Midterm Study Guide Combinatorics

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6.1 Block Designs 6.1

DEF: $(b, v, r, k, \lambda) - design$
b - subsets (blocks),
v - elements (varieties),
r - each element is in exactly r blocks,
k - each subset has k elements,
λ - each pair of elements appears in λ subsets

THM 6.1: In a block design each element lies in exactly r blocks, where

$$r(k-1) = \lambda(v-1)$$

THM 6.2: For a $(b, v, r, k, \lambda) - design$,

$$b \geq v$$

6.2 Square Block Designs 6.2

Properties $(v, k, \lambda) - design's$ incidence matrix has the following properties:

1. Any row contains k 1's.
2. Any column contains k 1's
3. Any pair of columns both have 1's in exactly λ rows
4. Any pair of rows both have 1's in exactly λ columns.

THM 6.3: If A is a square (0, 1)-matrix (i.e. a matrix all of whose entries are 0 or 1) and if A satisfies

$$A^T A = (k - \lambda)I - \lambda J$$

with $k > \lambda$, then

$$AA^T = (k - \lambda)I - \lambda J$$

also holds.

DEF: A finite projective plane of order q is defined to be a $(v, k, \lambda) - configuration$ with the properties:

1. $v = q^2 + q + 1$
2. $k = q + 1$
3. $\lambda = 1$

fpp properties: A fpp of order q has the following properties

1. Any line contains $q + 1$ points
2. Any point lies on $q + 1$ lines
3. Any pair of points are joined on exactly one line
4. Any pair of lines intersect in exactly one point

other knowledge about ffp's

a. A plane of order q definitely exists if $q \geq 2$ is a prime or a power of a prime.

b. No plane of any other order is known to exist.

c. There is definitely no plane of order 6, or in general of any order n, where n is of the form $(4k + 1)$ or $(4k + 2)$, and is divisible an odd number of times by a prime of the form $(4h + 3)$.

FACT: There is no finite projective plane of order 6.

6.3 Hadamard configurations

DEF: A $(v, k, \lambda) - configuration$ is called a Hadamard configuration when $v = 4m - 1$, $k = 2m - 1$, $\lambda = m - 1$ for some integer $m \geq 2$.
idea: Hadamard Matrix is formed from taking the incidence matrix of a Hadamard configuration and changing the 0's to 1's.
DEF: A nxn matrix is a Hadamard matrix of order n if:

1. $a_{ij} = \pm 1, \forall i, j$
2. $AA^T = nI$

FACT: Given a Hadamard matrix, it is permissible to interchange any two rows or any two columns, or to multiply any row or column by -1, for these operations do not effect the properties required by the definition.

THM 6.5: If A is an $n \times n$ Hadamard matrix with $n > 2$, then $n = 4m$ for some positive integer m. Further, each row has exactly $2m$ +1s and $2m$ -1s, and, for any two chosen rows, there are exactly m columns in which both rows have +1.

THM 6.6: Each normalized Hadamard matrix A of order $4m \geq 8$ yields a $(4m-1, 2m-1, m-1) - configuration$.

Process: A $mn \times mn$ Hadamard matrix can be formed by taking 2 Hadamard matrices, A and B, of order m and n respectively, and place A at every 1 in B and $-A$ for every -1 in B.

6.4 Error-correcting codes

THM 6.7: A code will detect all sets of h or fewer errors if any two words differ in at least $(h+1)$ places.

THM 6.8: A code will correct all sets of h or fewer errors if any two words differ in at least $(2h+1)$ places.

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