CP409 Problem Solving Block B – Week 1

In Block A we studied various problems associated with material balance, covering a number of useful modeling techniques.

This week we will consider a simple problem that requires energy rather than mass balances, but nevertheless allows us to formulate and then solve the problem with the same techniques as before. The problem I have chosen concerns camping, which allows us to relate our answers to personal experience; of course it could easily have been expressed in more practical engineering terms such as insulating a boiler or reactor.

Task B1 – Which Sleeping-bag?

Before a camping trip, you need to select which thickness of sleeping bag to take with you. Being an engineer, you decide to perform some calculations to help you decide. Being a wise engineer, you realize that it is best practice to solve your problem using symbols for various parameters, such as surface area of the sleeping bag, thermal conductivity of air, etc. Once you have your solutions in these terms, it is a simple matter to substitute in appropriate numbers, and see how the requirements change with external temperature.

(a) Sketch the steady-state temperature profile through the insulating sleeping bag, indicating some key parameters of the problem.
(b) You decide to neglect the boundary layer heat transfer coefficient at the outer surfaces of the sleeping bag; justify this.
(c) Hence derive an expression for the required thickness of the sleeping bag.

Hint: the human body loses heat at a rate of about 100W

(d) When you first get into the bag, it is cold. Sketch how the temperature profile through the insulation varies over time.
(e) Considering the differential energy balance, derive a first-order differential equation for the how the interior temperature of the sleeping bag varies over time.
(f) Find the analytical solution to this equation. How long do you have to wait for the sleeping bag to warm up?

Data for air: $\rho = 1.2$*kg.m*$^{-3}$*, heat capacity* $c_p \sim 1$*kJ/kg*$^{-1}$*.K*$^{-1}$*, thermal conductivity* $\kappa \sim 0.025$*W.m*$^{-1}$*.K*$^{-1}$

(g) Find the numerical solution to the equation in (e) using the Euler method in Matlab, and compare with your analytical solution.
(h) This is all very well, but what about your feet? Experience shows these take much longer to warm up, so you decide to heat up the sleeping bag with a hot-water bottle before you get in. Modify your Mathcad solutions to find how the temperature inside the sleeping bag varies over time once the hot-water bottle has been inserted (you can stay outside the sleeping bag for this exercise!).

Assume the hot water bottle has a 1L capacity, and initial temperature of 90^o*C, and an initial heat flux of 100W when first placed into the cold sleeping bag.*