Assignment: Section 5.3: 2 (a,b), 8, 24, 26a, 28a, 32a (7th edition)

2. Find $f(1)$, $f(2)$, $f(3)$, $f(4)$, and $f(5)$ if $f(n)$ is defined recursively by $f(0) = 3$ and for $n = 0, 1, 2, \ldots$

a) $f(n+1) = -2f(n)$.

-6, 12, -24, 48, -96

b) $f(n+1) = 3f(n) + 7$.

16, 55, 172, 523, 1576

8. Give a recursive definition of the sequence $\{an\}$, $n =$
1, 2, 3, . . . if

a) $a_n = 4n - 2$.

Basis Step: A(1) = 4 - 2 = 2
Recursive Step: A(n+1) = 4(n+1) - 2
= 4n + 4 - 2
= 4n - 2 + 4
= $\mathbf{a_n + 4}$

b) $an = 1 + (-1)^n$.

Basis Step: $\mathbf{A(1) = 1+(-1)^1 = 0}$
Recursive Step: $\mathbf{A(n+1) = 1 + (-1)^{n+1}}$
$\mathbf{= 1 + -1^n (-1^1)}$
$\mathbf{= 1 + (((-1)^n + 1) -1) -1))}$
$\mathbf{=1 + (a_n - 1)(-1)}$
$\mathbf{=2 - a_n}$

c) $an = n(n+1)$.

Basis Step: A(1) = 1(1+1) = 2
Recursive Step: A(n + 1) = n+1(n+1 + 1)
= n (n+1) + n + n+1 + 1
= $\mathbf{a_n}$ **2n + 2**

d) $an = n^2$.

Basis Step: $\mathbf{A(1) = (1)^2 = 1}$
Recursive Step: $\mathbf{A(n+1) = (n + 1)^2}$
$\mathbf{= n^2+2n+1}$
$\mathbf{=a_n + 2n + 1}$

24. Give a recursive definition of

a) the set of odd positive integers.

Basis Step: $\mathbf{1 \in S}$
Recursive Step: If $\mathbf{x \in S}$**, then** $\mathbf{x + 2}$ **is** $\mathbf{\in S}$

b) the set of positive integer powers of 3.

Basis Step: $\mathbf{3 \in S}$
Recursive Step: If x is $\mathbf{\in S}$**, then** $\mathbf{x^3 \in S}$

c) the set of polynomials with integer coefficients.

Basis Step: $\mathbf{0 \in S}$
Recursive Step: If $\mathbf{p(x) \in S}$**, then** $\mathbf{p(x) + cx^n \in S}$**, where** $\mathbf{c \in Z}$**,** $\mathbf{n \in Z}$**, and n >= 0**

26. Let S be the subset of the set of ordered pairs of integers defined recursively by

Basis step: $(0, 0) \in S$.

Recursive step: If $(a, b) \in S$, then $(a + 2, b + 3) \in S$ and $(a + 3, b + 2) \in S$.

a) List the elements of S produced by the first five applications of the recursive definition.

(2, 3) (3, 2)
(4, 6) (6, 4) (5, 5)
(6, 9) (7, 8) (8, 7) (9, 6)
(8, 12) (9, 11) (10, 10) (11, 9) (12, 8)
(10, 15) (11, 14) (12, 13) (13, 12) (14, 11) (15,10)

28. Give a recursive definition of each of these sets of ordered pairs of positive integers. [*Hint:* Plot the points in the set in the plane and look for lines containing points in the set.]

a) $S = \{(a, b) \mid a \in \mathbf{Z}^+, b \in \mathbf{Z}^+$, and $a + b$ is odd$\}$

Basis Step: a ∈ $\mathbf{Z}^+$ and b ∈ $\mathbf{Z}^+$

(a(1) + b(1)) + 2 is odd

Recursive Step: if (a, b) ∈ $\mathbf{Z}^+$ then (a + 1, b + 1) is odd, (a+2,b) is odd, (a ,b + 2) is odd

These conditions are all true if a + b is odd. If the sum of a and b is 3, which is the smallest case where a + b is odd and positive integers, then a =1 b =2, making them fitting of set S. The sum of (a+b) + 2 is 5 at its least, and (a -2, b) (a, b-2) (a-1, b-1) would have to have positive integers whose sum is odd and smaller than a+b, and therefore must be in S, and an application of the first recursive step would show (a, b) ∈ S

32. a) Give a recursive definition of the function *ones(s)*, which counts the number of ones in a bit string s (A bit string is a string of zeros and ones).

Let Σ = {0.1}
Basis Step: ones(λ) = 0 (empty string w/ no 1's or 0's)
Recursive Step: If x ∈ Σ, and w ∈ Σ*, then ones(wx) = ones(w) + x, where x is either 0 or 1.