Statement of Purpose
Application for PhD in Mathematics

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During my sophomore year, I often struggled to visualize algebraic arguments, a mental tension between algebra and geometry that had begun long before I switched my major from architecture to mathematics. Michael Atiyah's words, "Algebra is the offer made by the devil to the mathematician," deeply resonated with me, as he describes algebra as a Faustian bargain—solving problems at the cost of losing geometric intuition. I first encountered this dichotomy in my architectural practice: the golden ratio, an abstract algebraic concept, finds its beauty in geometric constructions, while the angle trisection problem, which I attempted repeatedly in floor plan drawings, is proven impossible through the algebraic extensions I later learned in my Galois theory course. Both subjects offer unique perspectives— Algebra provides precise quantitative descriptions, while geometry inspires a vivid, intuitive vision. This duality lies at the heart of my fascination with mathematics and draws me to the PhD program at UC Berkeley, where its strong foundation and rich history in both algebra and geometry will allow me to explore their intersections.

My preparation for a rigorous graduate program has been shaped by research experiences that bridge theoretical concepts and practical applications. In the 2023 Freiwald Scholars Program 2023, I studied Ollivier-Yau-Ricci curvature for metric measure spaces under Prof. Renato Feres. Inspired by Gromov's δ-hyperbolicity, I focused on the curvature κ of Cayley graphs of abelian groups. The main challenge is to find the graph optimal transport involved in the definition of κ. I taught myself the Kuhn-Munkres and Ford-Fulkerson linear optimization methods, used symmetries of Cayley graphs to improve efficiency of algorithms, and presented my findings at the Midstates Consortium for Math and Science 23 at the University of Chicago. By endowing algebraic objects with additional geometric structures, we can simplify algebraic processes, motivating me to seek further geometric applications.

As a fellow of the MIT Summer Geometry Initiative (SGI) last summer, I gained practical experience in geometric visualization and problem-solving techniques. Working with Prof. Oded Stein (USC) and Prof. Silvia Sellán (Columbia), I applied Gauss's lemma to prove that the Eikonal equation and the closest point condition together characterize two-dimensional signed distance functions, underscoring the importance of physical intuitions. Mentors also showcased unexpected ways in which applied mathematics can apply back to its origin. Bonnet once asked whether the metric and mean curvature function determine a unique smooth compact immersion. Bobenko et al. used discrete differential geometry to systematically construct counterexamples, resolving this 150-year-old problem. I followed their work to write a report, distributed on the MIT SGI webpage, utilizing visualization tools like Polyscope that I learned during SGI.

Beyond my focus on geometric applications, my interest in using algebraic computations to understand geometries has only grown stronger. During the summer of 2023, under the guidance of Prof. Renato Feres, I began reading *Geometry of Quantum States*, where objects like $\mathrm{SU}(2)$ and $\mathrm{SL}(n, \mathbb{C})$ sparked my journey. I was fortunate to attend the Noncommutative Geometry Festival 2023 to listen to inspiring talks on quantization and spectral triples. A pivotal moment was hearing Prof. Arthur Jaffe (Harvard) and Prof. Kaifeng Bu (OSU) present their work on the quantum central limit theorem. Inspired by their talk, I began weekly readings with Prof. Bu on Clifford algebras and spinors in quantum information, exploring their role in modeling state transformations and symmetries in multi-qubit systems. I recorded our discussions and calculations in detailed notes, deepening my appreciation for profound clarity and computational power of algebra, which I am eager to explore further at UCB.

Wishing to learn more algebraic tools for geometries, I took Math547 Theory of Polytopes with Prof. Laura Escobar. For the final project, I used Khovanskii and Pukhlikov's "Riemann-Roch

Theorem for Integrals and Sums of Quasipolynomials over Virtual Polytopes" as a foundation to present integer-point counting procedures of Delzant polytopes via Todd operators. This project introduced me to Delzant's classification of symplectic toric manifolds, inspiring my undergraduate thesis with Prof. Xiang Tang. Unlike Prof. Feres, who emphasized rapid learning across diverse theories, Prof. Tang encouraged a more meticulous approach, urging me to grasp every step and complete all exercises in Ana Cannas da Silva's *Lectures on Symplectic Geometry*. These readings introduced me to Arnold's work on Lagrangian Grassmannians, his conjecture on Hamiltonian diffeomorphisms, and various proofs through Floer homology. Recognizing that Lie groups and Lie algebras form the foundational language of abstract Hamiltonian actions, I seized the opportunity to take two Lie theory graduate courses to better prepare my thesis. In the Lie algebra and Representation theory course, I presented the Iwasawa decomposition of $\mathrm{GL}(n,\mathbb{C})$, which gives the symplectic orbit $\mathrm{U}(n)/\mathbb{T}^n$ of $\mathrm{U}(n)$-action on Hermitian matrices an algebraic characterization, i.e., isomorphic to the flag variety $\mathrm{GL}(n,\mathbb{C})/B$ where B consists of upper triangular matrices. Building on these experiences and tools, my thesis will focus on the Atiyah-Guillemin-Sternberg theorem, which describes the convexity properties of moment maps for $\mathbb{T}^n$-action on compact symplectic manifolds. In the coming months, I plan to study its generalizations in two directions: extending the theorem to action by semisimple Lie groups and to four-dimensional log-symplectic manifolds as base space. Through these studies, I aim to create a self-contained thesis that synthesizes the algebraic and geometric insights I have gained during my undergraduate journey, providing clear illustrations of these rich mathematical structures.

Teaching has been an important complement to my academic journey, aligning with Feynman's philosophy of "teaching as a method of learning." As a TA for Prof. Rachel Roberts' differential topology course and a grader for three others, I developed a deeper understanding of mathematical concepts through the process of explaining them to others. While research remains my primary focus, these experiences have prepared me to contribute to UCB's teaching mission, exemplified by programs like SURF SMART and the Directed Reading Program (DRP), while advancing my work as a researcher.

At UCB, I wish to do research in symplectic geometry and topology and its interplay with low-dimensional, quantum, and representation-theoretic aspects. Prof. Tang had enthusiastically highlighted his advisor Alan Weinstein's creed: "Everything is a Lagrangian manifold," which resonated with me as I have already seen its influence through Weinstein's tubular neighborhood theorem and the Arnold-Liouville theorem, and I am eager to explore this perspective further. I am thus particularly drawn to Prof. Michael Hutchings's research in symplectic and contact geometry, as well as Heegaard-Floer homology. I also wish to learn Gromov-Witten theory with Prof. Alexander Givental, whose *Introduction to Symplectic Field Theory*, with its novel treatment of Lagrangian submanifolds, I have begun exploring. Additionally, I am eager to engage with Prof. Constantin Teleman and Prof. David Nadler to delve deeper into algebraic topics such as mirror symmetry and geometric representation theory that arise from symplectic settings. I have also encountered Prof. John Lott's work during my research in optimal transport and Prof. Richard Bamler's work in my Riemannian geometry course, both of which have inspired me to broaden my understanding of these connections.

Ultimately, UC Berkeley offers an exceptional platform for me to grow as both a researcher and educator. The combination of strong research opportunities, a collaborative teaching environment, and its vibrant academic culture makes UCB the ideal place for me to pursue my PhD.