

1.3: 8, 14, 16, 20, 32

8. Use De Morgan's laws to find the negation of each of the following statements.

a) Kwame will take a job in industry or go to graduate school.

P: Kwame will take a job in industry

Q: Kwame will go to grad school

$\sim(p \vee q)$ | **Negation: $\sim p \wedge \sim q$**

Kwame will not take a job in the industry, and Kwame will not go to grad school.

b) Yoshiko knows Java and calculus.

P: Yoshiko knows Java

Q: Yoshiko knows calculus

$\sim(p \wedge q)$ | **Negation: $\sim p \vee \sim q$**

Yoshiko does not know Java or he does not know calculus.

c) James is young and strong.

P: James is young

Q: James is strong

$\sim(p \wedge q)$ | **Negation: $\sim p \vee \sim q$**

James is not young or he is not strong.

d) Rita will move to Oregon or Washington

P: Rita will move to Oregon

Q: Rita will move to Washington

$\sim(p \vee q)$ | **Negation: $\sim p \wedge \sim q$**

Rita will not move to Oregon and Rita will not move to Washington.

14. Determine whether $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$ is a tautology

p	q	$\sim p$	$\sim q$	$p \rightarrow q$	$\sim p \wedge (p \rightarrow q)$	$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$
T	T	F	F	T	F	T
T	F	F	T	F	F	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

$(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$ is a tautology, because it will always return a value of true.

Each of Exercises 16–28 asks you to show that two compound propositions are logically equivalent. To do this, either show that both sides are true, or that both sides are false, for exactly the same combinations of truth values of the propositional variables in these expressions (whichever is easier).

16. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

$p \leftrightarrow q$: **True when P is T and Q is T, True when P is F and Q is F**

$(p \wedge q) \vee (\sim p \wedge \sim q)$: **True when P and Q are true, or when P is false and Q is false ($\sim P$ and $\sim Q$ are true)**

p	q	$\sim p$	$\sim q$	$p \leftrightarrow q$	$p \wedge q$	$\sim p \wedge \sim q$	$(p \wedge q) \vee (\sim p \wedge \sim q)$
T	T	F	F	T	T	F	T
T	F	F	T	F	F	F	F
F	T	T	F	F	F	F	F
F	F	T	T	T	F	T	T

$p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent as they return the same truth values.

20. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

$\neg(p \oplus q)$: False when p is T and q is false

$p \leftrightarrow q$: False when p is T and q is false

p	q	$\neg p$	$\neg q$	$(p \oplus q)$	$\neg(p \oplus q)$	$p \leftrightarrow q$
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

$\neg(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent as they return the same truth values.

32. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent.

p	q	r	$p \wedge q$	$(p \wedge q) \rightarrow r$	$(p \rightarrow r)$	$(q \rightarrow r)$	$(p \rightarrow r) \wedge (q \rightarrow r)$
T	T	F	T	F	F	F	F
T	F	T	F	T	T	T	T
T	T	F	T	F	F	T	F
T	F	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T
F	T	F	F	T	T	F	F
F	F	T	F	T	T	T	T

$(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not logically equivalent because $(p \wedge q) \rightarrow r$ returns different truth values than $(p \rightarrow r) \wedge (q \rightarrow r)$.