cs208 HW 2

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Note: I include the code I wrote directly in line for some questions, such as for 2 *(a)* and *(b)* which are purely implementation problems. For other questions, my code can be found in the Appendix and on Github. I worked with Bhaven Patel, Lipika Ramaswamy, and Karina Huang for this assignment.

Question 1

(a) Both (i) and (iv) are (ϵ, 0)-differentially private. I demonstrate that below using two neighboring datasets, $x = [0, 0, ..., 0]$ and $x' = [0, 0, ..., 1]$.

(i)

$$\frac{P([\bar{x} + Z]_0^1 = r)}{P([\bar{x'} + Z]_0^1 = r)}$$

$$\frac{\frac{n}{4}exp(-\frac{n}{2}|r - \bar{x}|)}{\frac{n}{4}exp(-\frac{n}{2}|r - \bar{x'}|)}$$

$$exp(\frac{n}{2}|r - \bar{x'}| - |r - \bar{x}|)$$

By the triangle inequality, this expression is less than or equal to

$$exp(\frac{n}{2}|r - \bar{x'} - r + \bar{x}|) = exp(\frac{n}{2}|\bar{x} - \bar{x'}|)$$

Since $|\bar{x} - \bar{x'}|$ is just the global sensitivity, we get that

$$exp(\frac{n}{2}|\bar{x} - \bar{x'}|) = exp(\frac{n}{2}\frac{1}{n}) = exp(\frac{1}{2})$$

Thus, $M(x)$ is $(\epsilon, 0)$-differentially private for $\epsilon \geq 0.5$.

(iv)

$$\frac{exp(\frac{-n}{10}|y - \bar{x}|)}{exp(\frac{-n}{10}|y - \bar{x'}|)} * \frac{\int_0^1 exp(\frac{-n}{10}|z - \bar{x}|)dz}{\int_0^1 exp(\frac{-n}{10}|z - \bar{x'}|)dz}$$

I'll evaluate this term by term. First, the left term:

$$exp(\frac{n}{10}(-|y - \bar{x}| + |y - \bar{x'}|))$$

$$exp(\frac{n}{10}(-|y| + |y - \frac{1}{n}|))$$

By the triangle inequality, this is less than or equal to

$$exp(\frac{n}{10}(y - \frac{1}{n} - y)) = exp(\frac{1}{10})$$

Now, for the right term:

$$\frac{\int_0^1 exp(\frac{-n}{10}|z - \frac{1}{n}|)dz}{\int_0^1 exp(\frac{-nz}{10})dz}$$

$$\frac{\int_0^{\frac{1}{n}} exp(\frac{-n}{10}(z - \frac{1}{n}))dz + \int_{\frac{1}{n}}^1 exp(\frac{-n}{10}(z - \frac{1}{n}))dz}{\int_0^{\frac{1}{n}} exp(\frac{-nz}{10})dz + \int_{\frac{1}{n}}^1 exp(\frac{-nz}{10})dz}$$

$$\frac{exp(\frac{1}{10})\int_0^{\frac{1}{n}} exp(\frac{-nz}{10})dz + exp(\frac{1}{10})\int_{\frac{1}{n}}^1 exp(\frac{-nz}{10})dz}{\int_0^{\frac{1}{n}} exp(\frac{-nz}{10})dz + \int_{\frac{1}{n}}^1 exp(\frac{-nz}{10})dz}$$

This reduces to $exp(\frac{1}{10})$. Putting the two terms together, we have

$$exp(\frac{1}{10}) * exp(\frac{1}{10}) = exp(\frac{1}{5})$$

Thus, this mechanism is $(\epsilon, 0)$-differentially private for $\epsilon \geq 0.2$.

(b) Mechanisms (ii) and (iii) are not $(\epsilon, 0)$-differentially private. Below I'll provide a counterexample that demonstrates this and find a minimum value of δ for which they are (ϵ, δ)-differentially private.

(ii) Consider $x = [0, 0, ..., 0]$ and $x' = [0, 0, ..., 1]$. Now, $P(M(x) = -1) \geq 0$ while $P(M(x') = -1) = 0$. This violates $P(M(x) = -1) \leq exp(\epsilon)P(M(x') = -1)$, so this mechanism is not $(\epsilon, 0)$-differentially private. Now let's consider the minimum value of δ for which it is (ϵ, δ)-differentially private.

$$\delta \geq max_{x\ x'}[\int_y max(P(M(x) = y) - exp(\epsilon)P(M(x') = y), 0)]$$

Consider the worst-case scenario I defined above, where $x = [0, ..., 0]$ and $x' = [0, ..., 0, 1]$. The expression inside the integral will only be ≥ 0 for $y \in [-\infty, -1 + \frac{1}{n}]$ because $P(M(x') = y) = 0$ here.

$$max[\int_{-\infty}^{\infty} P(M(x) = y) - \exp(\epsilon)P(M(x') = y), 0]$$

$$\int_{-\infty}^{-1+\frac{1}{n}} P(M(x) = y)$$

$$\int_{-\infty}^{-1+\frac{1}{n}} P(\bar{x} + Z = y)$$

Since $\bar{x} = 0$,

$$\int_{-\infty}^{-1+\frac{1}{n}} P(Z = y)$$

$$\int_{-\infty}^{-1+\frac{1}{n}} \frac{n}{4} exp(\frac{-n|y|}{2})$$

$$\int_{-\infty}^{-1+\frac{1}{n}} \frac{n}{4} exp(\frac{ny}{2})$$

$$\frac{n}{4}\int_{-\infty}^{-1+\frac{1}{n}} exp(\frac{ny}{2})$$

After integrating we are left with

$$\frac{1}{2}[exp(\frac{ny}{2})]_{-\infty}^{-1+\frac{1}{n}}$$

$$\frac{1}{2}[exp(\frac{-n+1}{2}) - exp(\frac{-n\infty}{2})]$$

Since the second expression in the parenthesis goes to 0, we have

$$\frac{1}{2}exp(\frac{-n+1}{2})$$

Thus, this mechanism is (ϵ, δ)-differentially private for $\delta \geq \frac{1}{2}exp(\frac{-n+1}{2})$.

(iii) Consider $x = [0, 0, ..., 1]$ and $x' = [0, 0, ..., 0]$. Now, $P(M(x) = 1) = \frac{1}{n}$ while $P(M(x') = 1) = 0$. This clearly violates $P(M(x) = 1) \leq exp(\epsilon)P(M(x') = 1)$, so this mechanism is not $(\epsilon, 0)$-differentially private. Now let's consider the minimum value of δ for which it is (ϵ, δ)-differentially private.

$$\delta \geq max_{x\ x'}[\Sigma_y max(P(M(x) = y) - exp(\epsilon)P(M(x') = y), 0)]$$

$$\Sigma_{y\in[0,1]} max[(P(M(x) = y) - exp(\epsilon)P(M(x') = y), 0]$$
$$max[P(M(x) = 1) - exp(\epsilon)P(M(x') = 1), 0] + max[P(M(x) = 0) - exp(\epsilon)P(M(x') = 0), 0]$$

$$max[\frac{1}{n} - exp(\epsilon) * 0, 0] + max[1 - \frac{1}{n} - exp(\epsilon) * 1, 0] = \frac{1}{n} + 0 = \frac{1}{n}$$

Thus, this mechanism is (ϵ, δ)-differentially-private for $\delta \geq \frac{1}{n}$.

(c)

(i) For this mechanism, the minimum value of ϵ depends on the bounds. Since the data were bounded on [0,1], the minimum value of ϵ is 0.5, but if the data is on [a,b], then the minimum value of ϵ is $\frac{b-a}{n}$. From here the value of ϵ used for the mechanism can be tuned (increased above this minimum value) based on the data owner's standard.

(ii) The minimum value of δ will be $\frac{1}{2}exp(\frac{b-a-n}{2})$ for an arbitrary bound $[a, b]$. For a finite value of ϵ, δ can be increased between this value and 1 - if $\delta \geq 1$, then the mechanism violates privacy because the differential privacy guarantee fails with complete certainty.

(iii) In general, the minimum value of δ will be $\frac{b-a}{n}$. As with mechanism (ii), we can tune δ so long as it lives in the interval $[\frac{b-a}{n}, 1]$.

(iv) Similarly to (i), this minimum value of ϵ depends on the bounds of the data. Here, the minimum value would be $\frac{b-a}{5}$. Again, a data owner can increase ϵ to their desire by considering the utility gained from larger values of ϵ versus the loss of privacy incurred from larger values.

(d) I would consider mechanism (i) to be the best for releasing a mean. For one, adding noise from the Laplace distribution is a straightforward process and one that would be easy for an analyst to build into their analysis of data released in a differentially private fashion. Additionally, clamping the release value of the mean to the upper and lower bound is a sensible thing to do from a utility perspective - this means that the value of the released mean cannot be outside of the range of actual values that this variable could take on. While the minimum value of ϵ is not as low as it is for mechanism (iv), the benefits of the two features outlined above outweigh this.

Question 2

(a)

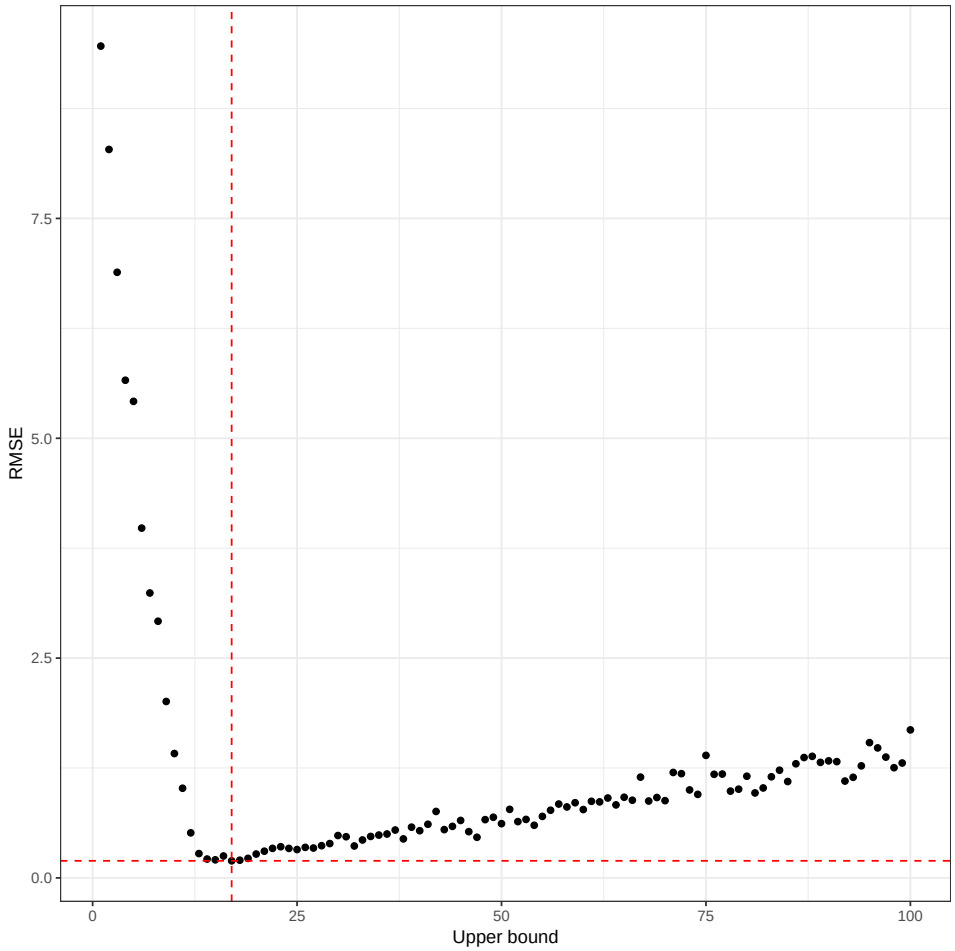
```
poissonDGP <- function(n){ return(rpois(n, lambda=10)) }
```

(b) I use the first mechanism from Question 1 to answer this question. From here on out I'll refer to it as the clamped Laplace mean release mechanism.

```
sgn <- function(x) {     # function borrowed from class
  return(ifelse(x < 0, -1, 1))
}

rlap = function(mu=0, b=1, size=1) {     # function borrowed from class
  p <- runif(size) - 0.5
  draws <- mu - b * sgn(p) * log(1 - 2 * abs(p))
  return(draws)
}

clip <- function(x, lower, upper){     # function borrowed from class
  x.clipped <- x
  x.clipped[x.clipped<lower] <- lower
  x.clipped[x.clipped>upper] <- upper
  return(x.clipped)
}

laplaceClampMeanRelease <- function(x, epsilon, a=0, b=1){
  n <- length(x)
  sensitivity <- (a - b)/n
  scale <- sensitivity / epsilon

  x.clipped <- clip(x, a, b)
  clipped.mean <- mean(x.clipped)
  noisy.mean <- clipped.mean + rlap(mu=0, b=scale, size=1)
  release.mean <- clip(noisy.mean, a, b)
  true.mean <- mean(x)

  return(list(release=release.mean, true=true.mean))
}
```

(c) I run 100 Monte Carlo simulations per upper bound value. In each simulation I draw 200 samples randomly drawn from a Poisson distribution and compute a noisy mean using the clamped Laplace mean mechanism. Then, I compare the root mean squared error between the true and noisy means for each upper bound value and plot these for every upper bound value. This is shown in the plot below.

Based on my analysis, the optimal value for the upper bound b - i.e., the one that minimizes RMSE - is 19. I will use that value for the remainder of this problem and for Question 3.



(d) This bootstrap approach is not a good mechanism to preserve privacy because we may leak information about outliers. Consider a dataset with a lot of observations between 0 and 10, a few less between 20 and 30, and a handful of observations between 35 and 40. When we bootstrap (resample without replacement) there is a possibility, albeit a small one, that our boostrapped sample will overrepresent these outliers. If we get one of those bootstrapped samples when we evaluate an upper bound of 20, it will appear that 20 is a poor upper bound to choose from a utility perspective, but if we get one of those bootstrapped samples when the upper bound in question is 40, then this cutoff will look like a good choice. Thus, the upper bound value may actually leak information about outliers in the dataset. Furthermore, in general, the bootstrap approach is specific to the dataset at hand and not the data universe that this dataset comes from. Thus, the bootstrap approach is akin to using local sensitivity and would leak information about this dataset.

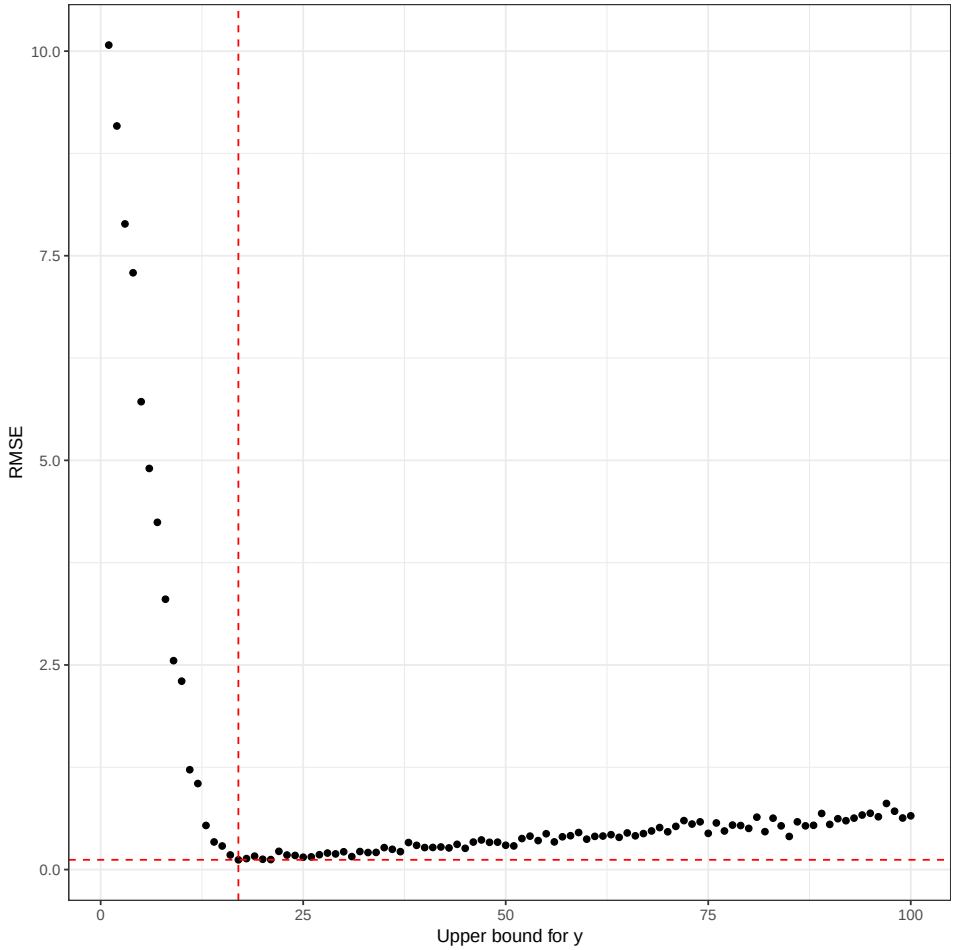
(e) The problem with the bootstrap method to determining the upper bound is that we are only considering the local attributes of the dataset and not the global attributes of the data universe that this dataset came from. A better approach to determining a bound would be to generate synthetic data from the data universe using the histogram approach we discussed in class. The synthetic data could then be used to develop an appropriate upper bound for data that comes from this data universe.

Question 3

(a) I break up the regression coefficient and intercept release mechanism into four differentially private subroutines.

First, I know that I can write the coefficient of the regression as $\frac{S_{xy}}{S_{xx}}$. We showed in class that simply computing a differentially private version of the coefficient directly would be useless because the global sensitivty approaches ∞. Thus, computing a differentially private version of the coefficient requires two subroutines: one for S_{xy} and another for S_{xx}. Next, to calculate the intercept, I can use the formula $\hat{\alpha} = \bar{y} - \hat{\beta}\bar{x}$. Since our β is now differentially private, we can use the post-processing property to use it in this calculation with no extra privacy loss. However, we also need to compute a differentially private version of $\bar{x}$, and $\bar{y}$. To do this, I'll use the clamped Laplace mean release mechanism.

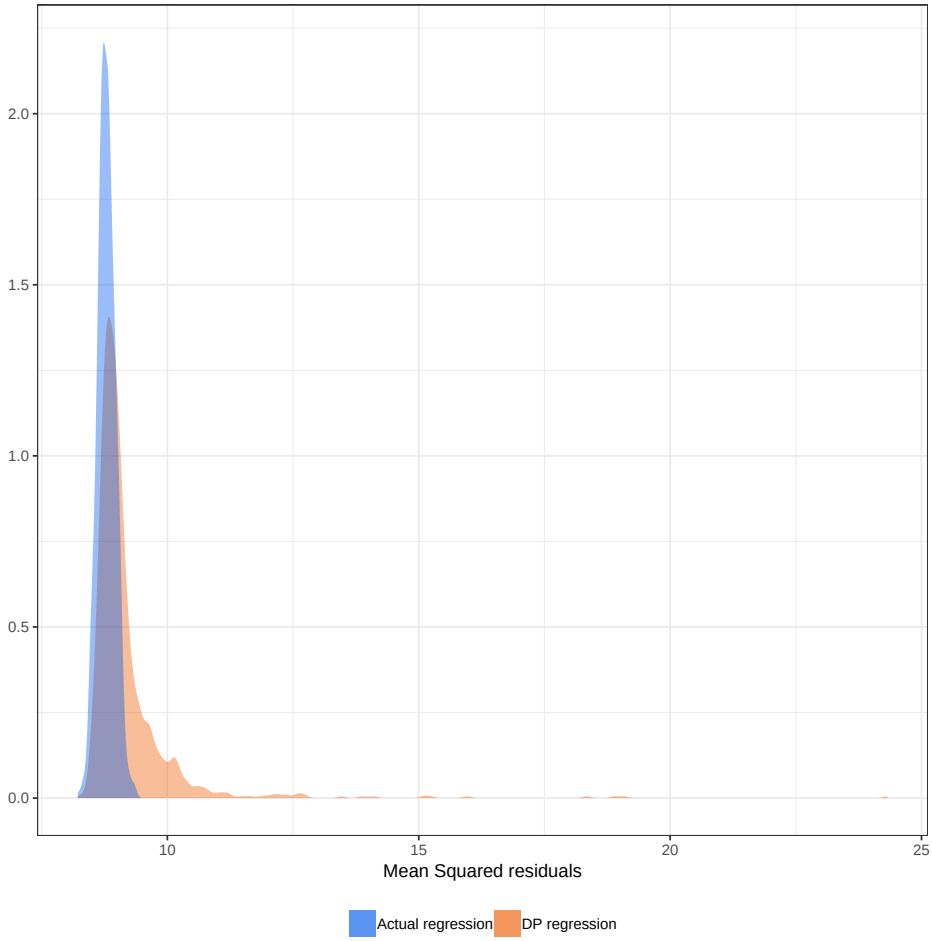
Furthermore, I will clamp both x and y as a part of this regression release mechanism. In Question 2 I found 19 to be a good upper bound value for x, and I'll continue to use that here. To get a good upper bound for y, I repeated the same process that I used for x but will also feed the data randomly drawn from a Poisson distribution into the noisy linear function and find the best upper bound for y that reduces the RMSE between the actual and noisy y values from the clamped Laplace mean release mechanism. As shown in the plot below, I find that 17 is the optimal upper bound for y.



(b) I generate 1,000 different datasets, perform my regression release on each dataset (splitting my budget evenly across the four subroutines), and then compare the performance of my noisy regression parameters to the performance of the true regression parameters by computing the mean-squared residuals for the original data. Here I consider the true regression to be the one computed on the clipped data so that my comparison

focuses specifically on the effect of the added noise and not on the clipping procedure.

The plot below shows the distributions of the mean-squared residuals for the private and non-private regressions (the plot shows kernel density estimates). In general, the distribution of the mean-squared residuals for the private regression has much longer tail, implying that this approach has a larger expected error and could potentially lead to an enormous, utility-diminshing amount of error.



(c) Now I run a grid search to attempt to identify a better partitioning of the privacy budget that leads to lower error. To set up my grid, I consider weights for each statistic ranging from 0.1 to 0.9 by steps of 0.1. I then apply a softmax function to ensure that each partition of epsilon adds up to 1 - the total privacy budget for this problem. In total, I consider 6,561 partitions, although some are redundant. For each partition in my grid, I run 20 simulations in which I generate fresh data from the Poisson and noisy linear data generating processes, compute the diffrentially private regression and finally take the average of the mean squared residuals across these 20 simulations.

I do not find that any of these partitions substantially improve the utility provided by the equal partition. I find that the minimum average of the mean squared residuals over 20 simulations is 8.25 and comes when the weights for S_{xx}, S_{xy}, $\bar{x}$, and $\bar{y}$ are 0.23, 0.35, 0.16, and 0.26, respectively. In comparison the minimum mean squared residual for the equal partition was 8.22 and the average was 9.19.

Question 4

First we can write out the expected fraction of bits reconstructed as the sum of the probabilities of reconstructing each bit.

$$\frac{\Sigma_{i=1}^{n} P[A(M(X_i, X_{-i})) = X_i]}{n}$$

By the definition of (ϵ, δ)-differential privacy and its post-processing property, we have

$$\frac{\Sigma_{i=1}^{n} P[A(M(X_i, X_{-i})) = X_i]}{n} \leq \frac{\Sigma_{i=1}^{n} exp(\epsilon) P[A(M(0, X_{-i})) = X_i] + \delta}{n}$$

The strategy that maximizes the attacker's expected fraction of bits reconstructed is to choose the most common value of the sensitive bit. With this in mind, the left hand term above is now equal to

$$\frac{exp(\epsilon)}{n} \Sigma_{i=1}^{n} max(P[A(M(0, X_{-i})) = 0], P[A(M(0, X_{-i})) = 1]) + \delta$$

$$\frac{exp(\epsilon)}{n} \Sigma_{i=1}^{n} max(p, 1-p) + \delta$$

$$\frac{exp(\epsilon)}{n} * n * max(p, 1-p) + \delta$$

Thus, we have that

$$E\left[\frac{\#i \in [n] : A(M(x))_i = X_i}{n}\right] \leq max(p, 1-p) * exp(\epsilon) + \delta$$

Question 5

I will be working with Bhaven Patel for the final project.

Appendix

I put the code for all of my analyses here. You can also find it on Github.

Question 2

```
require(plyr); require(dplyr); require(ggplot2)

# a
poissonDGP <- function(n){ return(rpois(n, lambda=10)) }

# b
sgn <- function(x) {     # function borrowed from class
  return(ifelse(x < 0, -1, 1))
}

rlap = function(mu=0, b=1, size=1) {     # function borrowed from class
  p <- runif(size) - 0.5
  draws <- mu - b * sgn(p) * log(1 - 2 * abs(p))
  return(draws)
}
```

```
clip <- function(x, lower, upper){     # function borrowed from class
  x.clipped <- x
  x.clipped[x.clipped<lower] <- lower
  x.clipped[x.clipped>upper] <- upper
  return(x.clipped)
}

laplaceClampMeanRelease <- function(x, epsilon, a=0, b=1){
  n <- length(x)
  sensitivity <- (a - b)/n
  scale <- sensitivity / epsilon

  x.clipped <- clip(x, a, b)
  clipped.mean <- mean(x.clipped)
  noisy.mean <- clipped.mean + rlap(mu=0, b=scale, size=1)
  release.mean <- clip(noisy.mean, a, b)
  true.mean <- mean(x)

  return(list(release=release.mean, true=true.mean))
}

rmse <- function(pred, true){ return(sqrt(mean((pred-true)^2))) }

# c
n = 200
epsilon = 0.5
b_vals = seq(from=1, to=100, by=1)
n_sims <- 100

results <- matrix(NA, nrow=(length(b_vals)*n_sims), ncol=4)
i = 1
for (b in b_vals){
  dat <- poissonDGP(n)
  for (j in 1:n_sims){
    DPrelease <- laplaceClampMeanRelease(dat, epsilon, a=0, b=b)
    results[i,1] <- b
    results[i,2] <- j
    results[i,3] <- DPrelease$release
    results[i,4] <- DPrelease$true
    i = i + 1
  }
}
results_df <- data.frame(results)
names(results_df) <- c("b", "sim", "release", "true")
avg_results_df <- results_df %>% group_by(b) %>% summarise(rmse = rmse(release, true))

q2_plot <- ggplot(data=avg_results_df, aes(x=b, y=rmse)) +
  geom_point() + geom_hline(yintercept = min(avg_results_df$rmse), col="red", lty=2) +
  geom_vline(xintercept = avg_results_df[which.min(avg_results_df$rmse), ]$b, col="red", lty=2) +
  labs(x="Upper bound", y="RMSE") + theme_bw()
pdf("plots/q2_plot.pdf", width=8, height=8)
q2_plot
dev.off()
```

Question 3

```
# a
poissonDGP <- function(n){ return(rpois(n, lambda=10)) }
noisyLinearDGP <- function(x, n, alpha, beta, mu=0, sd=1) { return(beta*x + alpha + rnorm(n, mu, sd)) }

sgn <- function(x) {     # function borrowed from class
  return(ifelse(x < 0, -1, 1))
}

rlap = function(mu=0, b=1, size=1) {     # function borrowed from class
  p <- runif(size) - 0.5
  draws <- mu - b * sgn(p) * log(1 - 2 * abs(p))
  return(draws)
}

clip <- function(x, lower, upper){     # function borrowed from class
  x.clipped <- x
  x.clipped[x.clipped<lower] <- lower
  x.clipped[x.clipped>upper] <- upper
  return(x.clipped)
}

rmse <- function(pred, true){ return(sqrt(mean((pred-true)^2))) }

laplaceClampMeanRelease <- function(x, epsilon, a=0, b=1){     # from q2
  n <- length(x)
  sensitivity <- (a - b)/n
  scale <- sensitivity / epsilon

  x.clipped <- clip(x, a, b)
  clipped.mean <- mean(x.clipped)
  noisy.mean <- clipped.mean + rlap(mu=0, b=scale, size=1)
  release.mean <- clip(noisy.mean, a, b)
  true.mean <- mean(x)

  return(list(release=release.mean, true=true.mean))
}

regressionRelease <- function(y, x, ylower=0, yupper=17, xlower=0, xupper=19, eplsilon, partition){
  x <- clip(x, xlower, xupper)
  y <- clip(y, ylower, yupper)

  n <- length(x)
  sens.Sxy <- ((xupper-xlower)*(yupper-ylower))
  sens.Sxx  <- ((xupper-xlower)^2)

  scale.Sxy <- sens.Sxy / (epsilon*partition$Sxy)
  scale.Sxx <- sens.Sxx / (epsilon*partition$Sxx)

  true.beta <- sum((x - mean(x))*(y - mean(y))) / sum((x - mean(x))^2)
  true.alpha <- mean(y) - true.beta*mean(x)

  release.Sxy <- sum((x - mean(x))*(y - mean(y)))  + rlap(mu=0, b=scale.Sxy, size=1)
```

```
  release.Sxx <- sum((x - mean(x))^2) + rlap(mu=0, b=scale.Sxx, size=1)
  release.beta <- release.Sxy/release.Sxx

  release.x.bar <- laplaceClampMeanRelease(x, epsilon*partition$x.bar, a=xlower, b=xupper)$release
  release.y.bar <- laplaceClampMeanRelease(y, epsilon*partition$y.bar, a=ylower, b=yupper)$release
  release.alpha <- release.y.bar - release.beta*release.x.bar

  release.mean.sq.residuals <- mean((y - release.beta*x - release.alpha)^2)
  true.mean.sq.residuals <- mean((y - true.beta*x - true.alpha)^2)

  return(list(release.beta=release.beta,
              release.alpha=release.alpha,
              true.beta=true.beta,
              true.alpha=true.alpha,
              release.mean.sq.residuals=release.mean.sq.residuals,
              true.mean.sq.residuals=true.mean.sq.residuals))
}

# get optimal upper bound for y
n = 200
epsilon = 1
b_vals = seq(from=1, to=100, by=1)
n_sims <- 100

results_y <- matrix(NA, nrow=(length(b_vals)*n_sims), ncol=4)
i = 1
for (b in b_vals){
  dat <- poissonDGP(n)
  y <- noisyLinearDGP(dat, n, alpha=1, beta=1, mu=0, sd=1)
  for (j in 1:n_sims){
    DPrelease <- laplaceClampMeanRelease(y, epsilon, a=0, b=b)
    results_y[i,1] <- b
    results_y[i,2] <- j
    results_y[i,3] <- DPrelease$release
    results_y[i,4] <- DPrelease$true
    i = i + 1
  }
}
results_y_df <- data.frame(results_y)
names(results_y_df) <- c("b", "sim", "release", "true")
avg_results_y_df <- results_y_df %>% group_by(b) %>% summarise(rmse = rmse(release, true))

q3_plot1 <- ggplot(data=avg_results_y_df, aes(x=b, y=rmse)) +
  geom_point() + geom_hline(yintercept = min(avg_results_y_df$rmse), col="red", lty=2) +
  geom_vline(xintercept = avg_results_y_df[which.min(avg_results_y_df$rmse), ]$b, col="red", lty=2) +
  labs(x="Upper bound for y", y="RMSE") + theme_bw()
pdf("plots/q3_plot1.pdf", width=8, height=8)
q3_plot1
dev.off()

# b
equal_partition <- list(Sxy=0.25, Sxx=0.25, x.bar=0.25, y.bar=0.25)
n = 1000
alpha = beta = epsilon = sd = 1
```

```
n_sims = 1000

results_reg <- matrix(NA, nrow=n_sims, ncol=6)
for (i in 1:n_sims){
  x <- poissonDGP(n)
  y <- noisyLinearDGP(dat, n, alpha=1, beta=1, mu=0, sd=1)
  DPrelease <- regressionRelease(y, x, ylower=0, yupper=17, xlower=0, xupper=19, epsilon, equal_partitio
  results_reg[i,1] <- DPrelease$release.beta
  results_reg[i,2] <- DPrelease$release.alpha
  results_reg[i,3] <- DPrelease$true.beta
  results_reg[i,4] <- DPrelease$true.alpha
  results_reg[i,5] <- DPrelease$release.mean.sq.residuals
  results_reg[i,6] <- DPrelease$true.mean.sq.residuals
}
results_reg_df <- data.frame(results_reg)
names(results_reg_df) <- c("release.beta", "release.alpha", "true.beta",
                           "true.alpha", "release.mean.sq.residuals", "true.mean.sq.residuals")

semi.blue <- rgb(0,90,239,50,maxColorValue=255)
semi.red  <- rgb(239,90,0,200,maxColorValue=255)
q3_plot2 <- ggplot(data=results_reg_df) +
  geom_density(aes(x=release.mean.sq.residuals, fill="DP regression"), alpha=0.4, colour=NA) +
  geom_density(aes(x=true.mean.sq.residuals, fill="Actual regression"), alpha=0.4, colour=NA) +
  labs(x="Mean Squared residuals", y="") + theme_bw() +
  scale_fill_manual(values=c(semi.blue, semi.red)) +
  theme(legend.position="bottom", legend.title=element_blank())
pdf("plots/q3_plot2.pdf", width=8, height=8)
q3_plot2
dev.off()

# c
# set up grid
softmax <- function(x) {
  return(exp(x)/sum(exp(x)))
}
weights = seq(0.1,0.9,by=0.1)
weights_list <- expand.grid(Sxx=weights, Sxy=weights, x.bar=weights, y.bar=weights)
partition_list<- t(apply(weights_list, 1, softmax))
partition_list_df <- data.frame(partition_list)
names(partition_list_df) <- c("Sxx", "Sxy", "x.bar", "y.bar")

# cross validation
n_sims <- 20

results_cv <- matrix(NA, nrow=nrow(partition_list), ncol=5)
for (i in 1:nrow(partition_list)){
  msr_sims <- c()
  for (j in n_sims){
    x <- poissonDGP(1000)
    y <- noisyLinearDGP(dat, 1000, alpha=1, beta=1, mu=0, sd=1)
    DPrelease <- regressionRelease(y, x, ylower=0, yupper=17, xlower=0, xupper=19, epsilon, partition_li
    msr_sims <- c(msr_sims, DPrelease$release.mean.sq.residuals)
  }
  results_cv[i,1] <- partition_list_df[i,]$Sxx
```

```
  results_cv[i,2] <- partition_list_df[i,]$Sxy
  results_cv[i,3] <- partition_list_df[i,]$x.bar
  results_cv[i,4] <- partition_list_df[i,]$y.bar
  results_cv[i,5] <- mean(msr_sims)
}
results_cv_df <- data.frame(results_cv)
names(results_cv_df) <- c("Sxx", "Sxy", "x.bar", "y.bar", "release.mean.sq.residuals")

print(results_cv_df[which.min(results_cv_df$release.mean.sq.residuals),])
print(min(results_reg_df$release.mean.sq.residuals))
print(mean(results_reg_df$release.mean.sq.residuals))
```