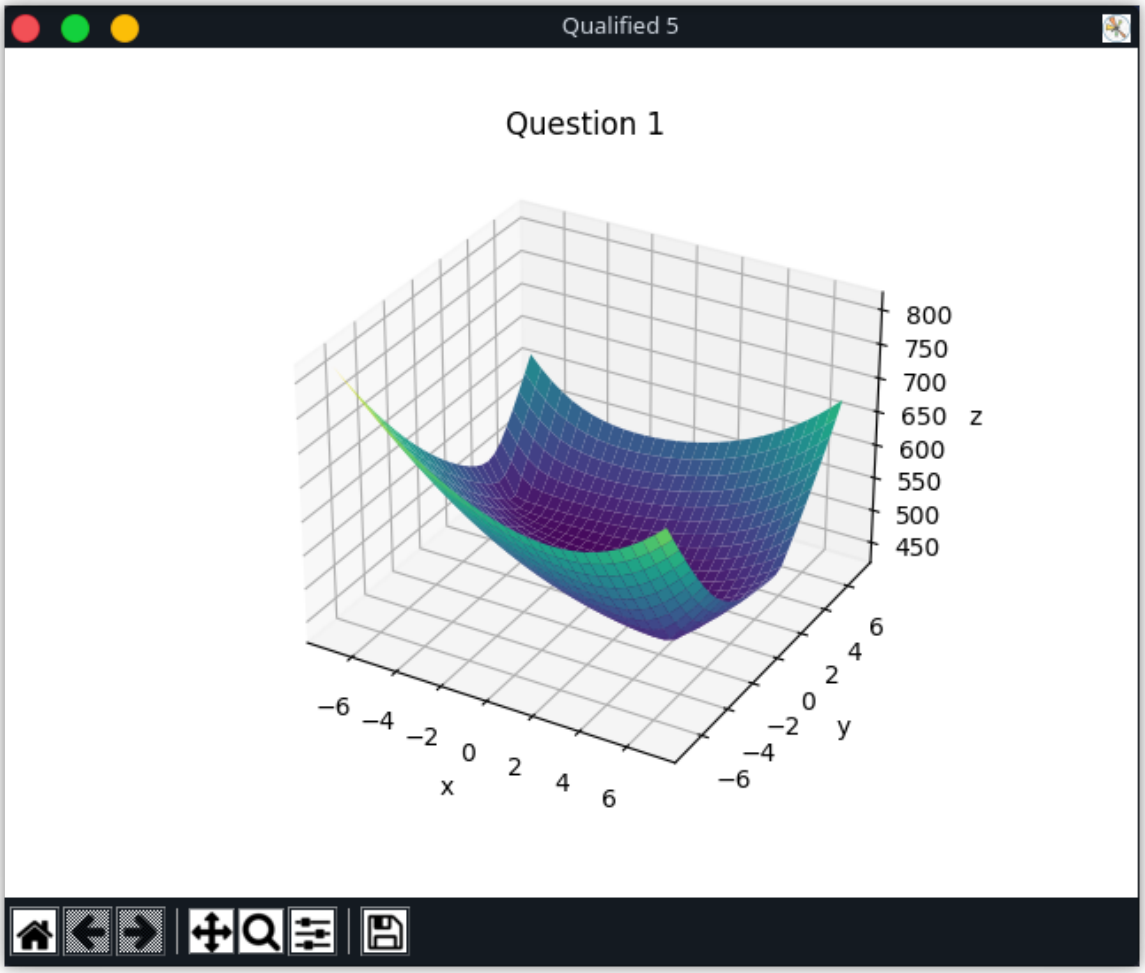
1.

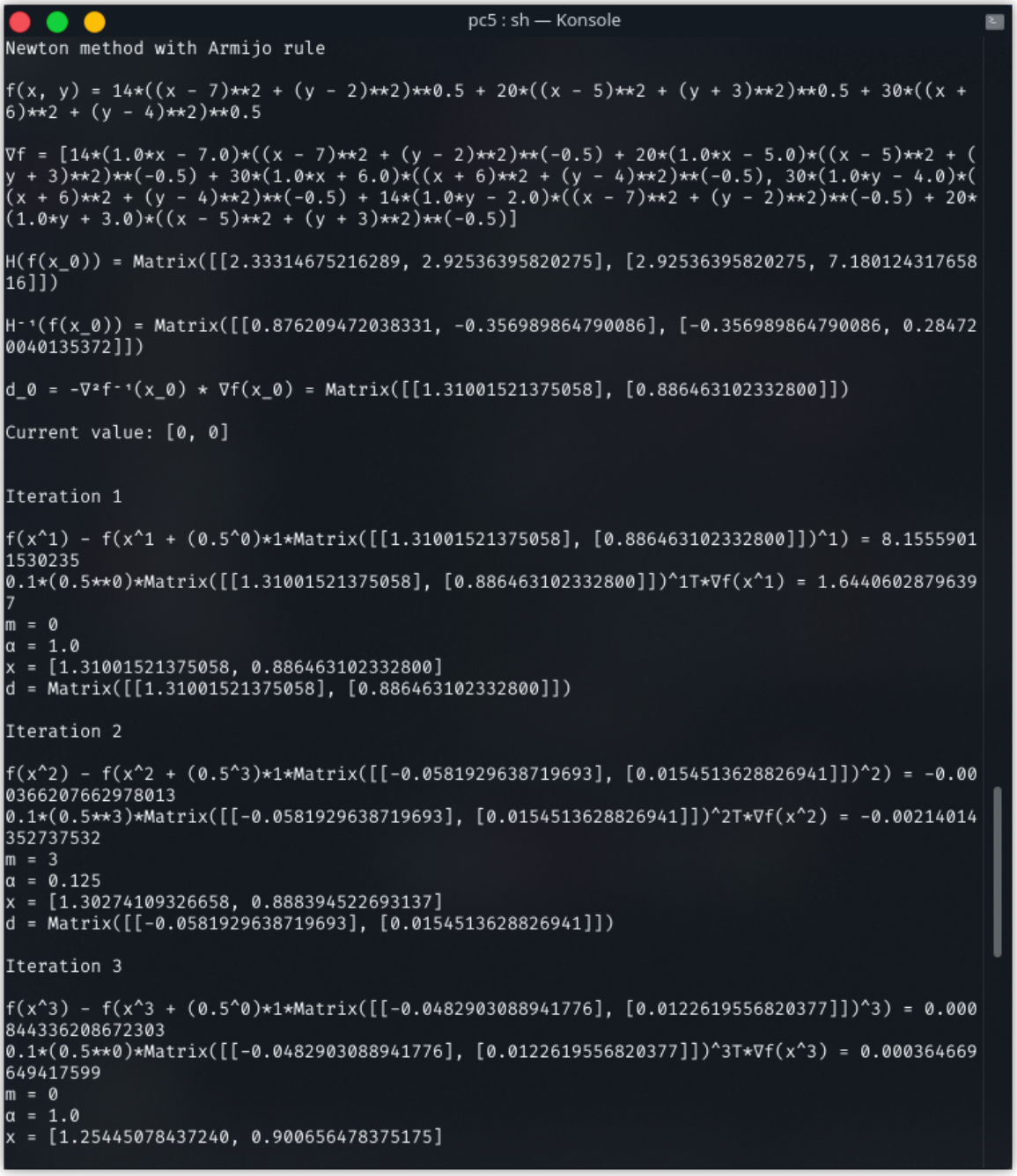

Qualified 5
Question 1
800
750
700
650
600
550
500
450
z
−6
−4
−2
0
2
4
6
x
6
4
2
0
−2
−4
−6
y

2.

a)

```
Steepest descent search

f(x, y) = 14*((x - 7)**2 + (y - 2)**2)**0.5 + 20*((x - 5)**2 + (y + 3)**2)**0.5 + 30*(
(x + 6)**2 + (y - 4)**2)**0.5

∇f = [14*(1.0*x - 7.0)*((x - 7)**2 + (y - 2)**2)**(-0.5) + 20*(1.0*x - 5.0)*((x - 5)**
2 + (y + 3)**2)**(-0.5) + 30*(1.0*x + 6.0)*((x + 6)**2 + (y - 4)**2)**(-0.5), 30*(1.0*
y - 4.0)*((x + 6)**2 + (y - 4)**2)**(-0.5) + 14*(1.0*y - 2.0)*((x - 7)**2 + (y - 2)**2
)**(-0.5) + 20*(1.0*y + 3.0)*((x - 5)**2 + (y + 3)**2)**(-0.5)]

H(f) = Matrix([[20*(5.0 - 1.0*x)*(1.0*x - 5.0)*((x - 5)**2 + (y + 3)**2)**(-1.5) + 14*
(7.0 - 1.0*x)*(1.0*x - 7.0)*((x - 7)**2 + (y - 2)**2)**(-1.5) + 30*(-1.0*x - 6.0)*(1.0
*x + 6.0)*((x + 6)**2 + (y - 4)**2)**(-1.5) + 14.0*((x - 7)**2 + (y - 2)**2)**(-0.5) +
 20.0*((x - 5)**2 + (y + 3)**2)**(-0.5) + 30.0*((x + 6)**2 + (y - 4)**2)**(-0.5), 20*(
5.0 - 1.0*x)*(1.0*y + 3.0)*((x - 5)**2 + (y + 3)**2)**(-1.5) + 14*(7.0 - 1.0*x)*(1.0*y
 - 2.0)*((x - 7)**2 + (y - 2)**2)**(-1.5) + 30*(-1.0*x - 6.0)*(1.0*y - 4.0)*((x + 6)**
2 + (y - 4)**2)**(-1.5)], [14*(2.0 - 1.0*y)*(1.0*x - 7.0)*((x - 7)**2 + (y - 2)**2)**(
-1.5) + 30*(4.0 - 1.0*y)*(1.0*x + 6.0)*((x + 6)**2 + (y - 4)**2)**(-1.5) + 20*(1.0*x -
 5.0)*(-1.0*y - 3.0)*((x - 5)**2 + (y + 3)**2)**(-1.5), 14*(2.0 - 1.0*y)*(1.0*y - 2.0)
*((x - 7)**2 + (y - 2)**2)**(-1.5) + 30*(4.0 - 1.0*y)*(1.0*y - 4.0)*((x + 6)**2 + (y -
 4)**2)**(-1.5) + 20*(-1.0*y - 3.0)*(1.0*y + 3.0)*((x - 5)**2 + (y + 3)**2)**(-1.5) +
14.0*((x - 7)**2 + (y - 2)**2)**(-0.5) + 20.0*((x - 5)**2 + (y + 3)**2)**(-0.5) + 30.0
*((x + 6)**2 + (y - 4)**2)**(-0.5)]])

d_0 = -∇f(x_0) = Matrix([[5.64968495108710], [10.1971865687696]])

Iteration 1:

d = Matrix([[5.64968495108710], [10.1971865687696]])
α = 0.115827110161577
x = [0.654386681207772, 1.18111065203904]

Iteration 2:

d = Matrix([[0.673460663951956], [-0.373126504317509]])
α = 0.852384370453681
x = [1.22843402527578, 0.863063451556777]

Iteration 3:

d = Matrix([[0.187415772774972], [0.338269057029242]])
α = 0.116869310826961
x = [1.25033717747809, 0.902596723125870]

Iteration 4:

d = Matrix([[0.0205405869951134], [-0.0113803787397793]])
α = 0.743129525261520
x = [1.26560149414036, 0.894139627675681]

Iteration 5:

d = Matrix([[0.00438606830408780], [0.00791646918143548]])
α = 0.116955268445853
x = [1.26611446793629, 0.895065500453939]
```

3.
b)


pc5 : sh — Konsole
Newton method with Armijo rule
f(x, y) = 14*((x - 7)**2 + (y - 2)**2)**0.5 + 20*((x - 5)**2 + (y + 3)**2)**0.5 + 30*((x + 6)**2 + (y - 4)**2)**0.5
∇f = [14*(1.0*x - 7.0)*((x - 7)**2 + (y - 2)**2)**(-0.5) + 20*(1.0*x - 5.0)*((x - 5)**2 + (y + 3)**2)**(-0.5) + 30*(1.0*x + 6.0)*((x + 6)**2 + (y - 4)**2)**(-0.5), 30*(1.0*y - 4.0)*((x + 6)**2 + (y - 4)**2)**(-0.5) + 14*(1.0*y - 2.0)*((x - 7)**2 + (y - 2)**2)**(-0.5) + 20*(1.0*y + 3.0)*((x - 5)**2 + (y + 3)**2)**(-0.5)]
H(f(x_0)) = Matrix([[2.33314675216289, 2.92536395820275], [2.92536395820275, 7.180124317658 16]])
H⁻¹(f(x_0)) = Matrix([[0.876209472038331, -0.356989864790086], [-0.356989864790086, 0.28472 0040135372]])
d_0 = -∇²f⁻¹(x_0) * ∇f(x_0) = Matrix([[1.31001521375058], [0.886463102332800]])
Current value: [0, 0]
Iteration 1
f(x^1) - f(x^1 + (0.5^0)*1*Matrix([[1.31001521375058], [0.8864631023328 00]])^1) = 8.1555901 1530235
0.1*(0.5**0)*Matrix([[1.31001521375058], [0.886463102332800]])^1T*∇f(x^1) = 1.6440602879639 7
m = 0
α = 1.0
x = [1.31001521375058, 0.886463102332800]
d = Matrix([[1.31001521375058], [0.886463102332800]])
Iteration 2
f(x^2) - f(x^2 + (0.5^3)*1*Matrix([[-0.0581929638719693], [0.0154513628826941]])^2) = -0.00 0366207662978013
0.1*(0.5**3)*Matrix([[-0.0581929638719693], [0.0154513628826941]])^2T*∇f(x^2) = -0.00214014 352737532
m = 3
α = 0.125
x = [1.30274109326658, 0.888394522693137]
d = Matrix([[-0.0581929638719693], [0.0154513628826941]])
Iteration 3
f(x^3) - f(x^3 + (0.5^0)*1*Matrix([[-0.0482903088941776], [0.0122619556820377]])^3) = 0.000 844336208672303
0.1*(0.5**0)*Matrix([[-0.0482903088941776], [0.0122619556820377]])^3T*∇f(x^3) = 0.000364669 649417599
m = 0
α = 1.0
x = [1.25445078437240, 0.900656478375175]

pc5 : sh — Konsole

```
Iteration 4

f(x^4) - f(x^4 + (0.5^2)*1*Matrix([[0.0164631699767993], [-0.0079128979920639
6]])^4) = -1.28662804854685E-8
0.1*(0.5**2)*Matrix([[0.0164631699767993], [-0.0079128979920639
6]])^4T*∇f(x^4) = -0.000021079363437879
7
m = 2
α = 0.25
x = [1.25856657686660, 0.898678253877159]
d = Matrix([[0.0164631699767993], [-0.0079128979920639
6]])

Iteration 5

f(x^5) - f(x^5 + (0.5^1)*1*Matrix([[0.0108448637002046], [-0.005206301810111
42]])^5) = 0.000050537833123021
0.1*(0.5**1)*Matrix([[0.0108448637002046], [-0.005206301810111
42]])^5T*∇f(x^5) = 0.0000105291774869670
m = 1
α = 0.5
x = [1.26398900871671, 0.896075102972103]
d = Matrix([[0.0108448637002046], [-0.005206301810111
42]])

Iteration 6

f(x^6) - f(x^6 + (0.5^1072)*1*Matrix([[0.00342552574117356], [-0.00163621010
025038]])^6) = 0
0.1*(0.5**1072)*Matrix([[0.00342552574117356], [-0.00163621010
025038]])^6T*∇f(x^6) = 0
m = 1072
α = 2e-323
x = [1.26398900871671, 0.896075102972103]
d = Matrix([[0.00342552574117356], [-0.00163621010
025038]])

Iteration 7

f(x^7) - f(x^7 + (0.5^0)*1*Matrix([[0.00342552574117356], [-0.00163621010
025038]])^7) = 0.00000433901738006170
0.1*(0.5**0)*Matrix([[0.00342552574117356], [-0.00163621010
025038]])^7T*∇f(x^7) = 0.000001380755617409
85
m = 0
α = 1.0
x = [1.26741453445788, 0.894438892871853]
d = Matrix([[0.00342552574117356], [-0.00163621010
025038]])

Iteration 8

f(x^8) - f(x^8 + (0.5^2)*1*Matrix([[-0.00127062158727062], [0.0006146008924
99585]])^8) = 0
0.1*(0.5**2)*Matrix([[-0.00127062158727062], [0.0006146008924
99585]])^8T*∇f(x^8) = -1.28371927101952E-7
m = 2
α = 0.25
x = [1.26709687906106, 0.894592543094978]
d = Matrix([[-0.00127062158727062], [0.0006146008924
99585]])
```