Empirical Asset Pricing

Part 3: Currencies

UCLA | Fall 2016

by Mikhail Chernov

This version: November 20, 2016

19. Exchange rate puzzles

The forward premium puzzle

- The forward contract pays you a unit of the foreign currency at time $t+1$ based on the price established at t
- Therefore, $rx_{t+1} = s_{t+1} - f_t$
- Of course, we have an expectations hypothesis; $E_t s_{t+1} = f_t$
- Bilson (1981); Fama (1984); Tryon (1979) run a regression

$$s_{t+1} - s_t = a_1 + a_2(f_t - s_t) + \varepsilon_{t+1}$$

- Main finding: a_2 is in the -1 to -2 range depending on the currency

The puzzle and risk premium

- Consider the decomposition:

$$f_t - s_t = (f_t - E_t s_{t+1}) + (E_t s_{t+1} - s_t) = p_t + q_t$$

  where p_t is the risk premium, or expected excess return.

- Note that

$$a_2 = \frac{cov(q, p+q)}{var(p+q)} = \frac{cov(q,p) + var(q)}{var(p+q)}$$

- As in FI, constant risk premium generates $a_2 = 1$. Otherwise, to get $a_2 < 0$ we need “Fama’s conditions:”
  1. $cov(p,q) < 0$
  2. $var(p) > var(q)$ [follows from an additional OLS]

- A common wisdom for a long time was that it is hard to get in “standard models”

Returns to carry trades

- Recall that FX returns are $R^{fx}_{t+1} = S_{t+1}/S_t \cdot R^*_{f,t}$
- The log-excess returns is $rx_{t+1} = s_{t+1} - s_t + r^*_t - r_t$; this is the carry trade if $r_t < r^*_t$
- The returns to carry trades are high: according to Brunnermeier, Nagel, and Pedersen (2008), they range from -0.004 (-1.6%/y, JPY) to 0.013 (5.2%/y NOK).
- Why? UIP tells us that it should be zero.
- A new puzzle?
  - By no-arbitrage, $F_t = S_t R_{f,t}/R^*_{f,t}$
  - So, $rx_{t+1} = s_{t+1} - f_t$
  - Conditioning on the sign of $r^*_t - r_t$ is new

20. Additional evidence

FX-specific risk factors

- Lustig, Roussanov, and Verdelhan (2011) systematically search for common risk factors in currency markets
- They sort currencies into 6 portfolios based on the interest rate
- They implement PC analysis
  1. 1st PC explains 70%; a level factor, loads equally on all currencies - a $ risk factor
  2. 2nd PC explains 12%; a slope factor, well-approximated by the carry-trade portfolio

Portfolio performance

Table 1: Currency Portfolios - US Investor

Portfolio	1	2	3	4	5	6	1	2	3	4	5
	Panel I: All Countries						Panel II: Developed Countries				
	Spot change: Δs^j						Δs^j				
Mean	-0.97	-1.33	-1.55	-2.73	-0.99	1.88	-1.86	-2.54	-4.05	-2.11	-1.11
Std	8.04	7.29	7.41	7.42	7.74	9.16	10.12	9.71	9.24	8.92	9.20
	Forward Discount: $f^j - s^j$						$f^j - s^j$				
Mean	-3.90	-1.30	-0.15	0.94	2.55	7.78	-3.09	-1.02	0.07	1.13	3.94
Std	1.57	0.49	0.48	0.53	0.59	2.09	0.78	0.63	0.65	0.67	0.76
	Excess Return: rx^j (without b-a)						rx^j (without b-a)				
Mean	-2.92	0.02	1.40	3.66	3.54	5.90	-1.24	1.52	4.11	3.24	5.06
Std	8.22	7.36	7.46	7.53	7.85	9.26	10.20	9.75	9.35	9.01	9.30
SR	-0.36	0.00	0.19	0.49	0.45	0.64	-0.12	0.16	0.44	0.36	0.54
	Net Excess Return: rx^j_{net} (with b-a)						rx^j_{net} (with b-a)				
Mean	-1.70	-0.95	0.12	2.31	2.04	3.14	-0.11	0.46	2.71	1.98	3.35
Std	8.21	7.35	7.43	7.48	7.85	9.25	10.20	9.75	9.32	9.02	9.30
SR	-0.21	-0.13	0.02	0.31	0.26	0.34	-0.01	0.05	0.29	0.22	0.36
	High-minus-Low: $rx^j - rx^1$ (without b-a)						$rx^j - rx^1$ (without b-a)				
Mean		2.95	4.33	6.59	6.46	8.83		2.75	5.35	4.47	6.29
Std		5.36	5.54	6.65	6.34	8.95		6.42	6.44	7.38	8.70
SR		0.55	0.78	0.99	1.02	0.99		0.43	0.83	0.61	0.72
	High-minus-Low: $rx^j_{net} - rx^1_{net}$ (with b-a)						$rx^j_{net} - rx^1_{net}$ (with b-a)				
Mean		0.75	1.82	4.00	3.73	4.83		0.57	2.82	2.09	3.46
Std		5.36	5.56	6.63	6.35	8.98		6.45	6.44	7.41	8.73
SR		0.14	0.33	0.60	0.59	0.54		0.09	0.44	0.28	0.40

Notes: This table reports, for each portfolio j, the average change in log spot exchange rates Δs^j, the average log forward discount $f^j - s^j$, the average log excess return rx^j without bid-ask spreads, the average log excess return rx^j_{net} with bid-ask spreads, and the average return on the long short strategy $rx^j_{net} - rx^1_{net}$ and $rx^j - rx^1$ (with and without bid-ask spreads). Log currency excess returns are computed as $rx^j_{t+1} = -\Delta s^j_{t+1} + f^j_t - s^j_t$. All moments are annualized and reported in percentage points. For excess returns, the table also reports Sharpe ratios, computed as ratios of annualized means to annualized standard deviations. The portfolios are constructed by sorting currencies into six groups at time t based on the one-month forward discount (i.e nominal interest rate differential) at the end of period $t-1$. Portfolio 1 contains currencies with the lowest interest rates. Portfolio 6 contains currencies with the highest interest rates. Panel I uses all countries, panel II focuses on developed countries. Data are monthly, from Barclays and Reuters (Datastream). The sample period is 11/1983 - 03/2008.

Principal Components

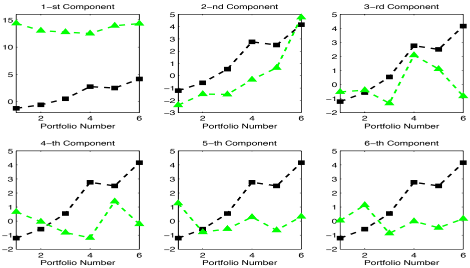


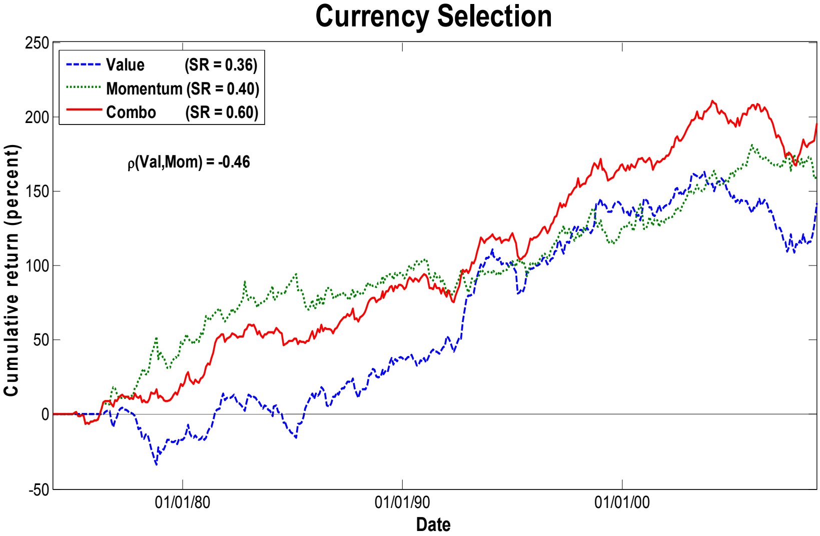
Figure 1: Mean Excess Returns and Covariances between Excess Returns and Principal Components - Developed and Emerging Countries

Each panel corresponds to a principal component. The upper left panel uses the first principal component. The black squares represent the average currency excess returns for the six portfolios. Each green triangle represents a covariance between a given principal component and a given currency portfolio. The covariances are rescaled (multiplied by 15,000). The average excess returns are annualized (multiplied by 12) and reported in percentage points. The sample is 11/1983 - 03/2008.

Connection to the common risk factors

- Asness, Moskowitz, and Pedersen (2013) study the performance of value and momentum strategies for all assets, and currencies in particular
- The momentum portfolio is the same as for stocks: 12-month cumulative raw return on the currency, skipping the most recent month's return; long the best performers, short the worst performers
- The value measure is "book" to price, where "book" is the average exchange rate 5 years ago adjusted for interest-rate differentials, i.e., excess return from $t-60$ to $t-1$ (deviation from PPP)
- H-L spread: Value = 3.8%/year; Momentum = 3.2%/year; 50-50 Combo = 3.4%
- Global CAPM alpha (SR): Value = 4.9% (0.54); Momentum = 2.7% (0.29); 50-50 Combo = 3.8% (0.72)

Value and Momentum in FX



21. Basic theory

Numeraire

- Suppose you have an asset whose price is (in US units of consumption)

$$P_t = E_t(M_{t+1}(P_{t+1} + D_{t+1}))$$

with return $R_{t+1} = (P_{t+1} + D_{t+1})/P_t$.

- You want to express this price in units of some index, I_t

$$\begin{aligned} P_t/I_t &= E_t(M_{t+1}(P_{t+1} + D_{t+1})/I_t) = E_t(M_{t+1} I_{t+1}/I_t \cdot (P_{t+1} + D_{t+1})/I_{t+1}) \\ &= E_t(M^I_{t+1}(P_{t+1} + D_{t+1})/I_{t+1}) \end{aligned}$$

with return $R^I_{t+1} = R_{t+1} I_t / I_{t+1}$.

- Examples:
  - Suppose the asset is the IBM stock and the index is the oil price. We've expressed prices and returns of IBM in units of oil.
  - Suppose that $I_t = CPI_t^{-1}$. We've expressed prices and returns on any assets in nominal terms (\$).
  - Suppose that $I_t = E_t$ (E_t is real exchange rate: relative value of the UK consumption vs the US consumption basket). We've expressed prices and returns on any assets in terms of the UK units of ocnsumption.
- In all these cases, $M^I_{t+1} = M_{t+1} I_{t+1}/I_t$ reflects the change in numeraire

Asset market view of FX

- This is mostly based on Backus, Foresi, and Telmer (2001) and Burnside and Graveline (2013)
- Domestic: $E_t(M_{t+1}R_{t+1}) = 1$, FX: S_t is the \$ price of £
- Important: R_{t+1} may include returns on foreign assets as long as they are denominated in \$
- Foreign: $E_t(M^*_{t+1}R^*_{t+1}) = 1$, $X_{t+1} = S_{t+1}/S_t$
- Dollar returns on this assets are $R_{t+1} = X_{t+1} \cdot R^*_{t+1}$, and

$$
\begin{aligned}
E_t(M_{t+1}X_{t+1} \cdot R^*_{t+1}) &= 1 \\
E_t(M^*_{t+1}R^*_{t+1}) &= E_t(M_{t+1}X_{t+1} \cdot R^*_{t+1})
\end{aligned}
$$

- $M^*_{t+1} = M_{t+1}X_{t+1}$ (looks like $\widehat{M}_{t+1}$)
- Rearrange to get

(log) depreciation rate $=$ foreign (log) PK $-$ domestic (log) PK

Risk premium

- Consider a forward contract:

$$
\begin{aligned}
0 &= E_t(M_{t+1}(F_t - S_{t+1})) \\
F_t/S_t \cdot E_t(M_{t+1}) &= E_t(M_{t+1}S_{t+1}/S_t) = E_t(M^*_{t+1}) \\
f_t - s_t &= \log E_t M^*_{t+1} - \log E_t M_{t+1} = r_t - r^*_t \\
q_t &= E_t(s_{t+1}) - s_t = E_t m^*_{t+1} - E_t m_{t+1}
\end{aligned}
$$

- Therefore, the risk premium is

$$
p_t = f_t - s_t - q_t = L_t(M^*_{t+1}) - L_t(M_{t+1})
$$

- In particular, if m, m^* are normal $p_t = (\kappa^*_{2,t} - \kappa_{2,t})/2$, so, we should not expect p_t to be equal to zero even under normality
- "Fama's conditions" imply ($q_t = \kappa^*_{1,t} - \kappa_{1,t}$)
    1. $cov(\kappa^*_{1,t} - \kappa_{1,t}, \kappa^*_{2,t} - \kappa_{2,t}) < 0$
    2. $var(\kappa^*_{2,t} - \kappa_{2,t})/4 > var(\kappa^*_{1,t} - \kappa_{1,t})$

Implications

- This leads many authors to believe that they derive exchange rates, but these expressions just reflect a change in numeraire
- One out of three random variables M_{t+1}, M^*_{t+1}, and X_{t+1} is redundant
- Empirically it is convenient to model M_{t+1}, and X_{t+1}
  - the forward premium puzzle is satisfied mechanically [to be shown later].
  - M^*_t is unique, mechanically
- However, if one starts with M and M^* that satisfy the respective pricing equations, it does not guarantee $M^*_{t+1} = M_{t+1}X_{t+1}$ unless markets are complete
- When markets are complete, the choice of M and M^* is unique. What does this mean?
  - It means that M is unique
  - Then, given the choice of numeraire S, M^* is unique
- When are markets complete?

Incomplete markets

- Consider all available assets with \$-denominated returns $\mathbf{R}_t \in \mathbb{R}^k$

- In general, there is not a unique M that satisfies $E_t(M_{t+1}\mathbf{R}_{t+1}) = 1$ for a given choice of assets

- If positive r.v. ξ_t is $E_t(\xi_{t+1}) = 1$, $\hat{M}_t = M_t\xi_t$ prices these assets

- One can think of M as the minimum-variance pricing kernel:
  - If $\mathbf{R}_t$ are non-redundant (linearly independent) then there is a unique pricing kernel that is linear in this return, a linear projection of a pricing kernel on these returns:

$$
\begin{aligned}
proj[M_t|\mathbf{R}_t] &= \beta \cdot \mathbf{R}_t = proj[\hat{M}_t|\mathbf{R}_t], \\
\beta &= \mathbf{1}(E_{t-1}\mathbf{R}_t^\top \mathbf{R}_t)^{-1}
\end{aligned}
$$

Change of numeraire

- Change numeraire on the projected kernel and get the "unique foreign" pricing kernel: $\beta \cdot \mathbf{R}_t \cdot X_t$

- Alternatively can construct MV pricing kernel by projecting it on the foreign-denominated asset returns:

$$\begin{aligned} proj[M_t X_t | \mathbf{R}_t / X_t] &= \beta^* \cdot \mathbf{R}_t / X_t = proj[\hat{M}_t X_t | \mathbf{R}_t / X_t], \\ \beta^* &= \mathbf{1}(E_{t-1} \mathbf{R}_t^\top \mathbf{R}_t X_t^{-2})^{-1} \end{aligned}$$

- Are the two the same:

$$\beta^* \cdot \mathbf{R}_{t+1} / X_{t+1} = \beta \cdot \mathbf{R}_{t+1} X_{t+1} ?$$

  - l.h.s. is linear in $\mathbf{R}_{t+1} / X_{t+1}$
  - r.h.s. is linear in $\mathbf{R}_{t+1} X_{t+1}$

Complete markets in reduced-form models

- Suppose there are only k distinct states, then there is a unique M in the information set generated by $\mathbf{R}_t$
- Then $\beta^* \cdot \mathbf{R}_{t+1} / X_{t+1}$ and $\beta \cdot \mathbf{R}_{t+1} X_{t+1}$ must be the same
- No-arb literature refers to this case as complete market
- Note that because continuous-time pure diffusion (no jumps) setting implies a complete market, the same result holds

Lessons from the cross-section

- Consider the \$ and HML factors:

$$\overline{rx}_{t+1} = \frac{1}{N}\sum_i rx^i_{t+1}, \; hml_{t+1} = \frac{1}{N_H}\sum_{i\in H} rx^i_{t+1} - \frac{1}{N_L}\sum_{i\in L} rx^i_{t+1}$$

- Each of the excess returns is

$$\begin{aligned} & rx^i_{t+1} = s^i_{t+1} - s^i_t + r^i_t - r_t \\ = \quad & m^i_{t+1} - m_{t+1} - \log E_t(e^{m^i_{t+1}}) + \log E_t(e^{m_{t+1}}) \text{ under log-normality} \\ = \quad & [m^i_{t+1} - E_t m^i_{t+1}] - [m_{t+1} - E_t m_{t+1}] - 0.5 Var_t m^i_{t+1} + 0.5 Var_t m_{t+1} \end{aligned}$$

- Therefore (assuming $N_H = N_L = N/6$),

$$\overline{rx}_{t+1} - E_t\overline{rx}_{t+1} = \frac{1}{N}\sum_i \text{shocks to } m^i - \text{shocks to } m$$

$$hml_{t+1} - E_t hml_{t+1} = \frac{6}{N}\sum_{i\in H} \text{shocks to } m^i - \frac{6}{N}\sum_{i\in L} \text{shocks to } m^i$$

- Want each of the above to be exposed to a separate shock

Constructing pricing kernels

- Imagine each pricing kernel m^i being hit by "country-specific" shocks u^i and "global" shocks u^w with exposures γ^i and δ^i, respectively.
- Then the \$ portfolio gives us

$$N^{-1}\sum_i \gamma^i u^i + \bar{\delta} u^w - \gamma u - \delta u^w,$$

and the carry portfolio

$$6N^{-1}\sum_{i \in H} \gamma^i u^i + \bar{\delta}_H u^w - 6N^{-1}\sum_{i \in L} \gamma^i u^i - \bar{\delta}_L u^w$$

- If the law of large numbers holds and
  - All δ^i are the same: global shocks u^w are eliminated from both sets of excess returns
  - All γ^i are the same: $\overline{rx}_{t+1} - E_t \overline{rx}_{t+1} = -\gamma u + (\bar{\delta} - \delta) u^w$, and $hml_{t+1} - E_t hml_{t+1} = (\bar{\delta}_H - \bar{\delta}_L) u^w$

Country-specific shocks?

- This setup implies:

$$
\begin{aligned}
s^{ij}_{t+1} - s^{ij}_{t} &= E_t(m^j_{t+1} - m^i_{t+1}) + \gamma(u^j_{t+1} - u^i_{t+1}) + (\delta^j - \delta^i)u^w_{t+1} \\
\tilde{rx}^{ij}_{t+1} &= \gamma(u^j_{t+1} - u^i_{t+1}) + (\delta^j - \delta^i)u^w_{t+1}
\end{aligned}
$$

- Thus, risk premium is (under log-normality)

$$
-cov_t(m^i_{t+1}, \tilde{rx}^{ij}_{t+1}) = \gamma^2 + \delta^i(\delta^i - \delta^j)
$$

- So, u^i is priced
- In a diversified portfolio vs the \$, $u^{\$}$ is priced:

$$
-cov_t(m_{t+1}, \bar{\tilde{rx}}_{t+1}) = \gamma^2 + \delta(\bar{\delta} - \delta)
$$

Economic restrictions implied by a change of numeraire

- The discussed papers do not model different pricing kernels
- Instead they model one and the same pricing kernel denominated in different units
- Nothing wrong about this, but there are no economic restrictions due to a change of numeraire
- In the affine literature, it is believed that FX impose economic restrictions on M and M^*

Affine example (I)

- Assume $k = 2$: two one-period default-free bank accounts in \$ (interest rate r) and in £(r^*)

- Then log asset returns are $\log \mathbf{R}_{t+1} = [r_t, r_t^* + s_{t+1} - s_t]$, or $\log \mathbf{R}_{t+1} - s_{t+1} + s_t = [r_t - s_{t+1} + s_t, r_t^*]$

- Assume $s_{t+1} - s_t + r_t^* - r_t = \mu_t + v^{1/2}\varepsilon_{t+1}$, e.g., $\mu_t = \alpha + \beta(r_t - r_t^*)$

- Construct a pricing kernel:

$$m_{t+1} = -\Gamma_t \cdot \log \mathbf{R}_{t+1} - c_t,$$

with $\Gamma_t = [0.5 - \mu_t/v, 0.5 + \mu_t/v] \equiv [1 - \gamma_t, \gamma_t]$ ($\Gamma_t \cdot \mathbf{1} = 1$) and $c_t = \gamma_t(1 - \gamma_t)v/2$ (from $E_t(M_{t+1}\mathbf{R}_{t+1}) = 1$)

Affine example (II)

- This setup implies:

$$m_{t+1} = -(1-\gamma_t)r_t - \gamma_t(r_t^* + s_{t+1} - s_t) - \gamma_t(1-\gamma_t)v/2$$

or

$$\begin{aligned} m_{t+1}^* &\equiv m_{t+1} + s_{t+1} - s_t = -\Gamma_t \cdot (\log \mathbf{R}_{t+1} - s_{t+1} + s_t) - c_t \\ &= -(1-\gamma_t)(r_t - s_{t+1} + s_t) - \gamma_t r_t^* - \gamma_t(1-\gamma_t)v/2 \end{aligned}$$

(this is why $\Gamma_t \cdot \mathbf{1} = 1$ is useful).

- Affine literature parametrizes m and m^* instead:
  - Consider $\lambda_t \equiv \mu_t/v^{1/2} + v^{1/2}/2 = \gamma_t v^{1/2}$ and $\lambda_t^* \equiv \mu_t/v^{1/2} - v^{1/2}/2 = -(1-\gamma_t)v^{1/2}$
  - Our pricing kernels become:

$$m_{t+1} = -r_t - \lambda_t^2/2 - \lambda_t \varepsilon_{t+1} \text{ and } m_{t+1}^* = -r_t^* - \lambda_t^{*2}/2 - \lambda_t^* \varepsilon_{t+1}$$

Affine example (III)

- Derive exchange rate:

$$s_{t+1} - s_t + r_t^* - r_t = (\lambda_t^2 - \lambda_t^{*2})/2 + (\lambda_t - \lambda_t^*)\varepsilon_{t+1} + \lambda_t^2/2 + \lambda_t \varepsilon_{t+1}$$

- But we've seen that we simply relabeled parameters in the pricing kernel
- What do we get from the "original" specification?

$$\begin{aligned} & s_{t+1} - s_t + r_t^* - r_t \\ = \quad & (1-\gamma_t)(r_t - r_t + s_{t+1} - s_t) + \gamma_t(r_t^* + s_{t+1} - s_t - r_t^*) \\ + \quad & \gamma_t(1-\gamma_t)v/2 - \gamma_t(1-\gamma_t)v/2 + r_t^* - r_t \end{aligned}$$

- The assumed dynamics of $s_{t+1} - s_t + r_t^* - r_t$ does not impose separate restrictions on M and M^*

Difficulties with "explaining" the Fama puzzle

- Backus, Foresi, and Telmer (2001) state that affine models have difficulties accounting for the Fama regression
- We just showed how easy it is with $\mu_t = \alpha + \beta(r_t - r_t^*)$
- BFT and many others make additional (implicit) assumption that markets are complete w.r.t. interest rates:
  - time is discrete
  - r_t and r_t^* are driven by two correlated normal shocks
  - M_t and M_t^* are driven by the same two shocks, so is exchange rate
  - Have at least 4 assets (in fact more): short-term rates and long-term rates in each country
  - After accounting for a numeraire, have at least 3 pieces of info for 2 shocks
  - The model is too restrictive, hence features informative restrictions
- Theoretically, there is nothing wrong with assuming this kind of completeness
- In practice, there is plenty of evidence that currencies do not strongly correlate with other assets

Back to the “KLV” model

- Add time-varying risk premiums

$$
\begin{aligned}
m_{t,t+1} &= \log\beta + \theta_m x_{1t} - (\lambda_0 + \lambda_1 x_{1t})^2/2 \\
&+ (\lambda_0 + \lambda_1 x_{1t}) w_{t+1} + \lambda_2 z^m_{t+1} \\
x_{1t+1} &= \varphi_1 x_{1t} + w_{t+1}
\end{aligned}
$$

- Depreciation rate

$$
\begin{aligned}
g_{t,t+1} &= \log\gamma + \theta x_{1t} + \theta_g x_{2t} + \eta_0 w_{t+1} + \eta_2 z^g_{t+1} \\
x_{2t+1} &= \varphi_2 x_{2t} + w_{t+1}
\end{aligned}
$$

- Transformed, aka foreign, pricing kernel

$$
\begin{aligned}
m^*_{t,t+1} = \; & (\log\beta + \log\gamma) + (\theta_m + \theta) x_{1t} + \theta_g x_{2t} - (\lambda_0 + \lambda_1 x_{1t})^2/2 \\
& + (\lambda_0 + \eta_0 + \lambda_1 x_{1t}) w_{t+1} + \lambda_2 z^m_{t+1} + \eta_2 z^g_{t+1}
\end{aligned}
$$

implies yields on foreign bonds

Back to the "KLV" model

- The nominal yield is $y_t^1 = \text{const} - \theta_m x_{1t}$
- The foreign yield is $\widehat{y}_t^1 = \text{const} - (\theta_m + \theta + \eta_0 \lambda_1) x_{1t} - \theta_g x_{2t}$
- The IR differential is $y_t^1 - \widehat{y}_t^1 = \text{const} + (\theta + \eta_0 \lambda_1) x_{1t} + \theta_g x_{2t}$
- The expected depreciation rate
  - Constant prices of risk, $\lambda_1 = 0$: $E_t g_{t,t+1} = \text{const} + y_t^1 - \widehat{y}_t^1$
  - Otherwise,

$$E_t g_{t,t+1} = \text{const} + \frac{\theta}{\theta + \eta_0 \lambda_1} (y_t^1 - \widehat{y}_t^1) + \frac{\eta_0 \lambda_1 \theta_g}{\theta + \eta_0 \lambda_1} x_{2t}$$

- Can calibrate θ to match Fama's regressions
- Calibrate the rest to match US and foreign yield curves

22. Structural models

International "risk sharing"

- Brandt, Cochrane, and Santa-Clara (2006) start with $s_{t+1} - s_t = m^*_{t+1} - m_{t+1}$, then

$$
\begin{aligned}
\underbrace{var(s_{t+1} - s_t)}_{0.15^2} &= \underbrace{var(m^*_{t+1}) + var(m_{t+1})}_{2 \cdot 0.5^2} \\
&- 2\underbrace{(var(m^*_{t+1})var(m_{t+1}))^{1/2}}_{2 \cdot 0.5^2} corr(m^*_{t+1}, m_{t+1})
\end{aligned}
$$

- Therefore, either $corr(m^*_{t+1}, m_{t+1}) = 0.955$ (perfect "risk sharing") or, if $corr(m^*_{t+1}, m_{t+1}) = 0$, then $var(s_{t+1} - s_t)$ should be 0.71^2 (exchange rates are too smooth)

- Puzzling from a naive perspective of two power pricing kernels $m^i_{t+1} = \log\beta - (1-\alpha)g^i_{t+1}$: $max[corr(g^i_{t+1}, g^j_{t+1})] = 0.41$

Long-run risk and high correlation of pricing kernels

- Colacito and Croce (2011) argue that long-run risk exposure can reconcile volatility of exchange rates and pricing kernels
- Consider the BY1 setup that yields:

$$m_{t+1} - E_t m_{t+1} = (\alpha - 1)v^{1/2} w_{gt+1} + (\alpha - \rho) b_1 (1 - b_1 \varphi_g)^{-1} v_x^{1/2} w_{xt+1}$$

- Assume that structure of the foreign economy is exactly the same, but with different shocks: $corr(w_{xt+1}, w^*_{xt+1}) = \rho_x$, $corr(w_{gt+1}, w^*_{gt+1}) = \rho_g$
- Juxtopose the two pricing kernels to get the exchange rate:

$$\begin{aligned}
\Delta e_{t+1} - E_t(\Delta e_{t+1}) &= (\alpha - 1)v^{1/2}(w^*_{gt+1} - w_{gt+1}) \\
&+ (\alpha - \rho) b_1 (1 - b_1 \varphi_g)^{-1} v_x^{1/2} (w^*_{xt+1} - w_{xt+1}) \\
Var(\Delta e_{t+1}) &= var(m^*_{t+1}) + var(m_{t+1}) \\
&- 2(\alpha - 1)^2 v \rho_g - 2(\alpha - \rho)^2 b_1^2 (1 - b_1 \varphi_g)^{-2} v_x \rho_x
\end{aligned}$$

Parameter sensitivity

$\rho_x^{hf} = \rho_x; \rho_x = \varphi_g$

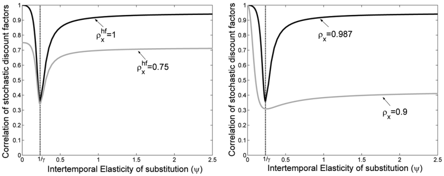


FIG. 1.—The role of intertemporal elasticity of substitution. In both panels, the dark line reports the correlation of stochastic discount factors when ψ changes. The grey line on the left panel is drawn for a smaller value of ρ_x^{hf}, everything else held equal; the grey line on the right panel is drawn for a lower value of ρ_x.

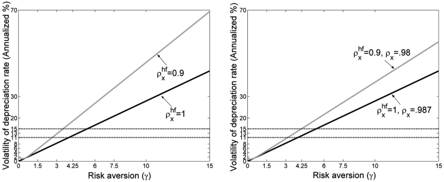


FIG. 2.—The role of risk aversion. In both panels, the dark line reports the volatility of the depreciation rate when γ changes. The grey line in the left panel is drawn for a smaller value of ρ_x^{hf}, everything else held equal; the grey line in the right panel is drawn for lower values of ρ_x and ρ_x^{hf}.

Real exchange rates

- Burnside and Graveline (2013) assume two agents A and B, who live anywhere in the world
  - could be two countries with different nominal currencies, e.g. US and UK (we will assume this scenario to be specific)
  - could be two countries with the same nominal currencies, e.g. France and Finland
  - could be different states in the US
- Let
  - $\tilde{P}_t$ be the \$ value of A's consumption basket
  - P_t^* be the £ value of B's consumption basket
  - P_t be the \$ cost of B's consumption basket
- The nominal exchange rate solves $P_t = P_t^* S_t$
- The real exchange rate is the relative value of the two baskets measured in the same units: $E_t = P_t / \tilde{P}_t$
- The depreciation rate is (repurpose notation from nominal to real):

$$x_t = e_{t+1} - e_t = p_{t+1}^* - p_t^* + s_{t+1} - s_t - [\tilde{p}_{t+1} - \tilde{p}_t]$$

Variation in real exchange rates

- If A and B have identical composition of consumption baskets and they face identical prices (when measured in common units), then E_t is constant
- E_t varies if
  - Composition of the two baskets is different and/or
  - A and B face different prices
- Empirically, both of these conditions are satisfied

Critique (I)

- Exchange rates vary only if there are frictions; asset market view is silent about that
- Let $\tilde{M}$ be A's IMRS over A's basket, and M^* be B's IMRS over B's basket; $M = M^*/X$ is B's IMRS in terms of A's basket
- Colacito and Croce (2011), and others, model the two economies separately, which makes $\tilde{M}$ and M^* exogenous, then they assume

$$M_t^* = X_t \tilde{M}_t, \text{ or } M_t = \tilde{M}_t$$

- Colacito-Croce feature a two-good economy with complete home bias. Thus, FX rate is indeterminate because households consume their endowments regardless of their relative price
- In single-good economies such as the Verdelhan (2010) version of Campbell-Cochrane $E_t = 1$ if trade is not ruled out, and is indeterminate if it is

Critique (II)

- IMRS's are always equalized if markets are complete and the agents are facing the same baskets, i.e., perfect risk sharing in this case
  - IMRS over different baskets could still differ and then exchange rate would move around
- If markets are incomplete, the IMRS won't equate in the unspanned states (imperfect risk sharing)
- If they are facing different prices, there must be a friction in the goods market (not the asset markets) that prevents the prices from equalizing; that same friction may cause imperfect risk sharing

	Composition of Consumption Baskets	
Asset Markets	Identical	Different
Complete	Yes	No
Incomplete	No	No

Does a variable real exchange rate directly reflect imperfect risk sharing?

Critique (III)

- Brandt, Cochrane, and Santa-Clara (2006) allow for market incompleteness by claiming that the "asset view" continues to hold for projections.
- Then

$$proj[M^*_{t+1}|\mathbf{R}_{t+1}/X_{t+1}] = \beta^* \cdot \mathbf{R}_{t+1}/X_{t+1}$$

  is the part of B's IMRS that is in the linear span of these particular assets denominated in the UK units of consumption
- Likewise

$$proj[\tilde{M}_{t+1}|\mathbf{R}_{t+1}] = \beta \cdot \mathbf{R}_{t+1}$$

  is the part of A's IMRS that is in the linear span of these particular assets denominated in the US units of consumption
- Therefore, if the two projections are equal to each other (the latter should be scaled by X_{t+1} to match the above claim) then observed X would appear to offer model-free insights into the differences between $\tilde{M}$ and M^*
  - e.g., Brandt & Co state: "We show that marginal utility growths must ... be highly correlated across countries in order to explain the relative smoothness of exchange rates."
  - Can't establish this because the two projections are not equal

Does consumption risk affect currency returns?

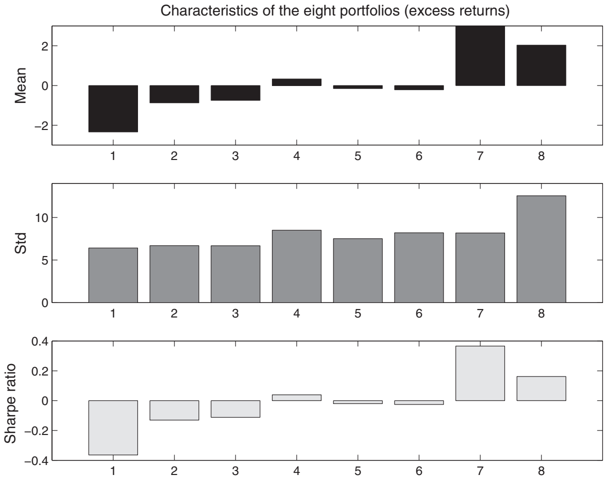
- Lustig and Verdelhan (2007) is an early paper on this topic, which documents a large spread in average returns of portfolios sorted on the interest rate differential. Is this due to exposure to consumption risk?

- Use the "observable" form of the pricing kernel

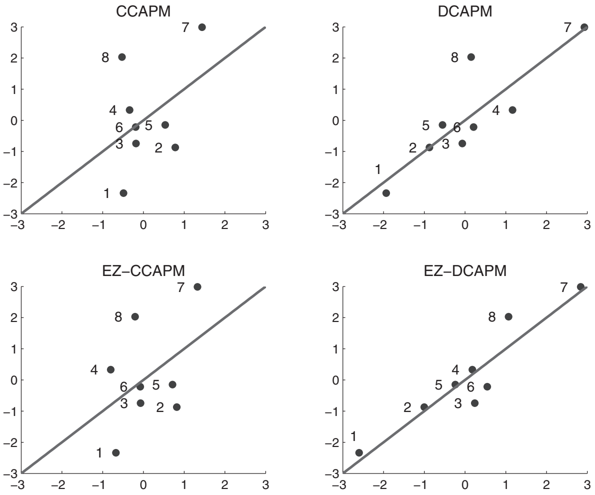
$$m_{t+1} = \log \beta \alpha/\rho + (\rho - 1)\alpha/\rho g_{t+1} + (\alpha - \rho)/\rho r_{w,t+1}$$

- This suggests the following strategy:
  - Form a beta representation
  - Run Fama-MacBeth. Are beta's significant?
  - Issues: time-series consistency, correlation between g_{t+1} and $r_{w,t+1}$

Properties of portfolios



Consumption CAPM



How does consumption risk and its prices change with horizon?

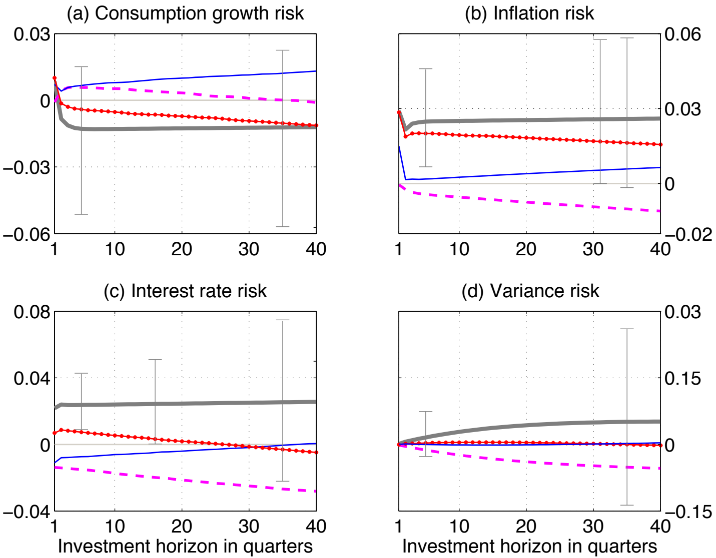
- Zviadadze (2015) pushes this further by exploring pricing effects at different horizons

- Define cashflow as $D_{t+1} = S_{t+1}/S_t \cdot P_t/P_{t+1}$, then

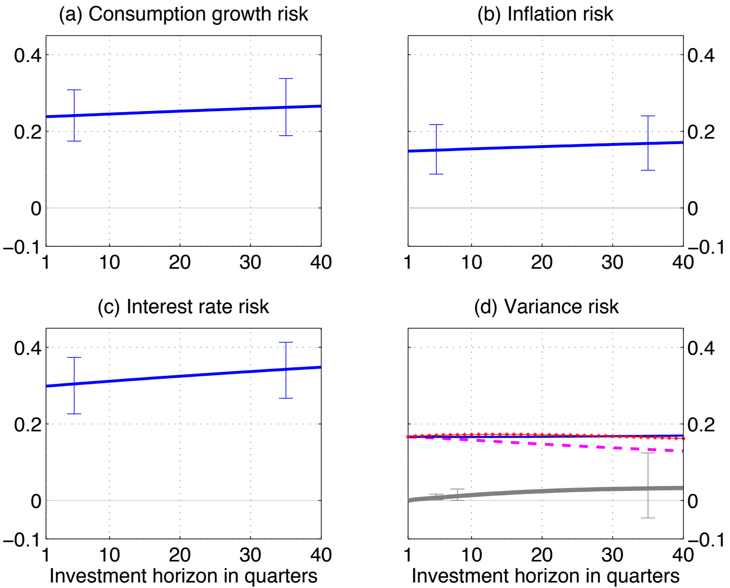
$$g^d_{i,t+1} = g^d_i + \eta_i^\top x_t + q_i^\top \sigma_t u_{t+1} + q_e \sigma_t e_{t+1}$$

- Once m is estimated jointly with x, we know all the variables, so can run an OLS, a nice byproduct of the Bayesian approach

Impulse responses

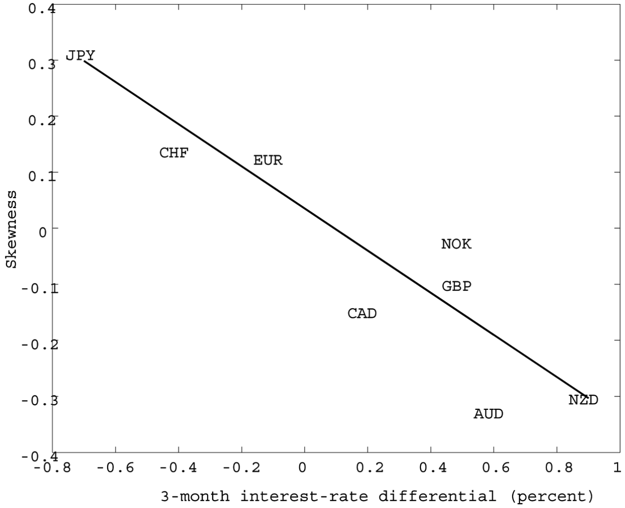


Marginal Sharpe ratios



23. FX Crash risk

Evidence from FX markets



Source: Brunnermeier, Nagel, and Pedersen (2008)

Evidence from FX markets

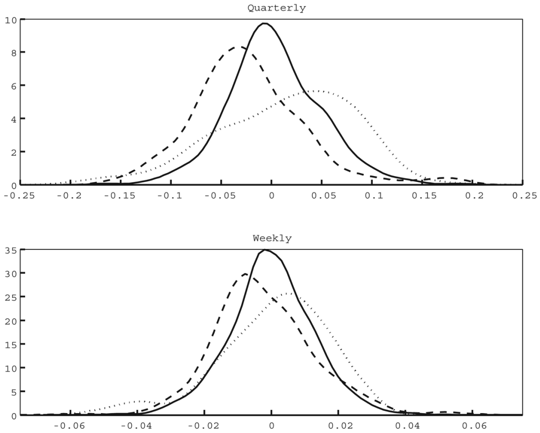


Fig. 3. Kernel density estimates of distribution of foreign exchange returns depending on interest rate differential. Interest rate differential groups after removing country fixed effects, quarterly (*top panel*): < −.005 (*dashed line*), −0.005 to 0.005 (*solid line*), > 0.005 (*dotted line*); weekly (*bottom panel*): < −0.01 (*dashed line*), −0.01 to 0.01 (*solid line*), > 0.01 (*dotted line*).

Source: Brunnermeier, Nagel, and Pedersen (2008)

Evidence from option markets

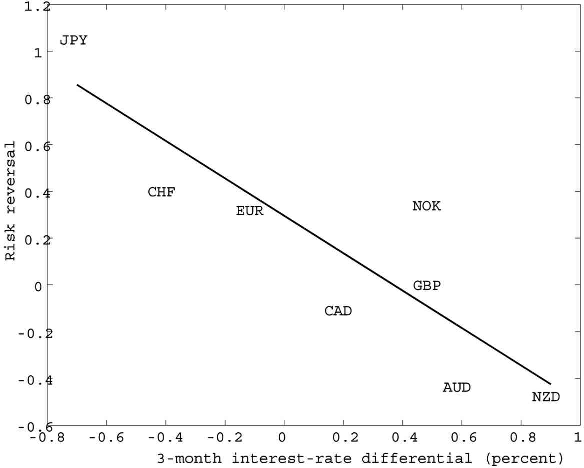
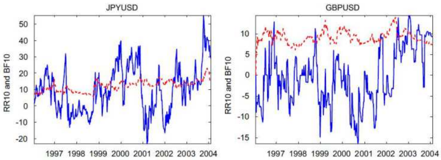


Fig. 2. Cross-section of empirical skewness (*top panel*) and of risk reversal (*bottom panel*), reflecting implied (risk-neutral) skewness, for different quarterly interest rate differentials $i^* - i$.

Evidence from option markets

- Carr and Wu (2007) document time variation in the risk reversals



- Jur (2014) wants to test UIP and hedges the downside of a carry position with options
- However, as we discussed,

$$p_t = f_t - E_t s_{t+1} = L_t(M^*_{t+1}) - L_t(M_{t+1})$$

- So, carry returns are not equal to zero even the pricing kernels are normal. The option-hedging strategy eliminates *all* risk.

A model?

- Take the Bilson-Fama-Tryon regression directly:

$$s_{t+1} - s_t = a_1 + a_2(f_t - s_t) + \varepsilon_{t+1} = a_1 + a_2(r_t - r_t^*) + \varepsilon_{t+1}$$

- Model:

$$\begin{aligned} s_{t+1} - s_t &= a_1 + a_2(r_t - r_t^*) + \text{any shock you like}, \\ -m_{t+1} &= r_t + \text{any shock you like} + 0.5 \cdot \text{variance of the shock}, \\ -m_{t+1}^* &= -m_{t+1} - (s_{t+1} - s_t) \end{aligned}$$

- What are the shocks?

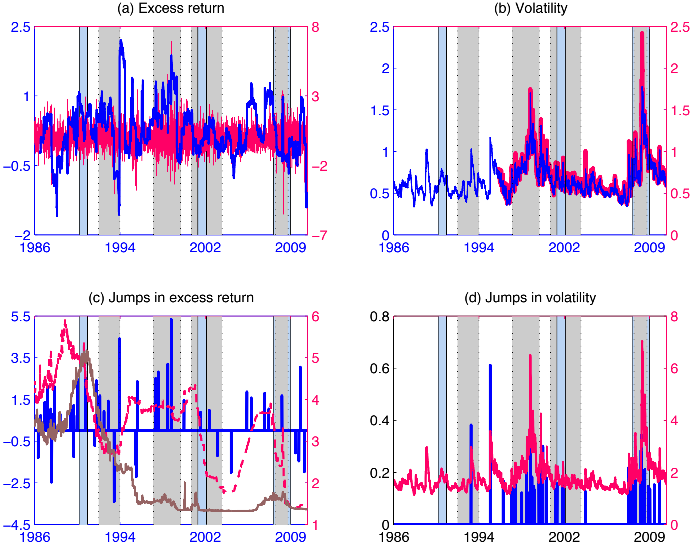
More direct evidence

- Chernov, Graveline, and Zviadadze (2016) use time series of currency returns and ATM IV to estimate

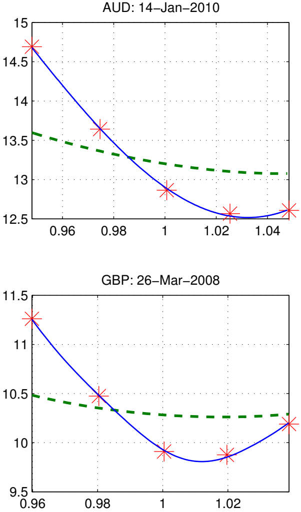
$$
\begin{aligned}
y_{t+1} &= \mu_0 + \mu_r(r_t - \tilde{r}_t) + \mu_v v_t + v_t^{1/2} w_{t+1}^s + z_{t+1}^u - z_{t+1}^d \\
v_{t+1} &= (1-\nu)v + \nu v_t + \sigma_v v_t^{1/2} w_{t+1}^v + z_{t+1}^v \\
h_t^u &= h_0 + h_r r_t,\ h_t^d = h_0 + h_r \tilde{r}_t,\ h_t^v = h_0^v + h_v v_t \\
z_t^{u,d} &\sim \mathcal{E}\mathrm{xp}(\theta),\ z_t^v \sim \mathcal{E}\mathrm{xp}(\theta_v)
\end{aligned}
$$

- Implications:
  - On average, 1.3 to 2.6 jumps in variance per year; average jump size increases vol by 20% to 40%
  - On average, 0.4 to 1.3 jumps in currencies per year; average jumps size is 1.2% to 1.6%
  - Third cumulant $\kappa_{3t}(s_{t+1} - s_t) = 6\theta^3 h_r(r_t - \tilde{r}_t)$
  - The loading $\mu_r \approx -3$ as in Fama's regression

JPY excess returns, estimated states, jump intensities



Is the jump risk priced?



Bibliography I

, 2014, Crash-neutral currency carry trades, *Journal of Financial Economics* 113, 325–347.

Asness, Cliff .S., Tobias Moskowitz, and Lasse H. Pedersen, 2013, Value and Momentum Everywhere, *Journal of Finance* 68, 929–986.

Backus, David K., Silverio Foresi, and Chris I. Telmer, 2001, Affine term structure models and the forward premium anomaly, *Journal of Finance* 56.

Bilson, John, 1981, The "speculative efficiency" hypothesis, *Journal of Business* 54, 435–451.

Brandt, Michael, John Cochrane, and Pedro Santa-Clara, 2006, International risk sharing is better than you think, or exchange rates are too smooth, *Journal of Monetary Economics* 53, 671–698.

Bibliography II

Brunnermeier, Markus K., Stefan Nagel, and Lasse H. Pedersen, 2008, Carry trades and currency crashes, *NBER Macroeconomics Annual* 23, pp. 313–348.

Burnside, Craig, and Jeremy Graveline, 2013, Exchange rate determination, risk sharing and the asset market view, Working paper, NBER.

Carr, Peter, and Liuren Wu, 2007, Stochastic skew in currency options, *Journal of Financial Economics* 86, 213–247.

Chernov, Mikhail, Jeremy Graveline, and Irina Zviadadze, 2016, Crash risk in currency returns, Journal of Financial and Quantative Analisys, forthcoming.

Colacito, Riccardo, and Mariano Croce, 2011, Risks for the long run and the real exchange rate, *Journal of Political Economy* 119, 153–182.

Bibliography III

Fama, Eugene F., 1984, Forward and spot exchange rates, *Journal of Monetary Economics* 14, 319–338.

Lustig, Hanno, and Adrien Verdelhan, 2007, The cross section of foreign currency risk premia and consumption growth risk, *American Economic Review* 97, 89–117.

Lustig, Hanno N., Nikolai Roussanov, and Adrien Verdelhan, 2011, Common risk factors in currency markets, *Review of FInancial Studies* 24, 3731–3777.

Tryon, Ralph, 1979, Testing for rational expectations in foreign exchange markets, Working paper, Federal Reserve.

Zviadadze, Irina, 2015, Term-structure of consumption risk premia in the cross-section of currency returns, Journal of Finance, forthcoming.