Assignment: Section 5.3: 12, 26c ("5 | a+b" meas that 5 divides a+b), 43, 44; (7th edition)

In Exercises 12–19 *fn* is the *n*th Fibonacci number.

12. Prove that $f^2_1 + f^2_2 + \cdot\cdot\cdot + f^2_n = f_n f_{n+1}$ when *n* is a positive integer.

P(n) = $f^2_1 + f^2_2 + \cdot\cdot\cdot + f^2_n = f_n f_{n+1}$

Basis Step: $f_1 \times f_2 = 1 \times 1 = 1$

Recursive Step: Assuming P(n) is true, show P(n+1) is also true.

This means $f^2_1 + f^2_2 + \cdot\cdot\cdot + f^2_n + f^2_{n+1} = f_{n+1} f_{n+2}$

$= f_n f_{n+1} + f^2_{n+2}$

$= f_{n+1}(f_n + f_{n+1})$

$= f_{n+1} f_{n+2}$

26. Let *S* be the subset of the set of ordered pairs of integers defined recursively by

Basis step: $(0, 0) \in S$.

Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

c) Use structural induction to show that $5 \mid a + b$ when $(a, b) \in S$.

By the basis step we know that 5| 0 + 0 = 0.

Recursive step: Show if 5|a + b, then 5|a+b for any elements obtained from a,b.

If a+b = 5k for an int k, where (a+2, b +3) then a+b +5 = 5k + 5 = 5(k+1), where k+1 is also an int.

In the same way, for (a + 3, b +2) = a + b + 5 = 5k + 5 = 5(k+1) where k + 1 is also an int.

43. Use structural induction to show that $n(T) \geq 2h(T) + 1$, where *T* is a full binary tree, $n(T)$ equals the number of vertices of *T*, and $h(T)$ is the height of *T*. The set of leaves and the set of internal vertices of a full binary tree can be defined recursively.

Basis step: The root *r* is a leaf of the full binary tree with exactly one vertex *r*. This tree has no internal vertices.

Recursive step: The set of leaves of the tree $T = T_1 \cdot T_2$ is the union of the sets of leaves of T_1 and of T_2. The internal vertices of *T* are the root *r* of *T* and the union of the set of internal vertices of T_1 and the set of internal vertices of T_2.

Basis Step is true because n(T) = 1 and h(T) = 0, and 1 >= 2 x 0 + 1 (when a tree has just a root node)

Recursive Step: Show n(T) >= 2h(T)+1 for the full binary tree T.

T is formed by two subtrees, T_1 and T_2 + the root node (where T_1 and T_2 are smaller than T) by the recursive def of a full binary tree.

T_1 and T_2 is holds true, and by the recursive definition of n(T) and h(T), we know that

$n(T) = 1 + n(T_1) + n(T_2)$ **and** $h(T) = 1 + \max(h(T_1), h(T_2))$

$n(T) = 1 + n(T_1) + n(T_2)$

$>= 1 + 2h(T_1) + 1 + 2h(T_2) + 1$ **Inductive Hyp**

$>= 1+2\max(h(T_1), h(T_2)) + 2$ **By** $2h(T_1) + 2h(T_2) >= 2\max(h(T_1), h(T_2))$

$= 1 + 2(\max(hT_1), h(T_2)) + 1)$ **Factor**

$= 1 + 2h(T)$ **Recursive Def of FBT**

44. Use structural induction to show that *l(T)*, the number of leaves of a full binary tree *T* , is 1 more than *i(T)*, the number of internal vertices of *T* .

Basis Step: The smallest full binary tree is single root r: l(T) = 1 = 1 + i(T)

Recursive Step: Show result holds for T

T is formed by two subtrees, T_1 and T_2 + the root node (where T_1 and T_2 are smaller than T) by the recursive def of a full binary tree

We know by the basis step that T_1 and T_2 hold and that $i(T) = i(T_1) + i(T_2) + 1$

$l(T) = l(T_1) + l(T_2)$ Rec Def of Full Binary Tree

$= i(T_1) + 1 + i(T_2) + 1$ Induction Hypoth

$= i(T) + 1$ Recursive Def of FBT

Completing the inductive step