

$$d) \sum_{j=0}^8 (2^{j+1} - 2^j) = 2^9 - 2^0 = 511$$

$$\sum_{j=0}^8 2^j = \frac{2^{8+1} - 2^0}{2-1} = 511$$

36) Compute each of these double sums

$$a) \sum_{i=1}^3 \sum_{j=1}^2 (i-j) = \sum_{i=1}^3 \left(\sum_{j=1}^2 i - \sum_{j=1}^2 j \right) = \sum_{i=1}^3 (2i - 3) = 2 \sum_{i=1}^3 i - \sum_{i=1}^3 3 = 2(1+2+3) - 9 = 6 - 9 = -3$$

$$b) \sum_{i=0}^3 \sum_{j=0}^2 (3i + 2j) = \sum_{i=0}^3 \left(3 \sum_{j=0}^2 i + 2 \sum_{j=0}^2 j \right) = \sum_{i=0}^3 (3(3i) + 2(3)) = \sum_{i=0}^3 (9i + 6) = 9(6) + 6(4) = 78$$

$$c) \sum_{i=1}^3 \sum_{j=0}^2 j = \sum_{i=1}^3 (0+1+2) = 3(3) = 9$$

$$d) \sum_{i=0}^3 \sum_{j=0}^3 i^2 j^3 = \sum_{i=0}^3 i^2 \left(\sum_{j=0}^3 j^3 \right) = \sum_{i=0}^3 i^2 (1+8+27) = \sum_{i=0}^3 36i^2 = 36(1+4) = 180$$

40) Use identity $\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$ & ex 19 to compute $\sum_{k=1}^n \frac{1}{k(k+1)}$

Telescoping 19 gives $\sum_{j=1}^n (a_j - a_{j+1}) = a_1 - a_{n+1}$ by telescoping def

for $\sum_{k=1}^n \frac{1}{k(k+1)} = \left(\frac{1}{1} - \frac{1}{n+1} \right)$ for the n th value or $a_k = \frac{1}{k}$

$$\text{then } \sum_{k=1}^n \frac{1}{k(k+1)} = 1 - \frac{1}{n+1} = \frac{n}{n+1}$$

48) Find $\sum_{k=99}^{200} k^3$

$$\sum_{k=1}^{200} k^3 - \sum_{k=1}^{98} k^3 = \frac{200^2 \cdot 201^2}{4} - \frac{98^2 \cdot 99^2}{4} = 38047749$$