Lab 06: OOP, Linked Lists, and Trees

1. Instructions

> Please download lab materials `lab06.zip` from our QQ group if you don't have one.

In this lab, you have one task:

- Complete the required problems described in section 3 and submit your code to our OJ website as instructed in lab00. The starter code for these problems is provided in `lab06.py`, which is distributed as part of the lab materials in the `code` directory.

Submission: As instructed above, you just need to submit your answer for problems described in section 3 to our OJ website. You may submit more than once before the deadline; only the final submission will be scored. See lab00 for more instructions on submitting assignments.

Readings: You might find the following reference to the textbook useful:

- Section 2.5
- Section 2.9

2. What Would Python Display?

In this section, you need to think about what python would display if the code below were input to a python interpreter.

Read over the `Link` class in `lab06.py`. Make sure you understand the doctests.

To check the correctness of your answer, you can start a python interpreter, input the code into it, and compare the output displayed in the terminal with yours. It is ok for the interpreter to output nothing or raise an error.

```
>>> link = Link(1000)
>>> link.first
_______

>>> link.rest is Link.empty
_______

>>> link = Link(1000, 2000)
_______

>>> link = Link(1000, Link())
_______
```

```
>>> link = Link(1, Link(2, Link(3)))
>>> link.first
```

```
_______

>>> link.rest.first
_______

>>> link.rest.rest.rest is Link.empty
_______

>>> link.first = 9001
>>> link.first
_______

>>> link.rest = link.rest.rest
>>> link.rest.first
_______

>>> link = Link(1)
>>> link.rest = link
>>> link.rest.rest.rest.rest.first
_______

>>> link = Link(2, Link(3, Link(4)))
>>> link2 = Link(1, link)
>>> link2.first
_______

>>> link2.rest.first
_______
```

```
>>> link = Link(5, Link(6, Link(7)))
>>> link                    # Look at the __repr__ method of Link
_______

>>> print(link)           # Look at the __str__ method of Link
_______
```

3. Required Problems

In this section, you are required to complete the problems below and submit your code to `Contest lab06` in our OJ website as instructed in lab00 to get your answer scored.

3.1 OOP

Problem 1: Mint (200pts)

Complete the `Mint` and `Coin` classes so that the coins created by a mint have the correct year and worth.

- Each `Mint` instance has a year stamp. The `update` method sets the year stamp to the `current_year` class attribute of the `Mint` class.
- The `create` method takes a subclass of `Coin` and returns an instance of that class stamped with the `mint`'s year (which may be different from `Mint.current_year` if it has not been updated.)
- A `Coin`'s `worth` method returns the `cents` value of the coin plus one extra cent for each year of age beyond 50. A coin's age can be determined by subtracting the coin's year from the `current_year` class attribute of the `Mint` class.

```
class Mint:
    """A mint creates coins by stamping on years.

    The update method sets the mint's stamp to Mint.current_year.

    >>> mint = Mint()
    >>> mint.year
    2020
    >>> dime = mint.create(Dime)
    >>> dime.year
    2020
    >>> Mint.current_year = 2100  # Time passes
    >>> nickel = mint.create(Nickel)
    >>> nickel.year     # The mint has not updated its stamp yet
    2020
    >>> nickel.worth()  # 5 cents + (80 - 50 years)
    35
    >>> mint.update()   # The mint's year is updated to 2100
    >>> Mint.current_year = 2175     # More time passes
    >>> mint.create(Dime).worth()    # 10 cents + (75 - 50 years)
    35
    >>> Mint().create(Dime).worth()  # A new mint has the current year
    10
    >>> dime.worth()     # 10 cents + (155 - 50 years)
    115
    >>> Dime.cents = 20  # Upgrade all dimes!
    >>> dime.worth()     # 20 cents + (155 - 50 years)
    125

    """
    current_year = 2020

    def __init__(self):
        self.update()
```

```
    def create(self, kind):
        "*** YOUR CODE HERE ***"

    def update(self):
        "*** YOUR CODE HERE ***"

class Coin:
    def __init__(self, year):
        self.year = year

    def worth(self):
        "*** YOUR CODE HERE ***"

class Nickel(Coin):
    cents = 5

class Dime(Coin):
    cents = 10
```

3.2 Linked Lists

Problem 2: Link to List (100pts)

Write a function `link_to_list` that takes in a linked list and returns the sequence as a Python list. You may assume that the input list is shallow; none of the elements is another linked list.

Try to find both an iterative and recursive solution for this problem!

```
def link_to_list(link):
    """Takes a linked list and returns a Python list with the same elements.

    >>> link = Link(1, Link(2, Link(3, Link(4))))
    >>> link_to_list(link)
    [1, 2, 3, 4]
    >>> link_to_list(Link.empty)
    []
    """
    "*** YOUR CODE HERE ***"
```

Problem 3: Store Digits (100pts)

Write a function `store_digits` that takes in an integer `n` and returns a linked list where each element of the list is a digit of `n`.

```
def store_digits(n):
    """Stores the digits of a positive number n in a linked list.

    >>> s = store_digits(1)
    >>> s
    Link(1)
    >>> store_digits(2345)
    Link(2, Link(3, Link(4, Link(5))))
    >>> store_digits(876)
    Link(8, Link(7, Link(6)))
    """
    "*** YOUR CODE HERE ***"
```

3.3 Trees

Problem 4: Cumulative Mul (100pts)

Write a function `cumulative_mul` that mutates the Tree `t` so that each node's label becomes the product of all labels in the subtree rooted at the node.

```
def cumulative_mul(t):
    """Mutates t so that each node's label becomes the product of all labels in
    the corresponding subtree rooted at t.

    >>> t = Tree(1, [Tree(3, [Tree(5)]), Tree(7)])
    >>> cumulative_mul(t)
    >>> t
    Tree(105, [Tree(15, [Tree(5)]), Tree(7)])
    """
    "*** YOUR CODE HERE ***"
```

Problem 5: Generate Paths (200pts)

Define a generator function `generate_paths` which takes in a Tree `t`, a value `value`, and returns a generator object which yields each path from the root of `t` to a node that has label `value`.

`t` is implemented with a class, not as the function-based ADT.

Each path should be represented as a list of the labels along that path in the tree. You may yield the paths in any order.

We have provided a (partial) skeleton for you. You do not need to use this skeleton, but if your implementation diverges significantly from it, you might want to think about how you can get it to fit the skeleton.

```
def generate_paths(t, value):
```

```
    """Yields all possible paths from the root of t to a node with the label
value
    as a list.

    >>> t1 = Tree(1, [Tree(2, [Tree(3), Tree(4, [Tree(6)]), Tree(5)]),
Tree(5)])
    >>> print(t1)
    1
      2
        3
        4
          6
        5
      5
    >>> next(generate_paths(t1, 6))
    [1, 2, 4, 6]
    >>> path_to_5 = generate_paths(t1, 5)
    >>> sorted(list(path_to_5))
    [[1, 2, 5], [1, 5]]

    >>> t2 = Tree(0, [Tree(2, [t1])])
    >>> print(t2)
    0
      2
        1
          2
            3
            4
              6
            5
          5
    >>> path_to_2 = generate_paths(t2, 2)
    >>> sorted(list(path_to_2))
    [[0, 2], [0, 2, 1, 2]]
    """

    "*** YOUR CODE HERE ***"

    for _________________ in _____________________:
        for _________________ in _____________________:

            "*** YOUR CODE HERE ***"
```

> *Hint*: If you're having trouble getting started, think about how you'd approach this problem if it wasn't a generator function. What would your recursive calls be? With a generator function, what happens if you make a "recursive call" within its body?

3.4 Extra Challenges

The problems below may be a bit difficult. Therefore, we just assign 100 points for each so that you will not lose much scores if you failed to solve them. Take your time to overcome them.

Problem 6: Deep Map (100pts)

Implement `deep_map`, which takes a function `f` and a `link`. It returns a *new* linked list with the same structure as `link`, but with `f` applied to any element within `link` or any `Link` instance contained in `link`.

The `deep_map` function should recursively apply `fn` to each of that `Link`'s elements rather than to that `Link` itself.

Hint: You may find the built-in `isinstance` function useful. You can also use the `type(link) == Link` to check whether an object is a linked list.

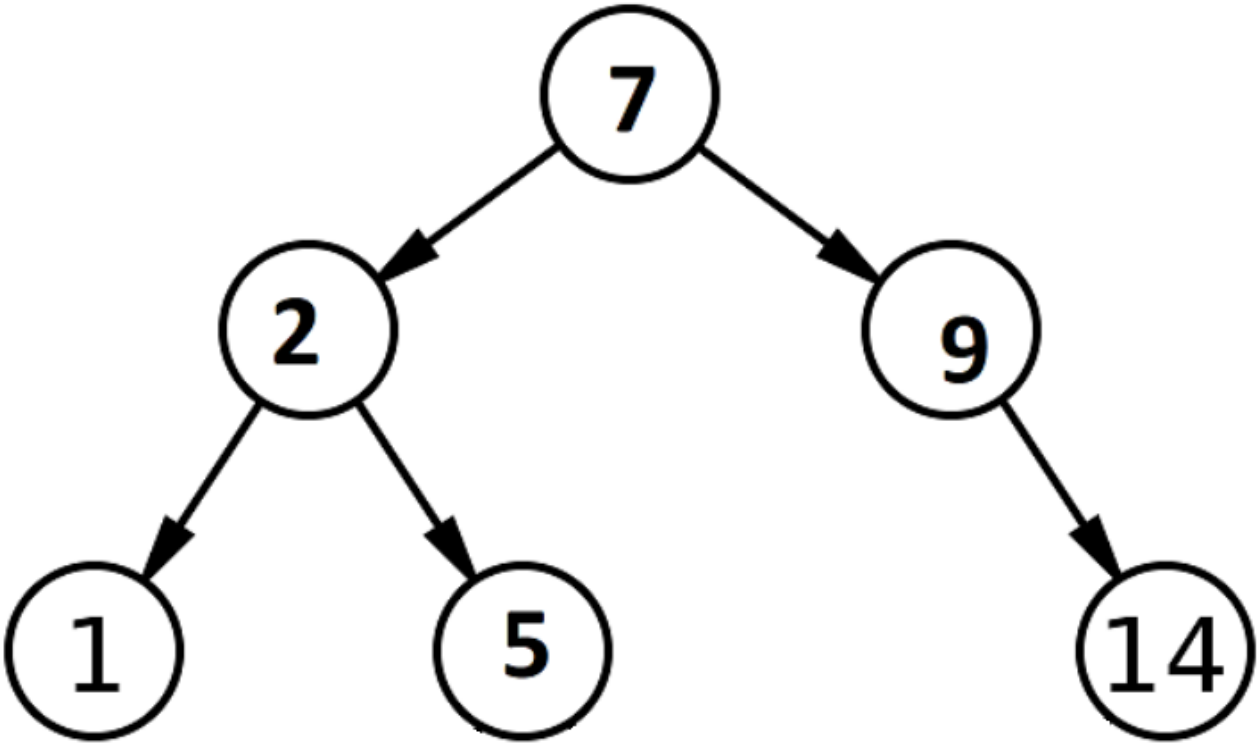
```
def deep_map(f, link):
    """Return a Link with the same structure as link but with fn mapped over
    its elements. If an element is an instance of a linked list, recursively
    apply f inside that linked list as well.

    >>> s = Link(1, Link(Link(2, Link(3)), Link(4)))
    >>> print(deep_map(lambda x: x * x, s))
    <1 <4 9> 16>
    >>> print(s) # unchanged
    <1 <2 3> 4>
    >>> print(deep_map(lambda x: 2 * x, Link(s, Link(Link(Link(5))))))
    <<2 <4 6> 8> <<10>>>
    """
    "*** YOUR CODE HERE ***"
```

Problem 7: Is BST (100pts)

Write a function `is_bst`, which takes a Tree `t` and returns `True` if, and only if, `t` is a valid binary search tree, which means that:

- Each node has at most two children (a leaf is automatically a valid binary search tree).
- The children are valid binary search trees.
- For every node, the entries in that node's left child are less than or equal to the label of the node.
- For every node, the entries in that node's right child are greater than the label of the node.

An example of a BST is:



Note that, if a node has only one child, that child could be considered either the left or right child. You should take this into consideration.

Hint: It may be helpful to write helper functions `bst_min` and `bst_max` that return the minimum and maximum, respectively, of a Tree if it is a valid binary search tree.

```
def is_bst(t):
    """Returns True if the Tree t has the structure of a valid BST.

    >>> t1 = Tree(6, [Tree(2, [Tree(1), Tree(4)]), Tree(7, [Tree(7),
Tree(8)])])
    >>> is_bst(t1)
    True
    >>> t2 = Tree(8, [Tree(2, [Tree(9), Tree(1)]), Tree(3, [Tree(6)]),
Tree(5)])
    >>> is_bst(t2)
    False
    >>> t3 = Tree(6, [Tree(2, [Tree(4), Tree(1)]), Tree(7, [Tree(7),
Tree(8)])])
    >>> is_bst(t3)
    False
    >>> t4 = Tree(1, [Tree(2, [Tree(3, [Tree(4)])])])
    >>> is_bst(t4)
    True
    >>> t5 = Tree(1, [Tree(0, [Tree(-1, [Tree(-2)])])])
    >>> is_bst(t5)
    True
    >>> t6 = Tree(1, [Tree(4, [Tree(2, [Tree(3)])])])
    >>> is_bst(t6)
    True
```

```
    >>> t7 = Tree(2, [Tree(1, [Tree(5)]), Tree(4)])
    >>> is_bst(t7)
    False
    """
    "*** YOUR CODE HERE ***"
```