Statement of Purpose (Draft)

Ten years ago, I could never have imagined living in a different country, speaking a foreign language, and studying subjects that were once beyond my reach. Growing up, I witnessed my parents' socio-economic advancement despite their limited educational backgrounds. By the time I reached secondary school, they could no longer comprehend the books I was reading. This disparity highlighted a broader issue of uneven development in China, prompting me to seek answers through quantitative analysis. To investigate, I delved into the literature, cleaned a stratified national dataset, and statistically justified a rapid decline in the intergenerational mobility of educational resources. My findings were published in IEEE MSIEID, marking my first experience using mathematics and computational tools to verify social observations.

This early research solidified my passion for data analysis and motivated me to pursue a joint major in Economics and Computer Science at WUSTL, with Mathematics as a second major. While this interdisciplinary study equipped me with strong analytical skills, the geometric and topological structures of data that I encountered made me realize the need to deepen my theoretical foundations and expand my toolbox. Columbia's emphasis on combining data analysis with geometric perspectives provides the perfect environment for me to further develop these skills and make meaningful contributions to the field.

My journey into computational geometry and data analysis truly began in the summer of 2023, when I joined the Freiwald Scholars Program and undertook an independent project with Prof. Renato Feres. Initially focused on verifying examples in Ollivier's Ricci Curvature of Markov Chains on Metric Spaces, I soon extended the definition to curvature on graphs. Central to this work was transforming the optimal transport problem into a linear optimization challenge solvable by algorithms like Kuhn-Munkres and Ford-Fulkerson. I became fascinated by the interplay between continuous and discrete mathematics and how computers can uncover geometric properties of graphs. This project culminated in a presentation at the Midstates Consortium for Math and Science 23 at UChicago, where I engaged with researchers across disciplines and discussed potential applications of geometric methods to physical models like point charge distributions.

This project also led to fruitful discussions with Dr. Kaifeng Bu at Ohio State University, whom I met at the Noncommutative Geometry Festival 23 at WashU. We explored information geometry and its connection to machine learning during weekly discussions. It was during one of these discussions that I realized Wasserstein distance could be used to quantify the deviation between population and empirical expectations in random processes, viewing a learning problem as a geometric movement. This intersection of learning and geometry resonated deeply with me—I still recall running out of the library in excitement when I encountered this connection while reading in high-dimensional probability.

Motivated by these research insights, I further expanded my understanding of optimization by enrolling in CSE543T Algorithms of Nonlinear Optimization. In the final project, my team and I applied gradient descent and numerical PDE solvers to optimal transport problems, implementing the Wasserstein distance for k-nearest neighbor image classification and comparing its effectiveness against other distance metrics. This experience reinforced my passion for using computational tools to tackle real-world challenges and solidified my interest in optimization and geometry.

Last summer, I was selected as a fellow for the MIT Summer Geometry Initiative (SGI), where I worked on four distinct projects that significantly broadened my understanding of computational geometry. In one project led by Dr. Nickolas Sharp from NVIDIA, I studied how Gromov-Hausdorff distance measures the dissimilarity of shapes as metric spaces. Another project, led by Prof. Oded Stein (USC) and Prof. Silvia Sellán (Columbia), involved designing signed distance functions (SDFs) and reconstructing zero level sets using the marching squares algorithm. Classical dissimilarity measures like Hausdorff and Chamfer variants can then be used to evaluate reconstructions. However, I found these measures limited, as they rely on the computation of original zero level sets as a priori. Thus, I was particularly thrilled when I derived an inherent measure independent of such a priori by applying Gauss's lemma to show that the Eikonal equation and the closest point condition together characterize SDFs on a plane (though Prof. Sellán later provided an elementary proof using the 1-Lipschitz property). My next project, mentored by Prof. Amir Vaxman (Edinburgh), continued analyzing reconstructions of 3-

dimensional SDFs from Poisson samples using shallow neural networks and adversarial modules. My final project at SGI with Prof. Edward Chien focused on generalizing the winding-number-implemented vector sketch colorization to different scenarios. Visualization tools like Adobe Illustrator and Blender played an essential role in representing those scenarios that were hard to construct otherwise.

SGI also exposed me to inspiring talks by industry leaders like Jesse Louis-Rosenberg, cofounder of Nervous System, on generative puzzle design, and Prof. Mina Konaković Luković on conformal geometry and auxetic materials for art and design. Motivated by these talks, I introduced conformal and hyperbolic geometries to a friend in architecture, who realized these concepts using CAD software. During this time, I realized the importance of bridging the gap between theorists and applied computational scientists. As Lloyd Nick Trefethen argued against Sir Michael Atiyah's dismissive view of applied mathematics in his autobiography, I have become increasingly passionate about using visualization to concretize abstract ideas and foster better interdisciplinary communication.

Teaching has also become a central part of my mission to promote understanding across disciplines. From explaining Fourier analysis in signal processing to my CMU friend, serving as a TA for differential topology, and creating educational videos on YouTube and Bilibili, I have applied Feynman's technique of teaching as learning to make mathematics more accessible. I aim to contribute to and thrive in an inclusive environment where theoretical, applied, and computational scientists can communicate seamlessly and benefit from each other's strengths. This commitment to creating a supportive and collaborative community aligns with Columbia's mission and is one of the reasons I am particularly drawn to this program.

Looking ahead, I am eager to further explore the intersection of geometry processing, machine learning, and information theory. I envision pursuing a research career that not only advances the theoretical foundations of computational geometry and machine learning but also applies these tools to solve real-world problems in areas like architectural modeling and optimizations in economics. Columbia's Computer Science department offers an ideal environment to pursue these goals. I am particularly drawn to the work of Prof. Silvia Sellán, whose research in applying stochasticity to geometry processing aligns with my interests, and Prof. Alexandr Andoni, whose expertise in high-dimensional computational geometry and metric embeddings could help me deepen my understanding of the algorithmic foundations of massive data. I am also excited about the possibility of collaborating with Professors Tal Malkin and Henry Yuen, whose research in information theory and complexity theory offers another avenue that aligns with my broader interests in the theoretical underpinnings of computation.

Beyond the department's academic resources, I am captivated by Columbia's vibrant research community that encourages interactions with scholars from a wide range of disciplines. The cultural and intellectual energy of New York City, with its museums, theaters, and concert halls, is another strong draw for me, as it would provide a stimulating backdrop for personal and professional growth. Columbia's combination of world-class faculty, interdisciplinary opportunities, and dynamic setting makes it the perfect place for me to pursue my long-term goal of contributing to both the theoretical and applied aspects of computer science.