1) a) SEA $M = (u \ v) = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

u y v DEBEN SER UNITARIOS ($u = (a, b)$)

$\Rightarrow \sqrt{a^2 + b^2} = 1$

$\Rightarrow a^2 + b^2 = 1$

SI ELEGIMOS $a = \pm \cos\theta$, $b = \pm \operatorname{Sen}\theta$

$\Rightarrow (\pm \cos\theta)^2 + (\pm \operatorname{Sen}\theta)^2 = \cos^2\theta + \operatorname{Sen}^2\theta$
$= 1$

$\Rightarrow$ PODEMOS ELEGIR $a = \pm\cos\theta$, $b = \pm \operatorname{Sen}\theta$

b) SEA $M = \begin{pmatrix} a & c \\ b & \pm a \end{pmatrix}$, $u = (a, b)$
$v = (c, \pm a)$

COMO u y v SON ORTOGONALES:

$u \cdot v = 0 \Rightarrow ac \pm ba = 0 \quad (a \neq 0)$

$c \pm b = 0$

$c = \mp b$

$\Rightarrow c = b \lor c = -b$

c)

1.- SEA UNA ROTACIÓN CUALQUIERA $R = \begin{pmatrix} \cos\theta & -\operatorname{Sen}\theta \\ \operatorname{Sen}\theta & \cos\theta \end{pmatrix}$,

ES EVIDENTE QUE LOS VECTORES $u = (\cos\theta, \operatorname{Sen}\theta)$
y $v = (-\operatorname{Sen}\theta, \cos\theta)$ SON UNITARIOS Y ORTOGONALES.

$\therefore$ UNA ROTACIÓN ES ORTOGONAL.

2.- LAS REFLEXIONES SOBRE X e Y VIENEN DADAS POR:
$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $M_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$; MATRICES EVIDENTEMENTE ORTOGONALES.

$\therefore$ UNA REFLEXIÓN ES ORTOGONAL.

3.- SEA $R = \begin{pmatrix} \cos\theta & -\operatorname{Sen}\theta \\ \operatorname{Sen}\theta & \cos\theta \end{pmatrix}$, $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$, $M_2 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

LUEGO: $R * M_1 = \begin{pmatrix} \cos\theta & -\operatorname{Sen}\theta \\ \operatorname{Sen}\theta & \cos\theta \end{pmatrix} * \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = P_1$

$P_1 = \begin{pmatrix} \cos\theta & \operatorname{Sen}\theta \\ \operatorname{Sen}\theta & -\cos\theta \end{pmatrix}$

DONDE P_1 ES EVIDENTEMENTE ORTOGONAL

FINALMENTE:

$R * M_2 = \begin{pmatrix} \cos\theta & -\operatorname{Sen}\theta \\ \operatorname{Sen}\theta & \cos\theta \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = P_2$

$P_2 = \begin{pmatrix} -\cos\theta & -\operatorname{Sen}\theta \\ -\operatorname{Sen}\theta & \cos\theta \end{pmatrix}$

DONDE P_2 ES EVIDENTEMENTE ORTOGONAL.

$\therefore$ LA ROTACIÓN, REFLEXIÓN Y EL PRODUCTO DE AMBAS POSEEN MATRICES ORTOGONALES.

d) YA QUE LA MATRIZ ES ORTOGONAL $\Rightarrow A A^T = I$

$\Rightarrow d(A A^T) = I$

$\Rightarrow d(A)\, d(A^T) = d(I)$

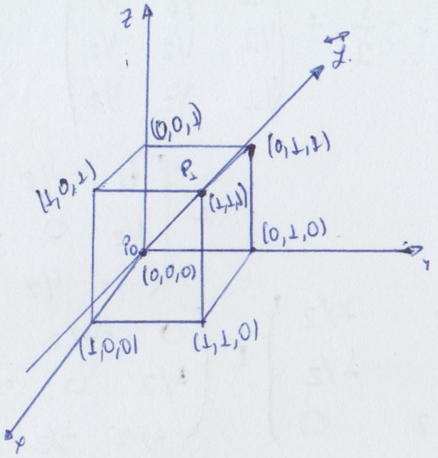
$\Rightarrow d(A) \cdot d(A) = 1$

$\Rightarrow d(A)^2 = 1$

$\Rightarrow d(A) = \pm 1$

CONSIDERANDO $A = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$

②



SEA $\vec{a} = \hat{u}_{P_1 - P_0} = \frac{1}{\sqrt{3}}(1,1,1)$

LUEGO, DEBEMOS ROTAR CADA PUNTO AL REDEDOR DE $\vec{a}$

$\Rightarrow$ DE ACUERDO A LA FÓRMULA (4.19) DEL JANKE:

a) $M_{ROT} = \left(I\cos\theta + (\vec{a}\otimes\vec{a})(1-\cos\theta) + C_a \operatorname{Sen}\theta\right)$

$$\Rightarrow M_{ROT} = \left(\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\cos\frac{\pi}{6} + \begin{pmatrix}1/3&1/3&1/3\\1/3&1/3&1/3\\1/3&1/3&1/3\end{pmatrix}\left(1-\cos\pi/6\right)\right.$$

$$\left. + \begin{pmatrix}0&-1/\sqrt{3}&1/\sqrt{3}\\1/\sqrt{3}&0&-1/\sqrt{3}\\-1/\sqrt{3}&1/\sqrt{3}&0\end{pmatrix}\operatorname{Sen}\frac{\pi}{6}\right)$$

$$= \begin{pmatrix}\sqrt{3}/2&0&0\\0&\sqrt{3}/2&0\\0&0&\sqrt{3}/2\end{pmatrix} + \frac{1}{3}\cdot\left(1-\frac{\sqrt{3}}{2}\right)\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix} + \frac{\sqrt{3}}{6}\begin{pmatrix}0&-1&1\\1&0&-1\\-1&1&0\end{pmatrix}$$

$$= \begin{pmatrix}\frac{1+\sqrt{3}}{3}&\frac{1-\sqrt{3}}{3}&\frac{1}{3}\\\frac{1}{3}&\frac{1+\sqrt{3}}{3}&\frac{1-\sqrt{3}}{3}\\\frac{1-\sqrt{3}}{3}&\frac{1}{3}&\frac{1+\sqrt{3}}{3}\end{pmatrix} = \frac{1}{3}\begin{pmatrix}1+\sqrt{3}&1-\sqrt{3}&1\\1&1+\sqrt{3}&1-\sqrt{3}\\1-\sqrt{3}&1&1+\sqrt{3}\end{pmatrix}$$

b) $\bullet\ T((0,0,0)) = M_{ROT}\times(0,0,0) = (0,0,0)$

$$\bullet\ T((1,0,0)) = \frac{1}{3}\begin{pmatrix}1+\sqrt{3}&1-\sqrt{3}&1\\1&1+\sqrt{3}&1-\sqrt{3}\\1-\sqrt{3}&1&1+\sqrt{3}\end{pmatrix}\begin{pmatrix}1\\0\\0\end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix}1+\sqrt{3}\\1\\1-\sqrt{3}\end{pmatrix}$$

$$\bullet\ T((1,1,0)) = \frac{1}{3}\begin{pmatrix}1+\sqrt{3}&1-\sqrt{3}&1\\1&1+\sqrt{3}&1-\sqrt{3}\\1-\sqrt{3}&1&1+\sqrt{3}\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix}2\\2+\sqrt{3}\\2-\sqrt{3}\end{pmatrix}$$

$$\bullet\ T((0,1,0)) = \frac{1}{3}\begin{pmatrix}1+\sqrt{3}&1-\sqrt{3}&1\\1&1+\sqrt{3}&1-\sqrt{3}\\1-\sqrt{3}&1&1+\sqrt{3}\end{pmatrix}\begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix}1-\sqrt{3}\\1+\sqrt{3}\\1\end{pmatrix}$$

$$\bullet\ T((0,1,1)) = \frac{1}{3}\begin{pmatrix}1+\sqrt{3}&1-\sqrt{3}&1\\1&1+\sqrt{3}&1-\sqrt{3}\\1-\sqrt{3}&1&1+\sqrt{3}\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}$$

$$= \frac{1}{3}\begin{pmatrix}2-\sqrt{3}\\2\\2+\sqrt{3}\end{pmatrix}$$

• $T((0,0,1)) = \frac{1}{3}\begin{pmatrix} 1+\sqrt{3} & 1-\sqrt{3} & 1 \\ 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$= \frac{1}{3}\begin{pmatrix} 1 \\ 1-\sqrt{3} \\ 1+\sqrt{3} \end{pmatrix}$

• $T((1,0,1)) = \frac{1}{3}\begin{pmatrix} 1+\sqrt{3} & 1-\sqrt{3} & 1 \\ 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

$= \frac{1}{3}\begin{pmatrix} 2+\sqrt{3} \\ 2-\sqrt{3} \\ 2 \end{pmatrix}$

• $T((1,1,1)) = \frac{1}{3}\begin{pmatrix} 1+\sqrt{3} & 1-\sqrt{3} & 1 \\ 1 & 1+\sqrt{3} & 1-\sqrt{3} \\ 1-\sqrt{3} & 1 & 1+\sqrt{3} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

$= \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

(3) La matriz de rotación viene dada por:

$$M_{ROT} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}\cos\frac{2\pi}{3} + \begin{pmatrix} 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}\left(1-\cos\frac{2\pi}{3}\right)$$

$$+ \begin{pmatrix} 0 & -1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{3} & 0 & -1/\sqrt{3} \\ -1/\sqrt{3} & 1/\sqrt{3} & 0 \end{pmatrix}\operatorname{Sen}\frac{2\pi}{3}$$

$$M_{ROT} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \times -\frac{1}{2} + \begin{pmatrix} 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \\ 1/2 & 1/2 & 1/2 \end{pmatrix}$$

$$+ \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1/2 & 1/2 \\ 1/2 & 0 & 1/2 \\ 1/2 & 1/2 & 0 \end{pmatrix} + \begin{pmatrix} 0 & -1/2 & 1/2 \\ 1/2 & 0 & -1/2 \\ -1/2 & 1/2 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$