

CH 1.7

2, 4, 6, 14, 16, 18a

2) Use a direct proof to show that the sum of two even integers is even.

-n is even if there exists an integer k such that  $n = 2k$

-we have two even integers  $k_1$  and  $k_2$

- $n_1 + n_2$  is then equal to  $2(k_1 + k_2)$

-thus by the definition of an even number,  $n = 2k$ , we can conclude that  $n_1 + n_2$ , when both numbers are even, will return an even number.

4) Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

-n is even if there exists an integer k such that  $n = 2k$

-when negative,  $n = -2k$  or  $2(-k)$

-regardless of the integer substituted for integer k, it is true that it will always return an even number.

6) Use a direct proof to show that the product of two odd numbers is odd.

-n is odd if there exists an integer k such that  $n = 2k + 1$

-we have two odd integers  $k_1$  and  $k_2$

- $n_1 * n_2$  is then equal to  $2(2k_1k_2 + k_1 + k_2) + 1$

-thus by the definition of odd numbers,  $n = 2k + 1$ , we can conclude that  $n_1 * n_2$ , when both numbers are odd, will return an odd number.

14) Prove that if x is rational and  $x \neq 0$ , then  $1/x$  is rational.

-A real number is rational if exists integers p and q with  $q \neq 0$  such that  $r = p/q$ .

-because  $x \neq 0$ , and x is a rational number, then  $1/x$  would return a rational number.

-thus by the definition of rational numbers, we can conclude that  $1/x$  is rational.

16) Prove that if m and n are integers and mn is even, then m is even or n is even.

-assume "if m and n are integers and mn is even, then m is even or n is even" is false.

-so assume that m or n is odd

-n is odd if there exists an integer k such that  $n = 2k + 1$

-sub  $2k + 1$  for n and m

- $(2k+1)(2k+1) = (2k+1)^2$

-For any value k that is plugged into  $(2k+1)^2$ , or  $4k^2 + 4k + 1$ , it will return an odd number, and therefore not an even.

-Thus, because the negation of the conclusion of the conditional statement is true, we can conclude that the original statement is true.

18a) Prove that if n is an integer and  $3n + 2$  is even, then n is even using a proof by contraposition.

-assume "if n is an integer and  $3n + 2$  is even, then n is even" is false.

-so assume that n is odd

-n is odd if there exists an integer k such that  $n = 2k + 1$

-substitute  $2k + 1$  into the equation  $3n + 2$ :  $3(2k + 1) + 2 = 6k + 3 + 2 = 6k + 5$

-For any value k substituted into the equation  $6k + 5$ , an odd number will be returned, and therefore not an even.

-Thus, because the negation of the conclusion of the conditional statement is true, we can conclude that the original statement is true.