HW 1.4

6, 8, 10, 12, 16, 24

6 Let N(x) be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.
a) ∃xN(x)
There is a student x, in my school, who has visited North Dakota.
b) ∀xN(x)
Every student in my school has visited North Dakota.
c) ~∃xN(x)
For every student in my school, none have been to North Dakota.
d) ∃x~N(x)
There is a student x, in my school, who has not visited North Dakota.
e) ~∀xN(x)
Not every student in my school has visited North Dakota.
f) ∀x~N(x)
For every student in my school, none have been to North Dakota.

8) Translate these statements into English, where R(x) is "x is a rabbit" and H(x) is "x hops" and the domain consists of all animals.
a) ∀x(R(x) -> H(x))
For all animals x, if x is a rabbit, then x hops.
b) ∀x(R(x)^H(x))
Every animal is a rabbit and hops.
c) ∃x(R(x) -> H(x))
There exists an animal x, for which if x is a rabbit, then it hops.
e) ∃x(R(x) ^ H(x))
There exists an animal x that is both a rabbit and hops.

10) Let C(x) be the statement "x has a cat," let D(x) be the statement "x has a dog," and let F(x) be the statement "x has a farret." Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.
a) A student in your class has a cat, a dog, and a ferret.
∃x (C(x)^D(x)^F(x))
b) All students in your class have a cat, a dog, or a ferret.
∀x(C(x)vD(x)vF(x))
c) Some student in your class has a cat and a ferret, but not a dog.
∃x(C(x) ^ F(x) ^ ~(D))
d) No student in your class has a cat, a dog, and a ferret.
~∃x(C(x)^D(x)^F(x))
e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has this animal as a pet.
∃xC(x) ^ ∃xD(x) ^ ∃xF(x))
12) Let Q(x) be the statement "x+1>2x." If the domain consists of all integers, what are these truth values?
a) Q(0)
True
b) Q(-1)
True

c) Q(1)
False
d) ∃xQ(x)
True, because it is sometimes true - Ex: Q(0)
e) ∀xQ(x)
False, because it is not always true - Ex: Q(1)
f) ∃x~Q(x)
True, there exists an x for which Q(x) is false - Ex: Q(1)
g) ∀x~Q(x)
False, it is not the case that for every x that Q(x) is false - Ex: Q(0)

16) Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.
a) $\exists x(x^2 = 2)$
True, there exists a number for integer x, that when squared, is equal to 2.
b) $\exists x(x^2 = -1)$
False, there is no number for integer x, that when squared, that will be negative.
c) $\forall x(x^2 + 2 \geq 1)$
True, for all integers x, when squared and added to two, they will be greater than or equal to 1.
d) $\forall x(x^2 \neq x)$
False, when integer x is the number 1, x^2 will be equal to x.

24) Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives. First, let the domain consist of the students in your class and second, let it consist of all people.
a) Everyone in your class has a cellular phone.
∀xC(x)
For every student x in my class, x has a cell phone.
∀x(I(x) -> C(x))
For every person x, if person x is a student in my class, then x has a cell phone.
b) Somebody in your class has seen a foreign movie.
∃xM(x)
There is a student x in my class having the property that x has seen a foreign movie.
∃x(I(x) ∧ M(x))
There is a person x, having the property x is a student in my class and has seen a foreign movie.
c) There is a person in your class who cannot swim.
∃x~S(x)
There is a person x in my class having the property that they cannot swim.
∃x(~S(x) ^ I(x))
There is a person x, having the property x is a student in my class and cannot swim.
d) All students in your class can solve quadratic equations.
∀xE(x)
For every student x in my class, x can solve quadratic equations.
∀x(I(x) -> E(x))
For every person x, if x is a person in my class, then x can solve quadratic equations.
e) Some student in your class does not want to be rich.
∃x~R(x)
There is a person x in my class having the property that they do not want to be rich.
∃x(~R(x)^I(x))

There is a person x having the property that person x is in my class and does not want to be rich.