IMPLIED VOLATILITY SURFACE AND MODEL OF VIX

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ABSTRACT.

1. INTRODUCTION

Black [1] first introduced the European futures options pricing based on Black-Scholes framework [2]. The common assumption for the process followed by future price F in risk-neutral framework is

$$dF = \sigma F dW, \tag{1}$$

where σ is a constant and W is the Wiener process. Similar to a non-dividend-paying stock, the differential equation satisfied by a derivative dependent on a futures price is

$$\frac{\partial f}{\partial t} + \frac{1}{2}\frac{\partial^2 f}{\partial F^2}\sigma^2 F^2 = rf.$$

Then the European call price c and the European put price p for the futures option are given by following equations,

$$\begin{aligned} c &= e^{-rT}[F_0 N(d_1) - KN(d_2)] \\ p &= e^{-rT}[KN(-d_2) - F_0 N(-d_1)] \end{aligned}$$

where

$$\begin{aligned} d_1 &= \frac{\log(F_0/K) + \sigma^2 T/2}{\sigma\sqrt{T}} \\ d_2 &= \frac{\log(F_0/K) - \sigma^2 T/2}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \end{aligned}$$

REFERENCES

[1] Black, Fischer. the pricing of commodity contracts. *Journal of Financial Economics*, 3:167–179, 1976.

[2] Black, Fischer and Scholes, Myron. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81:637–654, 1973.

APPENDIX: CODE