① $P_0 = (-3,0)$ | $P_1 = (-1,4)$ | $P_2 = (2,3)$ | $P_3 = (4,1)$

$$\alpha_0(t) = \frac{t-1}{0-1} \cdot \frac{t-2}{0-2} \cdot \frac{t-3}{0-3}$$

$$\alpha_1(t) = \frac{t-0}{1-0} \cdot \frac{t-2}{1-2} \cdot \frac{t-3}{1-3} =$$

$$\alpha_2(t) = \frac{t-0}{2-0} \cdot \frac{t-1}{2-1} \cdot \frac{t-3}{2-3}$$

$$\alpha_3(t) = \frac{t-0}{3-0} \cdot \frac{t-1}{3-1} \cdot \frac{t-2}{3-2}$$

$$\Rightarrow \alpha_0(t) = -(t-1) \cdot \frac{-(t-2)}{2} \cdot \frac{-(t-3)}{3} = -\frac{(t-1)(t-2)(t-3)}{6}$$

$$\alpha_1(t) = t \cdot -1(t-2) \cdot \frac{-(t-3)}{2} = \frac{t(t-2)(t-3)}{2}$$

$$\alpha_2(t) = \frac{t}{2} \cdot (t-1) \cdot -(t-3) = -\frac{t(t-1)(t-3)}{2}$$

$$\alpha_3(t) = \frac{t}{3} \cdot \frac{t-1}{2} \cdot (t-2) = \frac{t(t-1)(t-2)}{6}$$

$$\therefore \alpha_0(t) = -\frac{t^3 - 6t^2 + 11t - 6}{6}$$

$$\alpha_1(t) = \frac{t^3 - 5t^2 + 6t}{2}$$

$$\alpha_2(t) = -\frac{t^3 - 4t^2 + 3t}{2}$$

$$\alpha_3(t) = \frac{t^3 - 3t^2 + 2t}{6}$$

FINALMENTE:

$$P(t) = \sum_{i=0}^{m} \alpha_i(t) P_i = \alpha_0(t)P_0 + \alpha_1(t)P_1 + \alpha_2(t)P_2 + \alpha_3(t)P_3$$

$$= -\frac{(t^3 - 6t^2 + 11t - 6)}{6} \cdot (-3,0) + \frac{(t^3 - 5t^2 + 6t)}{2} \cdot (-1,4)$$

$$- \frac{(t^3 - 4t^2 + 3t)}{2} \cdot (2,3) + \frac{(t^3 - 3t^2 + 2t)}{6} \cdot (4,1)$$

$$= \left(-\frac{t^3}{3} + \frac{3t^2}{2} + \frac{5t}{6} - 18,\ \frac{2}{3}t^3 - \frac{9}{2}t^2 + \frac{47t}{6}\right)$$

②

a) INTERPOLAMOS DE P_0 A P_3 CON PUNTOS DE CONTROL P_1 Y P_2

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3 & -1 & 2 & 4 \\ 0 & 4 & 3 & 1 \end{bmatrix} \begin{bmatrix} (1-t)^3 \\ 3t(1-t)^2 \\ 3t^2(1-t) \\ t^3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -3(1-t)^3 - 3t(1-t)^2 + 6t^2(1-t) + 4t^3 \\ 12t(1-t)^2 + 9t^2(1-t) + t^3 \end{bmatrix}$$

$$= \begin{bmatrix} -2t^3 + 3t^2 + 6t - 3 \\ 4t^3 - 15t^2 + 12t \end{bmatrix}$$

b)

$$\vec{m} = \frac{d\begin{bmatrix} x \\ y \end{bmatrix}}{dt} = \begin{bmatrix} -6t^2 + 6t + 6 \\ 16t^2 - 30t + 12 \end{bmatrix} \Rightarrow \vec{m}(0) = \begin{bmatrix} 6 \\ 12 \end{bmatrix}$$

$$\vec{m}(1) = \begin{bmatrix} 6 \\ -6 \end{bmatrix}$$

$\therefore m(0) = 2$ y $m(1) = -1$

c) $P_0 = (-3,0)$, $P_1 = (-1,4)$, $P_2 = (2,3)$, $P_3 = (4,1)$

$$\Rightarrow m_{P_0P_1} = \frac{4-0}{-1-(-3)} = \frac{4}{2} = 2$$

$$m_{P_2P_3} = \frac{1-3}{4-2} = -1 \qquad \text{l.q.q.d.}$$

(3)

a) El primer segmento del B-spline cúbico va de $3 \leq t < 4$:

$$P(t) = \frac{1}{6}\left(-(t-3)^3 + 3(t-3)^2 - 3(t-3) + 1\right) P_0$$
$$+ \frac{1}{6}\left(3(t-3)^3 - 6(t-3)^2 + 4\right) P_1$$
$$+ \frac{1}{6}\left(-3(t-3)^3 + 3(t-3)^2 + 3(t-3) + 1\right) P_2$$
$$+ \frac{1}{6}(t-3)^3 P_3 = P_1(t)$$

el segundo segmento: $(4 \leq t < 5)$

$$P(t) = \frac{1}{6}\left(-(t-4)^3 + 3(t-4)^2 - 3(t-4) + 1\right) P_1$$
$$+ \frac{1}{6}\left(3(t-4)^3 - 6(t-4)^2 + 4\right) P_2$$
$$+ \frac{1}{6}\left(-3(t-4)^3 + 3(t-4)^2 + 3(t-4) + 1\right) P_3$$
$$+ \frac{1}{6}(t-4)^3 P_4 = P_2(t)$$

Luego:

$$P(t) = \begin{cases} P_1(t), & 3 \leq t < 4 \\ P_2(t), & 4 \leq t < 5 \end{cases}$$

$$P_1(t) = -\frac{-t^3 + 12t^2 - 48t + 64}{6} \cdot \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$
$$+ \frac{3t^3 - 33t^2 + 117t - 131}{6} \cdot \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$
$$+ \frac{-3t^3 + 30t^2 - 96t + 100}{6} \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$
$$+ \frac{t^3 - 9t^2 + 27 - 27}{6} \cdot \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$