

MAA Metro NY Section

Problem of the Month

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1 Problem

Imagine that you have three boxes. One contains two black marbles, one contains two white marbles, and the third contains one black marble and one white marble. Previously the boxes were labeled with their contents (BB, WW, and BW), but someone switched the labels in such a way that every box is now labeled incorrectly. You are allowed to draw one marble at a time out of any box, without looking inside, and by this process of sampling you are to determine the contents of all three boxes. **What is the smallest number of drawings needed for an algorithm to do this?**

2 Solution

1 drawing is the smallest number of drawings needed for an algorithm to do this.

Because each box is now labeled incorrectly, each permutation on the set of boxes (BB, WW, BW) will be a derangement. As such, there exist only two possible mappings for the set of 3 boxes:

Map 1	Map 2
$BW \mapsto WW$	$BW \mapsto BB$
$WW \mapsto BB$	$WW \mapsto BW$
$BB \mapsto BW$	$BB \mapsto WW$

The contents of all three boxes can be determined once you know which mapping was applied to the boxes.

Observe that the box labeled BW will always get mapped to a homogeneous box (WW or BB). Therefore, by drawing one marble from the box labeled BW, the marble's color determines the color of the other marble, and the contents of the box labeled BW become known. Afterwards, the mapping is known, and

the contents of the other boxes are known.

If a white marble is drawn from BW, then BW is mapped to WW, meaning Map 1 was applied. As such, the contents of the boxes labeled BW, WW, and BB will be WW, BB, BW respectively. If a black marble is drawn from BW, then BW is mapped to BB, meaning Map 2 was applied. As such, the contents of the boxes labeled BW, WW, and BB will be BB, BW, WW respectively.