Intermediate Financial Economics
Lecture V: Asset Pricing II

Dr. Zhang

A. Price of a Stock Index (Lucas' Tree)

- Suppose the representative agent is endowed with a stock index that is a claim to the entire resource of the economy.
  - Imagine Robinson Crusoe has only a tree that produces fruit. Also called the Lucas' tree.
- The stock index pays off the endowment e_t as dividends in each period.
- Since there is no trade in equilibrium, the rep. agent still holds the stock index and receives and consumes e_t.
- Since $c_t = e_t$, the price of the stock index is then

$$p_t^e = \beta E_t \left[\frac{u'(c_{t+1})}{u'(c_t)} (c_{t+1} + p_{t+1}^e) \right]$$

IID Log-Normal Consumption

- Applying (9), we have

$$p_t^e = E_t\left[\beta^j \Sigma_{j=1}^{\infty}\left(\frac{c_{t+j}}{c_t}\right)^{-a} c_{t+j}\right]$$

- Or in terms of the ratio of price-earnings:

(14) $$\frac{p_t^e}{c_t} = E_t\left[\beta^j \Sigma_{j=1}^{\infty}\left(\frac{c_{t+j}}{c_t}\right)^{1-a}\right]$$

- Suppose consumptions are i.i.d log-normally distributed
  - which implies that the geometric growth rate of consumption is i.i.d. normal.

$$g \equiv \ln c_{t+1} - \ln c_t \sim N(\mu, \sigma^2)$$

- The i.i.d. assumption simplifies the calculation:

$$\frac{p_t^e}{c_t} = E_t\left[\beta^j \Sigma_{j=1}^{\infty} \exp(j(1-a)g)\right]$$

Solving for Stock Return and Equity Risk Premium

- Let $E[R^e_{t+1}] = E\left[\frac{p^e_{t+1} + c_{t+1}}{c_t}\right]$ be the expected (gross) return of the stock index.

- Then the geometric expected return of the stock index can be obtained:

$$(15) \quad r^e \equiv \ln E[R^e_{t+1}] = -\ln\beta + a\mu + (a - a^2/2)\sigma^2$$

- The equity risk premium, the difference between the expected rate of return on equity and the riskfree rate of return, is obtained by using (12):

$$r^e - r^f = a\sigma^2$$

B. Confronting The Model with Data

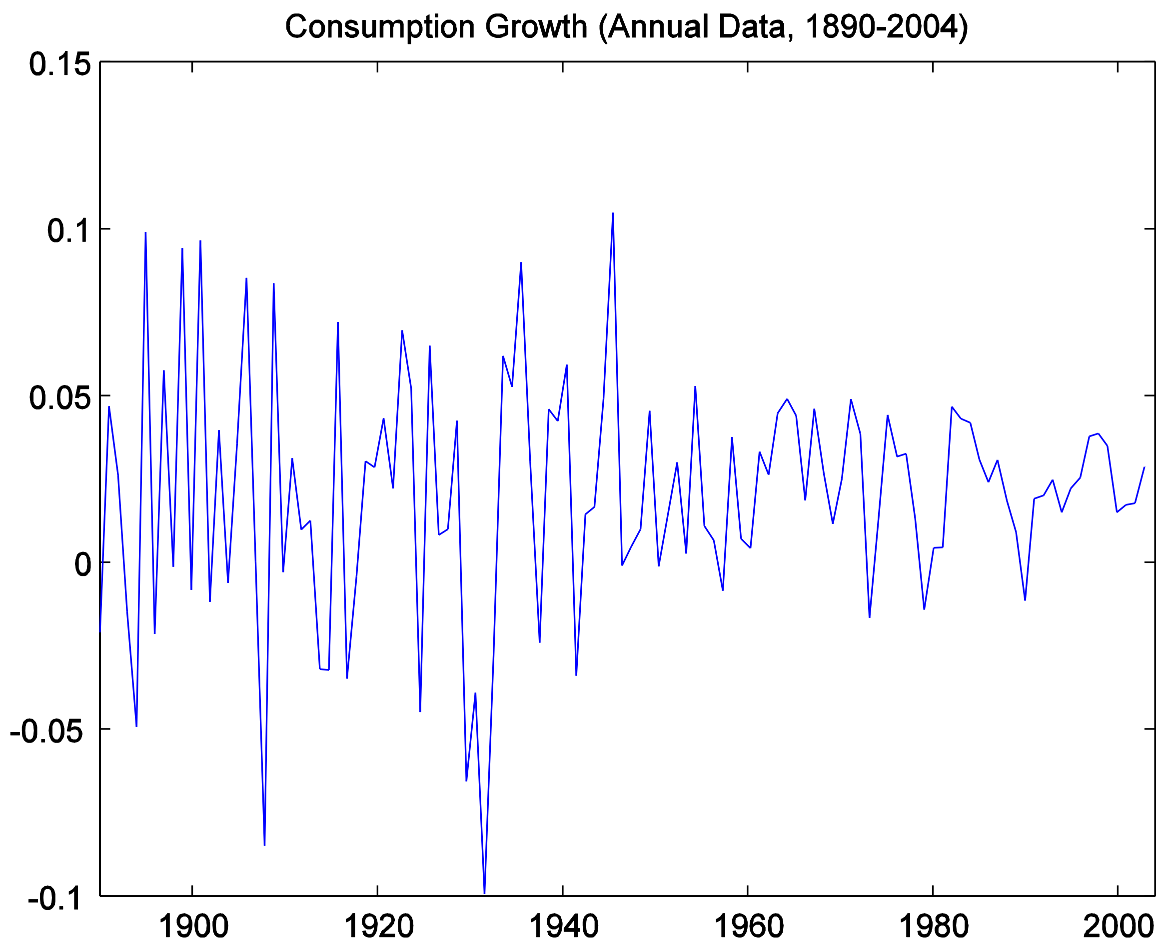
- The model has quantitative predictions about the expected return of an equity index and the riskfree rate of return.
- Goal: see whether these predictions match the average returns on the stock market and short-term government bonds in the data.
- The model does not make distinction between output, dividends, and consumption.
  - They are the same thing in the model.
- Since the model focuses on the demand side, we use the data on consumption.
  - Furthermore, we use the data on consumption of non-durable goods and services.
  - This corresponds to the assumption of perishable goods in the model.

Other Considerations

- Real v.s. nominal:
  - Since the model has no money, the returns are all real returns.
  - We need to compare real returns in reality with the model's predictions.
  - But real returns are not observable in reality.
  - According to the Fisher equation $r \approx i - \pi$, where π is the expected inflation rate, which is not observable.
  - We use realized inflation rate to approximate expected inflation. So the resulting real rates are also approximations of the true real rates.
- The (nominal) riskfree rate corresponds to yields on 1-month or 3-months T-bills.
- The (nominal) equity rate of return corresponds to historical average rates of return on a comprehensive stock index, such as S&P 500.
- Data source:
  - Consumption data: NIPA tables from the Bureau of Economic Analysis.
  - Inflation(CPI): Bureau of Labor and Statistics
  - T-bill returns: Federal Reserve
  - Stock returns: Economist Robert Shiller's website.

Stylized Facts

- The historical annual average of rate of return on equity, with inflation rate subtracted, is around 6%.
- The historical average of annualized real rate of return on 3-month T-bills is around 1%.
- The historical average of equity risk premium is 5%.
- Model inputs:
  - Annual consumption growth rate μ, around 2%.
  - Annual volatility of consumption growth rate σ, around 3%.
  - Time impatience, around 0.98.
  - Coefficient of relative risk aversion a, (0,6). Recall our experiment.


Consumption Growth (Annual Data, 1890-2004)
0.15
0.1
0.05
0
-0.05
-0.1
1900
1920
1940
1960
1980
2000

The Equity Premium Puzzle

- Model Output:

a	1	2	3	4	5	6	30	45
re	0.0407	0.06	0.078853	0.0966	0.113	0.1294	0.242	0.05
rf	0.0398	0.058	0.076153	0.093	0.109	0.124	0.215	0.01
re-rf	0.0009	0.002	0.0027	0.0036	0.004	0.0054	0.027	0.04

- Only with implausibly high risk aversion can the model produce sensible results.
- The equity premium is too low for a moderate degree of risk aversion (0, 5).
- This problem is robust to different model settings.
- This is termed the Equity Premium Puzzle.
- The consumption-based models cannot explain why risk-averse investors in the real world demand such a high risk premium for holding stocks.

UNDERSTANDING THE PUZZLE

- Equation (11) in the last lecture says

$$1 = E[m]E[R_i] + cov(m, R_i)$$

- By the definition of correlation coefficient,

$$\rho_{a,b} = cov(a, b)/\sigma_a \sigma_b$$

we have $\quad 1 = E[m]E[R^i] + \rho_{m,R^i}\sigma(R^i)\sigma(m)$

- Using (6), we have

(16) $$E[R^i] - R^f = -\rho_{m,R^i}\frac{\sigma(m)}{E[m]}\sigma(R^i)$$

- Since $|\rho| \leq 1$, we obtain

(17) $$\left|\frac{E[R^i] - R^f}{\sigma(R^i)}\right| \leq \frac{\sigma(m)}{E[m]}$$

- (17) says that the LHS, a.k.a. the Sharpe ratio cannot exceed $\sigma(m)/E(m)$.
- The historical average of the equity premium in the US is high relative to the volatility of equity returns.
- So the upper bound on the RHS should be high.
- However, the volatility of consumption growth is very low. For the case $m = (c_{t+1}/c_t)^{-a}$, the only way to make $\sigma(m)$ large is to assume that people are extremely risk averse (a=45), i.e. super sensitive to consumption changes.
- Yet empirical evidence suggests that people are only moderately risk averse ($a \sim (0,5)$).
- Therefore the essential problem is that consumption data do not vary enough to justify the observed large equity premium.

Attempts to Resolve The Puzzle

- Risk preferences under uncertainty: replacing or generalizing the theory of expected utility.
  - Expected utility may not describe human's behavior under uncertainty.
  - Famous challenge: the Allais Paradox.
- Problem with the data: finding new measures of consumption.
  - Consumption on luxury goods.
  - With data on garbage.
- Heterogeneous agents: introducing different degree of risk aversion, different wealth, and different idiosyncratic shocks to wealth.
  - There will be trade in equilibrium.
- Incomplete markets: making it impossible to completely share risks.
- Imperfect information: introducing learning about parameter and background uncertainty.
- Rare disasters.

The Expected Utility Theory

- A main stream theory describing individuals' behavior under uncertainty.
- Some economists attribute the failure of the asset pricing models to the expected utility (EU) theory.
- The EU theory assumes that individuals' choice under uncertainty follows the Independence Axiom.
- The Independence Axiom says that if an agent prefers Lottery L_1 to L_2, the agent also prefers L_1 mixed with an arbitrary simple lottery L_3 with any probability p to L_2 mixed with L_3 with the same probability.
  - If $L_1 \succ L_2$, then $pL_1 + (1-p)L_3 \succ pL_2 + (1-p)L_3$
  - That is, the preference shouldn't be affected by combining a third common lottery.

Allais Paradox

- A famous experiment that consistently yields contradictory results to the Independence Axiom.
- The possible prizes of some lottery are (0, 1m, 5m).
- Choose between two lotteries with different odds:
- A: (0, 1, 0); B: (.01, .89, .10)
- Choose again:
- A': (.89, .11, 0); B': (.90, 0, .10)
- These lotteries can be seen as the resulting combination of these simple lotteries: L_1= (0, 1, 0), L_2=(1/11, 0, 10/11), L_3=(0, 1, 0), and L_3' = (1, 0, 0).
- We have
  - A= .89L_1 + .11L_3, B=.89L_2 +.11L_3,
  - A'=.89L_1 + .11L_3', B'=.89L_2 +.11L_3'
- If you choose A over B, the expected utility theory predicts that you should choose A' over B', because they both reveal that L_1 is preferred to L_2.