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```
% AME 309
% WATER ROCKET PROJECT
% ROCKET ANALYTICS INC.

%%%\\\ CONSTANT INITIALIZATION ///%%%
T_atm     = 343;              % ambient temperature          [K]
ms        = 0.1265;           % mass of rocket shell         [kg]
ma        = 0;                % mass of compressed air       [kg]
rho_w     = 1000;             % density of water             [kg/
m^3]
g         = 9.81;             % gravitational constant       [m/s^2]
P_atm     = 101325;           % atmospheric pressure         [Pa]
gamma     = 1.401;            % ratio of specific heats      [Cp/Cv]
A_ex      = (pi/4)*(0.02178)^2;% area of exit nozzle          [m^2]
V_tot     = 0.000591471;      % total volume of bottle       [m^3]
Ra        = 287;              % gas constant of air          [kg*m/
K*s^2]
dt        = 0.001;            % time increment               [s]
z_0       = 0;                % initial height of rocket     [m]
Ta_0      = T_atm;            % initial air temperature      [K]
Cd        = .295;              % coefficient of drag          [N/A]
Ac        = (pi/4)*((2.5*0.0254)^2)+3*(.2*.0254)*(1.5*0.0254);% cross-
sectional area of rocket [m^2]
rho_atm   = P_atm/(Ra*T_atm); % density of atmoshpheric air    [kg/
m^3]
t_snap    = 0;

%%%\\\ FIRST ORDER PREDICTION VARIABLE INITIALIZATION ///%%%
Va_fop     = 0;               % air volume
Pa_fop     = 0;               % air pressure
w_avg_fop  = 0;               % average vertical velocity
Pa_avg_fop = 0;               % average air pressure
w_ex_fop   = 0;               % exit velocity first order prediction
opt_air_pressure = -1;        % optimal air pressure
opt_water_volume = -1;        % optimal water pressure

%%%\\\ ARRAY INITIALIZATION ///%%%
t     = 0:dt:3.5;                % time array
Va    = zeros(1,length(t));      % air volume
w_ex  = zeros(1,length(t));      % exhaust velocity
```

```
Pa    = zeros(1,length(t));      % air pressure
m     = zeros(1,length(t));      % mass
z_acc = zeros(1,length(t));      % z acceleration
w     = zeros(1,length(t));      % z velocity
z     = zeros(1,length(t));      % z height
Vw    = 2.3e-4:0.001e-4:2.5e-4;  % initial water volume for
 optimization
Pa_0  = 3*P_atm:P_atm/25:4*P_atm;% initial air pressure for
 optimization
max_height = zeros(length(Vw),length(Pa_0));
V_tol = 0.5e-5;
Pa_tol = 500;
t_tol = 0.0005;
cruise = 0;
%%%\\\ TIME VARIABLE INITIALIZATION ///%%%
t_thrust = 0;
t_index  = 0;


%%%LEILANI%%%
% PROCEED SIMULATION WITH DRAG FORCE CALCULATION? Yes = 1; No = 0
Drag = 0;
%%%\\\ SIMULATION ///%%%
for simulation = 1:2

    %%%\\\ OPTIMIZATION ///%%%
    for opt_air = 1:length(Pa_0)
        for opt_water = 1:length(Vw)

            cruise = 0;
            t_index = 0;
            t_thrust = 0;
            t     = 0:dt:3.5;              % time array
            Va    = zeros(1,length(t));   % air volume
            w_ex  = zeros(1,length(t));   % exhaust velocity
            Pa    = zeros(1,length(t));   % air pressure
            m     = zeros(1,length(t));   % mass
            z_acc = zeros(1,length(t));   % z acceleration
            w     = zeros(1,length(t));   % z velocity
            z     = zeros(1,length(t));   % z height
            Vw    = 2.3e-4:0.001e-4:2.5e-4;        % initial water
 volume for optimization
            Pa_0  = 3*P_atm:P_atm/25:4*P_atm;          % initial air
 pressure for optimization


            %%%\\\ TIME SIMULATION: THRUST PERIOD ///%%%
            for tau = 1:length(t)

                %%%\\\ INITIAL CONDITIONS ///%%%
                if tau == 1
                    Pa(1) = Pa_0(opt_air);
                    Va(1) = (V_tot - Vw(opt_water));
                    w_ex(1) = sqrt(2*(Pa_0(opt_air)-P_atm)/rho_w);
```

```
                ma = (Pa_0(opt_air)*Va(1))/(Ra*Ta_0);
                m(1) = ms + ma + rho_w*Vw(opt_water);
                z_acc(1) = (rho_w*(w_ex(1)^2)*A_ex)/m(1) - g;
                w(1) = 0;
                z(1) = z_0;
                t_thrust = t_thrust + 1;
                t_index = t_index + 1;

            %%%\\\ LIFT OFF ///%%%
            elseif tau ~= 1 && abs(Va(tau-1)-V_tot) > V_tol &&
 abs(Pa(tau-1)-P_atm) > Pa_tol && cruise == 0

                %%%\\\ THRUST PERIOD ///%%%
```

First-Order Predictions %%%

```
                    ma = (Pa(tau)*Va(tau))/(Ra*Ta_0);
                    Va_fop = Va(tau-1) + A_ex*w_ex(tau-1)*dt;
                    Pa_fop = Pa_0(opt_air)*((Va(1)/Va_fop)^gamma);
                    w_avg_fop = w(tau-1) + z_acc(tau-1)*dt/2;
                    Pa_avg_fop = (Pa(tau-1) + Pa_fop)/2;
                    w_ex_fop = sqrt(2*(Pa_avg_fop - P_atm)/rho_w);
```

Corrected Final Values %%%

```
                    Va(tau) = Va(tau-1) + A_ex*w_ex_fop*dt;
                    Pa(tau) = Pa_0(opt_air)*((Va(1)/
Va(tau))^gamma);

                    w_ex(tau) = sqrt(2*(Pa(tau) - P_atm)/rho_w);
                    m(tau) = ms + ma + rho_w*(V_tot - Va(tau));
                    z_acc(tau) = (rho_w*(w_ex(tau)^2)*A_ex)/m(tau)
 - rho_atm*Ac*Cd*(w_avg_fop^2) - g;
                    w(tau) = w(tau-1) + (z_acc(tau-1) +
 z_acc(tau))*dt/2;
                    z(tau) = z(tau-1) + (w(tau-1) + w(tau))*dt/2;
```

Time Indices %%%

```
                    t_thrust = t_thrust + 1;
                    t_index = t_index + 1;


                %%%\\\ CRUISE PERIOD ///%%%
            else
                if simulation == 2 && cruise == 0
                    t_snap = tau;
                end
                    cruise = 1;


                    %%%\\\ MAX TIME AND HEIGHT CALCULATION ///%%%
                    if tau == t_thrust + 1
```

Negligible Drag Force %%%

```
                                    if Drag == 0
                                        z_max = z(tau-1) + (w(tau-1)^2)/(2*g);
                                        t_max = t(tau-1) + w(tau-1)/g;
                                        max_height(opt_water,opt_air) =
 real(z_max);
```

Constant Drag Coefficient %%%

```
                                    elseif Drag == 1
                                        Beta = rho_atm*Ac*Cd/(2*m(tau-1));
                                        t_max = t(tau-1) +
 atan(w(tau-1)*sqrt(Beta/g))/sqrt(Beta*g);
                                        z_max = z(tau-1) -
 (Beta^-1)*log(cos((t_max - t(tau-1))*sqrt(Beta*g)));
                                        max_height(opt_water,opt_air) =
 real(z_max);
                                    end

                                end

                                %%%\\\ End Simulation? ///%%%


%                          if opt_air_pressure == opt_air &&
 opt_water_volume == opt_water
% %                          Plot Height vs Time to T_max
%                              fprintf('The optimal initial
 air pressure and water volume is %6.f and %5.7e,respectively.
\n',Pa_0(opt_air),Vw(opt_water_volume))
%                          end

                                if abs(t(tau)-t_max) < t_tol &&
 opt_air_pressure == opt_air && opt_water_volume == opt_water  %if
 abs(z(tau-1) - z_max) < z_tol;
                                          fprintf('The optimal initial air
 pressure and water volume is %5.fPa and %5.4gm^3,respectively.\nThe
 max height is %4.2fm\n',Pa_0(opt_air),Vw(opt_water_volume),z_max)

                                          figure
                                          plot(t(1:t_snap), z(1:t_snap))
                                          title('Thrust Phase')
                                          xlabel('Time [s]')
                                          ylabel('Height [m]')


                                          figure
                                          plot(t(t_snap:t_index),
 z(t_snap:t_index))

                                          title('Cruise Phase')
                                          xlabel('Time [s]')
```

```
                                        ylabel('Height [m]')

        %                               Plot Vertical Velocity vs Time to
T_max
                                        figure
                                        plot(t(1:t_index), w(1:t_index),'k')
                                        axis([0 t_max 0
round(w(t_thrust)+1)])
                                        title('Velocity vs Time to T_m_a_x')
                                        xlabel('Time [s]')
                                        ylabel('Velocity [w]')

        %                               Plot Mass Loss vs Time to T_max
                                        figure
                                        plot(t(1:t_index), m(1:t_index),'r')
                                        title('Mass vs Time to T_m_a_x')
                                        xlabel('Time [s]')
                                        ylabel('Mass [kg]')
                                        axis([0 t_max m(t_index)*.9
m(1)*1.1])
                                    %end
                                %end

                                    break
                                end
```

Kinematic Calculations %%%

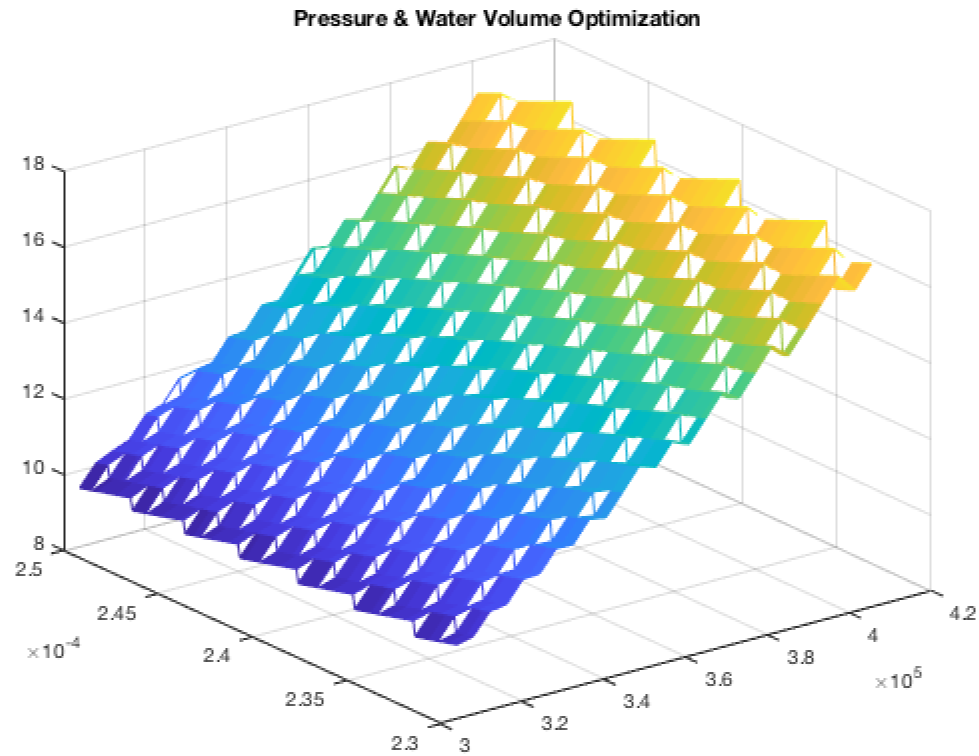
```
                                z(tau) = z(tau-1) + w(tau-1)*dt - 0.5*g*dt^2;
                                w(tau) = w(tau-1) - g*dt;
                                m(tau) = m(tau-1);
                                %z(tau) = (w(tau-1)+w(tau))*dt/2;
                                %w(tau) = sqrt(w(tau-1)^2 - 2*g*z(tau));
```

Time Index %%%

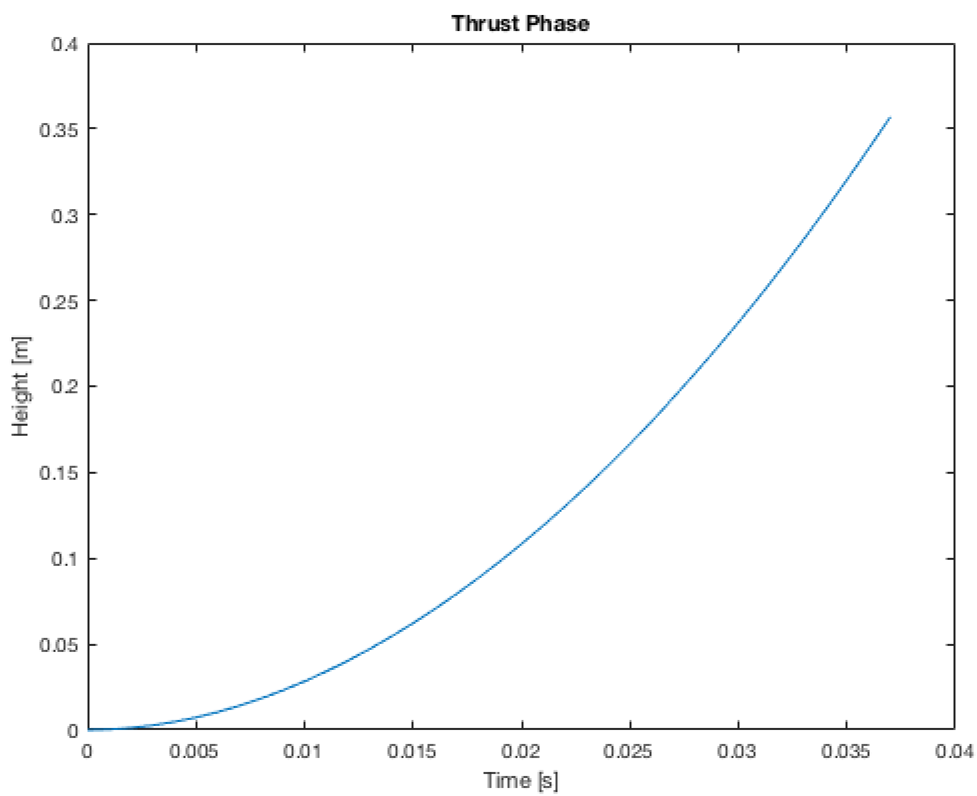
```
                                t_index = t_index + 1;

                        end


                    end

                end

            end




        % Determining optimal parameters
            if simulation == 1
```

```
        % 3D plot: Initial Water Volume vs Initial Air Pressure vs Max Height

        figure
        mesh(Pa_0,Vw,max_height)
        title('Pressure & Water Volume Optimization')

        max_value = max(max(max_height));

        for i = 1:size(max_height,1)

            for j = 1:size(max_height,2)

                if max_height(i,j) == max_value
                    opt_air_pressure = j;
                    opt_water_volume = i;

                    break
                end

            end

            if opt_air_pressure > 0
                break
            end

        end

    end

end
```

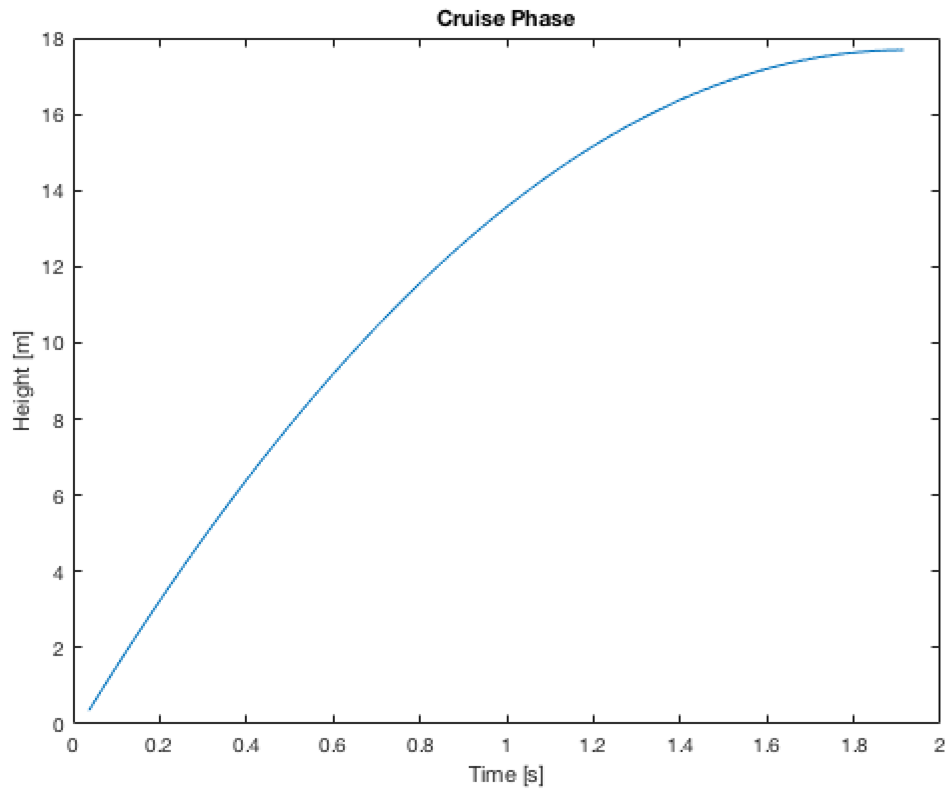
The optimal initial air pressure and water volume is 405300Pa and 0.0002415m^3,respectively.
The max height is 17.68m

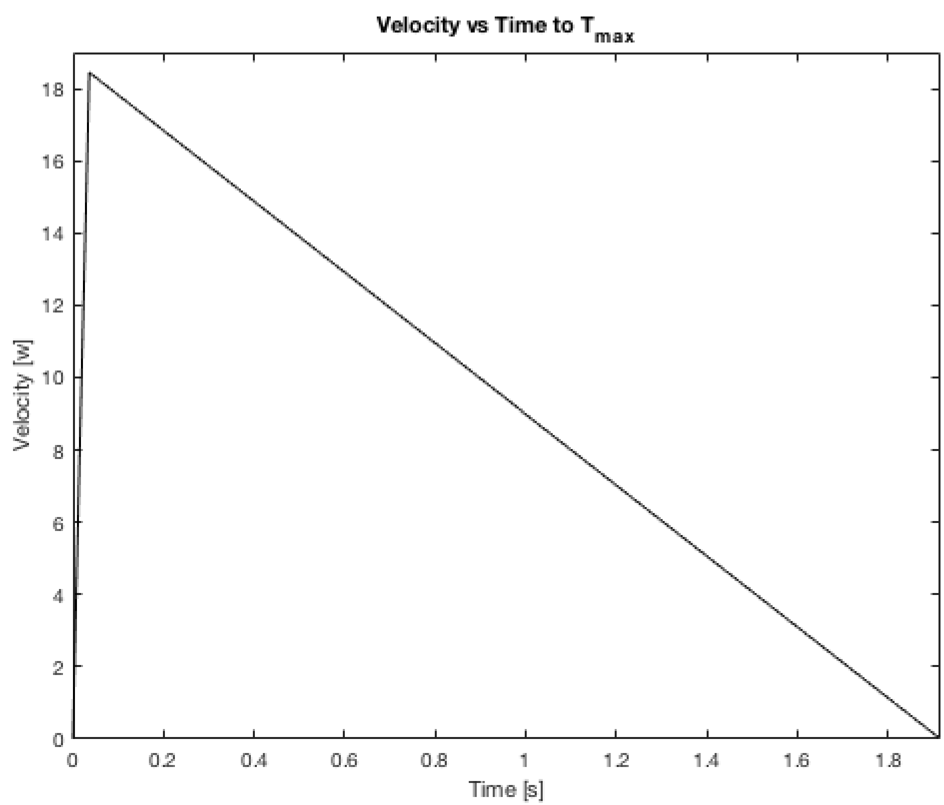


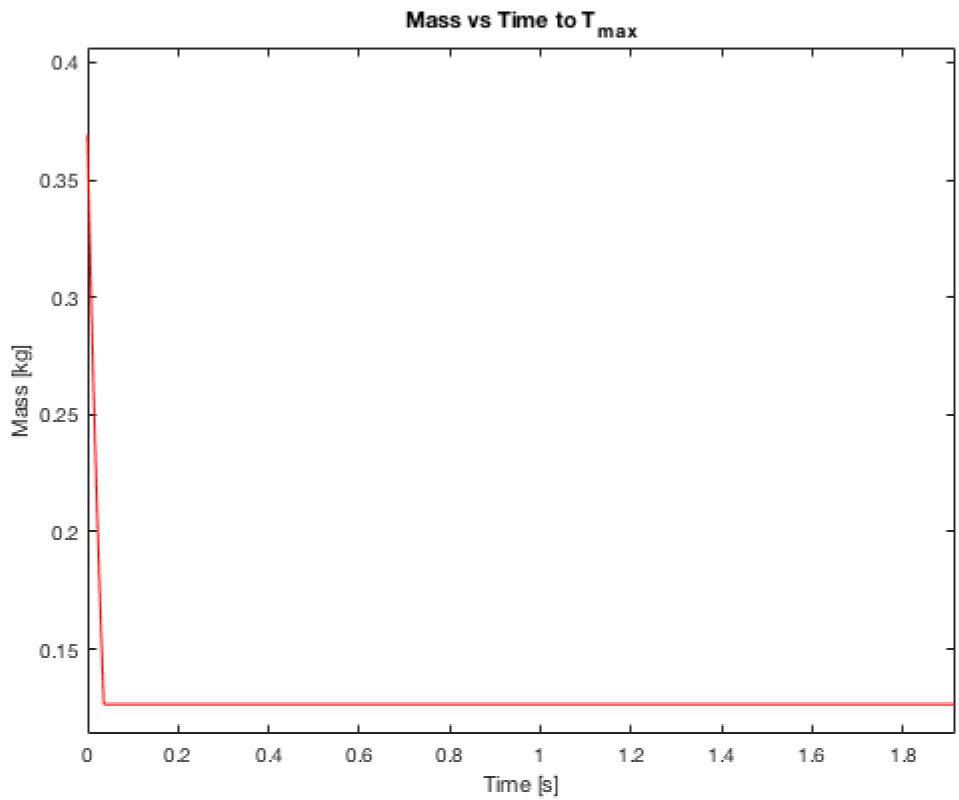
Pressure & Water Volume Optimization
18
16
14
12
10
8
2.5
2.45
$\times10^{-4}$
2.4
2.35
2.3
3
3.2
3.4
3.6
3.8
4
4.2
$\times10^{5}$



Thrust Phase
Height [m]
Time [s]
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
0.005
0.01
0.015
0.02
0.025
0.03
0.035
0.04


Cruise Phase
Height [m]
Time [s]
0
2
4
6
8
10
12
14
16
18
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2


Velocity vs Time to T_{max}
Velocity [w]
Time [s]
0
2
4
6
8
10
12
14
16
18
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8



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