The common factor in idiosyncratic volatility: Quantitative asset pricing implications ☆

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ABSTRACT

We show that firms' idiosyncratic volatility obeys a strong factor structure and that shocks to the common idiosyncratic volatility (CIV) factor are priced. Stocks in the lowest CIV-beta quintile earn average returns 5.4% per year higher than those in the highest quintile. The CIV factor helps to explain a number of asset pricing anomalies. We provide new evidence linking the CIV factor to income risk faced by households. Our findings are consistent with an incomplete markets heterogeneous agent model. In the model, CIV is a priced state variable because an increase in idiosyncratic firm volatility raises the average household's marginal utility. The calibrated model matches the high degree of co-movement in idiosyncratic volatilities, the CIV-beta return spread, and several other asset price moments.

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1. Introduction

We present new empirical evidence regarding the behavior of idiosyncratic risk and consider the implications of this behavior for asset prices. First, the idiosyncratic volatilities of US firms are synchronized. Second, this common idiosyncratic volatility (CIV) is correlated with various measures of household labor income risk. Third, exposure to CIV shocks is priced in the cross section of stocks.

We then propose a heterogeneous agent model with incomplete markets that offers an economic rationale and quantitatively accounts for our findings. The key novelty in the model is that households' consumption risk inherits the same factor structure of the idiosyncratic cash flow risk of firms. Common fluctuations in firm-level risk thus enter the pricing kernel of households and, as a result, CIV is a priced state variable. Stocks that tend to appreciate when CIV rises are valuable hedges to increases in households' marginal utility and earn relatively low average returns, consistent with our empirical findings.

We start by showing an extraordinary degree of co-movement among the idiosyncratic volatilities of more than 20 thousand Center for Research in Security Prices

(CRSP) stocks over a long sample spanning 1926–2010. A single factor explains 35% of the time variation firm-level idiosyncratic risk. At first sight, a factor structure in return volatilities might not appear surprising. After all, many finance theories posit that returns are linear functions of common factors, and, if the factors themselves have time-varying volatility, then firm volatility naturally inherits this factor structure.[1] However, we emphasize that this is co-movement in idiosyncratic volatility, defined as the standard deviation of residuals from factor model regressions. Volatility co-movement does not arise from omitted factors. Even after saturating the factor regression with up to ten principal components (and showing that model residuals are virtually uncorrelated), the residual firm volatilities continue to display the same co-movement seen in raw return volatilities.

We also show that co-movement in volatilities is a feature not only of returns, but also of the volatility of firm-level cash flows. We estimate volatilities of firm fundamentals such as sales or earnings growth using quarterly Compustat data. Although these volatility estimates are noisier and less frequently observed than those for stock returns, we again find a strong factor structure among firms' idiosyncratic cash flow volatilities. The common factor in fundamental volatility follows the same low frequency patterns as the common factor in idiosyncratic return volatilities. The two have a correlation of 65%. This suggests that return volatility patterns identified in this paper are not solely attributable to shocks to investor preferences or other sources of pure discount rate variation.[2] Instead, they measure the volatility of persistent idiosyncratic cash flow growth driven by firm-level productivity and demand shocks.

Persistent, idiosyncratic cash flow shocks that hit firms are an important source of undiversifiable risk to households. We present evidence showing that the CIV factor proxies for idiosyncratic risk faced by households. Individual income data from the US Social Security Administration from Guvenen, Ozkan, and Song (2014) show that the cross-sectional dispersion in household earnings growth rises and falls with CIV measured from stock returns. They share a correlation of nearly 60% in changes. Similarly, dispersion in firm-level employment growth (from Compustat) and in sector-level employment growth of both private and publicly firms (from the Federal Reserve) are also strongly correlated with CIV. Finally, CIV shocks are positively correlated with shocks to the dispersion in wage and house price growth across metropolitan areas.

How are persistent firm-level shocks transferred to households? Perhaps the main source of transmission is through the labor income that households derive from firms that employ them. For example, when workers possess firm-specific human capital, shocks to firm value are also shocks to workers' human wealth (Becker, 1962). Other transmission channels include under-diversified equity positions in own-employer stock and the influence of firm performance on local wages and residential real estate values. And while firms provide employees with some temporary insurance against idiosyncratic productivity shocks, workers have little protection against persistent shocks, which ultimately affect compensation through either wages or layoffs.[3] Because households cannot completely insulate their consumption from persistent shocks to their labor income (Blundell, Pistaferri, and Preston, 2008), the volatility of households' consumption growth distribution inherits the same factor structure as the volatility in firm-level returns and cash flow growth.[4] As the volatility of firm-level growth rates rises, investors face more idiosyncratic risk that is not fully hedged, increasing the dispersion of their consumption growth rates. Because increases in CIV represent an increase in consumption risk for the average household, they adversely affect its marginal utility.

This effect of a change in CIV on the marginal utility of the average investor is reflected in asset price data. Differences in firms' betas on CIV shocks are strongly associated with differences in expected returns. The top CIV-beta quintile earns average returns 5.4% per annum lower than firms in the bottom quintile. We show that this fact is not due to high CIV-beta firms having high exposure to the market return, a size or value factor, or a market variance factor. Instead, incorporating CIV innovations as a new asset pricing factor can account for the CIV-beta return spread and also helps to explain the cross-sectional differences in average returns on book-to-market, size, earning-to-price, and corporate bond portfolios. Replacing CIV with a factor based on the cross-sectional dispersion in household income growth or in firm size growth recovers many of the same asset pricing facts. This provides additional evidence for the connection between household risk and CIV.

Finally, we rationalize these empirical facts regarding idiosyncratic volatility co-movement and asset prices in a heterogeneous agent incomplete markets model. In our specification, households' equilibrium idiosyncratic consumption growth process possesses the same volatility factor structure as firm-level cash flow growth. We derive equilibrium asset prices on stocks with cash flow growth that features common idiosyncratic volatility and with different exposures to CIV shocks. In the model, CIV shocks carry a negative market price of risk. Our calibration shows that the return spread on high-minus-low CIV-beta stocks observed in the data is quantitatively consistent with the model, as are the return volatilities on the CIV-beta-sorted stocks. We also match the cross-sectional dispersion of household income growth, the mean and persistence of CIV, the cross-sectional spread in CIV betas, and the equity

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[1] Prominent factor models in finance include the capital asset pricing model (CAPM) (Sharpe, 1964), intertemporal CAPM (Merton, 1973), arbitrage pricing theory (APT) (Ross, 1976), and the Fama and French (1993) model.

[2] Pástor and Veronesi (2005, 2006) suggest that time variation in stochastic discount factor volatility (and hence market return volatility) can drive time variation in idiosyncratic stock return volatility.

[3] See, e.g., Berk, Stanton, and Zechner (2010), Lustig, Syverson, and Van Nieuwerburgh (2011), and Zhang (2015).

[4] Heathcote, Storesletten, and Violante (2014) estimate that over 40% of persistent labor income shocks are passed through to household consumption.

risk premium on the market portfolio, while respecting the properties of aggregate consumption and interest rates. One important reason that stocks have negative CIV-betas in the model is that their dividend growth rate is negatively predictable by the lagged CIV factor. We provide direct evidence for this cash flow predictability channel and calibrate the model to be consistent with it.

The connection between firm-level volatility, household-level risk, and asset prices established in our paper suggests that CIV is a plausible proxy for dispersion in consumption growth, which is the key ingredient in heterogenous agent asset pricing models. The advantage of CIV as a household risk proxy is that it is constructed from reliably measured stock return data and is observable at high frequencies, in contrast to the noisy and infrequent consumption survey data typically studied.

This paper relates to several strands of research. The arbitrage pricing theory (APT) of Ross (1976) shows that any common return factors are valid candidate asset pricing factors. The idiosyncratic return residuals are not priced, because they can be diversified away. This result breaks down in a world with non-traded assets, such as human wealth, in which case factors driving commonality in residual volatility can be valid asset pricing factors. We show that CIV is such an asset pricing factor.

Several representative agent models explore the role of aggregate consumption growth and market return volatility for explaining asset pricing stylized facts.[5] We focus on exposure to idiosyncratic volatility instead, which in our model is not fully diversifiable. To make this distinction clear, the model features a constant market variance. In our model, investors seek to hedge against idiosyncratic volatility shocks even if they are indifferent about the timing of uncertainty resolution, though a preference for early resolution of uncertainty magnifies the price of CIV risk. In our empirical work, we contrast the separate asset pricing roles of the CIV and market variance (MV) factors. While both help to explain the cross section of returns, the CIV factor appears to be the stronger pricing factor.

Our model builds on Mankiw (1986) and Constantinides and Duffie (1996), who explore counter cyclical dispersion in consumption growth as a mechanism to increase the equilibrium equity premium. As an empirical matter, we find that the market portfolio has little exposure to changes in CIV, making exposure to CIV unsuitable to explain the equity risk premium. In contrast, CIV exposure is useful to explain the cross section. Constantinides and Ghosh (2014) explore the asset pricing implications of counter cyclical left-skewness in the cross-sectional distribution of household consumption growth, but they do not study the association with the common idiosyncratic volatility of firms, which is the focus of our paper.[6]

Testing such consumption-based incomplete markets models is challenged by the poor quality of household-level consumption data.[7] Our incomplete markets model, which ties together idiosyncratic risks of firms and households, allows us to avoid consumption survey data. It suggests CIV, measured from daily stock returns, as a reliable high-frequency alternative to measure the consumption growth dispersion. This opens up new possibilities for empirical work that aims to test consumption-based asset pricing models and for research that studies the connection between inequality and financial markets. This proxy could be of independent interest to the consumption literature.[8]

Finally, our paper connects to a large empirical literature that studies the role of idiosyncratic return volatility. Campbell, Lettau, Malkiel, and Xu (2001) examine secular variation in average idiosyncratic return volatility but do not study its cross-sectional properties.[9] Ang, Hodrick, Xing, and Zhang (2006) show that stocks with high idiosyncratic volatility earn abnormally low average returns, while Chen and Petkova (2012) argue that this fact can be explained with an average volatility factor. Double-sorted portfolios on CIV-beta and idiosyncratic volatility display return spreads in both directions, so that both anomalies coexist. Gilchrist and Zakrajsek (2010) and Atkeson,

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Eisfeldt, and Weill (2013) study the firm volatility distribution to understand credit risk and debt prices. Kelly, Lustig, and Van Nieuwerburgh (2013) propose a network model in which firms are connected to other firms in a customer supplier network and show that size-dependent network formation generates a common factor in firm-level idiosyncratic volatility.

The rest of the paper is organized as follows. Section 2 describes the common idiosyncratic variance factor in US stock returns and firm-level cash flows. Section 3 provides evidence linking the CIV factor to dispersion in household income shocks. Section 4 demonstrates that CIV is a priced factor in the cross section of stock returns. Section 5 presents and calibrates the heterogeneous agent model with CIV as priced state variable. Section 6 concludes.

2. The factor structure in volatility

In this section, we study idiosyncratic volatility in the annual panel of US public firms. We first discuss data and how we construct volatilities and then describe the behavior of the volatility panel.

2.1. Data construction

We construct annual volatility of firm-level returns and cash flow growth. Return volatility is estimated using data from the CRSP daily stock file from 1926 to 2010. It is defined as the standard deviation of a stock's daily returns within the calendar year.[10] We refer to these estimates as total return volatility.

Idiosyncratic volatility is the focus of our analysis. Idiosyncratic returns are constructed within each calendar year τ by estimating a factor model using all observations within the year. Our factor models take the form

$$r_{i,t} = \gamma_{0,i} + \gamma_i' F_t + \varepsilon_{i,t} \tag{1}$$

where t denotes a daily observation in year τ. Idiosyncratic volatility is then calculated as the variance of residuals $\varepsilon_{i,t}$ within the calendar year. The result of this procedure is a panel of firm-year idiosyncratic volatility estimates. The first return factor model that we consider is the market model, specifying that F_t is the return on the CRSP value-weighted market portfolio. The second model specifies F_t as the 3×1 vector of Fama and French (1993) factors. The third return factor model we use is purely statistical and specifies F_t as the first five principal components (PCs) of the cross section of returns within the year.[11]

Total fundamental volatility in year τ is estimated for all CRSP and Compustat firms using the 20 quarterly year-on-year sales growth observations for calendar years $\tau - 4$ to τ.[12] We also estimate idiosyncratic volatility of firm fundamentals based on factor specifications. Because the literature has no predominant factor model for sales growth, we consider only principal component factors. The approach is the same as in Eq. (1), with the exception that the left-hand-side variable is sales growth and the data frequency is quarterly. F_t contains the first K principal components of growth rates within a five-year window ending in year τ, where K equals one or five and residual volatility in year τ is the standard deviation of model residuals over the five-year estimation period. The sales growth volatility panel covers 1975–2010.

2.2. The cross-sectional distribution of volatility

The cross-sectional distribution of estimated firm-level volatility is lognormal to a close approximation.[13] An attractive implication of this result is that dynamics of the entire cross-sectional distribution of firm volatility can be described with only two time-varying parameters: the cross-sectional mean and standard deviation of log volatility.[14] It also demonstrates that the average volatility levels calculated throughout this paper are not driven by extreme behavior in cross-sectional distributions.

2.3. Common secular patterns in firm-level volatility

We now study commonality in return-based and fundamentals-based measures of firm-level volatility.

2.3.1. Return volatility
Firm-level volatilities share an extraordinary degree of common time variation. Panel A of Fig. 1 plots annual firm-level total return volatility averaged within start-of-year size quintiles. Stocks of all sizes demonstrate very similar

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[10] A firm-year observation is included if the stock has a CRSP share code 10, 11 or 12 and the stock has no missing daily returns within the year.
[11] Robustness tests using ten PCs produce quantitatively similar results. Hence, we focus our presentation on five PCs. If the true underlying factor structure is nonlinear, then the linear regression models we estimate are misspecified. However, the principal components approach captures nonlinear dependencies to some extent through the inclusion of additional principal components. When using five or ten components, we find qualitatively identical results to those from lower-dimension factor models such as Fama and French, which suggests that model misspecification is unable to explain our findings. In addition, results are

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(footnote continued)
qualitatively unchanged when we allow for generalized autoregressive conditional heteroskedasticity (GARCH) residuals in factor model regressions.
[12] The data requirements for a non-missing sales growth volatility observation in year τ are analogous to those for returns. We use all Compustat firms linked to CRSP and possessing share code 10, 11 or 12, and we require a firm to have no missing observations in the 20-quarter window ending in year τ.
[13] Fig. A1 in the Appendix plots histograms of the empirical cross-sectional distribution of firm-level volatility (in logs). Panel A shows the distribution of total return volatility pooling all firm-years from 1926 to 2010. Panel B shows the distribution of total sales growth volatility (in logs) pooling all firm-years from 1975 to 2010. Panels C and D plot histograms of idiosyncratic return and sales growth volatility based on the five principal components factor model. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Log volatilities demonstrate only slight skewness (less than 0.4) and do not appear to be leptokurtic (kurtosis between 2.9 and 3.2). The cross-sectional volatility distribution also appears lognormal in one-year snapshots of the cross section, as shown in Appendix Fig. A2 for the 2010 calendar year.
[14] The distribution of idiosyncratic return volatility estimated from the market model and the Fama and French three-factor model are qualitatively identical to those shown in Fig. A1.

Fig. 1. Total and idiosyncratic return volatility by size and industry group. The figure plots annualized firm-level volatility averaged within size and industry groups. Within each calendar year, volatilities are estimated as the standard deviation of daily returns for each stock. Panel A shows firm-level total return volatility averaged within market equity quintiles. Panel B shows total return volatility averaged within the five-industry categorization based on the standard industry classification (SIC) provided by Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). Panels C and D report the same within-group averages of firm-level idiosyncratic volatility. Idiosyncratic volatility is the standard deviation of residuals from a five-factor principal components model for daily returns.

secular time series volatility patterns. The same is true of industry groups. Panel B reports average total return volatility among stocks in the five-industry categorization based on the standard industry classification (SIC) provided by Kenneth French's website.[15]

The common time series variation of total return volatilities by size and industry groups is perhaps unsurprising given that firm-level returns are believed to have a substantial degree of common return variation, as evidenced by the predominance of factor-based models of individual stock returns. If returns have common factors and the volatility of those factors varies over time, then firm-level variances also inherit a factor structure.

The fact that volatilities of residuals display the same degree of common variation after removing common factors from returns is surprising. Panels C and D of Fig. 1 plot average idiosyncratic volatility within size and industry groups based on residuals from a five principal components factor model for returns. The plots show the same dynamics for all groups of firms when considering idiosyncratic instead of total volatility. The correlation between average idiosyncratic volatility within size quintiles 1 and 5 is 81%. The minimum correlation among idiosyncratic volatilities of the five industry groups is 65%, which corresponds to the health care industry versus the "other" category (including construction, transportation, services, and finance).

Common variation in idiosyncratic volatility cannot be explained by co-movement among factor model residuals, for instance, due to omitted common factors. Panel A of

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[15] http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/

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B
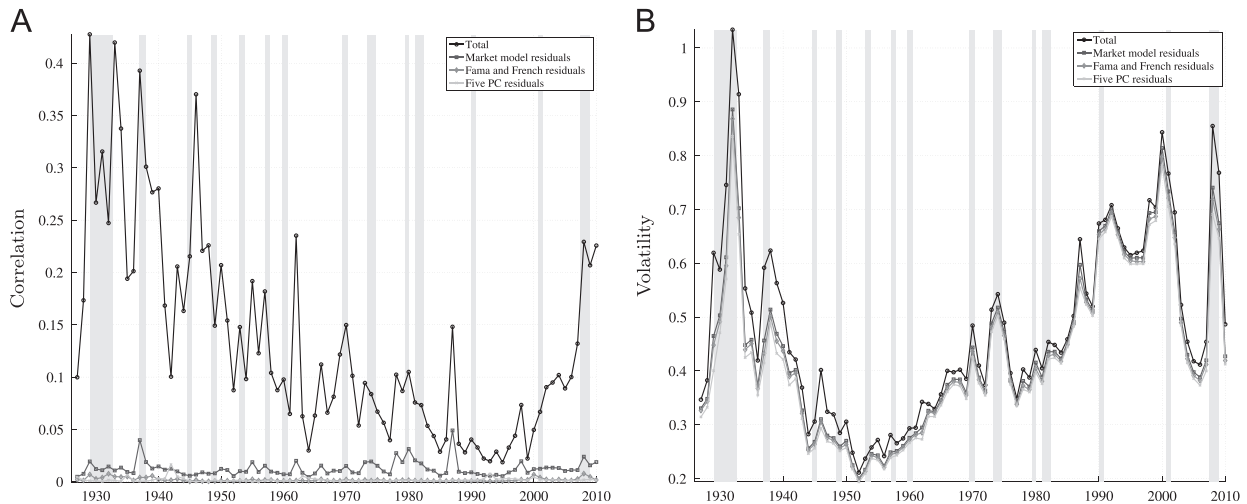
Fig. 2. Volatility and correlation of return factor model residuals. Panel A shows the average pairwise correlation for total and idiosyncratic returns within each calendar year. Panel B shows the cross-sectional average annualized firm-level volatility each year for total and idiosyncratic returns. Idiosyncratic volatility is the standard deviation of residuals from the market model, the Fama and French three-factor model, or a five-factor principal components (PCs) model for daily returns within each calendar year.

Fig. 2 shows that raw returns share substantial common variation, with an average pairwise correlation of 13% over the 1926–2010 sample (and occasionally exceeding 40%). However, the principal components model captures nearly all of this common variation at the daily frequency, as average correlations among its residuals are typically less than 0.2% and are never above 0.9% in a year. The same is true for the market and the Fama and French models. Moving to a higher number of principal components, such as ten, has no quantitative impact on these results. The Fama and French model and the five principal components model appear to absorb all of the co-movement in returns, making omitted factors an unlikely explanation for the high degree of commonality in idiosyncratic volatilities.

Despite the absence of co-movement among residual return realizations, Panel B of Fig. 2 shows that average idiosyncratic volatility from various factor models is nearly the same as average volatility of total returns. In the typical year, only 11% of average total volatility is accounted for by the five principal components factor model, with idiosyncratic volatility inheriting the remaining 89%. The same is true for the market model and the Fama and French model, with 8% and 9% of average volatility explained by common factors, respectively.[16]

The analyses in Figs. 1 and 2 use return volatility estimated from daily data within each year.[17] We find very similar results when estimating volatility from 12 monthly return observations within each year. First, firm-level total and idiosyncratic return volatilities share a high degree of co-movement. The average pairwise correlation of

idiosyncratic volatility among size quintiles is 84% annually, and the average correlation of idiosyncratic volatility for industry groups is 83% (based on residuals from the five principal components model). Second, the vast majority of correlation among monthly returns is absorbed by common factors. Pairwise stock return correlations are 30% on average based on monthly data, dropping to 0.4% for monthly Fama and French model residuals. Third, most of the average firm's volatility is left unexplained by common factors. The ratio of average Fama and French residual volatility to average total volatility is 67%. See Appendix Fig. A3 for details.

The strong co-movement of return volatility is also a feature of portfolio returns. Fig. A4 in the Appendix reports average volatility and average pairwise correlations for total and residual returns for one hundred Fama and French size and value portfolios. Portfolio return volatilities show a striking degree of co-movement across the size and book-to-market spectrum, even after accounting for common factors. Like the individual stock results, common idiosyncratic volatility patterns are unlikely to be driven by omitted common return factors.

2.3.2. Fundamental volatility

Fig. 3 reports average yearly sales growth volatility by size quintile and industry in Panels A and B, respectively. As in the case of returns, firm sales growth data display a high degree of volatility commonality. The average pairwise correlation among size and industry groups is 85% and 53%, respectively. Panels C and D show within-group average idiosyncratic volatility estimated from a five-factor principal components model for sales growth. Co-movement in residual sales growth volatility is nearly identical to total sales growth volatility. This is because total sales growth rates have very low average pairwise correlations (1.6% annually), as shown in Fig. 4. After accounting for one sales growth principal component, the average pairwise correlation drops to 0.7%.

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[16] We calculate the percent of average volatility explained by common factors as one minus the ratio of average factor model residual volatility to average total volatility, where averages are first computed cross-sectionally and then averaged over the full 1926–2010 sample.

[17] It is standard practice in the literature to estimate idiosyncratic volatility from daily data. See, for example, Ang, Hodrick, Xing, and Zhang (2006).
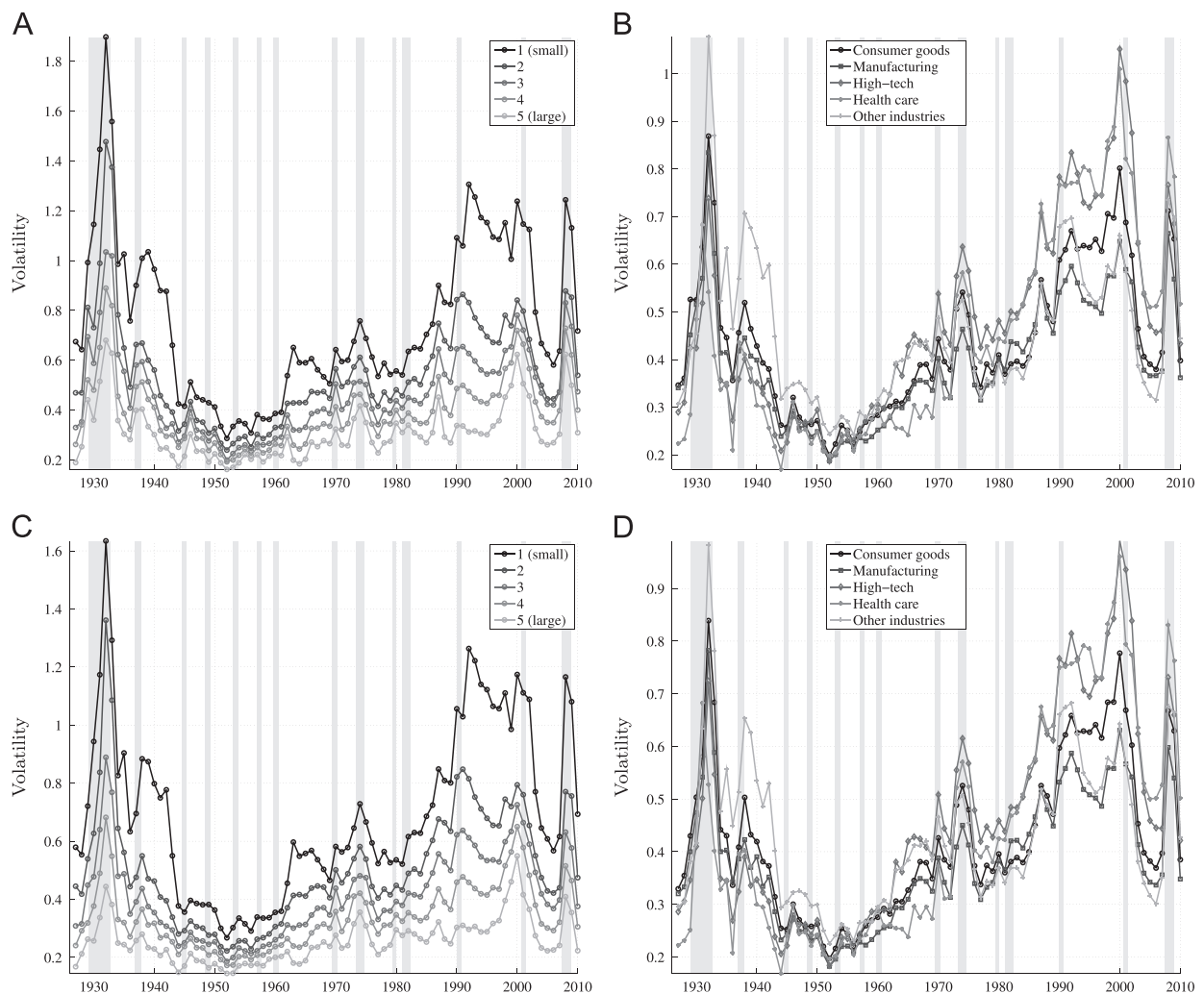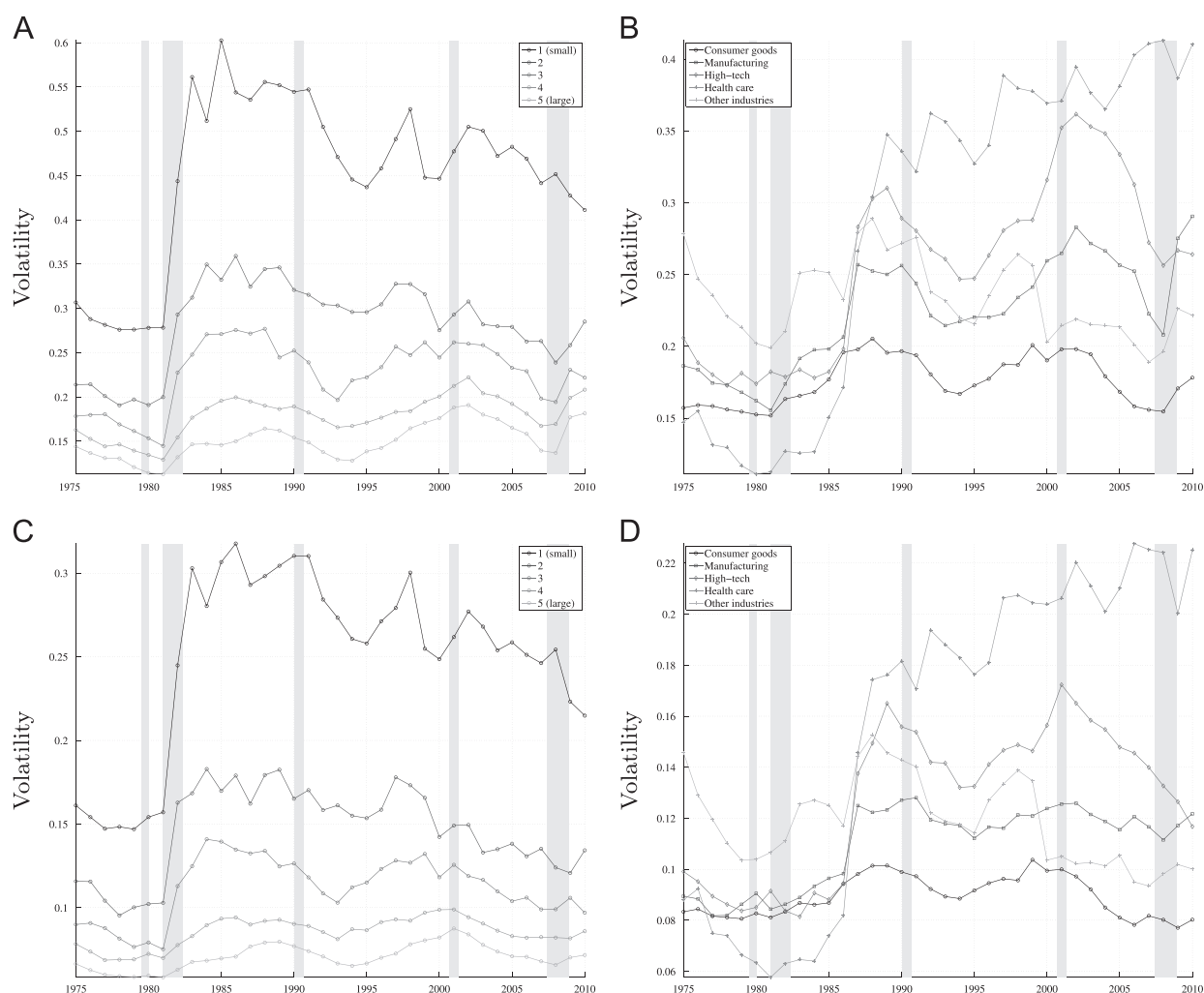
Fig. 3. Total and idiosyncratic sales growth volatility by size and industry group. The figures plot firm-level volatility averaged within size and industry groups. For each calendar year τ, volatilities are estimated as the standard deviation of 20 quarterly year-on-year sales growth observations in years $\tau - 4$ to τ for each firm. Panel A shows firm-level total volatility averaged within market equity quintiles. Panel B shows total volatility averaged within the five-industry categorization based on the standard industry classification (SIC) provided by Kenneth French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/). Panels C and D report the same within-group averages of firm-level idiosyncratic volatility. Idiosyncratic volatility is the standard deviation of residuals from a five-factor principal components model for quarterly sales growth. The components are estimated in the same 20-quarter window used to calculate volatility.

This fundamental volatility behavior is not only specific to sales growth but is also true of other measures of firm cash flows. For example, the average variance of firms' net income growth is 67.4% correlated with that of sales growth.[18] In summary, strong co-movement is not unique to return volatilities but appears to be a feature of fundamental volatility.

2.4. Volatility factor model estimates

We next estimate factor regression models for firm-level volatility. We consider total volatility as well as idiosyncratic volatility estimated from a Fama and French three-factor model or a five-factor principal components model. In all cases, time series regressions are run firm-by-firm with volatility as the left-hand-side variable. The factor in each set of regressions is defined as the equal-weighted average of the left-hand-size volatility measure. This is approximately equal to the first principal component of a given volatility panel but avoids principal components complications arising from unbalanced panels.

Panel A of Table 1 reports volatility factor model results for daily return volatilities.[19] Columns correspond to the method used to construct return residuals. The average univariate time series R^2 is 36.2% for the total volatility model and close to 35% for the idiosyncratic volatility

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[18] The common net income variance factor explains panel variation in firm-level net income variance with an R^2 of 28.5% on average. See Appendix Table A1, Panels B and C, for volatility factor model estimates based on net income growth and earnings before interests, taxes, depreciation, and amortization (EBITDA) growth.

[19] Average intercept and slope coefficients differ from zero and one due to unbalanced panel data.

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B
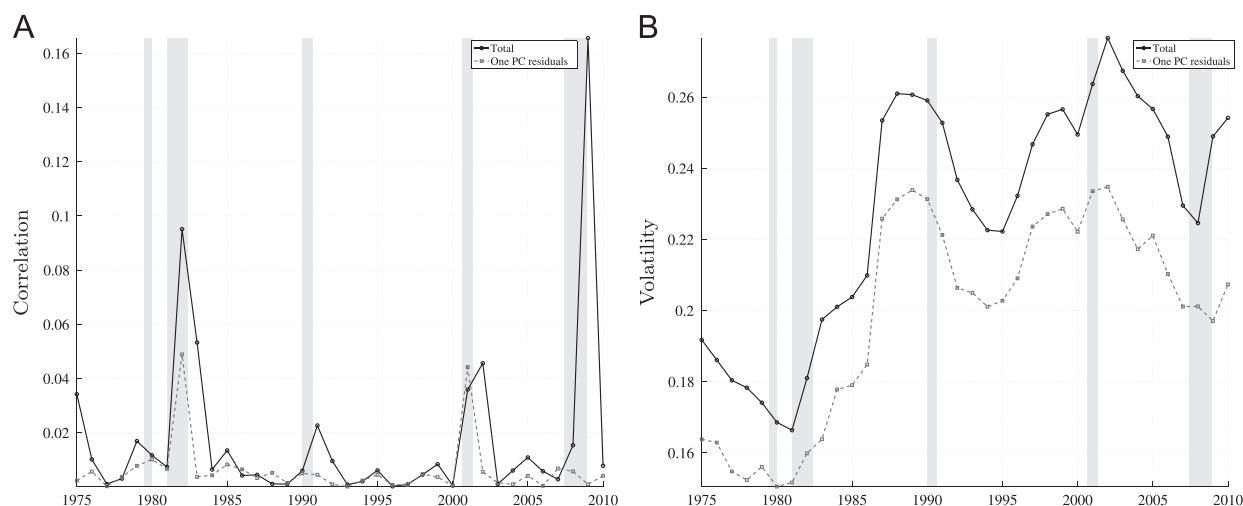
Fig. 4. Volatility and correlation of total and idiosyncratic sales growth. Panel A shows the average pairwise correlation for total and idiosyncratic sales growth within a 20-quarter window through the end of each calendar year. Panel B shows cross-sectional average firm-level volatility each year for total and idiosyncratic sales growth. Idiosyncratic volatility is the standard deviation of residuals from a one-factor principal components (PCs) model for quarterly sales growth. The components are estimated in the same 20-quarter window used to calculate volatility.

models. The pooled panel ordinary least squares (OLS) R^2 is between 33% and 35% (relative to a volatility model with only firm-specific intercepts).[20]

In Panel B of Table 1, we show volatility factor model estimates for sales growth volatility. The first three columns report results based on panels of total volatility and idiosyncratic volatility from one and five principal component models. The last column reports model estimates for an annual volatility panel that estimates volatility from only four quarterly year-on-year sales growth observations within each year.[21] Due to the small number of observations used to construct sales growth volatility, we could expect poorer fits in these regressions, yet the results are closely in line with those for return volatility. The time series R^2 for raw and idiosyncratic growth rate volatility ranges between 17.8% and 29.9% on average. The pooled R^2 reaches as high as 31.5%.

The common factor in fundamental volatility follows the same low frequency patterns as the common factor in idiosyncratic return volatilities, sharing a correlation of 64.6% with the common factor in Fama and French residual return volatility. This suggests that the return volatility patterns identified in this section are not attributable to discount rate shocks, instead, they measure the volatility of persistent idiosyncratic cash flow growth shocks at the firm level. If the shocks were largely transitory, they would have only a minor impact on returns.

Panel A of Fig. 5 plots the level of average annualized idiosyncratic volatility (labeled CIV) against the volatility of the value-weighted market portfolio (MV). The two series possess substantial common variation, particularly associated with deep recessions at the beginning and end of the sample (correlation of 63.8% in levels). Panel B reports changes in CIV, as well as residuals from a regression of CIV changes on changes in MV. The two sets of innovations share a correlation of 67.0%, indicating that the behavior of idiosyncratic volatility shocks is in large part distinct from shocks to market volatility. The asset pricing tests Section 3 show important additional differences in the behavior of CIV and MV.

3. Idiosyncratic risk of the firm and the household

The evidence presented in Section 2 indicates that firm-level idiosyncratic volatilities possess a high degree of comovement that is aptly described by a factor model. The commonality in firms' idiosyncratic risks hints at the possibility that income and consumption growth realizations experienced by households also possess common variation in their second moments. That is, households can face common fluctuations in their idiosyncratic risks, even though their individual consumption growth realizations can be (conditionally) uncorrelated. This seems plausible because shocks to households' labor income, human capital, and financial capital derive in large part from shocks to their employers. In this section, we investigate the empirical association between fluctuations in firm-level idiosyncratic volatility and idiosyncratic consumption and income risk faced by households.

3.1. Theoretical connection

Individual household income and individual firm performance are linked through a number of channels.

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[20] For portfolios, not individual stock returns, we find even higher volatility factor model R^2 values. Based on the Fama and French one hundred size and value portfolios, the average univariate R^2 is 70.8% for total volatility, 49.7% for market model residual volatility, and 39.4% for Fama and French model residual volatility (see Panel A of Appendix Table A1).

[21] This model avoids the issue of overlapping regression observations that arises from our rolling 20-quarter volatility used elsewhere. The similarity of these results with the first three columns of Panel B indicate that the strong factor structure in sales growth volatility is not an artifact of overlapping observations.

Table 1

Volatility factor model estimates.

   The table reports estimates of annual volatility one-factor regression models. In each panel, the volatility factor is defined as the equal-weighted cross-sectional average of firm volatilities within that year. That is, all estimated volatility factor models take the form: $\sigma_{i,t} = \text{intercept}_i + \text{loading}_i \cdot \bar{\sigma}_{.,t} + e_{i,t}$. Columns represent different volatility measures. For returns (Panel A), the columns report estimates for a factor model of total return volatility and idiosyncratic volatility based on residual returns from the market model (MM), the Fama and French model (FF), or the five principal components (PCs) models. For sales growth (Panel B), the columns report total volatility or idiosyncratic volatility based on sales growth residuals from the one and five principal components model (using a rolling 20-quarter window for estimation). The last column of Panel B reports results when using only the four quarterly growth observations within each calendar year to estimate total volatility. We report cross-sectional averages of loadings and intercepts as well as time series regression R^2 averaged over all firms. We also report a pooled factor model R^2, which compares the estimated factor model with a model with only firm-specific intercepts and no factor.

Panel A: Returns

	Total	MM	FF	Five PCs
Loading (average)	1.012	1.024	1.032	1.031
Intercept (average)	0.006	0.005	0.004	0.004
R^2 (average univariate)	0.362	0.347	0.346	0.348
R^2 (pooled)	0.345	0.337	0.339	0.347

Panel B: Sales growth

	Total (five years)	One PC (five years)	Five PCs (one year)	Total (one year)
Loading (average)	0.885	1.149	1.249	0.884
Intercept (average)	0.044	−0.018	−0.024	0.030
R^2 (average univariate)	0.293	0.299	0.299	0.178
R^2 (pooled)	0.303	0.315	0.304	0.168

First, households can be directly exposed to the equity risk of their employers. A large theoretical literature beginning with Jensen and Meckling (1976) predicts that management will hold under-diversified positions in their employers' stock for incentive reasons. This prediction is born out empirically.[22] Benartzi (2001), Cohen (2009), and Van Nieuwerburgh and Veldkamp (2006) show that non-manager employees also tend to overallocate wealth to equity of their employer and offer behavioral or information-based interpretations for this phenomenon.[23]

Firm-specific human capital (Becker, 1962) is a second channel tying household idiosyncratic outcomes to those of the firm. Hashimoto (1981, p. 475) notes: "The standard analysis of firm-specific human capital argues that the cost of and the return to the investment will be shared by the worker and the employer." The typical employee's wealth is dominated by her human capital (Lustig, Van Nieuwerburgh, and Verdelhan, 2013), implying that shocks to an employer's firm value and human wealth shocks of its employees move in tandem. This mechanism is empirically shown in Neal (1995) and Kletzer (1989, 1990). These studies also emphasize that the firm-specific human capital mechanism leads to protracted and potentially permanent income impairment following job

displacements, consistent with the evidence in Ruhm (1991) and Jacobson, LaLonde, and Sullivan (1993). Furthermore, the probability of job displacement, defined by Kletzer (1998, p. 116) as "a plant closing, an employer going out of business, a layoff from which he/she was not recalled," is directly tied to firm performance. In addition to job loss risk, the empirical findings of Brown and Medoff (1989) suggest that employees at larger firms enjoy a wage premium, presenting a mechanism through which employee income shocks can be correlated with idiosyncratic firm shocks. Theoretical work of skilled labor compensation in Harris and Holmstrom (1982), Berk, Stanton, and Zechner (2010), and Lustig, Syverson, and Van Nieuwerburgh (2011) finds that employers optimally insure some but not all productivity shocks, leaving employee compensation exposed to firm-level shocks.

A large literature finds that shocks to individual labor income growth translate into shocks to individual consumption growth because of incomplete risk sharing. Blundell, Pistaferri, and Preston (2008) and Heathcote, Storesletten, and Violante (2014) show that permanent shocks to labor income end up in consumption, while transitory shocks are partially insured.

3.2. Empirical evidence

Our empirical analysis suggests that common idiosyncratic return volatility is a plausible proxy for idiosyncratic risk faced by individual consumers. We present four new results consistent with this interpretation.

Our first result shows a significant association between shocks to firm-level idiosyncratic risk and data on idiosyncratic household income risk. Our measure of idiosyncratic firm risk is the equally weighted average of

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[22] See Demsetz and Lehn (1985), Murphy (1985), Morck, Shleifer, and Vishny (1988), Kole (1995), and others.

[23] A large literature including French and Poterba (1991), Coval and Moskowitz (1999), and Calvet, Campbell, and Sodini (2007) indicates that the typical household is exposed to idiosyncratic stock risk above and beyond that due to overallocation of savings to the equity of one's employer. Home and local bias represent additional ways through which domestic and local shocks that hit firms affect the financial income of households.
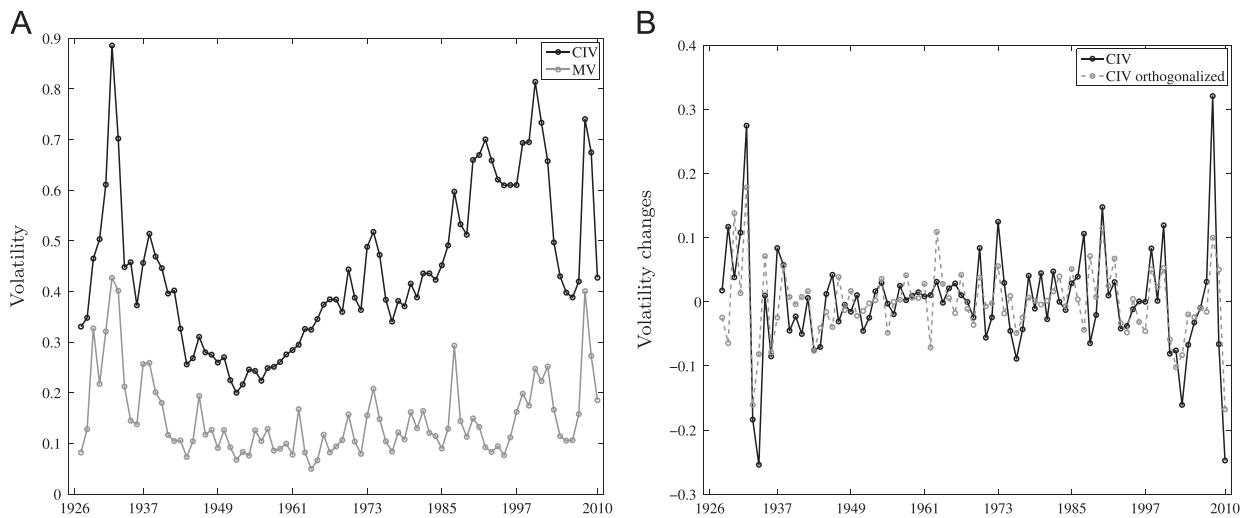
Fig. 5. Common idiosyncratic volatility (CIV): levels and changes. Panel A plots the average annual volatility of the Center for Research in Security Prices (CRSP) value-weighted market return (MV) and the cross-sectional average volatility of market model residuals (CIV). Panel B plots annual changes in CIV and CIV changes orthogonalized against annual changes in MV.

firm-level market model residual return variance.[24] The highest quality household income growth data come from the US Social Security Administration, Guvenen, Ozkan, and Song (2014) report cross-sectional summary statistics each year from 1978 to 2011. In Fig. 6, we plot yearly changes in CIV alongside yearly changes in the standard deviation and interdecile range of the cross-sectional earnings growth distribution for individuals. All series are standardized to have zero mean and unit variance for comparison. CIV innovations have a correlation of 57.9% ($t=3.8$) with changes in idiosyncratic income risk measured by the cross-sectional standard deviation of earnings growth and a correlation of 46.7% ($t=2.8$) with changes in the interdecile range.[25]

Our second finding is that changes in CIV are significantly associated with employment risk. We calculate firm-level employment growth rates growth for US publicly listed firms from 1952 to 2010 using Compustat data for number of employees. Then, to proxy for employment risk, we calculate the cross-sectional interquartile range of employment growth rates each year. Changes in CIV share

a correlation of 26.8% with changes in employment growth dispersion ($t=2.1$). The drawback of Compustat data is their restriction to the universe of public firms. The Federal Reserve reports monthly total employment for more than one hundred sectors beginning in 1991, aggregating both private and public firms, and we use these data to calculate log employment growth for each sector-year. Changes in CIV and changes in the cross-sectional standard deviation of sector-level employment growth have a correlation of 41.8% ($t=1.9$).

A large fraction of household wealth is invested in residential real estate, leaving individuals exposed to idiosyncratic wealth shocks deriving from fluctuations in the value of their homes.[26] Local house prices also reflect local labor market conditions (Van Nieuwerburgh and Weill, 2010). Our third and fourth findings relate CIV to the cross-sectional dispersion of house price growth and wage per job growth across metropolitan areas. House price data are from the Federal Housing Financing Agency, and wage data are the National Income and Product Accounts (NIPA) Regional Economic Information System. The merged data set contains annual information from 1969 to 2009 for 386 regions. The correlation between innovations in CIV and innovations in the cross-sectional standard deviation of house price growth is 23.2% per quarter ($t=2.7$), and the correlation with innovations in the cross-sectional standard deviation of per capita wage growth is 15.2% per quarter ($t=1.7$). This evidence offers further support of a link between the cross-sectional income distribution of firms and households.

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[24] Residuals are based on market model return regressions. Results are quantitatively similar and qualitatively the same if using an alternative definition such as variance of Fama and French model residuals. We compute a monthly, quarterly, and annual version of average idiosyncratic variance to conform with various data sources that are available at each of these frequencies.

[25] The Guvenen, Ozkan, and Song (2014) measures are also correlated with market variance innovations. For example, innovations in the cross-sectional standard deviation of household earnings growth have a correlation of 42.8% with MV innovations (t-statistic of 2.6). Because MV and CIV innovations are positively correlated, it makes sense to disentangle the contribution of each through two orthogonalizations. Innovations in the cross-sectional standard deviation of household earnings growth have a correlation of 38.7% (t-statistic of 2.3) with CIV innovations orthogonalized to MV innovations and a correlation of 4.3% (t-statistic 0.2) with MV innovations orthogonalized to CIV. Thus, the correlation is stronger with CIV than with MV. The same pattern holds for the other four correlations.

[26] For the median household with positive primary housing wealth in the 2010 wave of the Survey of Consumer Finance, the primary residence represents 61% of all assets. For 25% of households, it represents 90% or more of all assets.
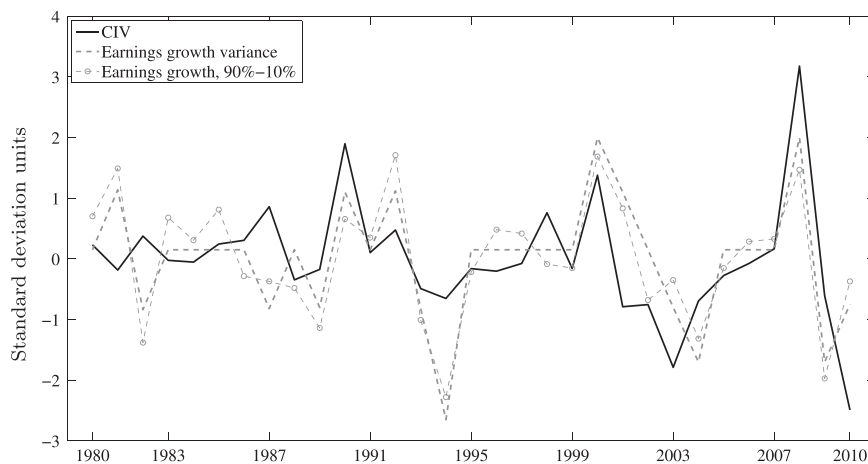
Fig. 6. Common idiosyncratic volatility (CIV) and dispersion in individual household income growth. The figure compares yearly changes in CIV with yearly changes in the standard deviation and interdecile range of the individual earnings growth distribution. CIV is the equal-weighted average of firm-level market model residual return variance each year. Individual earnings data are from the US Social Security Administration and summarized by Guvenen, Ozkan, and Song (2014). Each series is standardized to have equal mean and variance for ease of comparison.

4. CIV and expected stock returns

In this section, we show that stocks' exposure to CIV shocks helps explain cross-sectional differences in average stock returns. Then, Section 5 rationalizes these asset pricing findings and the empirical association between CIV and household income risk in an equilibrium incomplete markets model with heterogeneous agents.

We start by exploring average (abnormal) returns on portfolios sorted on stocks' CIV exposure. We then conduct a formal asset pricing test using a Fama and MacBeth analysis in which CIV shocks are the key asset pricing factor. Using this analysis, we ask whether CIV shocks also price other stock and corporate bond portfolios that have been deemed anomalous. Finally, we ask whether direct measures of household income dispersion price the cross section of stock returns.

4.1. CIV-beta-sorted portfolios

Our asset pricing analysis is conducted using monthly returns, so the results of this subsection use a monthly version of common idiosyncratic volatility (CIV).[27] We construct CIV shocks as monthly changes in CIV. Similarly, we construct a monthly measure of the market variance from daily value-weighted stock market returns, and we construct MV shocks as monthly changes in MV. For each month from January 1963 until December 2010, we regress monthly individual firm stock returns in excess of a risk-free rate on CIV innovations and MV innovations using a

trailing 60-month window.[28] We refer to a firm's exposure to the CIV shock as its CIV-beta and to its exposure to the MV shock as its MV-beta.

We sort stocks into quintiles based on their CIV-beta each month, form an equally weighted portfolio of the stocks in each quintile, and hold that portfolio for one month. Panel A of Table 2 reports the annualized average excess return of each portfolio, as well as the return on a strategy that goes long the highest CIV-beta quintile and short the lowest CIV-beta quintile. It shows that average returns are decreasing in CIV-beta. Stocks in the first quintile have negative CIV-betas and thus tend to lose value when CIV rises. In contrast, stocks in Q5 tend to hedge CIV increases, paying off in high volatility states. The long-short strategy has an average annualized return of -5.41% per year with a t-statistic of -3.94. The second and third rows report the abnormal return of each portfolio relative to the capital asset pricing model (CAPM) and Fama and French (1993) three-factor model, respectively. The spread portfolio has a CAPM alpha of -4.77% (t-statistic of -3.52) and three-factor alpha of -3.29% (t-statistic of -2.50). Panel B shows similar results for value-weighted portfolios.

In Panel C, we purge the CIV-beta quintile portfolios from their heterogeneous exposure to the MV shocks, following the approach in Ang, Hodrick, Xing, and Zhang (2006). We double-sort stocks first into MV-beta quintile portfolios and then, within each MV-beta quintile, into equally sized CIV-beta portfolios. For each CIV-beta decile, we combine all stocks in the five MV-beta quintiles, collapsing the double-sort back to a single-sort. We verify that this produces CIV-beta portfolios with essentially equal MV-betas. We call these CIV-beta-sorted portfolios

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[27] Each month, we estimate a regression of daily individual firm returns on the value-weighted market return for all CRSP firms with non-missing data that month. We then calculate CIV (in levels) as the equal-weighted average of market model residual variance across firms. The monthly nature of our return tests highlights an attractive feature of CIV estimated from returns. It is a plausible proxy for idiosyncratic household income risk while being easily observable at high frequencies.

[28] We use all CRSP stocks with share codes 10, 11, and 12 and include a stock in portfolio sorts if it had no missing monthly returns in the 60-month estimation window. The inclusion of MV shocks in a multivariate regression is equivalent to estimating univariate betas with respect to CIV shocks that have been orthogonalized to MV shocks.

Table 2

Portfolios formed on common idiosyncratic volatility (CIV)-beta.

The table reports average excess returns and alphas in annual percentages for portfolios sorted on the basis of monthly CIV-beta for the 1963–2010 sample. Panel A reports equally weighted average excess returns and alphas in one-way sorts using all Center for Research in Security Prices (CRSP) stocks. Panel B reports value-weighted averages in one-way sorts. Panel C shows equally weighted one-way sorts on CIV-beta that control for market variance (MV)-beta. Panel D shows equally weighted average excess returns in sequential two-way sorts on CIV-beta and MV-beta.

| | CIV beta | | | | | | |
	1 (low)	2	3	4	5 (high)	5 − 1	t(5 − 1)
Panel A: One-way sorts on CIV-beta							
$\mathbb{E}[R] - r_f$	12.08	10.88	9.96	8.70	6.68	− 5.41	− 3.94
α_{CAPM}	5.38	5.07	4.55	3.24	0.61	− 4.77	− 3.52
α_{FF}	1.06	1.07	0.78	− 0.07	− 2.23	− 3.29	− 2.50
Panel B: One-way sorts on CIV-beta (value-weighted)							
$\mathbb{E}[R] - r_f$	9.41	7.04	5.77	5.82	3.87	− 5.53	− 3.15
α_{CAPM}	2.84	1.34	0.49	0.74	− 1.72	− 4.56	− 2.65
α_{FF}	1.58	0.58	0.26	0.78	− 1.59	− 3.17	− 1.84
Panel C: One-way sorts on CIV-beta controlling for MV-beta							
$\mathbb{E}[R] - r_f$	11.71	11.08	9.57	8.57	7.36	− 4.35	− 3.10
α_{CAPM}	4.87	5.04	3.99	3.21	1.74	− 3.14	− 2.38
α_{FF}	0.66	1.22	0.31	− 0.19	− 1.39	− 2.05	− 1.64
Panel D: Two-way sorts on CIV-beta and MV-beta							
1 (low)	10.04	10.36	8.48	7.66	6.16	− 3.88	− 2.52
2	12.28	10.08	9.24	9.54	8.36	− 3.92	− 2.25
3	12.51	11.09	9.88	8.50	6.80	− 5.71	− 3.38
4	12.88	11.43	9.98	7.76	7.46	− 5.42	− 3.29
5 (high)	10.86	12.46	10.26	9.37	8.04	− 2.82	− 1.39
5 − 1	0.81	2.10	1.78	1.71	1.88	–	–
t(5 − 1)	0.48	1.17	1.01	0.90	0.84	–	–

controlling for MV-beta. The long-short strategy has an average return spread of −4.35% per year with t-statistic of −3.10. The spread portfolio has a CAPM (three-factor) alpha of −3.14% (−2.05%) with a t-statistic of −2.38 (−1.64). In unreported results, we find that sorting stocks into deciles instead of quintiles leads to similar simple CIV-beta spreads, but larger CIV-beta spreads controlling for MV-beta. The portfolio that goes long the tenth CIV-beta decile and short the first has an excess return of −5.76%, a CAPM alpha of −4.33%, and a three-factor alpha of −3.16%, all of which have t-statistics in excess of 2. They are also larger than for the simple CIV-beta decile sorts.

In Panel D, we report average excess returns on 25 double-sorted CIV-beta and MV-beta portfolios (5 by 5). They are the same portfolios as described in the sorting exercise, except that we do not collapse them back down to a single dimension. High CIV-beta stocks continue to earn substantially lower average returns within each MV-beta quintile. The Q5-minus-Q1 CIV-beta strategy has returns ranging from −2.8% to −5.7% per year depending on the MV-beta quintile and is significant for all MV-beta quintiles except for the fifth. The reverse is not true. Controlling for CIV-beta exposure, the return spread between the first and last MV-beta quintile is not statistically different from zero in any of the CIV-beta-sorted quintiles. We obtain similar results when we sort independently on CIV- and MV-betas. These double-sorts raise

an interesting question of whether exposure to MV risk is priced once exposure to CIV risk is controlled for. We return to this question in the Fama and MacBeth analysis in Section 4.2.

We conclude that stocks with more negative CIV exposure carry economically and significantly higher returns than stocks with less negative or positive exposure. This is true after accounting for their exposure to the market factor, the size and value factors, and their exposure to the market variance shocks.

The Appendix investigates the robustness of these results. Table A2 shows results for two subsamples as well as for single-sorts on CIV-beta, estimated without controlling for MV shocks in the firm-level regressions, and from single-sorts on MV-beta, estimated without controlling for CIV exposure. The results are present in both subsamples and stronger in the second half of the sample. The CIV sort is associated with a significant (abnormal) return spread, but not the MV sort. Together, they suggest that variation in CIV, as opposed to MV, is the primary driver of average return differences in Table 2. Table A3 studies several other double sorts in which we add log market equity, idiosyncratic variance, volatility index (VIX)-beta, and the (Pástor and Stambaugh, 2003) liquidity beta, always in addition to the CIV-beta. The results show that significant CIV-beta return spreads are present after accounting for betas on VIX and the Pastor and Stambaugh

Table 3

Fama and MacBeth analysis: common idiosyncratic volatility (CIV) single- and double-sorted portfolios.

　　The CIV-beta and market variance (MV)-beta for each stock are estimated in a multiple 60-month rolling window regression. The dependent variable is the excess return, and the independent variables are the CIV innovation and the MV innovation. The set of test assets in Columns 1–3 are decile portfolios sorted on CIV-beta. The test assets in Columns 4–6 are decile portfolios sorted on CIV-beta, controlling for the MV-beta. We first sort stocks into MV-beta deciles. Then, within each MV-beta decile, we sort stocks into CIV-beta deciles. For each CIV-beta decile, we then collect all the stocks in the ten MV-beta deciles. The set of test assets in Columns 7–9 are 25 portfolios double-sorted on CIV- and MV-betas. We first sort stocks into five MV-beta quintiles. Then, within each MV-beta quintile, we sort stocks into five CIV-beta quintiles. All portfolios are formed each month and held for one month. All dependent and independent variables in this table are expressed as monthly values (and not in percentage terms). The Fama and MacBeth estimation sample is 1963.01–2010.12. The model in Columns 1, 4, and 7 contains the excess market return as the factor. The model in Columns 2, 5, and 8 contains the excess market return and the CIV innovation as factors. The model in Columns 3, 6, and 9 contains the excess market return, the CIV innovation, and the MV innovation. The table reports monthly risk premia and Newey and West t-statistics (with one lag) and Shanken-corrected t-statistics from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2 and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. Statistics denoted with * are significant at 5%, with **, at 1%. Also reported is b_{MV}, the market price of risk of MV, and its associated t-statistic. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	Ten CIV-beta			Ten CIV-beta controling for MV-beta			25 double-sorted on CIV- and MV-betas		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	0.003	0.017	0.018	−0.003	0.017	0.016	0.006	0.008	0.006
NW t-statistic	0.552	23.922	7.321	−2.001	4.300	4.121	2.595	4.998	3.926
Shanken t-statistic	0.693	2.770	2.497	−0.848	2.135	1.987	1.752	2.199	1.603
Rm-Rf	0.005	−0.014	−0.015	0.010	−0.014	−0.014	0.002	−0.002	−0.000
NW t-statistic	1.125	−18.730	−5.912	7.784	−3.027	−2.869	1.124	−1.249	−0.312
Shanken t-statistic	1.179	−1.864	−1.767	2.394	−1.427	−1.336	0.615	−0.499	−0.105
CIV	–	−0.073	−0.071	–	−0.077	−0.083	–	−0.033	−0.039
NW t-statistic	–	−31.652	−22.352	–	−6.119	−5.994	–	−4.300	−4.950
Shanken t-statistic	–	−3.101	−3.053	–	−2.663	−2.552	–	−2.536	−2.586
MV	–	–	−0.012	–	–	−0.037	–	–	−0.029
NW t-statistic	–	–	−0.739	–	–	−3.653	–	–	−2.321
Shanken t-statistic	–	–	−0.551	–	–	−1.954	–	–	−2.411
b_{MV}	–	–	5.235	–	–	−10.186	–	–	−12.445
NW t-statistic	–	–	0.498	–	–	−1.625	–	–	−1.614
R^2	0.109	0.971	0.972	0.552	0.871	0.886	0.057	0.489	0.527
RMSE	1.759	0.317	0.310	1.141	0.611	0.575	1.752	1.289	1.241
χ^2	25.840**	4.029	3.394	18.791*	8.879	7.439	43.871**	36.272*	29.424

liquidity factor and for firm size and idiosyncratic variance.[29]

4.2. Pricing CIV-beta sorted portfolios

　　Next, we perform a two-stage Fama and MacBeth estimation to formally explore the ability of the CIV factor to explain the abnormal returns associated with CIV-beta-sorted portfolios. In the first stage, we estimate the factor betas by regressing monthly excess returns for each test asset on a constant and asset pricing factors. In the second stage, we estimate a single cross-sectional regression of the average excess test asset returns on the factor betas and a constant. Table 3 shows the risk premia estimates from that second stage. The estimation uses the same monthly portfolio returns, CIV and MV innovations, and sample as in the Section 4.1.

　　As a benchmark, the first column reports the results for the CAPM in which the market excess return is the sole asset pricing factor. The second column adds the CIV innovations as a second factor. These CIV innovations are the same ones used in the portfolio formation stage. This two-factor model is the model suggested by our theory in Section 5. The third

Table 4

Fama and MacBeth analysis: anomalies portfolios.

The set of test assets are decile portfolios sorted on book-to-market (BM) ratio in Columns 1–3, on size measured by market equity (ME) in Columns 4–6, and the book-to-market (BM) deciles, size deciles, and four corporate bond (CB) portfolios sorted by credit rating in Columns 7–9. The estimation sample is 1963.01–2010.12 in Columns 1–6 and 1980.01–2010.12 in Columns 7–9. The model in Columns 1, 4, and 7 contains the excess market return as the factor. The model in Columns 2, 5, and 8 contains the excess market return and common idiosyncratic volatility (CIV) innovation as factors. The model in Columns 3, 6, and 9 contains the excess market return, the CIV innovation, and the market variance (MV) innovation. The table reports the risk premia estimates (λ) associated with the factors and their Newey and West standard errors (with one lag) from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2 and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. * and ** denote significance at 5% and 1%, respectively. The table also reports b_{MV}, the market price of risk of MV, and its associated t-statistic. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	Ten BM			Ten ME			Ten BM, ten ME, four CB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	0.009	0.014	0.012	−0.008	−0.004	0.004	0.003	0.003	0.003
NW t-statistic	0.971	5.048	3.774	−4.816	−2.348	1.130	16.672	16.317	15.697
Shanken t-statistic	1.736	1.868	1.563	−1.223	−0.886	0.435	2.488	2.512	2.529
Rm-Rf	−0.003	−0.009	−0.007	0.013	0.009	0.001	0.004	0.004	0.004
NW t-statistic	−0.280	−3.292	−2.190	8.955	5.568	0.366	14.778	11.567	9.645
Shanken t-statistic	−0.517	−1.154	−0.912	1.943	1.836	0.139	1.452	1.347	1.342
CIV	–	−0.069	−0.069	–	−0.020	−0.033	–	−0.015	−0.015
NW t-statistic	–	−9.934	−8.855	–	−7.265	−6.777	–	−2.192	−2.251
Shanken t-statistic	–	−2.374	−2.291	–	−1.020	−1.722	–	−0.873	−0.868
MV	–	–	−0.005	–	–	−0.025	–	–	−0.006
NW t-statistic	–	–	−0.621	–	–	−4.286	–	–	−1.000
Shanken t-statistic	–	–	−0.357	–	–	−1.569	–	–	−0.537
b_{MV}	–	–	9.803	–	–	−10.370	–	–	−0.445
NW t-statistic	–	–	1.715	–	–	−3.065	–	–	−0.141
R^2	0.013	0.796	0.837	0.839	0.919	0.955	0.722	0.781	0.781
RMSE	1.886	0.857	0.768	0.543	0.386	0.287	1.031	0.914	0.913
χ^2	11.579	4.820	3.692	8.019	6.438	4.201	12.284	11.154	10.186

column adds the MV innovations to explore whether MV innovations help to price the cross section of test assets. The table reports monthly risk premia and Newey and West t-statistics, Shanken-corrected t-statistics, the R^2 statistic, and the root mean square pricing error among the test assets. The row labeled b_{MV} is the point estimate of the price of risk associated with MV in the stochastic discount factor (SDF). When factors are correlated, this risk price differs from the risk premium λ_{MV} reported in the top part of the table. Testing whether MV helps to price the test assets is done by testing the null hypothesis $b_{MV} = 0$. The t-statistic below b_{MV} is the relevant one for that test.[30]

The first column uses ten simple CIV-beta-sorted portfolios as test assets. We use decile instead of quintile portfolios to have a large enough cross section. It shows that the CAPM fails to price the CIV-beta-sorted portfolios. The cross-sectional R^2 is low and the pricing errors large. The second column shows that CIV is priced with a negative and highly statistically significant monthly risk premium λ_{CIV} of −0.073. Adding CIV as a pricing factor

increases the cross-sectional R^2 by 86% points and reduces the average pricing error to a mere 32 basis points (bps) per year. The third column shows that adding MV does not improve the fit any further. The pricing error falls by less than a basis point. And, while the point estimate on MV is negative, its t-statistic is only −0.74. MV does not help to price the test assets, as the t-statistic on b_{MV} indicates that we fail to reject the null that $b_{MV} = 0$. A chi-squared test indicates that the models with the CIV pricing factor in Columns 2 and 3 cannot be rejected.

Table 3 also shows the Fama and MacBeth results for ten CIV-beta-sorted portfolios that control for MV-beta. The table shows that, while the CAPM does better explaining these test assets (Column 4), CIV innovations still add substantially to the pricing performance (Column 5). Pricing errors are cut in half to 61 bps per year, and CIV comes in with a similar −0.077 risk premium estimate, which is highly significant. MV innovations now also carry a negative and significant risk premium (Column 6). However, we fail to reject the null hypothesis $b_{MV} = 0$. Furthermore, adding MV reduces pricing errors by only 4 basis points. Again, the pricing errors in the models with the CIV factor are not statistically different form zero according to the χ^2 test statistic.

The table also shows that CIV innovations price the 25 double-sorted portfolios on CIV-beta and MV-beta with a negative and significant risk premium (Column 8), again substantially improving on the pricing ability of the CAPM (Column 7). Adding the MV factor (Column 9) only marginally reduces the pricing error and marginally increases the cross-sectional R^2. While the risk premium on the MV

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[30] The asset pricing model has a log SDF $-m_{t+1} = r_t^f + \frac{1}{2}\mathbb{V}[m_{t+1}] + b'f_{t+1}$, where the asset pricing factors are collected in the vector f_{t+1} and the associated market prices of risk are collected in the vector b. The asset pricing factors are mean-zero with constant variance–covariance matrix $\Sigma = \mathbb{E}[ff']$. Risk premia on test asset j can be written as $\mathbb{E}[rx_{t+1}^j] = \text{Cov}(rx_{t+1}^j, f'_{t+1})b = \left(\text{Cov}(rx_{t+1}^j, f'_{t+1})\Sigma^{-1}\right)(\Sigma b) = \beta_j'\lambda$. When the factors are correlated, the λ's are linear combinations of the risk prices b. Cochrane (2005, pp. 260–261) explains that testing whether MV helps to price the test assets is done by testing the null hypothesis $b_{MV} = 0$. When asking the different question of whether MV is priced, one tests $\lambda_{MV} = 0$ instead.

Table 5

Benchmark calibration.

This table lists the parameter values of the benchmark calibration. The preferences parameters are intertemporal discount (δ), risk aversion (γ), and intertemporal elasticity of substitution (ψ). The aggregate consumption growth process, the consumption share process, and the dividend growth process are described by Eqs. (2)–(4). Finally, the table presents the calibration of the portfolios sorted from lowest common idiosyncratic volatility (CIV)-beta (Q1) to highest CIV-beta (Q5), and the market portfolio calibration is reported in the last column (M).

Panel A: Preferences, aggregate consumption growth, consumption share and dividend growth parameters							
Parameter	Calibration	Parameter	Calibration	Parameter	Calibration	Parameter	Calibration
δ	0.8747	μ_g	0.02	σ_g	0.1	σ_i	0.004
γ	15	σ_c	0.02469	ν_g	0.6	ν_i	0.15
ψ	2	ϕ_c	-0.0395	σ_w	0.01472	σ_{iw}	$1.5e-06$

Panel B: Portfolio-specific parameters						
Parameter	Q1	Q2	Q3	Q4	Q5	M
μ_i	6.58%	5.81%	4.92%	5.74%	4.55%	5.20%
φ	5.69	5.69	5.69	5.69	5.69	5.69
ϕ_i	-1.21	-0.68	-0.45	-0.22	0.20	-0.29
χ^i	-0.95	-0.25	0.06	-0.40	-1.42	-0.65
κ_i	2.76	2.07	1.88	1.88	2.27	–
ζ_i	141.12	106.73	96.95	97.25	118.92	–

innovations is negative and significant, we still (marginally) fail to reject the null hypothesis that $b_{MV} = 0$.

In conclusion, exposure to CIV shocks goes a long way toward reducing CAPM pricing errors associated with CIV-beta-sorted portfolios. We estimate a significant and negative risk premium associated with CIV shocks.

4.3. Pricing anomaly portfolios

It is natural to ask whether CIV is a risk factor that can help to explain other test assets whose returns are anomalous relative to benchmark asset pricing models. Table 4 summarizes our findings. For the standard ten book-to-market (BM)-sorted portfolios, Column 2 shows that adding CIV to the CAPM dramatically reduces pricing errors from 189 bps to 86 bps per year and increases the cross-sectional R^2 from 1% to 80%. Interestingly, the monthly risk premium associated with CIV innovations is -0.069, a similar point estimate to the one obtained on the CIV-beta-sorted portfolios in Table 3. It is highly statistically significant. In column 3, we add MV as a pricing factor and find that it is not priced and that it barely lowers pricing errors. The market price of MV risk, b_{MV} has the wrong sign, but is not different from zero.

The CAPM in Column 4 does substantially better for the ten size portfolios and leaves only 54 bps unexplained. Yet, adding CIV lowers the pricing errors further to 39 bps. CIV enters with the right sign and is significantly different from zero. Its point estimate is substantially smaller here than for the BM- or CIV-sorted portfolios, however. MV also prices the size portfolios with a significantly negative λ_{MV}. The estimate for b_{MV} is also significantly negative, so that we can reject the null that MV does not add pricing power to the other factors for size portfolios.

When adding corporate bond portfolios, we use data from Citibank's Yield Book for four investment-grade portfolios: AAA, AA, A, and BBB. Return data for these portfolios are available monthly from January 1980 until December 2010, which restricts our estimation to this

sample in Columns 7–9. Because we cannot identify risk prices of four test assets and because it is recommended to use large and diverse cross sections of test assets (Lewellen, Shanken, and Nagel, 2010), we combine the ten size, ten value, and four corporate bond portfolios. We find that CIV is priced with a significantly negative risk premium and MV is not. Inspection of the betas reveals that growth stocks have mildly positive CIV betas (0.005–0.009 for BM1–BM5) and that value stocks have negative CIV-betas (around -0.03 for BM6–BM9 and -0.09 for BM10). Small firms have negative CIV-betas (-0.13 for ME1, -0.07 for ME2), and large firms have less negative or even positive CIV-betas (0.01 for ME10). Finally, corporate bond portfolios of high credit quality have positive CIV-betas (0.013 for AAA), and corporate bonds of firms with lower credit quality have negative CIV-betas (-0.03 for the BBB portfolios). These patterns in CIV-betas help to explain why the CIV factor contributes to resolving the value, size, and credit spread anomalies.

We conduct several robustness checks in the Appendix. We use traded versions of the CIV and MV factors. The traded CIV (MV) factor is the return on a portfolio that goes long the tenth CIV-beta (MV-beta) decile and short the first CIV-beta (MV-beta) decile. Table A5 contains the results. The risk premium estimate on the traded CIV has the interpretation of a monthly excess return. We estimate an average value of around -0.005 or -6% annualized. This is consistent with the average return spread on the 10-minus-1 CIV-beta portfolio of -5.4% per year. In contrast, the risk premium on the traded MV factor is always positive, which is the wrong sign.

We also investigate the pricing ability of CIV and market variance for the same set of test assets as in Table 4, but using the Campbell, Giglio, Polk, and Turley (2014) measure of market variance, called N_V. The results in Table A6 show that CIV continues to carry a negative and significant risk premium for all sets of test assets, even after inclusion of the traded N_V factor.

Table 6
Calibration results.

This table reports moments from the model and compares them with the data. The first two rows report the average excess return. The next two rows split the equity risk premium into a contribution representing compensation for η risk and a compensation for w_g risk. Rows 5 and 6 report common idiosyncratic volatility (CIV)-betas, in which model betas have been scaled to ensure that the innovation volatility of CIV is the same in model and data. Rows 7 and 8 report stock return volatilities, followed by a breakdown of volatility into its five components in Rows 9–13 [see Eq. (7)]. Because the variance but not the volatility components are additive, we calculate the square root of each variance component and then rescale all components so they sum to total volatility. Rows 14 and 15 report the slope of a regression of $\mathbb{V}_t\left[r_{t+1}^{idio,i}\right]$ on CIV, multiplied by one hundred. Rows 16 and 17 report the R^2 of this regression, multiplied by one hundred. Rows 18 and 19 report the slope of a predictive regression of annual dividend growth on one-year lagged CIV. The model is simulated at annual frequency for 60 thousand periods. All moments in the data are expressed as annual quantities and computed from the 1963.01 to 2010.12 sample.

Row	Moment	Q1	Q2	Q3	Q4	Q5	M
Excess return							
1	Data	12.08	10.88	9.96	8.70	6.68	5.50
2	Model	8.65	6.65	5.74	4.89	3.25	5.50
3	η risk	5.21	5.21	5.21	5.21	5.21	5.21
4	w_g risk	3.45	1.44	0.53	−0.31	−1.96	0.29
Beta $\beta_{gs,i}$							
5	Data	−0.50	−0.21	−0.08	0.05	0.28	−0.04
6	Model	−0.50	−0.21	−0.08	0.05	0.28	−0.04
Return volatility							
7	Data	66.89	52.17	47.20	46.70	54.45	15.65
8	Model	65.00	49.62	45.34	45.45	54.78	14.08
9	η risk	8.42	8.50	8.64	8.71	8.51	13.39
10	w_g risk	5.15	2.18	0.82	0.48	2.96	0.69
11	e_i risk	16.52	12.51	11.53	11.65	13.73	–
12	ε_i risk	33.85	25.82	23.84	24.10	28.81	–
13	w_i risk	1.05	0.61	0.51	0.52	0.76	–
Regression of idiosyncratic variance on CIV: slope coefficient							
14	Data	1.54	0.87	0.71	0.72	1.04	–
15	Model	1.58	0.89	0.73	0.74	1.07	–
Regression of idiosyncratic variance on CIV: R^2							
16	Data	17.69	17.21	17.10	17.11	16.31	–
17	Model	17.69	17.21	17.10	17.11	16.31	–
Dividend predictability regression coefficient							
18	Data	−0.20	−0.05	0.01	−0.08	−0.29	−0.13
19	Model	−0.20	−0.05	0.01	−0.08	−0.29	−0.13

In addition, we investigate whether CIV is priced in the 25 portfolios double-sorted on book-to-market ratio and size and six portfolios double-sorted on their exposure to market returns and market variance innovations, in line with Campbell, Giglio, Polk, and Turley (2014) test assets. We construct the six risk-sorted portfolios as follows. In a 60-month trailing window, we regress stock returns on market return and MV innovations and use the coefficient estimates to form double-sorted stock portfolios, which we hold for the subsequent month. The six risk-sorted portfolios are independently sorted by terciles on the market exposure and into two groups based on the market variance innovations exposure. We report the Fama and MacBeth analysis for these test assets in Table A9 in the Appendix. Columns 1–3 use the 25 portfolios double-sorted on book-to-market ratio and size as test assets, Columns 4–6 add the six risk-sorted portfolios to a set of test assets, and Columns 7–9 add the four corporate bonds portfolios sorted by credit rating. The CIV price of risk is negative, and in five of the six specifications the point estimate is significantly different from zero.

Finally, we study earnings-price-sorted portfolios, idiosyncratic volatility-sorted portfolios, and momentum portfolios. The results for earnings-price portfolios look

very similar to those for book-to-market-sorted portfolios. In contrast, CIV has no pricing ability for the idiosyncratic volatility and momentum portfolios.

We conclude that CIV innovations have pricing ability for value, size, and corporate bond portfolios over and above the pricing ability of the market variance factor. This strengthens the appeal of CIV as a new asset pricing factor.

4.4. Household income growth dispersion as pricing factor

The positive correlation between CIV shocks and shocks to the dispersion in household income growth (Section 3), coupled with the significant asset pricing power of CIV, suggests that CIV proxies for partially uninsurable idiosyncratic consumption risk of households. To support this view, we show that direct measures of household income risk help price the cross section of returns. Unfortunately, our measures of income dispersion are available only at an annual frequency and for a relatively short sample. We explore two ways to overcome this difficulty.

First, we find a proxy for the cross-sectional dispersion of firm employment growth (such as the Compustat or Bureau of Economic Analysis measures used in

Section 3) that is available at high frequency. We use the market value of the firm's equity as a proxy for its number of employees. We calculate the cross-sectional standard deviation of the growth rate in firm size at the monthly frequency. We refer to monthly changes in firm size dispersion as the FSD factor. The correlation between the innovations in the CIV and FSD factors is 32.3%. We estimate a firm's FSD-beta in a multiple regression of excess returns on FSD and MV factors and then sort stocks into deciles by their FSD-beta. Stocks with low FSD-beta have excess returns that are 4.6% higher than stocks with a high FSD-beta. This spread is similar in magnitude to the CIV-beta spread (t-statistic of -2.86). Moreover, the CAPM and three-factor models leave the entire return spread unexplained. The α_{CAPM} spread is -5.8% (t-statistic of -3.7), and the α_{FF} spread is -4.6% (t-statistic of -3.02).[31]

Second, we construct a traded version of the shocks to the Guvenen, Ozkan, and Song (2014) household income growth dispersion measure, which we refer to as the GID factor.[32] The correlation between the annual innovations in the CIV and GID factors is 33.7%. We show that the GID factor prices the cross section of annual test asset excess returns.[33] Despite the short sample for the Guvenen measure, these results provide additional support for the mechanism in the paper.

5. Model with CIV in household consumption

Motivated by the three sets of facts presented in Sections 2–4, this section of the paper develops an incomplete markets asset pricing model in which a common idiosyncratic volatility factor is the key state variable driving both residual stock return volatility and dispersion in household income growth. Innovations to this factor represent bad news and carry a negative price of risk. Stocks with more negative exposure with respect to this innovation in turn earn a higher risk premium. We show a calibration that quantitatively accounts for the cross-sectional differences in average returns across CIV-beta-sorted portfolios. In the interest of space, the model details and parameter sensitivity analysis are relegated to the Appendix.

5.1. Model setup

There is a unit mass of atomless agents, each having Epstein-Zin preferences. There is a large number of securities in zero or positive net supply. Agents are endowed

with an equal fraction of these securities and with a labor income process of the form $I_{j,t} = S_t^j C_t - D_t$, where S^j denotes agent j's consumption share and per capita dividends are D_t. As in Constantinides and Ghosh (2014), given the symmetric and homogeneous preferences, households choose not to trade away from their initial endowments. That is, autarky is an equilibrium and individual j's equilibrium consumption is $C_{j,t} = I_{j,t} + D_t = S_t^j C_t^a$. Following Constantinides and Duffie (1996), we interpret the equilibrium consumption process $C_{j,t}$ as the post-trade consumption that obtains after households have exhausted all insurance options and the temporary innovations to labor income have been smoothed out. It reflects the permanent innovations to income that are not insurable and thus passed through to consumption.

Relative to the literature, and motivated by the empirical evidence in Section 3, the novel ingredient in the model is the link between the firms' dividend growth process and the households' individual income growth process. The common idiosyncratic volatility factor in firms cash flows is also the driver of the cross-sectional dispersion of household consumption growth rates:

$$\sigma_{g,t+1}^2 = \sigma_g^2 + \nu_g \left(\sigma_{gt}^2 - \sigma_g^2 \right) + \sigma_w \sigma_g w_{g,t+1} \tag{2}$$

and

$$\Delta c_{t+1}^j = \Delta c_{t+1}^a + \Delta s_{t+1}^j = \mu_g + \sigma_c \eta_{t+1} + \phi_c \sigma_g w_{g,t+1} + \sigma_{g,t+1} v_{t+1}^j - \frac{1}{2} \sigma_{g,t+1}^2. \tag{3}$$

All shocks are independent and identically distributed (i.i.d.) standard normal and mutually uncorrelated. The process $\sigma_{g,t+1}^2$ is the cross-sectional variance of the individual log consumption growth process. Eq. (2) assumes this dispersion follows an AR(1) with innovations $w_{g,t+1}$. The $w_{g,t+1}$ shock is the key source of aggregate risk in the model. It maps one-for-one to the CIV shock. The only other aggregate source of risk is the usual aggregate consumption growth shock η_{t+1} in Eq. (3). When $\phi_c < 0$, aggregate consumption growth is negatively correlated with shocks to the cross-sectional dispersion of individual consumption growth. This counter cyclical variation in idiosyncratic risk is a mechanism familiar from Mankiw (1986), Constantinides and Duffie (1996), and Krueger and Lustig (2010). Our main pricing predictions do not require this channel (i.e., we can allow $\phi_c = 0$), but it helps the quantitative fit of the model.

Eq. (4) describes the dividend growth process for a stock i:

$$\Delta d_{t+1}^i = \mu_i + \chi^i \left(\sigma_{gt}^2 - \sigma_g^2 \right) + \varphi \sigma_c \eta_{t+1} + \phi_i \sigma_g w_{g,t+1} + \kappa_i \sigma_{gt} e_{t+1}^i + \zeta_i \sigma_{it} \varepsilon_{t+1}^i \tag{4}$$

Dividend growth is affected by the two sources of aggregate risk. There is a traditional leverage effect with respect to the η_{t+1} shock ($\varphi > 1$) and a possible negative correlation with the CIV shock $w_{g,t+1}$ ($\phi_i < 0$). Dividend growth is also subject to two sources of idiosyncratic risk. The $\kappa_i e_{t+1}^i$ shock is orthogonal to all other shocks in the economy, but its variance σ_{gt}^2 is common across firms. In contrast, the $\zeta_i \varepsilon_{t+1}^i$ shock has a variance σ_{it}^2 that is firm-specific. Each stock's σ_{it}^2 process follows an AR(1), described in the Appendix. Thus, firm i's idiosyncratic cash flow growth risk

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[31] The Fama and MacBeth analysis in Table A7 shows that FSD prices the ten FSD-beta-sorted portfolios, the ten value portfolios, and the full cross section of 34 test assets that also includes ten size portfolios and four corporate bond portfolios with a negative and significant risk premium.

[32] First, we estimate GID-betas for each stock using firm-level regressions of annual returns that control for the MV factor. Second, we form the return on a portfolio that goes long the highest GID-beta decile of stocks and short the lowest GID-beta decile of stocks.

[33] Table A8 shows that the GID factor is significant and negative in seven out of eight specifications. MV innovations are significant in two out of four specifications.

is $\mathbb{V}_t\left[\kappa_i\sigma_{gt}e^i_{t+1}+\zeta_i\sigma_{it}\varepsilon^i_{t+1}\right]=\kappa_i^2\sigma_{gt}^2+\zeta_i^2\sigma_{it}^2$. This specification captures the factor structure in idiosyncratic variance shown in Section 2, in which the first term captures the common idiosyncratic volatility factor. Thus, $w_{g,t+1}$ shocks are perfectly correlated with innovations in CIV. Finally, the term $\chi^i\left(\sigma_{gt}^2-\sigma_g^2\right)$ allows for predictability of dividend growth between t and $t+1$ by the CIV level at time t. Apart from mean dividend growth μ_i and a common leverage parameter φ, the key cash flow parameters are $\Theta^i=(\chi_i,\phi_i,\kappa_i,\zeta_i)$.

The market portfolio follows a similar dividend growth process as in Eq. (4), except without the idiosyncratic risk terms that are diversified away. The conditional variance of the dividend growth on the market portfolio, just like that of aggregate consumption growth, is constant. This implies that the aggregate stock market variance is constant over time. We make this assumption to keep the model simple and to highlight the role of CIV.

5.2. The cross section of equity risk premia

The log stochastic discount factor innovations can be written as

$$-m_{t+1}+\mathbb{E}_t[m_{t+1}]=\lambda_\eta\sigma_c\eta_{t+1}+\lambda_w\sigma_g w_{g,t+1},\qquad(5)$$

where the market prices of risk are given by

$$\lambda_\eta=\gamma,\quad \lambda_w=-\frac{1}{2}\gamma(1+\gamma)\sigma_w+\gamma\phi_c+\frac{\gamma\nu_g\left(\frac{1}{\psi}-\gamma\right)}{2\left(\kappa_1^c-\nu_g\right)}\sigma_w.\qquad(6)$$

The Appendix provides the derivation and the expression for the risk-free rate r_t^f. Eq. (5) shows that there are two priced sources of aggregate risk: shocks to aggregate consumption growth carrying a price of risk, λ_η, equal to the coefficient of relative risk aversion γ, and shocks to the CIV factor carrying a market price of risk, λ_w. All three terms in the λ_w expression are negative, provided that the agent has a preference for early resolution of uncertainty ($\psi^{-1}<\gamma$), indicating that an increase in the cross-sectional volatility of consumption growth is bad news for investors. The first term in λ_w captures precautionary motives against changes in consumption risk sharing and is the main pricing effect of CIV shocks. The second term compensates for exposure to counter cyclical cross-sectional variation in idiosyncratic risk. Both terms appear when utility is time-additive. The third term arises from Epstein-Zin preferences, inducing investors to care not only about current but also about future cross-sectional dispersion of consumption growth. The size of this effect is governed by the persistence of the idiosyncratic volatility factor, ν_g.

We exploit the normality of the shocks to guess and verify that the log price–dividend ratio on a stock is affine in the CIV factor and in firm-specific variance: $pd_t^i=\mu_{pdi}+A_{gs}^i\left(\sigma_{gt}^2-\sigma_g^2\right)+A_{is}^i\left(\sigma_{it}^2-\sigma_i^2\right)$. The conditional variance of individual stock returns is given by

$$\mathbb{V}_t\left[r^i_{t+1}\right]=\beta_{\eta,i}^2\sigma_c^2+\beta_{gs,i}^2\sigma_g^2+\left(k_1^iA_{is}^i\right)^2\sigma_{iw}^2+\kappa_i^2\sigma_{gt}^2+\zeta_i^2\sigma_{it}^2\qquad(7)$$

and

$$CIV_t\equiv\mathbb{E}_i\left[\mathbb{V}_t\left[r^{idio,i}_{t+1}\right]\right]=const+\overline{\zeta}^2\sigma_i^2+\overline{\kappa}^2\sigma_{gt}^2,\qquad(8)$$

where

$$\beta_{\eta,i}\equiv\varphi,\quad \beta_{gs,i}\equiv k_1^i\frac{2\chi_i+\kappa_i^2+\left(1+\frac{1}{\psi}\right)\gamma\nu_g}{2\left(1-k_1^i\nu_g\right)}\sigma_w+\phi_i.\qquad(9)$$

The first two terms in Eq. (7) reflect the role of aggregate risks in a stock's return variance. The variance of idiosyncratic stock returns is given by the last three terms. Idiosyncratic return variance is driven by the common variance process σ_{gt}^2 and a firm-specific variance process σ_{it}^2.

Defining the CIV factor as the average of firms' idiosyncratic return variances, as we did in the data, Eq. (8) shows that CIV is affine in σ_{gt}^2. In Section 2, we show the presence of a large first principal component in both total and idiosyncratic return variance, which was the same component in both. We also show that total and idiosyncratic return variance at the firm level were nearly identical. This fact justifies our assumption of constant market variance. Finally, we show that a common component exists in firms' total and idiosyncratic cash flow (sales) growth and that the common component in cash flow growth and return volatilities is highly correlated. This model generates all these features. It associates the common component in idiosyncratic return variance with changes in the cross-sectional dispersion of consumption growth across agents, a connection for which we provide support in Section 3.

The equity risk premium on stock i is

$$\mathbb{E}_t\left[r^i_{t+1}-r^f_t\right]+0.5\mathbb{V}_t[r^i_{t+1}]=\mathrm{Cov}(-m_{t+1}+\mathbb{E}_t[m_{t+1}],r^i_{t+1}$$
$$-\mathbb{E}_t[r^i_{t+1}])=\beta_{\eta,i}\lambda_\eta\sigma_c^2+\beta_{gs,i}\lambda_w\sigma_g^2.\qquad(10)$$

The first term is the standard (consumption) CAPM term. The second term is a new term that compensates investors for movements in the cross-sectional consumption growth distribution, today and in the future. Stocks that have low returns when the cross-sectional dispersion of individual consumption growth increases, i.e., stocks with $\beta_{gs,i}<0$, are risky and carry high expected returns because $\lambda_w<0$. Because $\beta_{\eta,i}$ is constant across stocks, heterogeneous exposure to CIV shocks is the sole driver of differences in risk premia across stocks.

Eq. (9) shows the model-implied CIV-beta. Three mechanisms contribute to a negative $\beta_{gs,i}$. First, a sufficiently negative χ^i means that periods of high CIV predict low future dividend growth. Below, we show that CIV negatively forecasts dividend growth in the data and we choose χ^i to match the slope of the predictive relation. Second, if positive shocks to CIV coincide with low dividend growth realizations ($\phi_i<0$), then that stock has a more negative $\beta_{gs,i}$. Third, a stock with lower κ_i, or lower cash flow exposure to the common idiosyncratic risk term, will have a lower $\beta_{gs,i}$. Intuitively, when there is a positive shock to CIV, the quantity of idiosyncratic risk goes up more for stocks with greater exposure κ_i. As a result of convexity in the relation between growth and terminal value, this raises the price more for those high volatility stocks, increasing their $\beta_{gs,i}$, and lowering their equilibrium expected return.[34]

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[34] A similar convexity effect is explored by Pástor and Veronesi (2003, 2009).

5.3. Quantitative implications of the CIV model

We use our model to evaluate if the average return spreads across CIV-beta-sorted portfolios shown in Section 4 are quantitatively consistent with the extent of idiosyncratic volatility co-movement shown in Section 2. A detailed discussion of the calibration and a sensitivity analysis are in the Appendix. Our benchmark parameter values are in Table 5. The calibration matches the mean and volatility of aggregate consumption growth and the risk-free rate. The average cross-sectional standard deviation of consumption growth is set at 10% to reflect the partial pass-through of household income shocks to consumption.[35] The persistence of σ_{gt} is chosen to match that of CIV (annual autocorrelation of 0.6). Unlike other state variables in the asset pricing literature, this variable moves at business cycle frequencies. We also match the mean of the observed CIV process. The consumption leverage parameter φ and risk aversion parameter γ are chosen to match the equity risk premium on the market portfolio of 5.5%. Because the exposure of the aggregate market portfolio to the CIV shock, $\beta_{gs,M}$, is close to zero in the data, most of the equity risk premium on the market portfolio must come from the standard consumption-CAPM term. Given the low volatility of aggregate consumption growth, this requires a risk-aversion coefficient of 15. At this point, the market price of CIV risk λ_w is fully pinned down.

The main question is whether the model can generate the observed differences in average returns between stocks with low and high CIV-betas, shown in Section 4. To speak to the quintile portfolio evidence, we solve our model for a representative stock in each of the CIV-beta quintile portfolios. That is, we choose five sets of four cash flow growth parameters Θ^i and price the resulting cash flows inside the model. The four cash flow parameters are chosen to match four moments, for each quintile portfolio and the market portfolio. Those are the CIV-beta, $\beta_{gs,i}$ in Eq. (9), the slope of a regression of dividend growth on lagged CIV, and the slope and R^2 from a regression of stock return variance [in Eq. (7)] on the CIV factor [in Eq. (8)]. While these are four simultaneous equations, χ_i mostly affects the dividend growth predictability slope, κ_i governs the portfolio's return variance exposure to CIV, ζ_i affects the R^2 of that relation, and ϕ_i is chosen to match $\beta_{gs,i}$ given the other three parameters. Rows 5 and 6 of Table 6 show that the calibration matches the CIV-betas. Rows 14–17 shows that the model matches the slope and R^2 of the regression of return variance on CIV. Finally, Rows 18 and 19 show that the calibration matches the dividend growth predictability by lagged CIV. Dividend growth is much lower on the low CIV-beta stocks following an increase in CIV than it is for the high CIV-beta stocks.

The main result of the calibration exercise is that the model is able to match the 5.4% spread in excess returns on the CIV-beta-sorted portfolios. It generates a monotonically declining pattern in excess stock return from Q1 to Q5 (Row 2). While the average excess return levels are too low, the model exactly matches the return spread between Portfolio 5 and Portfolio 1 (Row 1). As Rows 3 and 4 make clear, the common level of the equity risk premium comes from compensation for η-risk, while the entire cross-sectional slope in excess returns is due to differential exposure to the w_g-risk. The stocks in Portfolio Q1 (Q5) have negative (positive) exposure to the CIV factor (Row 6). Their returns fall (rise) when the cross-sectional volatility increases, making them risky (a hedge). As a result, they carry the highest (lowest) risk premia.

The model accurately matches total return volatilities of the CIV-beta-sorted portfolios, shown in Rows 7 and 8 of Table 6. Annual return volatilities for the typical stock in each of the quintile Portfolios range from 45% to 65%. They are highest for Portfolios Q1 and Q5. The market portfolio has a volatility of 15.7% in the data and 14.1% in the model. Rows 9–13 break down total return volatility into its five components. As in the data, most of total return volatility is idiosyncratic return volatility. The common idiosyncratic and firm-specific idiosyncratic components contribute about one-third and two-thirds, respectively, to idiosyncratic firm volatility (Rows 11 and 12). The model matches the persistence of the various volatility components as well as the relative amount of variation that comes from the common and the firm-specific volatility components.

The model has additional testable implications. In the Appendix, we show that the log price–dividend ratio predicts CIV with a positive sign, consistent with the model. The point estimate in the data is significant for Quintiles 1–4 and the predictive R^2 ranges from 11% to 29%, which is substantial. Also, the model predicts a negative relation between real rates and CIV, resulting in a downward term structure of real rates. This is a prediction that it shares with the model of Bansal and Yaron (2004). Verdelhan (2010) shows a significant downward slope for 1983–1995 for the UK and a flat yield curve from 1995 to 2006. The evidence for the US shows a modest upward-sloping yield curve.

The Appendix explores several sensitivity analyses. In one exercise, we increase the mean of the σ_{gt} process, σ_g, thereby increasing the amount of labor income risk that is passed through to consumption. We increase σ_g from 0.10 to 0.42, thereby matching the cross-sectional dispersion of observed consumption growth. Interestingly, the model matches the asset pricing results as well as the benchmark calibration. We also consider a calibration with lower risk aversion, which implies a lower equity risk premium but a similar return spread on the high-minus-low CIV-beta portfolio.

6. Conclusion

We show strong co-movement of individual stock return volatilities. Removing all common variation in returns has little effect on volatility co-movement, as residual return volatility possesses effectively the same volatility factor structure as total returns. Volatility co-movement is also a prominent feature of firm-level cash

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[35] Blundell, Pistaferri, and Preston (2008) and Heathcote, Storesletten, and Violante (2014) provide empirical support for the partial pass-through of permanent income shocks to consumption. Given that the cross-sectional standard deviation of household income growth is 53% on average according to Guvenen, Ozkan, and Song (2014), this assumes that one-fifth of income shocks are uninsurable. We perform sensitivity with respect to this parameter in the Appendix, considering higher values that match the average cross-section dispersion in household-level consumption growth.

flows. We find a strong factor structure in total and idiosyncratic sales growth volatilities. Shocks to this common component of idiosyncratic volatility are priced. Sorting stocks on their CIV-beta results in a substantial return spread of about 6%. The spread survives inclusion of common risk factors. The CIV factor is also helpful to understand return differences on other stock and corporate bond portfolios. Finally, we establish an empirical connection between CIV shocks and shocks to the cross-sectional dispersion of household income growth.

We account for all three facts in a model with heterogeneous investors whose consumption risk is linked to firms' idiosyncratic cash flow risk. CIV is a priced state variable. Increases in CIV lead to an increase in the dispersion of consumption growth across households and are associated with high marginal utility for the average investor. Stocks whose returns rise with CIV hedge against deterioration in the investment opportunity set and thus earn low average returns. The calibrated model quantitatively matches the observed return spread and volatility facts.

Our work empirically shows a link between the volatility in firms' returns and cash flow growth and the cross-sectional volatility in household consumption growth. A large literature argues that shocks to firms have important effects on both the labor income and financial income of their employees. Another literature finds that households cannot or do not fully insure against their labor income shocks. One valuable direction for future work is to provide further evidence on the joint dynamics of the distributions of firm output and household income and consumption. A second direction is to use CIV as a measure of consumption growth dispersion in tests of consumption-based asset pricing models and in work that studies income and consumption inequality.

Appendix A. Empirical appendix

See Figs. A1–A4 and Tables A1–A9.
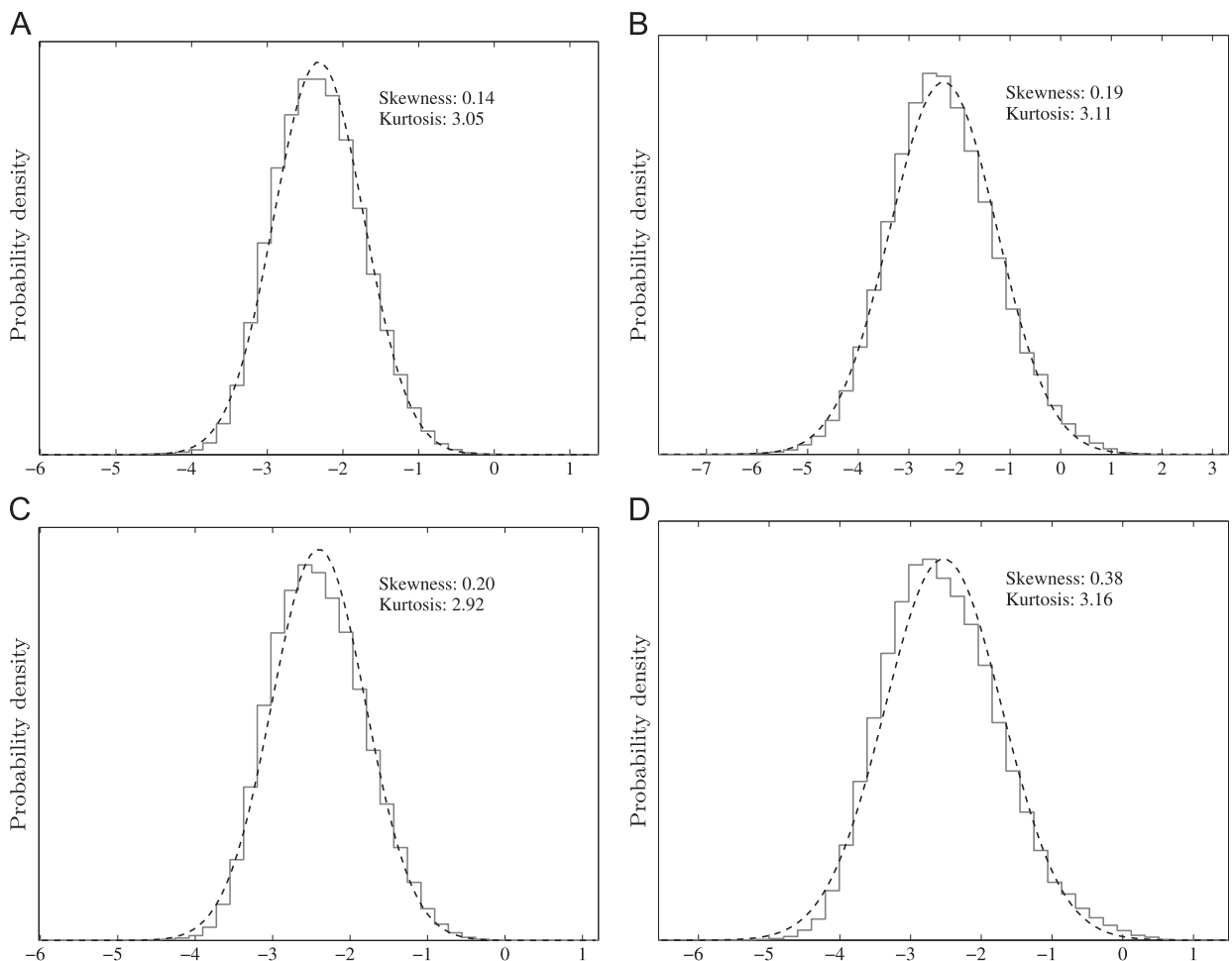
Fig. A1. Log volatility: empirical density versus normal density. The figure plots histograms of the empirical cross-section distribution of annual firm-level volatility (in logs) pooling all firm-year observations. Panel A shows total return volatility, calculated as the standard deviation of daily returns for each stock within a calendar year. Panel B shows total sales growth volatility, calculated as the standard deviation of quarterly year-on-year sales growth observations in a 20-quarter window. Panels C and D show idiosyncratic volatility based on the five principal components (PCs) factor model. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.
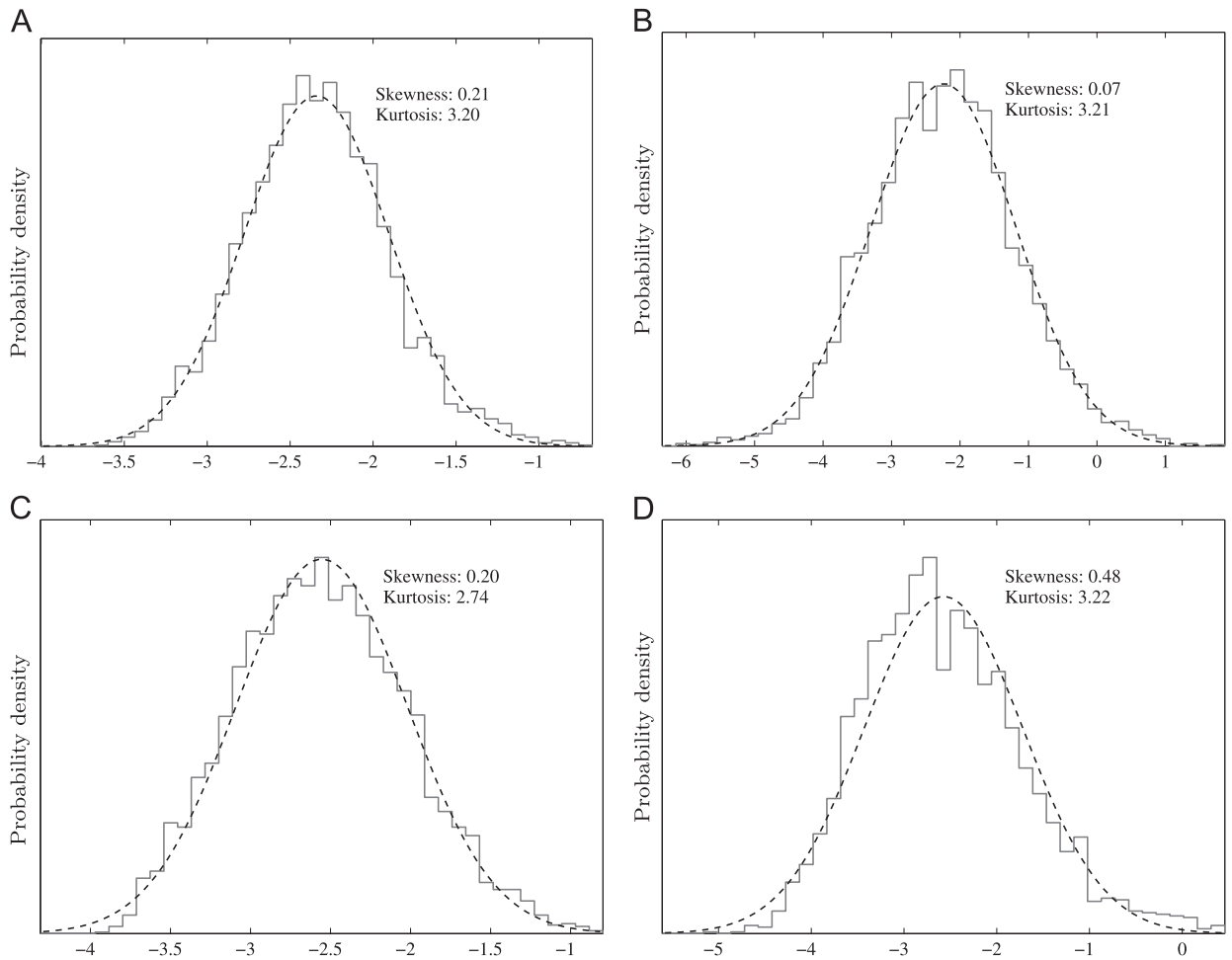
Fig. A2. Log volatility: empirical density versus normal density 2010 snapshot. The figure plots histograms of the empirical cross-section distribution of annual firm-level volatility (in logs) for the 2010 calendar year. Panel A shows total return volatility, calculated as the standard deviation of daily returns for each stock within a calendar year. Panel B shows total sales growth volatility, calculated as the standard deviation of quarterly year-on-year sales growth observations in a 20-quarter window. Panels C and D show idiosyncratic volatility based on the five principal components (PCs) factor model. Overlaid on these histograms is the exact normal density with mean and variance set equal to that of the empirical distribution. Each figure reports the skewness and kurtosis of the data in the histogram.
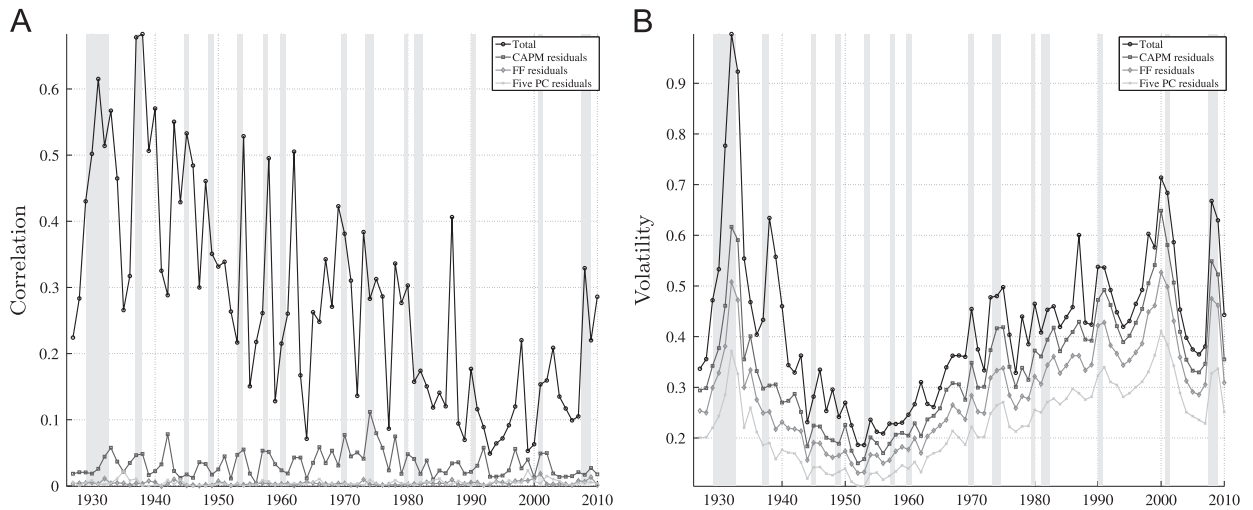
Fig. A3. Volatility and correlation of monthly returns. The figure repeats the analysis of Fig. 2 using monthly return observations within each calendar year, instead of daily. Panel A shows the average pairwise correlation for total and idiosyncratic returns within each calendar year. Panel B shows the cross-sectional average annualized firm-level volatility each year for total and idiosyncratic returns. Idiosyncratic volatility is the standard deviation of residuals from the capital asset pricing model (CAPM), the Fama and French (FF) three-factor model, or a five-factor principal components (PCs) model for daily returns within each calendar year.
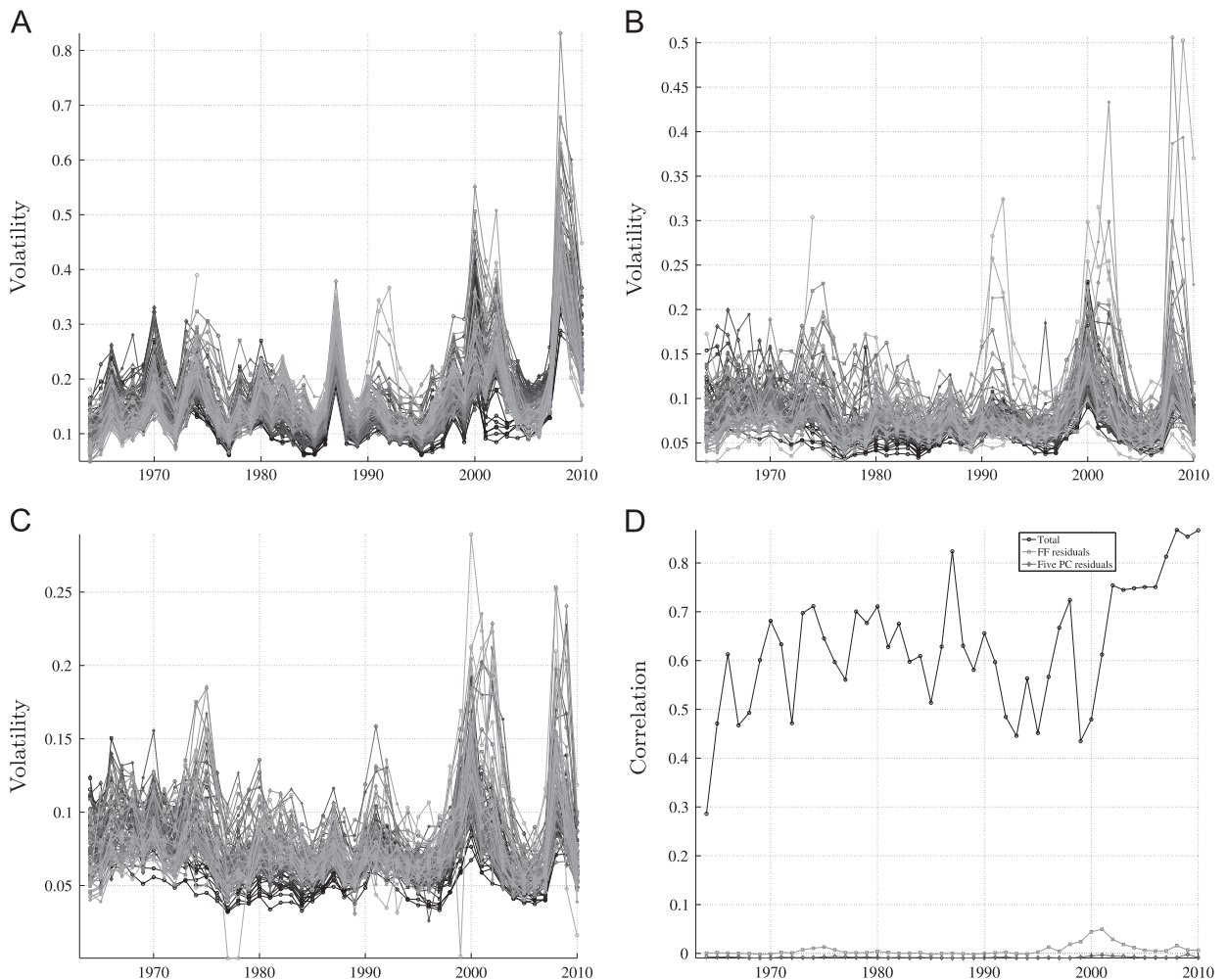
Fig. A4. Volatility of one hundred size and value portfolios. The figures plot volatility of total and idiosyncratic returns on one hundred size and value portfolios. Within each calendar year, total return volatilities are estimated from daily returns for each portfolio (Panel A). Idiosyncratic return volatility is the standard deviation of residuals from the three-factor Fama and French (FF) model (Panel B), or a five-factor principal components (PCs) model (Panel C), both estimated within each calendar year. Panel D shows average pairwise correlation for total and idiosyncratic returns on one hundred size and value portfolios within each calendar year.

Table A1

Volatility factor model estimates.

The table reports estimates of annual volatility one-factor regression models. In each panel, the volatility factor is defined as the equal-weighted cross section average of firm volatilities within that year. That is, all estimated volatility factor models take the form $\sigma_{i,t} = \text{intercept}_i + \text{loading}_i \cdot \bar{\sigma}_{\cdot,t} + e_{i,t}$. Columns represent different volatility measures. For the one hundred Fama and French (FF) returns (Panel A), the columns report estimates for a factor model of total return volatility and idiosyncratic volatility based on residual returns from the market model (MM), the Fama and French model, or the five principal components (PCs) models. For net income growth (Panel B) and earnings before interests, taxes, depreciation, and amortization (EBITDA) growth (Panel C), the columns report total volatility or idiosyncratic volatility based on sales growth residuals from the one and five principal components model (using a rolling 20-quarter window for estimation). The last column of Panels B and C reports results when using only the four quarterly growth observations within each calendar year to estimate total volatility. We report cross-sectional averages of loadings and intercepts as well as time series regression R^2 averaged over all firms. We also report a pooled factor model R^2, which compares the estimated factor model with a model with only firm-specific intercepts and no factor.

Panel A: Portfolio returns

	Total	MM	FF	Five PCs
Loading (average)	1.000	0.999	1.000	0.999
Intercept (average)	0.000	0.000	0.000	0.000
R^2 (average univariate)	0.708	0.497	0.394	0.450
R^2 (pooled)	0.691	0.454	0.375	0.470

Panel B: Net income growth

	Total (five years)	One PC (five years)	Five PCs (five years)	Total (one year)
Loading (average)	1.142	0.809	0.839	1.079
Intercept (average)	−0.053	−0.007	−0.011	−0.031
R^2 (average univariate)	0.285	0.270	0.269	0.199
R^2 (pooled)	0.273	0.257	0.252	0.169

Panel C: EBITDA growth

	Total (5 yr)	1 PC (5 yr)	5 PCs (5 yr)	Total (1 yr)
Loading (average)	0.842	0.934	0.884	0.990
Intercept (average)	0.065	−0.021	−0.009	0.017
R^2 (average univariate)	0.294	0.281	0.280	0.191
R^2 (pooled)	0.261	0.269	0.259	0.152

Table A2

Portfolios formed on common idiosyncratic volatility (CIV)-beta: additional single sorts.

The table reports annualized average excess returns, capital asset pricing model (CAPM) alphas, and three-factor Fama and French alphas for equally weighted portfolio sorts in annual percentages. Panels A and B report one-way sorts on CIV-beta using all Center for Research in Security Prices (CRSP) stocks in the 1986–2010 and 1963–1985 subsamples, respectively. Panel C reports sorts on CIV-beta, in which CIV-betas for stocks have been estimated from univariate regressions of monthly excess returns on CIV changes, without controlling for exposure to market variance (MV) shocks. Panel D reports sorts on MV-beta in the full 1963–2010 sample, in which MV-betas for stocks have been estimated from univariate regressions of monthly excess returns on MV changes, without controlling for exposure to CIV shocks.

	1 (low)	2	3	4	5 (high)	5 − 1	t(5 − 1)
Panel A: One-way sorts on CIV-beta, 1986–2010							
$\mathbb{E}[R] - r_f$	12.82	11.12	10.12	8.19	5.81	−7.00	−3.21
α_{CAPM}	4.82	4.34	4.01	2.25	−0.92	−5.73	−2.72
α_{FF}	2.74	2.11	1.69	0.26	−2.21	−4.94	−2.57
Panel B: One-way sorts on CIV-beta, 1963–1985							
$\mathbb{E}[R] - r_f$	11.29	10.63	9.79	9.26	7.62	−3.67	−2.29
α_{CAPM}	6.07	5.98	5.31	4.56	2.49	−3.57	−2.22
α_{FF}	−0.97	−0.08	−0.02	−0.11	−2.15	−1.18	−0.75
Panel C: One-way sorts on CIV-beta, no orthogonalization							
$\mathbb{E}[R] - r_f$	11.76	11.08	9.94	8.58	6.94	−4.82	−3.12
α_{CAPM}	4.77	5.03	4.44	3.32	1.30	−3.46	−2.39
α_{FF}	0.67	1.08	0.73	−0.11	−1.76	−2.43	−1.77
Panel D: One-way sorts on MV-beta							
$\mathbb{E}[R] - r_f$	9.98	10.42	10.39	9.26	8.25	−1.73	−0.94
α_{CAPM}	2.51	4.17	4.84	4.17	3.18	0.67	0.43
α_{FF}	−0.83	0.29	1.01	0.51	−0.38	0.45	0.33

Table A3
Portfolios formed on common idiosyncratic volatility (CIV)-beta: additional double sorts.

The table reports annualized average excess returns for several double-sorting exercises. In each panel, we sort stocks first into quintiles based on a factor and then, within each quintile, sort stocks in quintiles based on their CIV-beta. We form equally weighted average returns for all 25 portfolios, expressed in annual percentages. The second factor is size (log market equity) in Panel A, the level of idiosyncratic variance in Panel B, the volatility index (VIX)-beta in Panel C, and the Pastor and Stambaugh (PS) liquidity factor-beta in Panel D. The sample is 1963.01–2010.12, except for Panel C which is for 1990.1–2010.12.

| | CIV beta | | | | | $5-1$ | $t(5-1)$ |
	1 (low)	2	3	4	5 (high)		
Panel A: Two-way sorts on CIV-beta and log market equity							
1 (low)	14.77	14.22	12.67	11.86	9.97	−4.80	−2.80
2	10.40	11.03	11.66	10.64	6.89	−3.50	−2.45
3	11.56	11.14	10.07	8.93	7.60	−3.96	−2.72
4	10.39	9.89	9.48	8.44	6.35	−4.04	−2.88
5 (high)	8.23	7.62	6.69	6.02	5.00	−3.23	−2.33
$5-1$	−6.54	−6.60	−5.99	−5.84	−4.97	–	–
$t(5-1)$	−2.17	−2.42	−2.32	−2.35	−1.84	–	–
Panel B: Two-way sorts on CIV-beta and idiosyncratic variance							
1 (low)	9.52	9.50	7.92	7.66	7.43	−2.08	−2.09
2	13.20	10.99	10.12	9.09	8.65	−4.56	−4.24
3	14.49	13.12	11.69	11.27	8.97	−5.52	−4.25
4	14.32	12.44	11.12	10.44	9.34	−4.98	−3.42
5 (high)	8.31	7.01	7.21	5.24	3.36	−4.94	−2.70
$5-1$	−1.21	−2.49	−0.71	−2.42	−4.07	–	–
$t(5-1)$	−0.37	−0.81	−0.24	−0.84	−1.20	–	–
Panel C: Two-way sorts on CIV-beta and VIX-beta							
1 (low)	17.67	14.01	10.33	10.11	8.24	−9.43	−2.44
2	16.59	13.05	13.37	11.79	9.84	−6.75	−1.94
3	16.72	14.40	12.22	10.61	8.83	−7.89	−2.72
4	16.12	11.69	9.63	7.72	7.19	−8.93	−3.24
5 (high)	13.26	8.21	8.64	5.89	6.74	−6.52	−1.92
$5-1$	−4.41	−5.80	−1.69	−4.22	−1.49	–	–
$t(5-1)$	−0.95	−1.17	−0.34	−0.85	−0.28	–	–
Panel D: Two-way sorts on CIV-beta and PS liquidity beta							
1 (low)	11.89	9.76	8.02	6.31	5.20	−6.69	−3.51
2	11.27	9.66	8.57	7.93	5.53	−5.73	−3.59
3	11.99	10.85	9.40	8.17	6.63	−5.36	−3.48
4	11.85	10.94	10.41	8.19	6.11	−5.74	−3.86
5 (high)	10.30	9.81	9.90	8.83	6.25	−4.06	−2.45
$5-1$	−1.58	0.05	1.88	2.53	1.05	–	–
$t(5-1)$	−0.80	0.03	0.87	1.19	0.51	–	–

Table A4
Correlations among sorting variables.

For each month, we calculate the cross-sectional correlation between the sorting variables used in Panel E of Table 2 and in Table A3. These variables are common idiosyncratic volatility (CIV)-beta, market variance (MV)-beta, log market equity (ME), volatility index (VIX)-beta, and the Pastor and Stambaugh (PS) liquidity factor-beta. The table reports an average of these correlations over all months in the 1963–2010 sample. The CIV-beta and MV-beta are obtained from a multiple regression of individual stock excess returns on CIV shocks and MV shocks. All other betas are obtained from single regressions of individual stock excess returns on the asset pricing factors.

	CIV-beta	MV-beta	Log ME	Idiosyncratic Variance	VIX beta	PS liquidity beta
CIV-beta	1.00	–	–	–	–	–
MV-beta	−0.43	1.00	–	–	–	–
Log ME	0.13	0.03	1.00	–	–	–
Idiosyncratic Variance	−0.06	−0.04	−0.30	1.00	–	–
VIX-beta	−0.01	0.48	−0.09	−0.05	1.00	–
PS liquidity beta	−0.07	−0.26	−0.09	0.07	−0.23	1.00

Table A5

Fama and MacBeth analysis: traded factors.

The set of test assets are decile portfolios sorted on exposure to common idiosyncratic volatility (CIV) innovations in Columns 1–3, decile portfolios sorted on book-to-market (BM) ratio in Columns 4–6, decile portfolios sorted on size measured by market equity (ME) in Columns 7–9, and all of these 30 assets plus four corporate bond (CB) portfolios sorted on credit rating in Columns 10–12. The estimation sample is monthly from 1963.01–2010.12 in columns 1–9 and 1980.01–2010.12 in Columns 10–12. The model in Columns 1, 4, 7, and 10 contains the excess market return as the factor. The model in columns 2, 5, 8, and 11 contains the excess market return and the return on a portfolio that goes long in the tenth decile of CIV-beta stocks and short in the first decile of CIV-beta stocks (CIVtr) as factors. The model in Columns 3, 6, 9, and 12 contains the excess market return, CIVtr, and MVtr, where MVtr is the return a portfolio that goes long in the tenth decile of MV-beta stocks and short in the first decile of MV-beta stocks. The table reports monthly risk premia and Newey and West t-statistics (with one lag) and Shanken corrected t-statistics from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2, and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. * and ** denote significance at 5% and 1%, respectively. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	Ten CIV-beta			Ten BM			Ten ME			All 30 + four CB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.003	0.010	−0.026	0.009	0.007	0.014	−0.008	−0.005	−0.007	0.003	0.003	0.003
NW t-statistic	0.552	7.306	−2.706	0.971	3.598	2.952	−4.816	−2.670	−6.051	9.368	9.478	26.725
Shanken t-statistic	0.693	2.679	−1.516	1.736	1.386	1.885	−1.223	−0.997	−1.337	2.439	2.979	2.418
Rm-Rf	0.005	−0.004	0.036	−0.003	−0.002	−0.009	0.013	0.010	0.012	0.004	0.003	0.004
NW t-statistic	1.125	−2.709	3.476	−0.280	−1.214	−1.946	8.955	6.002	10.662	8.243	6.529	18.622
Shanken t-statistic	1.179	−0.798	1.908	−0.517	−0.419	−1.181	1.943	1.940	2.135	1.566	1.138	1.320
CIVtr	–	−0.005	−0.005	–	−0.014	−0.015	–	−0.005	−0.010	–	−0.005	−0.007
NW t-statistic	–	−12.607	−23.042	–	−23.828	−13.866	–	−7.499	−3.883	–	−4.815	−10.308
Shanken t-statistic	–	−3.913	−3.455	–	−2.815	−2.742	–	−1.003	−1.711	–	−2.022	−3.031
MVtr	–	–	0.011	–	–	0.003	–	–	0.005	–	–	0.007
NW t-statistic	–	–	6.844	–	–	1.062	–	–	1.535	–	–	9.168
Shanken t-statistic	–	–	2.441	–	–	0.418	–	–	0.701	–	–	2.144
b_{MV}	–	–	20.398	–	–	−7.637	–	–	3.839	–	–	3.785
NW t-statistic	–	–	4.072	–	–	−1.872	–	–	2.183	–	–	6.854
R^2	0.109	0.915	0.971	0.013	0.890	0.920	0.839	0.920	0.938	0.493	0.763	0.898
RMSE	1.759	0.543	0.315	1.886	0.628	0.535	0.543	0.384	0.336	1.653	1.130	0.741
χ^2	25.840**	14.955	3.097	11.579	4.413	3.424	8.019	5.109	4.250	44.783	37.128	32.736

Table A6

Fama and MacBeth analysis: Campbell, Giglio, Polk, and Turley measure.

The set of test assets are decile portfolios sorted on exposure to common idiosyncratic volatility (CIV) innovations in Columns 1–3, decile portfolios sorted on book-to-market (BM) ratio in Columns 4–6, decile portfolios sorted on size measured by market equity (ME) in Columns 7–9, and all of these 30 assets plus four corporate bond (CB) portfolios sorted on credit rating in Columns 10–12. The estimation sample is monthly from 1963.01–2010.12 in Columns 1–9 and 1980.01–2010.12 in Columns 10–12. The model in Columns 1, 4, 7, and 10 contains the excess market return as the factor. The model in Columns 2, 5, 8, and 11 contains the excess market return and the return on a portfolio that goes long in the tenth decile of CIV-beta stocks and short in the first decile of CIV-beta stocks (CIVtr) as factors. The model in Columns 3, 6, 9, and 12 contains the excess market return, CIVtr, and NVtr, where NVtr is the return a portfolio that goes long in the tenth decile of N_V-beta stocks and short in the first decile of N_V-beta stocks. N_V is the measure of market variance innovations proposed by Campbell, Giglio, Polk, and Turley (2014). The table reports monthly risk premia and Newey and West t-statistics (with one lag) and Shanken corrected t-statistics from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2, and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. * and ** denote significance at 5% and 1%, respectively. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	Ten CIV-beta			Ten BM			Ten ME			All 30 + four CB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	−0.001	0.012	0.028	0.009	0.006	0.007	−0.008	−0.003	−0.005	0.003	0.003	0.003
NW t-statistic	−0.311	5.827	3.779	0.971	4.678	2.333	−4.816	−1.649	−8.727	10.178	9.296	23.725
Shanken t-statistic	−0.182	3.005	2.702	1.736	1.072	1.177	−1.223	−0.695	−0.820	2.376	2.780	2.282
Rm-Rf	0.008	−0.007	−0.024	−0.003	−0.001	−0.003	0.013	0.008	0.009	0.005	0.003	0.004
NW t-statistic	3.978	−3.516	−3.087	−0.280	−0.973	−0.919	8.955	4.112	16.843	8.717	7.681	22.724
Shanken t-statistic	1.650	−1.387	−2.102	−0.517	−0.213	−0.442	1.943	1.495	1.503	1.615	1.195	1.381
CIVtr	–	−0.007	−0.007	–	−0.016	−0.017	–	−0.003	−0.010	–	−0.007	−0.008
NW t-statistic	–	−11.896	−13.125	–	−21.479	−6.927	–	−4.655	−7.147	–	−5.060	−8.027
Shanken t-statistic	–	−3.610	−3.559	–	−2.490	−2.083	–	−0.534	−1.718	–	−2.556	−3.124
MVtr	–	–	0.005	–	–	−0.002	–	–	−0.011	–	–	−0.003
NW t-statistic	–	–	1.132	–	–	−1.779	–	–	−3.908	–	–	−3.937
Shanken t-statistic	–	–	0.890	–	–	−0.445	–	–	−1.278	–	–	−0.771
b_{MV}	–	–	10.326	–	–	1.131	–	–	−7.544	–	–	−2.622
NW t-statistic	–	–	2.198	–	–	0.689	–	–	−5.759	–	–	−6.097
R^2	0.206	0.902	0.918	0.013	0.958	0.961	0.839	0.817	0.952	0.499	0.806	0.904
RMSE	2.147	0.755	0.688	1.886	0.360	0.348	0.543	0.409	0.209	1.686	1.048	0.739
χ^2	34.540**	22.693**	12.973	11.579	1.810	1.598	8.019	4.840	2.088	47.714*	38.044	36.174

Table A7
Fama and MacBeth analysis: firm size dispersion (FSD)-anomalies portfolios.

The set of test assets are decile portfolios sorted on exposure to innovations in the dispersion in firm size growth in Columns 1–3, decile portfolios sorted on book-to-market (BM) ratio in Columns 4–6, on size measured by market equity (ME) in Columns 7–9, and all of these 30 assets plus four corporate bond (CB) portfolios sorted by credit rating in Columns 10–12. The estimation sample is 1963.01–2010.12 in Columns 1–9 and 1980.01–2010.12 in Columns 10–12. The model in Columns 1, 4, 7, and 10 contains the excess market return as the factor. The model in Columns 2, 5, 8, and 11 contains the excess market return and the innovation in the cross-sectional dispersion of firm size growth as factors. The model in Columns 3, 6, 9, and 12 contains the excess market return, FSD, and the MV innovation. The table reports monthly risk premia and Newey and West t-statistics (with one lag) and Shanken corrected t-statistics from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2, and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. * and ** denote significance at 5% and 1%, respectively. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	Ten FSD-beta			Ten BM			Ten ME			All 30 + four CB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.011	0.007	0.009	0.009	−0.003	−0.001	−0.008	−0.003	0.006	0.003	0.003	0.003
NW t-statistic	4.798	2.516	3.486	0.971	−0.737	−0.132	−4.816	−0.688	1.968	6.908	13.595	7.871
Shanken t-statistic	3.490	1.225	1.794	1.736	−0.406	−0.068	−1.223	−0.600	0.811	2.764	2.255	2.291
Rm-Rf	−0.003	0.002	−0.003	−0.003	0.008	0.006	0.013	0.008	−0.002	0.004	0.004	0.003
NW t-statistic	−1.330	0.586	−0.618	−0.280	1.796	1.156	8.955	1.989	−0.465	6.917	11.094	7.543
Shanken t-statistic	−0.801	0.251	−0.475	−0.517	0.961	0.579	1.943	1.542	−0.186	1.423	1.376	1.109
FSD	–	−0.021	−0.016	–	−0.024	−0.021	–	0.007	0.002	–	−0.008	−0.010
NW t-statistic	–	−4.204	−3.468	–	−4.963	−4.696	–	1.320	0.891	–	−4.021	−8.404
Shanken t-statistic	–	−1.990	−1.911	–	−1.783	−1.904	–	0.686	0.176	–	−1.914	−2.185
MV	–	–	−0.033	–	–	−0.021	–	–	−0.024	–	–	−0.023
NW t-statistic	–	–	−0.952	–	–	−4.171	–	–	−5.631	–	–	−5.522
Shanken t-statistic	–	–	−1.440	–	–	−1.200	–	–	−1.579	–	–	−1.470
b_{MV}	–	–	−12.051	–	–	−4.317	–	–	−12.222	–	–	−4.838
NW t-statistic	–	–	−0.675	–	–	−1.628	–	–	−5.234	–	–	−3.954
R^2	0.121	0.407	0.450	0.013	0.656	0.669	0.839	0.876	0.957	0.492	0.695	0.812
RMSE	1.241	1.019	0.982	1.886	1.114	1.091	0.543	0.478	0.280	1.509	1.169	0.918
χ^2	15.727	6.356	7.004	11.579	3.268	3.568	8.019	7.076	3.988	39.105	30.578	26.128

Table A8
Fama and MacBeth analysis: Guvenen-anomalies portfolios.

The set of test assets are decile portfolios sorted on exposure to innovations in the dispersion of household income growth by Guvenen, Ozkan, and Song (2014) (GID) in Columns 1–3, decile portfolios sorted on book-to-market (BM) ratio in Columns 4–6, on size measured by market equity (ME) in Columns 7–9, and all of these 30 assets plus four corporate bond (CB) portfolios sorted by credit rating in Columns 10–12. The GID-beta-sorted portfolios are formed as follows. We estimate multiple regressions of annual excess returns of individual firms on annual GID innovations and annual market variance (MV) innovations. Annual MV innovations are formed as the difference between MV in the last month of the year and MV in the last month of the preceding year. We estimate rolling window factor betas using 15-year rolling windows. That implies that only firms with 15 years of return history are included. Once the betas estimated, we sort firms into deciles based on their GID-betas and calculate equally weighted portfolio returns. We form a traded risk factor, GIDtr, by going long the highest and short the lowest GID-beta decile portfolio. This traded risk factor return is available monthly, even though the underlying betas are estimated at annual frequency. In essence, the composition of the underlying decile portfolios is constant within the year. The estimation sample is 1978.01–2010.12 in Columns 1–9 and 1980.01–2010.12 in Columns 10–12. The model in Columns 1, 4, 7, and 10 contains the excess market return as the factor. The model in Columns 2, 5, 8, and 11 contains the excess market return and GIDtr as factors. The model in Columns 3, 6, 9, and 12 contains the excess market return, GIDtr, and MVtr, defined in Table A5. The table reports monthly risk premia and Newey and West t-statistics (with one lag) and Shanken corrected t-statistics from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2, and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. * and ** denote significance at 5% and 1%, respectively. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	Ten GID-beta			Ten BM			Ten ME			All 30 + four CB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
Constant	0.003	0.003	0.006	0.009	0.015	0.015	−0.008	0.005	0.006	0.003	0.004	0.003
NW t-statistic	4.841	2.631	2.517	0.971	5.252	2.748	−4.816	2.051	2.825	10.454	8.242	7.961
Shanken t-statistic	0.629	0.473	1.178	1.736	0.918	0.954	−1.223	0.459	0.529	2.628	3.397	3.162
Rm-Rf	0.007	0.007	0.002	−0.003	−0.008	−0.008	0.013	0.002	−0.000	0.005	0.004	0.004
NW t-statistic	10.058	6.900	0.653	−0.280	−2.787	−1.498	8.955	0.872	−0.058	11.270	7.098	7.349
Shanken t-statistic	1.024	1.060	0.299	−0.517	−0.516	−0.541	1.943	0.164	−0.010	1.241	1.061	1.063
GIDtr	–	−0.001	−0.001	–	−0.011	−0.011	–	−0.004	−0.012	–	−0.003	0.000
NW t-statistic	–	−4.157	−3.053	–	−2.959	−2.381	–	−8.474	−3.899	–	−4.580	0.040
Shanken t-statistic	–	−0.378	−0.444	–	−0.761	−0.718	–	−0.982	−1.332	–	−0.960	0.019
MVtr	–	–	−0.006	–	–	−0.005	–	–	0.005	–	–	−0.007
NW t-statistic	–	–	−2.244	–	–	−1.002	–	–	1.662	–	–	−6.601
Shanken t-statistic	–	–	−0.899	–	–	−0.481	–	–	0.436	–	–	−1.517
b_{MV}	–	–	−3.309	–	–	−0.152	–	–	8.698	–	–	−4.718
NW t-statistic	–	–	−1.540	–	–	−0.031	–	–	2.542	–	–	−4.747
R^2	0.602	0.606	0.652	0.013	0.479	0.480	0.839	0.809	0.874	0.549	0.631	0.719
RMSE	0.788	0.784	0.737	1.886	0.739	0.739	0.543	0.656	0.533	1.678	1.518	1.323
χ^2	9.676	9.633	9.269	11.579	2.015	2.009	8.019	5.500	4.198	27.570	27.354	24.745

Table A9

Fama and MacBeth analysis: additional anomalies portfolios.

The set of test assets are the 25 portfolios double sorted on book-to-market (BM) ratio and size measured by market equity (ME) in Columns 1–3. Then, six risk-sorted portfolios are added to test assets in Columns 4–6. We construct the 6 risk-sorted portfolios as follows. In a 60-month trailing window, we regress stock returns on market return and market variance (MV) innovations and use the coefficients (betas) to form double sorted portfolios. The six risk-sorted portfolios are independently sorted by terciles on the market exposure and into two groups based on the market variance innovations exposure. Then four corporate bond (CB) portfolios sorted by credit rating are added in Columns 7–9. The estimation sample is 1963.01–2010.12 in Columns 1–6 and 1980.01–2010.12 in Columns 7–9. The model in Columns 1, 4, and 7 contains the excess market return as the factor. The model in Columns 2, 5, and 8 contains the excess market return and common idiosyncratic volatility (CIV) innovation as factors. The model in Columns 3, 6, and 9 contains the excess market return, the CIV innovation and the MV innovation. The table reports monthly risk premia and Newey and West t-statistics (with one lag) and Shanken corrected t-statistics from a cross-sectional regression of average monthly excess portfolio returns on factor exposures. The third-to-last row reports the cross-sectional R^2, and the second-to-last row reports the root mean square pricing error, expressed as an annual return. The last row reports an asymptotic χ^2 (Wald) testing whether all pricing errors are jointly zero. * and ** denote significance at 5% and 1%, respectively. The market prices of risk b are estimated from a cross-sectional regression of average excess returns on the test assets on the (univariate) covariances of the returns with the factors.

	25 BM-ME			5 BM-ME, six risky			25 BM-ME, six risky, four CB		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Constant	0.012	0.018	0.020	0.009	0.012	0.012	0.005	0.005	0.005
NW t-statistic	2.926	5.665	9.162	3.560	4.543	4.478	3.516	3.495	3.459
Shanken t-statistic	2.964	3.574	3.171	3.496	4.288	3.977	4.001	4.077	3.686
Rm-Rf	−0.004	−0.012	−0.015	−0.002	−0.006	−0.007	0.002	0.001	0.001
NW t-statistic	−1.238	−4.346	−6.881	−0.776	−2.424	−2.514	1.497	0.831	0.564
Shanken t-statistic	−0.984	−2.350	−2.368	−0.543	−1.904	−2.059	0.797	0.473	0.332
CIV	–	−0.060	−0.065	–	−0.042	−0.049	–	−0.019	−0.026
NW t-statistic	–	−4.802	−7.343	–	−3.683	−4.790	–	−1.654	−2.429
Shanken t-statistic	–	−2.672	−2.338	–	−2.564	−2.529	–	−1.210	−1.445
MV	–	–	−0.048	–	–	−0.031	–	–	−0.025
NW t-statistic	–	–	−6.678	–	–	−4.839	–	–	−3.671
Shanken t-statistic	–	–	−3.152	–	–	−2.659	–	–	−2.013
b_{MV}	–	–	−21.273	–	–	−12.329	–	–	−8.278
NW t-statistic	–	–	−3.849	–	–	−2.903	–	–	−2.389
R^2	0.078	0.535	0.727	0.024	0.366	0.456	0.103	0.168	0.233
RMSE	2.691	1.911	1.465	2.541	2.049	1.898	2.791	2.688	2.580
χ^2	68.800**	41.993**	24.390	94.425**	75.304**	59.608**	148.731**	141.250**	119.360**

Appendix B. Model

This appendix solves the model sketched in the main text. It then discusses the benchmark model calibration. Next, we test the model's implication that price–dividend ratios predict CIV in the data. Finally, it provides sensitivity analysis to the benchmark calibration.

B.1. Model setup and solution

B.1.1. Setup

There is a unit mass of atomless agents, each having Epstein-Zin preferences. Let $U_t(C_t)$ denote the utility derived from consuming C_t. The value function of each agent takes the following recursive form:

$$U_t(C_t) = \left[(1-\delta)C_t^{(1-\gamma)/\theta} + \delta\left(\mathbb{E}_t U_{t+1}^{1-\gamma}\right)^{1/\theta} \right]^{\theta/(1-\gamma)}, \quad (11)$$

where $\theta \equiv (1-\gamma)/\left(1-\frac{1}{\psi}\right)$. The time discount factor is δ, the risk-aversion parameter is $\gamma \geq 0$, and the intertemporal elasticity of substitution (IES) is $\psi \geq 0$. When $\psi > 1$ and $\gamma > 1$, then $\theta < 0$ and agents prefer early resolution of uncertainty.

Aggregate labor income is defined as I_t. There is a large number of securities in zero or positive net supply. The combined total (and per capita) dividends are D_t. Aggregate dividend income plus aggregate labor income equals aggregate consumption: $C_t = I_t + D_t$. Individual consumption is given by $S_t^j C_t$, where S^j denotes agent j's

consumption share, and individual labor income is defined by

$$I_{j,t} = S_t^j C_t - D_t. \quad (12)$$

All agents can trade in all securities at all times and are endowed with an equal number of all securities at time zero. Labor income risk is, however, uninsurable. As in Constantinides and Ghosh (2014), given the symmetric and homogeneous preferences, households choose not to trade away from their initial endowments. That is, autarky is an equilibrium and individual j's equilibrium consumption is $C_{j,t} = I_{j,t} + D_t = S_t^j C_t$.

We use lowercase symbols to denote logs. We impose the same idiosyncratic volatility factor structure on investor consumption growth and firm dividend growth by adopting the following specification for consumption growth in aggregate and for each agent j:

$$\Delta c_{t+1}^j = \Delta c_{t+1}^a + \Delta s_{t+1}^j, \quad (13)$$

$$\Delta c_{t+1}^a = \mu_g + \sigma_c \eta_{t+1} + \phi_c \sigma_g w_{g,t+1}, \quad (14)$$

$$\Delta s_{t+1}^j = \sigma_{g,t+1} v_{t+1}^j - \tfrac{1}{2}\sigma_{g,t+1}^2, \quad (15)$$

and

$$\sigma_{g,t+1}^2 = \sigma_g^2 + \nu_g\left(\sigma_{gt}^2 - \sigma_g^2\right) + \sigma_w \sigma_g w_{g,t+1}. \quad (16)$$

All shocks are i.i.d. standard normal and mutually uncorrelated. While aggregate consumption growth is

Table B1

Predictability analysis.

The table reports results from predictive regressions of the log price–dividend ratio for future values of common idiosyncratic volatility (CIV) (Panels A and D), market variance (MV) (Panels B and E), and employment growth dispersion (Panels C and F). The sample is 1964–2010 in Panels A, B, and C, 1926–2010 in Panels D and E, and 1950–2010 in Panel F. CIV and MV are the levels (not the innovations) of the common idiosyncratic stock return variance and market return variance. The regressions predict one-month-ahead in Panels A, B, D, and E and one-year-ahead in Panels C and F. The Compustat data used to construct the employment growth dispersion are available only annually from 1950 until 2010. Q1 to Q5 refer to the five CIV-beta-sorted quintile portfolios. For each portfolio, we calculate the log price–dividend ratio from the cum-dividend and ex-dividend portfolio returns, which are constructed from the cum-dividend and ex-dividend returns of the individual stocks in that portfolio. Dividends are summed over 12 months (no reinvestment within the year). The regression controls for the idiosyncratic variance of the portfolio return. This idiosyncratic variance is constructed by regressing each individual stock's variance on CIV and averaging the residual among all the stocks in the portfolio. The t-statistics in Panels A, B, D, and E (C and F) use Newey and West standard errors with 12 (1) lags.

	Q1	Q2	Q3	Q4	Q5
Panel A: Predicting CIV (1964–2010)					
Coefficient	0.131	0.207	0.222	0.177	0.058
t-Statistic	5.02	5.61	4.02	2.92	1.16
R^2	0.29	0.26	0.20	0.11	0.04
Panel B: Predicting MV (1964–2010)					
Coefficient	0.004	0.010	0.014	0.015	0.010
t-Statistic	1.18	2.34	1.89	2.18	2.22
R^2	0.04	0.02	0.04	0.04	0.13
Panel C: Predicting employment growth dispersion (1964–2010)					
Coefficient	0.031	0.038	0.048	0.043	0.022
t-Statistic	5.03	4.24	3.97	3.21	2.09
R^2	0.37	0.38	0.29	0.21	0.11
Panel D: Predicting CIV (1926–2010)					
Coefficient	0.172	0.249	0.227	0.176	0.079
t-Statistic	6.18	7.32	4.85	2.97	1.49
R^2	0.37	0.32	0.15	0.09	0.09
Panel E: Predicting MV (1926–2010)					
Coefficient	0.011	0.017	0.012	0.012	0.003
t-Statistic	2.62	2.90	1.50	1.65	0.43
R^2	0.05	0.06	0.02	0.08	0.01
Panel F: Predicting employment growth dispersion (1950–2010)					
Coefficient	0.039	0.051	0.063	0.059	0.039
t-Statistic	8.39	6.66	6.12	5.35	4.39
R^2	0.55	0.52	0.44	0.39	0.30

Table B2

Calibration parameters: higher σ_g.

This table lists the parameter values for a calibration using a higher σ_g, the average of the cross-sectional variance of the individual log consumption growth process. The preferences parameters include intertemporal discount (δ), risk aversion (γ), and intertemporal elasticity of substitution (ψ). The aggregate consumption growth process, the consumption share process and the dividend growth process are described by Eqs. (2)–(4). Finally, the table also presents the calibration of the portfolios sorted from lowest common idiosyncratic volatility (CIV)-beta (Q1) to highest CIV-beta (Q5), and the market portfolio calibration is reported in the last column (M).

Panel A: Preferences, aggregate consumption growth, consumption share and dividend growth parameters

Parameter	Calibration	Parameter	Calibration	Parameter	Calibration	Parameter	Calibration
δ	0.12974	μ_g	0.02	σ_g	0.424	σ_i	0.004
γ	15	σ_c	0.01856	ν_g	0.6	ν_i	0.15
ψ	2	ϕ_c	-0.0395	σ_w	0.002841	σ_{iw}	1.5e−06

Panel B: Portfolio-specific parameters

Parameter	Q1	Q2	Q3	Q4	Q5	M
μ_i	6.58%	5.81%	4.92%	5.74%	4.55%	5.20%
φ_{di}	10.07	10.07	10.07	10.07	10.07	10.07
ϕ_i	-0.26	-0.14	-0.08	-0.03	0.07	-0.06
χ^i	-0.23	-0.06	0.01	-0.10	-0.35	-0.16
κ_i	1.37	1.03	0.93	0.93	1.12	–
ζ_i	63.24	47.83	43.45	43.58	53.29	–

Table B3

Calibration results: higher σ_g.

This table reports moments from the model calibrated with higher σ_g according to Table B2 and compares them with the data. The first two rows report the average excess return. The next two rows split out the equity risk premium into a contribution representing compensation for η risk and a compensation for w_g risk. Rows 5 and 6 report common idiosyncratic volatility (CIV)-betas, in which model betas have been scaled to ensure that the innovation volatility of CIV is the same in model and data. Rows 7 and 8 report stock return volatilities, followed by a breakdown of volatility into its five components in Rows 9–13 [see Eq. (7)]. Because the variance but not the volatility components are additive, we calculate the square root of each variance component and then rescale all components so they sum to total volatility. Rows 14 and 15 report the slope of a regression of $\mathbb{V}_t\left[r_{t+1}^{idio,i}\right]$ on CIV, multiplied by one hundred. Rows 16 and 17 report the R^2 of this regression, multiplied by one hundred. Rows 18 and 19 report the slope of a predictive regression of annual dividend growth on one-year-lagged CIV. The model is simulated at annual frequency for 60 thousand periods. All moments in the data are expressed as annual quantities and computed from the 1963.01 to 2010.12 sample.

Row	Moment	Q1	Q2	Q3	Q4	Q5	M
Excess return							
1	Data	12.08	10.88	9.96	8.70	6.68	5.50
2	Model	8.65	6.65	5.74	4.89	3.25	5.50
3	η risk	5.21	5.21	5.21	5.21	5.21	5.21
4	w_g risk	3.45	1.44	0.53	− 0.31	− 1.96	0.29
Beta $\beta_{gs,i}$							
5	Data	− 0.50	− 0.21	− 0.08	0.05	0.28	− 0.04
6	Model	− 0.50	− 0.21	− 0.08	0.05	0.28	− 0.04
Return volatility							
7	Data	66.89	52.17	47.20	46.70	54.45	15.65
8	Model	66.50	51.16	46.97	47.09	55.67	18.71
9	η risk	11.21	11.24	11.41	11.49	11.22	18.01
10	w_g risk	5.15	2.16	0.81	0.48	2.93	0.70
11	e_i risk	34.74	26.13	24.04	24.30	28.59	–
12	ε_i risk	15.18	11.50	10.60	10.71	12.79	–
13	w_i risk	0.21	0.12	0.10	0.10	0.15	–
Regression of idiosyncratic variance on CIV: slope coefficient							
14	Data	1.54	0.87	0.71	0.72	1.04	–
15	Model	1.58	0.89	0.73	0.74	1.07	–
Regression of idiosyncratic variance on CIV: R^2							
16	Data	17.69	17.21	17.10	17.11	16.31	–
17	Model	17.69	17.21	17.10	17.11	16.31	–
Dividend predictability regression coefficient							
18	Data	− 0.20	− 0.05	0.01	− 0.08	− 0.29	− 0.13
19	Model	− 0.20	− 0.05	0.01	− 0.08	− 0.29	− 0.13

Table B4

Calibration parameters: lower γ.

This table lists the parameter values for a calibration using a lower risk aversion (γ). The preferences parameters include intertemporal discount (δ), risk aversion (γ), and intertemporal elasticity of substitution (ψ). The aggregate consumption growth process, the consumption share process, and the dividend growth process are described by Eqs. (2)–(4). Finally, the table also presents the calibration of the portfolios sorted from lowest common idiosyncratic volatility (CIV)-beta (Q1) to highest CIV-beta (Q5), and the market portfolio calibration is reported in the last column (M).

Panel A: Preferences, aggregate consumption growth, consumption share and dividend growth parameters

Parameter	Calibration	Parameter	Calibration	Parameter	Calibration	Parameter	Calibration
δ	0.33086	μ_g	0.02	σ_g	0.424	σ_i	0.004
γ	8	σ_c	0.01856	ν_g	0.6	ν_i	0.15
ψ	2	ϕ_c	− 0.0395	σ_w	0.01472	σ_{iw}	1.5e − 06

Panel B: Portfolio-specific parameters

Parameter	Q1	Q2	Q3	Q4	Q5	M
μ_i	6.58%	5.81%	4.92%	5.74%	4.55%	5.20%
φ_{di}	5.69	5.69	5.69	5.69	5.69	5.69
ϕ_i	− 0.35	− 0.23	− 0.17	− 0.12	− 0.02	− 0.15
χ^i	− 0.14	− 0.04	0.01	− 0.06	− 0.21	− 0.09
κ_i	1.05	0.79	0.72	0.72	0.87	–
ζ_i	111.04	83.98	76.28	76.52	93.57	–

Table B5
Calibration results: lower γ.

This table reports moments from the model calibrated with higher γ according to Table B4 and compares them with the data. The first two rows report the average excess return. The next two rows split out the equity risk premium into a contribution representing compensation for η risk and a compensation for w_g risk. Rows 5 and 6 report common idiosyncratic volatility (CIV)-betas, in which model betas have been scaled to ensure that the innovation volatility of CIV is the same in model and data. Rows 7 and 8 report stock return volatilities, followed by a breakdown of volatility into its five components in Rows 9–13 [see Eq. (7)]. Since the variance but not the volatility components are additive, we calculate the square root of each variance component and then rescale all components so they sum to total volatility. Rows 14 and 15 report the slope of a regression of $\mathbb{V}_t\left[r_{t+1}^{idio,i}\right]$ on CIV, multiplied by one hundred. Rows 16 and 17 report the R^2 of this regression, multiplied by one hundred. Rows 18 and 19 report the slope of a predictive regression of annual dividend growth on one-year-lagged CIV. The model is simulated at annual frequency for 60 thousand periods. All moments in the data are expressed as annual quantities and computed from the 1963.01 to 2010.12 sample.

Row	Moment	Q1	Q2	Q3	Q4	Q5	M
Excess return							
1	Data	12.08	10.88	9.96	8.70	6.68	5.50
2	Model	4.91	2.97	2.09	1.27	−0.33	1.85
3	η risk	1.57	1.57	1.57	1.57	1.57	1.57
4	w_g risk	3.34	1.40	0.52	−0.30	−1.90	0.28
Beta $\beta_{gs,i}$							
5	Data	−0.50	−0.21	−0.08	0.05	0.28	−0.04
6	Model	−0.50	−0.21	−0.08	0.05	0.28	−0.04
Return volatility							
7	Data	66.89	52.17	47.20	46.70	54.45	15.65
8	Model	64.50	48.76	44.37	44.49	53.75	10.59
9	η risk	6.23	6.29	6.40	6.44	6.28	9.92
10	w_g risk	5.07	2.14	0.80	0.48	2.90	0.68
11	e_i risk	26.35	19.96	18.39	18.60	21.85	–
12	ε_i risk	26.20	20.00	18.47	18.66	22.25	–
13	w_i risk	0.64	0.37	0.31	0.32	0.46	–
Regression of idiosyncratic variance on CIV: slope coefficient							
14	Data	1.54	0.87	0.71	0.72	1.04	–
15	Model	1.58	0.89	0.73	0.74	1.07	–
Regression of idiosyncratic variance on CIV: R^2							
16	Data	17.69	17.21	17.10	17.11	16.31	–
17	Model	17.69	17.21	17.10	17.11	16.31	–
Dividend predictability regression coefficient							
18	Data	−0.20	−0.05	0.01	−0.08	−0.29	−0.13
19	Model	−0.20	−0.05	0.01	−0.08	−0.29	−0.13

homoskedastic, household consumption growth is not. The cross-sectional mean and variance of the consumption share process are

$$\mathbb{E}_j\left[\Delta s_{t+1}^j\right] = -\frac{1}{2}\sigma_{g,t+1}^2, \quad \mathbb{V}_j\left[\Delta s_{t+1}^j\right] = \sigma_{g,t+1}^2, \qquad (17)$$

where $\mathbb{E}_j[\cdot]$ and $\mathbb{V}_j[\cdot]$ are expectation and variance operators over the cross section of households. Thus, the process $\sigma_{g,t+1}$ measures the cross-sectional standard deviation of consumption share growth. The mean consumption share in levels is one ($\mathbb{E}_j[S_t^j] = 1$). As an aside, our results are robust to changes in the timing of the consumption share growth process in Eq. (15). The consumption share growth dispersion could be modeled either as $\sigma_{g,t}$ or as $\sigma_{g,t+1}$. This changes the expressions in the model as well as the calibration, but it does not affect our quantitative results. We prefer the current timing because it allows for shocks to the σ_g process that occur between t and $t+1$ to affect the cross-sectional dispersion of consumption growth between t and $t+1$. Because CIV is the state variable that forecasts future dividend growth, σ_{gt} shows up in the dividend growth Eq. (4).

Dividend growth of firm i is given by

$$\Delta d_{t+1}^i = \mu_i + \chi^i\left(\sigma_{gt}^2 - \sigma_g^2\right) + \varphi\sigma_c\eta_{t+1} + \phi_i\sigma_g w_{g,t+1} \\ + \kappa_i\sigma_{gt}e_{t+1}^i + \zeta_i\sigma_{it}\varepsilon_{t+1}^i \qquad (18)$$

and

$$\sigma_{i,t+1}^2 = \sigma_i^2 + \nu_i\left(\sigma_{it}^2 - \sigma_i^2\right) + \sigma_{iw}w_{i,t+1}. \qquad (19)$$

The firm i's idiosyncratic dividend growth risk is

$$\mathbb{V}_t\left[\kappa_i\sigma_{gt}e_{t+1}^i + \zeta_i\sigma_{it}\varepsilon_{t+1}^i\right] = \kappa_i^2\sigma_{gt}^2 + \zeta_i^2\sigma_{it}^2. \qquad (20)$$

The market portfolio's dividend growth process is given by

$$\Delta d_{t+1}^M = \mu_M + \chi^M\left(\sigma_{gt}^2 - \sigma_g^2\right) + \varphi\sigma_c\eta_{t+1} + \phi_m\sigma_g w_{g,t+1}. \qquad (21)$$

B.1.2. Claim to individual consumption stream

We start by pricing a claim to individual consumption growth, using the individual's own intertemporal marginal rate of substitution. We conjecture that the log wealth-consumption ratio of agent j is linear in the state variable

σ_{gt}^2 and does not depend on any agent-specific characteristics: $wc_t^j = \mu_{wc} + W_{gs}\left(\sigma_{gt}^2 - \sigma_g^2\right)$. We verify this conjecture evaluating the Euler equation for the consumption claim of agent j: $\mathbb{E}_t[M_{t+1}^j R_{t+1}^j] = 1$, where M_{t+1}^j is agent j's stochastic discount factor. Under symmetric preferences, this conjecture implies that the individual wealth-consumption ratio does not depend on agent-specific attributes, only on aggregate objects.

The beginning-of-period (or cum-dividend) total wealth W_t^j that is not spent on consumption C_t^j earns a gross return R_{t+1}^j and leads to beginning-of-next-period total wealth W_{t+1}^j. The return on a claim to consumption, the total wealth return, can be written as

$$R_{t+1}^j = \frac{W_{t+1}^j}{W_t^j - C_t^j} = \frac{C_{t+1}^j}{C_t^j} \frac{WC_{t+1}^j}{WC_t^j - 1}. \tag{22}$$

We use the Campbell (1991) approximation of the log total wealth return $r_t^j = \log(R_t^j)$ around the long-run average log wealth-consumption ratio $\mu_{wc} \equiv \mathbb{E}[w_t^j - c_t^j]$:

$$r_{t+1}^j = \kappa_0^c + \Delta c_{t+1}^j + wc_{t+1}^j - \kappa_1^c wc_t^j, \tag{23}$$

where the linearization constants κ_0^c and κ_1^c are non-linear functions of the unconditional mean log wealth-consumption ratio μ_{wc}:

$$\kappa_1^c = \frac{e^{\mu_{wc}}}{e^{\mu_{wc}} - 1} > 1 \quad \text{and} \quad \kappa_0^c = -\log(e^{\mu_{wc}} - 1) + \frac{e^{\mu_{wc}}}{e^{\mu_{wc}} - 1}\mu_{wc}. \tag{24}$$

The return on a claim to the consumption stream of agent j, R^j, satisfies the Euler equation under her stochastic discount factor:

$$1 = \mathbb{E}_t\left[M_{t+1}^j R_{t+1}^j\right] \tag{25}$$

$$1 = \mathbb{E}_t\left[\mathbb{E}_j\left[M_{t+1}^j R_{t+1}^j\right]\right] \tag{26}$$

$$1 = \mathbb{E}_t\left[\mathbb{E}_j\left[\exp\{m_{t+1}^j + r_{t+1}^j\}\right]\right] \tag{27}$$

and

$$1 = \mathbb{E}_t\left[\exp\{\mathbb{E}_j\left(m_{t+1}^j + r_{t+1}^j\right) + \tfrac{1}{2}\mathbb{V}_j\left(m_{t+1}^j + r_{t+1}^j\right)\}\right] \tag{28}$$

where the second equality applies the law of iterated expectations, and the last equality applies the cross-sectional normality of consumption share growth.

We combine the approximation of the log total wealth return with our conjecture for the wealth-consumption ratio of agent j, $wc_t^j = \mu_{wc} + W_{gs}\left(\sigma_{gt}^2 - \sigma_g^2\right)$, and solve for the coefficients μ_{wc} and W_{gs} by imposing the Euler equation for the consumption claim.

First, using the conjecture, we compute the individual log total wealth return r_{t+1}^j:

$$r_{t+1}^j = \kappa_0^c + \Delta c_{t+1}^j + wc_{t+1}^j - \kappa_1^c wc_t^j = r_0^c + \left[W_{gs}(\nu_g - \kappa_1^c)\right.$$
$$\left. - \tfrac{1}{2}\nu_g\right]\left(\sigma_{gt}^2 - \sigma_g^2\right) + \sigma_c\eta_{t+1} + (\phi_c + W_{gs}\sigma_w$$
$$- \tfrac{1}{2}\sigma_w)\sigma_g w_{g,t+1} + \sigma_{g,t+1}\nu_{t+1}^j, \tag{29}$$

where $r_0^c = \kappa_0^c + \mu_g + (1 - \kappa_1^c)\mu_{wc} - \tfrac{1}{2}\sigma_g^2$.

Second, Epstein and Zin (1989) show that the log real stochastic discount factor is

$$m_{t+1}^j = \theta \log \delta - \frac{\theta}{\psi}\Delta c_{t+1}^j + (\theta - 1)r_{t+1}^j$$
$$= \mu_s + \left[(\theta - 1)W_{gs}(\nu_g - \kappa_1^c) + \gamma\tfrac{1}{2}\nu_g\right]\left(\sigma_{gt}^2 - \sigma_g^2\right)$$
$$- \gamma\sigma_c\eta_{t+1} - \gamma\sigma_{g,t+1}\nu_{t+1}^j + \left[(\theta - 1)W_{gs}\sigma_w - \gamma\phi_c\right.$$
$$\left. + \tfrac{1}{2}\gamma\sigma_w\right]\sigma_g w_{g,t+1}, \tag{30}$$

where $\mu_s = \theta \log \delta - \gamma\mu_g + (\theta - 1)\left[\kappa_0^c + (1 - \kappa_1^c)\mu_{wc}\right] + \gamma\tfrac{1}{2}\sigma_g^2$ is the unconditional mean log SDF.

Third, using the individual stochastic discount factor and total wealth return expressions, we can compute elements of Eq. (28). We have that

$$\mathbb{E}_j\left(m_{t+1}^j\right) = \mu_s + \left[(\theta - 1)W_{gs}(\nu_g - \kappa_1^c) + \gamma\tfrac{1}{2}\nu_g\right]\left(\sigma_{gt}^2 - \sigma_g^2\right)$$
$$- \gamma\sigma_c\eta_{t+1} + \left[(\theta - 1)W_{gs}\sigma_w - \gamma\phi_c\right.$$
$$\left. + \tfrac{1}{2}\gamma\sigma_w\right]\sigma_g w_{g,t+1}, \tag{31}$$

$$\mathbb{E}_j\left(r_{t+1}^j\right) = r_0^c + \left[W_{gs}(\nu_g - \kappa_1^c) - \tfrac{1}{2}\nu_g\right]\left(\sigma_{gt}^2 - \sigma_g^2\right) + \sigma_c\eta_{t+1}$$
$$+ (\phi_c + W_{gs}\sigma_w - \tfrac{1}{2}\sigma_w)\sigma_g w_{g,t+1}, \tag{32}$$

and

$$\mathbb{V}_j\left[m_{t+1}^j + r_{t+1}^j\right] = (1 - \gamma)^2\left(\sigma_g^2 + \nu_g\left(\sigma_{gt}^2 - \sigma_g^2\right) + \sigma_w\sigma_g w_{g,t+1}\right). \tag{33}$$

Finally, we can use Eqs. (31)–(33) to solve the Euler equation (28). Using log normal properties, we can take the expected value conditional on time t information and compute the Euler equation. Applying method of undetermined coefficients, the following equalities hold:

$$0 = \mu_s + r_0^c + \tfrac{1}{2}(1 - \gamma)^2\sigma_g^2 + \tfrac{1}{2}(1 - \gamma)^2\sigma_c^2 + \tfrac{1}{2}\left[\theta W_{gs}\sigma_w\right.$$
$$\left. + (1 - \gamma)\phi_c + \tfrac{1}{2}\gamma(\gamma - 1)\sigma_w\right]^2\sigma_g^2, \tag{34}$$

and

$$W_{gs} = \frac{\nu_g\gamma(\gamma - 1)}{2\theta(\kappa_1^c - \nu_g)} = -\frac{\gamma\nu_g\left(1 - \dfrac{1}{\psi}\right)}{2(\kappa_1^c - \nu_g)}. \tag{35}$$

If the IES ψ exceeds one, then $W_{gs} < 0$. Plugging the W_{gs} expression as well as κ_0^c and κ_1^c back into Eq. (34) implicitly defines a nonlinear equation in one unknown (μ_{wc}), which can be solved for numerically, characterizing the average log wealth-consumption ratio.

B.1.3. Aggregate SDF

Because all agents can invest in all risky assets, the Euler equation has to be satisfied for any two agents j and j' and for every stock i:

$$1 = \mathbb{E}_t\left[M_{t+1}^j R_{t+1}^i\right], \quad \forall i, j. \tag{36}$$

This implies that the average SDF must also price all financial assets i:

$$1 = \mathbb{E}_t\left[\mathbb{E}_j\left(M_{t+1}^j R_{t+1}^i\right)\right] = \mathbb{E}_t\left[\mathbb{E}_j\left(M_{t+1}^j\right)R_{t+1}^i\right]$$

$$= \mathbb{E}_t\left[M_{t+1}^a R_{t+1}^i\right], \quad \forall i. \tag{37}$$

Once we solve for the individual stochastic discount factors, the common log real stochastic discount factor can be derived:

$$
\begin{aligned}
m_{t+1}^a &= \mathbb{E}_j\left[m_{t+1}^j\right] + \tfrac{1}{2}\mathbb{V}_j\left[m_{t+1}^j\right] = \mu_s + \tfrac{1}{2}\gamma^2\sigma_g^2 \\
&\quad + s_{gs}\left(\sigma_{gt}^2 - \sigma_g^2\right) - \lambda_\eta \sigma_c \eta_{t+1} - \lambda_w \sigma_g w_{t+1},
\end{aligned}
\tag{38}
$$

where the loadings are given by

$$
s_{gs} \equiv (\theta-1)W_{gs}(\nu_g - \kappa_1^c) + \tfrac{1}{2}\gamma(1+\gamma)\nu_g = \tfrac{1}{2}\gamma\nu_g\left(\tfrac{1}{\psi}+1\right),
\tag{39}
$$

$$
\lambda_\eta \equiv \gamma,
\tag{40}
$$

and

$$
\begin{aligned}
\lambda_w &\equiv (1-\theta)W_{gs}\sigma_w + \gamma\phi_c - \tfrac{1}{2}\gamma(1+\gamma)\sigma_w \\
&= \frac{\gamma\nu_g\left(\tfrac{1}{\psi}-\gamma\right)}{2(\kappa_1^c - \nu_g)}\sigma_w + \gamma\phi_c - \tfrac{1}{2}\gamma(1+\gamma)\sigma_w.
\end{aligned}
\tag{41}
$$

The risk-free rate is given by

$$
\begin{aligned}
r_t^f &= -\mathbb{E}_t\left[m_{t+1}^a\right] - \tfrac{1}{2}\mathbb{V}_t\left[m_{t+1}^a\right] = -\mu_s - \tfrac{1}{2}\gamma^2\sigma_g^2 - \tfrac{1}{2}\lambda_\eta^2\sigma_c^2 \\
&\quad - \tfrac{1}{2}\lambda_w^2\sigma_g^2 - s_{gs}\left(\sigma_{gt}^2 - \sigma_g^2\right).
\end{aligned}
\tag{42}
$$

Interest rates contain the usual impatience and intertemporal substitution terms. They also capture the precautionary savings motive: When idiosyncratic risk is high, agents increase savings, thereby lowering interest rates.

The maximum Sharpe ratio in the economy is

$$
\max SR_t = \sqrt{\lambda_\eta^2\sigma_c^2 + \lambda_w^2\sigma_g^2}.
\tag{43}
$$

It is larger when these risk prices are higher and shocks more volatile.

B.1.4. Firm stock return

Turning to the pricing of the dividend claim defined by Eq. (18), we guess and verify that its log price–dividend ratio is affine in the common and idiosyncratic variance terms:

$$
pd_t^i = \mu_{pdi} + A_{gs}^i\left(\sigma_{gt}^2 - \sigma_g^2\right) + A_{is}^i\left(\sigma_{it}^2 - \sigma_i^2\right).
\tag{44}
$$

As usual, returns are approximated as

$$
r_{t+1}^i = \Delta d_{t+1}^i + k_0^i + k_1^i pd_{t+1}^i - pd_t^i,
\tag{45}
$$

where $k_1^i = \frac{\exp(\mu_{pdi})}{1+\exp(\mu_{pdi})}$ and $k_0^i = \log(1+\exp(\mu_{pdi})) - k_1^i\mu_{pdi}$ are approximation constants. Plugging in the dividend growth equation as well as the price dividend expression, we get

$$
\begin{aligned}
r_{t+1}^i &= r_0^i + \left[\chi_i - A_{gs}^i\left(1-k_1^i\nu_g\right)\right]\left(\sigma_{gt}^2 - \sigma_g^2\right) \\
&\quad - A_{is}^i\left(1-k_1^i\nu_i\right)\left(\sigma_{it}^2 - \sigma_i^2\right) + \varphi\sigma_c\eta_{t+1} \\
&\quad + \left(\phi_i + A_{gs}^i k_1^i\sigma_w\right)\sigma_g w_{t+1} + \kappa_i\sigma_{gt}e_{t+1}^i + \zeta_i\sigma_{it}\varepsilon_{t+1}^i \\
&\quad + A_{is}^i k_1^i\sigma_{iw}w_{i,t+1},
\end{aligned}
\tag{46}
$$

where $r_0^i = \mu_i + k_0^i + (k_1^i - 1)\mu_{pdi}$.

Innovations in individual stock market return and individual return variance reflect the additional sources of idiosyncratic risk:

$$
\begin{aligned}
r_{t+1}^i - \mathbb{E}_t\left[r_{t+1}^i\right] &= \beta_{\eta,i}\sigma_c\eta_{t+1} + \beta_{gs,i}\sigma_g w_{g,t+1} + \kappa_i\sigma_{gt}e_{t+1}^i \\
&\quad + \zeta_i\sigma_{it}\varepsilon_{t+1}^i + k_1^i A_{is}^i\sigma_{iw}w_{i,t+1}
\end{aligned}
\tag{47}
$$

and

$$
\mathbb{V}_t\left[r_{t+1}^i\right] = \beta_{\eta,i}^2\sigma_c^2 + \beta_{gs,i}^2\sigma_g^2 + \left(k_1^i A_{is}^i\right)^2\sigma_{iw}^2 + \kappa_i^2\sigma_{gt}^2 + \zeta_i^2\sigma_{it}^2,
\tag{48}
$$

where

$$
\beta_{\eta,i} \equiv \varphi
\tag{49}
$$

and

$$
\beta_{gs,i} \equiv k_1^i A_{gs}^i\sigma_w + \phi_i.
\tag{50}
$$

The expression for the equity risk premium on an individual stock is

$$
\mathbb{E}_t\left[r_{t+1}^i - r_t^f\right] + 0.5\mathbb{V}_t[r_{t+1}^i] = \beta_{\eta,i}\lambda_\eta\sigma_c^2 + \beta_{gs,i}\lambda_w\sigma_g^2.
\tag{51}
$$

The coefficients of the price–dividend equation are obtained from the Euler equation:

$$
A_{gs}^i = \frac{2s_{gs} + 2\chi_i + \kappa_i^2}{2\left(1 - k_1^i\nu_g\right)} = \frac{2\chi_i + \kappa_i^2 + \left(1+\tfrac{1}{\psi}\right)\gamma\nu_g}{2\left(1 - k_1^i\nu_g\right)}
\tag{52}
$$

and

$$
A_{is}^i = \frac{\zeta_i^2}{2(1 - k_1^i\nu_i)},
\tag{53}
$$

and the constant μ_{pdi} is the mean log pd ratio, which solves the nonlinear equation

$$
\begin{aligned}
0 &= r_0^i + \mu_s + \tfrac{1}{2}\gamma^2\sigma_g^2 + \tfrac{1}{2}(\beta_{gs,i} - \lambda_w)^2\sigma_g^2 + \tfrac{1}{2}(\beta_{\eta,i} - \lambda_\eta)^2\sigma_c^2 \\
&\quad + \tfrac{1}{2}\kappa_i^2\sigma_g^2 + \tfrac{1}{2}\zeta_i^2\sigma_i^2 + \tfrac{1}{2}\left(k_1^i A_{is}^i\right)^2\sigma_{iw}^2.
\end{aligned}
\tag{54}
$$

The equity risk premium on the market portfolio is derived using the same procedure. It is given by

$$
\mathbb{E}_t\left[r_{t+1}^M - r_t^f\right] + 0.5\mathbb{V}_t[r_{t+1}^M] = \beta_{\eta,M}\lambda_\eta\sigma_c^2 + \beta_{gs,M}\lambda_w\sigma_g^2,
\tag{55}
$$

where

$$
\beta_{\eta,M} \equiv \varphi,
\tag{56}
$$

$$
\beta_{gs,M} \equiv \kappa_1^M A_{gs}^M\sigma_w + \phi_m,
\tag{57}
$$

and

$$
A_{gs}^M = \frac{2s_{gs} + 2\chi_M}{2\left(1 - \kappa_1^M\nu_g\right)} = \frac{2\chi_M + \left(1+\tfrac{1}{\psi}\right)\gamma\nu_g}{2\left(1 - \kappa_1^M\nu_g\right)}.
\tag{58}
$$

B.2. Calibration

Table 5 shows our benchmark parameter choices. The model is calibrated to match moments of the data for the 1963–2010 period and simulated at an annual frequency. Risk-aversion γ is set to 15 and the inter-temporal elasticity of substitution ψ is set to 2. The high value for γ is

needed to generate a high equity risk premium for the market portfolio. A richer model with time variation in market variance or with a slow-moving component in expected consumption growth as in Bansal and Yaron (2004) would allow us to match the equity risk premium with lower risk aversion, but at the expense of higher model complexity. We perform sensitivity with respect to the parameter γ. The time discount factor $\delta = 0.875$ is set to produce a mean real risk-free rate of 1.5% per year, given all other parameters. The model produces a risk-free rate with modest volatility of 1.25% per year due to the high elasticity of intertemporal substitution. Mean consumption growth μ_g is 2% per year and σ_c is 0.0247 per year. We set ϕ_c equal to -0.04 to capture the negative correlation between aggregate consumption growth and the cross-sectional volatility of consumption growth. Aggregate consumption growth volatility is modest at 2.5% per year.

We set the mean of the cross-sectional dispersion in consumption growth, σ_g, to 10%. One valid interpretation of σ_g in the model is as the mean dispersion of those household income growth shocks that are uninsurable and that end up in household consumption growth. The labor economics literature has found that transitory income shocks are more easily insured than permanent shocks (Blundell, Pistaferri, and Preston, 2008). In the same vein, Heathcote, Storesletten, and Violante (2014) find that 40% of persistent wage fluctuations end up in consumption, while the rest are effectively smoothed. Incomplete pass-through of income shocks to consumption therefore motivates values for σ_g that are lower than household income growth dispersion of 53%. Our approach is to consider two extreme cases. In our benchmark case, we assume that all but one-fifth of the income growth shocks are insurable. In our sensitivity analysis, we assume that none of the income growth shocks is insurable. We find that the asset pricing implications are very similar for both extreme cases and for the intermediate cases we study as well.

The persistence of the cross-sectional dispersion process, ν_g, is set to 0.6 per year, a value equal to the annual persistence of the CIV factor in the data. This choice implies that our main state variable moves at business cycle instead of at much lower frequencies. We set σ_w to 1.47%. This ensures that σ_{gt}^2 remains positive. The time series standard deviation of σ_{gt} is 0.94%.

The model results in a market price of CIV risk of -4.82. This market price of risk is completely pinned down by the preference parameters and the dynamics of CIV. The first component of λ_w contributes -1.77, the second one, -0.59; and the Epstein and Zin term, -2.46. The model has a substantial maximum conditional Sharpe ratio of 0.61 per year.

To represent the typical stock in each of the CIV-beta-sorted quintile portfolios, we solve our model for five assets that differ in terms of their cash flow growth process [Eq. (18)]. We also consider the market portfolio, which is an asset whose cash flow growth has no idiosyncratic shocks. We set mean dividend growth μ_i equal to the values observed for the CIV-beta-sorted portfolios and the market portfolio in the data.

We set the consumption leverage parameter φ equal to 5.69 for all portfolios. By setting this parameter equal for all portfolios, we impose that all differences in risk premia across portfolios arise from differences in exposure to the $w_{g,t+1}$ shocks. The choice is such that the model matches the equity risk premium for the market portfolio of 5.50% exactly, given all other parameters. The contribution to the equity risk premium from the η-term is 5.21% per year.

The other parameters that we hold fixed across portfolios are the parameters governing the σ_{it} process in Eq. (19). We set σ_i to 0.4%, ν_i to 0.15, and σ_{iw} to $1.5e-6$. The persistence of σ_{it} is much lower than that of σ_{gt}, consistent with the data. We choose σ_{iw} such that σ_{it} never becomes negative in simulations. Finally, the value for σ_i is chosen to match the mean of the observed annual CIV process of 0.254, given all other parameters.

The four key cash flow parameters for each quintile portfolio are ϕ_i, χ_i, κ_i, and ζ_i. We pin down these four parameters to match four moments. The first is the CIV-beta, $\beta_{gs,i}$, in Eq. (9). The second is the slope of a regression of dividend growth on lagged CIV, ensuring that the model respects the dividend growth predictability patterns observed in the data. The third and fourth moments are the slope and the R^2 from a regression of idiosyncratic stock return variance on the CIV factor:

$$\mathbb{V}_t \left[r_{t+1}^{idio,i} \right] = a^i + b^i CIV_t + \nu_t^i. \tag{59}$$

While these are four simultaneous equations, χ_i mostly affects the dividend growth predictability slope, κ_i governs the portfolios' return variance exposure to CIV, ζ_i affects the R^2 of the regression in (59), and ϕ_i is chosen to match $\beta_{gs,i}$ given the other three parameters. The last four rows of Table 5 show the chosen values for these four parameters for each portfolio.

B.3. Stock prices predict CIV

In the spirit of Beeler and Campbell (2012), we test whether the price–dividend ratio on stocks predicts CIV or idiosyncratic household risk. In the structural model, the log price–dividend ratio on a stock of firm i, pd_t^i is linear in the cross-sectional dispersion in idiosyncratic income growth σ_{gt}^2 as well as in the firm-specific idiosyncratic variance σ_{it}^2 [Eq. (44)]. Because σ_{gt}^2 is persistent, the pd_t^i ratio today should forecast future $\sigma_{g,t+1}^2$. In the model, the relation between pd_t^i and σ_{gt}^2 is given by A_{gs}^i. Because A_{gs}^i is positive, we expect a positive sign in the predictive relation. Finally, because the log price–dividend ratio also depends on the firm-specific idiosyncratic variance σ_{it}^2, the regression should control for idiosyncratic variance.

To implement this test, we start by constructing the log price–dividend ratio for each CIV-beta-sorted quintile portfolio from cum-dividend and ex-dividend portfolio returns, aggregated up from cum- and ex-dividend returns of the stocks in the portfolio. The dividend is a cumulative 12-month dividend, and monthly dividends are not reinvested. We calculate the return variance of each stock in the portfolio and orthogonalize it to CIV to obtain a measure of σ_{it}^2. We average this residual variance across the stocks in each portfolio. We use this residual variance as a

control in the predictive regression, as suggested by the theory. Omitting this control has little effect on the results. Panel A of Table B1 shows the results of predicting CIV one-month-ahead. The sample is 1963–2010, for consistency with the other asset pricing results. We find that the log price–dividend ratio on all portfolios predicts CIV with a positive slope and the point estimate is significant for quintiles 1–4. For those portfolios, the predictive R^2 ranges from 11% to 29%, which is substantial. Panel D shows that the predictive ability is even stronger in the full 1926–2010 sample with higher point estimates, t-statistics, and R^2 values.

It is instructive to compare the predictability of CIV with the predictability of MV. The evidence is reported in Panels B and E. We find some evidence for predictability of MV by the log pd ratio in portfolios 2, 4, and 5 in Panel B, but the t-statistics and R^2 values are substantially lower than for CIV. The same is true for the longer sample in Panel E.

Finally, we look at whether the log pd ratio also forecasts employment growth dispersion. This is the series with the longest available sample: annual data from 1950 until 2010. Panels C and F show that pd_t^i significantly predicts employment growth dispersion with t-statistics ranging from 2.1 to 5.0 in the shorter sample and 4.4–8.4 in the longer sample. The R^2 values range from 11% to 38% in the shorter sample and 30% to 55% in the longer sample. This predictability evidence provides further support for the mechanism and the model we propose.

B.4. Sensitivity analysis

The first robustness exercise is with respect to the parameters of the key state variable σ_{gt}^2, which is proportional to CIV. We explore increasing the mean of this process, σ_g to 0.42. This value is implied by the cross-sectional labor income growth dispersion in Guvenen, Ozkan, and Song (2014) and a labor income share of 80% (Lustig, Van Nieuwerburgh, and Verdelhan, 2013). When labor income makes up 80% of total income, log consumption growth dispersion equals log labor income growth dispersion divided by 1.25. This results in a cross-sectional consumption growth dispersion of 0.424. This 42% is close to the 38% value for the cross-sectional dispersion in consumption growth we measure in the Consumer Expenditure Survey (CEX) for the period 1984–2011. The quantitative results are similar if we calibrate to a value of 38%. However, we prefer not to rely on the CEX due to the measurement error issues.

When increasing σ_g, we lower the innovation volatility σ_w to keep the risk premium λ_w associated with w_g shocks unchanged. This change lowers the volatility of the model-implied CIV process. We also change the time discount factor δ to continue to match the observed mean risk-free rate. Higher values for σ_g results in lower values for δ. The market price of CIV risk is -1.41. The first component of λ_w contributes -0.34; the second one, -0.59; and the Epstein-Zin term, -0.48. The model has a maximum conditional Sharpe ratio of 0.66 per year. We recalibrate the cash-flow parameters $(\varphi_{di}, \phi_i, \chi_i, \kappa_i, \zeta_i)$ following the

same procedure outlined in the main text. The parameters are listed in Table B2.

The results are reported in Table B3. This calibration generates the same CIV-beta return spread as in the benchmark case and continues to match the CIV-betas and the moments in Rows 14–19 relating to regressions of idiosyncratic return volatility on CIV and the regression slope of a dividend growth predictability regression on CIV. The only difference with the benchmark model is in the return volatilities of the portfolios. The overall volatilities are very similar (about 1–2% difference on a baseline volatility of 45–65%). A larger fraction of the return volatility now comes from the common idiosyncratic risk term (e_i) and a smaller fraction from the firm-specific idiosyncratic risk term (ε_i). The model continues to match the observed mean CIV by virtue of a higher $\bar{\kappa}$. Therefore, this exercise shows the same asset pricing performance for a calibration in which the average amount of idiosyncratic household risk is higher but its innovations are less volatile.

Our second robustness exercise is with respect to the coefficient of relative risk aversion γ. We explore lowering it from 15 to 8. We keep the consumption leverage parameter and the aggregate consumption growth process the same as in the benchmark model. As a result, the equity risk premium on the market portfolio drops from 5.5% to 1.2%. We keep the average consumption growth dispersion at $\sigma_g = 0.42$, and keep the innovation volatility at the benchmark value of $\sigma_w = 1.47\%$. The parameters are listed in Table B4 while the calibration results are in Table B5, and the model no longer matches the equity risk premium, it still generates a large spread of -5.23% between the high and the low CIV-beta quintile portfolio. It also continues to match return volatilities, dividend growth predictability slopes, and slopes and R^2 of idiosyncratic volatilities of the quintile portfolios on CIV.

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