HW 1.7: 8(use contradiction), 22, 24, 26,30

8. Prove that if *n* is a perfect square, then *n* + 2 is not a perfect square.
-Assume ~P (n is not a perfect square)
-n = m^2
-There exists an integer k such that n+2 = k^2
-if m = 0, then n + 2 = 0 + 2, which is not a perfect square.
-m therefore must be ≥ 1, or m + 1.
- $(m + 1)^2 = m^2 + 2m + 1 = n + 2m + 1 \geq n + 2(1) + 1 > n + 2$.
Because when m ≥ 1 n cannot be a perfect square, we have a contradiction of the original hypothesis, so we can conclude the statement is true.

22. Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.
-Use exhaustive proof
-Our possible options are:
(black, black, black): True, we would end up with a pair of black socks
(black, black, blue): True, we would end up with a pair of black socks
(black, blue, blue): True, we would end up with a pair of blue socks
(blue, blue, blue): True, we would end up with a pair of blue socks
Because in each case it is true that we will get either a pair of blue socks or a pair of black socks, we can conclude that the statement is true.

24. Show that at least three of any 25 days chosen must fall in the same month of the year.
-Show that the statement is false, or that ~P is true.
-Prove that 2 of any 25 days chosen must fall in the same month of the year.
-Because each year contains 12 months, there can only be a maximum of 24 days chosen for 2 days to fall within the same month of the year.
-Because this contradicts the hypothesis, we've shown that ~P is true, and we can conclude that p is true, and that for 3 of any 25 days chosen will fall in the same month.

26. Prove that if *n* is a positive integer, then *n* is even if and only if 7*n* + 4 is even.
-Show that the bi conditional statement is true by proving each condition.
P= ""n is even"
Q = "7n + 4 is even"
P → Q, Q → P
- n is even when an integer k exists such that n = 2k. 7(2k) + 4 = 14k + 4 = 2(7k + 2). If we make 7k + 2 equal to M, then n = 2(m), which is our definition of an even integer. We can thus conclude that P → Q is true.
-n is odd when an integer k exists such that n = 2k + 1. For 7n + 4 to be odd, n must be equal to 2k + 1. 7(2k + 4) + 1 = 14k + 5. Because 14k + 5 will always return an odd number, then if 7n + 4 is odd, then n will be odd. We can thus conclude that when 7n + 4 is even, n will be even (Q → P).

30. Show that these three statements are equivalent, where *a* and *b* are real numbers: (*i*) *a* is less than *b*, (*ii*) the average of *a* and *b* is greater than *a*, and (*iii*) the average of *a* and *b* is less than *b*.
P: A < B
Q: (A + B)/2 > A
R: (A + B)/2 < B

-Show that P → Q, P→ R, Q → R, and R → P to prove true
- If A < B, then B ≥ (A + 1). A + A + 1/2 will produce (2A + 1)/2 or A + .5 which is > A. Therefore, if B > A, then the average of a and b is greater than a. P → Q is true.
- If A < B, then A ≤ (B - 1). B + B -1/2 will produce (2B - 1)/2 or B - .5 which is < B. Therefore, if B > A, then the average of a and b is less than b. P → R is true.
-If (A + B)/2 > A is true, then (A + B)/2 < B is true. We showed in our P → Q proof that for the average of a and b to be greater than A, then B must be greater than A. We similarly showed for our P → R proof that for the average of a and b to be less than b that B must be less than A. Therefore, each can be true only if B > A. So if one is true, the other is as well. Q → R is true.
-Show ~R (The average of A and B is greater than B) to prove that if the average of A and B is less than B, then A is less than B. (A+B/2 >B) For this to be true, then A ≥ (B + 1) where B + B + 1/2 returns B + .5 > B. By showing the contradiction is true, we can also conclude that the original conditional, R → P is true.