t)\Delta t + \sigma \epsilon_{t+1} \\
r_{t+1} &= ab\Delta t + (1 - a\Delta t) r_t + \sigma \epsilon_{t+1}
\end{aligned}$$

Change the notation from b to $\bar{X}$ to represent the mean level of rates, from $(1 - a\Delta t)$ to ϕ simplifying the parameter. Use X_t to denote r_t, we yield

$$X_{t+1} = \bar{X}(1 - \phi) + \phi X_t + \sigma \epsilon_{t+1}$$

which has the form of an AR(1) process.

1.3. **Likelihood of the AR(1) Process.**

REFERENCES

[1] Vasicek O. An equilibrium characterization of the term structure. *Journal of Financial Economics*, 5:177–188, 1977.

[1] https://fred.stlouisfed.org/series/DTB3

Appendix: Code

```r
setwd('C:\\Users\\ranzhao\\Documents\\Empirical Asset Pricing\\Assignment 1')
setwd('D:\\PhD FE\\Empirical-Asset-Pricing\\Assignment 1')

# Data loading
require(ggplot2)
spx_index_values = read.csv('spx_index_values.csv', header = TRUE)
t_bill_3M_values = read.csv('TB3MS.csv', header = TRUE)
plot(as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y"), t_bill_3M_values$TB3MS,
     type='l',
     main='3-Month Treasury Bill, from 1954 to 2015',
     xlab='year', ylab='rate (in percentage)')
# add the moving average of the rates to the plot, ggplot?


# Data segments
ir_full = t_bill_3M_values
ir_1954_1975 = t_bill_3M_values[
  as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") >= as.Date('1954-01-01') &
  as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") <= as.Date('1975-12-31'), ]
ir_1976_1981 = t_bill_3M_values[
  as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") >= as.Date('1976-01-01') &
    as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") <= as.Date('1981-12-31'),
         ]
ir_1982_2005 = t_bill_3M_values[
  as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") >= as.Date('1982-01-01') &
    as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") <= as.Date('2005-12-31'),
         ]
ir_2006_2015 = t_bill_3M_values[
  as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") >= as.Date('2006-01-01') &
    as.Date(as.character(t_bill_3M_values$DATE), "%m/%d/%Y") <= as.Date('2015-12-31'),
         ]
```

assignment1.R