HW 10.4: 12(a, b, c), 16

10.4

12) Determine whether each of these graphs is strongly connected and if not, whether it is weakly connected.

a) Weakly connected, and not strongly
b) Strongly connected
c) Not weakly or strongly connected

16) Show that if $G = (V, E)$ is a directed graph and u, v, and w are vertices in V for which u and v are mutually reachable and v and w are mutually reachable, then u and w are mutually reachable.

If there is a direct path from a to b (where a and b are nodes in the strongly connected component), then the vertices visited a, $v_{1....}v_n$, b. Show that v_i, i = 1... , n is in the strong componenet to show that for any vertices u in the strong component, there exists a path from v_i to u and from u to v_i. There is a path from v_i to u and from u to v_i, meaning that v_i is also in the strong component. From v_i we can reach b, and we have already shown that from b we can reach u, since they both exist within the strong component. From u we can also reach a (also in strong comp), and from a we can reach v_i by the directed path a, v_1, ... v_i. Because we have shown a path from v_i to u and from u to v_1, we've shown that v_i is also in the strongly connected component.