Life-Cycle exercises with taxes

Setup

Define the objective
u[c, l] is utility function
β is discount factor

```
obj := Sum[β^(t-1) u[c[t], l[t]], {t, 1, T}]
```

Specify a tax function
rinc is interest income, tauk is tax rate on interest income
winc is wage income, taul is tax rate on wage income
consump is consumption, tauc is consumption tax rate

```
Tax[rinc_, winc_, consump_] := tauk rinc + taul winc + tauc consump
```

Define constraints:
a[t] is assets at beginning of period t.
R is gross return on assets (R-1 is paid right after period t begins)
w[t] is wage in period t
c[t] is consumption in period t
l[t] is labor supply in period t

Budget constraint in period t

```
bc := Table[
  a[t + 1] - (R a[t] + w[t] l[t] - c[t] - Tax[(R - 1) a[t], w[t] l[t], c[t]]) ≤ 0, {t, 1, T}]
```

Initial wealth is fixed

```
bc0 := {a[1] == ainit}
```

Terminal wealth must be nonnegative

```
bcT := {a[T + 1] ≥ 0}
```

Borrowing constraints

```
abnds := Table[a[t] ≥ amin, {t, 2, T}]
```

List the variables

```
varsc := Table[c[t], {t, 1, T}]
varsl := Table[l[t], {t, 1, T}]
varsa := Table[a[t], {t, 2, T + 1}]
```

We should also add the lower bound constraints

```
lbndsc := Table[c[t] ≥ 0.0001, {t, 1, T}]
```

```
lbndsl := Table[l[t] ≥ 0.0001, {t, 1, T}]
```

Define present values of various taxes

```
PVtauc := Sum[tauc c[t] R^(-t+1), {t, 1, T}]
PVtaul := Sum[taul w[t] l[t] R^(-t+1), {t, 1, T}]
PVtauk := Sum[tauk R a[t] R^(-t+1), {t, 1, T}]
```

Assignment:

Write programs that solve these lifecycle problems for various values of
parameters: T, β, R, w[t]
tax rates: tauk, taul, tauc
utility function (additively separable, Cobb-Douglas, CES)

Compute:
consumption, asset, and labor supply paths
present values of the three kinds of taxes