

HW 6.1: 8, 12, 16, 26, 28, 48, 52, 72

- 8) How many different three-letter initials with none of the letters repeated can people have?

$$26 \times 25 \times 24 = 15,600$$

- 12) How many bit strings are there of length six or less, not counting the empty string?

$$\text{-----} 2^6 + 2^5 + 2^4 + 2^3 + 2^2 + 2 + 2^0 = 127$$

- 16) How many strings are there of four lowercase letters that have the letter x in them?

$$\text{-----} 25 \times 25 \times 25 \times 4 = 62500$$

$$\text{-----} 25 \times 25 \times 6 = 3750$$

$$\text{-----} 25 \times 4 = 100$$

$$\text{-----} 1$$

$$\text{Total } 66,351$$

- 26) How many strings of four decimal digits

- a) do not contain the same digit twice?

$$\text{-----} 10 \times 9 \times 8 \times 7 = 5040$$

- b) end with an even digit?

$$\text{-----} 10 \times 10 \times 10 \times 4 = 4000 \text{ (if zero is counted, then } 10 \times 10 \times 10 \times 5) = 5000$$

- c) have exactly three digits that are 9s?

$$\text{-----} 9 \times 4 = 36$$

- 28) How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

$$\text{###UUU: } 10 \times 10 \times 10 \times 26 \times 26 \times 26 = 17576000$$

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$$\text{Total: } 35,152,000$$

- 48) How many bit strings of length seven either begin with two 0s or end with three 1s?

$$00 \text{ -----: } 2^5 = 32$$

$$\text{-----} 111: 2^4 = 16$$

$$00 \text{ --} 111: 2^2 = 4$$

$$\text{Total: } 48 - 4 = 44$$

- 52) Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?

$$38 + 23 - 7 = 54$$

- 72) Use mathematical induction to prove the product rule for  $m$  tasks from the product rule for two tasks.

- Let  $P(M)$  be the product rule for  $m$  tasks

- Basis Step:  $P(2)$  is true because we would  $n_1$  and  $n_2$  ways to complete the task, giving us  $n_1 n_2$  total ways to complete the task.

-Inductive Step: If we assume  $P(k)$  is true, where  $k$  is an int  $\geq 2$ , then show that  $P(k+1)$  is also true.

-If we show that the task for  $k+1$  requires more than one task, then we can show that  $n_{k+1}$ , where  $n_1, n_2, \dots, n_{k+1}$

-This will give us  $n_1 n_2 \dots n_k (n_{k+1})$  ways to complete the task.

- We have thus proven the inductive step and the basis step, showing that when  $k \geq 2$ , then  $P(k)$  and  $P(k+1)$  is true.