HW 10.1: 24a, 28 / 10.2" 6, 16, 18 (think about pigeons), 26(a, b, c)

10.1

24A) Explain how graphs can be used to model electronic mail messages in a network. Should the edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

Graphs can model electronic mail messages in a network by using a directed graph where for each vertex, an email address is represented, and where an edge would begin at one address and end at another address (for a message sent from one to another). Since more than a single message can be sent from one email address to another, there would be multiple directed edges that would connect the two address. Loops could be allowed since email addresses are capable of messaging their selves.

28) Describe a graph model that represents a subway system in a large city. Should edges be directed or undirected? Should multiple edges be allowed? Should loops be allowed?

A graph for a subway system would be a directed graph with stops shown as vertexes, and where an edge begins at one station stop and ends at another if there is a subway line that goes between the two. There can be multiple edges allowed as it is possible that multiple subway lines could travel from one subway station to another. Loops in this case would not be allowed as a station does not travel directly to itself.

10.2

6) Show that the sum, over the set of people at a party, of the number of people a person has shaken hands with, is even. Assume that no one shakes his or her own hand.

The set of people can be shown with an undirected graph G(V, E) where each person is a vertex and two people are connected by an edge if they've shaken hands. The number of people someone has shaken hands with is then the degree of v (a person). To find the sum, we use the hand shaking theorem which we know is equal to 2e, where e represents the edges in the graph. The sum therefore is always going to be an even number.

16) What do the in-degree and the out-degree of a vertex in theWeb graph, as described in Example 5 of Section 10.1, represent?

The in-degree represent the number of webpages with a link that are pointing to a, while the out-degree of a is the number of web pages that a has a link pointing to.

18) Show that in a simple graph with at least two vertices there must be two vertices that have the same degree.

If G is a simple graph with V vertices, then the highest degree of a vertex v is V - 1, where edges exist between all V - 1 verts and v, and the lowest degree of a vertex v is 0, where there are no edges.

If all vertices have different degrees, then it could be the case that a vertex v has degree of 0, AND a vertex u has degree of v - 1, but for u to have a degree of v - 1, there must be an edge between u and v, which would contradict the fact that v has a degree of 0. By contradiction, we have then shown that the possible degrees of all vertices in the graph are from 1 to v - 1, and since there are v vertices in the graph, by applying the pigeonhole principle, there would have to be two vertices that share the same degree.

26) For which values of *n* are these graphs bipartite?

a) K_n

For any K_n, n >= 3

n = 1, n = 2
We cannot assign one of two different colors for each vertex because our vertices are connected by an edge.

b) C_n

If n is even, we can use one of two different colors for each vertex of C_n. If we color them in order, we will have the first colored one way, and then we color the next with our remaining color. If we continue coloring this way for as long as n is even, we will continue to color all the way until the end where the last two vertices that have been colored has the common adjacent index, and we can color the final vertex in the same manner.

c) W_n

W_n is not bipartite at any value.
Since we will have to color two edges, those in C_n and those of an additional vertex, we can color the additional vertex red. We must then color all the C_n vertices another color, say green, as they are adjacent to the additional vertex. Because all the C_n vertices are adjacent to one another, we will inevitably end up with edges that connect two green vertices.
Starting by coloring C_n vertices will also show to be impossible as coloring two vertices in C_n adjacent to u red, and the additional vertex green will give us c adjacent with v and w, and two edges that connect two green vertices.