Problem Solving Block B: Iceberg Challenge

In this set of tasks we are going to consider the possibility of towing icebergs from the arctic region to the south-east of England which increasingly suffers from water shortages. If we could sell iceberg water (mmm, refreshing and pure) to a distributer at 1p per litre, perhaps a profit can be made. The challenge is how to go about estimating whether or not the whole idea is feasible, using basic engineering knowledge and some simple empirical facts. Note that in the first instance we require fairly crude calculations, just to see if the scheme is worthy of more serious investment of time and effort. Over the next 3 weeks we will look at different aspects of the problem and attempt to come some reasonable conclusions.

Task B2.1

You place a 4cm diameter ice-cube into a large bowl of water at constant temperature 15°C, and find that it melts in 10 minutes.

(a) Predict the size of the ice-cube as a function of time. You need to consider the rate of heat flow into the ice-cube through its surface, and therefore the rate at which its mass changes with time. Try to express the rate of change of ice-cube size as a simple first-order differential equation.
(b) What is the heat transfer coefficient between the ice-cube and the water?
(c) Estimate how long it takes the ice-cube to melt in water of different constant temperature T.

Hints: Treat the ice-cube as a sphere fully immersed in the water. Assume the water is well-mixed and remains at its constant temperature of 15°C throughout the melting, so that the rate-limiting process is the heat flow at the ice-water boundary. Assume the temperature of the un-melted ice is 0°C. The ice has density 916 kg/m^3, and the latent heat of melting ice is 334 kJ/kg.

Task B2.2

You place another 4cm diameter ice-cube into a smaller glass of water of volume 0.3 litre, initially at the temperature 15°C.

(a) What is the final temperature of the water in the glass when the ice-cube has melted?
(b) Sketch how you expect the size of the ice-cube to vary over time, alongside your expected variation of water temperature over time assuming the water remains well-mixed.
(c) Write down first-order, coupled differential equations for the water temperature and ice-cube size as a function of time.
(d) Solve the coupled differential equations numerically using the Euler Method in Matlab.

Hints: The mass of water initially in the ice-cube is small compared to that of the water in the glass, so treat the latter as constant. Assume that the total system of water and ice-cube is adiabatic (isolated from the surroundings). The heat capacity of liquid water is 4.22 kJ/(kg.K), and its density is 1000 kg/m^3.