Computacional Mathematics Entrance Test

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Problem 1

Solution:

- $x(u, v) = u \cos v, \;\; y(u, v) = u \sin v$
- $f(x, y) = e^{x^2+y^2}$

Then:

$$\begin{aligned}
\int_B f(x, y)dxdy &= \int_A f(x(u, v), y(u, v))|JF(u, v)|dudv \\
&= \int_A e^{u^2\cos^2 v+u^2\sin^2 v} \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix} dudv \\
&= \int_A e^{u^2\cos^2 u+u^2\sin^2 u} \begin{vmatrix} \frac{\partial(u\cos v)}{\partial u} & \frac{\partial(u\cos v)}{\partial v} \\ \frac{\partial(u\sin v)}{\partial u} & \frac{\partial(u\sin v)}{\partial v} \end{vmatrix} dudv \\
&= \int_A e^{u^2\cos^2 u+u^2\sin^2 u} \begin{vmatrix} \frac{\partial(u\cos v)}{\partial u} & \frac{\partial(u\cos v)}{\partial v} \\ \frac{\partial(u\sin v)}{\partial u} & \frac{\partial(u\sin v)}{\partial v} \end{vmatrix} dudv \\
&= \int_A e^{u^2} \begin{vmatrix} \cos v & -u\sin v \\ \sin v & u\cos v \end{vmatrix} dudv \\
&= \int_A e^{u^2}(u\cos^2 v + u\sin^2 v)dudv \\
&= \int_A e^{u^2} ududv \\
&= \frac{1}{2}\int_{\pi/6}^{\pi/3} dv \int_2^3 e^{u^2} du^2 \\
&= \frac{\pi}{12}(e^9 - e^4)
\end{aligned}$$

Problem 2

a) $P(A \cap B) = 0$, due the fact that: $A \cap B = \emptyset$.

So then, if A and B are independent:

$$P(A \cap B) = P(A)P(B)$$
$$0 = P(A)P(B)$$
$$\implies P(A) = 0 \vee P(B) = 0$$

b) $P((A \cup B)/C) = \frac{P((A \cup B) \cap C)}{P(C)}$
$$= \frac{P((A \cap C) \cup (B \cap C))}{P(C)}$$
$$= \frac{P((A \cap C) + P(B \cap C) - P(A \cap B \cap C))}{P(C)}$$
$$= P(A/C) + P(B/C) - P((A \cap B)/C)$$

c) $P(\bigcup_{i=1}^{\infty} A_i/C) = \frac{P((A_1 \cup A_2 \cup \cdots \cup A_n)/C)}{P(C)}$
$$= \frac{P((A_1 \cap C) \cup (A_2 \cap C) \cup \cdots \cup (A_n \cap C))}{P(C)}$$
$$= \frac{P(A_1 \cap C) + P(A_2 \cap C) + \cdots + P(A_n \cap C)}{P(C)}$$
$$= \frac{\sum_{j=1}^{\infty} P(A_j \cap C)}{P(C)}$$
$$= \sum_{j=1}^{\infty} P(A_j/C)$$

d) $P(A) = P((A \cap B) \cup (A \cap B^c))$
$$= P(A \cap B) + P(A \cap B^c)$$
$$= P(A/B)P(B) + P(A/B^c)P(B^c)$$