School of Computer Science
Faculty of Science
National University of Engineering

Test 4

Topics: convex sets; convex functions; optimality conditions
Subject: Computational Mathematics **Period: 2020.1**

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1. Let $a \in \mathbb{R}^{\mathrm{n}}$, let $d \in \mathbb{R}^{\mathrm{n}} \setminus \{0\}$ and let $c \in \mathbb{R}$. Show that the following sets are convex:

   (a) (1 pt.) $\mathcal{L}(a,d) := \{a + td \ ; \ t \in \mathbb{R}\}$;

   (b) (2 pts.) $\mathcal{H}(d,c) := \{v \in \mathbb{R}^n \ ; \ \langle v, d \rangle = c\}$;

   (c) (2 pts.) $B(a,r) := \{v \in \mathbb{R}^n \ ; \ |v - a| < r\}$.

2. Let $f : \mathbb{R} \to \mathbb{R}$ be such that $f(x) = \ln(1 + e^x)$.

   (a) (2 pts.) Determine whether f is convex. The assertion must be proven.

   (b) (2 pts.) Determine whether f has a global minimum. The assertion must be proven.

   (c) (1 pt.) Sketch the graph of f.

3. Let $C := (-\infty, 1]$ and consider the function $f : C \to \mathbb{R}$ given by

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1, \\ 2 & \text{for } x = 1. \end{cases}$$

   (a) (2 pts.) Determine whether f is convex. The assertion must be proven.

(b) (2 pts.) Determine whether f has a unique global minimum over C. The assertion must be proven.

(c) (1 pt.) Sketch the graph of f.

4. Let $C := (0, \infty)$ and let $f : C \to \mathbb{R}$ be such that $f(x) = x + \frac{4}{x}$.

   (a) (2 pts.) Determine whether f is convex. The assertion must be proven.

   (b) (2 pts.) Determine whether f has a unique global minimum over C. The assertion must be proven.

   (c) (1 pt.) Sketch the graph of f.

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