朱浩泽 1911530
智能计算系统作业一
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智能计算系统第一次作业

Question 1. 假设只有一个隐层的多层感知机, 其输入、隐层、输出层的神经元个数分别为 33、512、10, 那么这个多层感知机总共有多少个参数是可以被训练的?

答: 有 $33 \times 512 + 512 \times 10 + 2 = 22018$ 个参数是可以被训练的。

Question 2.

$$W^{(1)} = \begin{bmatrix} w_{1,1}^{(1)} & w_{1,2}^{(1)} & w_{1,3}^{(1)} \\ w_{2,1}^{(1)} & w_{2,2}^{(1)} & w_{2,3}^{(1)} \\ w_{3,1}^{(1)} & w_{3,2}^{(1)} & w_{3,3}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.25 & 0.15 & 0.30 \\ 0.25 & 0.20 & 0.35 \\ 0.10 & 0.25 & 0.15 \end{bmatrix}$$

假定输入数据 $x_1 = 0.02, x_2 = 0.04, x_3 = 0.01$
截距 $b_1 = 0.4, b_2 = 0.7$
期望输出 $y_1 = 0.9, y_2 = 0.5$

$$W^{(1)} = \begin{bmatrix} w_{1,1}^{(1)} & w_{1,2}^{(1)} & w_{1,3}^{(1)} \\ w_{2,1}^{(1)} & w_{2,2}^{(1)} & w_{2,3}^{(1)} \\ w_{3,1}^{(1)} & w_{3,2}^{(1)} & w_{3,3}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.25 & 0.15 & 0.30 \\ 0.25 & 0.20 & 0.35 \\ 0.10 & 0.25 & 0.15 \end{bmatrix} W^{(2)} = \begin{bmatrix} w_{1,1}^{(1)} & w_{1,2}^{(1)} \\ w_{2,1}^{(1)} & w_{2,2}^{(1)} \\ w_{3,1}^{(1)} & w_{3,2}^{(1)} \end{bmatrix} = \begin{bmatrix} 0.40 & 0.25 \\ 0.35 & 0.30 \\ 0.01 & 0.35 \end{bmatrix}$$

答: 输入到隐藏层计算

$$v = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = w^{(1)^T} x + b_1 = \begin{bmatrix} 0.25 & 0.15 & 0.30 \\ 0.25 & 0.20 & 0.35 \\ 0.10 & 0.25 & 0.15 \end{bmatrix} \begin{bmatrix} 0.02 \\ 0.04 \\ 0.01 \end{bmatrix} + 0.4 = \begin{bmatrix} 0.416 \\ 0.4135 \\ 0.4215 \end{bmatrix}$$

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix} = 1 + \frac{1}{1 + e^{-v}} = \begin{bmatrix} 1 + \frac{1}{1+e^{-0.416}} \\ 1 + \frac{1}{1+e^{-0.4135}} \\ 1 + \frac{1}{1+e^{-0.4215}} \end{bmatrix} = \begin{bmatrix} 0.6025 \\ 0.6019 \\ 0.6038 \end{bmatrix}$$

隐含层到输出层计算

$$\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = W^{(2)^T} h + b_2 = \begin{bmatrix} 0.40 & 0.35 & 0.01 \\ 0.25 & 0.30 & 0.35 \end{bmatrix} \begin{bmatrix} 0.6025 \\ 0.6019 \\ 0.6038 \end{bmatrix} + 0.7 = \begin{bmatrix} 1.1577 \\ 1.2425 \end{bmatrix}$$

$$\hat{y} = \begin{bmatrix} \hat{y_1} \\ \hat{y_2} \end{bmatrix} = \frac{1}{1 + e^{-z}} = \begin{bmatrix} \frac{1}{1+e^{-1.1577}} \\ \frac{1}{1+e^{-1.2425}} \end{bmatrix} = \begin{bmatrix} 0.7609 \\ 0.7760 \end{bmatrix}$$

误差

$$\begin{aligned} L(w) =& L_1 + L_2 = \frac{1}{2}(y_1 - \hat{y_1})^2 + \frac{1}{2}(y_2 - \hat{y_2})^2 \\ =& \frac{1}{2}(0.7609 - 0.9)^2 + \frac{1}{2}(0.7760 - 0.5)^2 \\ =& 0.0478 \end{aligned}$$

误差相对较大，进行反向传播更新
记 $w_{2,2}$ 为 w

$$L(w) = L_1 + L_2 = \frac{1}{2}(y_1 - \hat{y}_1)^2 + \frac{1}{2}(y_2 - \hat{y}_2)^2$$

$$\begin{aligned} \frac{\partial L(w)}{\partial w} &= \frac{\partial L(w)}{\partial \hat{y}_1} \cdot \frac{\partial \hat{y}_1}{\partial t_1} \cdot \frac{\partial z_1}{\partial h_2} \cdot \frac{\partial h_2}{\partial v_2} \cdot \frac{\partial v_1}{\partial w} + \frac{\partial L(w)}{\partial \hat{y}_2} \cdot \frac{\partial \hat{y}_2}{\partial z_2} \cdot \frac{\partial z_2}{\partial h_2} \cdot \frac{\partial h_2}{\partial v_2} \cdot \frac{\partial v_2}{\partial w} \\ &= 0.6019 \times 0.2987 \times 0.04 \times (-0.3191 \times 0.1819 \times 0.35 + 0.2760 \times 0.7760 \times 0.2240 \times 0.3) \\ &= 0.00005307 \end{aligned}$$

$$w = w - \partial \times \frac{\partial L(w)}{\partial w} = 0.2 - 0.00005 \approx 0.20$$