Prompt: Please describe your aptitude and motivation for graduate study in your area of specialization, including your preparation for this field of study, your academic plans or research interests, and your future career goals. Please be specific about why UC Berkeley would be a good intellectual fit for you.

Statement of Purpose (1148 words)

Stepping out of my dorm at three in the morning, the cold air cleared my clouded mind that has been wrestling with mathematical concepts. This mental struggle between algebra and geometry had begun long before I switched my major from architecture to mathematics. Michael Atiyah's words, "Algebra is the offer made by the devil to the mathematician," resonated deeply with me. He describes algebra as a Faustian bargain—solving the problem but at the cost of losing geometric intuition.

I first felt this tension in my architectural practice: the golden ratio, an abstract algebraic concept, finds its beauty and meaning only through the elegance of geometric forms. The angle trisection, a geometric problem that I tried on floor plan drawings many times, turns out to be impossible by the algebraic extensions I learned in my algebra course. It revealed that algebra and geometry each bring a unique perspective—algebra offers rigor and structure, while geometry breathes vision and meaning. This is why I am now drawn to the PhD program at UC Berkeley, for its strong foundation and rich history in both algebra and geometry that will allow me to delve deeper into their interactions.

My preparation for a rigorous graduate program in mathematics has been shaped by research experiences that bridge theoretical concepts and practical applications. In the summer of 2023, I focused on Riemannian geometry and Markov chains to understand Yau et al.'s graph analogue of Ollivier-Ricci curvature. Inspired by Gromov's theory, I applied this framework to study Cayley graphs of abelian and nilpotent groups, exploring how their geometric properties reflect underlying algebraic structures. One of the main challenges was devising linear optimization algorithms to compute the Wasserstein distance, a key component in Ollivier-Ricci curvature. To address this, I taught myself the Kuhn-Munkres and Ford-Fulkerson methods and took relevant algorithm course. I presented my findings at the Midstates Consortium for Math and Science 23 at the University of Chicago. These discussions underscored my ability to rapidly learn new topics, independently research, and effectively communicate complex mathematical ideas, motivating me to find applications of geometric methods.

As a fellow of the MIT Summer Geometry Initiative (SGI) last summer, I gained practical experience in geometric visualization and problem-solving techniques. Among the four projects I participated in, two focused on signed distance functions (SDFs). Working with Prof. Oded Stein (USC) and Prof. Silvia Sellán (Columbia), I explored both the theoretical and computational aspects of SDFs. I applied Gauss's lemma to prove that the Eikonal equation and the closest point condition together characterize SDFs on a plane. This project, along with others using neural networks to model surfaces, underscored the importance of physical intuition in tackling geometric problems. With the help of Polyscope, a visualization software introduced at SGI, I have since created educational videos on YouTube and Bilibili to communicate the beauty of mathematics. Additionally, the large community of researchers at SGI, coupled with frequent invited talks and tutorials, demonstrated how computational methods can systematically discover examples and counterexamples in pure mathematical problems, further enriching my perspective.

Beyond my focus on applying pure mathematics to solve practical problems, my interest in theoretical areas has only grown stronger. During the summer of 2023, under the guidance of Prof. Renato Feres, I began reading *Geometry of Quantum States*, which sparked my interest in noncommutative theories. This led me to attend the Noncommutative Geometry Festival 2023, where I had the opportunity to listen to inspiring talks on quantization and spectral triples. My curiosity in this field deepened through discussions during MIT SGI with Prof. Keenan Crane at CMU about discrete Dirac operators. A pivotal moment was hearing Prof. Arthur Jaffe and Dr.

Kaifeng Bu present their work on the quantum central limit theorem. Inspired by their talk, I began weekly readings in information theory with Dr. Bu, exploring entropy-based proofs and the geometric structures arising from the Fisher information metric. These experiences reinforced my fascination with abstract theories that I am eager to explore further at UCB.

My project in Math547 Theory of Polytopes was a turning point in my undergraduate journey, guiding me into the depths of symplectic geometry and geometric quantization. Using Khovanskii and Pukhlikov's "Riemann-Roch Theorem for Integrals and Sums of Quasipolynomials over Virtual Polytopes" as a foundation, I presented a theorem on the integer-point count of Delzant polytopes using Todd operators. This project introduced me to Delzant's classification theorem for symplectic toric manifolds, and I began developing my undergraduate thesis around these ideas under the mentorship of Prof. Xiang Tang. Unlike Prof. Feres, who emphasizes rapid learning across diverse theories, Prof. Tang encouraged a more meticulous approach, urging me to grasp every intermediate step and complete all exercises in Ana Cannas da Silva's *Lectures on Symplectic Geometry*. These readings introduced me to the proof of the Arnold conjecture through Floer homology and the Kähler geometry of toric varieties. The balance—between progressing through broad concepts and thoroughly filling in each detail—has deepened my understanding of complex geometric structures and will undoubtedly enhance the insights in my thesis as I integrate concepts from upcoming courses in index theory and operator theory.

Alongside mathematics, teaching has been another central aspect of my academic journey. Feynman's philosophy of "teaching as a method of learning" has deeply influenced me. Inspired by Prof. Quo-Shin Chi's assignment of explaining Brouwer's theorem to high-schoolers through the Hex game, I served as a TA for Prof. Rachel Roberts' differential topology course. I also helped a high school friend at CMU with harmonic analysis in signal processing. These early teaching opportunities solidified my commitment to making complex mathematics more approachable. I am eager to continue developing as an educator at UCB, where the university's emphasis on teaching is exemplified by the SURF SMART and Directed Reading Program (DRP).

While my interests span several fields, geometry remains central to my mathematical journey. I am particularly fascinated by how different perspectives—analytic, topological, algebraic, and physical—intertwine through geometry. For example, I am drawn to the dynamics of low-dimensional manifolds, the classical and quantum aspects of symplectic geometry, and the interplay between large-scale geometry and group theory. UCB is an ideal match with its renowned faculty and strong research groups. Prof. Tang had enthusiastically highlighted his advisor Alan Weinstein's creed: "Everything is a Lagrangian manifold," which resonated with me deeply. I am eager to engage with Prof. John Lott's work on geometric analysis, as well as with experts like Michael Hutchings and David Nadler in symplectic and low-dimensional topology. I also look forward to collaborating with Constantin Teleman and Alexander Givental, whose research in physical and quantum aspects of geometry.

In summary, UC Berkeley offers an exceptional platform for me to grow as both a researcher and educator. The combination of strong research opportunities, a collaborative teaching environment, and its vibrant academic culture makes UCB the ideal place for me to pursue my PhD.