Prompt: Please upload a letter (not to exceed 3 pages) indicating why you want to study mathematics here, what has brought you to this decision, and what are your goals in the future.

Letter (1106 words)

To the OSU Admissions Committee,

My name is Anthony Hong, and I am applying to the PhD program in mathematics at Ohio State University. With significant research experience in both theoretical and applied mathematics, I aim to explore the intersection of geometry and other fields. I believe my interdisciplinary background, coupled with my passion for teaching and collaborative research, will allow me to make meaningful contributions to OSU's mathematics department as well as the broader university community. Ohio State's commitment to fostering a diverse research environment, as well as its dedication to cultivating future leaders in academia and professional development, aligns perfectly with my academic and professional aspirations.

My preparation for a rigorous graduate program in mathematics has been shaped by research experiences that bridge theoretical concepts and practical applications. In the summer of 2023, I focused on Riemannian geometry and Markov chains to understand Yau et al.'s graph analogue of Ollivier-Ricci curvature. Inspired by Gromov's theory, I applied this framework to study Cayley graphs of abelian and nilpotent groups, exploring how their geometric properties reflect underlying algebraic structures. One of the main challenges was devising linear optimization algorithms to compute the Wasserstein distance, a key component in Ollivier-Ricci curvature. To address this, I taught myself the Kuhn-Munkres and Ford-Fulkerson methods and took relevant algorithm course. I presented my findings at the Midstates Consortium for Math and Science 23 at the University of Chicago. These discussions underscored my ability to rapidly learn new topics, independently research, and effectively communicate complex mathematical ideas, motivating me to find applications of geometric methods.

As a fellow of the MIT Summer Geometry Initiative (SGI) last summer, I gained practical experience in geometric visualization and problem-solving techniques. Among the four projects I participated in, two focused on signed distance functions (SDFs). Working with Prof. Oded Stein (USC) and Prof. Silvia Sellán (Columbia), I explored both the theoretical and computational aspects of SDFs. I applied Gauss's lemma to prove that the Eikonal equation and the closest point condition together characterize SDFs on a plane. This project, along with others using neural networks to model surfaces, underscored the importance of physical intuition in tackling geometric problems. With the help of Polyscope, a visualization software introduced at SGI, I have since created educational videos on YouTube and Bilibili to communicate the beauty of mathematics. Additionally, the large community of researchers at SGI, coupled with frequent invited talks and tutorials, demonstrated how computational methods can systematically discover examples and counterexamples in pure mathematical problems, further enriching my perspective.

Beyond my focus on applying pure mathematics to solve practical problems, my interest in theoretical areas has only grown stronger. During the summer of 2023, under the guidance of Prof. Renato Feres, I began reading *Geometry of Quantum States*, which sparked my interest in noncommutative theories. This led me to attend the Noncommutative Geometry Festival 2023, where I had the opportunity to listen to inspiring talks on quantization and spectral triples. My curiosity in this field deepened through discussions during MIT SGI with Prof. Keenan Crane at CMU about discrete Dirac operators. A pivotal moment was hearing Prof. Arthur Jaffe and Dr. Kaifeng Bu present their work on the quantum central limit theorem. Inspired by their talk, I began weekly readings in information theory with Dr. Bu, exploring entropy-based proofs and the geometric structures arising from the Fisher information metric. These experiences reinforced my fascination with abstract theories that I am eager to explore further at OSU.

My project in Math547 Theory of Polytopes was another significant turning point in my undergraduate journey. Using Khovanskii and Pukhlikov's "Riemann-Roch Theorem for Integrals and Sums of Quasipolynomials over Virtual Polytopes" as a foundation, I presented a theorem on the integer-point count of Delzant polytopes using Todd operators. This project not only introduced me to Delzant's classification theorem for symplectic toric

manifolds but also led me to explore proof of Arnold conjecture by Floer homology and Kähler geometry of toric varieties. Under the guidance of Prof. Xiang Tang, I started working on my undergraduate thesis on this topic. I also presented a project on the irreducible representation of compact Lie groups arising from geometric quantization in a graduate-level topic course. The thesis is ongoing, and I believe that with upcoming courses in index theory and operator theory, I will be able to produce a thesis that presents some of the most intriguing geometric phenomena.

Alongside mathematics, teaching has been another central aspect of my academic journey. Feynman's philosophy of "teaching as a method of learning" has deeply influenced me. Inspired by Prof. Quo-Shin Chi's assignment of explaining Brouwer's theorem to high-schoolers through the Hex game, I served as a TA for Prof. Rachel Roberts' differential topology course. I also helped a high school friend at CMU with harmonic analysis in signal processing. These early teaching opportunities solidified my commitment to making complex mathematics more approachable. I am eager to continue developing as an educator at OSU, where the university's emphasis on teaching is exemplified by the Phil Huneke Distinguished Graduate Teaching Associate Award.

Ohio State University's vibrant mathematical community provides an ideal environment for both research and teaching. I am particularly drawn to OSU's strong seminars in geometry and topology, including the Topology, Geometry, and Data Analysis (TGDA) Seminar and the Math 2 Industry Seminar, which align perfectly with my research interests. The Math-Stats Learning Center (MSLC), a free tutoring service for students, also offers a fantastic opportunity for me to contribute to the undergraduate learning environment while honing my teaching skills.

Moreover, I am excited about the opportunity to collaborate with faculty members at OSU whose research aligns well with my own. I have already been in contact with Dr. Kaifeng Bu, whose work in quantum information theory complements my background in geometric structures. I am also eager to work with Prof. Nathan Broaddus and Prof. James Fowler, whose research in geometric group theory resonates with my interests. Additionally, Prof. Jingyin Huang's work in metric geometry and low-dimensional topology and Prof. Beibei Liu's work in hyperbolic geometry offer exciting avenues for potential collaboration. Lastly, I am also excited about the opportunity to work with Prof. Andrey Gogolev, whose research on Anosov flows directly aligns with the work of my current advisor Prof. Steven Frankel.

In summary, Ohio State University offers an exceptional platform for me to grow as both a researcher and educator. The combination of strong research opportunities, a collaborative teaching environment, and the vibrant academic culture in Columbus—known for its robust support for STEM initiatives, dynamic arts scene, and highly livable environment—makes OSU the ideal place for me to pursue my PhD.

Sincerely,
Anthony Hong