A. Explanation on the effective conductivity/permeability

Suppose we have a flux

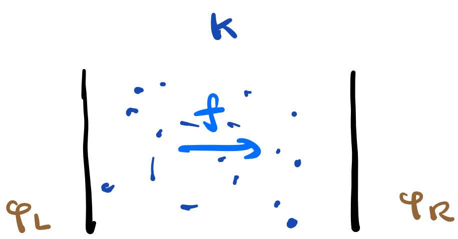
$$f = K \nabla \varphi$$

potential, could be pressure in the permeability case

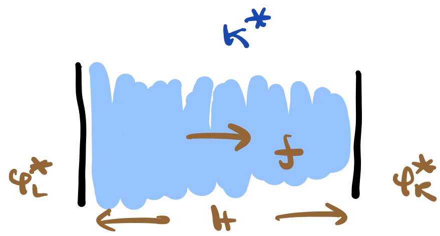
effective

$$\text{and} \quad \nabla \cdot f = 0 \Rightarrow \nabla \cdot (K \nabla \varphi) = 0 \rightarrow K^* \nabla^2 \varphi^* = 0$$

Laplace's equation for φ



non-homogeneous



homogeneous

$$f \approx K^* \frac{\varphi_R - \varphi_L}{H}$$

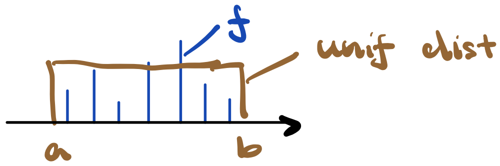
$$\Downarrow$$

$$K^* \approx \frac{\langle f \rangle}{\underbrace{\varphi_R - \varphi_L}_{\text{pressure drop}}} \cdot H$$

$$f = K \nabla \varphi$$

we use $\langle f \rangle \approx \left(\int_{\text{left}} K \frac{\partial \varphi}{\partial x} + \int_{\text{right}} K \frac{\partial \varphi}{\partial x} \right) / 2$

B. MLE



Approximation error

$E = \int_{-\infty}^{\infty} \left(f(x) - \frac{1}{b-a} I_{[a,b]} \right)^2 dx$ = function of a and b

$\frac{\partial E}{\partial a} = 0$ and $\frac{\partial E}{\partial b} = 0$ $\Rightarrow$ Equations fn a and b

$\rightarrow$ solutions give the MLE estimate fn a and b