School of Computer Science
Faculty of Science
National University of Engineering

Test 1

Subject: Computational Mathematics **Period: 2020.1**

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1. The 2×2 matrix M can be thought of as having two vectors: $u := (a, b)$ and $v := (c, d)$ forming the columns. If the vectors are unit vectors and they are perpendicular, then the matrix is called **orthogonal**.
   (a) (1 pt.) Show that we can set $a = \pm\cos(\theta)$ and $b = \pm\sin(\theta)$.
   (b) (1 pt.) Show further that $d = a$ or $d = -a$, which implies $c = -b$ or $c = b$.
   (c) (2 pts.)Explain why rotation and reflection matrices are examples of orthogonal matrices and that products of these two types are also orthogonal.
   (d) (1 pt.) Intuitively, we know that rotations and reflections should not change the area of triangles and, indeed, (show that) the determinant of an orthogonal matrix is ± 1.
2. The unit cube with vertices (a, b, c), where each component is 0 or 1, is rotated by $\pi/6$ counterclockwise around the diagonal through $P_0 := (0, 0, 0)$ and $(1, 1, 1)$.
   (a) (3 pts.) Find the transformation matrix.
   (b) (2 pts.) Find the coordinates of the transformed cube.

   (Hint: read pages 115-117 of [1].)
3. (5 pts.) Show that a rotation of $2\pi/3$ clockwise around the line from $P_0 := (0, 0, 0)$ to $(1, 1, 1)$ is the product of two rotations around coordinate axes.
4. Write a program to present
   (a) (3 pts.) the unit cube of question two;
   (b) (2 pts.) the transformed cube of question two.

   (Hint: read pages 115-117 of [1].)

June 17, 2020

Bibliography

[1] Janke, S. J. *Mathematical Structures for Computer Graphics.* Wiley, 2015.