

# Linear Algebra from Scratch: Introduction

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# Matrices are not Digital Constructions of the Environment



# A Matrix is a collection of elements

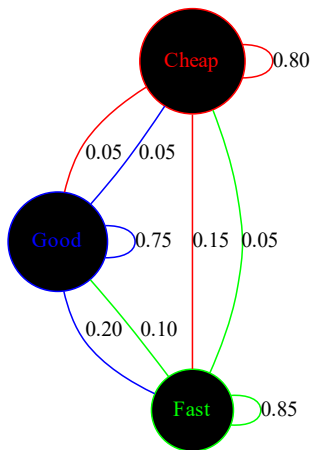
①  $a_1 \sim \text{Cheap}$

②  $a_2 \sim \text{Good}$

③  $a_3 \sim \text{Fast}$

The probability of changing to the  $i_{th}$  state to the  $j_{th}$  state can be expressed as:

$$A' = \begin{bmatrix} 0.80 & 0.05 & 0.10 \\ 0.05 & 0.75 & 0.05 \\ 0.15 & 0.20 & 0.85 \end{bmatrix}$$





# A Matrix is a rectangular array of elements

## Overview.

- 1 Systems of Linear Equations
- 2 Row Echelon Form



Determinants can be calculated for square matrices.

### Overview.

- 1 Determinant of a matrix
- 2 Properties of determinants





# Vector Spaces preserve linearity

## Overview.

- 1 Definitions and Examples
- 2 Subspaces
- 3 Linear Independence
- 4 Basis and Dimension
- 5 Change of Basis
- 6 Row and Column Spaces



# A Linear Transformation is a mapping

## Definition.

- 1 Definitions and Examples
- 2 The Matrix representation of a linear transformation
- 3 Similarity between Matrices



# Orthogonality is the geometric generalization of perpendicular

## Definition.

- 1 Inner Product
- 2 Orthogonal Subspaces
- 3 Least Squares
- 4 Inner Product Spaces
- 5 Gram-Schmidt Orthogonalization



# Solving the eigenvalue problem is useful to many applications

## [Overview]

- 1 Eigenvalues and Eigenvectors
- 2 Diagonalization
- 3 Hermitian Matrices
- 4 Singular Value Decomposition
- 5 Quadratic Forms

# Coding as an additional exercise for ambitious scholars

## Overview.

The Matlab, Python, and other relevant course related content can be found at: **GitHub Link**



# References I

- [1] David Harville. *Matrix Algebra From a Statistician's Perspective*. New York: Springer-Verlag, 1997.
- [2] Leon Stephen. *Linear Algebra with Applications (9th Edition)* (Featured Titles for Linear Algebra. London, England: Pearson, 2014.