cs208 HW 3

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Question 1

(a) To prove that this mechanism is ϵ-DP, I will show that (i) the percentile trimming transformation is 1-Lipschitz, (ii) that the Laplace noise injection mechanism is ϵ-DP, and (iii) that this implies that the entire mechanism $M(x)$ is $(1 * \epsilon)$-DP.

(i) A mapping T from dataset to dataset is c-Lipschitz iff $\forall x, x'$ $d(T(x), T(x')) \leq c * d(x, x')$. Here let's consider that x and x' only differ on one element. It follows that $d(x, x') = 1$.

Now consider the percentile trimming transformation in this mechanism. It again follows that $d(T(x), T(x')) = 1$ since the maximum number of rows that these two datasets will differ on is 1. Returning the inequality in the definition of a Lipschitz constant, we see that this transformation is 1-Lipschitz.

(ii) First, we observe that

$$\frac{1}{.9n} \sum_{P_{.05} \leq x \leq P_{0.95}} x_i$$

is simply an estimator for the mean of x after trimming the bottom and top 5% of the data. For simplicity, replace $.9n$ with n' and call this mechanism M'. Note that the global sensitivity of this query is $GS_q = D/n'$. Since the Laplace noise is scaled by $\frac{GS_q}{\epsilon}$, M' is ϵ-DP.

(iii) In class, we discussed a lemma that states that if M is ϵ-DP and T is c-Lipschitz, then $M \circ T$ is $(c * \epsilon)$-DP. Following from (i) and (ii), we then have that $M = M' \circ T$ is $(1 * \epsilon)$-DP.

Below is the implementation of this mechanism.

```
sgn <- function(x) {     # function borrowed from class
  return(ifelse(x < 0, -1, 1))
}

rlap = function(mu=0, b=1, size=1) {     # function borrowed from class
  p <- runif(size) - 0.5
  draws <- mu - b * sgn(p) * log(1 - 2 * abs(p))
  return(draws)
}

trimmedMean <- function(x, d, n, epsilon) {
  scale <- d/(epsilon*0.9*n)
  quants <- quantile(x, c(0.05,0.95))
  x_trimmed <- x[x>quants[1] && x<quants[2]]
  mean_trimmed <- (1/(0.9*epsilon*n))*sum(x_trimmed)
  mean_release <- mean_trimmed + rlap(mu=0,b=scale)
  return(mean_release)
}
```

(b) Let's first consider the Lipschitz constant of the transformation $[x]_{P_{05}}^{P_{95}}$.

- this must be 2-Lipschitz

(c)

- describe mechanism
- implment with code

(d)

- code

(e)

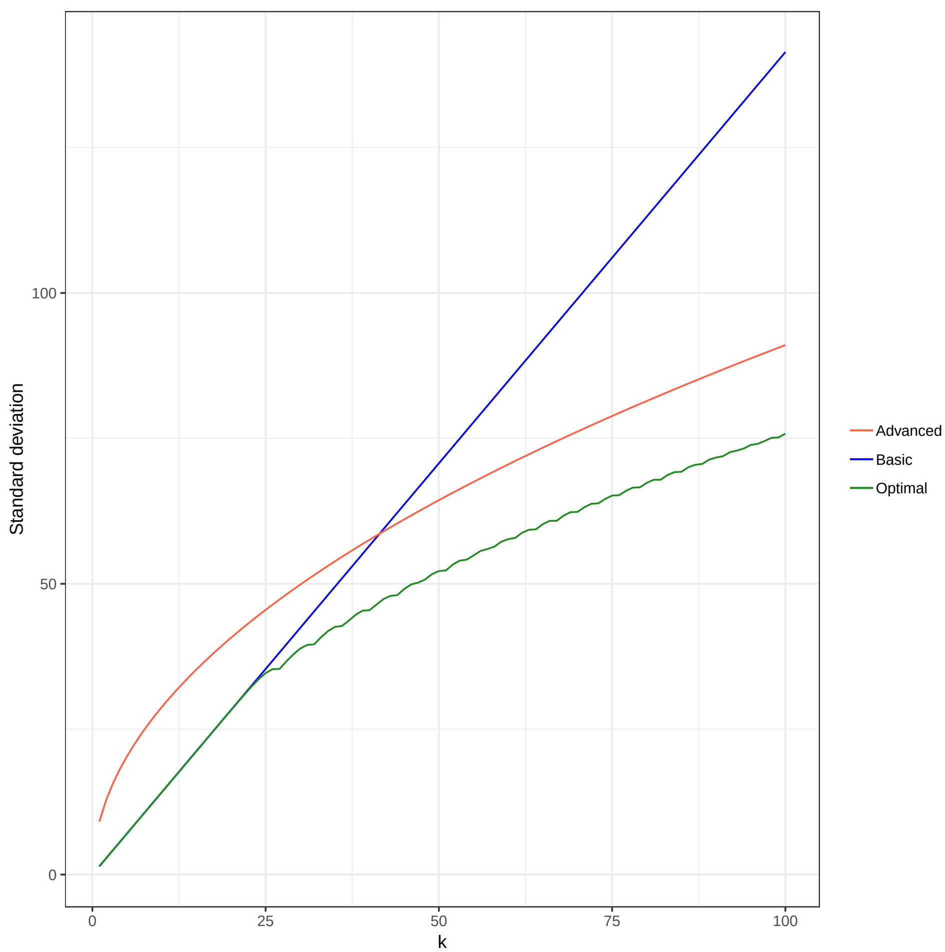
- informal sketch of proof

(f)

- have to re-implement laplace clamped mean from last hw
- code + plots
- description of results/ situation in which this would be a good approach

Question 2

The "optimal" composition theorem strictly improves upon the standard deviation of the injected noise from the basic composition theorem when $k \geq 17$ and advanced composition strictly improves upon basic composition when $k \geq 42$. Up to $k = 17$, basic and "optimal" composition correspond to roughly the same standard deviation. The standard deviation from advanced composition is strictly larger than the standard deviation from "optimal"" composition, but the ratio between the two appears to remain constant as $k \to \infty$.



Question 3

- clip data to small positive value (1) and log it as a pre-process
- income value i use from here on out is now the $log(clip(income))$
- print out main results in a table

X	Metric	Education	Age
1	DP MSE	0.0000379	0.0004556
2	DP Variance	0.0000010	0.0000758
3	DP Bias^2	0.0000370	0.0003797
4	Sampling MSE	0.0000242	0.0010317

Age DP MSE: 0.00003791765 DP Variance: 0.0000009574045 DP Bias^2: 0.00003696024 Sampling MSE: 0.00002420438

Educ DP MSE: 0.0004555563 DP Variance: 0.00007583785 DP Bias^2: 0.0003797184 Sampling MSE: 0.001031675

BONUS

Appendix

I put the code for all of my analyses here. You can also find it on Github.

Question 1

Question 2

Question 3