HW 1.8: 8, 30, 36

8. Prove that there is a positive integer that equals the sum of the positive integers not exceeding it. Is your proof constructive or nonconstructive?

-We can conclude there is a positive integer that equals the sum of the positive integers not exceeding it by providing the example of 1. Because the only positive integer that does not exceed 1 is itself, it proves the existence and the statement as true. This is a constructive proof.

30. Prove that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

$-1 \leq y \leq 1$ (-1, 0, 1)
$-2 \leq x \leq 2$ (-2, -1, 0, 1, 2)
for (y, x)
(-1 , -2) = 13 **F**, (-1, -1) = 7 **F**, (-1, 0) = 5 **F**, (-1, 1) = 7 **F**, (-1, 2) = 13 **F**

(0, -2) = 8 **F**, (0, -1) = 2 **F**, (0, 0) = 0 **F**, (0, 1) = 2 **F**, (0, 2) = 8 **F**

(1, -2) = 13 **F**, (1, -1) = 7 **F**, (1, 0) = 5 **F**, (1, 1) = 7 **F**, (1, 2) = 13 **F**

By exhausting all possibilities, we can conclude that there are no solutions in integers x and y to the equation $2x^2 + 5y^2 = 14$.

36. Prove that between every rational number and every irrational number there is an irrational number.

-A real # r is rational if exists integers p and q with q ≠ 0 such that r = p/q.
- A real # r that is not rational is called irrational.
-x is an irrational number, and y is a rational number
-To find a value between two numbers, take their average. (x+y)/2
- y = s/t where t is not equal 0
To contradict:
- x = p/q where q is not equal to 0
-x+y/2 is equivalent to ((s/ø) + (p/ ø))/2
- Because our denominators (q and t) are not equal to zero, our averaged number will also be a rational number, due to the definition of a rational number.
- We thus have a contradiction of the initial proposition that x is an irrational number, proving that (x + y)/2 would result in an irrational number.