5.2: 2, 4, 12, 30

2) Use strong induction to show that all dominoes fall in an infinite arrangement of dominoes if you know that the first three dominoes fall, and that when a domino falls, the domino three farther down in the arrangement also falls. (n, n+1, n+2)

**Basis Case:**
**P(1): 1, 2, 3. The basis case is thus true.**
**Inductive Case:**
**Show that if P(k) then P(k+1) must also be true.**
**In the case of k+1, if k=2, then we know the next two dominoes also fall (2, 3, 4).**
**For any domino that falls, we can say that dominoes k, k-1, and k-2 fall as well.**
**Because the third domino down always falls, we have shown that the inductive hypothesis is true, and then that dominoes at n, n+1, and n+2 will fall, proving the original statement true.**

4) Let P(n) be the statement that a postage of n cents can be formed using just 4-cent stamps and 7-cent stamps. The parts of the exercise outline a strong induction proof that P(n) is true for $n \geq 18$.

**a)** Show statements *P(*18*)*, *P(*19*)*, *P(*20*)*, and *P(*21*)* are true, completing the basis step of the proof.

**P(18): Two 4 cent stamps, one 7 cent stamp**
**P(19): Three 4 cent stamps, one 7 cent stamp**
**P(20): Five 4 cent stamps**
**P(21): Three 7 cent stamps.**

**b)** What is the inductive hypothesis of the proof?

**The inductive hypothesis is that we can form x cents postage for all x where $18 \leq x \leq k$, where $k \geq 18$**

**c)** What do you need to prove in the inductive step?

**To prove the inductive step, we need to show we can form K+1 cent postage while using just 4 and 7 cent stamps**

**d)** Complete the inductive step for $k \geq 21$.

**We know from our basis case that P(21) can be solved with three 7 cent stamps, and to solve for any value > 21, we will need to use k+1 stamps.**

**e)** Explain why these steps show that this statement is true whenever $n \geq 18$.

**Because we have completed the basis and induction steps, we have shown that the original statement is true.**

12) Use strong induction to show that every positive integer n can be written as a sum of distinct powers of two, that is, as a sum of a subset of the integers $2^0 = 1$; $2^1 = 2$; $2^2 = 4$, and so on. [Hint: For the inductive step, separately consider the case where k + 1 is even and where it is odd. When it is even, note that k + ½ is an integer.]

**Basis Step:**
**P(1): $1 = 2^0$, this shows proves the basis step**

**Inductive step: Assume P(x) is true for all x ≤ k, where k is a positive integer.**
**When k + 1 is even, k+1/2 is an integer and is ≤ k for any pos int k. Because it's ≤ k and p(x) is true for all pos int ≤ k, we can conclude k+1/2 can be shown to be a sum of distinct powers of two.**
**K+1 then = 2 x k+1/2, which is k+1.**
**If an even number, P(k+1) is true.**

**When k+1 is odd, then k is even. For this to be the case, k+1 must be $2^0$, because it is the only power of two that results in an odd number. K+1 = k+$2^0$ which is a sum of powers of two.**

**We have thus shown that the inductive hypothesis is true, proving that both P(k) is true, and then that P(k+1) is also true. By completing the basis and inductive steps, we have shown the original statement is true.**

30) Find the flaw with the following "proof" that $a^n = 1$ for all nonnegative integers $n$, whenever $a$ is a nonzero real number.
*Basis Step:* $a^0 = 1$ is true by the definition of $a^0$.
*Inductive Step:* Assume that $a^j = 1$ for all nonnegative integers $j$ with $j \leq k$. Then note that $a^{k+1} = a^k \cdot a^k / a^{k-1} = 1 \cdot 1/1 = 1$.

**In the basis step, 0 is used. The original statement clearly indicates that a must be a nonzero real number. Because of this, we can't make the assumption that $a^1$=1.**