School of Computer Science
Faculty of Science
National University of Engineering

Midterm Exam

Topics: transformations; curves; linear programming - Simplex method
Subject: Computational Mathematics **Period: 2020.1**

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1. Let U be the unit cube with vertices (a, b, c), where each component is 0 or 1.
   (a) (2 pts.) Find the rotation of U of $2\pi/3$ clockwise around the line from $P_0 := (0, 0, 0)$ to $(1, 1, 1)$.
   (b) (1 pt.) Print the transformed cube of the item above.
2. Let $P_0 := (-2, 1)$, $P_1 := (0, -4)$, $P_2 := (3, 2)$ and $P_3 := (5, 0)$.
   (a) (2 pts.) Find the parametric description $P(t)$ of the cubic Bézier curve with control points: P_0, P_1, P_2 and P_3.
   (b) (1 pt.) Print the Bézier curve above with its control points.
   (c) (2 pts.) Find the parametric description $P(t)$ of the uniform quadratic B-spline using control points: P_0, P_1, P_2 and P_3.
   (d) (1 pt.) Print the B-spline above with its control points.
3. In a Python program implement:
   (a) (2 pts.) A function that receives a 'tableau', a basic `b` and a non-basic element `n`, and returns the resulting 'tableau' of conducting the process: `pivot(b,n)`.
   (b) (2 pts.) A function called `iterate` that receives a 'tableau' with $\tilde{b} \geq 0$, and returns the final 'tableau' of conducting the iteration process of the Simplex method.
   (c) (1 pt.) Pass Table 1 to the `iterate` function above and print the final 'tableau'.

	1	x_1	x_3
z	4.30	-0.2	6
x_2	1.05	-1.2	1
x_4	0.05	-0.2	1

Table 1: 'Tableau' for iteration process of Simplex method.

4. Considering the following linear optimization problem:

   "Jose builds electrical cable using two types of metallic alloys. Alloy 1 is 55% aluminum and 45% copper, while alloy 2 is 75% aluminum and 25% copper. Market prices for alloys 1 and 2 are \$5 and \$4 per ton, respectively. Formulate a linear optimization problem to determine the cost-minimizing quantities of the two alloys that Jose should use to produce 1 ton of cable that is at least 30% copper."

   do the following:

   (a) (2 pts.) Formulate in the general form of a LPP.

   (b) (2 pts.) Conduct a geometric analysis: find (sketch) the feasible region, find the level sets, find the optimal solution and optimal value.

   (c) (2 pts.) Conduct the Simplex method with $K = 6$ for the regularization step.

July 29, 2020