Statement of Purpose

Application for PhD in Mathematics

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During my sophomore year, I often struggled to visualize algebraic arguments, a mental tension between algebra and geometry that had begun long before I switched my major from architecture to mathematics. Michael Atiyah's words, "Algebra is the offer made by the devil to the mathematician," deeply resonated with me, as he describes algebra as a Faustian bargain—solving problems at the cost of losing geometric intuition. I first encountered this dichotomy in my architectural practice: the golden ratio, an abstract algebraic concept, finds its beauty in geometric constructions, while the angle trisection problem, which I attempted repeatedly in floor plan drawings, is proven impossible through the algebraic extensions I later learned in my Galois theory course. Both subjects offer unique perspectives—algebra provides precise quantitative descriptions, while geometry inspires a vivid, intuitive vision. This duality lies at the heart of my fascination with mathematics and draws me to the PhD program at University of Chicago, where its strong foundation and rich history in both algebra and geometry will allow me to explore their intersections.

My preparation for a rigorous graduate program has been shaped by research experiences that bridge theoretical concepts with practical applications. During the 2023 Freiwald Scholars Program, I studied Ollivier-Yau-Ricci curvature for metric measure spaces under the guidance of Prof. Renato Feres. Inspired by Gromov's δ-hyperbolicity, I focused on the curvature κ of Cayley graphs of abelian groups. A key challenge was computing the graph optimal transport involved in defining κ. To address this, I taught myself the Kuhn-Munkres and Ford-Fulkerson linear optimization methods and leveraged the symmetries of Cayley graphs to improve algorithmic efficiency. I presented these findings at the Midstates Consortium for Math and Science 23 at the University of Chicago. This project also introduced me to foundational ideas about the interplay between local curvature and global topology, which I encountered through the work of Cédric Villani, John Lott, and Jeff Cheeger. These ideas deeply resonated with me and are now topics I have explored further in Prof. Feres' Math5047 Differential Geometry. Since then, I have been captivated by geometric group theory and metric geometry for its diverse tools and applications.

Last summer, I joined the MIT Summer Geometry Initiative (SGI) led by Prof. Justin Solomon, who has done extensive research on optimal transportation, I gained practical experience in geometric visualization and problem-solving techniques. Among the four projects I participated in, two focused on signed distance functions (SDFs). Working with Prof. Oded Stein (USC) and Prof. Silvia Sellán (Columbia), I applied Gauss's lemma to prove that the Eikonal equation and the closest point condition together characterize SDFs in $\mathbb{R}^2$. In Prof. Amir Vaxman's group, I used shallow neural networks to build SDFs in $\mathbb{R}^3$ from the Poisson-sampled point clouds of their zero-level-surfaces. These projects underscored the importance of physical intuition in tackling geometric problems. SGI mentors also showcased unexpected ways in which applied mathematics can apply back to its origin. Bonnet once asked whether the metric and mean curvature function determine a unique smooth compact immersion. Bobenko et al. used discrete differential geometry to systematically construct counterexamples, resolving this 150-year-old problem. I followed their work to write a report, distributed on the MIT SGI webpage, utilizing visualization tools like Polyscope that I learned during SGI.

Beyond my focus on geometric applications, my interest in using algebraic computations to understand geometries has only grown stronger. During the summer of 2023, under the guidance of Prof. Renato Feres, I began reading *Geometry of Quantum States*, where objects like $\mathrm{SU}(2)$ and $\mathrm{SL}(n, \mathbb{C})$ sparked my journey. I was fortunate to attend the Noncommutative Geometry Festival 2023 to listen to inspiring talks on quantization and spectral triples. A pivotal moment was hearing Prof. Arthur Jaffe (Harvard) and Prof. Kaifeng Bu (OSU) present their work on the quantum central limit theorem. Inspired by their talk, I began weekly readings with Prof. Bu on Clifford algebras and spinors in quantum information, exploring their role in modeling state transformations and symmetries in multi-qubit systems. I recorded our discussions and calculations in detailed notes, deepening my appreciation for profound clarity and computational power of algebra, which I am eager to explore further at UChicago.

Wishing to learn more algebraic tools for geometries, I took Math547 Theory of Polytopes with Prof. Laura Escobar. For the final project, I used Khovanskii and Pukhlikov's "Riemann-Roch Theorem for Integrals and Sums of Quasipolynomials over Virtual Polytopes" as a foundation to present integer-point counting procedures of Delzant polytopes via Todd operators. This project introduced me to Delzant's classification of symplectic toric manifolds, inspiring my undergraduate thesis with Prof. Xiang Tang. Unlike Prof. Feres, who emphasized rapid learning across diverse theories, Prof. Tang encouraged a more meticulous approach, urging me to grasp every step and complete all exercises in Ana Cannas da Silva's *Lectures on Symplectic Geometry*. I was also introduced to fascinating laws in the physical world, including variational principles and Noether's theorem. The exercises and readings also provided paths for me to explore Morse and Floer theory and Guillemin's treatment of Kähler geometry of toric varieties. In the coming months, I plan to study two generalizations, for action by semisimple Lie groups and for base space four-dimensional log-symplectic manifolds. Through these studies, I aim to create a self-contained thesis that synthesizes the algebraic and geometric insights I have gained during my undergraduate journey.

At UChicago, I wish to do research in geometric group theory and dynamics. I am drawn to the work of professors like Prof. Shmuel Weinberger and Prof. Danny Calegari. I had the pleasure of meeting Prof. Weinberger at the Noncommutative Geometry Festival 23, where I was inspired by his insights on quantitative cobordism theory. I am equally excited by Prof. Calegari's research in low-dimensional dynamics, especially given his two visually appealing talks at WashU on laminations, $K(\pi, 1)$ and $CAT(0)$ properties. I am excited about the opportunity to work with Prof. Amie Wilkinson, whose research in group actions and dynamics connects closely with concepts I studied in my Dynamics and Chaos course and found deeply intriguing. I am also eager to collaborate with Prof. Benson Farb, whose work in geometric group theory and Teichmüller theory relates to my experience with quasi-conformality and quasi-isometries in my graph curvature project.

Geometry, to me, is more than a toolset—it is a rendering program contextualizing knowledge and a bridge guiding imagination, where a "metric" is analytical, and a "topology" reveals shapes. Algebra, by contrast, smashes mountains, breaking down complex structures into fundamental forms. Through my work on Cayley graphs of groups and the structure theory of Lie groups to understand their actions over geometries, I have seen how geometry and algebra together allow us to move forward with both vision and precision. Determined to learn mathematics with this spirit and inspired by Plato's adage, "Let no one ignorant of geometry enter," I strive to make mathematics inviting and accessible to all. My teaching experiences and the creation of educational videos reflect this passion, and while research remains my primary focus, I am eager to contribute to teaching programs at UChicago. Joining University of Chicago's vibrant academic culture offers the ideal environment to further explore the interplay between geometry and algebra as both a researcher and an educator.