2. Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B.

a) the set of sophomores taking discrete mathematics in your school

A ∩ B

b) the set of sophomores at your school who are not taking discrete mathematics

A - B

c) the set of students at your school who either are sophomores or are taking discrete mathematics

A U B

d) the set of students at your school who either are not sophomores or are not taking discrete mathematics

$\overline{\mathbf{A \cup B}}$

4. Let A = {a, b, c, d, e} and B = {a, b, c, d, e, f, g, h}.
Find

a) A ∪ B.

{a, b, c, d, e, f, g, h}

b) A ∩ B.

{a, b, c, d, e}

c) A − B.

none ϕ

d) B − A.

{f, g, h}

12. Prove the first absorption law from Table 1 by showing that if A and B are sets, then A ∪ (A ∩ B) = A.

Show each side is a subset of the other.

-In a union, x ∈ A or x ∈ A∩B.

-Intersection states that either x ∈ A or x ∈ A and x ∈ B, so x ∈ A.

-We can then say that AU(A∩B)⊆A

-if x ∈ A then x by the definition of union, ∈AU(A∩B), because x ∈ A can also be x ∈ A and x ∈ B, which also means that A⊆AU(A∩B)

- We can therefore conclude that A ∪ (A ∩ B) = A

16. Let A and B be sets. Show that

a) (A ∩ B) ⊆ A.

Show x∈(A∩B)

- x∈A and x∈B by intersection

-because x∈A, we can then conclude (A∩B)⊆A, proving the statement

b) A ⊆ (A ∪ B)

Show x ∈ A

-x ∈ A or x ∈

-by union we know that x ∈ AUB,

- because x ∈ AUB we can thus conclude that A ⊆ (A ∪ B)

c) A − B ⊆ A.

- x ∈ A - B

-x ∈ A and x ∉B because of their difference, so x ∈ A

because x ∈ A, we can conclude A − B ⊆ A.

d) A ∩ (B − A) = ∅.

Show contradiction

- A ∩ (B − A) does not = ∅

-x ∈ A and x ∈ (B−A) by union
- through difference, we can find that x ∈ A, x ∈ B, but also x ∉ A.
-This contradiction proves that A ∩ (B − A) = ∅.

e) A U(B − A) = AU B.

A	B	B - A	A U (B - A)	A U B
1	1	0	**1**	**1**
1	0	0	**1**	**1**
0	1	1	**1**	**1**
0	0	0	**0**	**0**

Because A U (B - A) and A U B have the same values, we can conclude that they are equal.

18. Let A, B, and C be sets. Show that
a) (A ∪ B) ⊆ (A ∪ B ∪ C).
-if x ∈ (A∪B), then by union x ∈ A or x ∈ B.
-x then is also ∈ C
-Again using union, we can conclude then that because x ∈ (A u B u C) that (A ∪ B) ⊆ (A ∪ B ∪ C).

b) (A ∩ B ∩ C) ⊆ (A ∩ B).
- by interstection, x ∈ A, x ∈ B, x ∈ c, which would also give that x ∈ (A ∩ B)
-We can then conclude that (A ∩ B ∩ C) ⊆ (A ∩ B).

c) (A − B) − C ⊆ A − C.
- To show the left, we can assume x ∈ (A−B)−C.
-Through difference, this returns x ∈ A, while x ∈ B and x ∉ C.
-Intersection then gives us x ∈ (A ∩ B).
-We can thus conclude that (A ∩ B ∩ C) ⊆ (A ∩ B).

d) (A − C) ∩ (C − B) = ∅.
Show the contradiction to prove. (That there is a value that exists so that (A − C) ∩ (C − B) ≠ ∅.)
-By intersection, we can show x ∈ (A - C) and x ∈ (C - B).
-By difference, x ∈ A and x ∉ C
-By difference, x ∈ C and x ∉ B
-Because x ∉ C and x ∈ C contradict eachother, we can conclude that (A − C) ∩ (C − B) = ∅.
e) (B − A) ∪ (C − A) = (B ∪ C) − A.

A	B	C	B - A	C - A	B U C	(B - A) U (C - A)	(B U C) - A
1	1	1	0	0	1	**0**	**0**
1	1	0	0	0	1	**0**	**0**
1	0	1	0	0	1	**0**	**0**
1	0	0	0	0	0	**0**	**0**
0	1	1	1	1	1	**1**	**1**
0	1	0	0	1	1	**1**	**1**
0	0	1	1	0	1	**1**	**1**
0	0	0	0	0	0	**0**	**0**

Because the values for (B - A) U (C - A) are equal to (B U C) - A, we can conclude that (B − A) ∪ (C − A) = (B ∪ C) − A.

20. Show that if A and B are sets with A ⊆ B, then

a) A ∪ B = B.

-Assume that A ⊆ B

-if x ∈ A ∪ B then x ∈ A or x ∈ B

-Assume x ∈ A

-if x ∈ A, then because A ⊆ B, x must also be ∈ B

- This shows B ⊆ A U B, and thus that B = A U B.

-Together, this proves B = A U B

b) A ∩ B = A.

-Assume that A ⊆ B

- If x ∈ A ∩ B, then x ∈ A and x ∈ B by intersection, so x ∈ A.

-This shows A ∩ B ⊆ A.

-if x ∈ A, then x ∈ B, too, so x ∈ A ∩ B.

-This shows A ⊆ A ∩ B, and thus that A = A ∩ B.

- If x ∈ A, then x ∈ A ∩ B, so x ∈ A and x ∈ B.

- This proves A ⊆ B, so we can conclude that if A ⊆ B, where A and B are sets, then A ∩ B = A.