① a) $u = \frac{\sqrt{3}}{3}(1,1,1) = (a_x, a_y, a_z)$, $\theta = -\frac{2\pi}{3}$ (PORQUE ESTÁ EN SENTIDO HORARIO)

Luego:

$$M_{ROT} = \cos\theta\, I + (1-\cos(\theta))\, u u^T + \operatorname{sen}\theta \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$= -\frac{1}{2}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \left(1-\left(-\frac{1}{2}\right)\right) u u^T - \frac{\sqrt{3}}{2}\begin{bmatrix} 0 & -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} & 0 & -\frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} + \frac{3}{2}\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \\ \frac{\sqrt{3}}{3} \end{bmatrix} - \begin{bmatrix} 0 & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & 0 & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

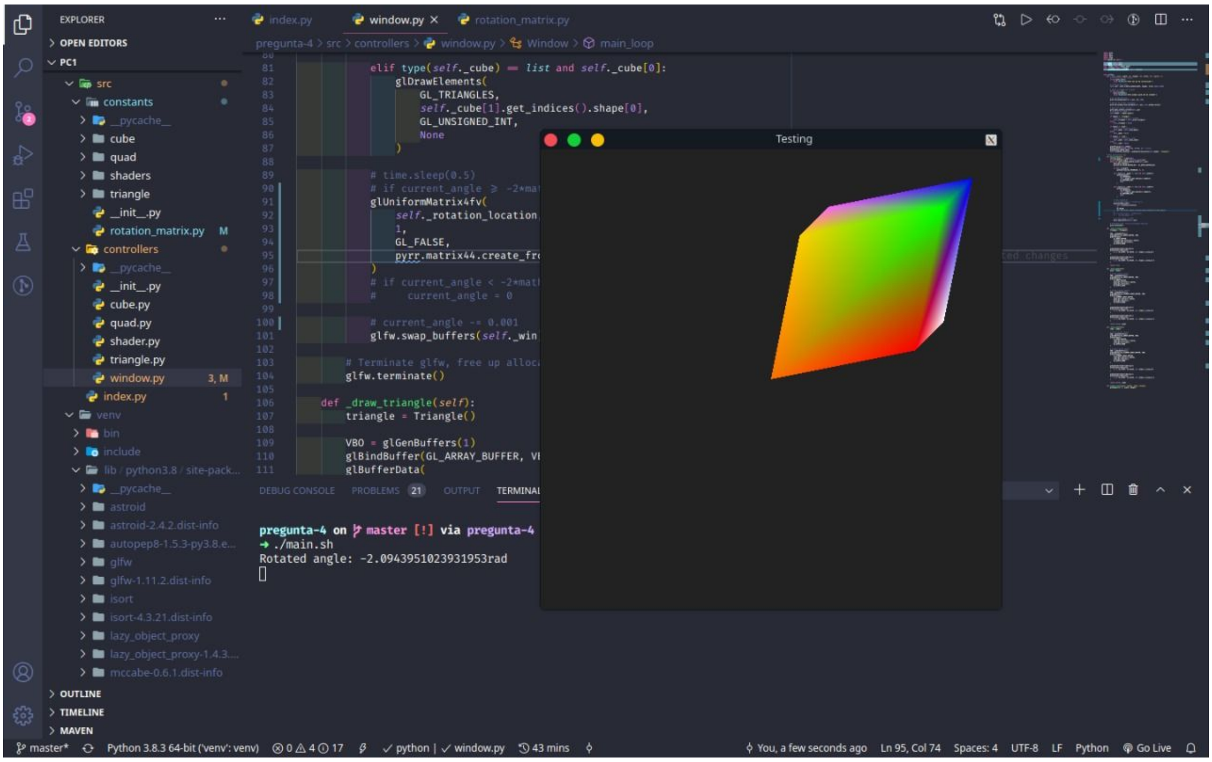
$$= \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -0.5 & 0.5 \\ 0.5 & 0 & -0.5 \\ -0.5 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.5 & 0 & 0 \\ 0 & -0.5 & 0 \\ 0 & 0 & -0.5 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} - \begin{bmatrix} 0 & -0.5 & 0.5 \\ 0.5 & 0 & -0.5 \\ -0.5 & 0.5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$$

- $T((0,0,0)) = M_{rot_x} [0 \ 0 \ 0]^T = (0, 0, 0)$
- $T((1,0,0)) = M_{rot_x} [1 \ 0 \ 0]^T = (0, 0, 1)$
- $T((1,1,0)) = M_{rot_x} [1 \ 1 \ 0]^T = (1, 0, 1)$
- $T((0,1,0)) = M_{rot_x} [0 \ 1 \ 0]^T = (1, 0, 0)$
- $T((0,1,1)) = M_{rot_x} [0 \ 1 \ 1]^T = (1, 1, 0)$
- $T((0,0,1)) = M_{rot_x} [0 \ 0 \ 1]^T = (0, 1, 0)$
- $T((1,0,1)) = M_{rot_x} [1 \ 0 \ 1]^T = (0, 1, 1)$
- $T((1,1,1)) = M_{rot_x} [1 \ 1 \ 1]^T = (1, 1, 1)$

b)



② a) $P_0 = (-2,1)$, $P_1 = (0,-4)$, $P_2 = (3,2)$, $P_3(5,0)$

Forma cúbica de Bézier:

$$P_B(t) = \begin{bmatrix} P_0 & P_1 & P_2 & P_3 \end{bmatrix} \begin{bmatrix} (1-t)^3 \\ 3t(1-t)^2 \\ 3t^2(1-t) \\ t^3 \end{bmatrix}$$

$$P_B(t) = P_0(1-t)^3 + P_1\,3t(1-t)^2 + P_2\,3t^2(1-t) + P_3 t^3$$

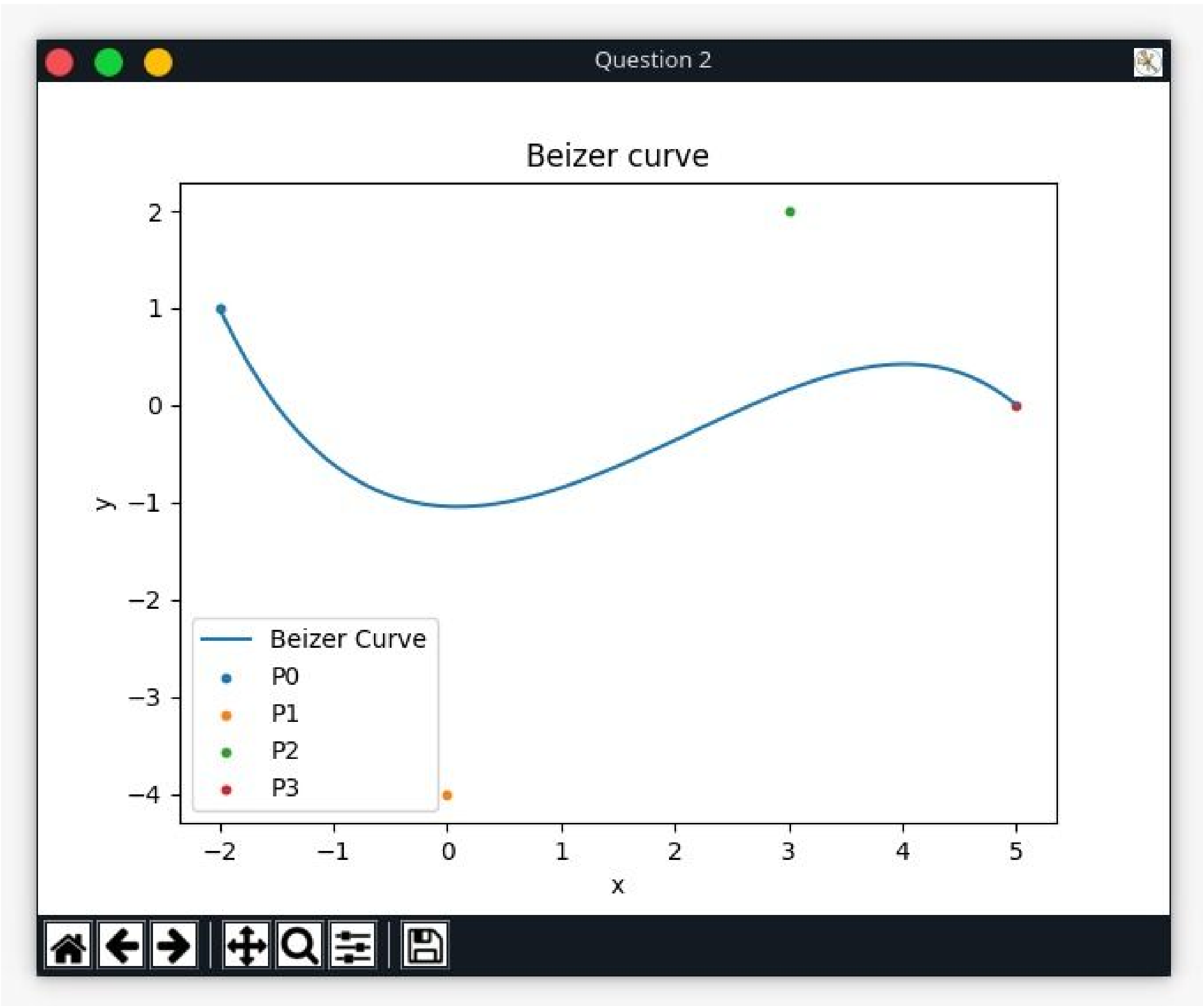
$$\Rightarrow X(t) = -2(1-t)^3 + 3.3t^2(1-t) + 5.t^3$$

$$Y(t) = (1-t)^3 + (-4).3t^2(1-t) + 2.3t^2(1-t)$$

$$\Rightarrow X(t) = -2t^3 + 3t^2 + 6t - 2$$

$$Y(t) = -19t^3 + 33t^2 - 15t + 1$$

b)



c) $P(\tau) = N_{(0,2)}(\tau)\begin{bmatrix}-2\\1\end{bmatrix} + N_{(1,2)}(\tau)\begin{bmatrix}0\\-4\end{bmatrix} + N_{(2,2)}(\tau)\begin{bmatrix}3\\2\end{bmatrix} + N_{(3,2)}(\tau)\begin{bmatrix}5\\0\end{bmatrix}$

PARA EL PRIMER SEGMENTO $2 \leq \tau < 3$

$$P(\tau) = \left(\frac{1}{2} - (\tau-2) + \frac{1}{2}(\tau-2)^2\right)\begin{bmatrix}-2\\1\end{bmatrix} + \left(\frac{1}{2} + (\tau-2) - (\tau-2)^2\right)\begin{bmatrix}0\\-4\end{bmatrix}$$

$$+ \frac{1}{2}(\tau-2)^2\begin{bmatrix}3\\2\end{bmatrix} + 0\cdot\begin{bmatrix}5\\0\end{bmatrix}$$

$$\Rightarrow P(\tau) = \begin{bmatrix}0.5\tau^2 - 3\\ -5.5\tau^2 - 27\tau + 30.5\end{bmatrix}$$

PARA EL SEGMENTO $3 \leq \tau < 4$

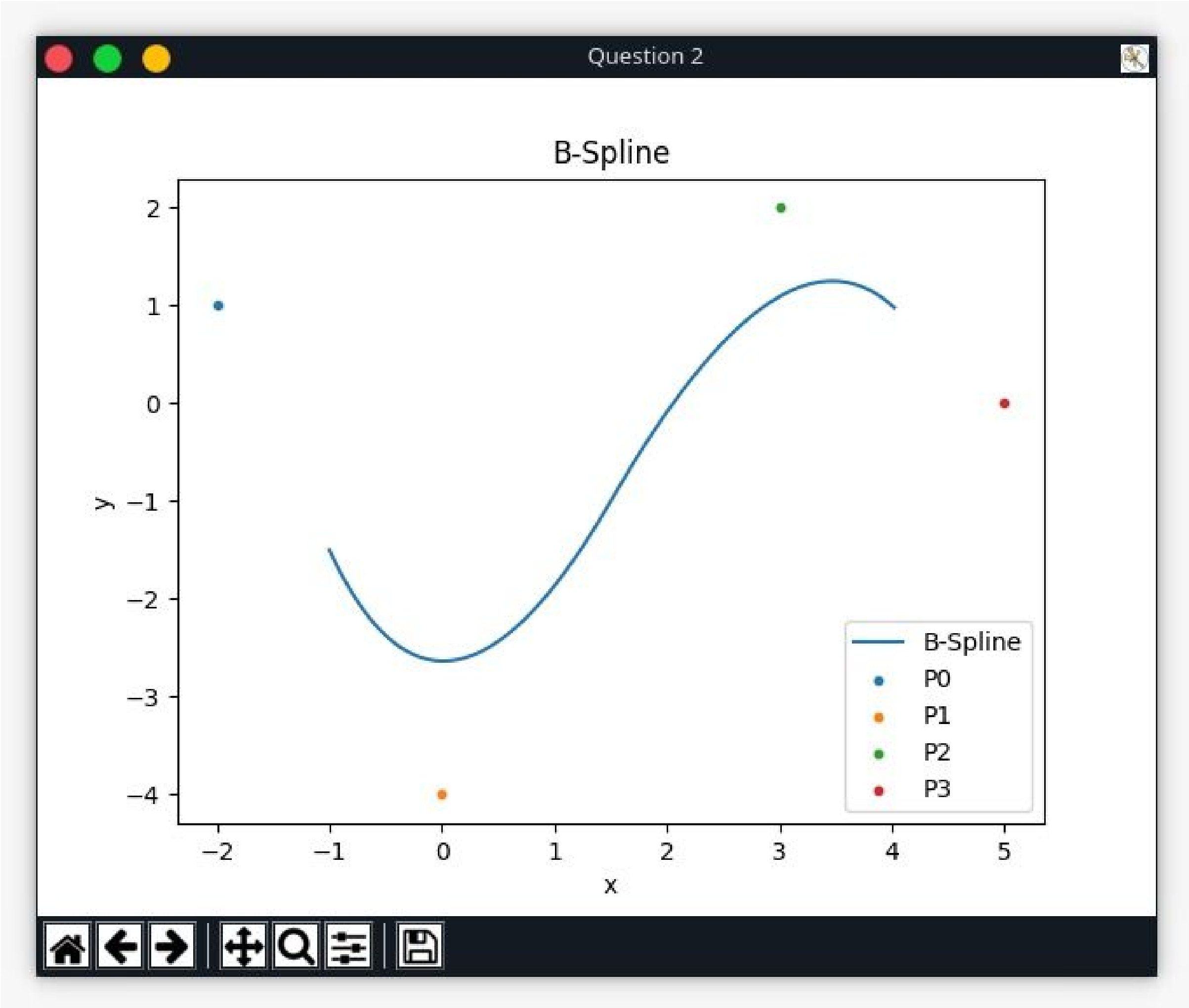
$$P(\tau) = 0\cdot\begin{bmatrix}-2\\1\end{bmatrix} + \left(\frac{1}{2} - (\tau-3) + \frac{1}{2}(\tau-3)^2\right)\begin{bmatrix}0\\-4\end{bmatrix} + \left(\frac{1}{2} + (\tau-3) - (\tau-3)^2\right)\begin{bmatrix}3\\2\end{bmatrix}$$

$$+ \frac{1}{2}(\tau-3)^2\begin{bmatrix}5\\0\end{bmatrix}$$

$$\Rightarrow P(\tau) = \begin{bmatrix}-0.5\tau^2 + 6\tau - 12\\ -4\tau^2 + 30\tau - 55\end{bmatrix}$$

$$\therefore\ P(\tau) = \begin{cases}\begin{bmatrix}0.5\tau^2 - 3\\ 5.5\tau^2 - 27\tau + 30.5\end{bmatrix}, & \tau \in [2,3\rangle \\ \begin{bmatrix}-0.5\tau^2 + 6\tau - 12\\ -4\tau^2 + 30\tau - 55\end{bmatrix}, & \tau \in [3,4\rangle\end{cases}$$

d)



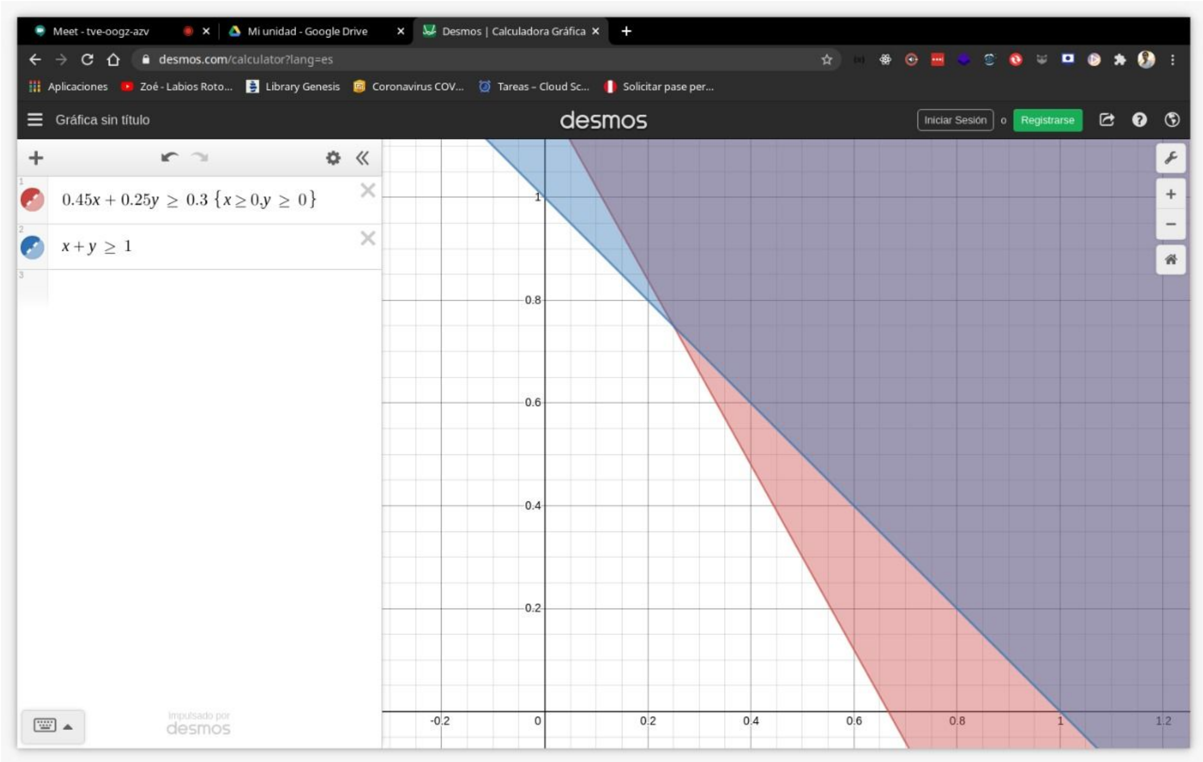
(c)

a) $\min\limits_{x_1, x_2} \; 4x_1 + 5x_2$

s.t. $0.45x_1 + 0.25x_2 \geq 0.3$

$x + y \geq 1$

$x, y \geq 0$



b) Los puntos factibles son:

(1,0) , (0.25, 0.75) y (0,1)