IB Diploma HL Physics Internal Assessment 2021

Investigating the Effect of Temperature on a RC Oscillator Circuit Utilising CMOS Inverters

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The aim of this experiments is to investigate how changing the operating temperature of a RC oscillator will effect it's oscillation frequency. In order to investigate this relationship the rate at which an LED "blinks" on the RC oscillator circuit was recorded with a camera at different temperatures. This blinking rate was analysed using a software called Tracker to find the oscillation frequency of the circuit. It was determined that when graphing Temperature against this Average Oscillation Frequency and plotting a line of best fit, that there exists a strong negative linear correlation between the two. This result supports the idea that decreasing temperature decreases propagation delay for a CMOS inverter in an RC circuit by reducing its threshold voltage as a result of increasing charge carrier mobility.

Abstract Word Count: 128

Word Count: 4558

Introduction:

In my grade 10 computer technology, I had the opportunity to create a christmas tree circuit. The idea behind it was that it had 9 LEDs that would blink on and off like a christmas tree. At the time I had no idea how the circuit actually functioned. It was after learning in HL physics about RC circuits (circuits using resistors and capacitors) and the time constant for charging and discharging a capacitor that I once again became interested in the mechanics behind the blinking Christmas Tree. I quickly learned that my intuition was right and that RC circuits played a big role in causing the LEDs on the christmas tree circuit to blink, or “oscillate”. I soon learned that apparently temperature has an affect on the frequency of this oscillation. I became interested in exploring this phenomenon as I knew from class that temperature had only a minor effect on resistance and capacitance. Hence I define the following research question.

Research Question:

What is the correlation between the temperature of an RC oscillator and it’s oscillation frequency?

Background:

An oscillator is any circuit that alternates between voltages, it has a frequency equal to how many times in one second it outputs a high voltage. For our purposes a high voltage is called simply high, and a low voltage is called low. This high voltage than creates a current which is in our case is detected by an LED, where it would show up as “blinking”, off at low and on at high. This relationship between current and voltage is described elegantly in ohms law: $I = \frac{V}{R}$.

Figure 1: A blinking LED, No Current Left, Current Right

The way such an oscillator works is using a device know as an inverter. This device outputs high when it receives low and outputs low when it receives high. This receiving voltage is called the input of the inverter. If you were to string these inverters together so that the output of one is the input of the other, you would get alternating voltage or oscillation.

Figure 2: Simple Inverter, A is Input, out is Output [13]

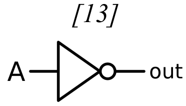
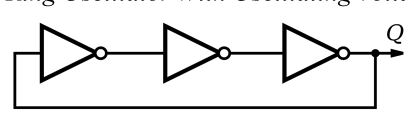


Figure 3: Ring Oscillator With Oscillating Voltage Q [14]



In the set up in figure 3. the voltage will constantly oscillate. If the first inverter had a input voltage of high, it will output low to the second inverter who would output high to the third inverter who would output low to the first inverter meaning Q changes from high to low after 3 inversions. It makes sense than that after another 3 inversions Q becomes high again and the circuit will continue loop in this matter. This means that every 6 inversions, Q is high. If each inversions takes a length of time τ_D, also called propagation delay, than we can define the frequency of the oscillator as $f = \frac{1}{2n\tau_D}$ where n is the number of inverters, and $2n$ is the amount of inversions required to get another high, in this case 6.

While our circuit is **not** a ring oscillator (it is actually a differential ring oscillator), the idea that oscillation frequency is based solely on the number of inverters and two times their associated propagation delay holds true. Thus, as we have 4 inverters in our circuit: $f = \frac{1}{2(4)\tau_D} = \frac{1}{8\tau_D}$

The next question is then what is the propagation delay, τ_D. Well typically the CMOS inverters in our circuit have a delay of $120\ ns$ at 10V [9]. If we assume all 4 inverter in our oscillator are the same and have a propagation delay of $120\ ns$, than the oscillation frequency is:

$$f = \frac{1}{8(120\times10^{-9})} = 1.04 \times 10^{6}\ Hz$$

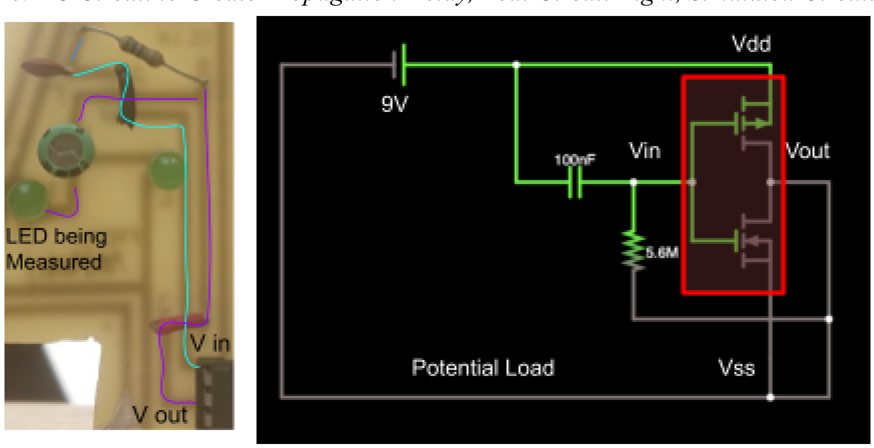
However, we know that a frequency of $1.04 \times 10^{6}\ Hz$ is way too fast to see a blinking LED. Thus, we need some way to get a smaller frequency. The only way to do this is to increase the propagation time. We can accomplish this using a RC circuit. We know that when charging or discharging a capacitor, the voltage across it (V_c) changes with time in proportion to the source voltage (V_s), as shown:

$$\textbf{Charging:}\ \ V_c = V_s(1 - e^{\frac{-t}{RC}})$$

$$\textbf{Discharging:}\ \ V_c = V_s(e^{\frac{-t}{RC}})$$

If the inverter only propagates after the voltage across a capacitor reaches a certain value, than we have created a delay equivalent to the length of time it takes for the capacitor to charge / discharge. This is exactly what is done in our circuit to achieve propagation delay. We can examine this effect in isolation by simulating a small part of of our real circuit. [7] Modelling off the values of our real circuit [3], we have a capacitor of 100 nF, a resistor of 5.6MΩ and a potential load which are connected to an integrated circuit which houses our CMOS inverter.

Figure 4: RC Circuit to Create Propagation Delay, Real Circuit Right, Simulated Circuit Left.



While the simulated circuit at first looks intimidating, the part highlighted in red functions exactly how we defined an inverter to work. When the input voltage (V_{in}) is high, like it is right now, no voltage is outputted (V_{out}). Now, there are two parts of the inverter that we haven't discussed, the part on the top called the source voltage for high, V_{dd} ,and the part on the bottom called the source voltage for low,

V_{ss}. These two additional inputs in the inverter elegantly accomplishes two things, it sets what voltage is outputted at high and what voltage is outputted at low and, as well, sets what value the input voltage must be to be considered high and what input voltage is considered low.

These input voltages are set according to the voltage threshold (V_T) of the inverter, which states by how much the input voltage needs to vary from V_{dd} to be considered low/high. For low, V_T is $1.5V$ (which is not the same as in our real circuit) and V_{dd} is 9V, meaning the input voltage is defined as low when:

$$V_{in} < 9V - 1.5$$
$$V_{in} < 7.5V$$

Thus, we get a output of high, set to V_{dd} (9V), whenever $V_{in} < 7.5V$. This can be detected as current by our simulation at the wire marked "potential load". The reason we define it in terms of current is simply because there is no resistance between the output of the inverter and the ground of the battery. Given $V = IR$ and $R = 0$, we get $V = 0$, but there is still electron flow to ground and thus current. Luckily for us, we can calculate how long it takes for this input voltage V_{in} to reach 7.5V, and thus generate a current, using what we know of RC circuits.

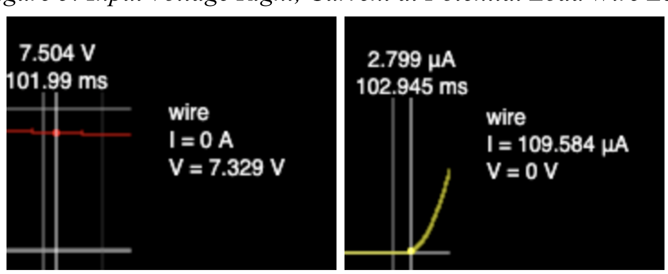
In the circuit we have a capacitor, V_{in} and a resistor in series. This means V_{in} must be the source voltage subtracted by the voltage difference across the capacitor. Mathematically, where V_c is voltage across capacitor, we get: $V_s - V_c = V_{in}$ If we want to find how large the voltage across the capacitor must be for V_{in} to be considered low, we can solve the inequality:

$$9V - V_c < 7.5V$$
$$-V_c < -1.5V$$
$$V_c > 1.5V \quad \text{\{Flip sign due to division by (-)\}}$$

This is curious as 1.5V is the same as our threshold voltage meaning we can define V_c when V_{in} is low as $V_c > V_T$. As we have a formula for V_c of a charging capacitor, we can calculate the time it takes for this inequality to be reached:

$$V_s(1 - e^{\frac{-\tau_D}{RC}}) = V_T$$
$$9(1 - e^{-\frac{\tau_D}{(100\times10^{-9})(5.6\times10^{6})}}) = 1.5$$
$$\tau_D = 0.102\ s$$
$$\tau_D = 102\ ms$$

Figure 5: Input Voltage Right, Current at Potential Load Wire Left



Verifying this result using our simulation, we get current flowing as an output when time is around 102ms! Thus, we have shown with a simulation that , $V_T = V_s(1 - e^{\frac{-\tau_D}{RC}})$. We can than rewrite this in terms of propagation delay: $\tau_D = -RCln(\frac{-V_T + V_s}{V_s})$ (Appendix 2).

Analysing this formula we find that only 4 variables affect propagation delay, resistance, capacitance, voltage threshold and voltage of source. We will now halve all of these variables individually and calculate to what extent this affects propagation time, than use our simulation to verify our calculations.

Table 1: Analysis of $\tau_D = -RCln(\frac{-V_T + V_s}{V_s})$

Resistance (R): $2.8M\Omega$ $\tau_D = -RCln(\frac{-V_T+V_s}{V_s})$ $\tau_D = -(2.8 \times 10^6)(100 \times 10^{-9})ln(\frac{-1.5+9}{9})$ $\tau_D = -(-0.0511)s$ $\tau_D = 51.1ms$	**Capacitance** (C): 50 nF $\tau_D = -RCln(\frac{-V_T+V_s}{V_s})$ $\tau_D = -(5.6 \times 10^6)(50 \times 10^{-9})ln(\frac{-1.5+9}{9})$ $\tau_D = -(-0.0511)s$ $\tau_D = 51.1ms$
Voltage Threshold (V_T): $0.75V$ $\tau_D = -RCln(\frac{-V_T+V_s}{V_s})$ $\tau_D = -(5.6 \times 10^6)(100 \times 10^{-9})ln(\frac{-0.75+9}{9})$ $\tau_D = -(5.6 \times 10^6)(100 \times 10^{-9})ln(\frac{-0.75+9}{9})$ $\tau_D = -(-0.0487)s$ $\tau_D = 48.7ms$	**Voltage Threshold** (V_s): $4.5V$ $\tau_D = -RCln(\frac{-V_T+V_s}{V_s})$ $\tau_D = -(5.6 \times 10^6)(100 \times 10^{-9})ln(\frac{-1.5+4.5}{4.5})$ $\tau_D = -(5.6 \times 10^6)(100 \times 10^{-9})ln(\frac{-1.5+4.5}{4.5})$ $\tau_D = -(-0.227)s$ $\tau_D = 227ms$

Figure 6: Output Current, Resistance Halved Left, Capacitance Halved Right

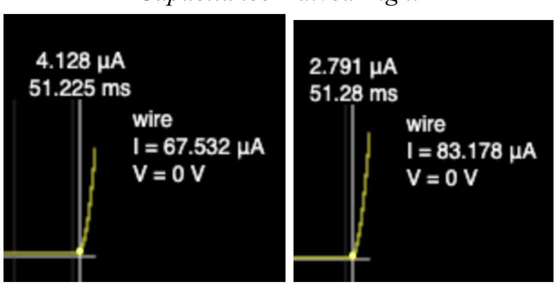
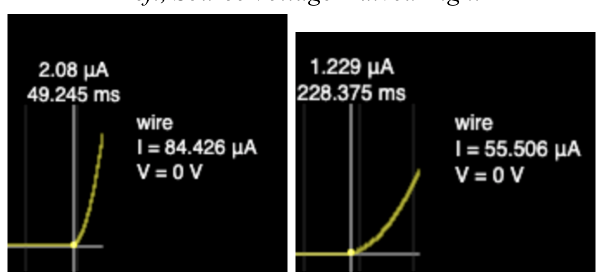


Figure 7: Output Current, Voltage Threshold Halved Left, Source Voltage Halved Right



These results line up really well with our calculations, at most being one millisecond off. Thus, if temperature has an affect on oscillation frequency it must affect one or more of these variables. We can rewrite this in terms of oscillation frequency to get our final equation:

$$f = \frac{1}{-8RCln(\frac{-V_T+V_s}{V_s})} \text{ (appendix 2)}$$

Methodology:

Materials:

Table 2: Materials List (see Appendix 1 for pictures)

Circuit Oscillator	- Composed of 9 LEDs, 4 ceramic capacitors, 4 electrolytic capacitors, 4 resistors, and an integrated circuit, code: CD4093BC[9] - 9V battery code: 6LR61[15]
Samsung Family Hub 36" 26.5 Cu. Ft. French Door Refrigerator (RF27T5501SR/AC)	- Two different controllable areas, a fridge area and a freezer area - The range of temperature values were chosen as the maximum and minimum temperatures allowed for both fridge and freezer area
Recording Tools	- Sony Xperia Phone Camera (30fps) - Tracker Physics Software

Independent Variable:

The independent variable is the operating temperature of the circuit in kelvin. Due to the lack of means to predictably heat the circuit, the analysis will focus primarily on cooling the circuit. This will be done by utilising a refrigerator with adjustable internal temperature. For room temperature an thermostat will be used. The temperatures used will be 20, 7, 3, -15, -19. Due to imprecise temperature measurement tools and issues related to newton's law of cooling, the uncertainty of the measured operating temperature of the circuit will be ± 2 .

Dependent Variable:

The dependent variable is the oscillation frequency as measured by the blinking frequency of the bottom left green LED in Hertz (Hz). This will be determined by finding the length of time it takes for the LED to turn on 10 times than using the formula $f = \frac{10}{T}$ to find the frequency. Due to my phone recording the LED blinking at 30 frames per second, the video can only be analyzed within 0.033ms steps. Thus the error for the time it takes for LED to blink 10 times is rounded up to ± 0.04 ms.

Control Variables:

Table 3: Control Variables and Method of Control

Control Variable	Method of Control
Time taken for the circuit to acclimate to its environment	- Give the circuit 5 minutes to acclimate to its environment - After each trial reacclimate the circuit to room temperature
The rapid loss of temperature due to introducing a camera into a cold environment	- When introducing a camera into a cold environment the circuit, reintroduce the circuit with the camera back into the cold environment and only video recorded after 30 seconds in the desired operating temperature was used to gather raw data
Humidity of the freezer might increase the conductivity of the circuit	- The circuit was wrapped in plastic while conducting the trials in the freezer

Safety Precautions:

The circuit uses a 9 Volt Battery which is generally not dangerous. There is, however, the possibility of excessive heat or circuit failure, in order to protect against this the datasheet for the integrated circuit as well as the battery was read carefully for the operating temperature range and voltage range. [9] [15] The experiment also had to be conducted at home due to safety concerns around Covid-19.

Procedure:

Place the circuit in an environment of recorded temperature. Then wait 5 minutes for the circuit to acclimate to its environment. After waiting, introduce a camera into the environment, quickly, so as to minimize temperature change of the circuit. Then, let the camera record the bottom left green LED for 1 minute. After 1 minute take the circuit and camera out of the environment and let the circuit reacclimate to room temperature. Do this process for each desired operating temperature, each for 5 trials.

After recording all the footage import the videos into tracker. For each video use tracker to record the time it takes for the LED to turn on 10 times using the luminosity of each frame as a reference. Its important to note that only footage after the 30 seconds were analysed. Record this raw data into google sheets.

Calculate the oscillation frequency of the circuit for each trial by dividing 10 by the time taken for the LED to turn on 10 times. Than use this data to calculate the average and uncertainty of the oscillation frequency. This processed data was than graphed into Logger Pro and the line of best fit was calculated.

Figure 8: A Recording from -3 Degrees in Tracker, the Luminosity is Tracked Throughout the Recording

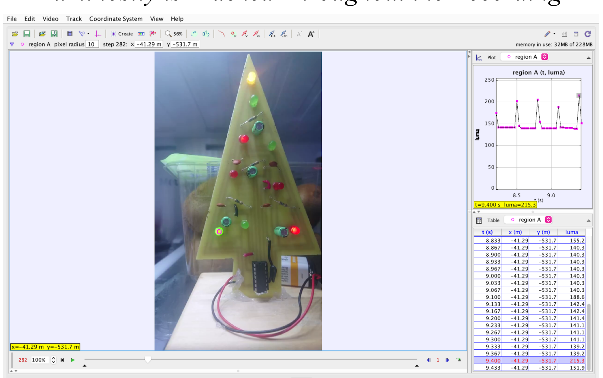
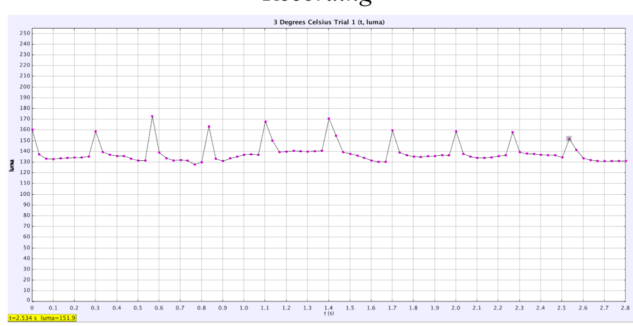


Figure 9: An Example of the Raw Data extracted from the Recording



Qualitative Observations:

The LED blinked at a visually greater blinking rate in colder temperatures. When letting the circuit adjust back to room temperature from colder temperatures, the LED blinking rate remained constant until suddenly falling off after about a minute. The temperature of the integrated circuit felt the coldest to touch out of the entire circuit.

Raw Data:

Table 4: Time for Green LED in Circuit Oscillator to turn on 10 times, extracted from Tracker

	Time for Green LED to blink on 10 times (t, ± 0.04 s)				
Circuit Operating Temperature T / C° $\Delta C^{\circ} = \pm 2$	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
-19	2.33	2.27	2.27	2.40	2.13
-15	2.47	2.53	2.43	2.43	2.37
3	2.53	2.53	2.50	2.53	2.60
7	2.70	2.70	2.67	2.67	2.63
20	2.93	3.04	3.04	3.00	3.00

Processed Data:

Table 5: Average Oscillation Frequency and Uncertainty in a Circuit Oscillator, calculated from Table 2

Circuit Operating Temperature T / K $\Delta K \pm 2$	Average Oscillation Frequency f / Hz	Uncertainty of Oscillation Frequency
254	4.39	0.1
258	4.09	0.05
276	3.94	0.04
280	3.74	0.03
293	3.33	0.03

Sample Calculation for Average Oscillation Frequency and its Associated Uncertainty

Calculating Average Oscillation Frequency (f_m) for Temperature, 254 Kelvin.

Formula for mean:

$$f_m = \frac{1}{5}\sum_{i=1}^{5} f_i$$

Where frequency (f_i) is the amount of blinks per second. So if we get the amount of time it takes for a LED to blink once using $\frac{10}{T_i}$ and divide this by one second we get frequency:

$$f_i = 1/\frac{T_i}{10},\ f_i = \frac{10}{t_i}$$

Substituting Values:

$$f_m = \frac{1}{5}\left[\frac{10}{2.33} + \frac{10}{2.27} + \frac{10}{2.27} + \frac{10}{2.40} + \frac{10}{2.13}\right]$$

$$f_m \approx 4.39$$

Calculating Average Oscillation Frequency Uncertainty (f_u) for Temperature, 254 Kelvin.

First calculate variance of the sample, $(\sigma_{f_m})^2$

$$(\sigma_{f_m})^2 = \frac{1}{5}\sum_{i=1}^{5}(f_i)^2 - (f_m)^2$$

$$(\sigma_{f_m})^2 = \frac{1}{5}\left[\left(\frac{10}{2.33}\right)^2 + \left(\frac{10}{2.27}\right)^2 + \left(\frac{10}{2.27}\right)^2 + \left(\frac{10}{2.40}\right)^2 + \left(\frac{10}{2.13}\right)^2\right] - (4.39)^2$$

$$(\sigma_{f_m})^2 \approx 0.0549$$

As the absolute error of each measurement is ± 0.04 at worst the absolute error of frequency is:

$$\frac{0.04}{2.13} = 1.8\%,\ \frac{10}{2.13} * 1.8\% = 0.085 \approx 0.1$$

We can add that to the variance as we can say each frequency varies from the mean by an additional 0.1

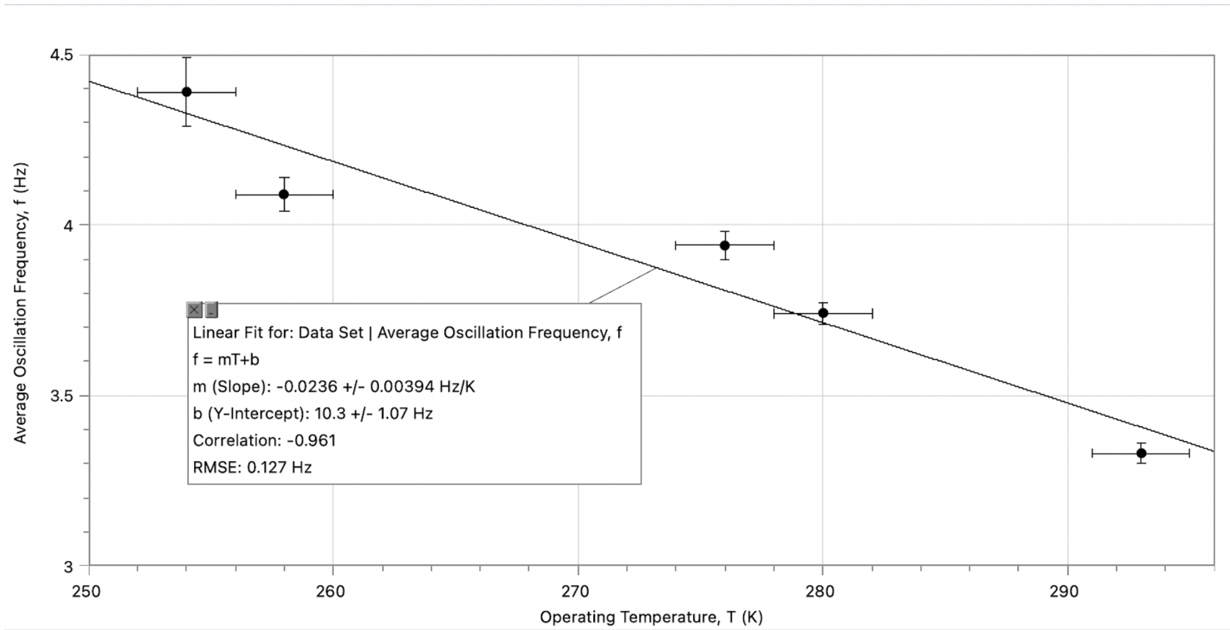
$$(\sigma_{f_m})^2 = 0.0549 + 0.1^2$$

$$(\sigma_{f_m})^2 = 0.0559$$

$$\sigma_{f_m} \approx 0.236$$

	Hence the Standard Error is $f_u = \frac{\sigma_{f_m}}{\sqrt{n}} = \frac{0.236}{\sqrt{5}} \approx 0.106 \approx \pm 0.1$

Figure 10: A scatter plot of Average Oscillation Frequency against Operating Temperature of an Oscillator Circuit with Lines of Best Fit.



Analysis:

Initial Observations:

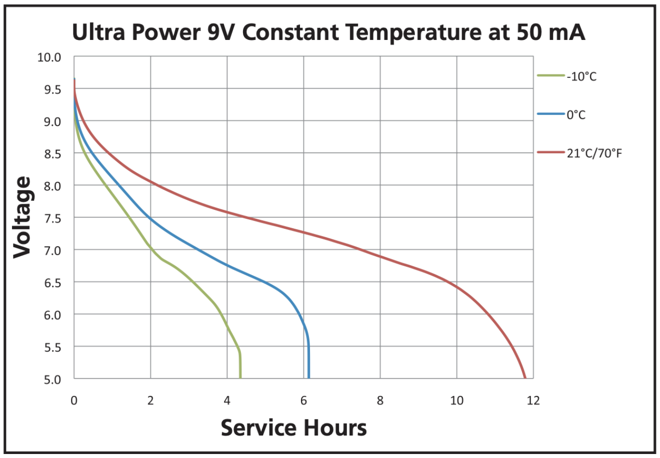
In this analysis linear regression was used by logger pro to calculate and plot the line of best fit. The uncertainty in the equation for the line of best fit was computed to be ± 0.004 for the slope and ± 1 for the y-intercept. The full equation is: $f = -0.0236T \pm 0.004 + 10 \pm 1$. This reveals that the temperature has a negative correlation with the frequency of oscillation. This is also shown in the correlation coefficient of -0.961. This linear correlation is reasonably confident with the percentage uncertainty for the slope of the line of best fit being, $\frac{0.004}{-0.0236} = 17\%$.

In addition, the uncertainty of the average oscillation frequency seems to increase with decreasing temperature. This could be due to a greater disparity between measured temperature and the operating temperature of the circuit. As shown in Newtons law of cooling, the rate at which the temperature of a system changes is proportional to the difference between its current temperature and that of its environment, this might have created more noticeable error for temperatures further from room temperature.

Interpretation of Result:

As oscillation frequency changes with temperature we know from $f = \frac{1}{-8RC\ln(\frac{-V_T+V_s}{V_s})}$ that one or multiple of either R, C, V_T, V_s must change in response to a temperature change. Doing external research the ceramic capacitors used in our circuit are class I capacitors, meaning it’s capacitance changes very little with a change in temperature. [2] This leaves our resistors, which also changes very little with respect to temperature. [1] Both are measured in terms of how temperature affects them as parts per million. This is way too small to account for what was recorded in our experiment. However, the voltage outputted by of our battery, the source voltage (V_s), is affected by temperature. Looking at the data sheet for our battery, the effect is given in the follow graph:

Figure 11 [15]



It is difficult to interpret the effect of temperature on the battery as it seems to be affected over a prolonged period of time. While it was not constant, the battery was subjected to 2 hours sub room temperature. Looking at the graph this means that at worst the source voltage at cold temperatures is 7V instead of 9V. We can see this effect by comparing it to our calculations all the way back to table 1, when $V_s = 9\text{V}$, $\tau_D = 102ms$, and $f = \frac{1}{8(0.102)} = 1.23\ Hz$. If we set the source voltage to $7V$ instead:

$$f = \frac{1}{-8RC\ln(\frac{-V_T+V_s}{V_s})}$$

$$f = \frac{1}{-8(5.6\times10^6)(100\times10^{-9})\ln(\frac{-1.5+7}{7})}$$

$$f = 0.926\ Hz$$

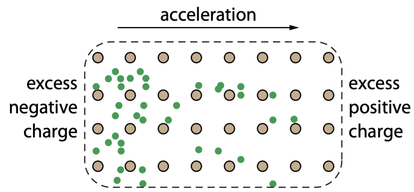
We find surprisingly that oscillation frequency decreases with decreased source voltage, meaning that the effect of temperature on the battery can not account for the increase in oscillation frequency with decreasing temperature, and actually opposes it.

The last variable we have yet to discuss is V_T. As we saw in table 1, decreasing V_T meant a decrease in τ_D which means an increase in oscillation frequency. As shown when $V_T = 0.75$, $\tau_D = 48.7ms$, we get $f = \frac{1}{8(0.0487)} = 2.57\ Hz$. This increase in oscillation frequency compared to the baseline $1.23\ Hz$ is exactly what we are looking for. It follows than if our results are accurate and ruling out all other possible explanations, $f \propto T$, $f \propto V_T$, therefore, $V_T \propto T$.

Why is this? To understand we must first understand the inner workings of an inverter. The following is from research gathered[6][5][11][16][17]. In our circuit we use a CMOS inverter. This CMOS inverter is made with a semiconductor. A semiconductor is any element, in our case silicon, that has a conductivity in between a conductor and insulator. On their own semiconductors are pretty uninteresting, however, their behavior can be predictably modified by changing the concentration of what are called “charge carriers”. An illustration of these charge carriers can be seen in the Tolman-Stewart experiment, in which a piece of metal was accelerated

.

Figure 12: The Tolman–Stewart experiment [4]



As shown above they recorded excess negative charge against the direction of acceleration, showing that negatively charged electrons can move freely in the metal. On the other hand, positively charged cations stayed in place and created excess positive charge where the electrons had been. This shows how because positively charge cations are fixed, purely the **lack** of electron causes excess positive charge. For our purposes we can aptly name these areas that lack electrons as “holes”

This idea of mobile charge carriers is important as you can affect the behavior of a semiconductor by changing which charge carrier is dominant, either majority holes or majority electrons, through a process called doping. Electron dominant semiconductors are called n-type (n for negative), hole dominant semiconductors are called p-type (p for positive). Now consider if we arrange these n-type and p-type semiconductors like below, where n-type is yellow and p-type is blue.

Figure 13: Two n-type Semiconductor Terminals with a p-type Semiconductor Body With Applied Voltage

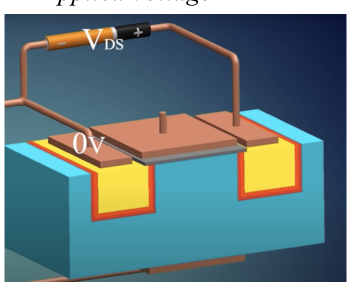
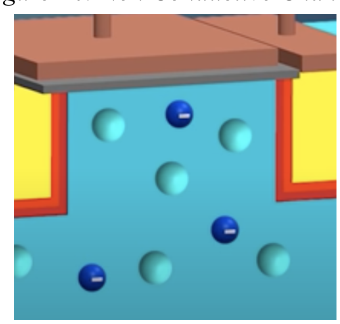
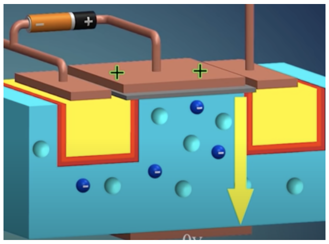
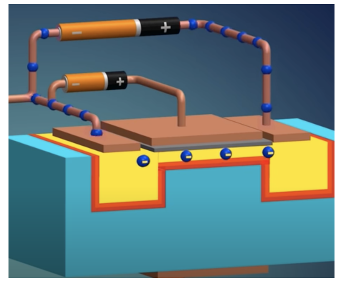


Figure 14: Non Conductive Channel



As is pretty clear there is no current flowing between the two n-type terminals due to the opposing charge of the p-type body. There is, however, a way to change this. As is seen in figure 14 there is a metal plate above an insulator right on top of the space between the two terminals, called the “channel”. If you were to by some method attract enough electrons to this channel than you can conduct electricity and complete the circuit. The way we can accomplish this is by forming an electric field.

We can accomplish this by adding another voltage source that connects from the left terminal to the metal plate on top of the channel, called the gate. What we end up with is a positive voltage on the gate above and a negative voltage on the source terminal below. We know from topic 11 that this difference in voltage creates an electric field as described by $E = \frac{V}{d}$. This positive electric field then attracts electrons to the channel and creates a conductive channel. This is shown below:

Figure 15: Electric Field Attracting Electrons to the Channel	*Figure 16: Conductive Channel Between n-type Terminals*
	

It is from this that we can define V_T as as how large the voltage difference between the source terminal and gate must be in order to form a positive enough electric field that can attract electrons to form a conductive channel. The opposite of this is two p-type terminals with a n-type body that requires a negative enough electric field to attract holes to form a conductive channel.

We can use this to understand how V_T varies with temperature. Essentially, decreased temperature increases the ability for an electron/hole to be attracted to an electric field. While deriving this complex relationship is outside of the scope of this paper. Using this idea, the equation for V_T given temperature T is:

Figure 17: V_T as a function of Temperature T [17]

$$V_t = \underbrace{2\phi_b + \frac{\sqrt{2\varepsilon_{Si}qN_A 2\phi_b}}{C_{ox}}}_{\text{Ideal threshold voltage}} + \underbrace{V_{fb}}_{\text{Flat band voltage}} \qquad \text{where} \quad \underbrace{\phi_b = \frac{kT}{q}\ln\left(\frac{N_A}{N_i}\right)}_{\text{Bulk potential}}$$

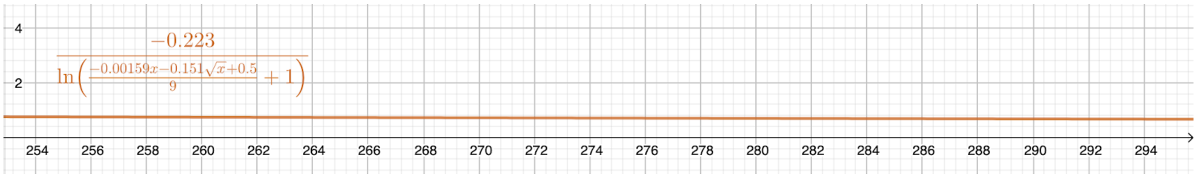
What we end up with is a formula composed of a lot of constants and Temperature T. We can greatly simplify this formula by approximating the constants, so as to help us just visualize the relationship between frequency and temperature:

Approximation of Constants (Appendix 2)

$A = 2\frac{k}{q}ln(\frac{N_A}{N_i}) \approx 1.59 \times 10^{-3}$	$B = \frac{\sqrt{4\varepsilon_{si}qN_A}}{c_{ox}} \approx 0.151$	$C = V_{fb} \approx -0.5$

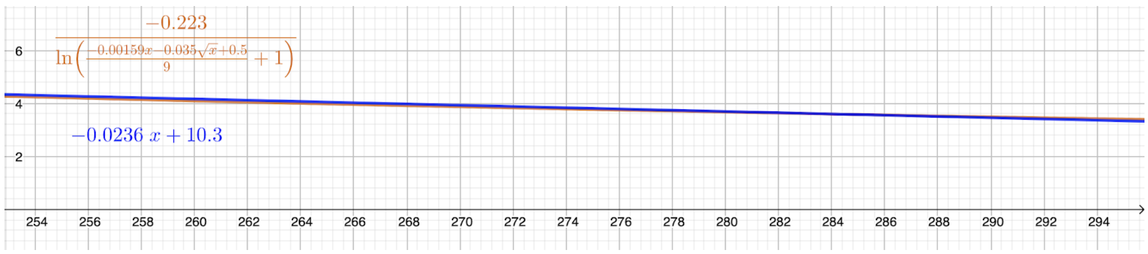
This gives us the relationship that $V_T = AT + B\sqrt{T} + C$, rewriting this in terms of oscillation frequency, we get: $f = \frac{-0.223}{ln(\frac{-AT-B\sqrt{T}-C)}{9}+1)}$ (Appendix) Graphing this using geogebra [21] for our approximated values of the constants, gives us:

Figure 18: An Approximation of the Oscillation Frequency as a Function of Temperature



We see that despite not being a linear function, in our data range, it takes on a linear trend. In fact, if we change the constant B to 0.035 we get a linear line that very closely matches our line of best fit.

Figure 19: An Approximation of the Oscillation Frequency as a Function of Temperature
- With Constants That Yield a Similar Line to Our Line of Best Fit



Conclusion:

In conclusion, the outcome of this exploration is that if, as shown in figure 10 , the oscillation frequency increases with decreasing temperature, than as explained, this means decreasing temperature decreases propagation delay. From this we can conclude that decreasing temperature decreases threshold voltage. If temperature decreases threshold voltage, than this means that temperature is inherently linked to the ability of an electron (or hole) to be attracted by an electric field. For our temperature range of $254K$ to $293K$, this relationship with oscillation frequency takes on the approximation of a linear trend line, $f = -0.0236T + 10$, despite the true equation being a reciprocal function.

Strengths & Weaknesses:

The biggest strength in this experiment was the ability to conduct 5 trials. This along with the strong negative correlation coefficient gives me confidence that the correlation, and its consequences, revealed in the data is accurate. A weakness is in the limited data range, which doesn't truly allow us to see if the relationship between frequency and temperature is in fact a reciprocal function. It was only shown that we can have linear trend in our data but the true relationship still be a reciprocal function.

Extensions & Limitations:

One big limitation of the experiment is that the change in temperature affected the whole circuit. This introduces unknowns, which while significant work was done to thoroughly present that only voltage threshold can account for the negative correlation, the affect of temperature on the battery could have skewed the results. One possible method to solve this is to place just the integrated circuit into a bowl of cold pure water, measuring the exact temperature of the water using a thermometer. The reason this couldn't be done originally was because I lacked a thermometer and pure water.

One big extension to this experiment is to measure the affect warmer temperature has on the oscillation frequency. This can also help establish a larger data range, which could provide proof of the reciprocal nature of the relationship between frequency and temperature.

Work Cited

https://eepower.com/resistor-guide/resistor-fundamentals/temperature-coefficient-of-resistance/[1]

https://product.tdk.com/en/contact/faq/capacitors-0044.html [2]

http://kiloohm.info/ [3]

Tsokos, K. A., and Mark Farrington. *Physics for the IB Diploma*. Cambridge University Press, 2014. [4]

https://courses.engr.illinois.edu/ece110/sp2021/content/courseNotes/files/?MOSFETs [5]

https://www.electronicsforu.com/resources/mosfet-basics-working-applications [6]

http://www.falstad.com/circuit/ [7]

https://commons.wikimedia.org/wiki/File:CMOS_inverter.svg [8]

https://pdf1.alldatasheet.com/datasheet-pdf/view/50861/FAIRCHILD/CD4093BCN.html [9]

https://www.physics-and-radio-electronics.com/electronic-devices-and-circuits/semiconductor/electron-and-hole-mobility.html [10]

https://www.vlsiinterviewquestions.org/2012/07/21/inverted-temperature-dependence/ [11]

http://nopr.niscair.res.in/bitstream/123456789/7244/1/IJPAP%2048%282%29%20136-145.pdf [12]

https://commons.wikimedia.org/wiki/File:Not-gate-en.svg [13]

By Inductiveload - Own work, Public Domain, https://commons.wikimedia.org/w/index.php?curid=8570936 [14]

http://www.farnell.com/datasheets/1842389.pdf [15]

http://web.mit.edu/6.012/www/SP07-L3.pdf [16]

https://ece-research.unm.edu/jimp/vlsi/slides/chap2_1.html [17]

https://youtu.be/rkbjHNEKcRw [18]

https://ecee.colorado.edu/~bart/book/ex007.htm [19]

https://inst.eecs.berkeley.edu/~ee105/fa05/handouts/discussions/Discussion1.pdf [20]

https://www.geogebra.org/calculator [21]

Appendix 1 - Equipment :

Figure 20: Refrigerator Temperature Control and Measurement

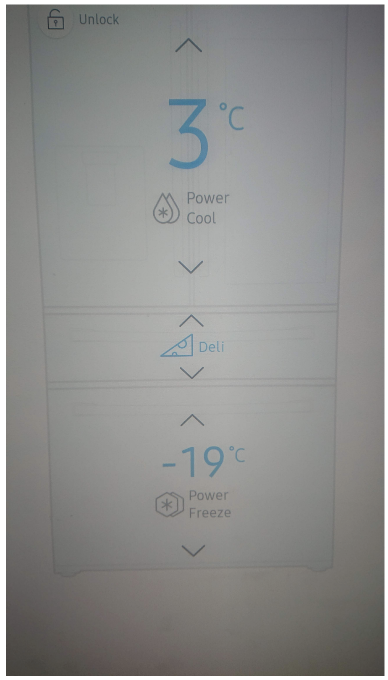


Figure 21: 100 nF Ceramic Capacitor [3]

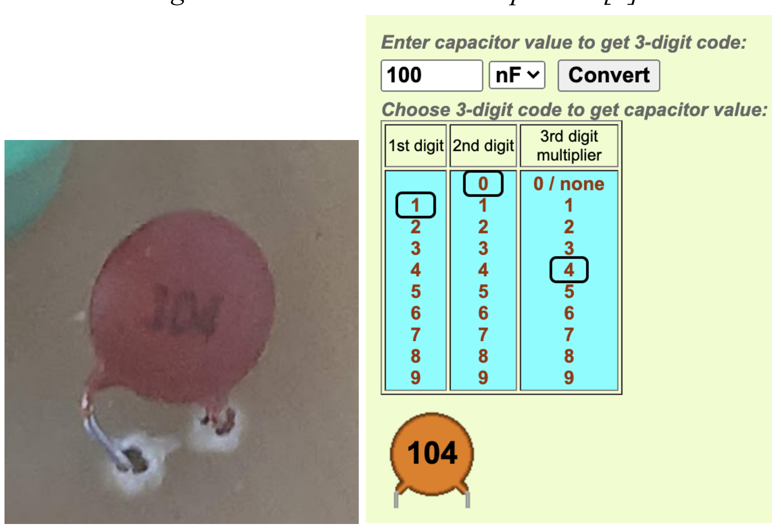


Figure 22: 5.6MΩ Resistor [3]

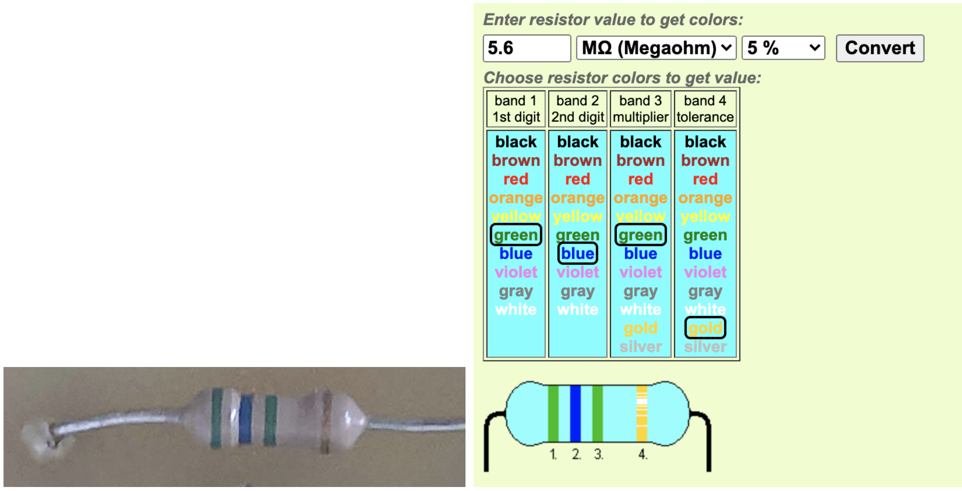


Figure 23: IC, Code: CD4093BC, and Associated Connection Diagram, Notch Indicates Pin 1 [9]

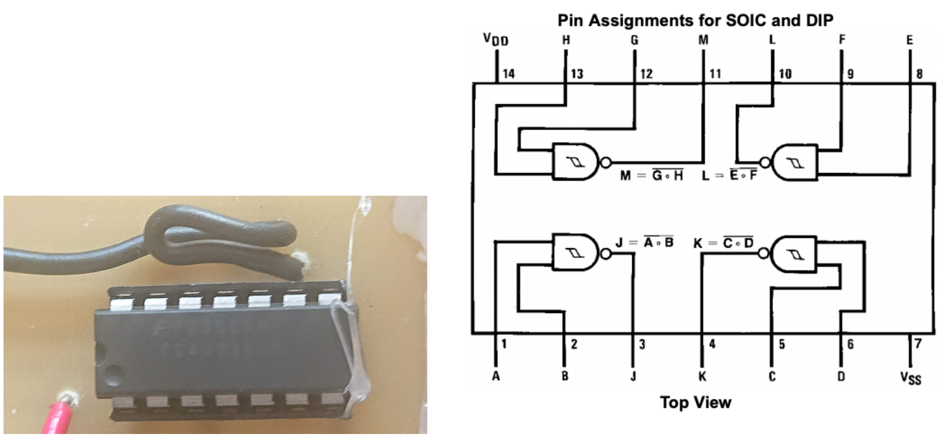
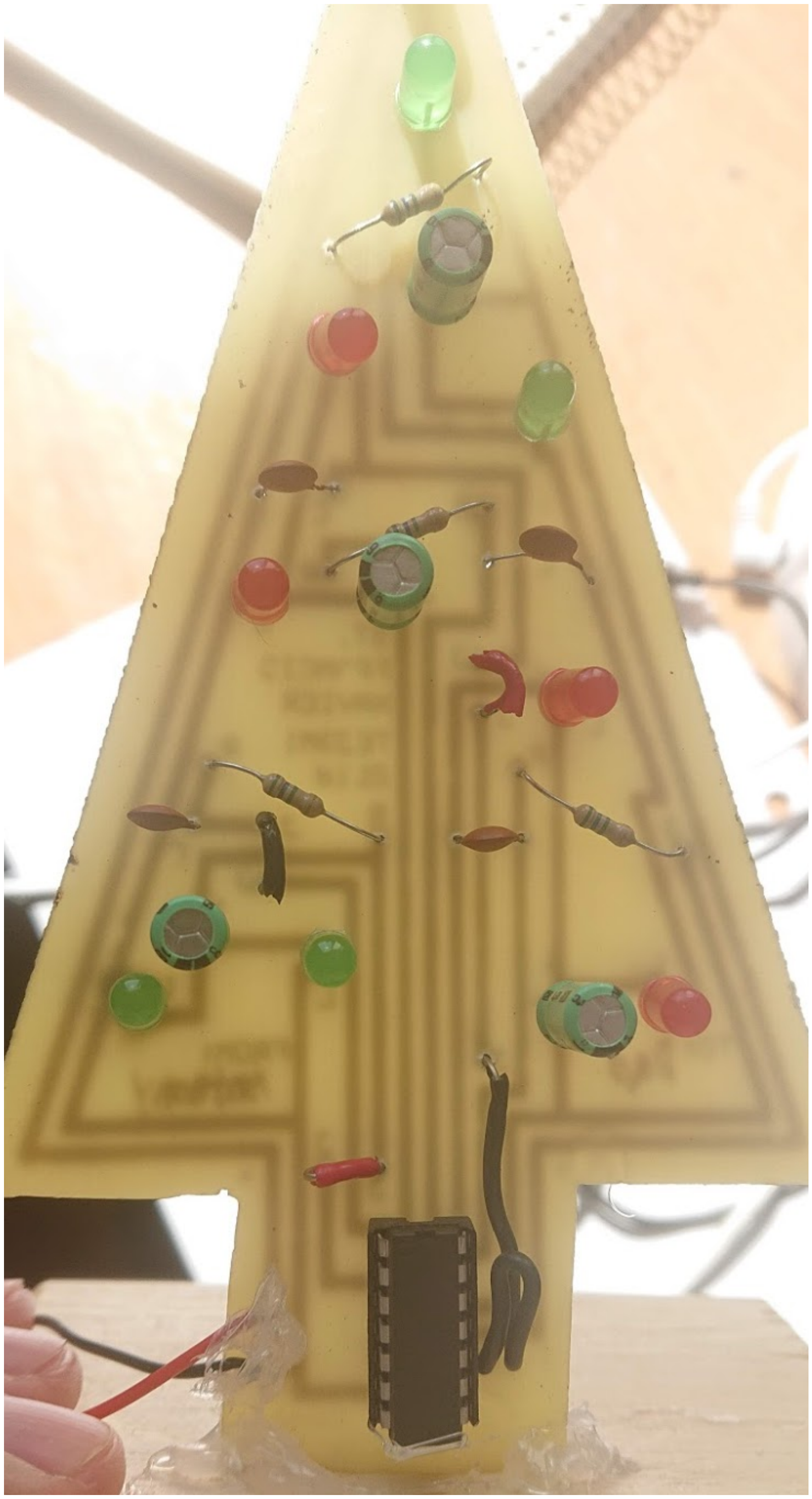


Figure 24: Battery, Code: 6LR61

Figure 25: Full Circuit



Appendix 2 - Derivations:

For τ_D:

$$V_s(1 - e^{\frac{-\tau_D}{RC}}) = V_T$$
$$(1 - e^{\frac{-\tau_D}{RC}}) = \frac{V_T}{V_s}$$
$$- e^{\frac{-\tau_D}{RC}} = \frac{V_T - V_s}{V_s}$$
$$e^{\frac{-\tau_D}{RC}} = \frac{-V_T + V_s}{V_s}$$
$$\frac{-\tau_D}{RC} = ln(\frac{-V_T + V_s}{V_s})$$
$$-\tau_D = RC ln(\frac{-V_T + V_s}{V_s})$$
$$\tau_D = - RC ln(\frac{-V_T + V_s}{V_s})$$

For f in terms of V_T:

$$f = \frac{1}{8\tau_D}$$
$$f = \frac{1}{-8RC ln(\frac{-V_T + V_s}{V_s})}$$

For f in terms of T:

$$\tau_D = - RC ln(\frac{-V_T + V_s}{V_s})$$
$$\tau_D = - (100 \times 10^{-9})(5.6 \times 10^{6}) ln(\frac{-(AT + B\sqrt{T} + C) + 9}{9})$$
$$\tau_D = - 0.56 ln(\frac{-AT - B\sqrt{T} - C)}{9} + 1)$$
$$f = \frac{1}{8\tau_D}$$
$$f = \frac{1}{8(-0.56) ln(\frac{-AT - B\sqrt{T} - C)}{9} + 1)}$$
$$f = \frac{-0.223}{ln(\frac{-AT - B\sqrt{T} - C)}{9} + 1)}$$

Solving for constants, A, B:

$A = 2\frac{k}{q} ln(\frac{N_A}{N_i})$ $k = 1.38 \times 10^{-23}$, $q = 1.60 \times 10^{-19}$, $\frac{N_A}{N_i} \approx 10^4$ $A = 2(\frac{1.38 \times 10^{-23}}{1.60 \times 10^{-19}})(ln(10^4))$ $A = (\frac{1.38 \times 10^{-23}}{1.60 \times 10^{-19}})18.42$ $A \approx 1.59 \times 10^{-3}$	$B = \frac{\sqrt{\varepsilon_{si} q N_A A}}{c_{ox}}$ $\varepsilon_{si} = 1.06 \times 10^{-12}$, $c_{ox} \approx 1.73 \times 10^{-7}$, $q = 1.60 \times 10^{-19}$, $N_A \approx 10^{15}$, $A \approx 1.59 \times 10^{-3}$ {C_{ox} and N_A are best guesses derived from university example questions and so are almost certainly not accurate}[19][20] $B = \frac{\sqrt{2(1.06 \times 10^{-12})(1.60 \times 10^{-19})(10^{15})(1.59 \times 10^{-3})}}{1.73 \times 10^{-7}}$ $B = \frac{\sqrt{6.784 \times 10^{-16}}}{1.73 \times 10^{-7}}$ $B = \frac{2.605 \times 10^{-8}}{1.73 \times 10^{-7}}$ $B = \frac{2.605 \times 10^{-8}}{1.73 \times 10^{-7}}$ $B = 0.151$

C was also found from example questions. [19][20].