CS 383 HW 1

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July 16, 2022

1 Theory

1. Consider the following supervised dataset:

$$X = \begin{bmatrix} 1 & -2 \\ 1 & -5 \\ 1 & -3 \\ 1 & 0 \\ 1 & -8 \\ 1 & -2 \\ 1 & 1 \\ 1 & 5 \\ 1 & -1 \\ 1 & 6 \end{bmatrix}, Y = \begin{bmatrix} 1 \\ -4 \\ 1 \\ 3 \\ 11 \\ 5 \\ 0 \\ -1 \\ -3 \\ 1 \end{bmatrix}$$

   (a) $\theta = (X^T X)^{-1} X^T Y$

   $\theta = \begin{bmatrix} 10 & -9 \\ -9 & 169 \end{bmatrix}^{-1} \begin{bmatrix} 14 \\ -79 \end{bmatrix} = \begin{bmatrix} 0.10503418 & 0.00559354 \\ 0.00559354 & 0.00621504 \end{bmatrix} \begin{bmatrix} 14 \\ -79 \end{bmatrix} = \begin{bmatrix} 1.0285919 \\ -0.41267868 \end{bmatrix}$

   (b) Using sklearn:

```
    from sklearn.linear_model import LinearRegression
    X = np.array(
    [
        [1, -2],
        [1, -5],
        [1, -3],
        [1, 0],
        [1, -8],
        [1, -2],
        [1, 1],
        [1, 5],
        [1, -1],
        [1, 6],
    ]
)
    Y = np.array([1, -4, 1, 3, 11, 5, 0, -1, -3, 1])
```

```python
reg = LinearRegression().fit(X, Y)
for idx, x in enumerate(reg.coef_):
    print(f'x_{idx}: {x:0.4f}')
print(f'intercept: {reg.intercept_:0.4f}')

# x_0: 0.0000
# x_1: -0.4127
# intercept: 1.0286

# y = 1.0286 - 0.4127x_1
```

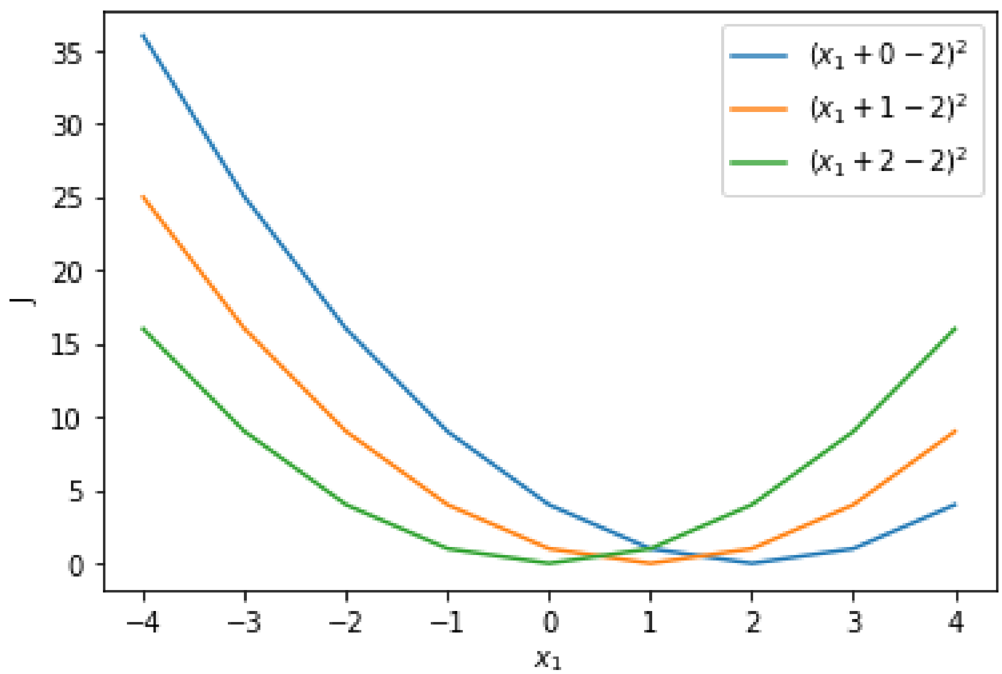
---

2. (a)

$$\frac{\partial J}{\partial x_1} = \frac{\partial}{\partial x_1}(x_1 + x_2 - 2)^2 = 2(x_1 + x_2 - 2)\frac{\partial}{\partial x_1}(x_1 + x_2 - 2) = 2(x_1 + x_2 - 2)$$

$$\frac{\partial J}{\partial x_2} = \frac{\partial}{\partial x_2}(x_1 + x_2 - 2)^2 = 2(x_1 + x_2 - 2)\frac{\partial}{\partial x_2}(x_1 + x_2 - 2) = 2(x_1 + x_2 - 2)$$

(b) Create a 2D plot of x_1 vs J matplotlib, for fixed values of x_2 at 0,1, and 2.



(c) The (x_1, x_2) pairs that minimize J are $(0, 2), (1, 1), (2, 0)$, which is given by the function $x_1 + x_2 - 2 = 0$.

2 Closed Form Linear Regression

$$y = 3275.6667 + 1097.6031x_1 - 259.3279x_2$$

$$RMSE = 601.9303$$

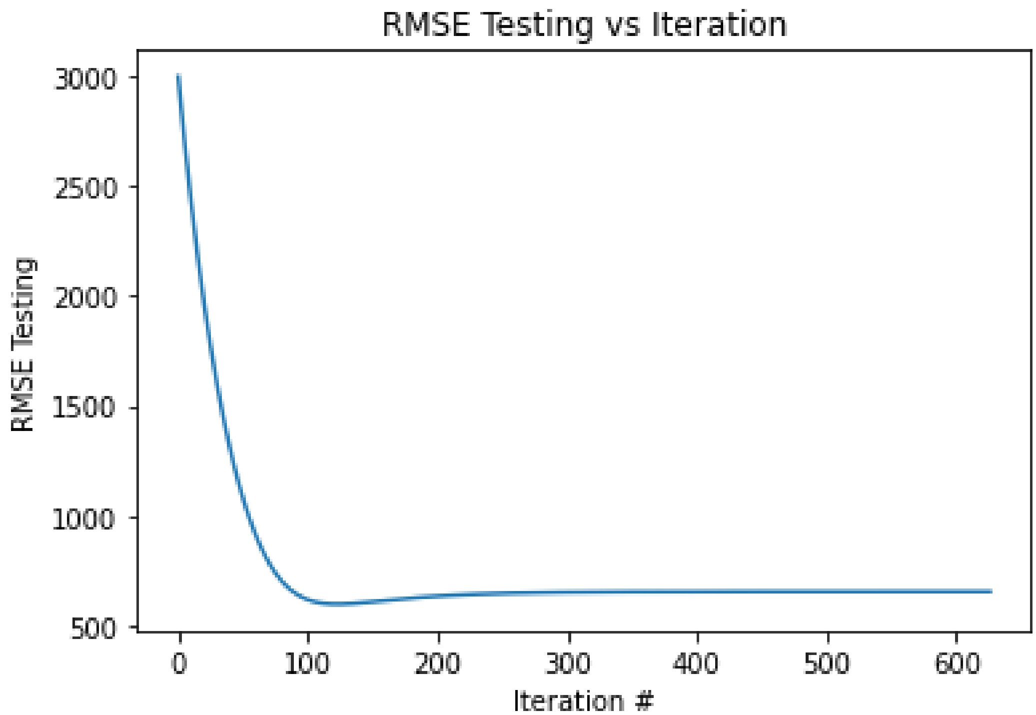
3 Locally-Weighted Linear Regression

$$RMSE = 323.1185$$

4 Gradient Descent

$$y = 3343.2651 + 1036.6251x_1 - 295.6675x_2$$

$$RMSE = 653.7564$$


RMSE Testing vs Iteration
RMSE Testing
3000
2500
2000
1500
1000
500
0
100
200
300
400
500
600
Iteration #