Exercise 11: Networks 3

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Problem 11.1.1. - SIS model

Building the SIS model is relatively straight forward. First of all we require $S + I = k$ where k is some fixed constant. This restraints the dynamic to fixed populations. We also take some initial levels S_0, I_0 as given. Then, the number of infected at any specific time will change due to *i)* new infections βSI with the number of new infections being proportional in both the number of susceptible and the number of currently infected, and *ii)* the number if recoveries from infected to suceptible (again), γI. Thus in total

$$\frac{dI}{dt} = \beta SI - \gamma I \tag{1}$$

Since there are only two states and the population is of a fixed size naturally it must be that

$$\frac{dS}{dt} = -\frac{dI}{dt} \tag{2}$$

Problem 11.1.1. - SIRS model

In the SIRS model the removed don't nessecarily stay removed but instead transition into becoming suceptible again with a fixed probability. Note that this model does not include deaths because all of the removed can reenter the suceptible population. If death was possible the reentry rate would depend on the composition of the removed. In any case, this model looks very much like the SIR model but with an added ϕR term, where ϕ is the reentry rate from the recoved to suceptible populations

$$\begin{aligned} \frac{dI}{dt} &= \beta SI - \gamma I \\ \frac{dR}{dt} &= \gamma I - \phi R \\ \frac{dS}{dt} &= -\beta SI + \phi R \end{aligned} \tag{3}$$

That is in words

- The number of infected increase by βSI at any given time (i.e. is proportional to the number of suceptible "candidates" and to the number of current infected "donators"), while it decreases by γI which is the constant fraction being removed.
- The number of removed increase by the constant fraction entering from the infected compartment and decreases by the constant fraction reentering succeptibility.
- The number of suceptible decrease by the amount that becomes infected, and increase by the amount that reenters from having recovered.