HW 6.2: 4, 6, 14, 32, 36

4) A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

a) How many balls must she select to be sure of having at least three balls of the same color?

5

b) How many balls must she select to be sure of having at least three blue balls?

13

6) Let *d* be a positive integer. Show that among any group of *d* + 1 (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by *d*.

If we let P = d + 1, and Q = d, then for P/Q, we will have two values with the same remainder by the pigeon hole principle.

14) **a)** Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.

-For the first 10 positive integers, the first 7 are 1, 2, 3, 4, 5, 6, 7

-We can show that there exist at least two pairs of integers in this range whose sum is 11

-5+6 = 11 and 7 + 4 = 11

-Because there are at least 2 pairs of these integers whose sum is 11, we have proven the original statement.

b) Is the conclusion in part (a) true if six integers are selected rather than seven?

No, we can at most select 1 pair of integers (5+6) whose sum would be equal to 11 within this range.

32) Show that if there are 100,000,000 wage earners in the United States who earn less than 1,000,000 dollars (but at least a penny), then there are two who earned exactly the same amount of money, to the penny, last year.

-If there were 100,000,000 earners last year, and each earned less than 1,000,000, but more than 1 cent, then there are 99,999,999 possible earnings (.01 - 999,999.99)

-Because there are 100,000,000 earners and 99,999,999 ways for them to earn money, it necessitates that two of the earners earned the same amount by the pigeonhole priniciple.

36) A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

-If we have 6 computers, and each is connected to at least one other computer, then we have 5 connections ranging between each of the six computers.

-Because we have 6 computers and only 5 connections available, then by the pigeon hole principle, we know that we must have at least 2 computers that are connected to the same number of other computers