

Statement of Purpose (Draft)

During my sophomore year, I often stepped out of my dorm at three in the morning, letting the cold air clear my mind as I wrestled with mathematical concepts. This internal struggle between algebra and geometry began long before I switched my major from architecture to mathematics. Michael Atiyah's words, "Algebra is the offer made by the devil to the mathematician," resonated deeply with me. He describes algebra as a Faustian bargain—giving quick answers but at the cost of losing geometric intuition.

I felt this tension in my architectural practice: the golden ratio, an abstract algebraic concept, finds its beauty and meaning only through the elegance of geometric forms. I vividly remember standing in front of the facade of Notre-Dame de Paris, where the golden ratio seemed to fit the scale and vision of visitors perfectly. The angle trisection, a geometric problem that I tried on floor plan drawing many times, turns out to be impossible by algebraic extensions I learned in my graduate algebra course. These experiences taught me that algebra and geometry each bring a unique perspective—algebra offers rigor and structure, while geometry breathes vision and meaning into those structures. This is why I am now drawn to programs like UCB's, where a strong foundation and rich history in both algebra and geometry will allow me to delve deeper into their interactions.

My preparation for a rigorous graduate program in mathematics has been shaped by diverse research experiences that bridge theoretical concepts and practical applications. In the summer of 2023, I began with intensive readings in Riemannian geometry and Markov chains to understand Yau et al.'s graph analogue of Ollivier-Ricci curvature. Inspired by geometric group theory, I applied this framework to study Cayley graphs of abelian and nilpotent groups to explore how geometric properties reflect underlying algebraic structures. The main challenge was devising linear optimization algorithms to compute the Wasserstein distance, a key component in Ollivier-Ricci curvature. To tackle this, I quickly taught myself the Kuhn-Munkres algorithm and Ford-Fulkerson method, which proved effective for solving this problem. I consolidated my findings into a Beamer presentation that I was honored to present at the Midstates Consortium for Math and Science 23 at UChicago. The presentation was well received, leading to engaging discussions with the audience and a correspondence with a chemistry professor interested in applying these methods to compute point charge distributions on curved spaces. This experience demonstrated my ability to learn rapidly, my passion for research, and my strong communication skills in presenting complex ideas.

During the same summer, I was fortunate to attend the Noncommutative Geometry Festival 2023 at WashU, where I witnessed talks by renowned mathematicians such as Mikhail Gromov and Alain Connes. Their lifelong dedication to advancing mathematics deeply inspired me. One particularly memorable talk on the quantum central limit theorem by Prof. Arthur Jaffe and Dr. Kaifeng Bu at Harvard led me to start weekly discussions in information theory with Dr. Bu. The elegance of the entropy-based proof of Brégman's Theorem and the geometric structures arising from the Fisher information metric captured my imagination, solidifying my interest in blending abstract theory with real-world applications.

My research interests also extend to symplectic geometry, which I first explored in Math547 Theory of Polytopes. Using Khovanskii and Pukhlikov's paper "Riemann-Roch Theorem for Integrals and Sums of Quasipolynomials over Virtual Polytopes" as a foundation, I presented a theorem on integer-point counting of Delzant polytopes. This project introduced me to Delzant's classification theorem for symplectic toric manifolds, and I further developed this research under the guidance of Prof. Xiang Tang for my undergraduate thesis. Prof. Tang encouraged me to tackle each skipped step in Ana Cannas da Silva's Lectures on Symplectic Geometry, pushing me to master every detail. This process, though challenging, provided a concrete base to connect with broader Lie theory and introduced me to Morse theory and Hodge theory—areas I am eager to explore further in graduate school.

Last summer, as a fellow of the MIT Summer Geometry Initiative (SGI), I gained practical experience in geometric visualization and problem-solving techniques. Among the four projects I participated in, two focused on signed distance functions (SDFs), where I explored the theoretical and computational aspects of SDFs with Prof. Oded Stein and Prof. Silvia Sellán. I was particularly excited to apply Gauss's lemma to prove that the Eikonal equation and the closest point condition

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together characterize SDFs on a plane. This project, along with another using neural networks to model SDFs of surfaces, underscored the importance of physical intuition in tackling geometric problems. The visualization tools I used at SGI, like Polyscope and Adobe Illustrator, empowered me to communicate complex ideas more effectively. I have since created educational videos on YouTube and Bilibili to share these insights and inspire others to appreciate the beauty of mathematics.

The roots of this passion trace back to my freshman year when I first embraced Feynman's philosophy of teaching as a method of learning. Inspired by Prof. Quo-Shin Chi's assignments of explaining Brouwer's theorem to high-schoolers via the Hex game, I served as a TA for Prof. Rachel Roberts' differential topology course and helped a high school friend at CMU with Fourier analysis in signal processing. Revisiting and reorganizing problems from a higher-level perspective deepened my understanding. The experiences at SGI, combined with these early teaching opportunities, have shaped my commitment to making complex mathematics approachable. UCB's strong emphasis on teaching, exemplified by its prestigious Graduate Student Instructor (GSI) awards, and its thriving Directed Reading Program (DRP) offer the perfect environment for me to continue pursuing this passion. Having participated in two DRPs at WashU, I know firsthand how transformative they can be in building meaningful connections between students and mentors. I look forward to contributing to these initiatives at UCB while developing as both an active researcher and a passionate communicator of mathematical ideas.

While my interests span several fields, geometry remains central to my mathematical journey. I am particularly fascinated by how different perspectives—analytic, topological, algebraic, and physical—intertwine through geometry. For example, I am drawn to the dynamical and visual landscape of low-dimensional manifolds, the classical and quantum aspects of symplectic geometry, and the interplay between large-scale geometry and group theory. UCB is an ideal environment for pursuing these interests, with its renowned faculty and strong research groups in geometry and analysis. During a weekly meeting, Professor Tang enthusiastically highlighted the creed of her advisor, Alan Weinstein: "everything is a Lagrangian manifold," which resonated with me deeply. I am drawn to the rich tradition of symplectic geometry at UCB, which continues to shape the field today. I look forward to engaging with the work of Professor John Lott, whose research on Wasserstein geometry has deeply influenced my own, as well as with experts like Michael Hutchings and David Nadler, whose research in symplectic and low-dimensional topology aligns with my interests. I am also excited about the potential to work with Constantin Teleman and Alexander Givental, whose work in the physical and quantum aspects of geometry has sparked my curiosity.

In preparation for a seamless transition to UCB's PhD program, I plan to revisit the graduate courses I took at WashU and address any gaps in my understanding. I am particularly eager to enroll in UCB's regular courses on several complex variables and complex manifolds—topics that were only briefly covered at WashU. Additionally, I am enthusiastic about the many opportunities for academic engagement at UCB, from Cal Day and annual talks to seminars and guest lectures. I believe that these interactions will be invaluable in refining my research interests and finding potential advisors.

With UCB's rich academic environment and the dynamism of its mathematical community, I am confident that my strong academic foundation and research experiences have prepared me to succeed as a graduate student. I look forward to thriving as both a researcher and an active contributor to the department's vibrant intellectual life.